Mechanics

Mechanics

- Kinematics
- Dynamics

Kinematics deals with the concepts that are needed to describe motion. How?

Dynamics deals with the effect that forces have on motion. Why?

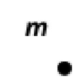
1st simplification; *Blind the Force*

Mechanics

There are mainly 3 types of motion

1. Translational Motion
2. Rotational Motion
3. Vibrational Motion

A point mass is an idealization of a real solid body. It possesses mass, but its dimensions are assumed to be so small that its location can be sufficiently accurately defined by the position of a point.



Geometric point (0- dimensional)

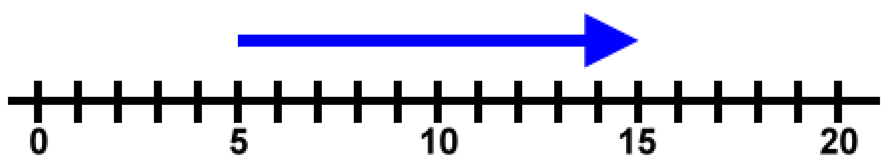
Two them will be excluded;
1- Rotational Motion
2- Vibrational Motion

2nd simplification; *only translation*

Types of Motion as Per Directions are:

1. One Dimensional Motion
2. Two Dimensional Motion
3. Three Dimensional Motion

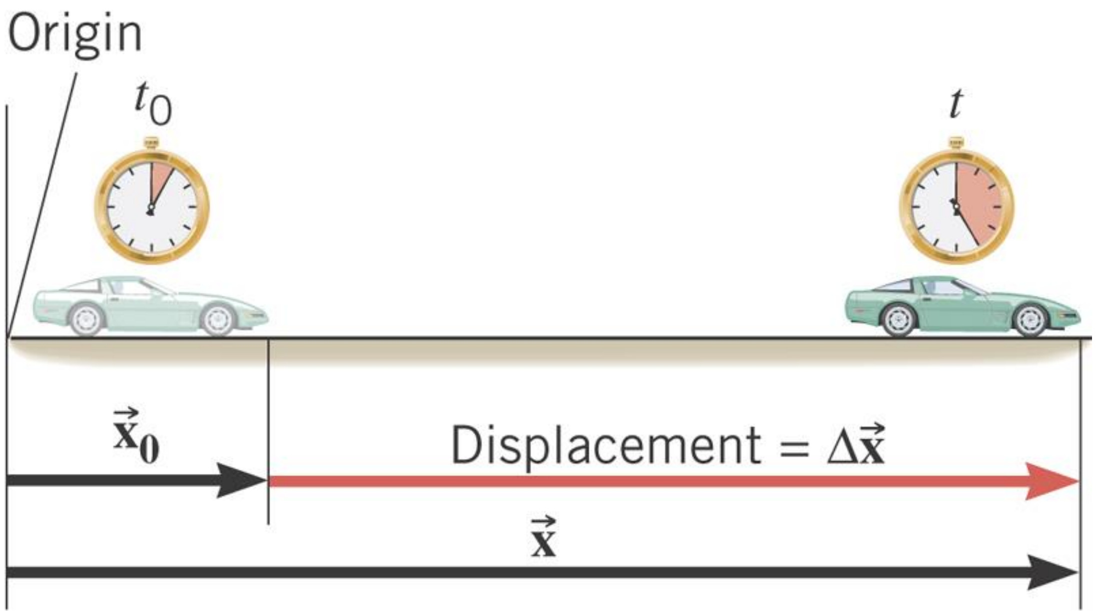
Kinematics in One Dimension



3rd simplification; *One Dimension*

There are three parameters of kinematics in addition to time:
Displacement, Velocity, and Acceleration

Displacement: ***the change in position of a particle***

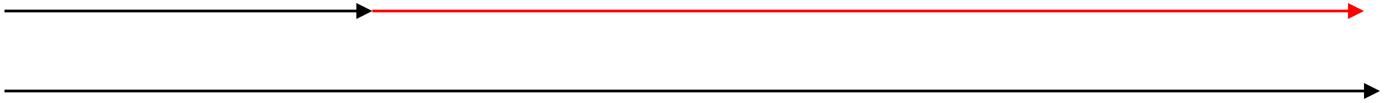


$$\vec{\mathbf{x}}_o = \text{initial position} \qquad \vec{\mathbf{x}} = \text{final position}$$

$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = \text{displacement}$$

Displacement

$\vec{\mathbf{x}}_o = 2.0\,\mathrm{m}$ $\Delta\vec{\mathbf{x}} = 5.0\,\mathrm{m}$

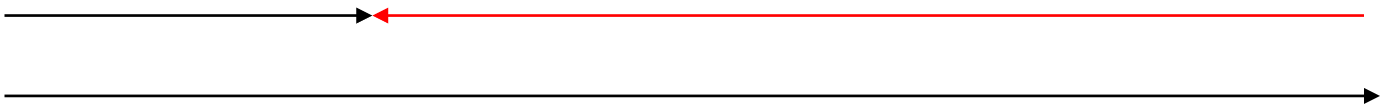


$\vec{\mathbf{x}} = 7.0\,\mathrm{m}$

$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = 7.0\,\mathrm{m} - 2.0\,\mathrm{m} = 5.0\,\mathrm{m}$$

Displacement

$\vec{\mathbf{x}} = 2.0\,\mathrm{m}$ $\qquad$ $\Delta\vec{\mathbf{x}} = -5.0\,\mathrm{m}$

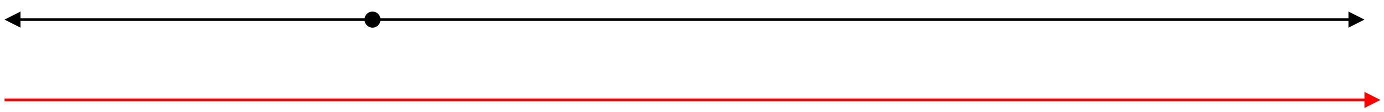


$\vec{\mathbf{x}}_o = 7.0\,\mathrm{m}$

$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = 2.0\,\mathrm{m} - 7.0\,\mathrm{m} = -5.0\,\mathrm{m}$$

Displacement

$$\vec{\mathbf{x}}_o = -2.0\,\mathrm{m} \qquad \vec{\mathbf{x}} = 5.0\,\mathrm{m}$$



$$\Delta\vec{\mathbf{x}} = 7.0\,\mathrm{m}$$

$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = 5.0\,\mathrm{m} - \left(-2.0\right)\mathrm{m} = 7.0\,\mathrm{m}$$

Velocity

Average velocity is the displacement divided by the elapsed time.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Elapsed time}}$$

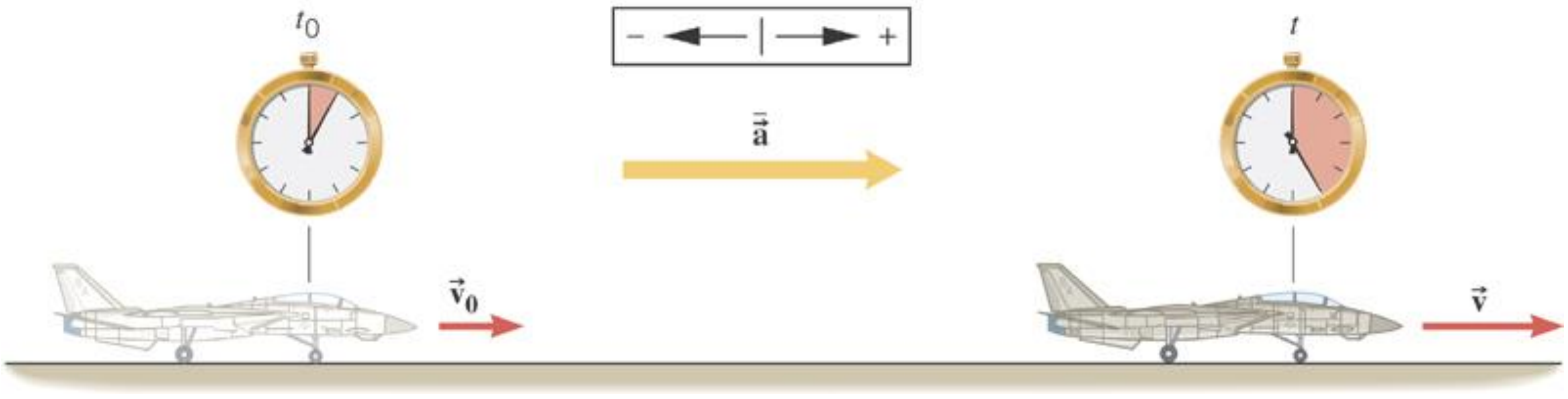
$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{x}} - \vec{\mathbf{x}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{x}}}{\Delta t}$$

