DEPARTMENT OF ENGINEERING AND TECHNOLOGY

QUESTION BANK

Course Name: Engineering Mathematics- -1

Course code: BTEM101

Batch: 2024 -2025

COURSE OUTCOME	
On completion of this course, students are able to	
CO1	Explain the fundamental concepts and principles of discrete mathematics, including logic, sets, functions, and relations.
CO2	Summaries mathematical ideas and solutions clearly and effectively, both orally and in writing.
CO3	Outline discrete mathematics concepts to create mathematical models for real-world problems, such as scheduling and network optimization
CO4	Apply MATLAB to solve systems of linear equations and compute eigenvalues/vectors of matrices
CO5	Make use of discrete mathematics concepts to solve problems in computer science.

Engineering Mathematics-1 (BTEM101) Syllabus	
SL.NO	**MODULES**
1	**Module-1: Linear Algebra** **(9 hours)** Rank of a matrix, Echelon form, Consistency and Solution of a system of linear equations-Gauss-elimination method. Eigenvalues and eigenvectors of a square matrix, Rayleigh's power method to find the dominant Eigenvalue and Eigenvector.
2	**Module-2: Logics and Quantifiers** **(12 hours)** Fundamentals of Logic: Propositions- Logical connectives, Tautologies, contradictions. Logical equivalence-The Laws of Logic, inverse, converse and contrapositive. Logical Implication Rules of Inference, Quantifiers-Types and uses of quantifiers. Mathematical Induction.
3	**Module-3: Graph Theory** **(9 hours)** Basic Terminologies, Types of graph, Hand shaking property, Walk, Sub graph, Spanning sub graphs, Havel- Hakimi's theorem(Statement only)-problems, Complementary graph, Isomorphism of graphs.
4	**Module-4: Number Theory** **(8 hours)** Set theory- Types, subsets, Operations on sets, Laws of set theory, Member ship table. Counting, Relations and function, Permutation and combination**.**
5	**Module-5: Probability Theory** **(7 hours)** Basic definitions, Events, Types of events, Conditional probability, Baye's theorem.

PART-A (3 Marks questions)

MODULE –1: LINEAR ALGEBRA

1. Find the rank of the matrix using elementary transformation $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$
2. Find the rank of the matrix using elementary transformation $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & -2 \end{bmatrix}$
3. Write the conditions to check the consistency of the system of equations.
4. Test the consistency of the system of equations: $x+2y+3z=1,\ 2x+3y+8z=2,\ x+y+z=3$
5. Test the consistency of the system of equations: $5x+3y+7z=4,\ 3x+26y+2z=9,\ 7x+2y+10z=5$
6. Test the consistency of the system of equations: $x+y+z=-3,\ 3x+y-2z=-2,\ 2x+4y+7z=7$
7. Solve system of equations using Gauss elimination method : $x-2y+3z=2,\ \ 3x-y+4z=4,\ \ x+y-2z=5$
8. Solve system of equations using Gauss elimination method: $x+y+z=9,\ x-2y+3z=8,\ 2x+y-z=3$
9. Using Rayleigh's power method, Find the largest Eigen value and corresponding Eigen vector of the matrix $\begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$
10. Using Rayleigh's power method, Find the largest Eigen value and corresponding Eigen vector of the matrix $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$

MODULE –2: LOGIC AND PREPOSITION

11. Define tautology, contradiction and contingency.
12. Check whether statement $\neg p \to (p \to q)$ is a tautology or not.
13. Write the inverse, converse and contrapositive of the following statement " If it rains then match will be cancelled"
14. Write the inverse, converse, and contrapositive of the following statement "If a number is even, then it is divisible by 2''
15. Explain briefly about quantifiers and its types?
16. Provide the steps and reasons to establish the logical equivalence of the following statements:

    $i)\ p \vee [p \wedge (p \vee q)] \Leftrightarrow p \quad ii)\ [(\neg p \vee \neg q) \to (p \wedge q \wedge r) \Leftrightarrow p \wedge q$
17. Symbolize using quantifiers and write converse, inverse and contrapositive of the implications:

    For each x, if $x^2+4x>21$ then $x \geq 3\, or\, x<-7$.
18. For all $n \in z^+$, prove that $\sum_{i=1}^{n} i = 1+2+3+....+n = \frac{n(n+1)}{2}$ using Principle of mathematical induction method.

19. For all $n \in z^+$ prove that $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ using Principle of mathematical induction method.

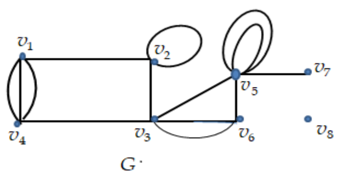
20. For all $n \in Z^+$, show that if $n \geq 24$, then n can be written as a sum of 5's and | or 7's using Mathematical Induction method.

