Collisions

Linear Momentum and Collisions

- Momentum
- Impulse
- Conservation of Momentum
- 1-D Collisions
- 2-D Collisions
- The Center of Mass

Conservation of Energy

- $\Delta E = \Delta K + \Delta U = 0$ if conservative forces are the only forces that do work on the system.
- The total amount of energy in the system is constant.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

- $\Delta E = \Delta K + \Delta U = -f_k d$ if friction forces are doing work on the system.
- The total amount of energy in the system is still constant, but the change in mechanical energy goes into "internal energy" or heat.

$$-f_k d = \left(\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2\right) - \left(\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2\right)$$

Momentum and Energy

- Two objects with masses m_1 and m_2 have equal kinetic energy. How do the magnitudes of their momentum compare?

(A) $p_1 < p_2$

(B) $p_1 = p_2$

(C) $p_1 > p_2$

(D) Not enough information is given

Linear Momentum, cont'd

- Linear momentum is a vector quantity $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
  - Its direction is the same as the direction of the velocity
- The dimensions of momentum are ML/T
- The SI units of momentum are kg m / s
- Momentum can be expressed in component form:

$$p_x = mv_x \quad p_y = mv_y \quad p_z = mv_z$$

Newton's Law and Momentum

- Newton's Second Law can be used to relate the momentum of an object to the resultant force acting on it

$$\vec{F}_{net} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$$

- The change in an object's momentum divided by the elapsed time equals the constant net force acting on the object

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

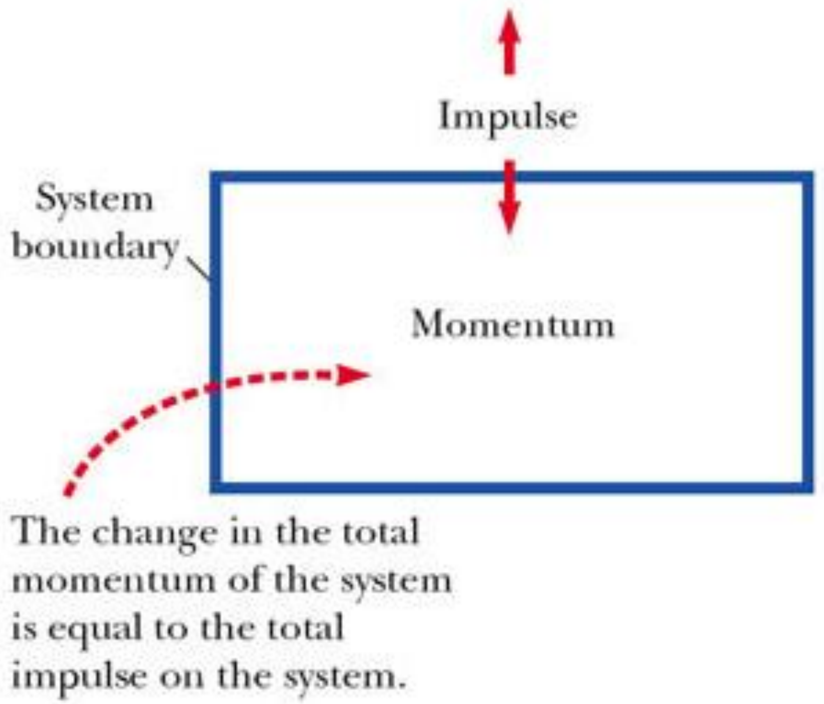
Impulse

- When a single, constant force acts on the object, there is an **impulse** delivered to the object
  -
  - $\vec{\mathbf{I}}$ is defined as the *impulse* $\vec{I} = \vec{F}\Delta t$
  - The equality is true even if the force is not constant
  - Vector quantity, the direction is the same as the direction of the force

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse-Momentum Theorem

- The theorem states that the impulse acting on a system is equal to the change in momentum of the system