The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

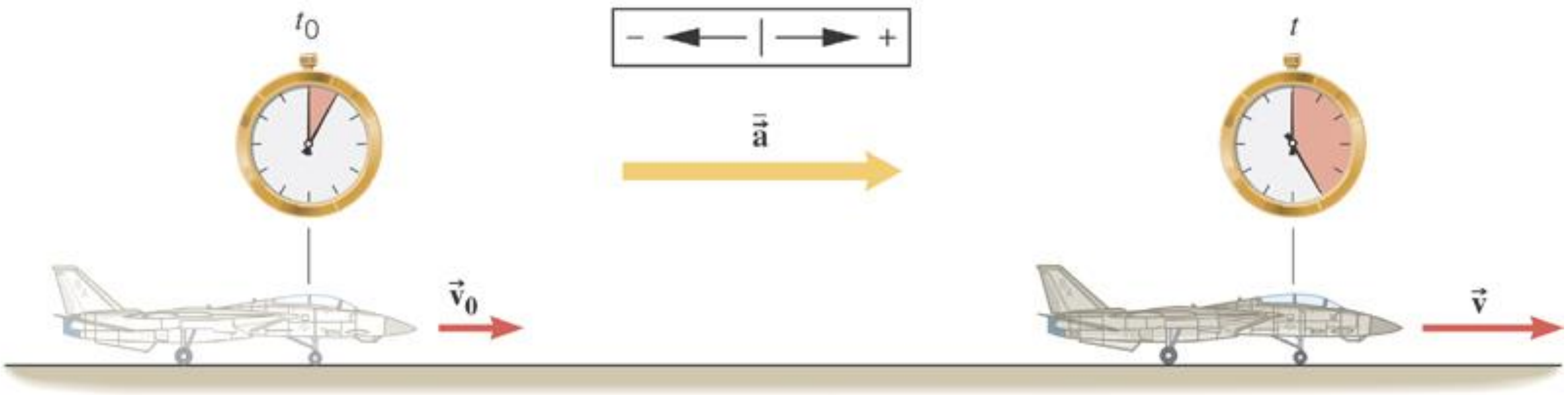
$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{x}}}{\Delta t}$$

Acceleration

The notion of *acceleration* emerges when a change in velocity is combined with the time during which the change occurs.



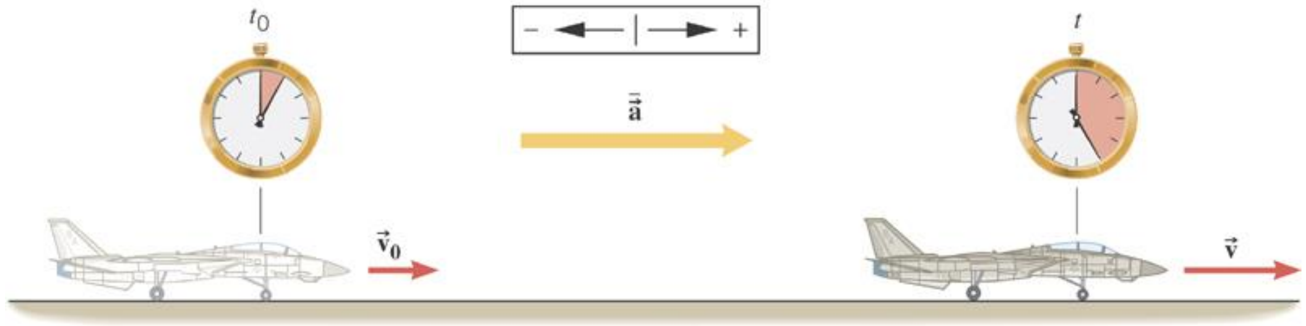
Acceleration



DEFINITION OF AVERAGE ACCELERATION

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Acceleration



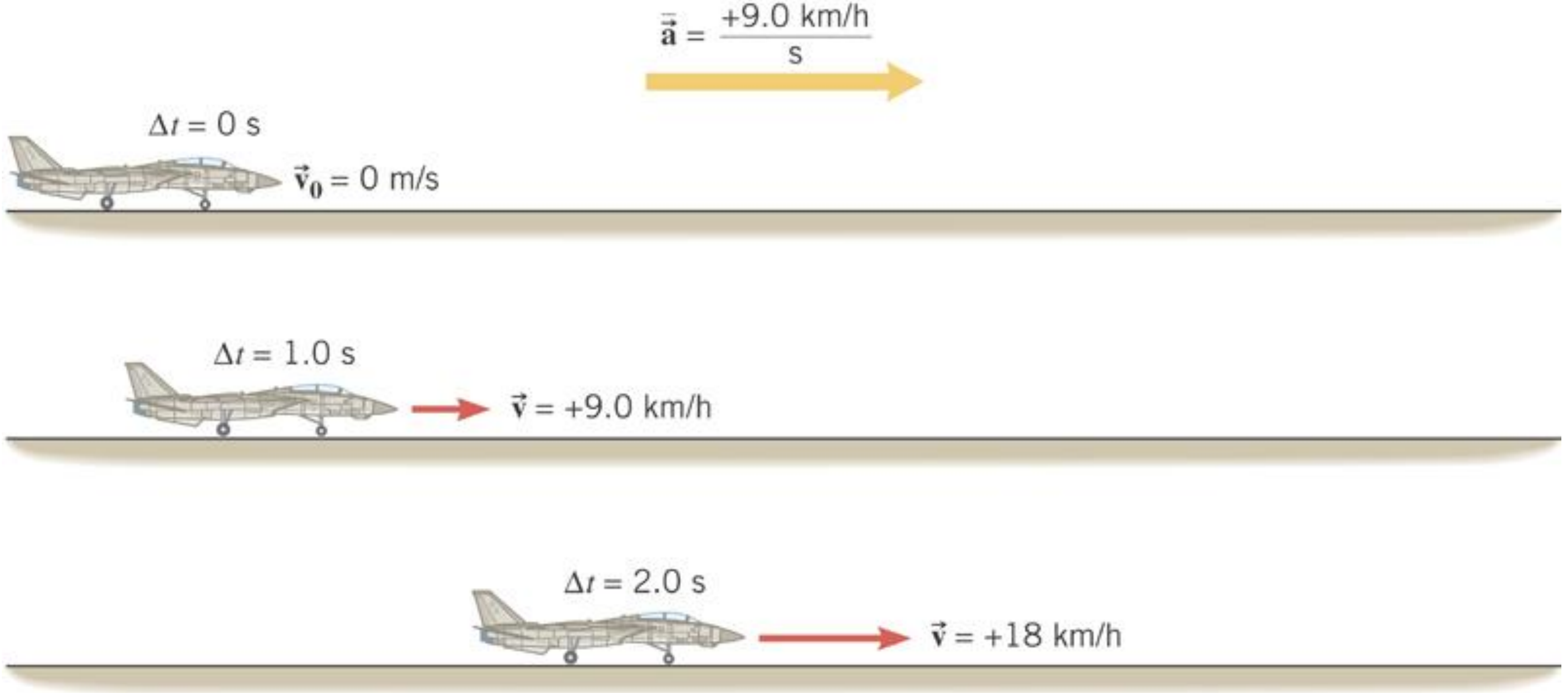
Example 1- Acceleration and Increasing Velocity

Determine the average acceleration of the plane.

$\vec{\mathbf{v}}_o = 0\,\mathrm{m/s}$ $\quad\quad$ $\vec{\mathbf{v}} = 260\,\mathrm{km/h}$ $\quad\quad$ $t_o = 0\,\mathrm{s}$ $\quad\quad$ $t = 29\,\mathrm{s}$

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{260\,\mathrm{km/h} - 0\,\mathrm{km/h}}{29\,\mathrm{s} - 0\,\mathrm{s}} = +9.0\,\frac{\mathrm{km/h}}{\mathrm{s}}$$

