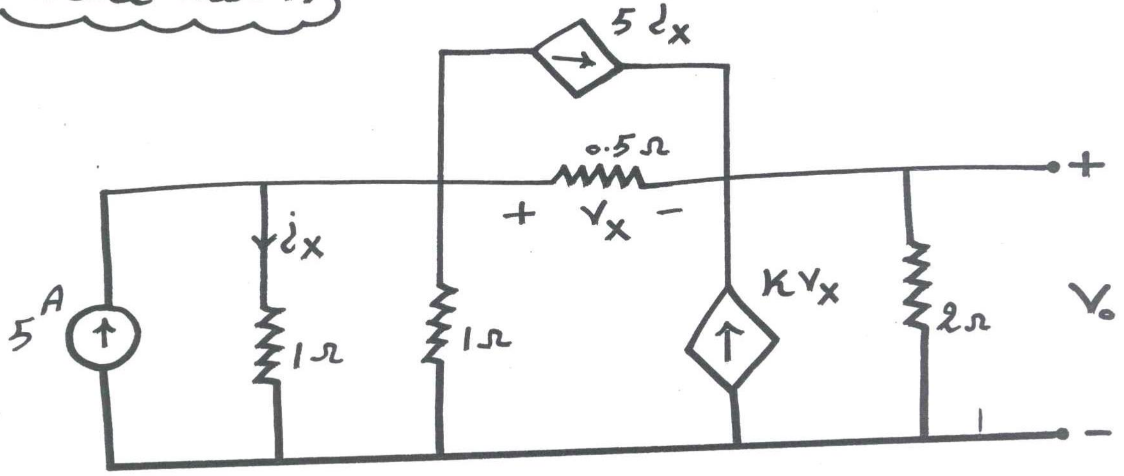
Electric Circuits I

DC Circuits

Midterm Revision (2)

Final (2014)

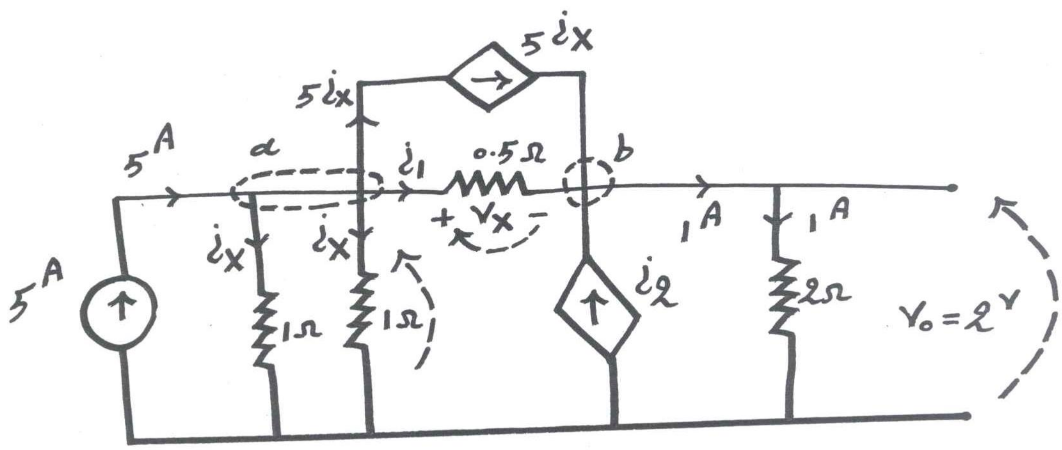


for the shown circuit, find the value of K so that the output voltage $V_o = 2^V$.

solution

↳ let $i_2 = KV_x$

↳ $1\Omega \parallel 1\Omega$ ---> equal and parallel resistors ---> the same current will pass through them.



↳ i_1 ---> assumed current.

↳ using KCL and KVL:

↳ KCL at node (a):

$$\therefore 5 = i_x + i_x + 5 i_x + i_1$$

$$\therefore 7 i_x + i_1 = 5 \dashrightarrow \{1\}$$

↳ KCL at node (b):

$$\therefore 1 = i_1 + i_2 + 5 i_x$$

$$\therefore 5 i_x + i_1 + i_2 = 1 \dashrightarrow \{2\}$$

↳ KVL: $(1\Omega + 0.5\Omega + 2\Omega)$ loop:

$$\therefore (1) i_x - 0.5 i_1 - 2 = 0$$

$$\therefore i_x - 0.5 i_1 = 2 \dashrightarrow \{3\}$$

solving {1}, {2} and {3}

$$\therefore i_x = 1^A$$

$$\therefore i_1 = -2^A$$

$$\therefore i_2 = -2^A$$

$\hookrightarrow V_x = 0.5\,(i_1)$

$= 0.5\,(-2) = -1^{V}$

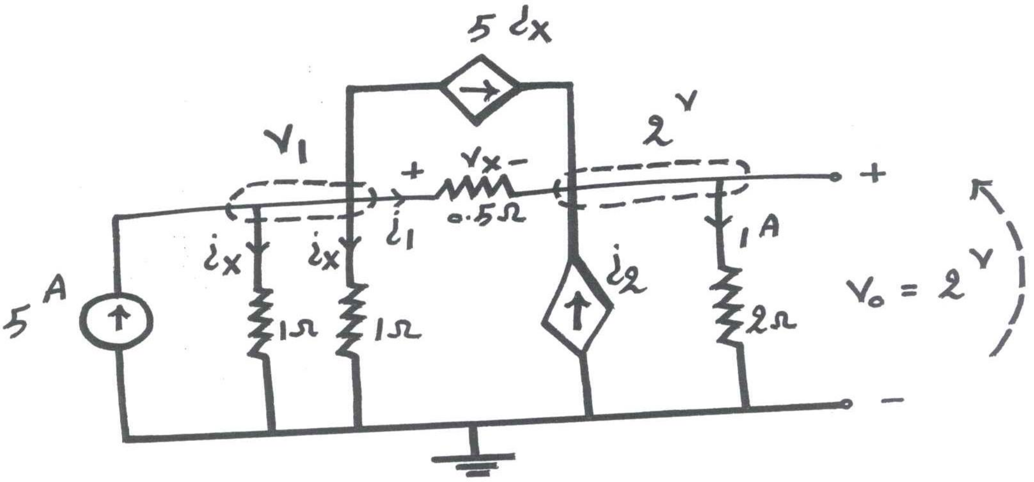
$\hookrightarrow i_2 = K\,V_x$

$\therefore -2 = K\,(-1)$

$\therefore \quad K = 2$

$\hookrightarrow$ another method :

$\dashrightarrow$ using nodal analysis :



Kcl at node (1)

$$\therefore i_x + i_x + 5\,i_x + \frac{V_1 - 2}{0.5} = 5$$

$$\hookrightarrow i_x = \frac{V_1}{1} = V_1$$

$$\therefore 7\,V_1 + 2\,V_1 - 4 = 5$$

$$\therefore 9\,V_1 = 9$$

$\therefore\ V_1 = 1^{V}$

$\therefore\ i_x = 1^{A}$

$$\therefore\ i_1 = \frac{V_1 - 2}{0.5} = \frac{1-2}{0.5} = -2^{A}$$

$\hookrightarrow V_x = 0.5\, i_1 = 0.5(-2) = -1^{V}$

<u>Kcl at node (2)</u>

$\therefore\ 5 i_x + i_1 + i_2 = 1$

$\therefore\ 5(1) + (-2) + i_2 = 1$

$\therefore\ 3 + i_2 = 1$

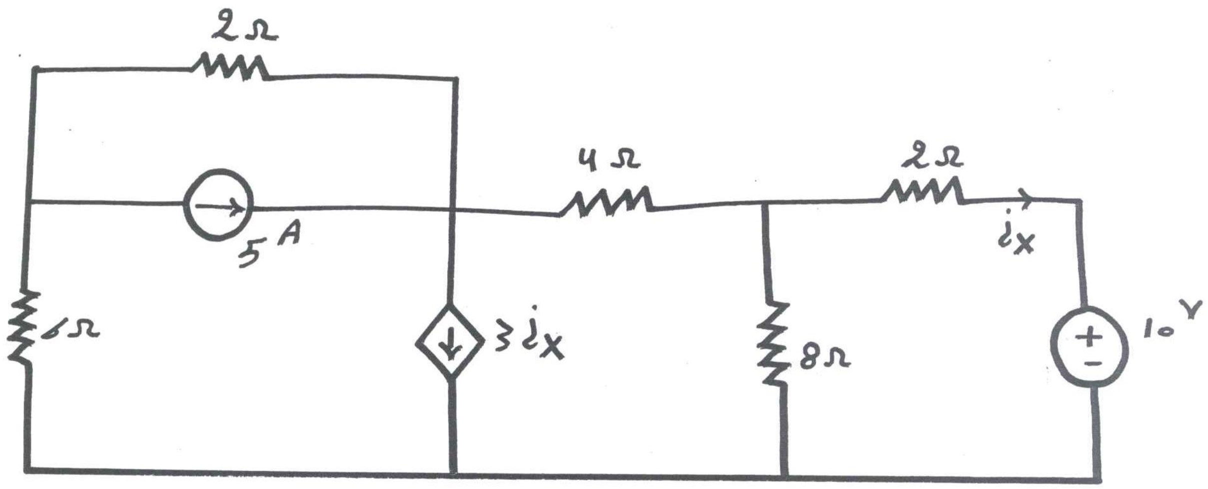
$\therefore\ i_2 = -2^{A}$

$\hookrightarrow i_2 = K\, V_x$

$\therefore\ -2 = K(-1)$

$\therefore\ K = 2$

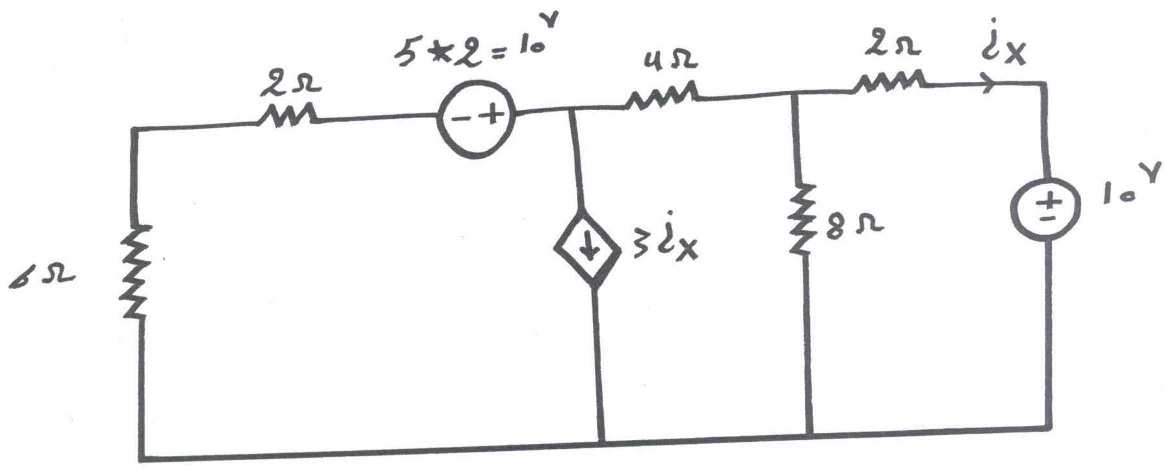
Final (2014)



