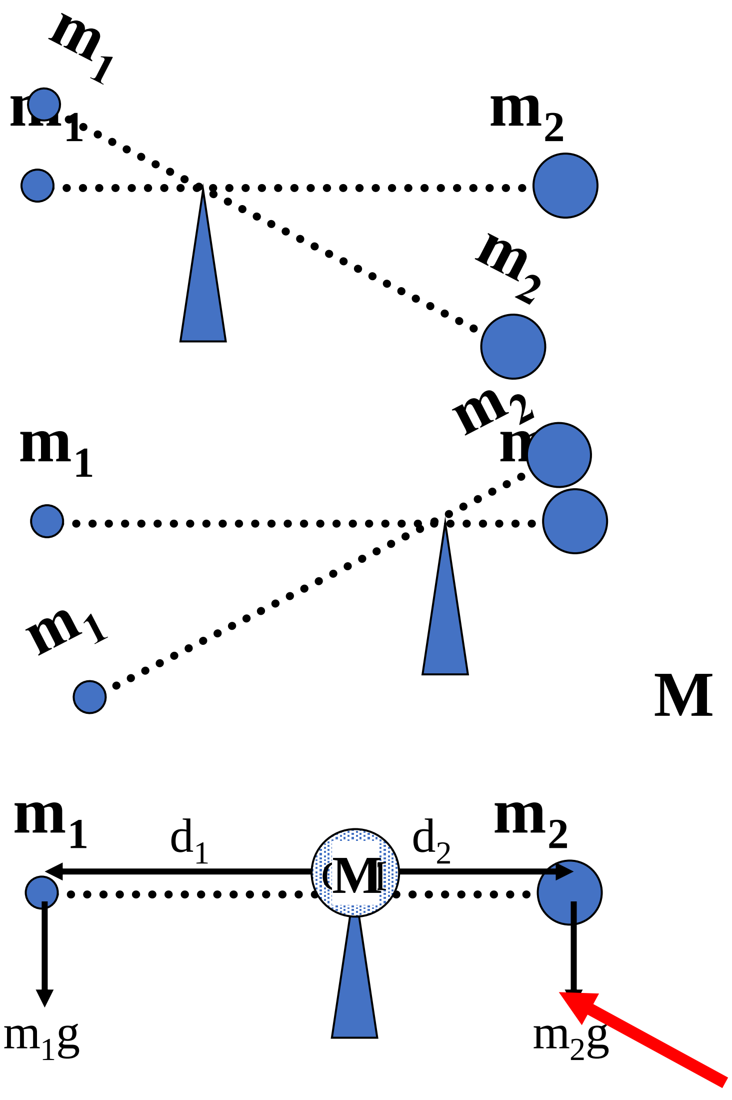
System of particles

The Centre of Mass

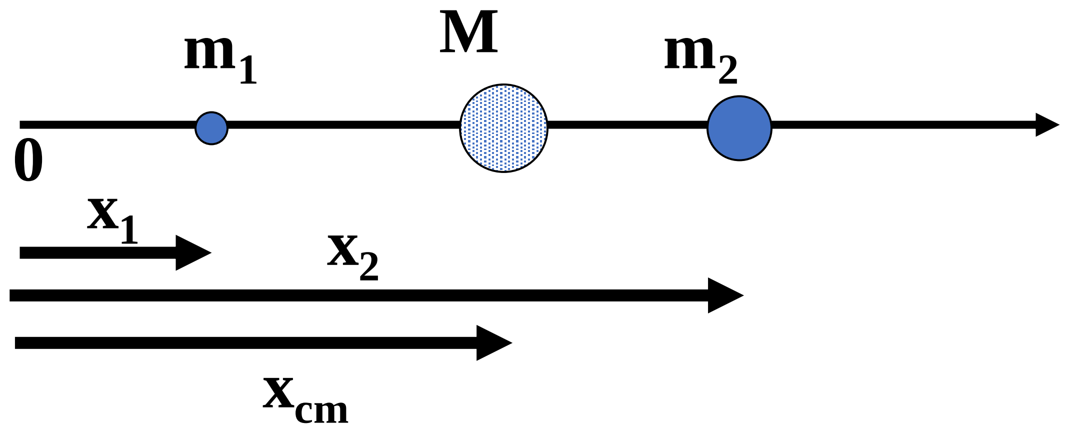


$$M = m_1 + m_2$$

$$m_1g \times d_1 = m_2g \times d_2$$

Centre of Mass (1D)

$$M = m_1 + m_2$$



$$M\,x_{cm} = m_1\,x_1 + m_2\,x_2$$

moment of M = moment of individual masses

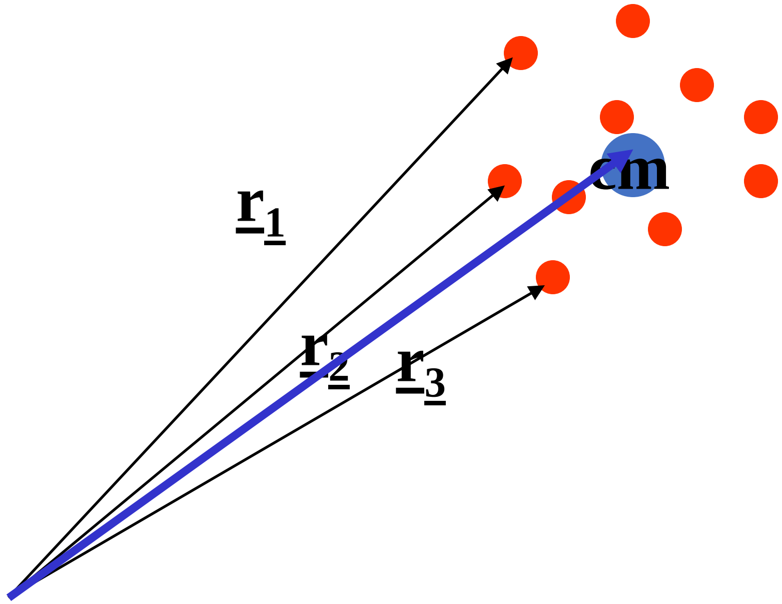
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{M}$$