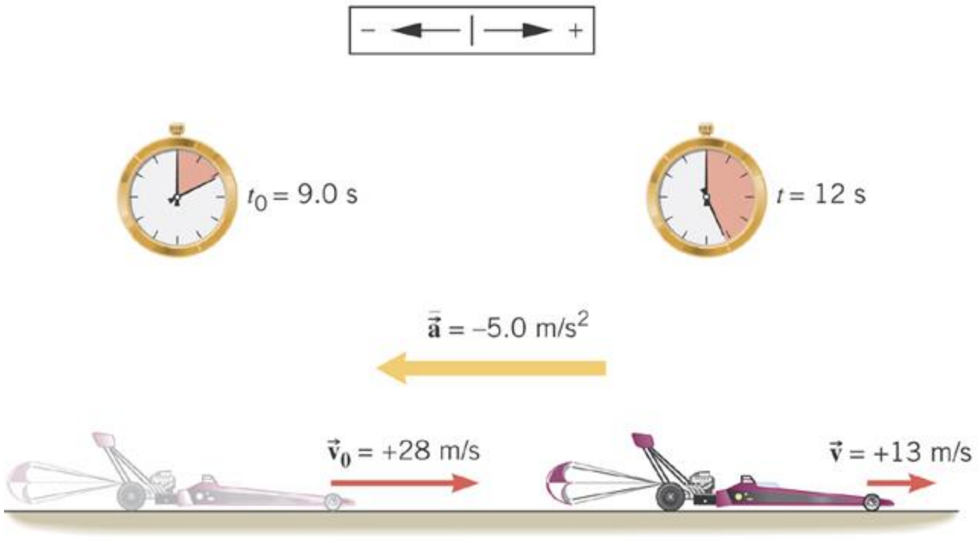
Acceleration



Acceleration

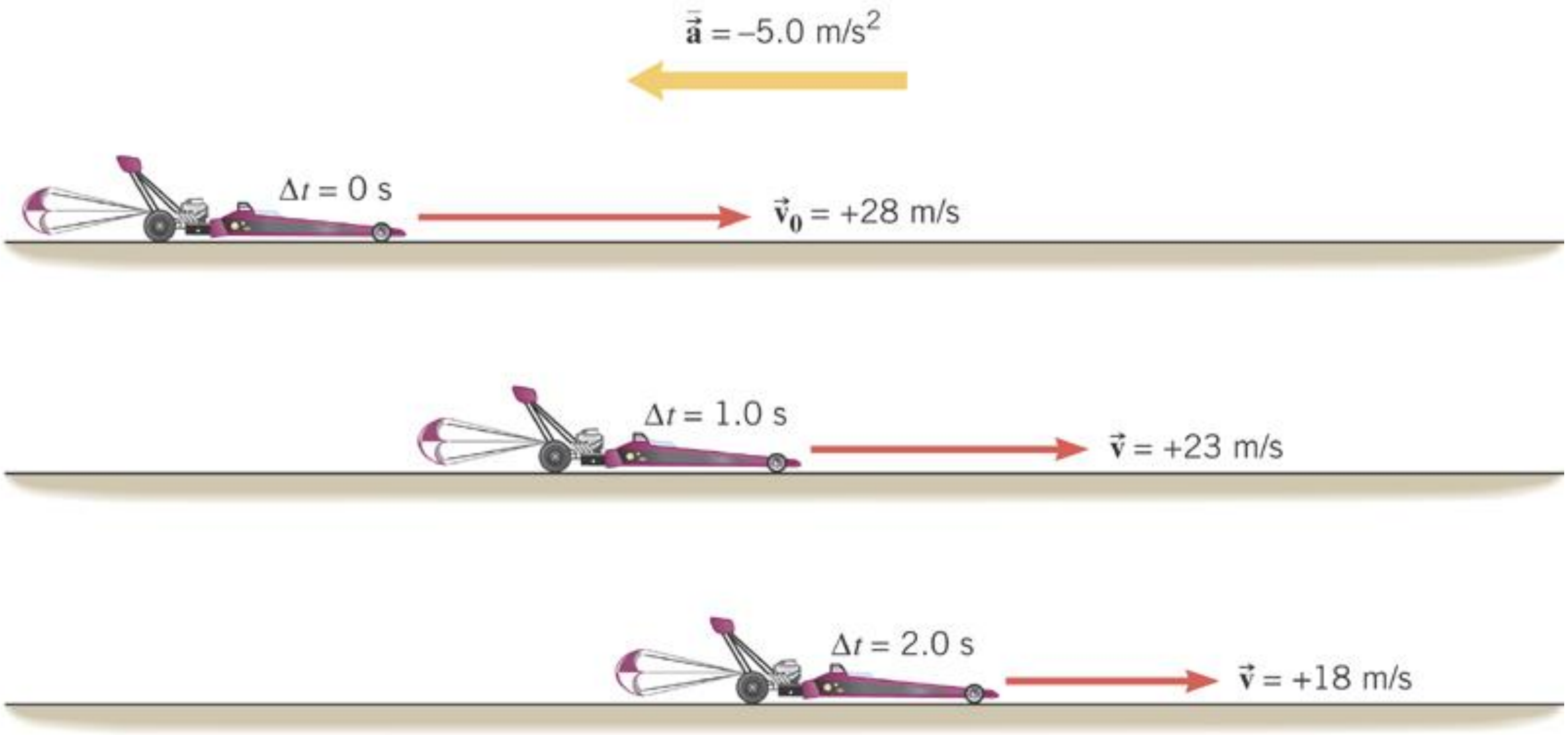
Example 1- Acceleration and Decreasing Velocity

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{13\,\text{m/s} - 28\,\text{m/s}}{12\,\text{s} - 9\,\text{s}} = -5.0\,\text{m/s}^2$$



(*b*)

Acceleration



Equations of Kinematics for Constant Acceleration

$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{x}} - \vec{\mathbf{x}}_o}{t - t_o} \qquad \overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o}$$

It is usually to dispense with the use of boldface symbols overdrawn with arrows for the displacement, velocity, and acceleration vectors. We will, however, continue to convey the directions with a plus or minus sign.

$$v = \frac{x - x_o}{t - t_o} \qquad a = \frac{v - v_o}{t - t_o}$$

Equations of Kinematics for Constant Acceleration

Let the object be at the origin when the clock starts.

$$x_o = 0 \qquad t_o = 0$$

$$\bar{v} = \frac{x - x_o}{t - t_o} \quad \Rightarrow \quad \bar{v} = \frac{x}{t}$$

$$x = \bar{v}t = \tfrac{1}{2}\left(v_o + v\right)t$$

Equations of Kinematics for Constant Acceleration

$$a = \frac{v - v_o}{t - t_o} \Rightarrow a = \frac{v - v_o}{t}$$

$$\Downarrow$$

$$at = v - v_o$$

$$\Downarrow$$

$$v = v_o + at$$

Equations of Kinematics for Constant Acceleration

Five kinematic variables:

1. displacement, x

2. acceleration (constant), a

3. final velocity (at time t), v

4. initial velocity, v_o

5. elapsed time, t

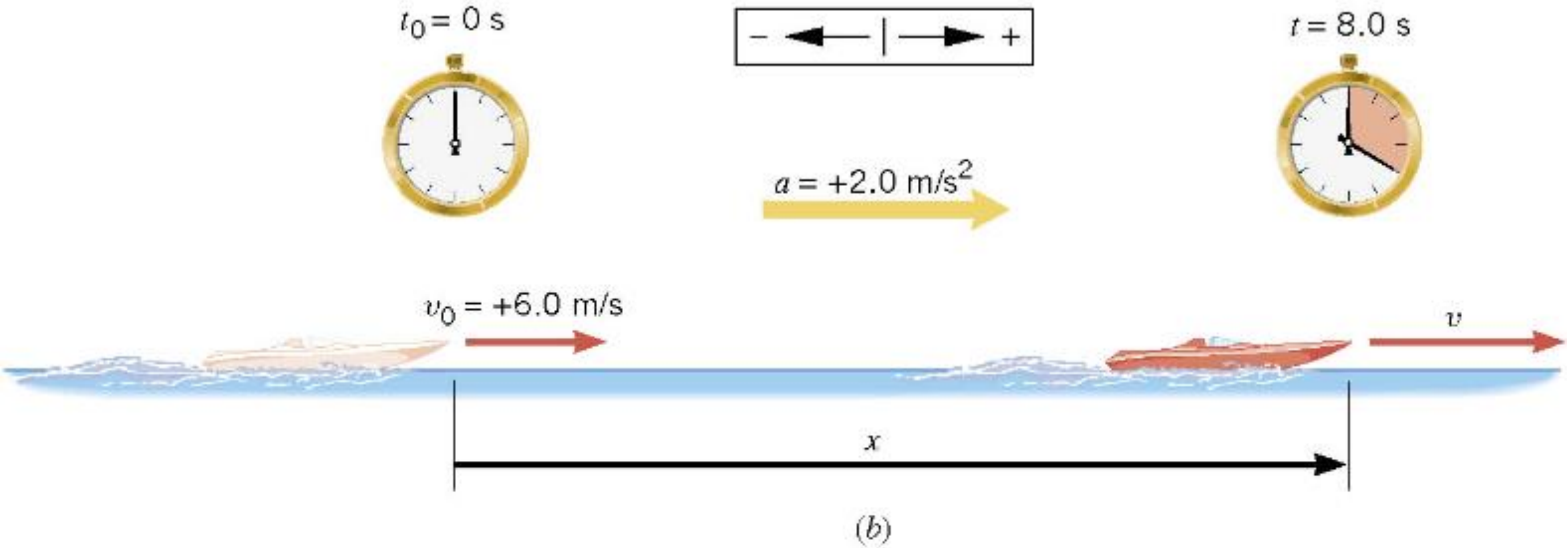
Equations of Kinematics for Constant Acceleration