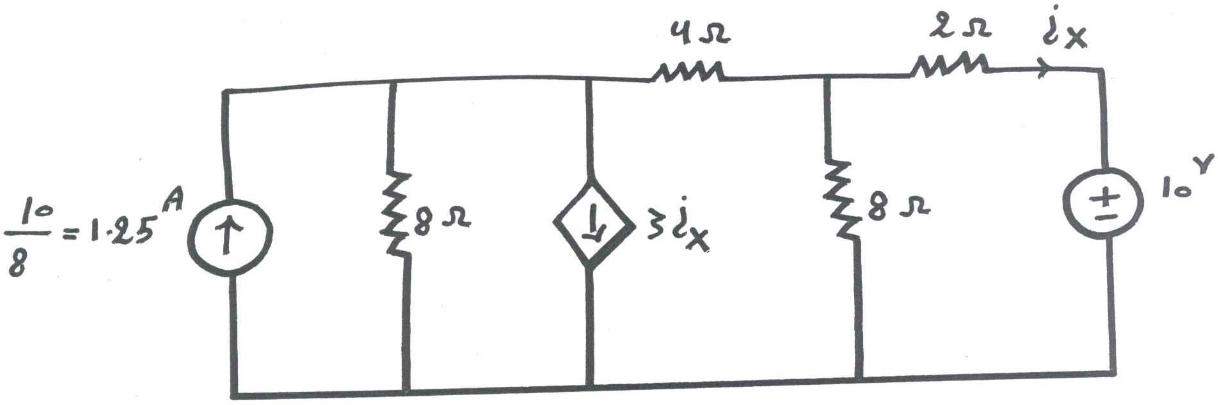
use source transformation to find i_x in the shown circuit. Then find the power of each source.

Solution

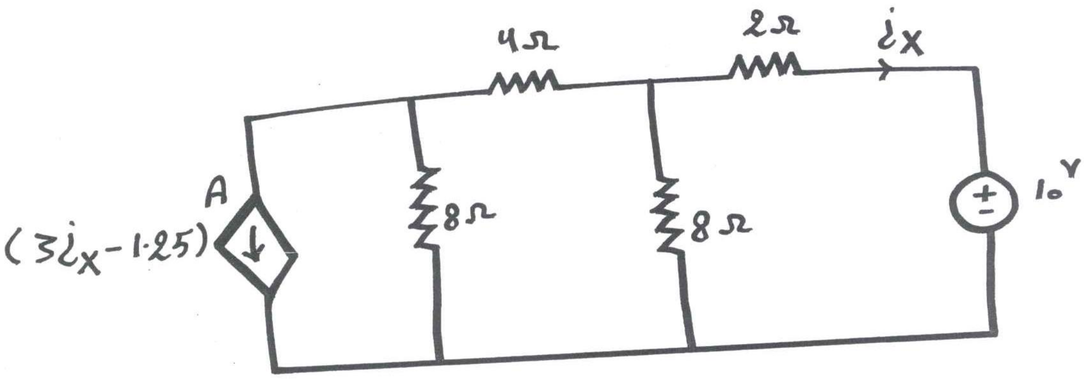
$\hookrightarrow 5^A \parallel 2^{\Omega}$ ---> Source transformation:



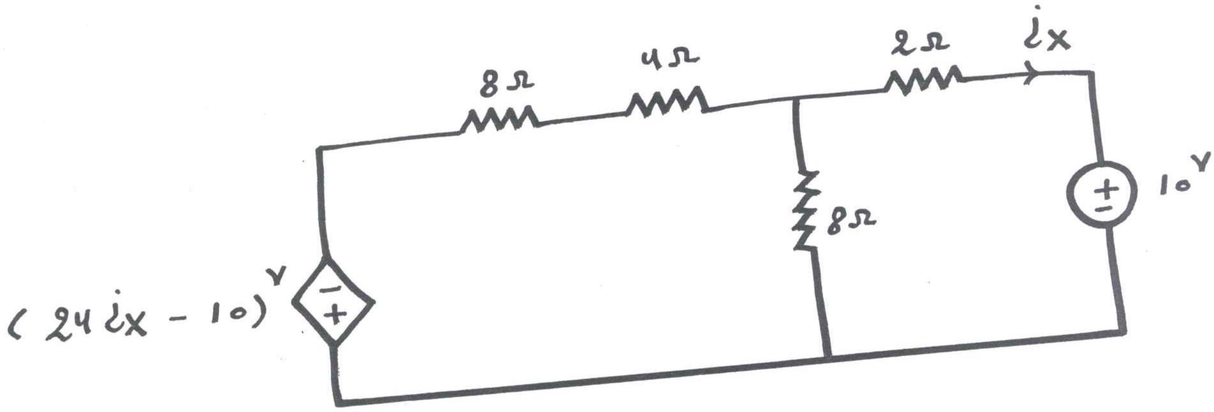
$\hookrightarrow 6+2 = 8^{\Omega}$ ---> series with 10^V source:



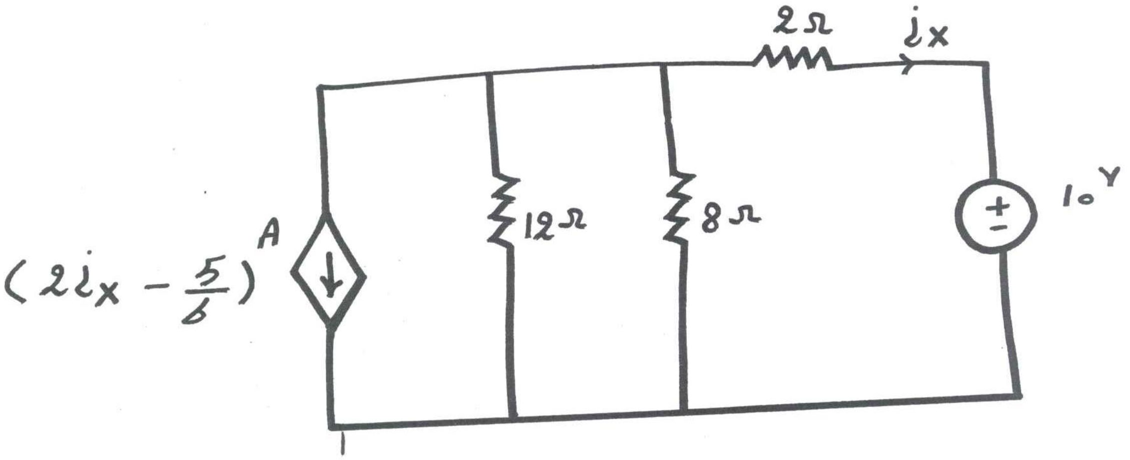
↳ 1.25^A , $3\,i_x^A$ ---> Parallel current sources :



↳ $(3i_x - 1.25)^A \parallel 8\,\Omega$ ---> Source transformation:

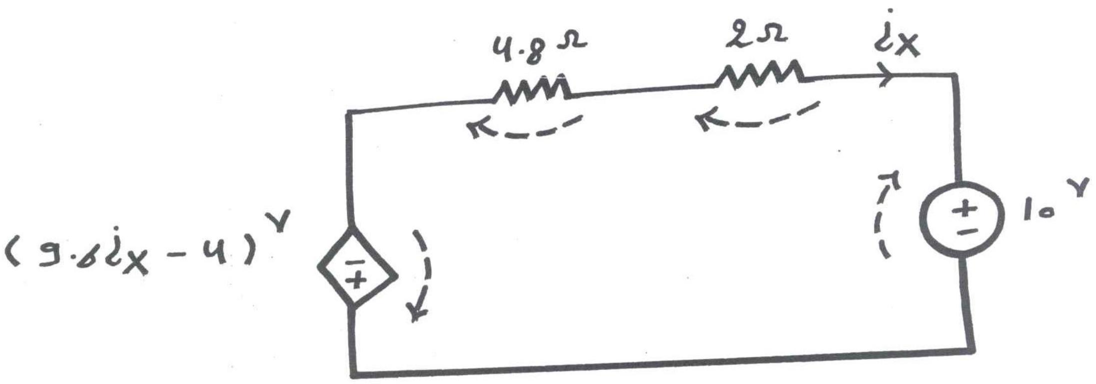


↳ $8 + 4 = 12\,\Omega$ ---> series with $(24\,i_x - 10)^V$ source :



$\hookrightarrow$ $12 \parallel 8 = \frac{12 * 8}{12 + 8} = 4.8\ \Omega$

$\hookrightarrow$ $(2i_x - \frac{5}{6})^A \parallel 4.8^{\Omega}$ ---> Source transformation:



KVL

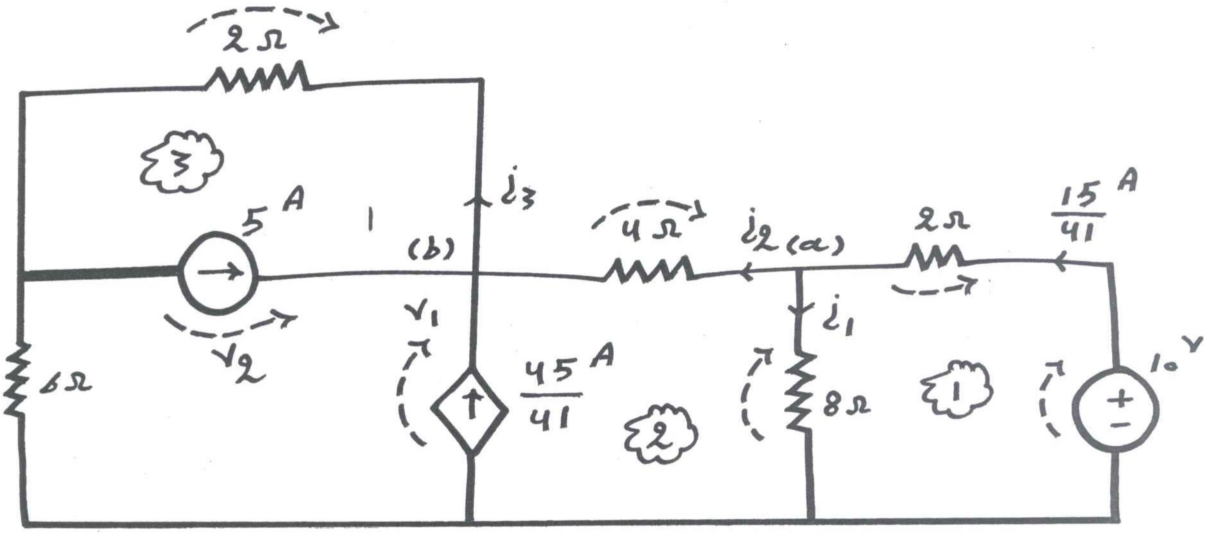
$\therefore\ 9.6\, i_x - 4 + 10 + 2\, i_x + 4.8\, i_x = 0$