In general

$$\boxed{x_{cm} = \frac{1}{M}\sum m_i x_i}$$

Centre of Mass (2Dims)

For a collection of masses in 2D

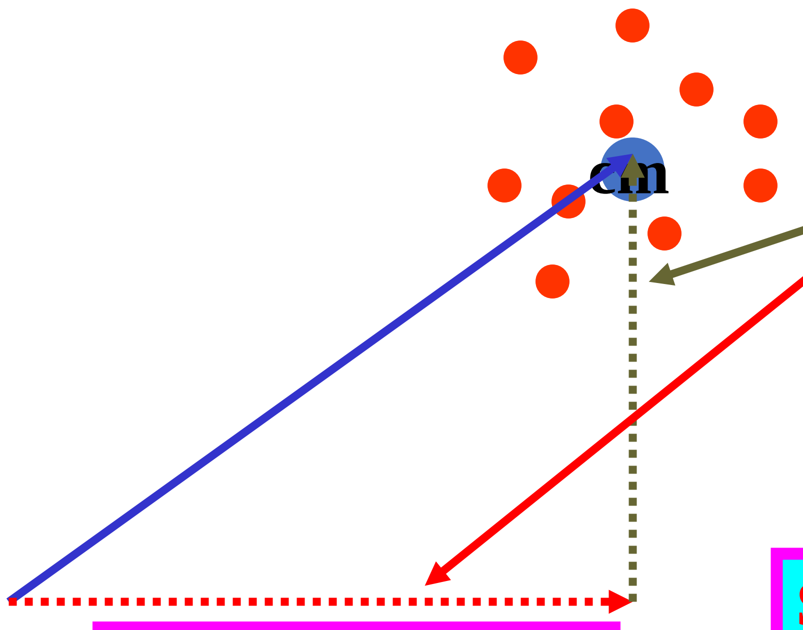


$$\underline{r}_{cm} = \frac{1}{M}\sum m_i \underline{r}_i$$

Centre of Mass (3D)

For a collection of masses in 3D

$$\underline{\mathbf{r}}_{cm} = \underline{\mathbf{i}}x_{cm} + \underline{\mathbf{j}}y_{cm}$$



$$x_{cm} = \frac{1}{M}\sum m_i x_i$$

$$y_{cm} = \frac{1}{M}\sum m_i y_i$$

$$\boxed{\underline{\mathbf{r}}_{cm} = \frac{1}{M}\sum m_i \underline{\mathbf{r}}_i}$$

So in a solid body we can find the CM by finding x_{cm} and y_{cm}

$$\boxed{\underline{r}_{cm} = \frac{1}{M}\sum m_i \underline{r}_i}$$

$$x_{cm} = \frac{1}{M}\sum m_i x_i$$

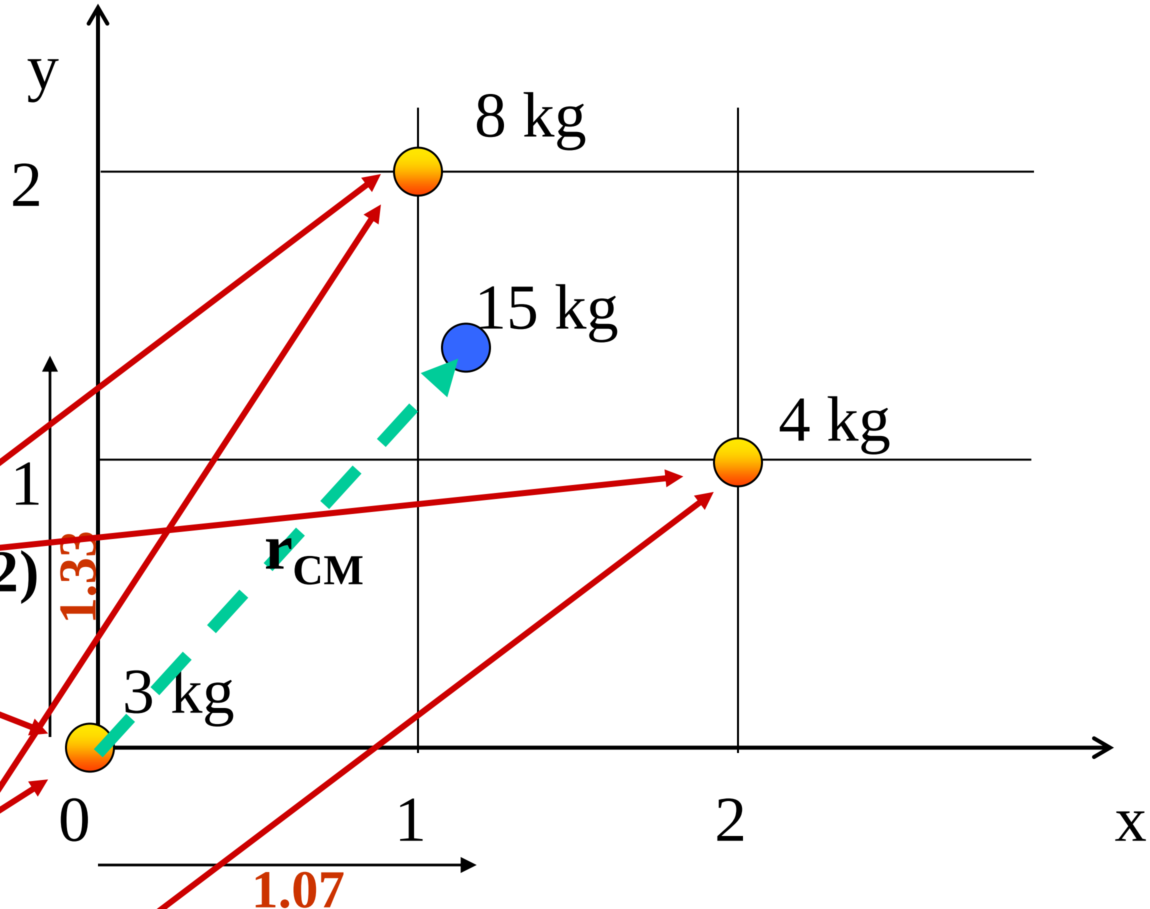
$$x_{cm} = \frac{1}{15}(3x0 + 8x1 + 4x2)$$