$$\Delta\vec{p} = \vec{F}_{net}\Delta t = \vec{I}$$

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

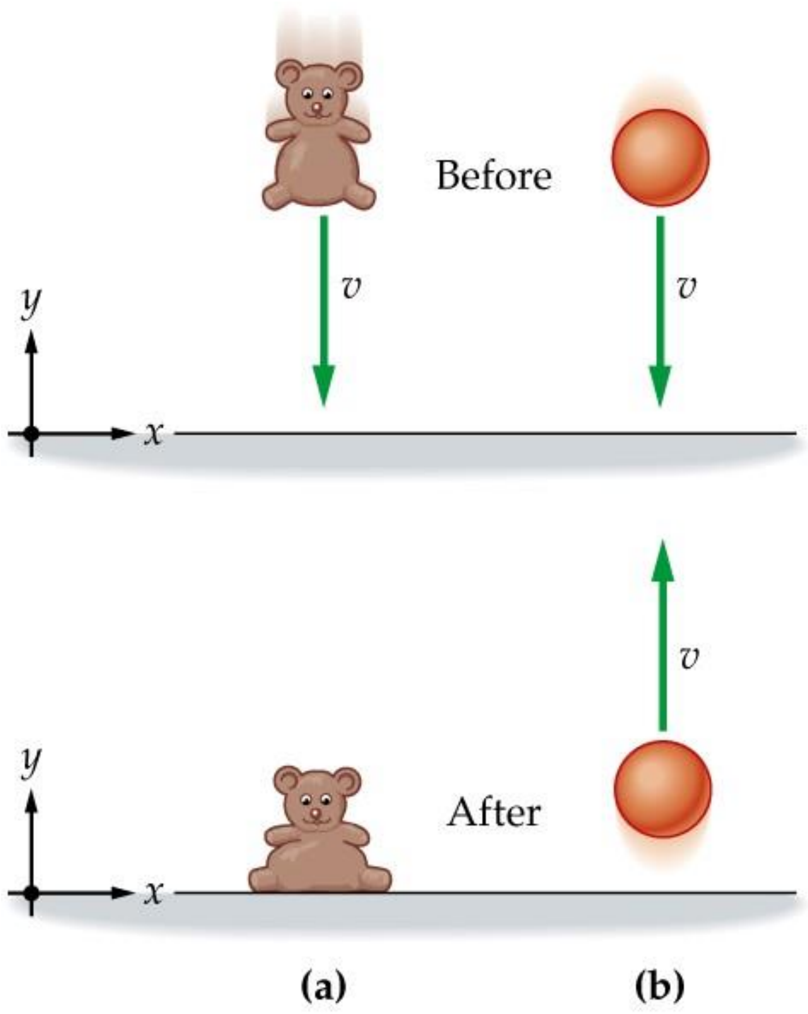
Calculating the Change of Momentum

$$\boxed{\begin{aligned}\Delta\vec{p} &= \vec{p}_{after} - \vec{p}_{before}\\ &= mv_{after} - mv_{before}\\ &= m(v_{after} - v_{before})\end{aligned}}$$

For the teddy bear $\Delta p = m[0-(-v)] = mv$

For the bouncing ball

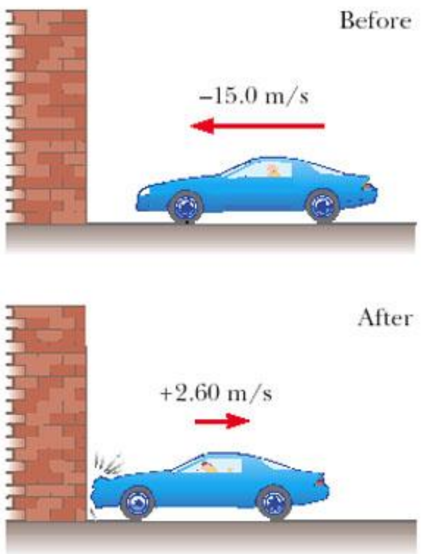
$$\Delta p = m[v-(-v)] = 2mv$$



January 26, 2022

How Good Are the Bumpers?

In a crash test, a car of mass 1.5×10^3 kg collides with a wall and rebounds as in figure. The initial and final velocities of the car are v_i=-15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car



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(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car

$$p_i = mv_i = (1.5\times10^3 kg)(-15 m/s) = -2.25\times10^4 kg\cdot m/s$$

$$p_f = mv_f = (1.5\times10^3 kg)(+2.6 m/s) = +0.39\times10^4 kg\cdot m/s$$

$$\begin{aligned} I &= p_f - p_i = mv_f - mv_i \\ &= (0.39\times10^4 kg\cdot m/s) - (-2.25\times10^4 kg\cdot m/s) \\ &= 2.64\times10^4 kg\cdot m/s \end{aligned}$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{2.64\times10^4 kg\cdot m/s}{0.15 s} = 1.76\times10^5 N$$

Impulse-Momentum Theorem

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the impulse imparted to the superball?

(A) 0
(B) 2 kg-m/s
(C) 20 kg-m/s
(D) 200 kg-m/s
(E) 2000 kg-m/s

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

Impulse-Momentum Theorem 2

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the force between the sidewalk and the superball?