$$v = v_o + at$$

$$x = \tfrac{1}{2}\left(v_o + v\right)t = \tfrac{1}{2}\left(v_o + v_o + at\right)t$$

$$x = v_o t + \tfrac{1}{2}at^2$$

Equations of Kinematics for Constant Acceleration

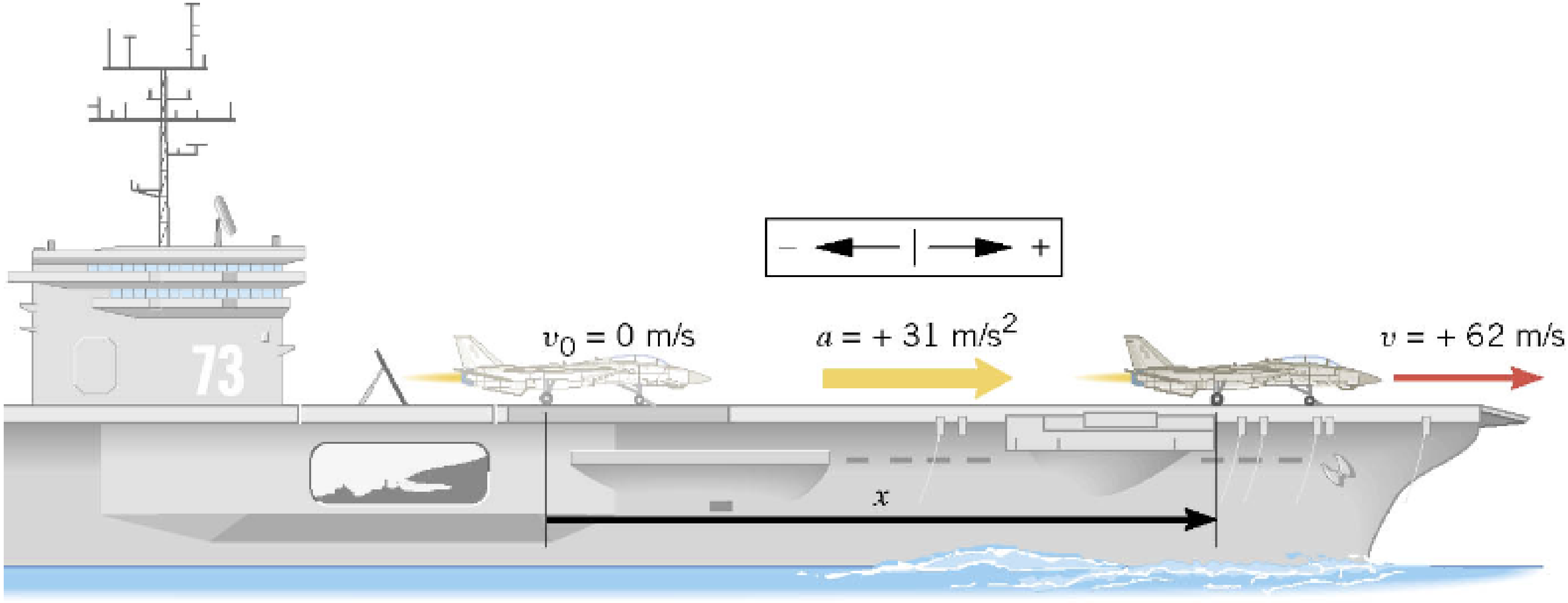


$$x = v_o t + \tfrac{1}{2} a t^2$$

$$= (6.0\,\mathrm{m/s})(8.0\,\mathrm{s}) + \tfrac{1}{2}\left(2.0\,\mathrm{m/s^2}\right)(8.0\,\mathrm{s})^2$$

$$= +110\,\mathrm{m}$$

Equations of Kinematics for Constant Acceleration



(*b*)

Example 2- Catapulting a Jet

Find its displacement.

$$v_o = 0\,\text{m/s} \qquad a = +31\,\text{m/s}^2$$

$$x = ?? \qquad v = +62\,\text{m/s}$$

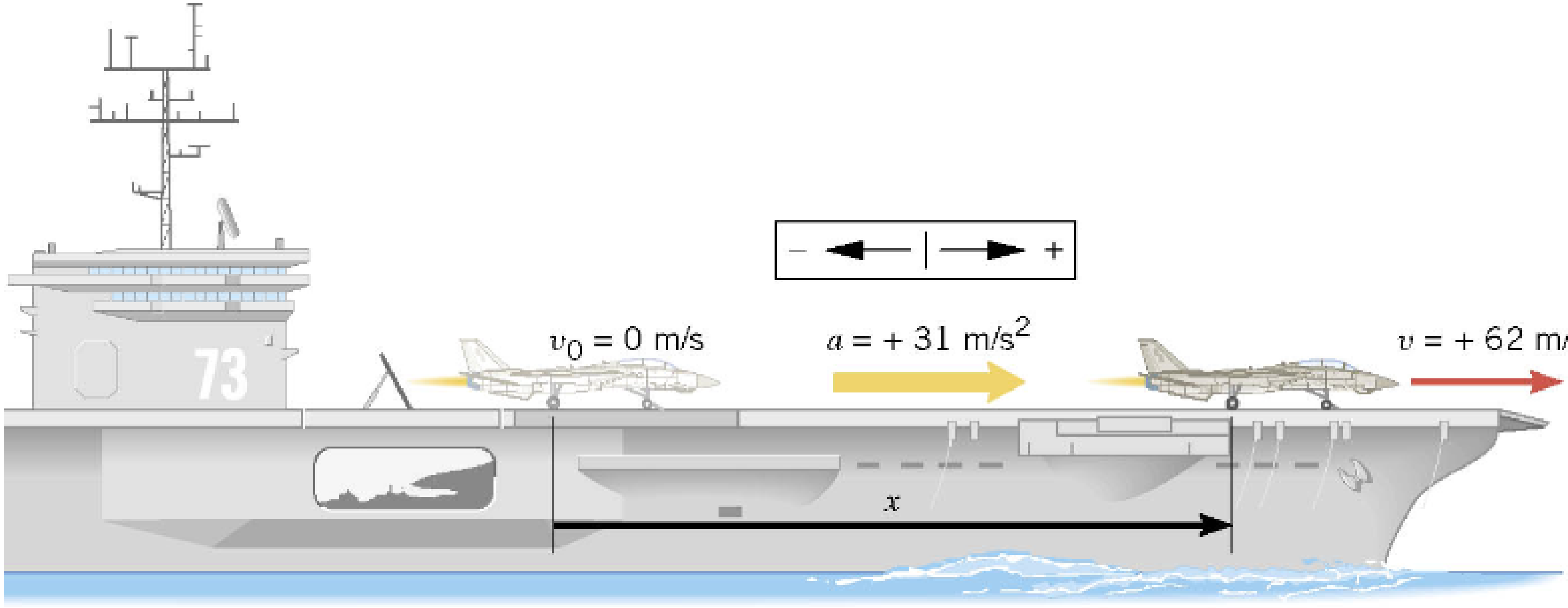
Equations of Kinematics for Constant Acceleration

$$a = \frac{v - v_o}{t} \quad \Rightarrow \quad t = \frac{v - v_o}{a}$$

$$x = \tfrac{1}{2}\left(v_o + v\right)t = \tfrac{1}{2}\left(v_o + v\right)\frac{\left(v - v_o\right)}{a}$$

$$x = \frac{v^2 - v_o^2}{2a}$$

Equations of Kinematics for Constant Acceleration



(b)

$$x = \frac{v^2 - v_o^2}{2a} = \frac{\left(62\,\mathrm{m/s}\right)^2 - \left(0\,\mathrm{m/s}\right)^2}{2\left(31\,\mathrm{m/s^2}\right)} = +62\,\mathrm{m}$$