$\therefore\ 16.4\, i_x = -6$

$\therefore\ i_x = -\frac{15}{41}\ A = -0.366^A$

↳ Find the Power of each Source:

back to the original circuit:



↳ $P_{10V} = 10^V * \frac{15}{41}$

$$\therefore P_{10V} = \frac{150}{41} W = 3.66^W$$

Generated Power

KVL : Loop (1) :

$$\therefore 8 i_1 + 2\left(\frac{15}{41}\right) - 10 = 0$$

$$\therefore 8 i_1 = 10 - \frac{30}{41}$$

$$\therefore i_1 = \frac{95}{82} A$$

KCL : (a):

$$\therefore i_2 = \frac{15}{41} - i_1 = -\frac{65}{82} A$$

KVL : Loop (2) :

$$\therefore V_1 + 4 i_2 - 8 i_1 = 0$$

$$\therefore V_1 = 8 i_1 - 4 i_2$$

$$= 8\left(\frac{95}{82}\right) - 4\left(-\frac{65}{82}\right)$$

$$= \frac{510}{41} V$$

$$\therefore P_{i_x} = \frac{510}{41} V * \frac{45}{41} = 13.65 W$$

dependent Source

Generated Power

KCL : (b) :

$$\therefore i_3 = 5 + \frac{45}{41} + i_2$$

$$= 5 + \frac{45}{41} - \frac{65}{82}$$

$$= \frac{435}{82} A$$

KVL : Loop (3) :

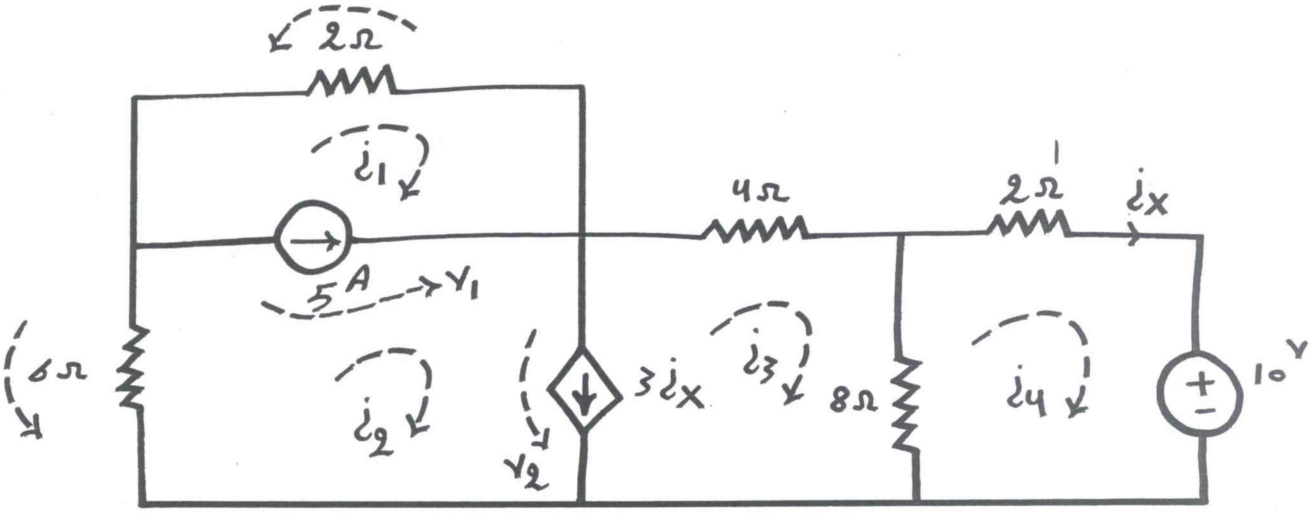
$$\therefore V_2 = 2 i_3 = 2\left(\frac{435}{82}\right) = \frac{435}{41} V$$

$$\therefore P_{5A} = \frac{435}{41} V * 5 A = 53.049 W$$

Generated Power

Example

Solve the previous problem using mesh analysis :



Solution

mesh (4)

$$\therefore 10 i_4 - 8 i_3 = -10$$

$$\therefore -8 i_3 + 10 i_4 = -10 \quad (1)$$

5 A current source

$$\therefore i_2 - i_1 = 5$$

$$\therefore -i_1 + i_2 = 5 \quad (2)$$

$3i_x$ A current source

$\therefore i_2 - i_3 = 3i_x$

$\hookrightarrow i_x = i_4$

$\therefore i_2 - i_3 - 3i_4 = 0 \dashrightarrow (3)$

supermesh equation

$\dashrightarrow$ mesh (1) + mesh (2) + mesh (3) :

$\therefore 2i_1 + 6i_2 + 12i_3 - 8i_4 = 0 \dashrightarrow (4)$

From (2)

$\therefore i_1 = i_2 - 5$

$\dashrightarrow$ sub in (4) :

$\therefore 2(i_2 - 5) + 6i_2 + 12i_3 - 8i_4 = 0$

$\therefore 2i_2 - 10 + 6i_2 + 12i_3 - 8i_4 = 0$

$\therefore 8i_2 + 12i_3 - 8i_4 = 10 \dashrightarrow (5)$

Solving 1, 3 and 5

$$\therefore i_2 = -\frac{25}{82} \text{ A}$$

$$\therefore i_3 = \frac{65}{82} \text{ A}$$

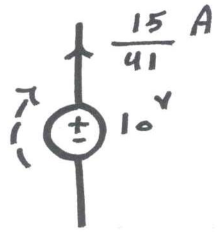
$$\therefore i_4 = -\frac{15}{41} \text{ A} \quad \therefore \rightarrow \quad \boxed{i_x = i_4 = -\frac{15}{41} \text{ A}}$$

$$\therefore i_1 = i_2 - 5$$

$$= -\frac{25}{82} - 5 = -\frac{435}{82} \text{ A} = -5.3 \text{ A}$$

Find the Power of each source:

10 V Source



$$\therefore P_{10V} = 10^V * \frac{15}{41} \text{ A} = \boxed{\frac{150}{41} \text{ W} = 3.66 \text{ W}}$$

Generated Power

5 A Source

KVL at mesh (1):

$$\therefore V_1 = -2\, i_1 = -2\left(-\frac{435}{82}\right)$$

$$\therefore V_1 = \frac{435}{41} \text{ V} = 10.61 \text{ V}$$

$$\therefore P_{5A} = 5^{A} * \frac{435}{41}^{V}$$

$$\boxed{\therefore P_{5A} = \frac{2175}{41} W = 53.049 W}$$

Generated Power

3i_x Source

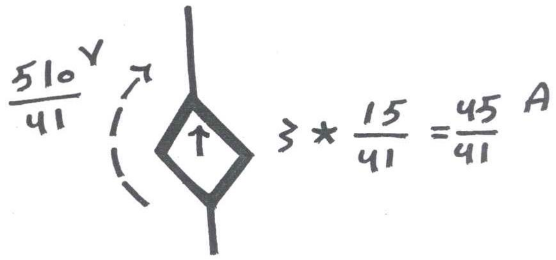
KVL at mesh (2):

$$\therefore V_1 + V_2 - 6 i_2 = 0$$

$$\therefore V_2 = 6 i_2 - V_1$$

$$= 6\left(-\frac{25}{82}\right) - \frac{435}{41}$$

$$= -\frac{510}{41}^{V}$$



$$\therefore P_{3i_x} = \frac{510}{41}^{V} * \frac{45}{41}$$

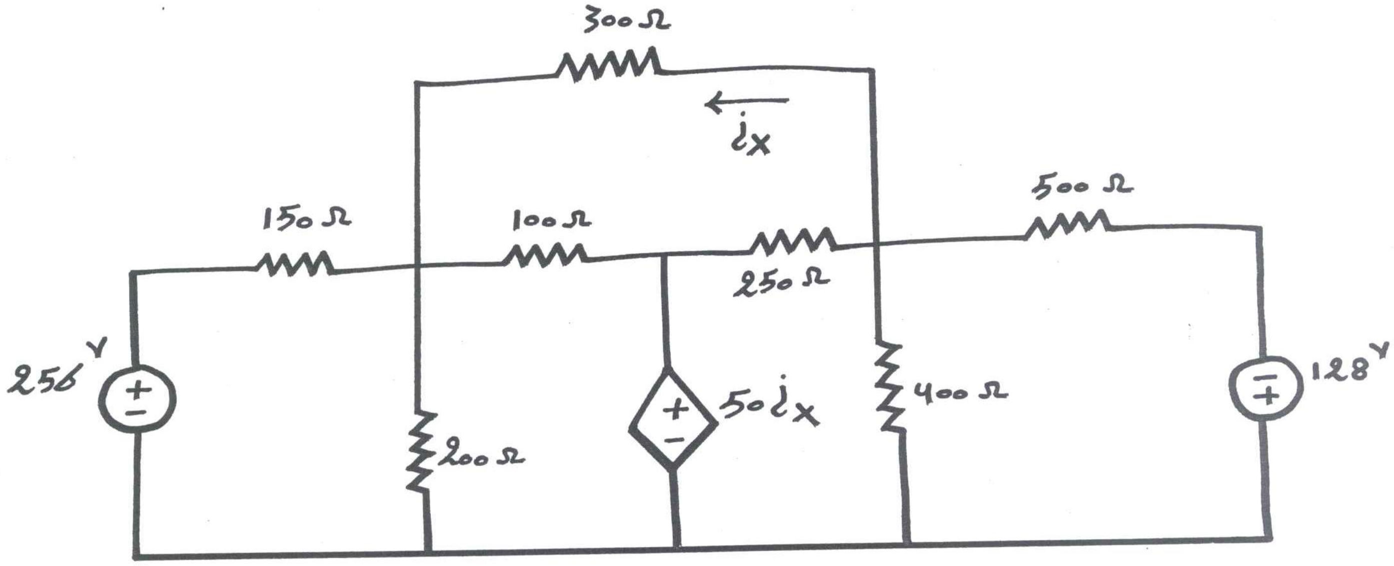
$$\boxed{\therefore P_{3i_x} = 13.65 W}$$

Generated Power

↳ check Power balance to verify your answer.