(A) 0

(B) $2\,\mathrm{N}$

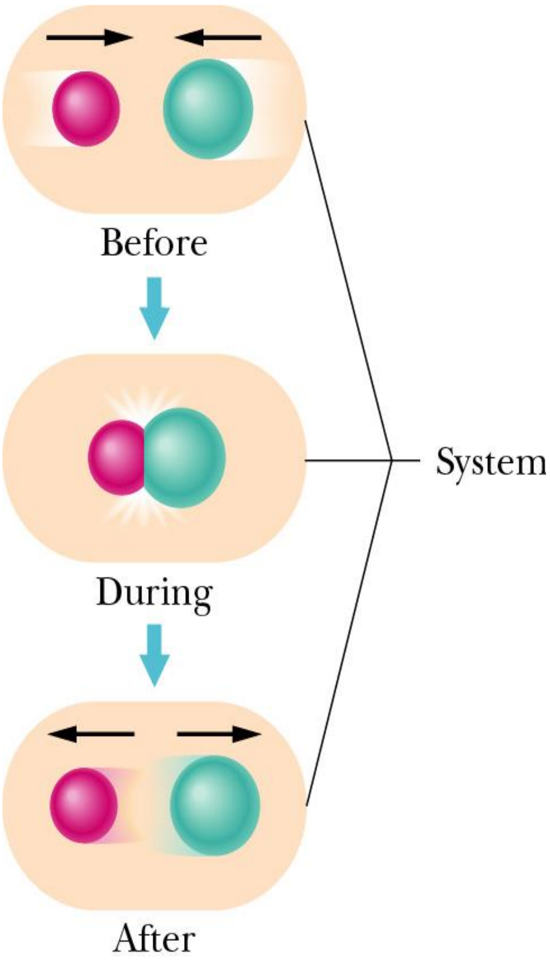
(C) $20\,\mathrm{N}$

(D) $200\,\mathrm{N}$

(E) $2000\,\mathrm{N}$

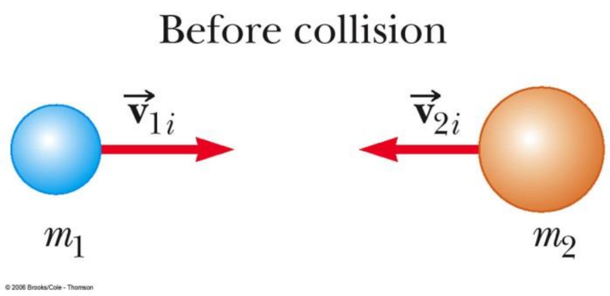
$$\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

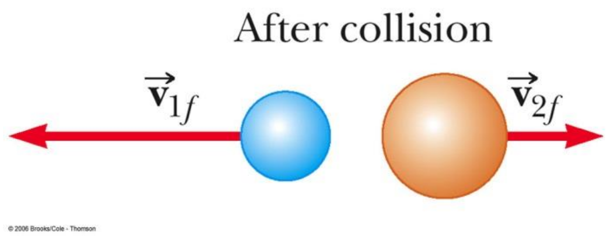
Conservation of Momentum

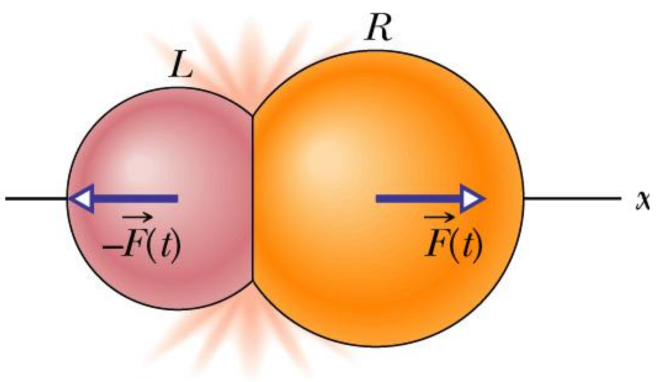


- In an isolated and closed system, the total momentum of the system remains constant in time.
  - Isolated system: no external forces
  - Closed system: no mass enters or leaves
  - The linear momentum of each colliding body may change
  - The total momentum P of the system cannot change.

Conservation of Momentum







- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since

$$\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$$

- Then

$$m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$$

- So

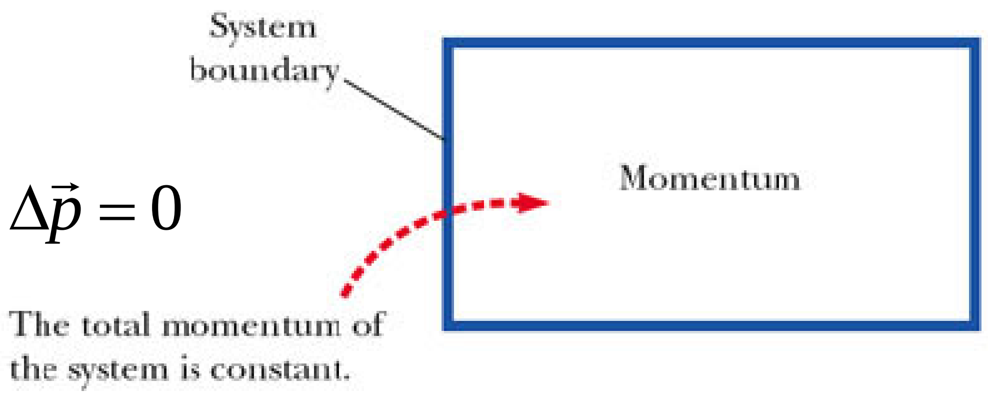
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Conservation of Momentum

- When no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

When $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$



- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Conservation of Momentum

- A 100 kg man and 50 kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1 m/s, at what speed does she recoil?

  (A) 0

  (B) 0.5 m/s

  (C) 1 m/s

  (D) 1.414 m/s

  (E) 2 m/s

Types of Collisions

- Momentum is conserved in any collision
- **Inelastic collisions:** *rubber ball and hard ball*
  - Kinetic energy is not conserved
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved
- **Actual collisions (Entangled)**
- **Perfectly inelastic** collisions occur when the objects stick together
  - Most collisions fall between elastic and perfectly inelastic collisions

Collisions Summary

- In an elastic collision, both **momentum** and **kinetic energy** are conserved
- In a non-perfect inelastic collision, **momentum** is conserved but kinetic energy is **not**. Moreover, the objects do not stick together
- In a perfectly inelastic collision, **momentum** is conserved, kinetic energy is **not**, and the two objects **stick together** after the collision, so their final velocities are the same
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types
- **Momentum** is conserved in all collisions

