Some example of 2-Dims.

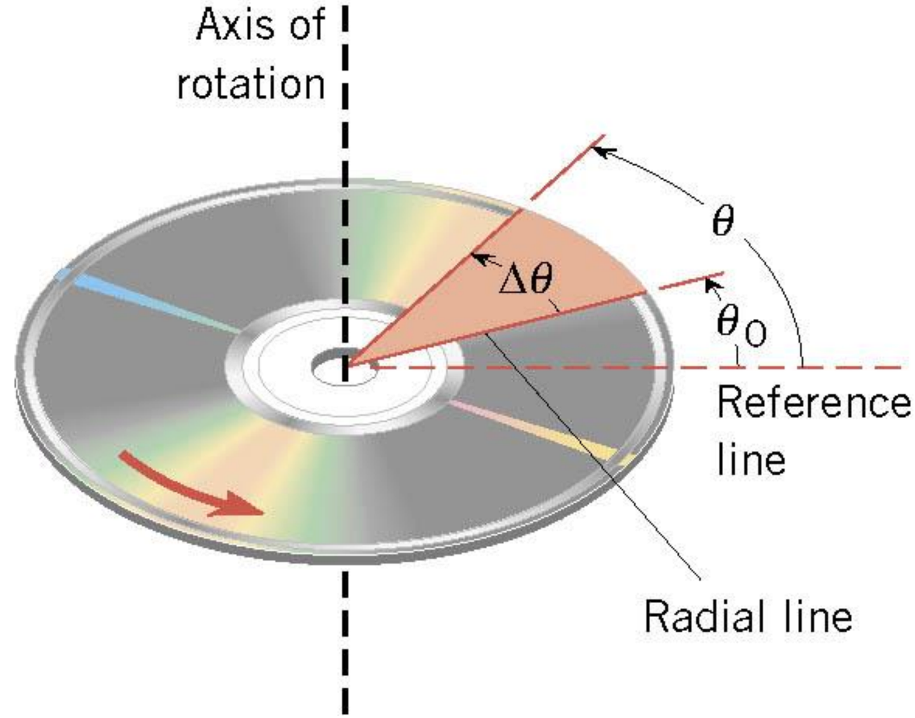
Circular Motion
Special Case of Two Dims

Benefit of Circular!

3.4 Angular Displacement

The angle through which the object rotates is called the ***angular displacement.***

$$\Delta\theta = \theta - \theta_o$$

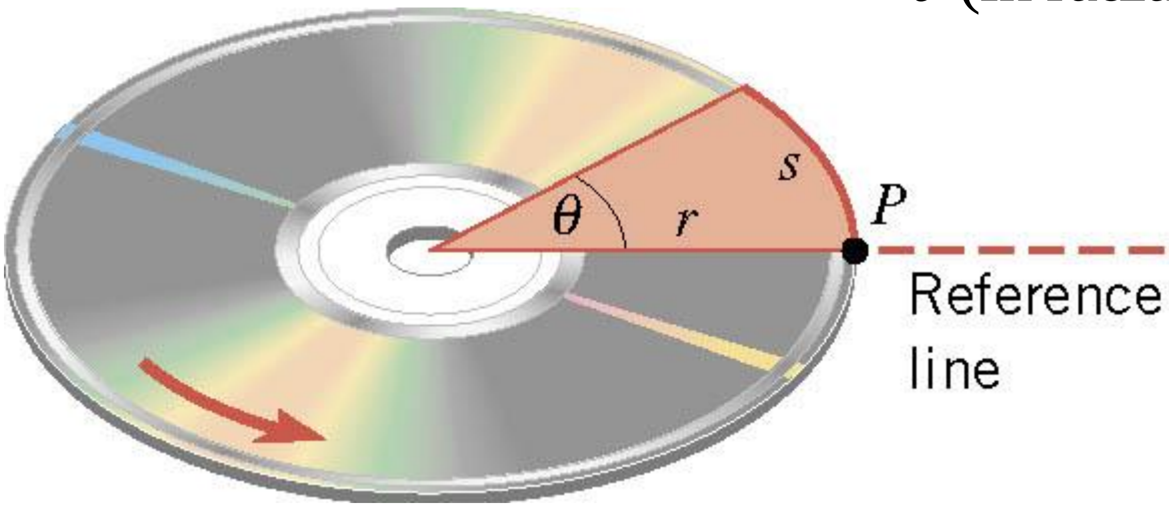


is the angle swept out by a line passing through any point on the body and away the axis of rotation.