Final (2016)

For the shown circuit:



(a) is it better to use nodal analysis or mesh analysis to solve this circuit? give detailed reasons to your answer.

(b) based on your (choice) preference in part (a), solve the shown circuit and find the value of the dependent voltage source $50\, i_x$.

(c) find the power of each source in this circuit and state whether this power is supplied or consumed by the source.

Solution

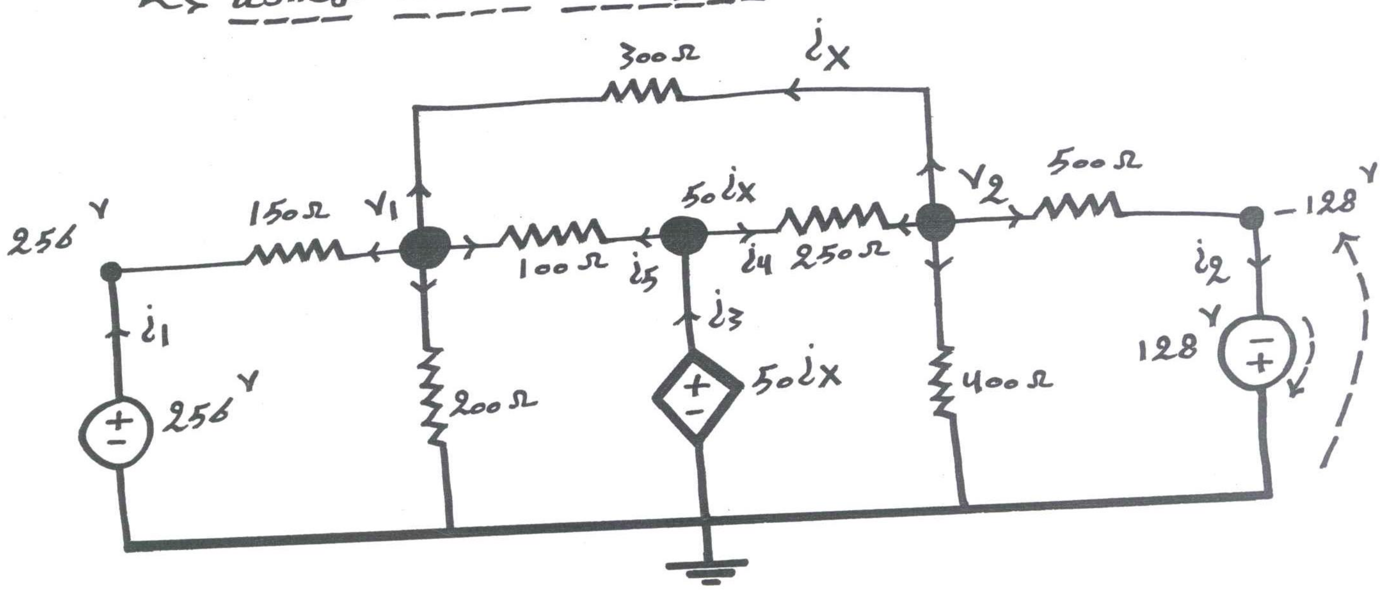
(a)

↳ using nodal analysis is better to solve this circuit because we have many voltage sources in this circuit and by selecting suitable reference node, the voltage at many nodes will be known and we will have only two unknowns.

↳ using mesh analysis is a bad choice because we will have 5 unknowns for the 5 meshes so we will have 5 equations to solve.

(b)

↳ using nodal analysis:



Kcl at node (1)

$$\therefore \frac{V_1-256}{150}+\frac{V_1-50\,i_x}{100}+\frac{V_1-V_2}{300}+\frac{V_1}{200}=0$$

$$\hookrightarrow \boxed{i_x=\frac{V_2-V_1}{300}}$$

$$\therefore V_1\left(\frac{1}{150}+\frac{1}{100}+\frac{1}{300}+\frac{1}{200}\right)-\frac{1}{2}i_x-\frac{1}{300}V_2=\frac{256}{150}$$

$$\therefore V_1\left(\frac{1}{40}\right)-\frac{1}{300}V_2-\frac{1}{600}V_2+\frac{1}{600}V_1=\frac{256}{150}$$

$$\therefore V_1\left(\frac{1}{40}+\frac{1}{600}\right)-\left(\frac{1}{300}+\frac{1}{600}\right)V_2=\frac{256}{150}$$

$$\therefore \frac{2}{75}V_1-\frac{1}{200}V_2=\frac{256}{150} \qquad * 300$$

$$\boxed{\therefore 8\,V_1-1.5\,V_2=512} \dashrightarrow (1)$$

Kcl at node (2)

$$\therefore \frac{V_2-V_1}{300}+\frac{V_2-50\,i_x}{250}+\frac{V_2+128}{500}+\frac{V_2}{400}=0$$

$$\therefore V_2\left(\frac{1}{300}+\frac{1}{250}+\frac{1}{500}+\frac{1}{400}\right)-\frac{1}{300}V_1-\frac{1}{5}i_x=-\frac{128}{500}$$

$$\hookrightarrow i_x=\frac{1}{300}V_2-\frac{1}{300}V_1$$

$$\hookrightarrow \therefore \frac{1}{5}i_x=\frac{1}{1500}V_2-\frac{1}{1500}V_1$$

$$\therefore \frac{71}{6000} V_2 - \frac{1}{300} V_1 - \frac{1}{1500} V_2 + \frac{1}{1500} V_1 = -\frac{128}{500}$$

$$\therefore -\frac{1}{375} V_1 + \frac{67}{6000} V_2 = -\frac{128}{500} \qquad * 6000$$

$$\boxed{\therefore -16 V_1 + 67 V_2 = -1536} \dashrightarrow \boxed{2}$$

Solving $\boxed{1}$ and $\boxed{2}$

$$\boxed{\therefore V_1 = 62.5^V}$$

$$\boxed{\therefore V_2 = -8^V}$$

$$\hookrightarrow \therefore i_x = \frac{V_2 - V_1}{300} = \frac{-8 - 62.5}{300} = -0.235^A$$

$\hookrightarrow \therefore$ dependent voltage source $= 50\, i_x$

$$= 50(-0.235)$$

$$= \boxed{-11.75^V}$$

(c)

$$\hookrightarrow i_1 = \frac{256 - V_1}{150} = \frac{256 - 62.5}{150} = 1.29^A$$

$$\hookrightarrow \therefore P_{256^V} = 256^V * 1.29^A = \boxed{330.24^W}$$

supplied Power

$$\hookrightarrow i_2 = \frac{V_2 + 128}{500} = \frac{-8+128}{500} = 0.24^{A}$$

$$\hookrightarrow \therefore P_{128^V} = 128^V * 0.24^A = \boxed{30.72^W}$$

Supplied Power

$$\hookrightarrow i_3 = i_4 + i_5$$

$$= \frac{-11.75+8}{250} + \frac{-11.75-62.5}{100}$$

$$= -0.015 - 0.7425 = -0.7575^{A}$$

$$\hookrightarrow \therefore P_{50 i_x} = (11.75)(0.7575) = \boxed{8.9\ W}$$

Supplied Power

Example

2.55 Find I_o in the network in Fig. P2.55.

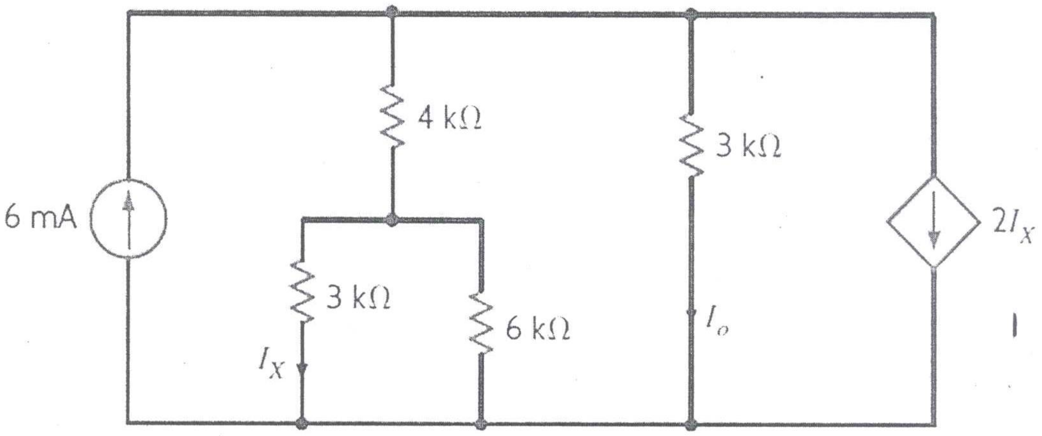
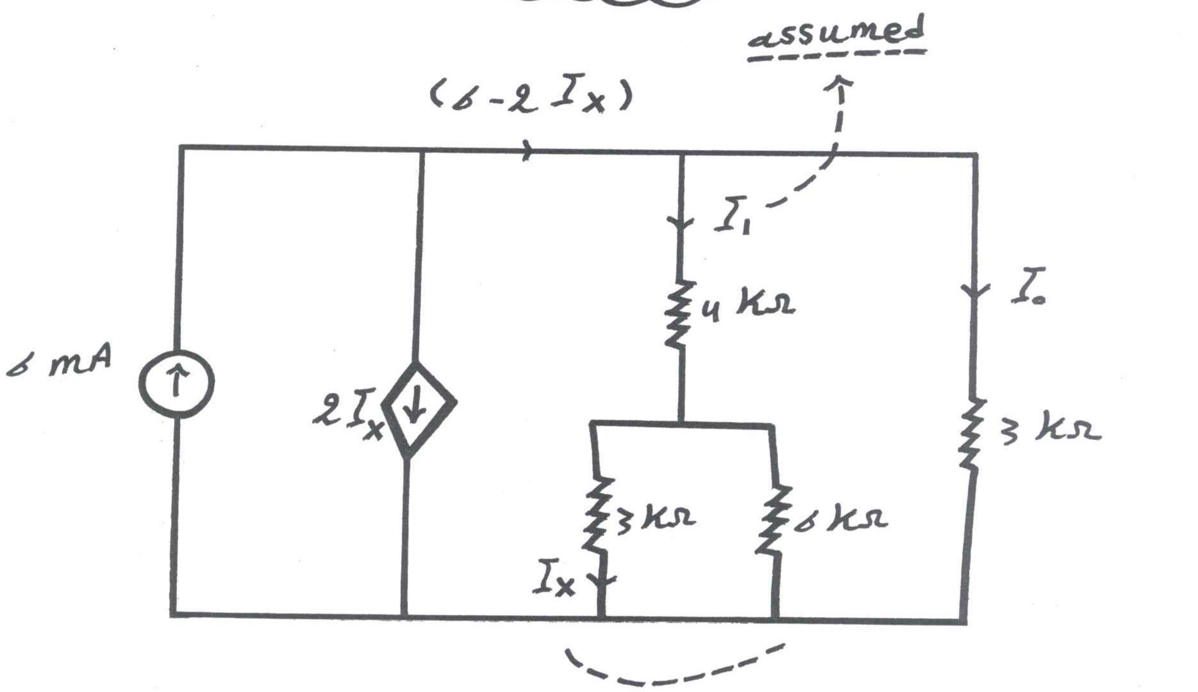