MODULE –3: GRAPH THEORY

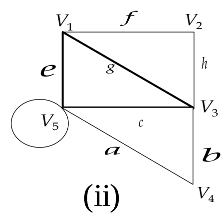
21. Define the following with suitable example.

(i) simple graph (ii) Pseudo graph (iii) Degree of a vertex (iv) Regular graph

22. Verify the Hand shaking property for the following graph:

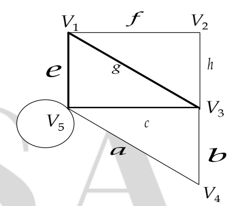


(i)



(ii)

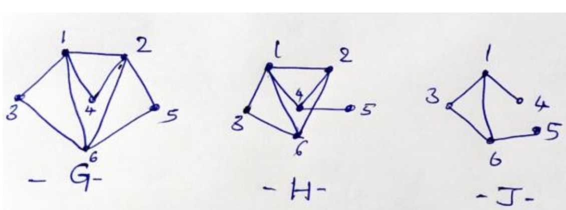
23. Define a walk. For the following graph 'G' , write the length of walk between (i) V_1 to V_5 (ii) V_1 to V_3 (iii) Identify the terminal vertices of G



24. State Hakimi's thorem. Verify which of the following represents a degree sequence of a graph using Hakimi's theorem.
i) 1, 1,1, 2, 2, 3, 4 ii) 1, 2, 2, 3, 3, 3, 5

25. Define Isomorphic graph stating all necessary conditions.

26. Define sub graph. If $G = (V, E)$ is graph, H = (V,E) and J = (V,E) are sub graphs of G then verify which of the following is a sub graph of G.



27. Define self-complementary graphs. Write the complementary graph $\overline{G}$ of the following graph.

28. Define spanning subgraph with example.
29. Write the five major applications of graph theory in the field of computer science and engineering.
30. Define: Acyclic graph, tree and Forest.

MODULE –4: SET THEORY

31. Apply Membership table to establish the following: $\overline{(A\cap B)\cup(\overline{A}\cap C)} = (A\cap \overline{B})\cup(\overline{A}\cap\overline{C})$.
32. Apply Membership table to establish the following: $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

33. Let $f, g: R \to R$, $f(x)=2x+5$, $g(x)=\dfrac{x-5}{2}$, show that f and g are invertible functions

34. Let $f, g: R\to R$, $f(x)=x^2$, $g(x)=x+5$, $h(x)=\sqrt{x^2+2}$, then that show $(hog)of = ho(gof)$.

35. Given $A=\{x/x \text{ is an integer } 5<x\le 11\}$, $B=\{3,4,5,6\}$ and $C=\{2,4,6,8,10\}$ Find
    $i)\ A\bigcup B$ $\quad ii)\ A\bigcap B$ $\quad iii)\ A-B$ $\quad iv)\ A-(B\bigcap C)$ $\quad v)\ A\Delta B$

36. Define Invertible function. Prove that $f: A\to B$ and $g: B\to C$ are invertible functions.

37. If U= {1,2,3,4…..10}, A= {1,2,3,8,9,10} and B= {3,4,5,7,8}. Verify that (A U B)'= A' ∩ B'

38. If $f: A\to B$, $g: B\to C$ and $h: C\to A$ are any functions , then prove that $(hog)of = ho(gof)$

39. Four different mathematics books, five different computer science books and two different control theory books are to be arranged in a shelf. How many different arrangements are possible if (a) the books in each particular subject must all be together (b) only the mathematical books must be together?

40. How many committees of five with a given chairperson can be selected from 12 persons?

MODULE – 5: PROBABILITY

41. Define the following with examples: (i) Random experiment (ii) Sample space

42. Define events and types of events.

43. Define axiomatic probability.

44. Define Conditional Probability.
45. Ten numbered cards are there from 1 to 15 and two cards a chosen at random such that the sum of the numbers on both the cards is even. Find the probability that the chosen cards are odd-numbered.
46. Let E and F are events of a experiment such that P(E) = 3/10 P(F) =1/2 and P(F|E) = 2/5. Find the value of (i) P(E ∩ F) (ii) P(E|F) (iii) P(E U F)
47. The probability of a student passing in science is 4/5 and the of the student passing in both science and math is 1/2. What is the probability of that student passing in math knowing that he passed in science?
48. In a survey among few people, 60% read Hindi newspaper, 40% read English newspaper and 20% read both. If a person is chosen at random and if he already reads English newspaper find the probability that he also reads Hindi newspaper.