X_{cm} = 16/15 = 1.07 m

$$y_{cm} = \frac{1}{M}\sum m_i y_i$$

$$y_{cm} = \frac{1}{15}(3x0 + 8x2 + 4x1)$$

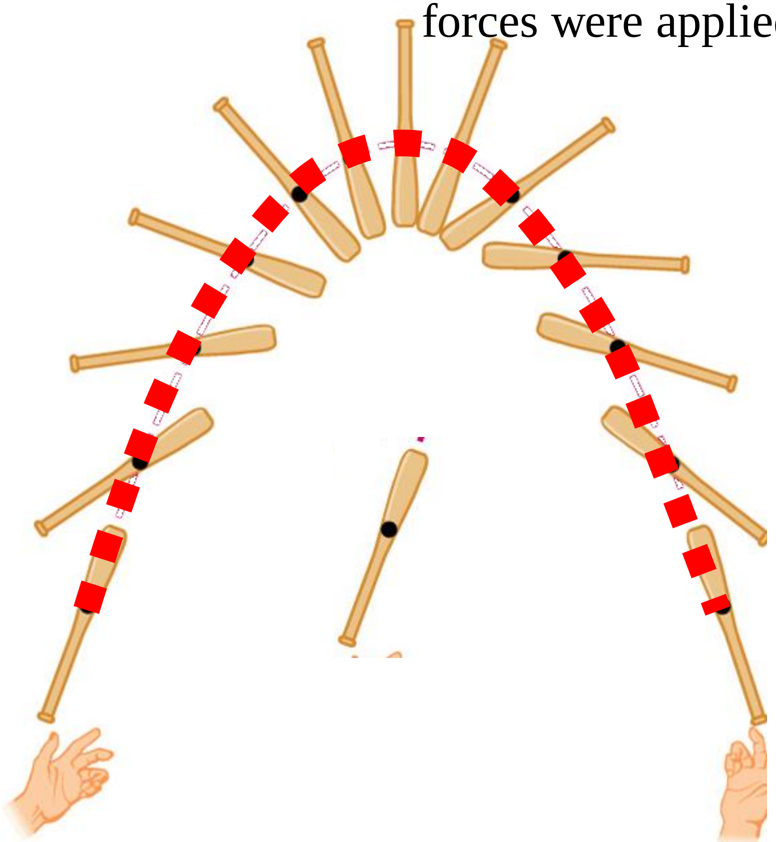
y_{cm} = 20/15 = 1.33 m

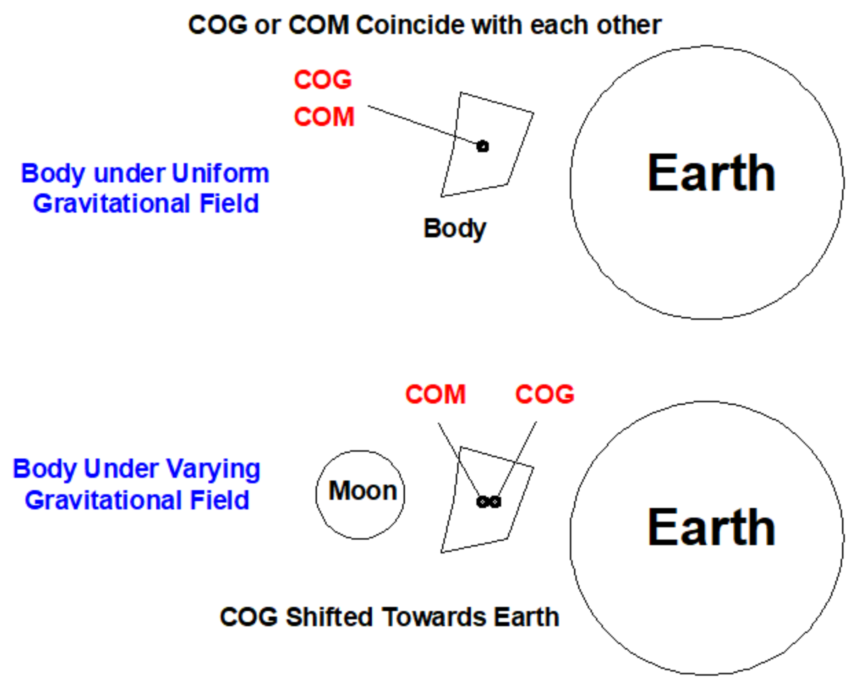


The Centre of Mass

The Difference Between Centre of Mass and Centre of Gravity

The center of mass of a body or a system of bodies is the point that moves as though all the mass were concentrated there and all external forces were applied there.