Figure P2.55

Solution



↳ $4 + (6 \parallel 3) = 4 + 2 = 6\ k\Omega$

↳ current divider:

$$\therefore I_x = I_1 * \frac{6}{6+3}$$

$$\therefore I_x = \frac{2}{3} I_1$$

$$\therefore I_1 = 1.5\, I_x \quad \text{---} \rightarrow (1)$$

current divider

$$\therefore I_1 = (6-2I_x) * \frac{3}{3+6}$$

$$\therefore I_1 = (6-2I_x) * \frac{1}{3}$$

$$\therefore I_1 = 2 - \frac{2}{3} I_x \dashrightarrow (2)$$

From (1) and (2)

$$\therefore \frac{3}{2} I_x = 2 - \frac{2}{3} I_x$$

$$\therefore \left(\frac{3}{2} + \frac{2}{3}\right) I_x = 2$$

$$\therefore I_x = \frac{12}{13} \text{ mA}$$

current divider

$$\therefore I_o = (6-2I_x) * \frac{6}{6+3}$$

$$= \left(6 - 2 * \frac{12}{13}\right) * \frac{2}{3}$$

$$\therefore I_o = \frac{36}{13} \text{ mA} = 2.77 \text{ mA}$$

Example

2.67 Find R_{AB} in the network in Fig. P2.67.

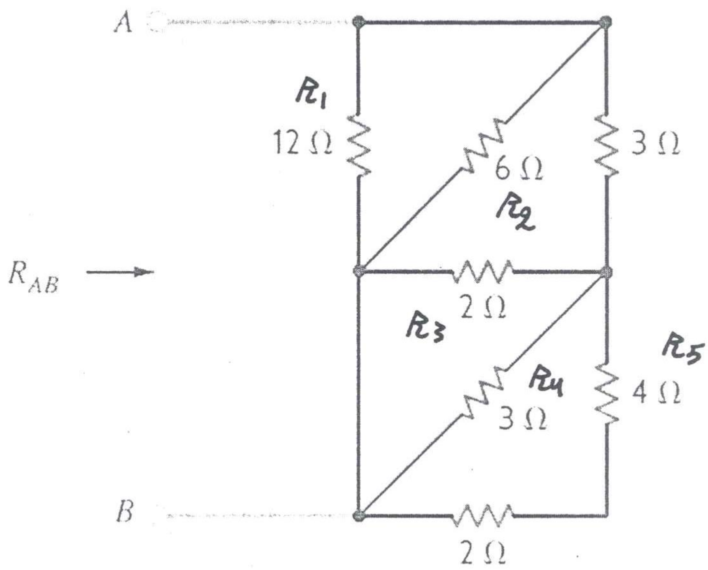


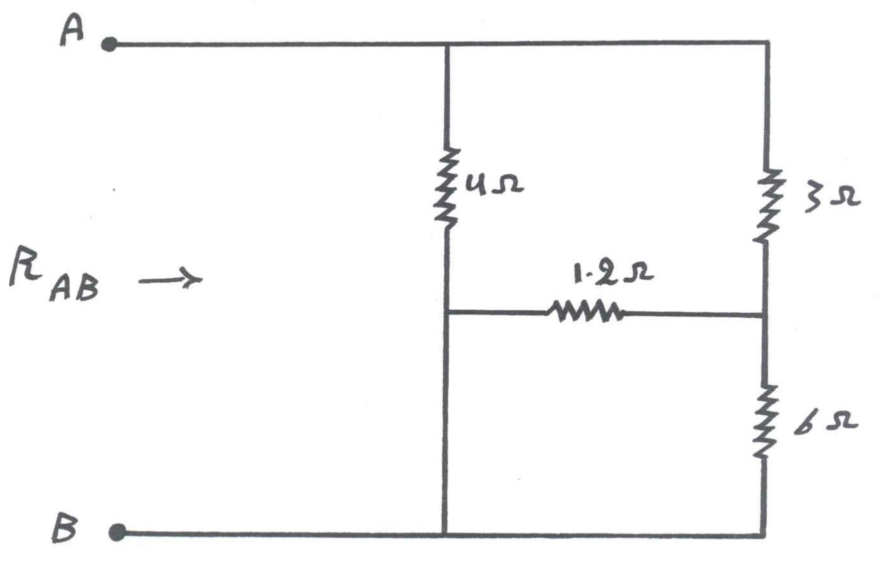
Figure P2.67

Solution

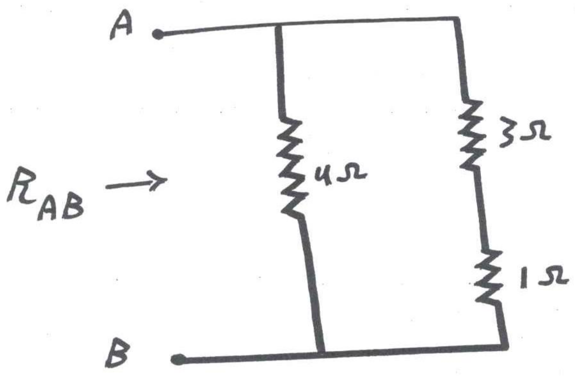
↳ $R_1 \parallel R_2 \dashrightarrow 12 \parallel 6 = 4\,\Omega$

↳ $R_3 \parallel R_4 \dashrightarrow 2 \parallel 3 = 1.2\,\Omega$

↳ $R_5 + R_6 \dashrightarrow 4 + 2 = 6\,\Omega$



$\hookrightarrow 6 \parallel 1.2 = 1\Omega$



$\hookrightarrow \therefore R_{AB} = 4 \parallel 4$

$\therefore R_{AB} = 2\Omega$

Example

2.69 Determine the total resistance, R_T, in the circuit in Fig. P2.69.

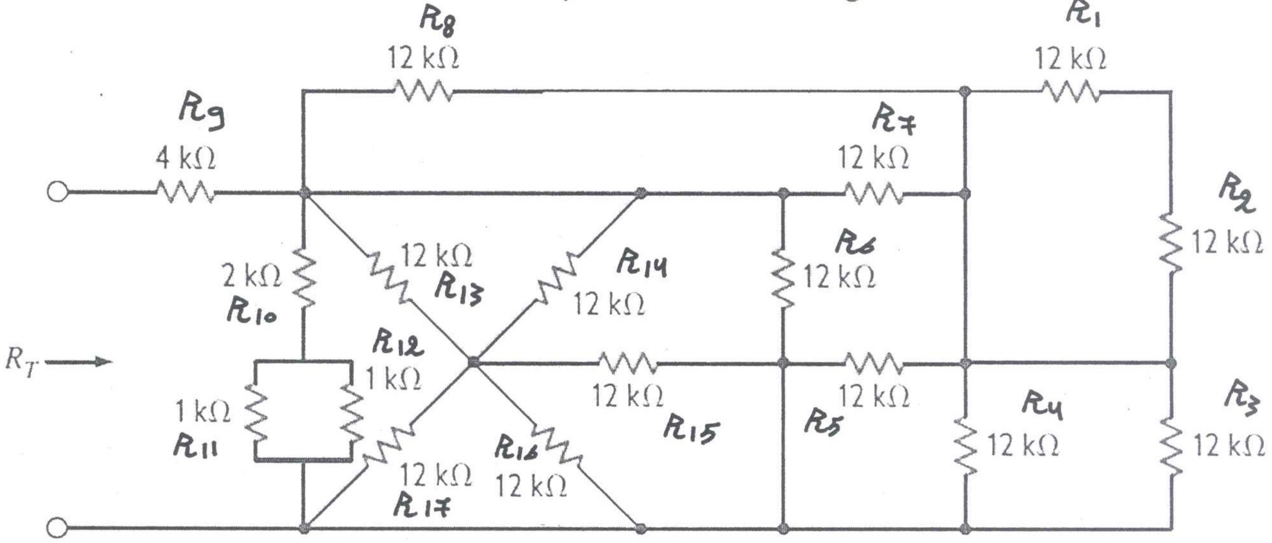


Figure P2.69

Solution

$\hookrightarrow R_1 + R_2 = 12 + 12 = 24 \ K\Omega$

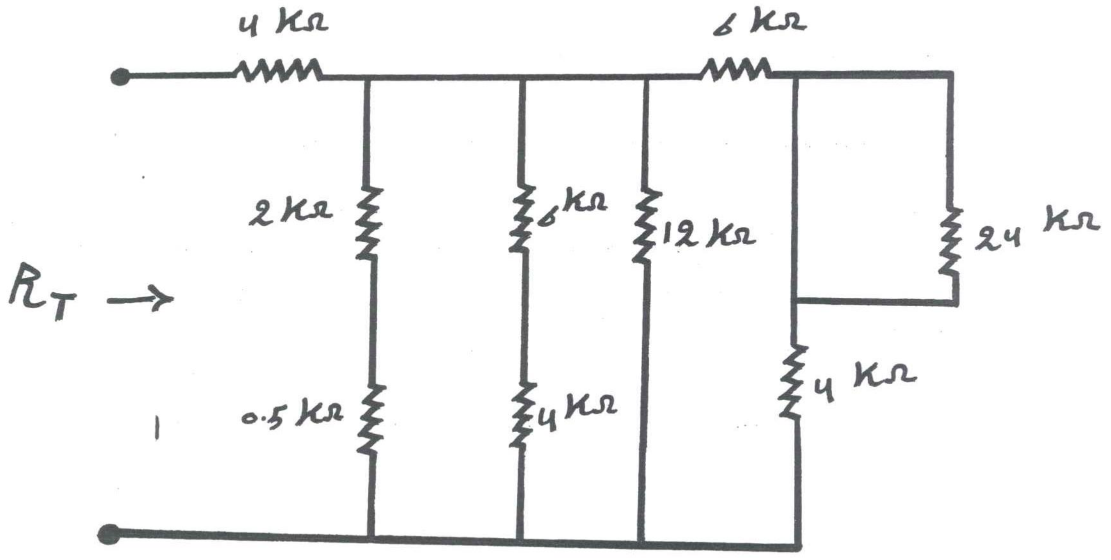
$\hookrightarrow R_3 \parallel R_4 \parallel R_5 = 12 \parallel 12 \parallel 12 = \frac{12}{3} = 4 \ K\Omega$