By convention, angular displacement is
- positive if it is counterclockwise and
- negative if it is clockwise.

SI Unit of Angular Displacement: radian (rad)

3.4 Angular Displacement



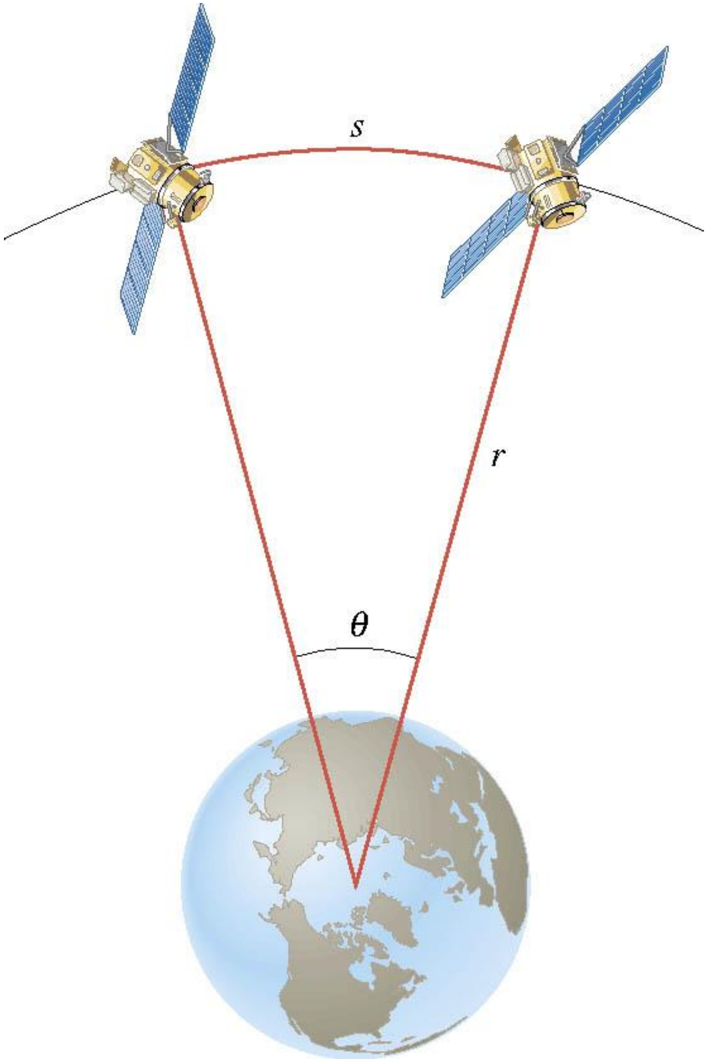
$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

For a full revolution:

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \quad \Rightarrow \quad 2\pi \text{ rad} = 360^\circ$$

Example 1 - Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^7m.



If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.

$$\theta\,(\text{in radians}) = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

$$2.00\,\text{deg}\left(\frac{2\pi\,\text{rad}}{360\,\text{deg}}\right) = 0.0349\,\text{rad}$$

$$s = r\theta = \left(4.23\times10^{7}\,\text{m}\right)\left(0.0349\,\text{rad}\right)$$

$$= 1.48\times10^{6}\,\text{m}\ \ (920\,\text{miles})$$

3.5 Angular Velocity

$$\text{Average angular velocity} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

$$\overline{\omega} = \frac{\theta - \theta_o}{t - t_o} = \frac{\Delta\theta}{\Delta t}$$

SI Unit of Angular Velocity: radian per second (rad/s)

INSTANTANEOUS ANGULAR VELOCITY

$$\omega = \lim_{\Delta t \to 0} \overline{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