Velocity and Acceleration of the Center of Mass

- If its particles are moving, the CM of a system can also move.
- Suppose we know the position $\boldsymbol{r}_i$ of every particle in the system as a function of time.

$$\boldsymbol{R}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i \boldsymbol{r}_i \qquad \left(M = \sum_{i=1}^{N} m_i \right)$$

So: $$\boldsymbol{V}_{CM} = \frac{d\boldsymbol{R}_{CM}}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \frac{d\boldsymbol{r}_i}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \boldsymbol{v}_i$$

And: $$\boldsymbol{A}_{CM} = \frac{d\boldsymbol{V}_{CM}}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \frac{d\boldsymbol{v}_i}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \boldsymbol{a}_i$$

- The velocity and acceleration of the CM is just the weighted average velocity and acceleration of all the particles.

Linear Momentum:

- **Definition:** For a single particle, the momentum $\boldsymbol{p}$ is defined as:

$$\boxed{\boldsymbol{p} = m\boldsymbol{v}}$$

($\boldsymbol{p}$ is a vector since $\boldsymbol{v}$ is a vector).

So $p_x = mv_x$ etc.

- Newton's 2nd Law:

$$\boldsymbol{F} = m\boldsymbol{a}$$

$$= m\frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}(m)\boldsymbol{v} \Rightarrow \boxed{\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}}$$

- Units of linear momentum are *kg m/s*.

Linear Momentum:

- For a system of particles the total momentum *P* is the vector sum of the individual particle momenta:

$$\boldsymbol{P} = \sum_{i=1}^{N} \boldsymbol{p}_i = \sum_{i=1}^{N} m_i \boldsymbol{v}_i$$

But we just showed that

$$\sum_{i=1}^{N} m_i \boldsymbol{v}_i = M\boldsymbol{V}_{CM}$$

$$\left(\boldsymbol{V}_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i \boldsymbol{v}_i \right)$$

So

$$\boxed{\boldsymbol{P} = M\boldsymbol{V}_{CM}}$$

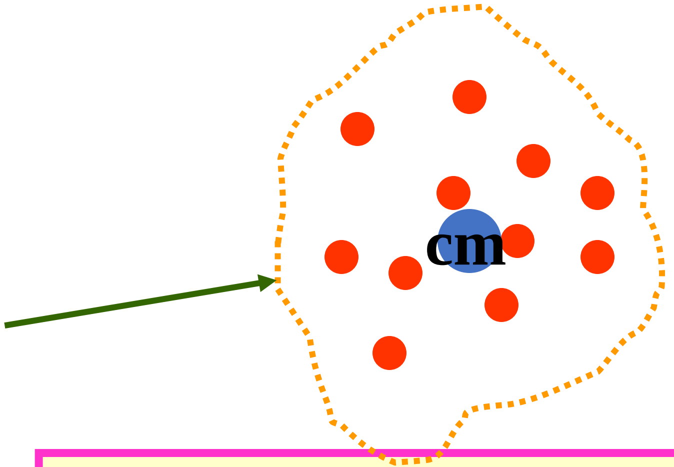
Linear Momentum:

- So the total momentum of a system of particles is just the total mass times the velocity of the center of mass.

$$\boldsymbol{P} = M\boldsymbol{V}_{CM}$$

- Observe: $$\frac{d\boldsymbol{P}}{dt} = M\frac{d\boldsymbol{V}_{CM}}{dt} = M\boldsymbol{A}_{CM} = \sum_i m_i \boldsymbol{a}_i = \sum_i \boldsymbol{F}_{i,net}$$

- We are interested in $\frac{d\boldsymbol{P}}{dt}$ so we need to figure out $\sum_i \boldsymbol{F}_{i,net}$



For a system of particles, the dynamics of the Centre of Mass obeys Newton 2.

$$\Sigma F = Ma_{cm}$$

Sum of all EXTERNAL forces acting on system

The total mass of the system

The acceleration of the CM of the system

$$\Sigma F_{ext-z} = Ma_{cm-z}$$

$$\Sigma F_{ext-y} = Ma_{cm-y}$$

$$\Sigma F_{ext-x} = Ma_{cm-x}$$

This also applies to a solid body, where the individual particles are rigidly connected. The dynamics of the Centre of Mass obeys Newton 2

$$\Sigma F = Ma_{cm}$$

$$\Sigma F_{ext-z} = Ma_{cm-z}$$

$$\Sigma F_{ext-y} = Ma_{cm-y}$$

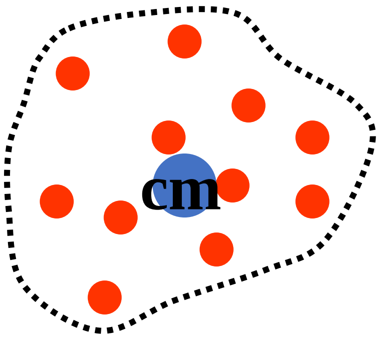
$$\Sigma F_{ext-x} = Ma_{cm-x}$$

Linear Momentum of system of particles

$$\boxed{\sum F_{ext} = ma}$$

You will recall that $\sum F = \frac{dp}{dt} = ma$

Where $\underline{p} = m\underline{v}$ is the momentum of each particle



<u>For a system of particles</u>

$$\underline{P} = M\underline{v}_{cm}$$

$$\boxed{\sum F_{ext} = \frac{dP}{dt}}$$

for system of particles

This also applies to extended objects

Conservation of Linear Momentum

$$\sum F_{ext} = \frac{dP}{dt}$$

NO EXTERNAL forces act on the system

If $\underline{F}_{ext} = 0$

$$\frac{d\underline{P}}{dt} = 0 \Rightarrow \underline{P} \text{ is a constant}$$