$\hookrightarrow R_7 \parallel R_8 = 12 \parallel 12 = 6 \ K\Omega$

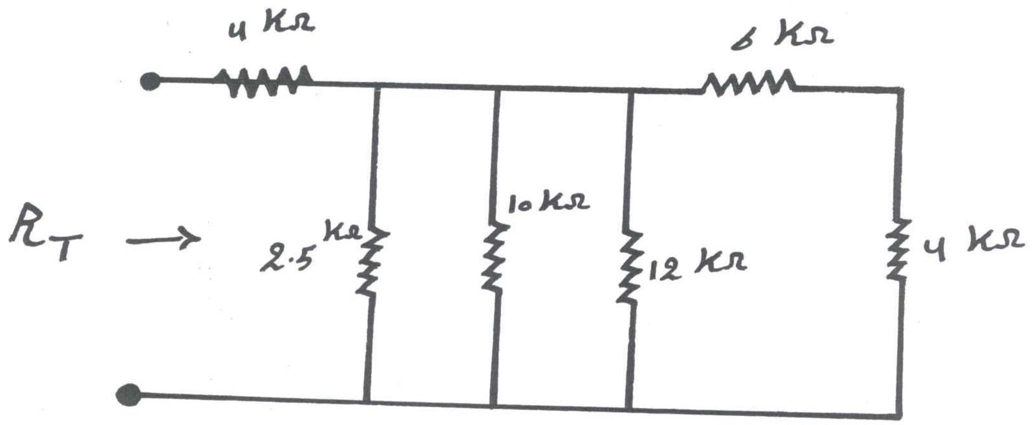
$\hookrightarrow R_{11} \parallel R_{12} = 1 \parallel 1 = 0.5 \ K\Omega$

$\hookrightarrow R_{13} \parallel R_{14} = 12 \parallel 12 = 6 \ K\Omega$

$\hookrightarrow R_{16} \parallel R_{17} \parallel R_{15} = 12 \parallel 12 \parallel 12 = \frac{12}{3} = 4 \ K\Omega$



↳ 24 KΩ ∥ short circuit --∴--> remove 24 KΩ resistor.



$\therefore R_T = 4 + (2.5 \parallel 10 \parallel 10 \parallel 12)$

↳ $2.5 \parallel 10 \parallel 10 \parallel 12 = 2.5 \parallel 12 \parallel 5$

$= 12 \parallel \frac{5}{3} = \frac{60}{41}$ KΩ

$\therefore R_T = 4 + \frac{60}{41} = \frac{224}{41}$ KΩ $= 5.46$ KΩ

Example

2.70 Determine the total resistance, R_T, in the circuit in Fig. P2.70.

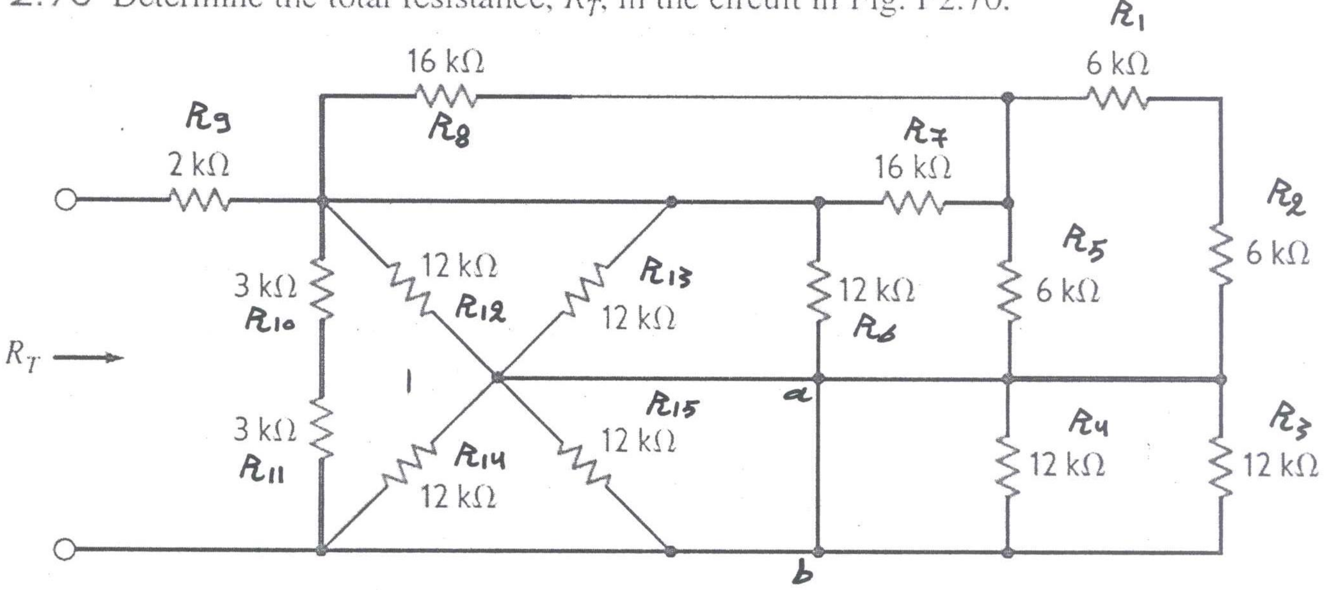


Figure P2.70

Solution

↳ $R_1 + R_2 = 6\ k\Omega + 6\ k\Omega = 12\ k\Omega$

↳ $(R_1 + R_2) \parallel R_5 = 12 \parallel 6 = 4\ k\Omega$

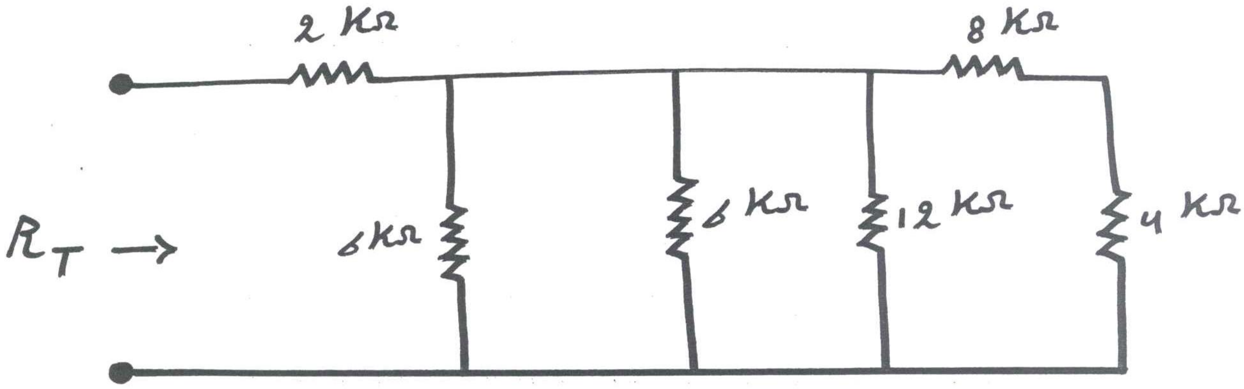
↳ $R_7 \parallel R_8 = 16 \parallel 16 = 8\ k\Omega$

↳ $R_{10} + R_{11} = 3 + 3 = 6\ k\Omega$

↳ $R_{12} \parallel R_{13} = 12 \parallel 12 = 6\ k\Omega$

↳ $(R_3 \parallel R_4 \parallel R_{14} \parallel R_{15}) \parallel$ short circuit (wire ab)

$\therefore$ R_3, R_4, R_{14}, R_{15} ---> removed.



$$\therefore R_T = 2 + (6 \parallel 6 \parallel 12 \parallel 12)$$

$$= 2 + (3 \parallel 6)$$

$$= 2 + 2$$

$$\therefore R_T = 4\ K\Omega$$

Example

1. *For the in circuit shown in Fig.(1), Find v_0 and i_0*

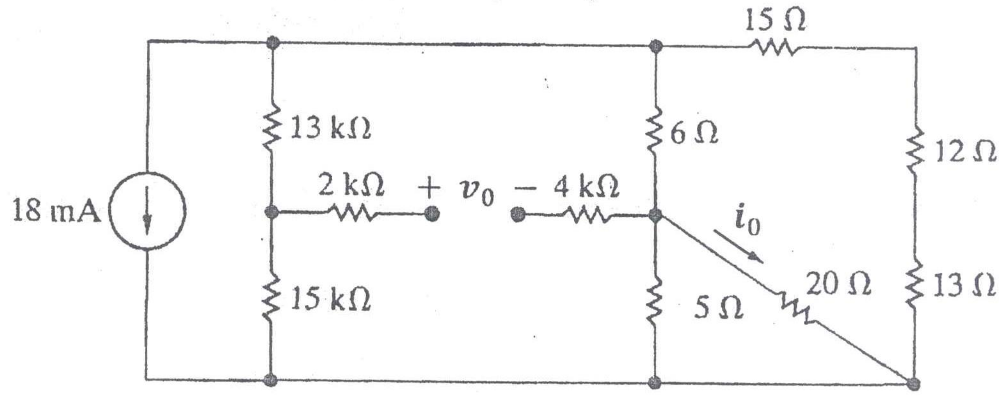


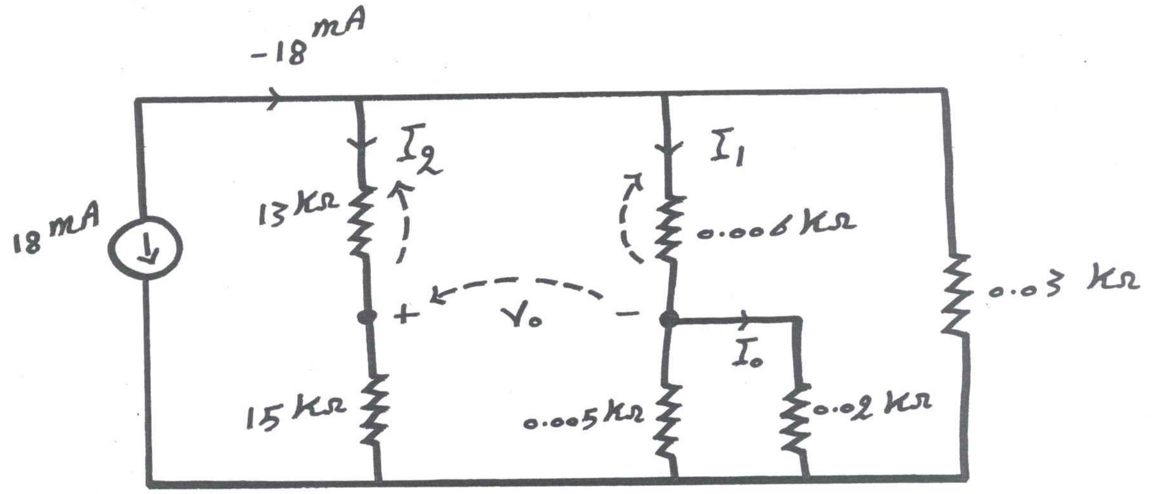
Fig.(1)

Solution

↳ current through 2 kΩ and 4 kΩ = Zero (open circuit)

∴ 2 kΩ and 4 kΩ resistors will be removed.