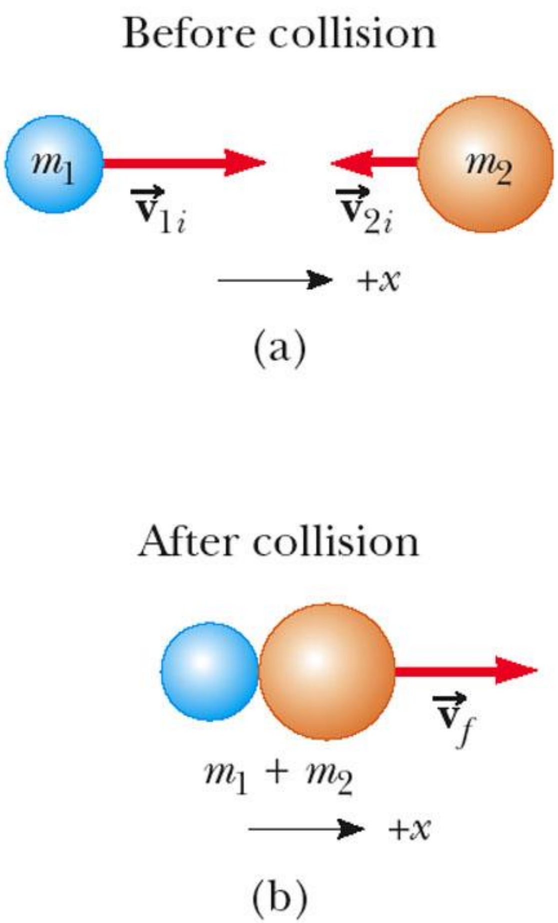
More about Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

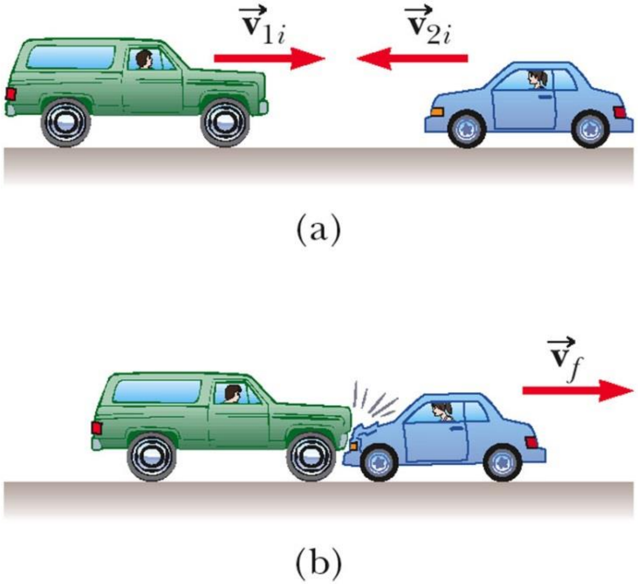
- Kinetic energy is NOT conserved



An SUV Versus a Compact

An SUV with mass 1.80×10^3 kg is travelling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is travelling westbound at -15.0 m/s. The cars collide head-on, becoming entangled.

A- Find the speed of the entangled cars after the collision.
B- Find the change in the velocity of each car.
C- Find the change in the kinetic energy of the system consisting of both cars.



An SUV Versus a Compact

Find the speed of the entangled cars after the collision.

$$m_1 = 1.80 \times 10^3 kg, v_{1i} = +15 m/s$$

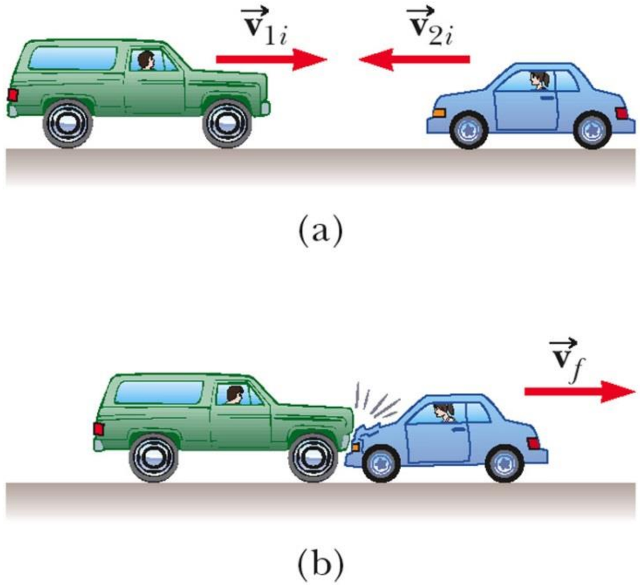
$$m_2 = 9.00 \times 10^2 kg, v_{2i} = -15 m/s$$

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$v_f = +5.00 m/s$$



An SUV Versus a Compact

Find the change in the velocity of each car.

$$m_1 = 1.80\times10^3 kg, v_{1i} = +15 m/s$$
$$m_2 = 9.00\times10^2 kg, v_{2i} = -15 m/s$$

$$v_f = +5.00 m/s$$

$$\Delta v_1 = v_f - v_{1i} = -10.0 m/s$$
$$\Delta v_2 = v_f - v_{2i} = +20.0 m/s$$

$$m_1\Delta v_1 = m_1(v_f - v_{1i}) = -1.8\times10^4 kg\cdot m/s$$
$$m_2\Delta v_2 = m_2(v_f - v_{2i}) = +1.8\times10^4 kg\cdot m/s$$

$$m_1\Delta v_1 + m_2\Delta v_2 = 0$$

An SUV Versus a Compact

Find the change in the kinetic energy of the system consisting of both cars.