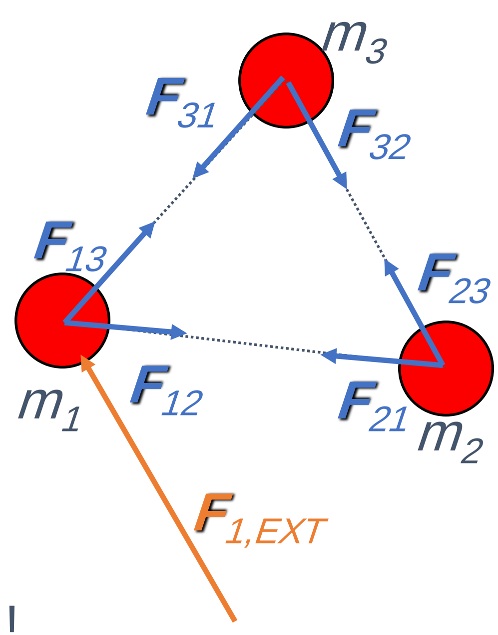
That is: $\underline{P}_x$, $\underline{P}_y$ and $\underline{P}_z$ remain constant if $\underline{F}_{ext\text{-}x}$, $\underline{F}_{ext\text{-}y}$ and $\underline{F}_{ext\text{-}z}$ are zero

In an isolated system, momentum is conserved.

Linear Momentum:

- Suppose we have a system of three particles as shown. Each particle interacts with every other, and in addition there is an external force pushing on particle 1.

$$\sum_i \boldsymbol{F}_{i,NET} = \left(\boldsymbol{F}_{13} + \boldsymbol{F}_{12} + \boldsymbol{F}_{1,EXT}\right)$$
$$+ \left(\boldsymbol{F}_{21} + \boldsymbol{F}_{23}\right)$$
$$+ \left(\boldsymbol{F}_{31} + \boldsymbol{F}_{32}\right)$$
$$= \boldsymbol{F}_{1,EXT}$$



(since the other forces cancel in pairs...Newton's 3rd Law)

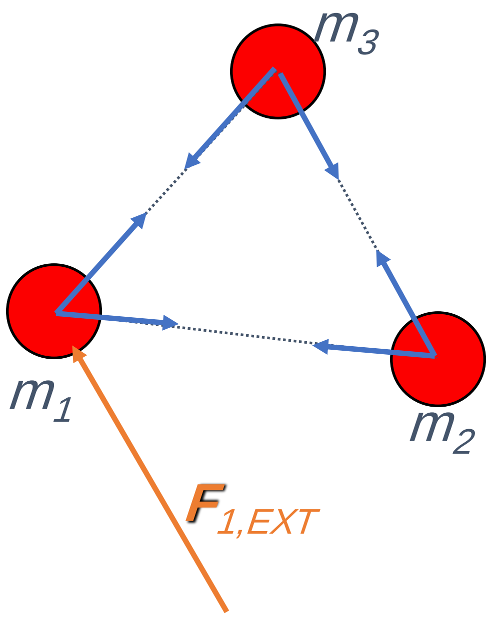
All of the **"internal"** forces **cancel** !!

Only the **"external"** force **matters** !!

Linear Momentum:

- Only the total external force matters!

$$\frac{d\boldsymbol{P}}{dt} = \sum_i \boldsymbol{F}_{i,EXT} = \boldsymbol{F}_{NET,EXT}$$



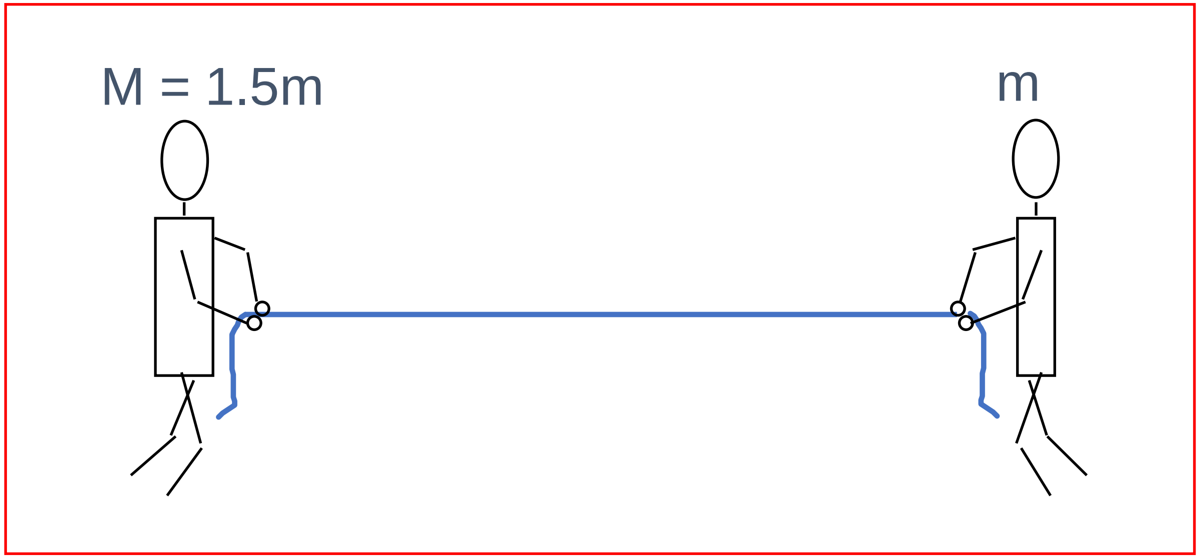
Which is the same as:

$$\boldsymbol{F}_{NET,EXT} = \frac{d\boldsymbol{P}}{dt} = M\boldsymbol{A}_{CM}$$

Newton’s 2nd law applied to systems!

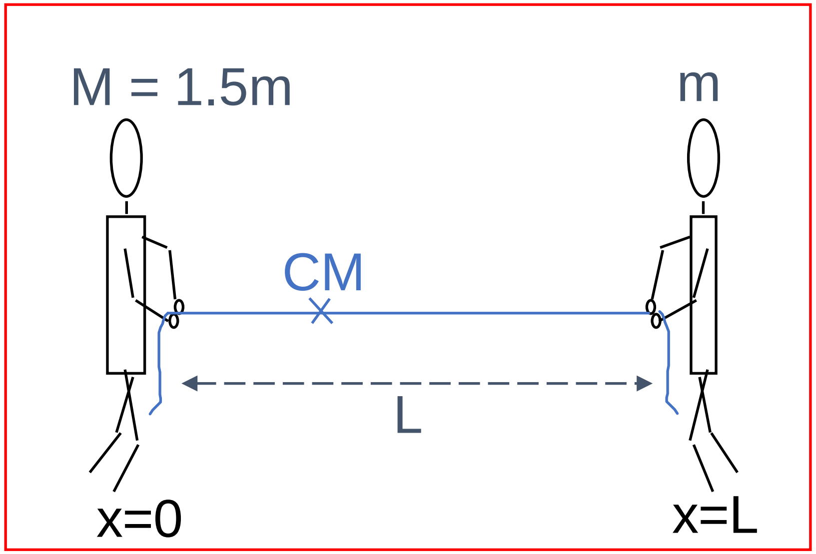
Example: Astronauts & Rope

- Two astronauts at rest in outer space are connected by a light rope. They begin to pull towards each other. Where do they meet?



Example: Astronauts & Rope...

- They start at rest, so $V_{CM} = 0$.
- V_{CM} remains zero because there are no external forces.
- So, the CM does not move!
- They will meet at the CM.



Finding the CM:

If we take the astronaut on the left to be at x = 0:

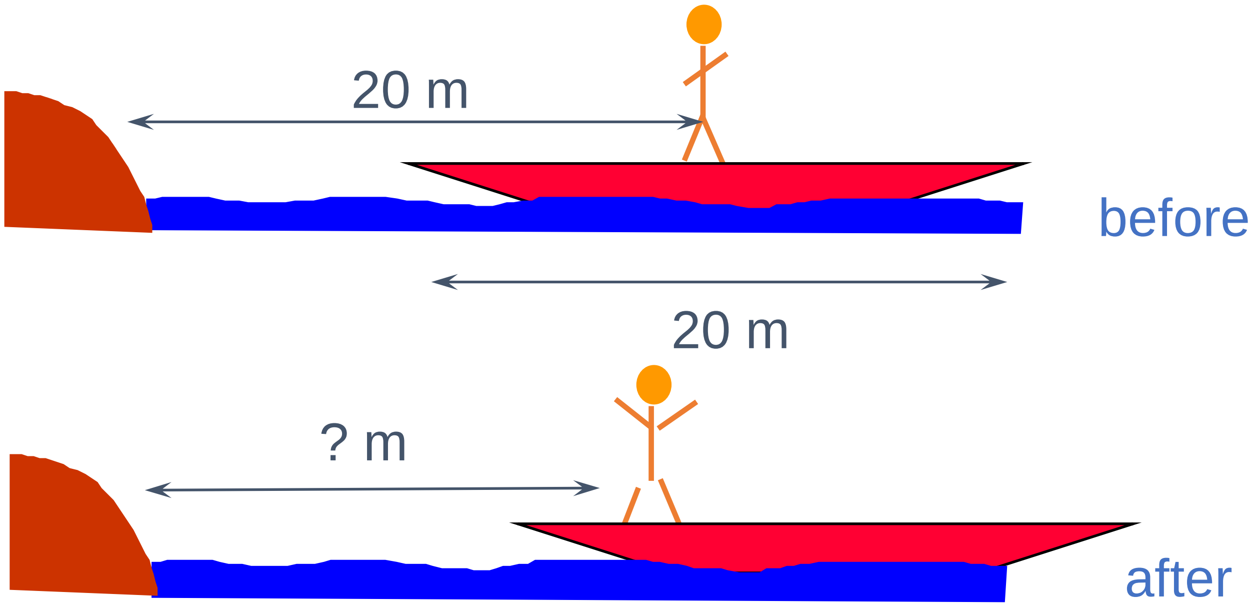
$$x_{cm} = \frac{M(0) + m(L)}{M + m} = \frac{m(L)}{2.5m} = \frac{2}{5}L$$

Example

Center of Mass Motion

- A man weighing 80 kg is standing at the center of a flatboat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weighs 200 kg. How far is he from the shore at the end of this time?

$$\mathbf{x_{cm}} = \frac{\mathbf{1}}{\mathbf{M}} \sum \mathbf{m_i x_i} \qquad 20 = \frac{80 \times X_1 + 200 \times (8 + X_1)}{280} \qquad X_1 = 14.3\, m$$



(a) 11.2

(b) 14.3

(c) 13.8

Overall Translational Motion of a System of Particles

The Center of Mass of a system of particles moves according to Newton's law, as though the entire mass of the system were concentrated at it and the net external force were applied to it.