↳ $15 + 12 + 13 = 30\ \Omega = 0.03\ k\Omega$



↳ 13 kΩ + 15 kΩ

= 28 kΩ

↳ 0.006 kΩ + (0.005 ∥ 0.02)

= 0.006 + 0.004

= 0.01 kΩ

<u>current divider</u>

$\hookrightarrow \therefore I_1 = -18 * \frac{(28 \parallel 0.03)}{(28 \parallel 0.03) + 0.01}$

$\therefore I_1 = -13.496 \text{ mA}$

$\hookrightarrow \therefore I_o = I_1 * \frac{0.005}{0.005 + 0.02}$

$\therefore I_o = -2.7 \text{ mA}$

$\hookrightarrow 0.03 \parallel 0.01 = 0.0075 \text{ k}\Omega$

<u>current divider</u>

$\hookrightarrow \therefore I_2 = -18 * \frac{0.0075}{0.0075 + 28}$

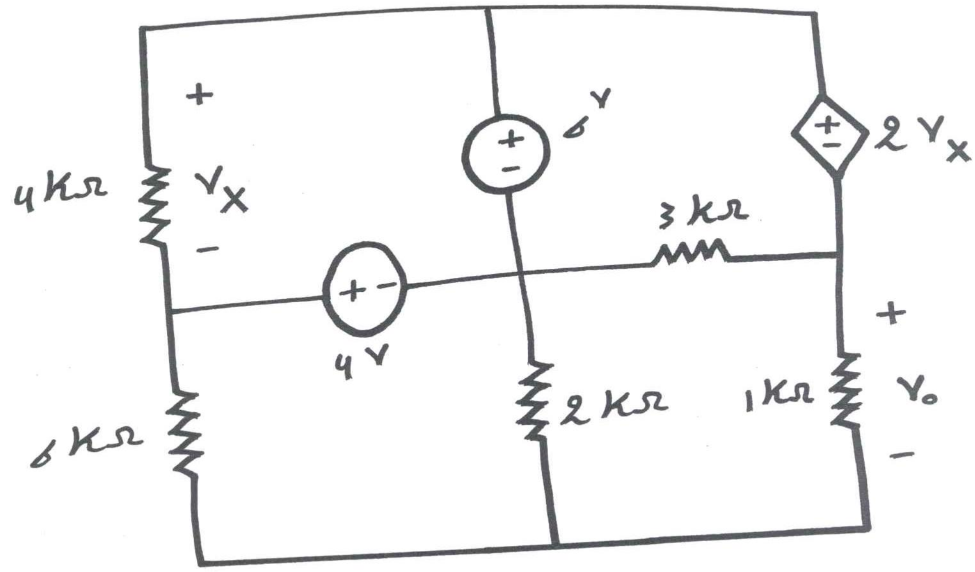
$\therefore I_2 = -0.0048 \text{ mA}$

<u>KVL</u>

$\therefore V_o + 13(-0.0048) - 0.006(-13.496) = 0$

$\therefore V_o = -0.018 \text{ V}$

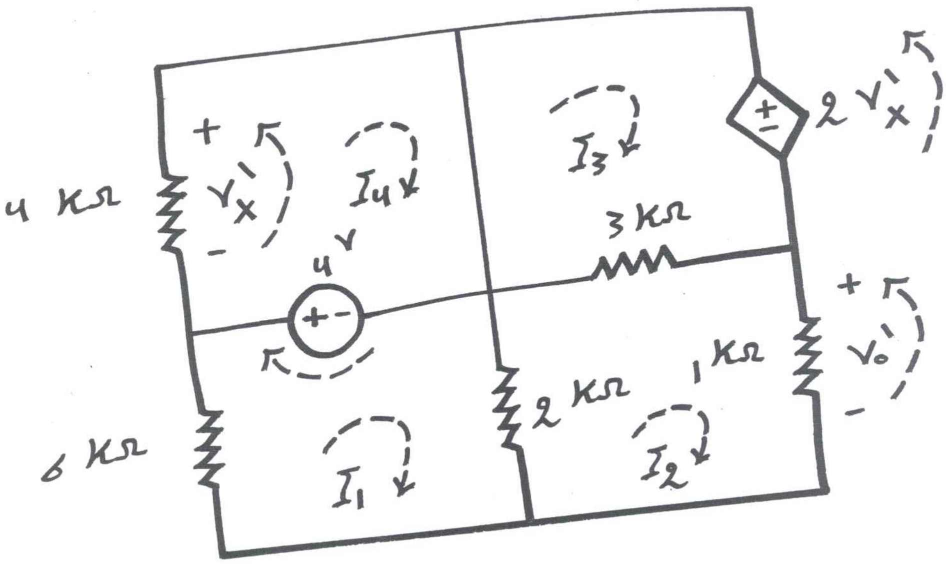
Example



find V_o using superposition.

solution

4 V source alone



↳ using mesh analysis :

mesh (1)

$$\therefore 8 I_1 - 2 I_2 = -4 \dashrightarrow \{1\}$$

mesh (2)

$$\therefore -2 I_1 + 6 I_2 - 3 I_3 = 0 \dashrightarrow \{2\}$$

mesh (3)

$$\therefore -3 I_2 + 3 I_3 = -2 V_x'$$

↳ $V_x' = -4 \dashrightarrow$ KVL at mesh (4).

$$\therefore -3 I_2 + 3 I_3 = 8 \dashrightarrow \{3\}$$

solving {1} , {2} and {3}

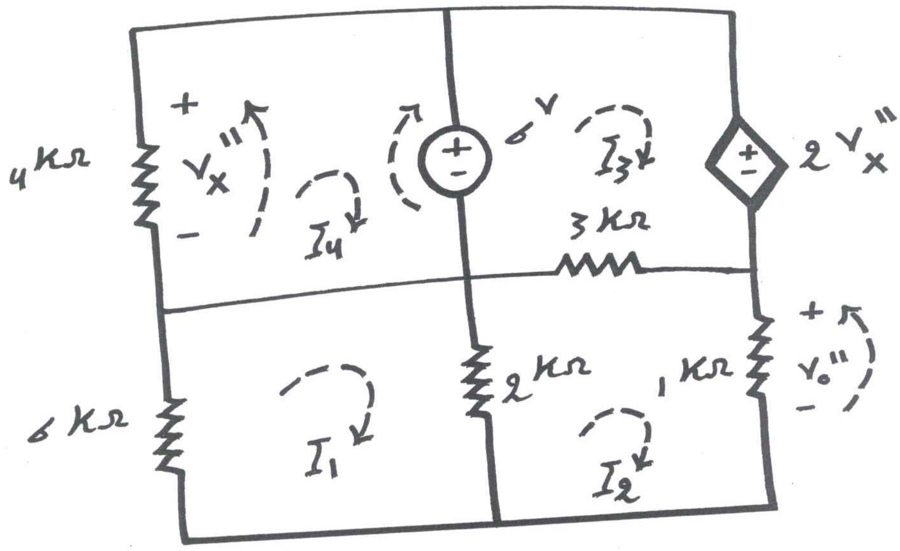
$$\therefore I_1 = 0.2 \, mA$$

$$\therefore I_2 = 2.8 \, mA \dashrightarrow V_o' = (1) I_2$$

$$\therefore V_o' = 2.8 \, V$$

$$\therefore I_3 = \frac{82}{15} \, mA$$

6^V source alone



↳ using mesh analysis:

mesh (1)

$$\therefore 8 I_1 - 2 I_2 = 0 \quad \text{---> } (1)$$

mesh (2)

$$\therefore -2 I_1 + 6 I_2 - 3 I_3 = 0 \quad \text{---> } (2)$$

mesh (3)

$$\therefore -3 I_2 + 3 I_3 = 6 - 2 V_x''$$

↳ $V_x'' = 6^V$ ---> KVL at mesh (4).

$$\therefore -3I_2 + 3I_3 = 6 - 12$$

$$\therefore -3I_2 + 3I_3 = -6 \dashrightarrow (3)$$

<u>solving (1), (2) and (3)</u>

$$\therefore I_1 = -0.6 \ mA$$

$$\therefore I_2 = -2.4 \ mA$$

$$\therefore I_3 = -4.4 \ mA$$

$$\hookrightarrow \therefore V_o'' = (1)\, I_2$$

$$\therefore V_o'' = -2.4 \ V$$

$$\hookrightarrow \therefore V_o = V_o' + V_o''$$

$$= 2.8 - 2.4$$

$$\therefore V_o = 0.4 \ V$$

Example

5.2 Using linearity and the assumption that $I_o = 1$ mA, find the actual value of I_o in the network use Fig. P5.2.

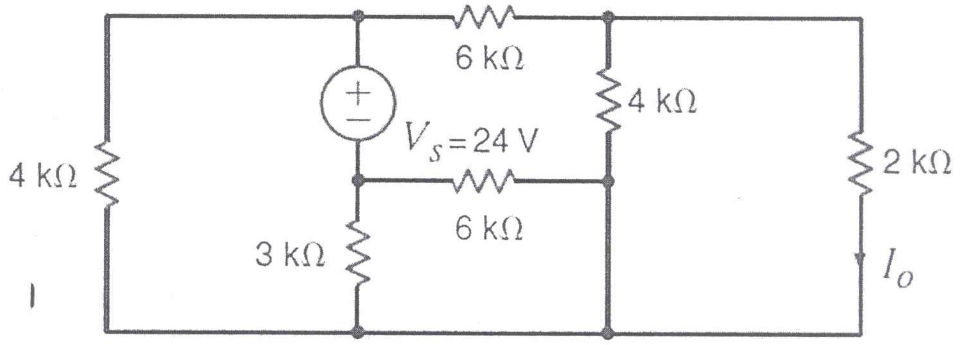
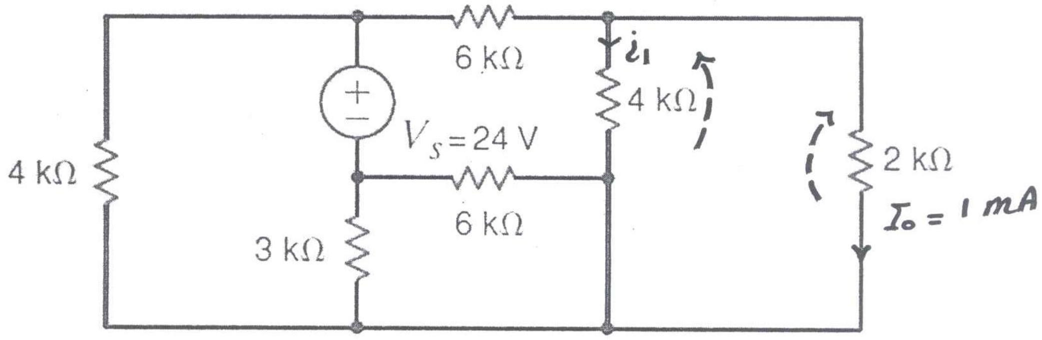