Equations of Kinematics for Constant Acceleration

$$v = v_o + at$$

$$x = \tfrac{1}{2}\left(v_o + v\right)t$$

$$v^2 = v_o^2 + 2ax$$

$$x = v_o t + \tfrac{1}{2}at^2$$

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

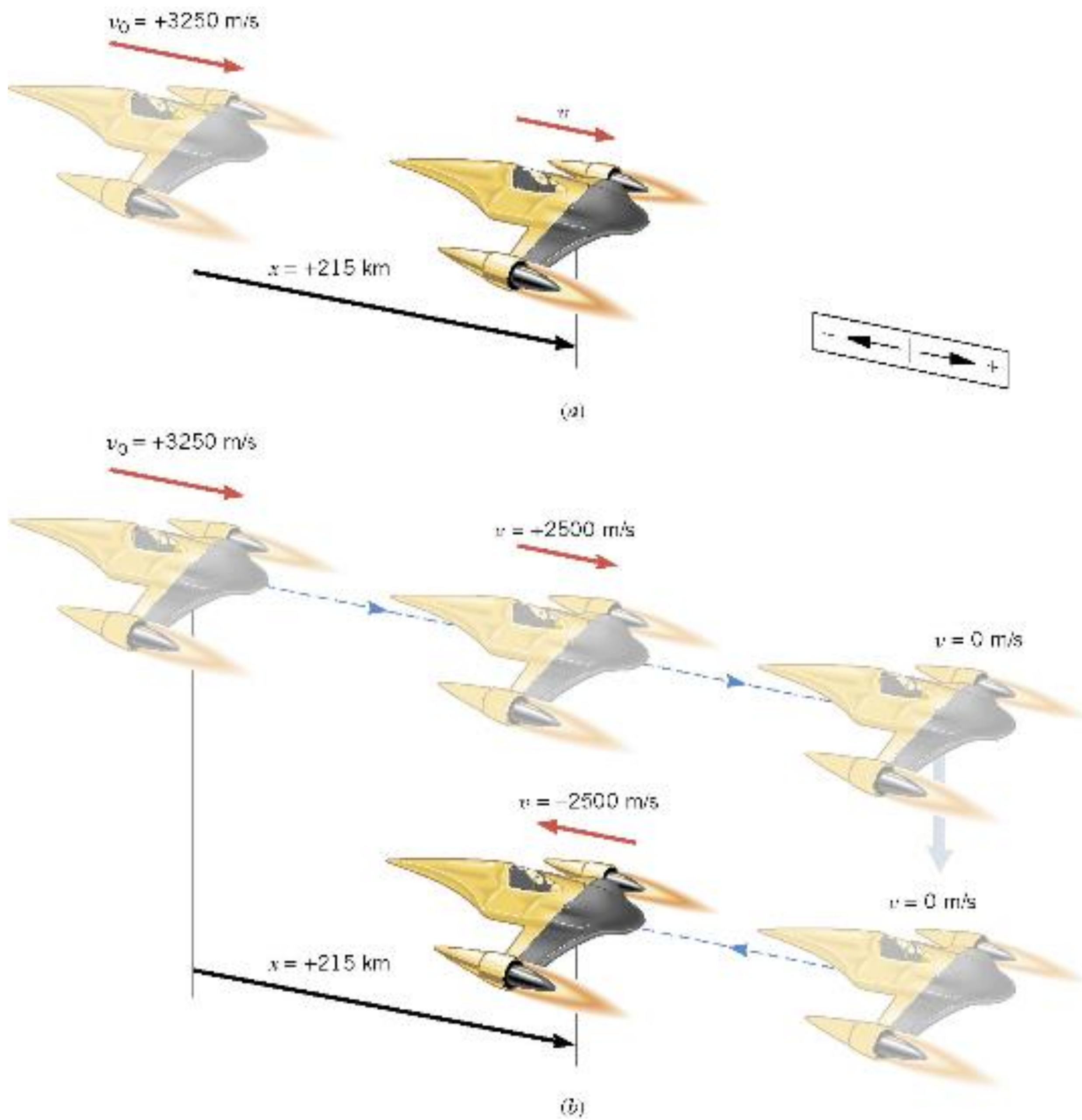
6. Keep in mind that there may be two possible answers to a kinematics problem.

Example 3- An Accelerating Spacecraft

A spacecraft is traveling with a velocity of +3250 m/s. Suddenly the retrorockets are fired, and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. What is the velocity of the spacecraft when the displacement of the craft is +215 km, relative to the point where the retrorockets began firing?

x	a	v	v_o	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

Applications of the Equations of Kinematics



Applications of the Equations of Kinematics

x	a	v	v_o	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

$$v^2 = v_o^2 + 2ax \longrightarrow v = \sqrt{v_o^2 + 2ax}$$

$$v = \pm\sqrt{(3250\,\text{m/s})^2 + 2(10.0\,\text{m/s}^2)(215000\,\text{m})}$$

$$= \pm 2500\,\text{m/s}$$

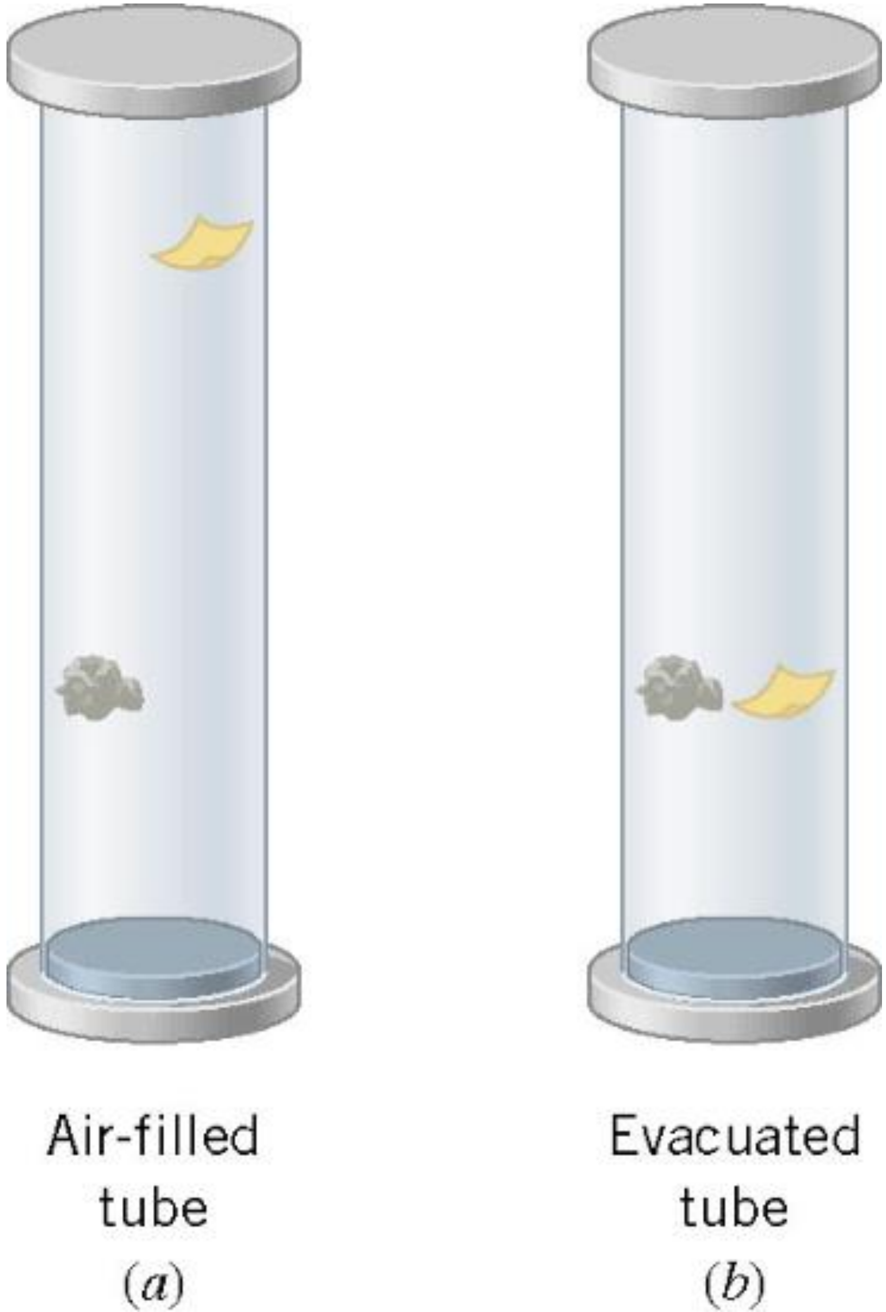
Freely Falling Bodies

In the absence of air resistance, it is found that all bodies at the same location above the Earth fall vertically with the same acceleration. If the distance of the fall is small compared to the radius of the Earth, then the acceleration remains essentially constant throughout the descent.