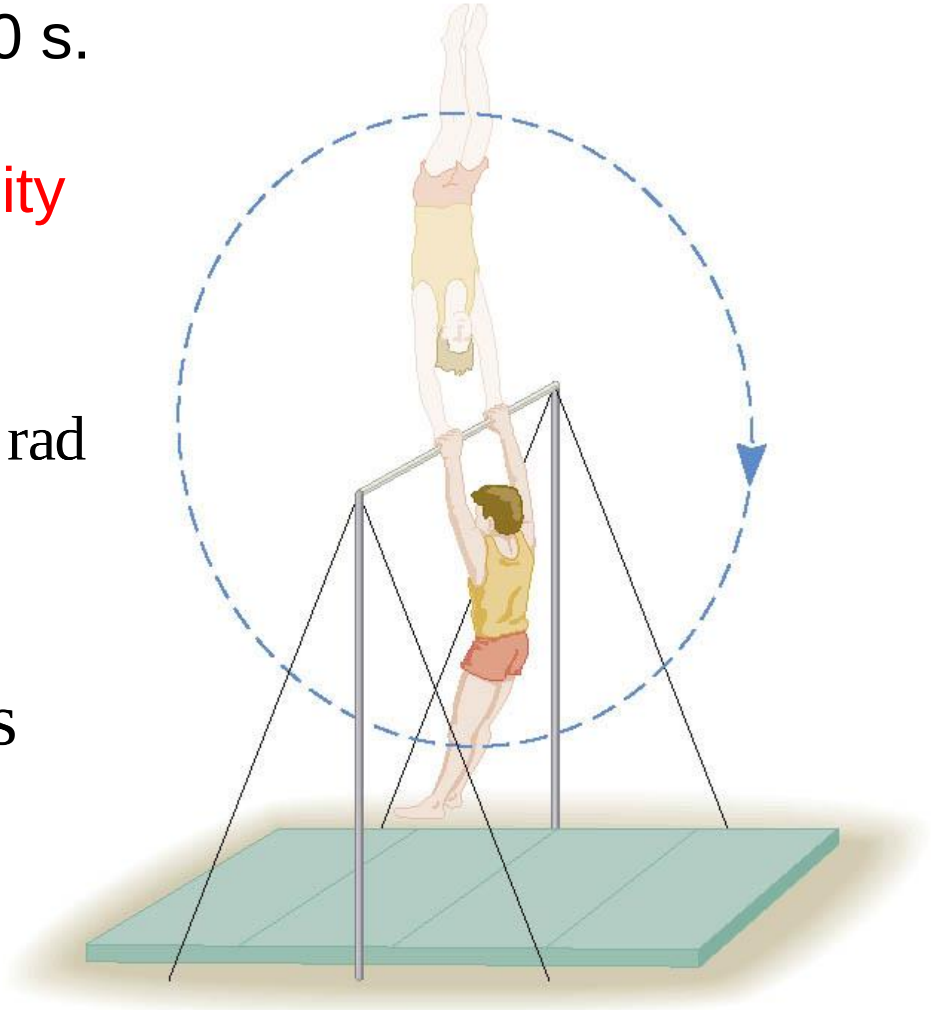
Example 2- Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s.

Find the average angular velocity of the gymnast.

$$\Delta\theta = -2.00\,\text{rev}\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = -12.6\,\text{rad}$$

$$\overline{\omega} = \frac{-12.6\,\text{rad}}{1.90\,\text{s}} = -6.63\,\text{rad/s}$$



3.6 Angular Acceleration

Changing angular velocity means that an ***angular acceleration*** is occurring.

$$\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Elapsed time}}$$

$$\overline{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta\omega}{\Delta t}$$

SI Unit of Angular acceleration: radian per second squared (rad/s^2)