$$m_1 = 1.80\times 10^3 kg, v_{1i} = +15 m/s$$

$$m_2 = 9.00\times 10^2 kg, v_{2i} = -15 m/s$$

$$v_f = +5.00 m/s$$

$$KE_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = 3.04\times 10^5 J$$

$$KE_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = 3.38\times 10^4 J$$

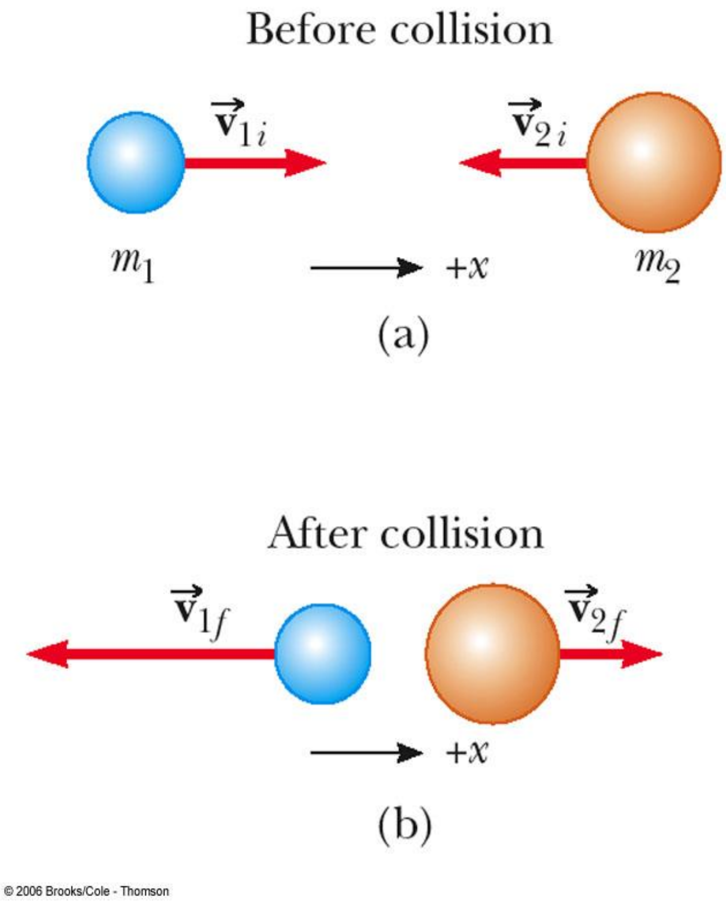
$$\Delta KE = KE_f - KE_i = -2.70\times 10^5 J$$

More About Elastic Collisions

- Both momentum and kinetic energy a conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$



- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Solve the equations simultaneously

Elastic Collisions

- A simpler equation can be used in place of the KE equation

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$\boxed{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}}$$

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

Special cases

- On solving the all equations we get value of v_2 as

$$\mathbf{v}_2 = \left(\frac{2m_1}{m_1 + m_2}\right)\mathbf{u}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{u}_2$$

- On solving the all equations we get value of v_1 as

$$\mathbf{v}_1 = \left(\frac{2m_2}{m_1 + m_2}\right)\mathbf{u}_2 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{u}_1$$

Case I: When the mass of both the particles are equal i.e., $m_1 = m_2$ then from the two equations, .

$v_2=u_1$ and $v_1=u_2$.

head on elastic collision then the particles will exchange their velocities.

Exchange of momentum between two particles suffering head on elastic collision is maximum when mass of both the particles is same

Case II: when the target particle is at rest i,e $u_2=0$, From equations

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 \qquad \text{---(10)}$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \qquad \text{---(11)}$$

Special cases

- On solving the all equations we get value of v_2 as

$$\mathbf{v}_2 = \left(\frac{2m_1}{m_1+m_2}\right)\mathbf{u}_1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)\mathbf{u}_2$$

- On solving the all equations we get value of v_1 as

$$\mathbf{v}_1 = \left(\frac{2m_2}{m_1+m_2}\right)\mathbf{u}_2 + \left(\frac{m_1-m_2}{m_1+m_2}\right)\mathbf{u}_1$$

Case III:

if $m_2 >>>> m_1$ and $u_2=0$ then

$v_1 \cong -u_1$ and $v_2=0$

For example when a ball thrown upwards collide with earth

Case IV:

if $m_1 >>>> m_2$ and $u_2=0$ then

$v_1 \cong u_1$ and $v_2=2u_1$

Conservation of Momentum

- An object of mass m moves to the right with a speed v. It collides head-on with an object of mass $3m$ moving with speed $v/3$ in the opposite direction. If the two objects stick together, what is the speed of the combined object, of mass $4m$, after the collision?

