Units & Dimensional Analysis

Units

Physics experiments involve the measurement of a variety of quantities.

These measurements should be accurate and reproducible.

The first step in ensuring accuracy and reproducibility is defining the units in which the measurements are made.

SI units

- *meter* (m): unit of length
- *kilogram* (kg): unit of mass
- *second* (s): unit of time

SI Units

SI Base quantities	Unit	Abbreviation
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

These units are used in combination to define additional units for other important physical quantities such as force and energy

Prefixes

- Depending on the scale one often likes to use prefixes.
- Example, for length it is convenient to use km = 1000 m when traveling by car,

  or nm=10^{-9} m when discussing molecular scale objects.

Prefiks	Symbol	Multiplying factor
yotta	Y	1 000 000 000 000 000 000 000 000 = 10^{24}
zetta	Z	1 000 000 000 000 000 000 000 = 10^{21}
exa	E	1 000 000 000 000 000 000 = 10^{18}
peta	P	1 000 000 000 000 000 = 10^{15}
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1 000 = 10^{3}
hecto	h	100 = 10^{2}
deka	da	10 = 10^{1}
deci	d	0,1 = 10^{-1}
centi	c	0,01 = 10^{-2}
milli	m	0,001 = 10^{-3}
mikro	μ	0,000 001 = 10^{-6}
nano	n	0,000 000 001 = 10^{-9}
piko	p	0,000 000 000 001 = 10^{-12}
femto	f	0,000 000 000 000 001 = 10^{-15}
atto	a	0,000 000 000 000 000 001 = 10^{-18}
zepto	z	0,000 000 000 000 000 000 001 = 10^{-21}
yocto	y	0,000 000 000 000 000 000 000 001 = 10^{-24}

The universe by orders of magnitude

Table 1-3 The Universe by Orders of Magnitude

Size or Distance	(m)	Mass	(kg)	Time Interval	(s)
Proton	10^{-15}	Electron	10^{-30}	Time for light to cross nucleus	10^{-23}
Atom	10^{-10}	Proton	10^{-27}	Period of visible light radiation	10^{-15}
Virus	10^{-7}	Amino acid	10^{-25}	Period of microwaves	10^{-10}
Giant amoeba	10^{-4}	Hemoglobin	10^{-22}	Half-life of muon	10^{-6}
Walnut	10^{-2}	Flu virus	10^{-19}	Period of highest audible sound	10^{-4}
Human being	10^{0}	Giant amoeba	10^{-8}	Period of human heartbeat	10^{0}
Highest mountain	10^{4}	Raindrop	10^{-6}	Half-life of free neutron	10^{3}
Earth	10^{7}	Ant	10^{-4}	Period of Earth's rotation	10^{3}
Sun	10^{9}	Human being	10^{2}	Period of Earth's revolution around the Sun	10^{7}
Distance from Earth to the Sun	10^{11}	Saturn V rocket	10^{6}		
		Pyramid	10^{10}	Lifetime of human being	10^{9}
Solar system	10^{13}	Earth	10^{24}	Half-life of plutonium-239	10^{12}
Distance to nearest star	10^{16}	Sun	10^{30}	Lifetime of mountain range	10^{15}
Milky Way galaxy	10^{21}	Milky Way galaxy	10^{41}	Age of Earth	10^{17}
Visible universe	10^{26}	Universe	10^{52}	Age of universe	10^{18}

Units Conversion

- How does dimensional analysis work?
- It will involve some easy math (Multiplication & Division)
- In order to perform any conversion, you need a ***conversion factor***.
- Conversion factors are made from any two terms that describe the **same** or **equivalent** "amounts" of what we are interested in.

For example, we know that:

1 inch = 2.54 centimeters

1 dozen = 12

Conversion Factors

- So, conversion factors are nothing more than equalities or ratios that equal to each other. In "math-talk" they are equal to one.
- In mathematics, the expression to the left of the equal sign is equal to the expression to the right. They are equal expressions.
- For Example

  12 inches = 1 foot

  Written as an "equality" or "ratio" it looks like

$$\frac{12\,inches}{1\,foot} = 1 \qquad \text{or} \qquad \frac{1\,foot}{12\,inches} = 1$$

Conversion Factors

$$\frac{12\,inches}{1\,foot} \quad \text{or} \quad \frac{1\,foot}{12\,inches}$$

Hey! These look like fractions!