Centripetal Acceleration and Tangential Acceleration

$$a_T = r\alpha = r\frac{d\omega}{dt}$$

Tangential Acceleration-Change in magnitude of Velocity

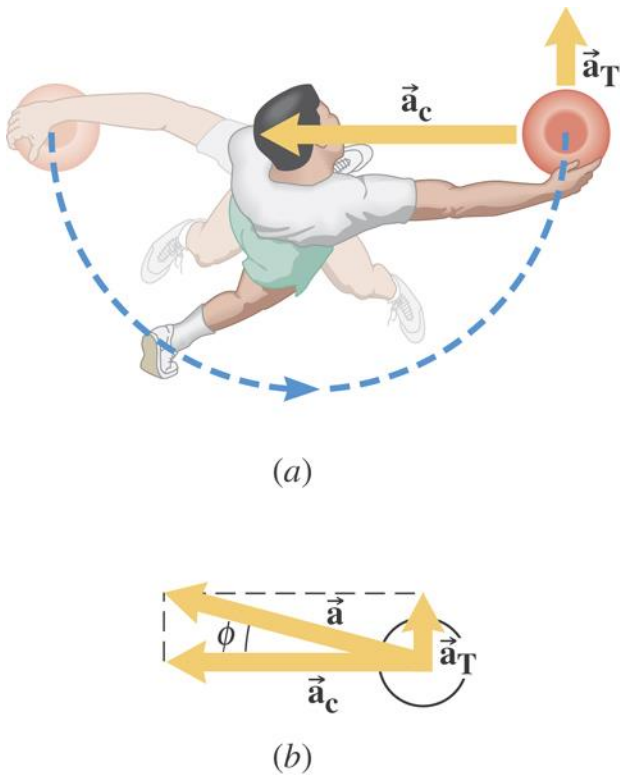
$$a_c = \frac{v_T^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 \quad (\omega \text{ in rad/s})$$

Centripetal Acceleration-Change in direction of Velocity

Example 3- A Discus Thrower

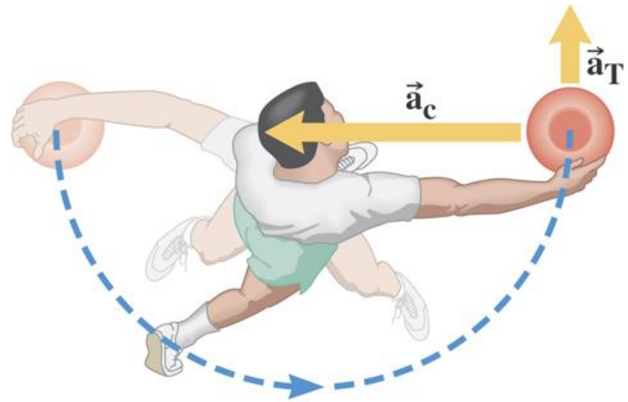
Starting from rest, the thrower accelerates the discus to a final angular speed of +15.0 rad/s in a time of 0.270 s before releasing it. During the acceleration, the discus moves in a circular arc of radius 0.810 m.

Find the magnitude of the total acceleration.



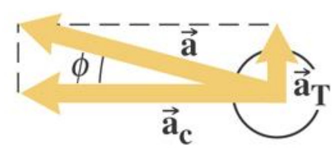
3.6 Centripetal Acceleration and Tangential Acceleration

$$a_c = r\omega^2 = (0.810\,\mathrm{m})(15.0\,\mathrm{rad/s})^2$$
$$= 182\,\mathrm{m/s^2}$$



(*a*)

$$a_T = r\alpha = r\frac{\omega\text{-}\omega_o}{t} = (0.810\,\mathrm{m})\left(\frac{15.0\,\mathrm{rad/s}}{0.270\,\mathrm{s}}\right)$$
$$= 45.0\,\mathrm{m/s^2}$$



(*b*)

$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{\left(182\,\mathrm{m/s^2}\right) + \left(45.0\,\mathrm{m/s^2}\right)} = 187\,\mathrm{m/s^2}$$

Recall the equations of kinematics for constant acceleration.

Five kinematic variables:

1. displacement, *x*

2. acceleration (constant), *a*

3. final velocity (at time *t*), *v*

4. initial velocity, v_o

5. elapsed time, t

$$v = v_o + at$$

$$x = \tfrac{1}{2}(v_o + v)t$$

$$v^2 = v_o^2 + 2ax$$

$$x = v_o t + \tfrac{1}{2}at^2$$

3.7 The Equations of Circular Kinematics

The equations of Circular kinematics for constant angular acceleration:

ANGULAR ACCELERATION

ANGULAR VELOCITY

$$\omega = \omega_o + \alpha t$$

$$\theta = \tfrac{1}{2}\left(\omega_o + \omega\right)t$$

TIME

ANGULAR DISPLACEMENT

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

Example 4- Blending with a Blender

The blades are whirling with an angular velocity of +375 rad/s when the “puree” button is pushed in.

When the “blend” button is pushed, the blades accelerate and reach a greater angular velocity after the blades have rotated through an angular displacement of +44.0 rad.

The angular acceleration has a constant value of +1740 rad/s^2.

Find the final angular velocity of the blades.