49. For three events A, B, and C, we know that, A and C are independent, B and C are independent, A and B are disjoint, P(A∪C)=2/3, P(B∪C)=3/4, P(A∪B∪C)=11/12, P(A∪C)=2/3, P(B∪C)=3/4, P(A∪B∪C)=11/12 Find P(A), P(B) and P(C).
50. If Aand B are independent events then prove that $\overline{A}$ *and* $\overline{B}$ are independent.

PART-B

MODULE –1: LINEAR ALGEBRA

1. Define Rank of a matrix and find the Rank of the matrix by reducing into echelon form :
$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$
2. Define the rank of a matrix and Find the Rank of the matrix by reducing into echelon form :
$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$
3. Test the consistency and solve the system of equation:
$x+2y+2z=5,\quad 2x+y+3z=6,\quad 3x-y+2z=4\ ,\ x+y+z=-1$
4. Test the consistency and solve the system of equation:
$x+3y+2z=0,\quad 2x-y+3z=0,\quad 3x-5y+4z=0,\quad x+17y+4z=0$
5. Find the value of λ and μ for which the system $2x+3y+5z=9,\quad 7x+3y-2z=8,\quad 2x+3y+\lambda z=\mu$ has (i) Unique solution (ii) Infinitely many solutions (iii) No solution.
6. Find the value of λ and μ for which the system $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ has (i) Unique solution (ii) Infinitely many solutions (iii) No solution.
7. Apply Gauss elimination method to solve the system of equations
$3x+y+2z=3,\ 2x-3y-z=-3,\ x+2y+z=4$
8. Apply Gauss elimination method to solve the system of equations
$2x+y+2=10,\ 3x+2y+3z=18,\ x+4y+9z=16$
9. Using Rayleigh's power method, find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by taking $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ as initial eigen vector. Carry out 4 iterations.
10. Using Rayleigh's power method, find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ as initial eigen vector. Carry out 5 iterations.

MODULE –2: LOGIC AND PREPOSITION

11. Let $p(x), q(x), r(x)$ and $s(x)$ are the open statements given by $p(x): x \ge 0, q(x): x^2 \ge 0$, $r(x): x^2 - 3x - 4 = 0$ *and* $s(x): x^2 - 3 > 0$. Determine the truth value of the following $i)\ \exists x[p(x) \wedge r(x)]$ ii) $\forall x[p(x) \to q(x)]$ iii) $\forall x[q(x) \to s(x)]$

12. Establish the validity of the argument:

$$\begin{array}{l} \forall x\ [\ p(x) \vee q(x)\ ] \\ \exists x\ [\neg p(x)] \\ \forall x\ [\neg q(x) \vee r(x)\ ] \\ \underline{\forall x\ [\ s(x)\ \to\ \neg r(x)\ ]} \\ \therefore\ \exists x\ \neg s(x) \end{array}$$

13. Establish the validity of the following argument:

$$\begin{array}{l} p \to r \\ r \to s \\ t \to \neg s \\ \neg t \to u \\ \underline{\quad\ \neg u} \\ \therefore \neg p \end{array}$$

14. Symbolize using quantifiers and negate the following statements.
   i) For all integers x, if x is odd then $x^2 - 1$ is even.
   ii) For some integer x; $2x + 1 = 5$ and $x^2 = 9$

15. Show that 2^n greater than $2^n > n^2$ whenever 'n' is positive integer greater than 4 using mathematical induction.

16. Prove using **Mathematical Induction** that for all n∈N, $n^5 - n$ is divisible by 5

17. State and prove the principle of Mathematical induction.

18. For $n \ge 0$, Let F_n denote the n^{th} Fibonacci number, Prove that

$$F_0 + F_1 + \cdots + F_n = \sum_{i=0}^{n} {F_i}^2 = F_n \times F_{n+1}$$

19. Using principal of Mathematical induction prove that:

$$F_0 + F_1 + \cdots + F_n = \sum_{i=0}^{n} F_i = F_{n+2} - 1$$

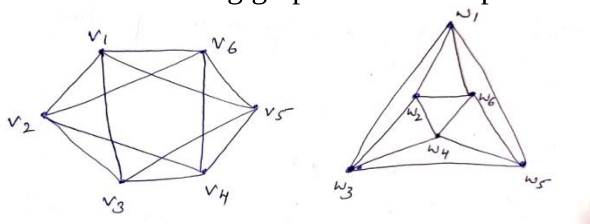
20. On an island, there are only two types of people: **truth-tellers**, who always tell the truth, and **liars**, who always lie. You meet **three people**: Alice, Bob, and Charlie.

   i) Alice says: *"Bob is a liar."*
   ii) Bob says: *"Charlie is a liar."*
   iii) Charlie says: *"Alice and I are of the same type."*

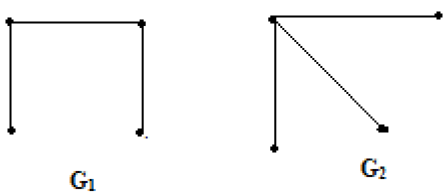
   Who is a truth-teller and who is a liar?