This idealized motion is called *free-fall* and the acceleration of a freely falling body is called the *acceleration due to gravity*.

$$g = 9.80\,\mathrm{m/s^2} \quad \text{or} \quad 32.2\,\mathrm{ft/s^2}$$

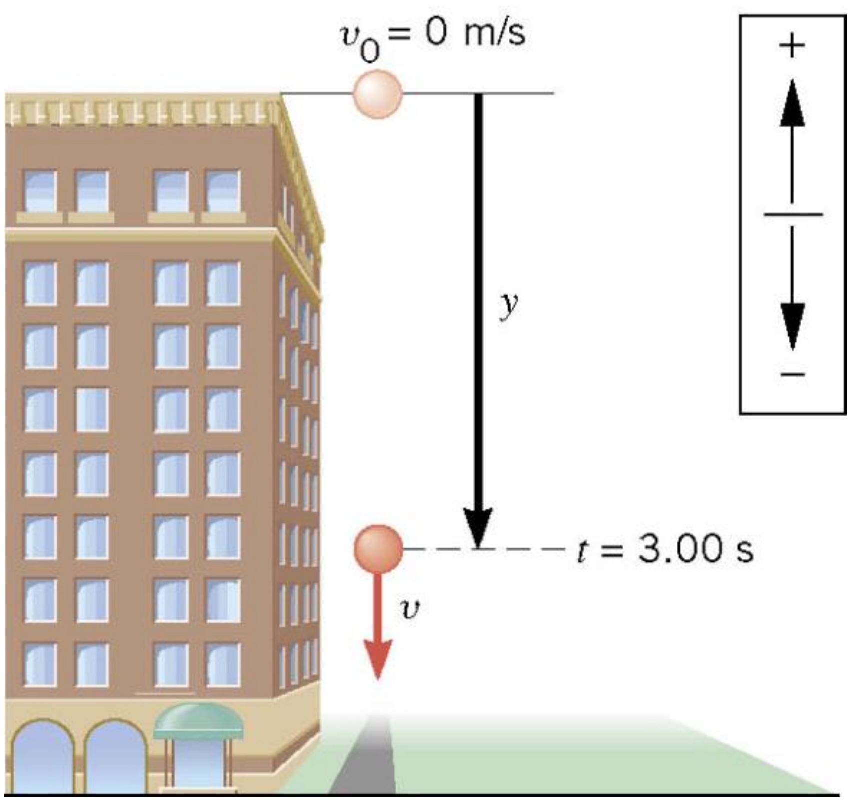
Freely Falling Bodies



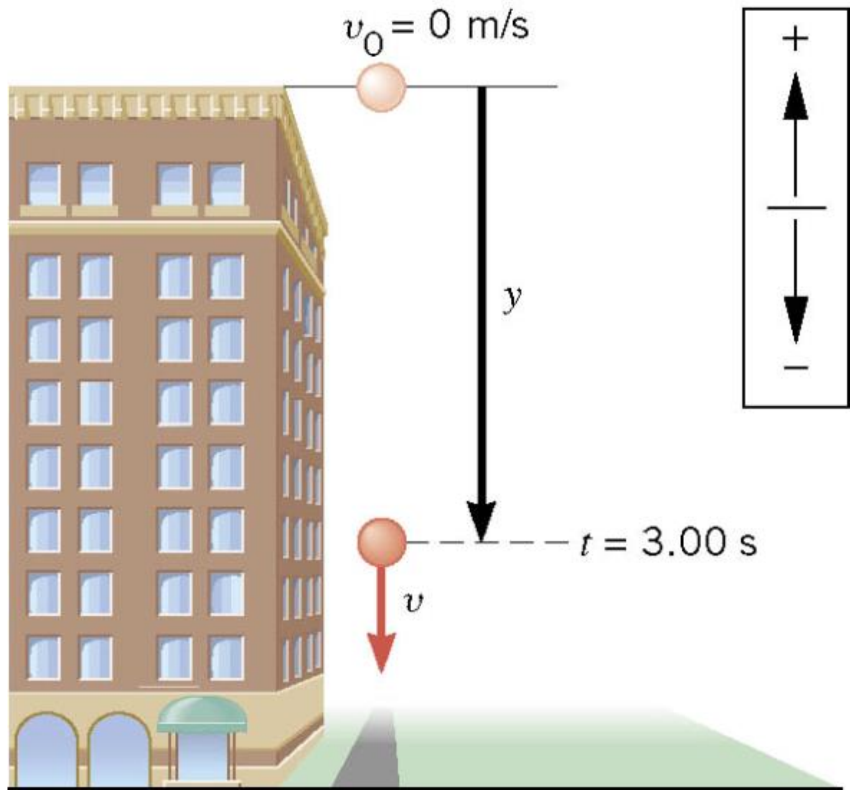
$$g = 9.80\,\mathrm{m/s^2}$$

Example 4- A Falling Stone

A stone is dropped from the top of a tall building. After 3.00s of free fall, what is the displacement *y* of the stone?



Freely Falling Bodies



y	a	v	v_o	t
?	-9.80 m/s^2		0 m/s	3.00 s

Freely Falling Bodies

y	a	v	v_o	t
?	-9.80 m/s^2		0 m/s	3.00 s

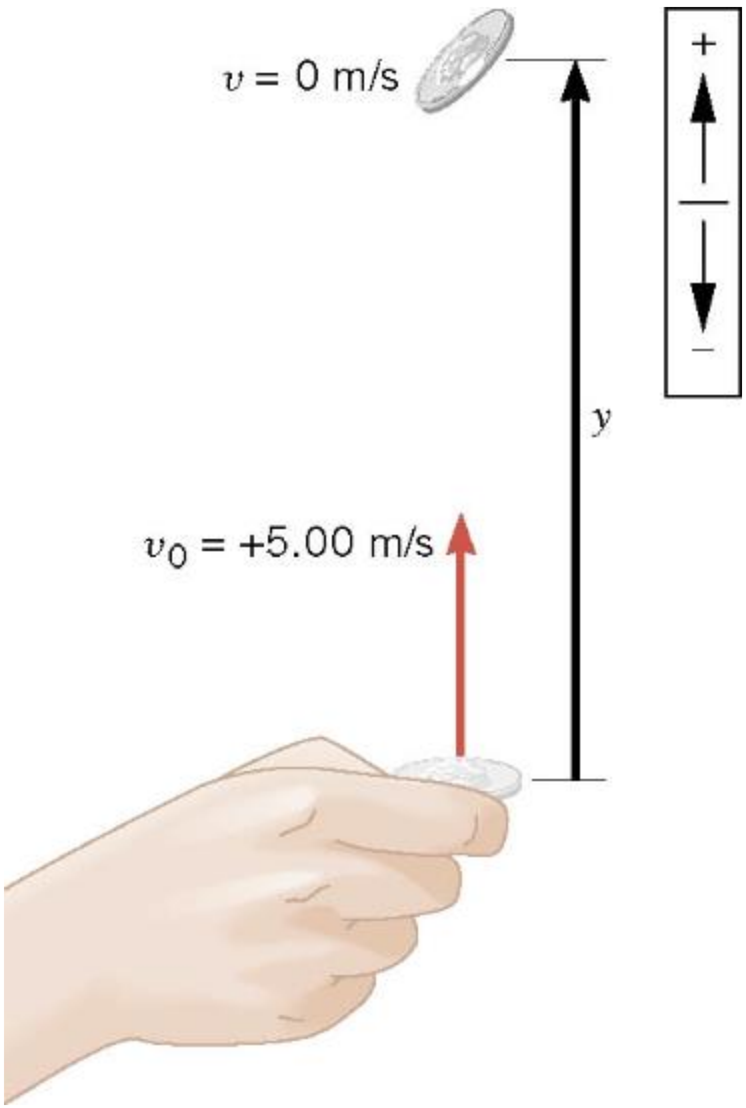
$$y = v_o t + \tfrac{1}{2} a t^2$$

$$= (0\,\mathrm{m/s})(3.00\,\mathrm{s}) + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)(3.00\,\mathrm{s})^2$$

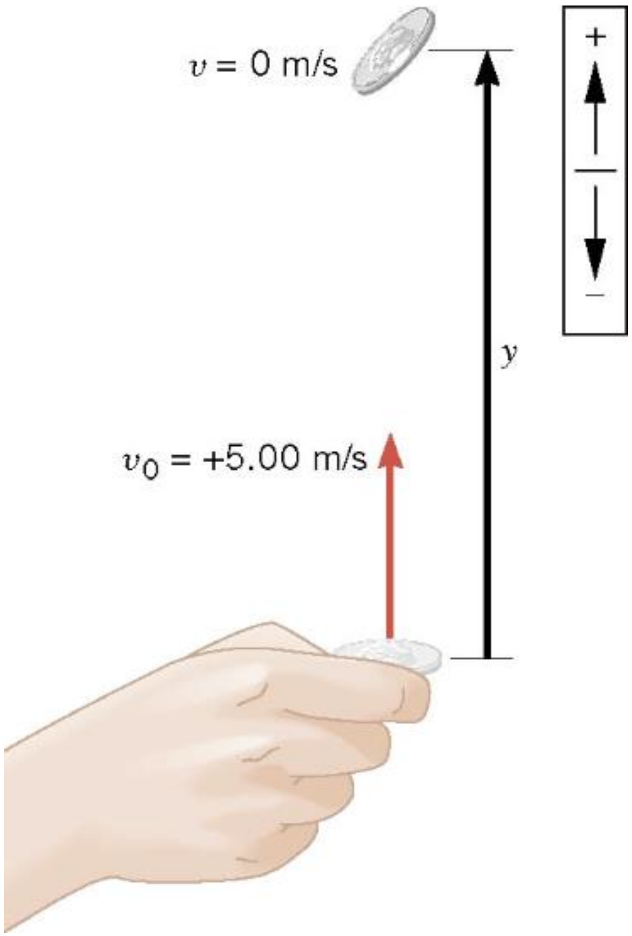
$$= -44.1\,\mathrm{m}$$

Example 5- How High Does it Go?