θ	α	ω	ω_o	t
+44.0 rad	+1740 rad/s^2	?	+375 rad/s	

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

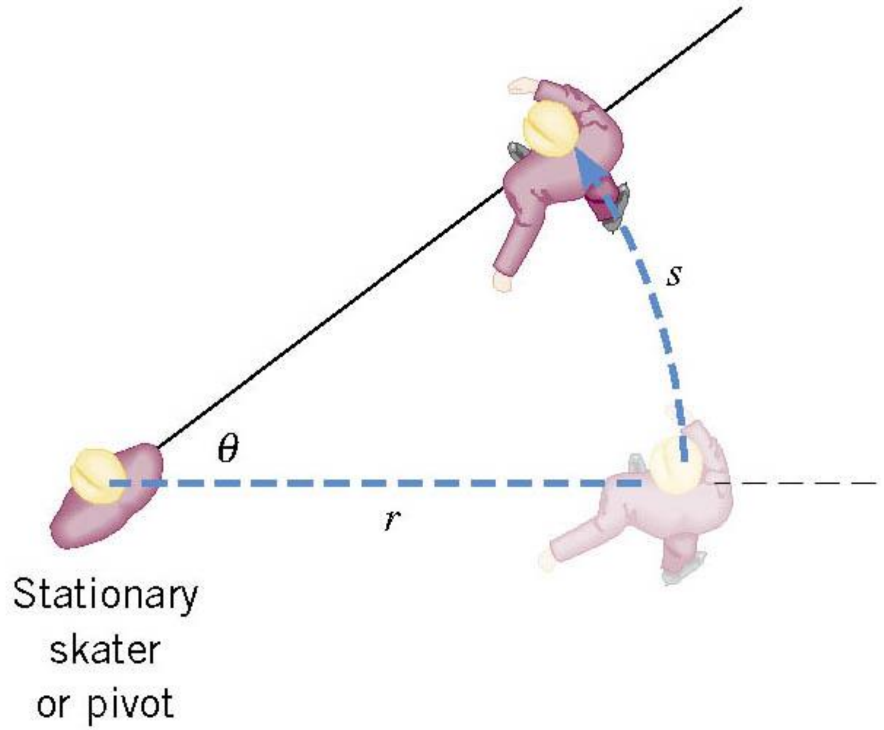
$$\omega = \sqrt{\omega_o^2 + 2\alpha\theta}$$

$$= \sqrt{(375\,\text{rad/s})^2 + 2(1740\,\text{rad/s}^2)(44.0\,\text{rad})} = +542\,\text{rad/s}$$

$$v_T = \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right)$$

$$\omega = \frac{\theta}{t}$$

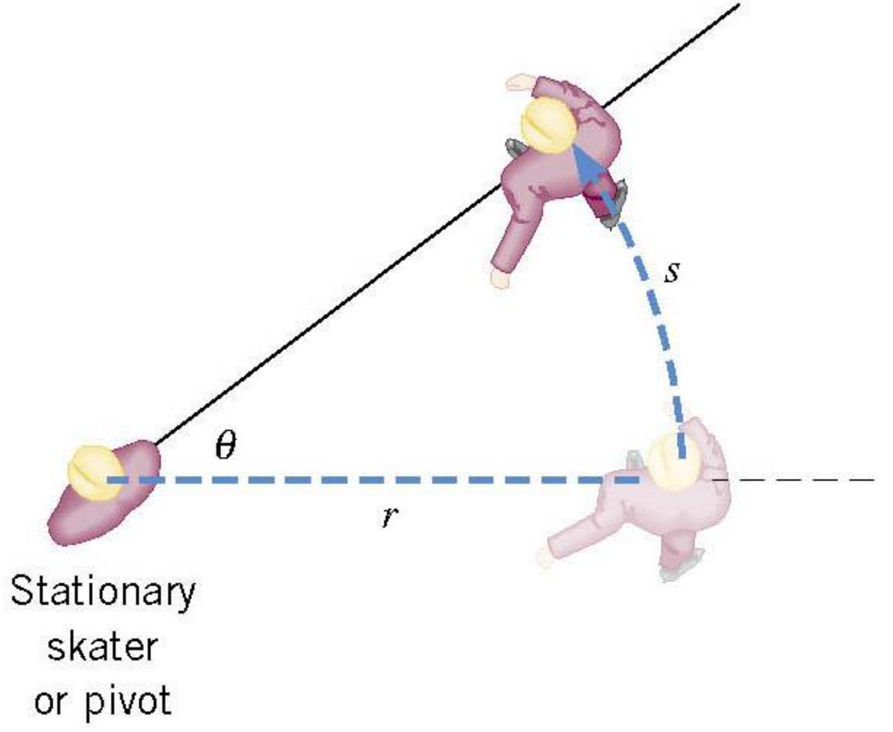
$$v_T = r\omega \quad (\omega \,\text{in rad/s})$$