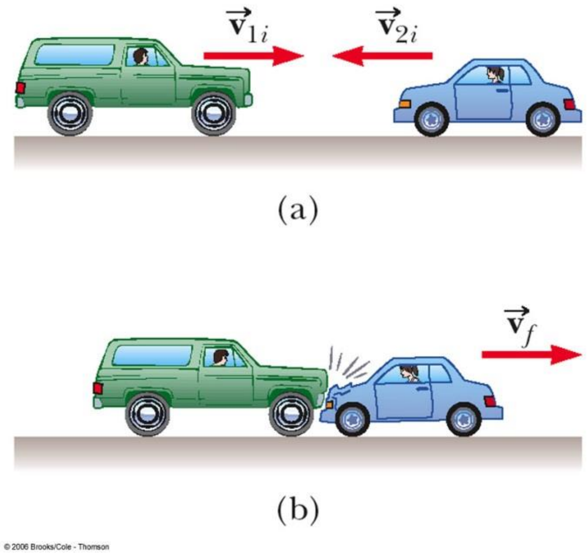
(A) 0

(B) v/2

(C) v

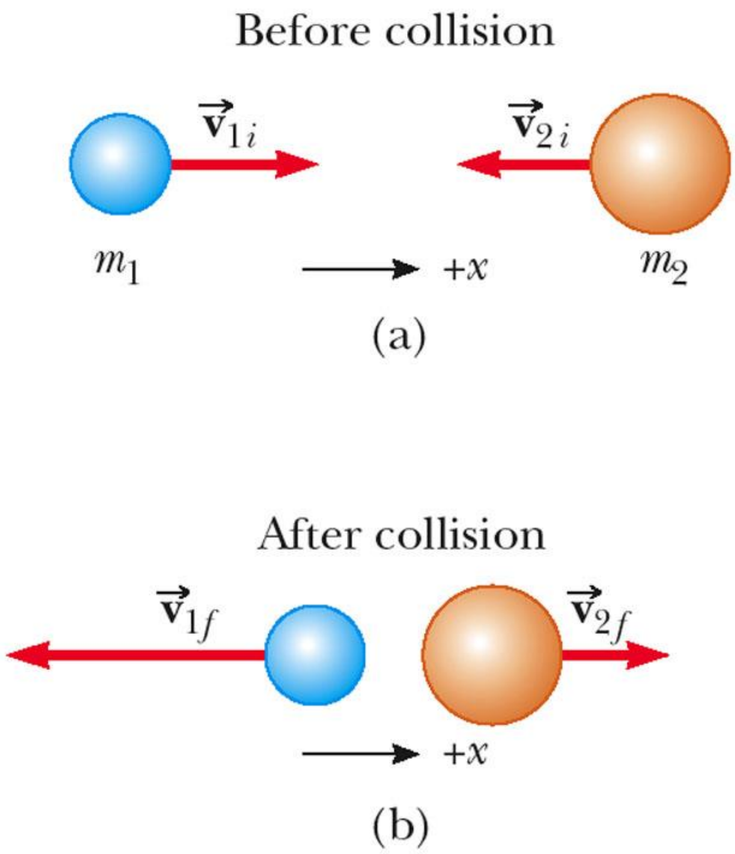
(D) 2v

(E) 4v



Problem Solving for 1D Collisions, 1

- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Problem Solving for 1D Collisions, 2

•Elastic collision can be further divided into head on collision (i.e collision in one dimension) and opaque collision (i.e collision in two dimension)

- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

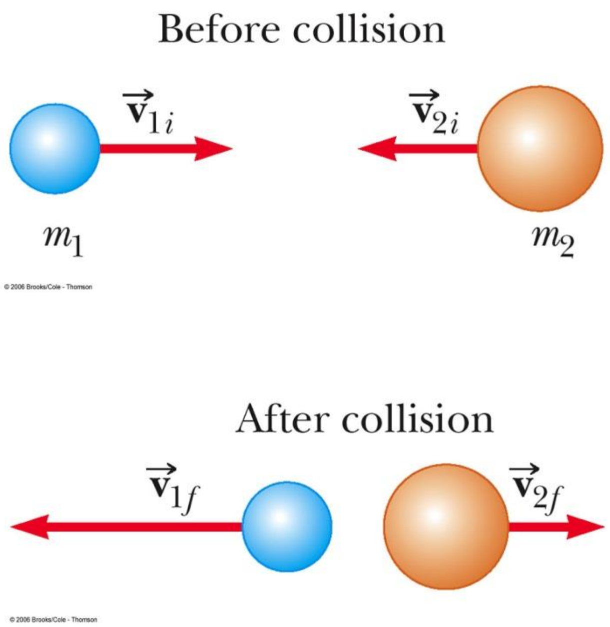
Problem Solving for 1D Collisions, 3

- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

- **Solve:** the resulting equations simultaneously

One-Dimension vs Two-Dimension

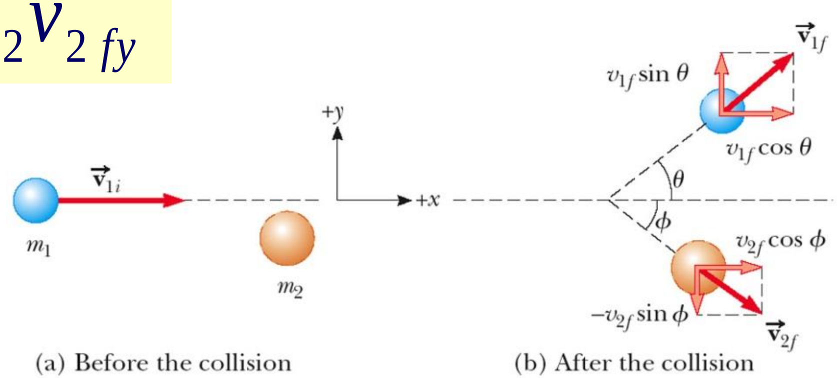


Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



Two-Dimensional Collisions

- The momentum is conserved in all directions
- Use subscripts for
  - Identifying the object
  - Indicating initial or final values
  - The velocity components