The referee tosses the coin up with an initial speed of 5.00m/s. In the absence if air resistance, how high does the coin go above its point of release?



Freely Falling Bodies



y	a	v	v_o	t
?	-9.80 m/s^2	0 m/s	+5.00 m/s	

Freely Falling Bodies

y	a	v	v_o	t
?	-9.80 m/s^2	0 m/s	+5.00 m/s	

$$v^2 = v_o^2 + 2ay \quad \Rightarrow \quad y = \frac{v^2 - v_o^2}{2a}$$

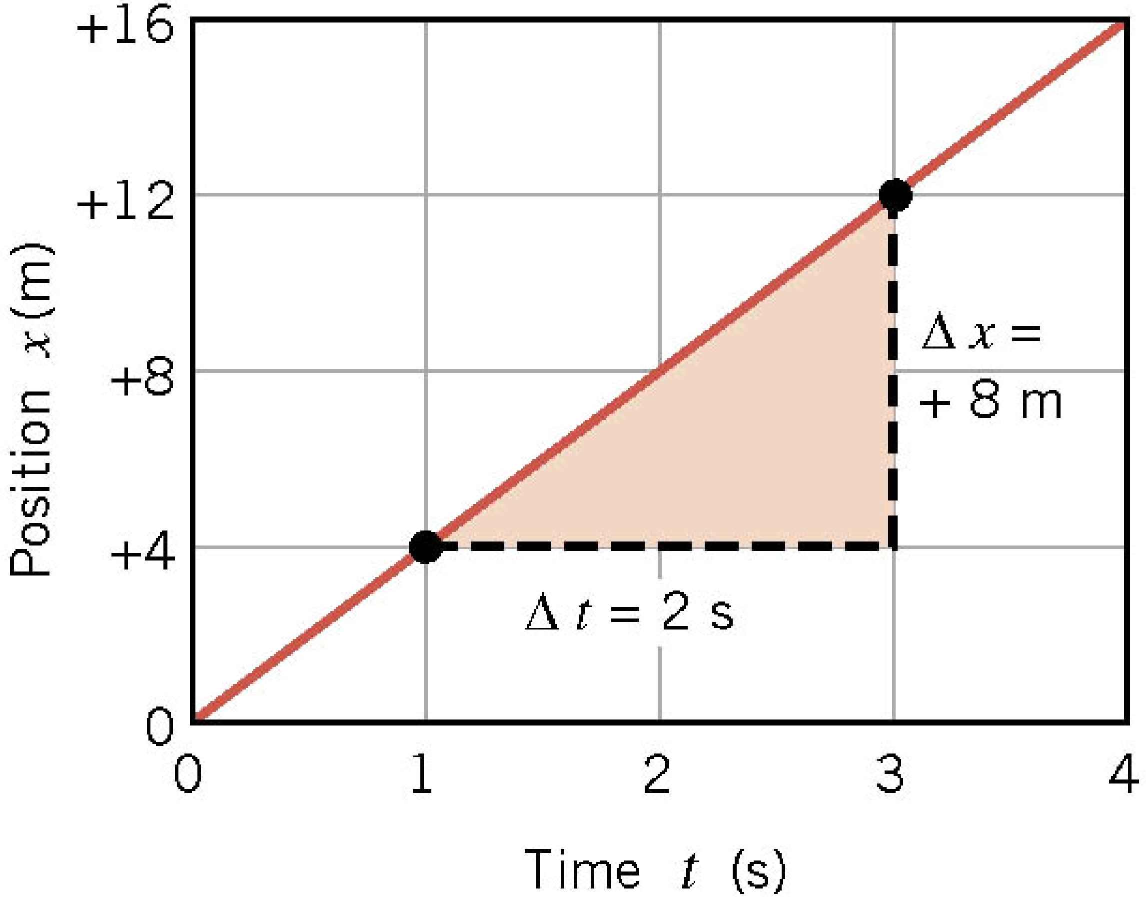
$$y = \frac{v^2 - v_o^2}{2a} = \frac{(0\,\mathrm{m/s})^2 - (5.00\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 1.28\,\mathrm{m}$$

Conceptual Example 6- Acceleration Versus Velocity

There are three parts to the motion of the coin. On the way up, the coin has a vector velocity that is directed upward and has decreasing magnitude. At the top of its path, the coin momentarily has zero velocity. On the way down, the coin has downward-pointing velocity with an increasing magnitude.

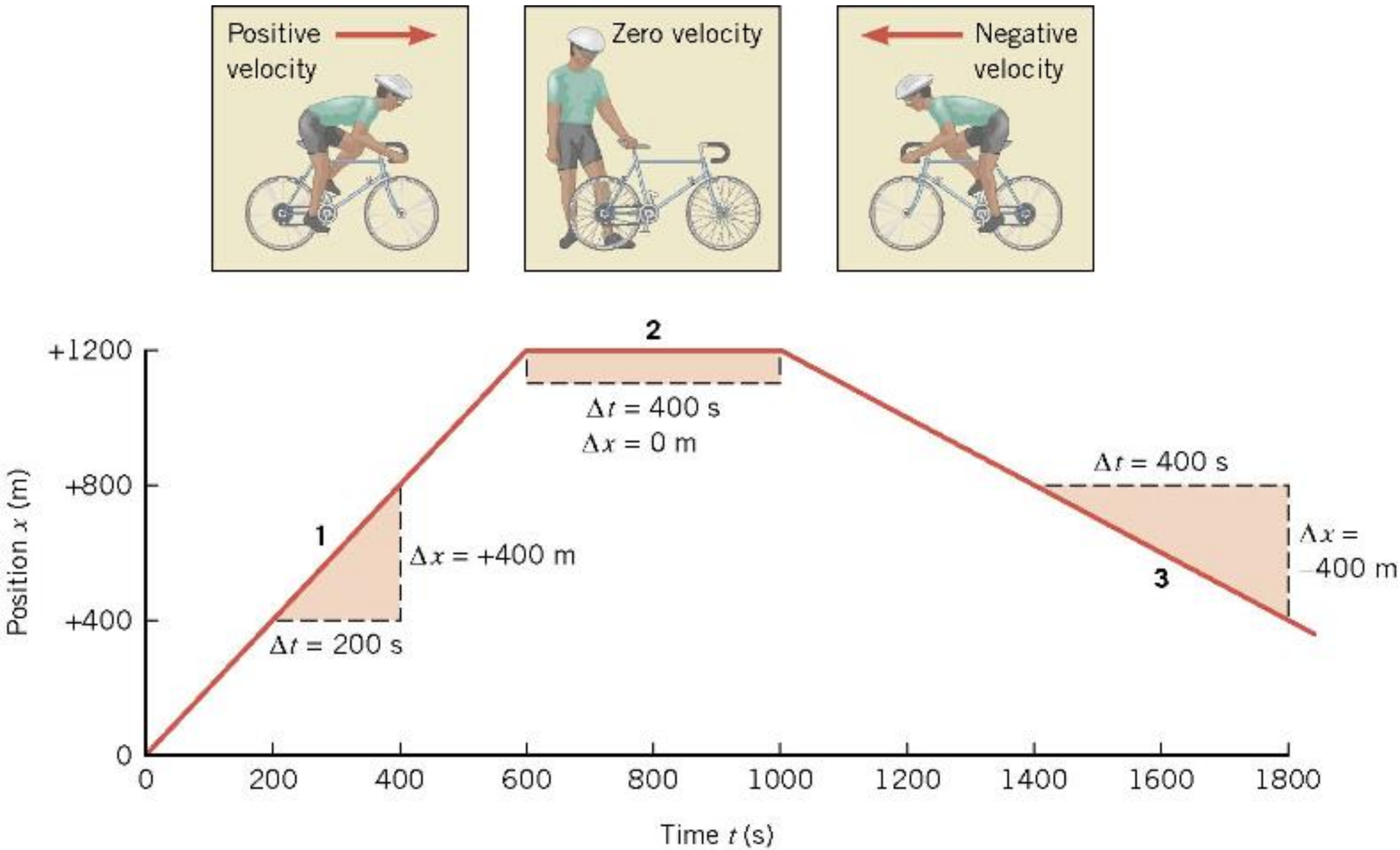
In the absence of air resistance, does the acceleration of the coin, like the velocity, change from one part to another?