$$\alpha = \frac{\omega - \omega_o}{t}$$

$$a_T = \frac{v_T - v_{To}}{t} = \frac{(r\omega) - (r\omega_o)}{t} = r\,\frac{\omega - \omega_o}{t}$$

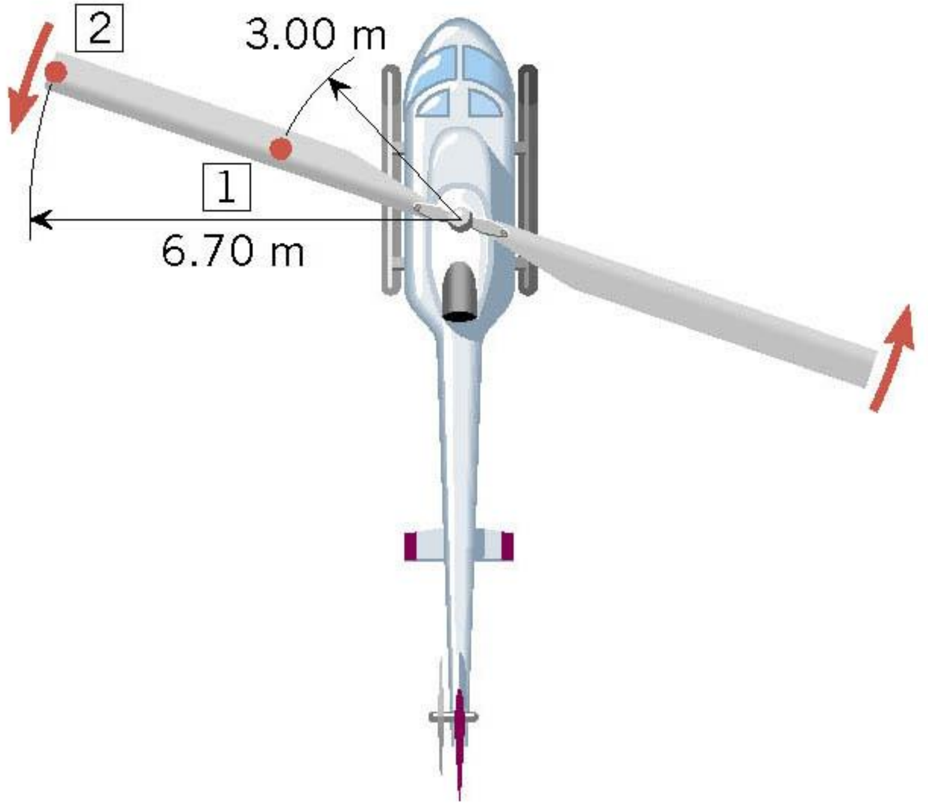
$$a_T = r\alpha \qquad (\alpha \text{ in rad/s}^2)$$