Principle of momentum conservation for a system of particles

We have, $M\vec{v}_{cm} = \sum_i m_i \vec{v}_i = \sum_i \vec{P}_i = \vec{P}$

where $\vec{P}$ is the total momentum of the system

That is, the total linear momentum of a system of particles is the total mass times the velocity of center of mass.

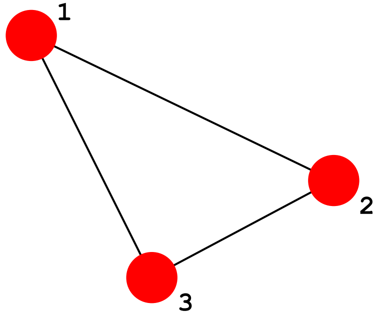
Differentiating w.r.t time,

$$\frac{d\vec{P}}{dt} = M\vec{a}_{cm} = \vec{F}_{tot,ext}$$

That is, the rate of change of total momentum is the net external force acting on the system

$\Rightarrow$ In the absence of external forces, the total momentum of a system of particles is conserved.

System of Particles



- Consider a system of several particles
  - Each particle obeys Newton's Laws:
  - This force can be split into internal and external parts

$$\mathbf{f}_i = \mathbf{f}_i^{\text{ext}} + \mathbf{f}_i^{\text{int}}$$

  - Now sum up the forces on all the particles

$$\mathbf{F} = \sum_i \mathbf{f}_i = \sum_i \mathbf{f}_i^{\text{ext}} + \sum_i \mathbf{f}_i^{\text{int}}$$

  - For the internal forces we have

$$\mathbf{f}_i^{\text{int}} = \sum_{j \neq i} \mathbf{f}_{ij} \qquad \mathbf{f}_{ij} = -\mathbf{f}_{ji}$$

  - Thus

$$\sum_i \mathbf{f}_i^{\text{int}} = 0$$

Systems of Particles

- The total moment of momentum (angular momentum) around the center of mass is:

$$\mathbf{L}_{cm} = \sum \mathbf{r}_i \times \mathbf{p}_i$$

$$\mathbf{L}_{cm} = \sum \left(\mathbf{x}_i - \mathbf{x}_{cm} \right) \times \mathbf{p}_i$$

Torque in a System of Particles

$$\mathbf{L}_{cm} = \sum \mathbf{r}_i \times \mathbf{p}_i$$

$$\boldsymbol{\tau}_{cm} = \frac{d\mathbf{L}_{cm}}{dt} = \frac{d\sum \mathbf{r}_i \times \mathbf{p}_i}{dt}$$

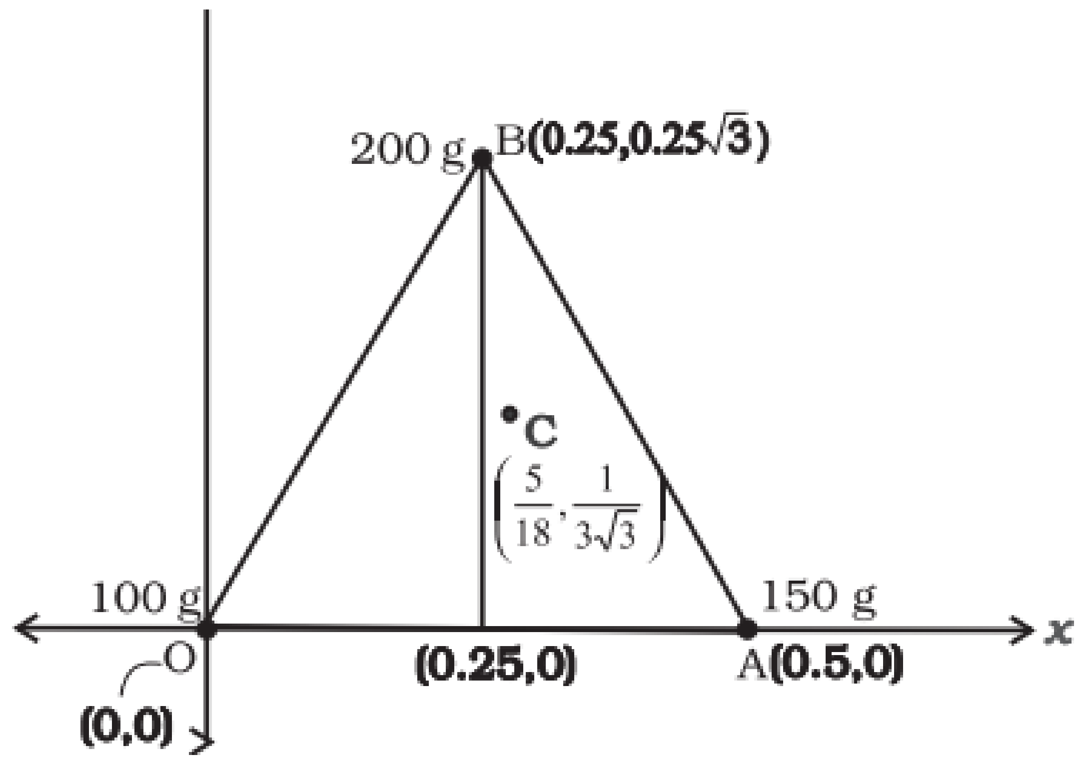
$$\boldsymbol{\tau}_{cm} = \frac{\sum d\left(\mathbf{r}_i \times \mathbf{p}_i\right)}{dt}$$