Graphical Analysis of Velocity and Acceleration

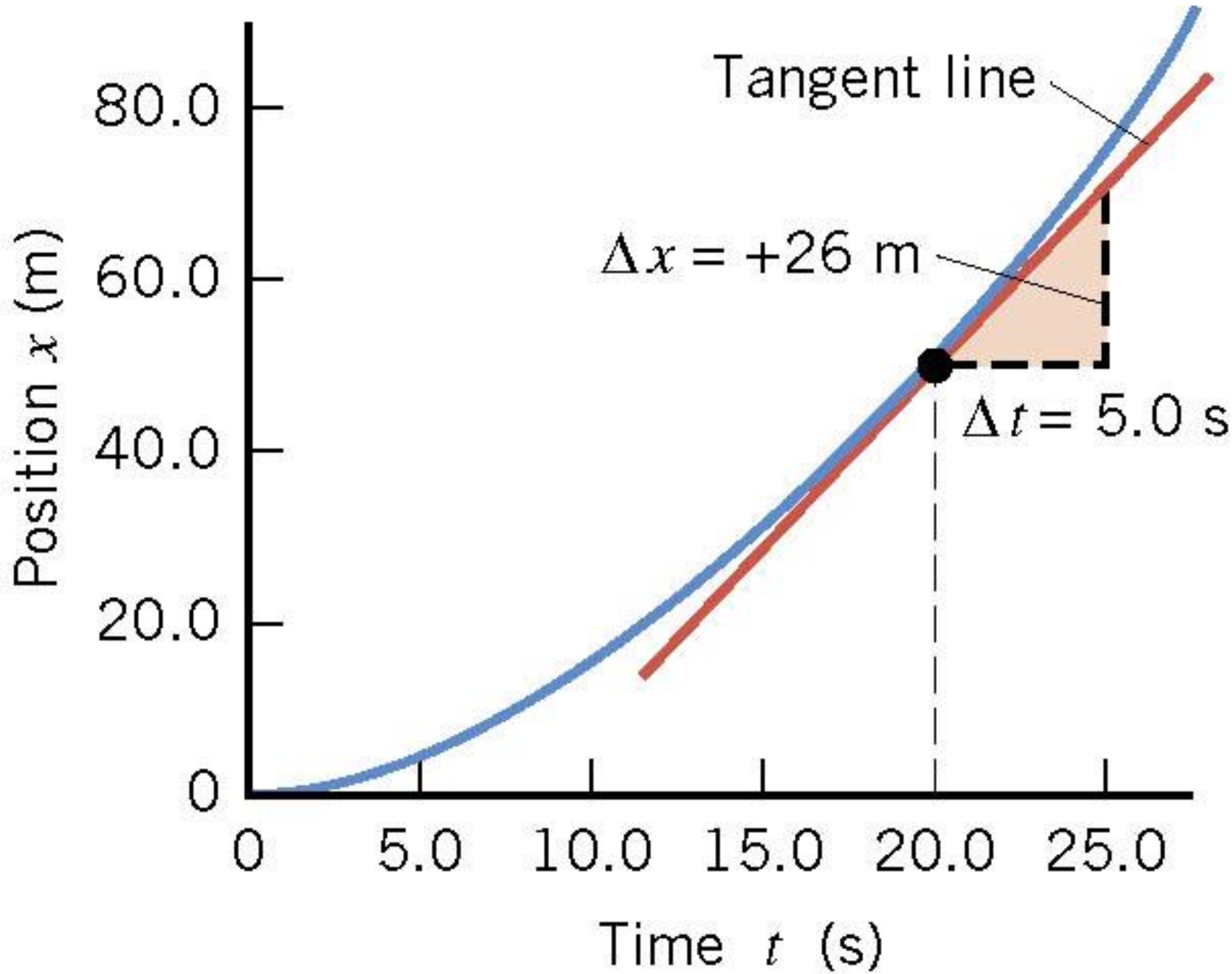


$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\,\text{m}}{2\,\text{s}} = +4\,\text{m/s}$$

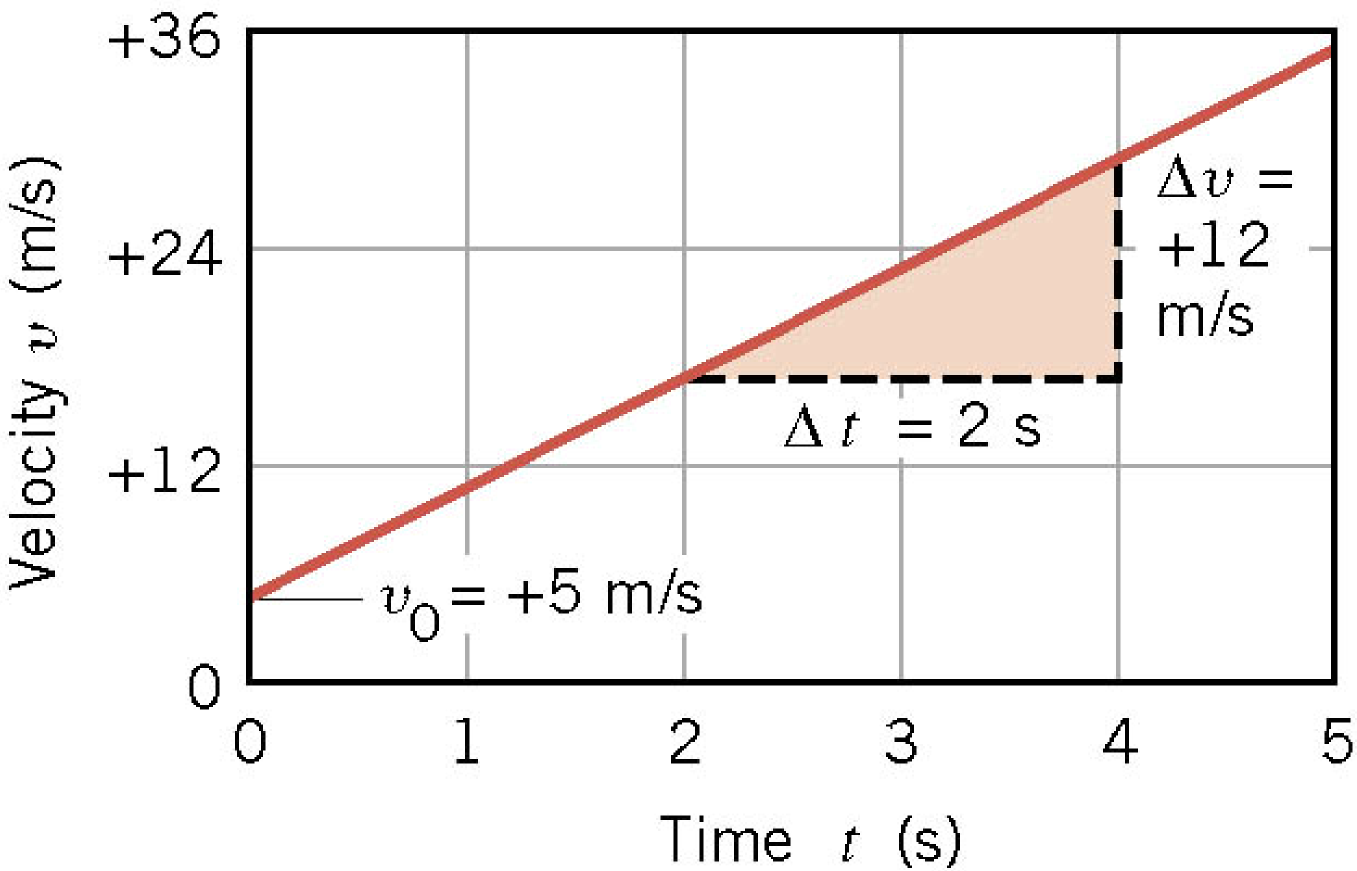
Graphical Analysis of Velocity and Acceleration



Graphical Analysis of Velocity and Acceleration



Graphical Analysis of Velocity and Acceleration



$$\text{Slope} = \frac{\Delta v}{\Delta t} = \frac{+12\,\text{m/s}}{2\,\text{s}} = +6\,\text{m/s}^2$$

Summery

$$x$$

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$x = \int_{t_1}^{t_2} v\,dt$$

$$v = \int_{t_1}^{t_2} a\,dt$$

$$a$$

Differentiation

Integration

Example-7

- An airplane accelerates down a runway at 3.20 m/s^2 for 32.8 s until is finally lifts off the ground. Determine the distance traveled before takeoff.
- Given:
- a = +3.2 m/s^2 t = 32.8 s v_i = 0 m/s
- Find: d = ??

$$y = v_i t + \tfrac{1}{2} a t^2$$

- d = (0 m/s)*(32.8 s)+ 0.5*(3.20 m/s2)*(32.8 s)^2
- d = 1720 m

Example-8

- A stone is dropped into a deep well and is heard to hit the water 3.41 s after being dropped. Determine the depth of the well.
- Given:
- a = -9.8 m/s^2 t = 3.41 s v_i = 0 m/s
- Find: d = ??

$$y = v_i t + \tfrac{1}{2} a t^2$$

- d = (0 m/s)*(3.41 s)+ 0.5*(-9.8 m/s^2)*(3.41 s)^2
- d = 0 m+ 0.5*(-9.8 m/s^2)*(11.63 s^2)
- d = -57.0 m