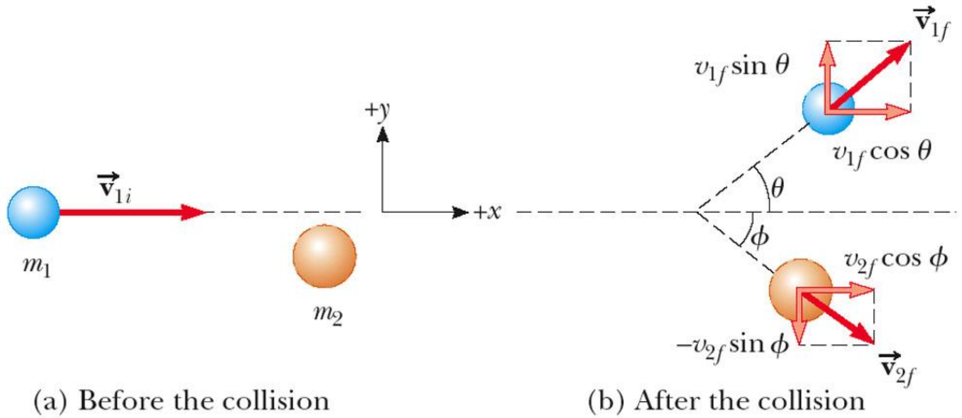
$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

$$\cancel{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

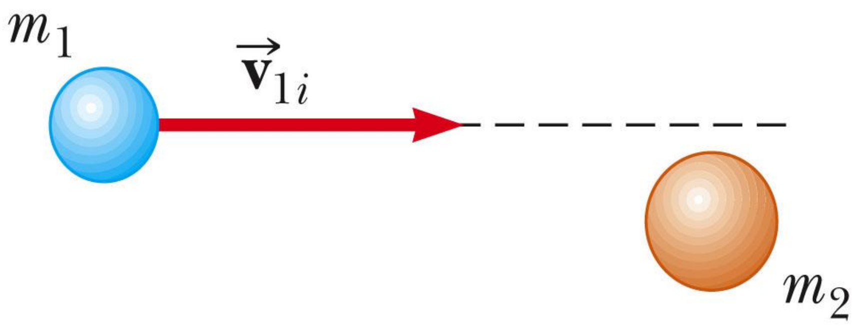
Glancing Collisions



- The "after" velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction

2-D Collision, example

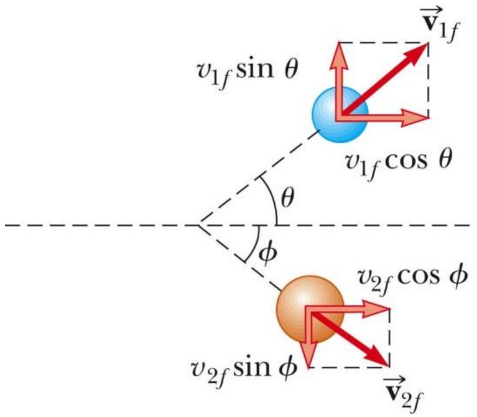
- Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest
- In the *x*-direction, the initial momentum is $m_1 v_{1i}$
- In the *y*-direction, the initial momentum is 0



(a) Before the collision

2-D Collision, example cont

- After the collision, the momentum in the ***x*-direction** is
- $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- After the collision, the momentum in the ***y*-direction** is
- $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$



(b) After the collision

$$m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

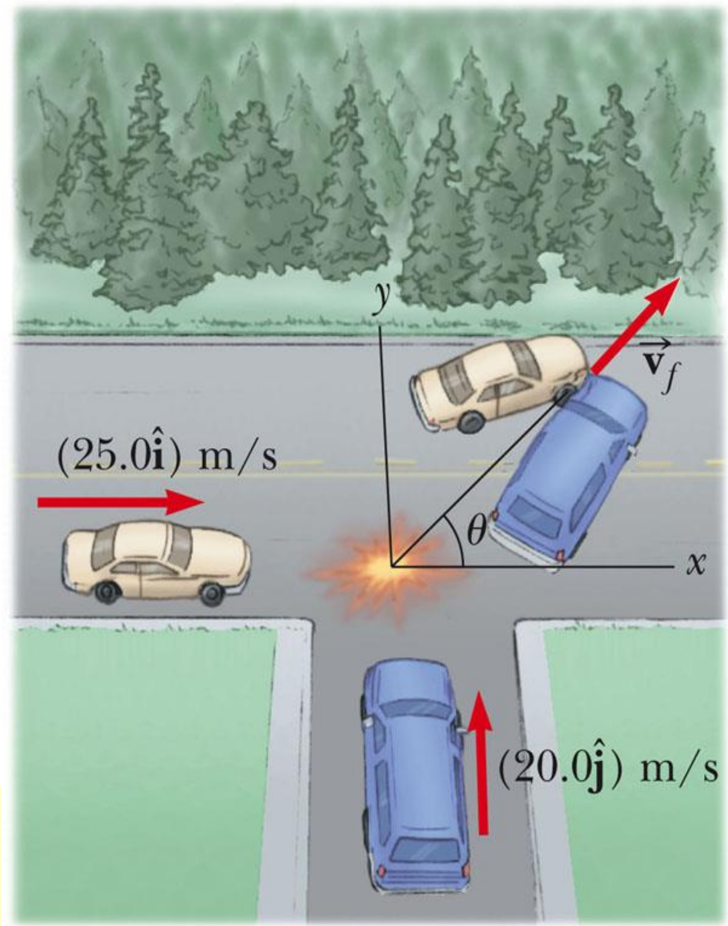
$$0 + 0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