$$\boldsymbol{\tau}_{cm} = \sum \left(\mathbf{r}_i \times \mathbf{f}_i\right)$$

Example Find the center of mass of three particles at the vertices of an equilateral triangle. The masses of the particles are 100g, 150g, and 200g respectively. Each side of the equilateral triangle is 0.5m long.

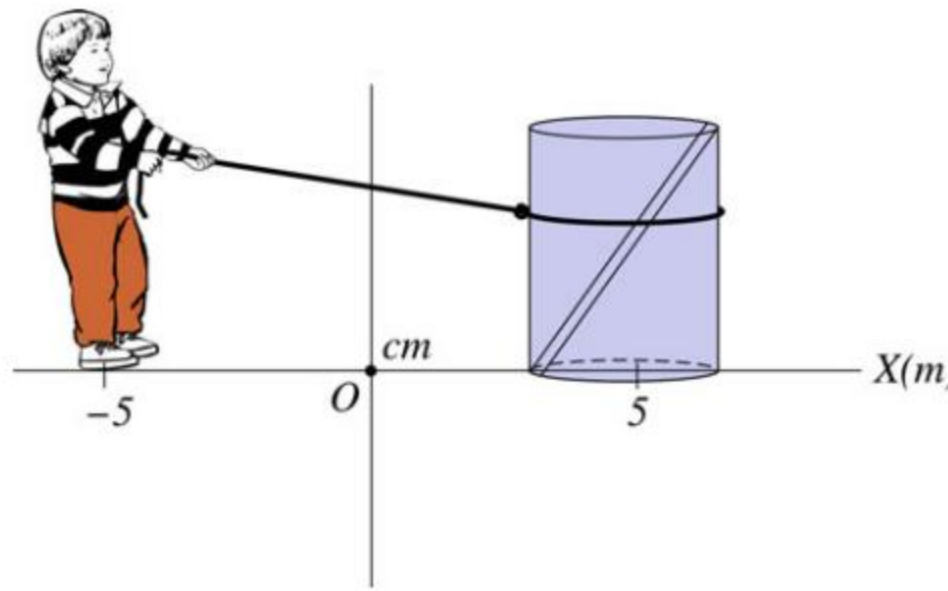
$$X_{cm} = \frac{5}{18}\ m$$

$$Y_{cm} = \frac{1}{3\sqrt{3}}\ m$$



A boy standing on a smooth ice surface wants to bring a container that is at a distance of 10 m away from him.

To do that, he throws a rope around the container and starts to pull. Because the surface is smooth, both the boy and the container will move until they meet. If the masses of the boy and of the container are 40 kg and 70 kg respectively, how far will the container move when the boy has moved a distance of 2 m?



$$x_{cm} = \frac{\Sigma_i m_i x_i}{\Sigma_i m_i} = \frac{(70\,\text{kg})(5\,\text{m}) + (40\,\text{kg})(-5\,\text{m})}{(110\,\text{kg})} = 1.36\,\text{m}$$

$$(1.36\,\text{m}) = \frac{(70\,\text{kg})x_c + (40\,\text{kg})(-3\,\text{m})}{(110\,\text{kg})}$$

$$x_c = 3.86\ m$$

How to love college?

Stay focused

Be on time. Make sure you go to class. If you have a learning disability speak with your professor so they can accommodate your needs. Use the syllabus & course book. Create study hours for yourself.

Study tips

Always ask. Study groups are the best. Don’t panic about midterm grades. Don’t cram for exams. Always treat yourself after taking an exam to reward your efforts.

Get involved

Get to know your campus. Attend a lot of events. Because your college years are the best years of your life, so make sure your squeeze the most from them.

Talk to the staff

Get to know staff and faculty members. Be friendly.

Be social

KEEP YOUR FRIENDS AND FAMILY CLOSE, ENJOY YOUR COWORKERS

Need income?

1) An ideal rigid body is the body with a

a) Imperfect shape

b) Perfectly definite and unchanging shape

c) Both a and b

d) None

2) In case of rigid body the distance between all pairs of particles of such body

a) Changes

b) Do not changes

c) Both a and b

d) None

3) In pure translational motion at any instant of time, all particles of the body have

a) Different velocity

b) Changing velocity

c) Same velocity

d) None

5) The point at which whole mass of body is supposed to be concentrated is called as

a) Centre of gravity

b) Centre of mass

c) Both a and b

d) None

6) The total momentum of the system of particle is equal to the product of the total mass of the system and the

a) Velocity

b) Acceleration

c) Velocity of its centre of mass

d) None

7) When the total external force acting on the system of particles is zero then the total

a) Angular momentum of the system is constant

b) Linear momentum of the system is constant

c) Both a and b

d) None

8) When the external force on the system is zero then _______ remains constant.

a) Velocity of body

b) Velocity of centre of mass

c) Both a and b

d) None