Figure P5.2

Solution

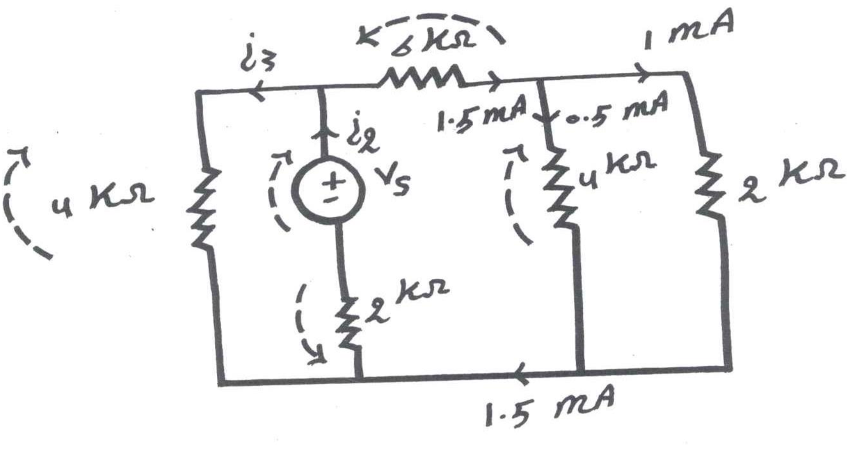


$6 \parallel 3 = 2$ kΩ

KVL

$\therefore 4 i_1 = 2 I_o = 2$

$\therefore i_1 = 0.5$ mA



KVL

$\therefore 4 i_3 = 6(1.5) + 4(0.5)$

$\therefore i_3 = 2.75$ mA

$$\therefore\ i_2 = i_3 + 1.5$$

$$= 2.75 + 1.5$$

$$= 4.25\ ^{mA}$$

<u>KVL</u> : <u>to get</u> <u>V_S</u> :

$$\therefore\ V_S = 4\, i_3 + 2\, i_2$$

$$= 4(2.75) + 2(4.25)$$

$$\therefore\ V_S = 19.5\ ^{V}$$

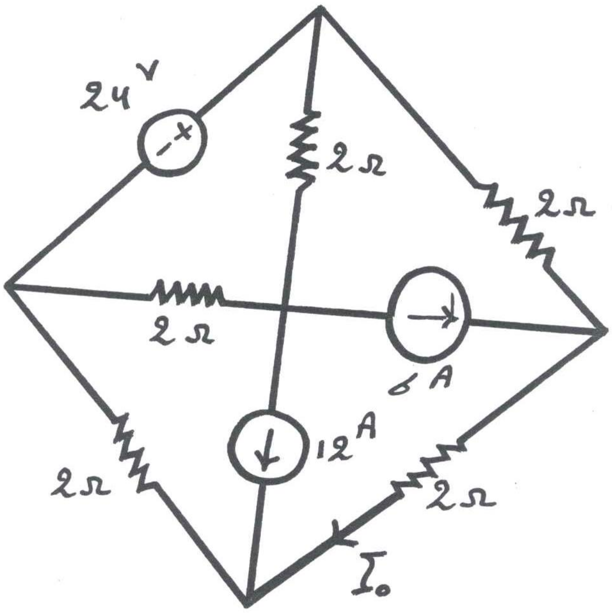
$$\rightarrow \therefore\ 24 = \alpha * 19.5$$

$$\therefore\ \alpha = \frac{24}{19.5} = \frac{16}{13}$$

$$\rightarrow \therefore\ I_o = 1^{mA} * \alpha$$

$$\boxed{\therefore\ I_o = \frac{16}{13}^{mA} = 1.23^{mA}}$$

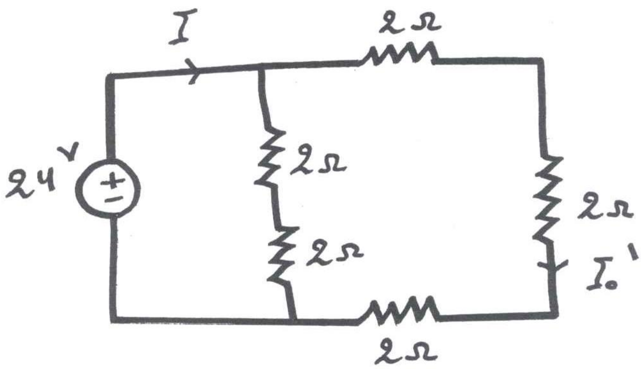
Example



↳ find I_o using superposition theorem.

Solution

24^V source alone



$$\therefore R_{eq} = 4 \parallel 6 = 2.4\ \Omega$$

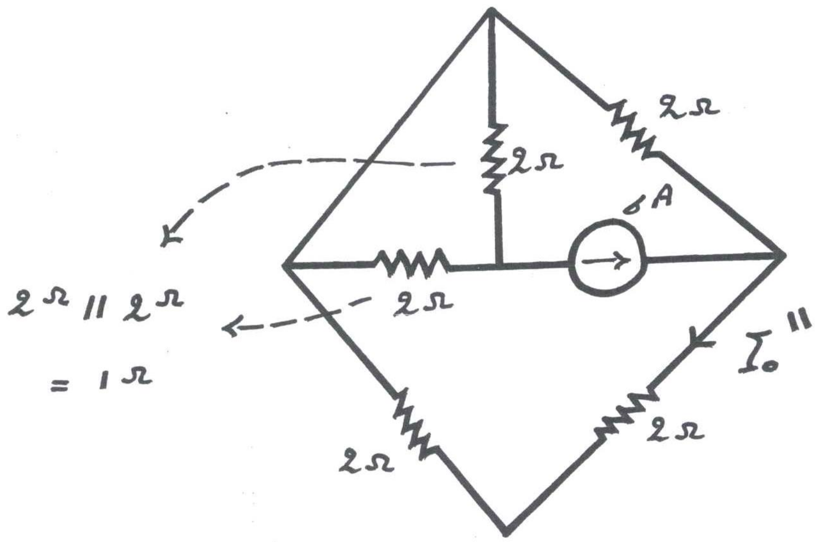
$$\therefore I = \frac{24}{2.4} = 10\ A$$

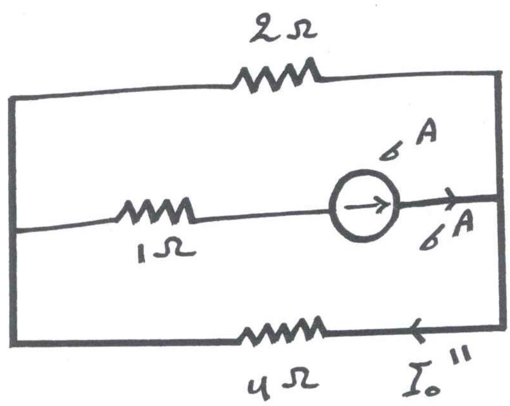
current divider

$$\therefore I_o' = 10 * \frac{4}{4+6}$$

$$\therefore I_o' = 4 \text{ A}$$

6 A source alone



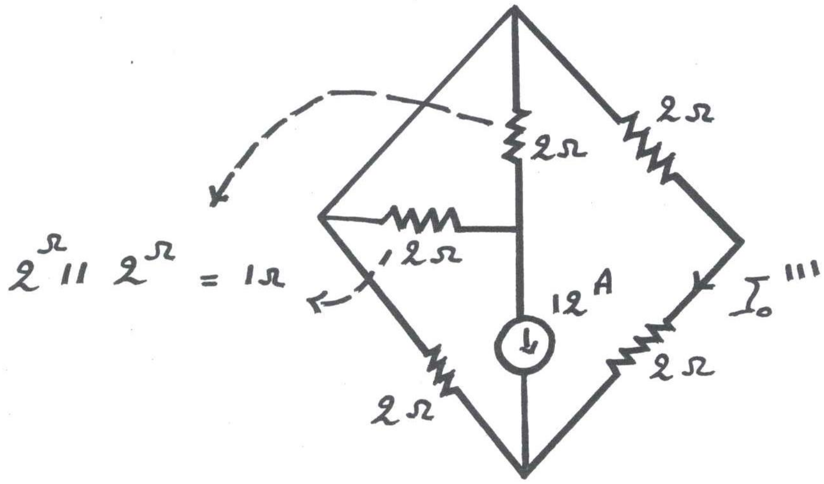


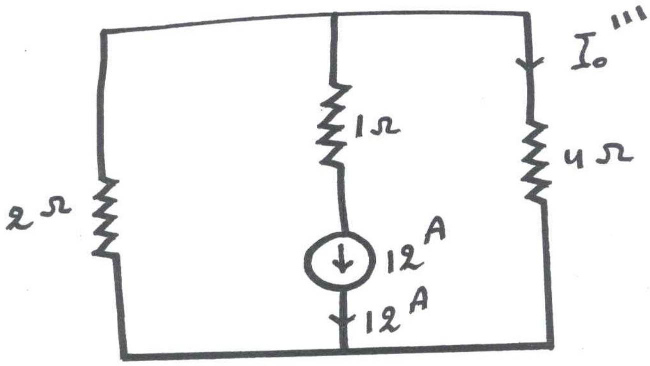
current divider

$$\therefore I_o'' = 6 * \frac{2}{2+4}$$

$$\therefore I_o'' = 2 \text{ A}$$

12 A source alone





current divider

$$\therefore I_o''' = -12 * \frac{2}{2+4}$$

$$\therefore I_o''' = -4\ A$$

$$\therefore I_o = I_o' + I_o'' + I_o'''$$

$$= 4 + 2 - 4$$

$$\therefore I_o = 2\ A$$