- If the collision is elastic, apply the kinetic energy equation

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Collision at an Intersection

A car with mass 1.5×10^3 kg traveling east at a speed of 25 m/s collides at an intersection with a 2.5×10^3 kg van traveling north at a speed of 20 m/s. Find the magnitude and direction of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision and assuming that friction between the vehicles and the road can be neglected.



$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$

$$v_{cix} = 25 m/s, v_{viy} = 20 m/s, v_f = ?\ \theta = ?$$

Collision at an Intersection

$$m_c = 1.5\times10^3\,\text{kg}, m_v = 2.5\times10^3\,\text{kg}$$

$$v_{cix} = 25\,\text{m/s}, v_{viy} = 20\,\text{m/s}, v_f = ?\,\theta = ?$$

$$\sum p_{xi} = m_c v_{cix} + m_v v_{vix} = m_c v_{cix} = 3.75\times10^4\,\text{kg}\cdot\text{m/s}$$

$$\sum p_{xf} = m_c v_{cfx} + m_v v_{vfx} = (m_c + m_v) v_f \cos\theta$$

$$3.75\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \cos\theta$$

$$\sum p_{yi} = m_c v_{ciy} + m_v v_{viy} = m_v v_{viy} = 5.00\times10^4\,\text{kg}\cdot\text{m/s}$$

$$\sum p_{yf} = m_c v_{cfy} + m_v v_{vfy} = (m_c + m_v) v_f \sin\theta$$

$$5.00\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \sin\theta$$

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$$3.75\times10^4 \text{kg}\cdot\text{m/s} = (4.00\times10^3 \text{kg}) v_f \cos\theta$$

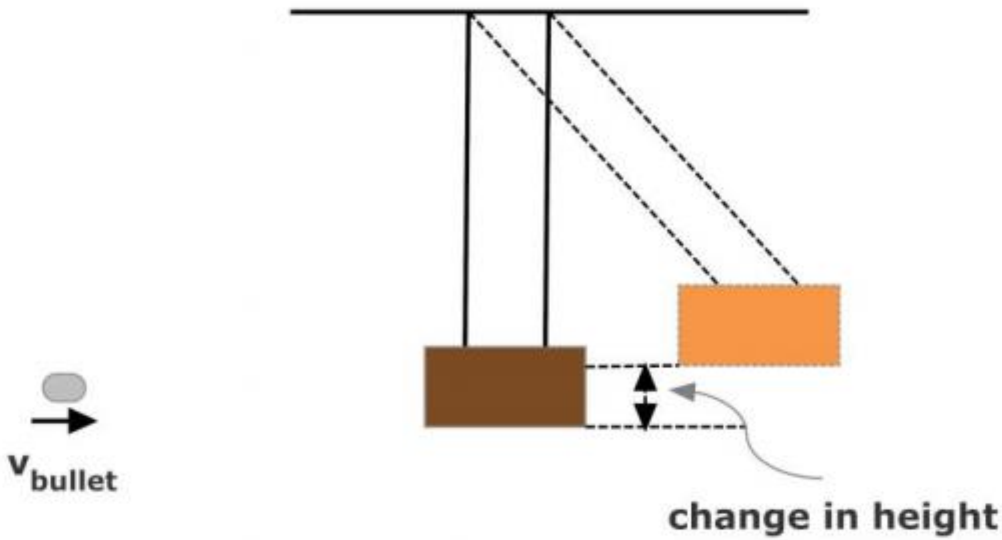
$$\tan\theta = \frac{5.00\times10^4 kg\cdot m/s}{3.75\times10^4 kg\cdot m/s} = 1.33$$

$$\theta = \tan^{-1}(1.33) = 53.1^\circ$$

$$v_f = \frac{5.00\times10^4 \text{kg}\cdot\text{m/s}}{(4.00\times10^3 \text{kg})\sin 53.1^\circ} = 15.6\,\text{m/s}$$

Example

A ballistic pendulum can be used to measure the speed of bullets. A bullet (mass of 5.0 g) is fired into a block of wood (mass of 4.995 kg) hanging from two cords. The block with the embedded bullet swings up to a maximum vertical height of 7 cm.



Assuming that the kinetic energy of the block at the bottom of the swing is converted to gravitational energy at the top of the swing, calculate:

a. The potential energy at the top of the swing.

b. The velocity of the block plus bullet at the bottom of the swing.

c. The momentum of the block plus bullet at the bottom of the swing.

d. Given that momentum is conserved in the collision of the bullet with the block, calculate the velocity of the bullet just before it strikes the block.