Example 5- **A Helicopter Blade**

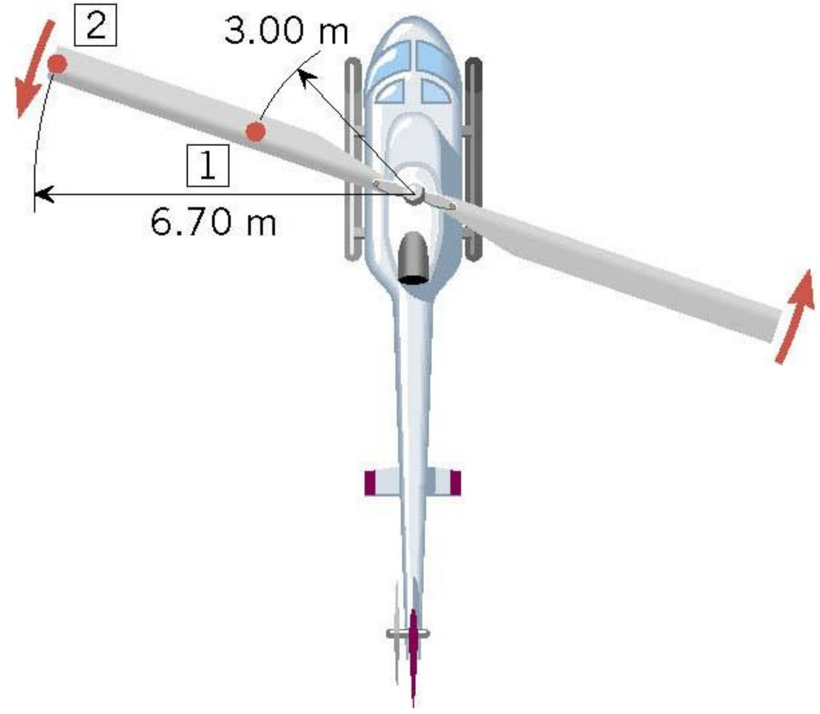
A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2.

For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



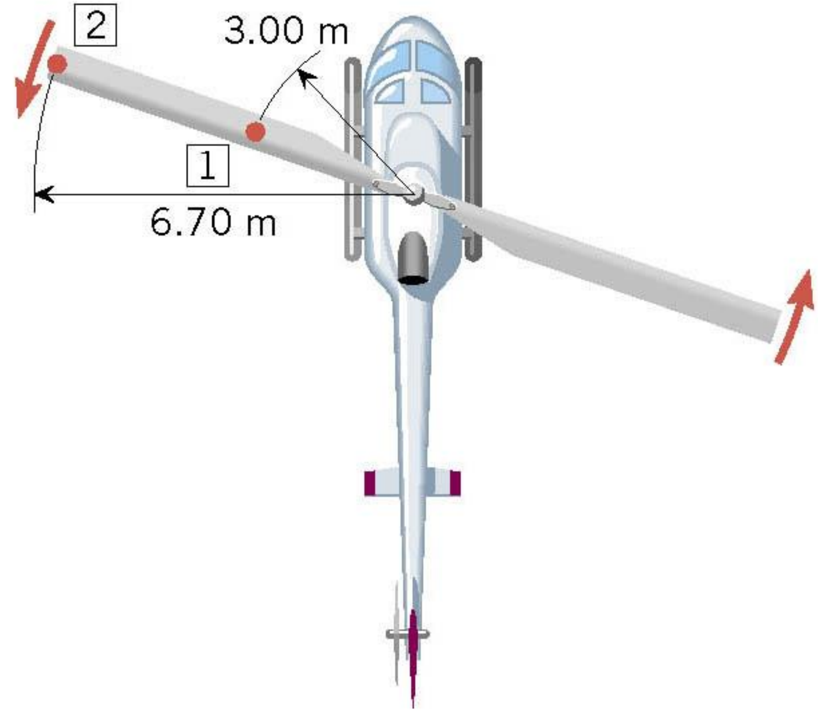
$$\omega = \left(6.50\,\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = 40.8\,\text{rad/s}$$

$$v_T = r\omega = (3.00\,\text{m})(40.8\,\text{rad/s}) = 122\,\text{m/s}$$



$$\alpha = \left(1.30\,\frac{\text{rev}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\,\text{rev}}\right) = 8.17\,\text{rad/s}^2$$

$$a_T = r\alpha = (3.00\,\text{m})\left(8.17\,\text{rad/s}^2\right) = 24.5\,\text{m/s}^2$$



3.7 The Equations of Circular Kinematics

Circular Motion	Linear Motion
$\omega = \omega_0 + \alpha t$	$v = v_o + at$
$\theta = \omega_0 t + 1/2\alpha t^2$	$x = v_o t + 1/2at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_o^2 + 2ax$

Foucault pendulum

"Challenging questions"

Q1- How do we know that the Earth isn't standing still and the Universe is rotating?

Q2- concerning human being, which velocity is greater, the linear or angular velocity? and why?

Q3- Can a body have acceleration with constant velocity? Explain

Q4- Can there be a motion in two dimensions with an acceleration only in one direction?