MODULE –3: GRAPH THEORY

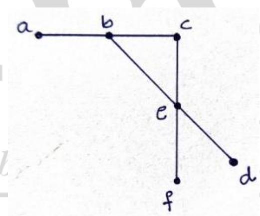
21. Prove that the number of vertices with odd degree in a graph is always even.
22. Prove that the following graphs are isomorphic.



23. Check whether the following graphs are Isomorphic or not?

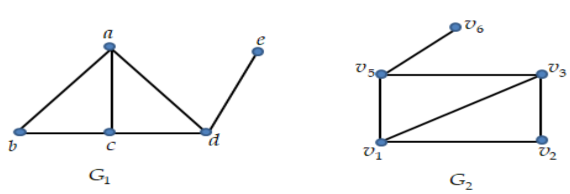


24. Write down all walks of length 5 between the vertices a and d in the graph G .



25. Which of the following is a degree sequence of a graph. Justify your answer with relevant theory. In possible cases construct at least one graph corresponding to the sequence.
    (i) 1, 1, 2, 2, 3, 3, 4
    (ii) 1, 2, 2, 3, 3, 3, 5
    (iii) 1, 1, 2, 2, 2, 3, 7

26. Show that the graphs G1 and G2 are isomorphic to each other.



27. In a party, each person shakes hands with every other person exactly once. If there are 10 people at the party:
    (a) How many handshakes took place?
    (b) Model the situation as a graph and describe the type of graph formed.
    (c) Is this graph connected? Justify.

MODULE –4: SET THEORY

28. . For any universe U and any sets $A,B\subseteq U$, prove that the following statements are equivalent; a) $A\subseteq B$ b) $A\bigcup B=B$
29. Using the laws of set theory, simplify the following:
    i) $\overline{A\Delta B}=\overline{A}\Delta B=A\Delta\overline{B}$
    ii) $(A-B)\bigcup(A\bigcap B)=A$.

30. State and prove Pigeonhole Principle. Using the principle show that in any group of 29 persons must have been born on the same day of the week.
31. In how many ways can the letters of the word ' **ÉNGINEERING** 'be arranged if the
    (i) Words start with 'E' and end with 'E'. (ii) All E's are together.

32. Find the number of 4-digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5,6 if no digit is repeated. How many of these will be odd?
33. A certain question paper contains three parts A, B, C, with four questions in Part A, five questions in Part B and six questions in Part C. It is required to answer seven questions selecting at least two questions from each part. In how many different ways can a student select his seven questions for answering?
34. A woman has 11 close relatives and she wishes to invite five of them to dinner. In how many ways can she invite them in the following situations:

    (i) There is no restriction on the choice.

    (ii) Two particular persons will not attend separately.

    (iv) Two particular persons will not attend together.

35. Let U={x∈Z:1≤x≤30} be the universal set. Define the following sets:

A={x∈U: x is a multiple of 3},B={x∈U: x is a multiple of 5}, C={x∈U: x is a multiple of 4} then

(i) Find A∪B, A∩C and$(A\cup B)^c$.
(ii) Verify whether the following statement is true or false:(A∪B)∩C=(A∩C)∪(B∩C)

MODULE – 5 : PROBABILITY

36. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student strength,
    (i) what is the probability that mathematics being studied
    (ii) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl.

37. If A and B are two events with $p(A)=\frac{1}{2},\ p(\text{B})=\frac{1}{3},\ p(A\cap B)=\frac{1}{4}$ then

Find $p\left(A/B\right),\ p\left(B/A\right), p\left(\overline{A}/\overline{B}\right), p\left(\overline{B}/\overline{A}\right).$

38. Among 20 engineers working on a project, 5 are post graduates. If 3 of them are selected at random, what is the probability that (i)they are all post graduates (ii) at least one is post graduate
39. Three machines A, B and C produces 50%,30% and 20% of the items in a factory. The percentages of defective outputs are 3%, 4% and 5% respectively. If an item is selected at random, what is the probability that it is defective? What is the probability that it is from A.
40. State and Prove Bay's theorem.
41. In a college, 40% of students play cricket, and 30% play football. 20% of the students play both cricket and football. What is the probability that a student plays football **given** that they play cricket?
42. In a class of 100 students: 45 students passed in Mathematics,50 passed in Physics,30 passed in both subjects. Then

    (i) What is the probability that a randomly selected student passed in at least one subject?
    (ii) What is the probability that a student passed in Mathematics **given** they passed in Physics?

43. Let A and B be two events such that P(A)=0.5, P(B)=0.6 and P(A∩B)=0.3.Find:
    a) P(A∪B) b) $P(A^c \cap B)$

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