- *Conversion Factors* look a lot like fractions, but they are **<u>not</u>**!
- The critical thing to note is that *the units behave like numbers do when you multiply fractions.* That is, the inches (or foot) on top and the inches (or foot) on the bottom can cancel out. Just like in algebra,

Example #1

- How many feet are in 60 inches? Solve using dimensional analysis.
- All dimensional analysis problems are set up the same way. They follow this same pattern:

What units you have x $\left(\dfrac{\textbf{What units you want}}{\text{What units you have}} \right)$ = **What units you want**

The number & units you start with

The conversion factor
(The equality that looks like a fraction)

The units you want to end with

Example #1 (cont)

- You need a conversion factor. Something that will change inches into feet.
- Remember

12 inches = 1 foot

Written as an "equality" or "ratio" it looks like $\frac{1\,foot}{12\,inches}$

$$60\ \cancel{\text{inches}} \times \left(\frac{1\,foot}{12\,\cancel{inches}}\right) = 5\text{ feet}$$

(Mathematically all you do is: 60 x 1 ÷ 12 = 5)

$$\text{What units you have} \times \left(\frac{\textbf{What units you want}}{\text{What units you have}}\right) = \textbf{What units you want}$$

Example #1 (cont)

- The previous problem can also be written to look like this:

- $$\begin{array}{c|c|c} 60 \text{ inches} & 1 \text{ foot} & = 5 \text{ feet} \\ \hline & 12 \text{ inches} & \end{array}$$

- This format is more visually integrated, more bridge like, and is more appropriate for working with factors. In this format, the horizontal bar means "divide," and the vertical bars mean "multiply".

Example #2

- You need to put gas in the car. Let's assume that gasoline costs \$3.35 per gallon and you've got a twenty dollar bill. How many gallons of gas can you get with that twenty? Try it!

- $$\frac{\cancel{\$}20.00}{} \Bigg| \frac{1 \text{ gallon}}{\cancel{\$}3.35} \Bigg| = 5.97 \text{ gallons}$$

(Mathematically all you do is: 20 x 1 ÷ 3.35 = 5.97)

Example #3

- What if you had wanted to know not how many gallons you could get, but how many miles you could drive assuming your car gets 24 miles a gallon? Let's try building from the previous problem. You know you have 5.97 gallons in the tank. Try it!

- $$\frac{5.97\ \cancel{\text{gallons}}}{} \left| \frac{24\ \text{miles}}{1\ \cancel{\text{gallon}}} \right| = 143.28\ \text{miles}$$

(Mathematically all you do is: 5.97 x 24 ÷ 1 = 143.28)

Example #3

- There's another way to do the previous two problems. Instead of chopping it up into separate pieces, build it as one problem. Not all problems lend themselves to working them this way but many of them do. It's a nice, elegant way to minimize the number of calculations you have to do. Let's reintroduce the problem.

Example #3 (cont)

- You have a twenty dollar bill and you need to get gas for your car. If gas is \$3.35 a gallon and your car gets 24 miles per gallon, how many miles will you be able to drive your car on twenty dollars? Try it!

- $$\frac{\cancel{\$}\,20.00}{} \;\Big|\; \frac{1\ \cancel{\text{gallon}}}{\cancel{\$}\,3.35} \;\Big|\; \frac{24\ \text{miles}}{1\ \cancel{\text{gallon}}} \;\Big|\; = \; 143.28\ \text{miles}$$

(Mathematically all you do is: 20 x 1 ÷ 3.35 x 24 ÷ 1 = 143.28)

Example #4

- Try this expanded version of the previous problem.
- You have a twenty dollar bill and you need to get gas for your car. Gas currently costs $3.35 a gallon and your car averages 24 miles a gallon. If you drive, on average, 7.1 miles a day, how many weeks will you be able to drive on a twenty dollar fill-up?

Example #4 (cont)

- $$\begin{array}{l|l|l|l|l|} \cancel{\$}\,20.00 & 1\ \cancel{\text{gallon}} & 24\ \cancel{\text{miles}} & 1\ \cancel{\text{day}} & 1\ \text{week} \\ \hline & \cancel{\$}\,3.35 & 1\ \cancel{\text{gallon}} & 7.1\ \cancel{\text{miles}} & 7\ \cancel{\text{days}} \end{array}$$

$$= \ 2.88 \text{ weeks}$$

(Mathematically : 20 x 1 ÷ 3.35 x 24 ÷ 1 x 1 ÷ 7.1 x 1 ÷ 7 = 2.88)

H.W #1

- You're throwing a pizza party for 15 and figure each person might eat 4 slices. How much is the pizza going to cost you? You call up the pizza place and learn that each pizza will cost you $14.78 and will be cut into 12 slices. You tell them you'll call back. Do you have enough money?

HW #2

- You have come down with a bad case of the eye pollution, but fortunately your grandmother knows how to cure the eye pollution. She sends you an eyedropper bottle labeled:
- Take 1 drop per 10 Kg. of body weight per day divided into 4 doses until the eyes are clean.
- This problem is a bit more challenging, *but don't panic*. Break the problem down into a bunch of small problems, and tackle each one by one.

Dimensional Analysis

[L] = length   [M] = mass   [T] = time

Is the following equation dimensionally correct?

$$x = \frac{1}{2} v t^2$$

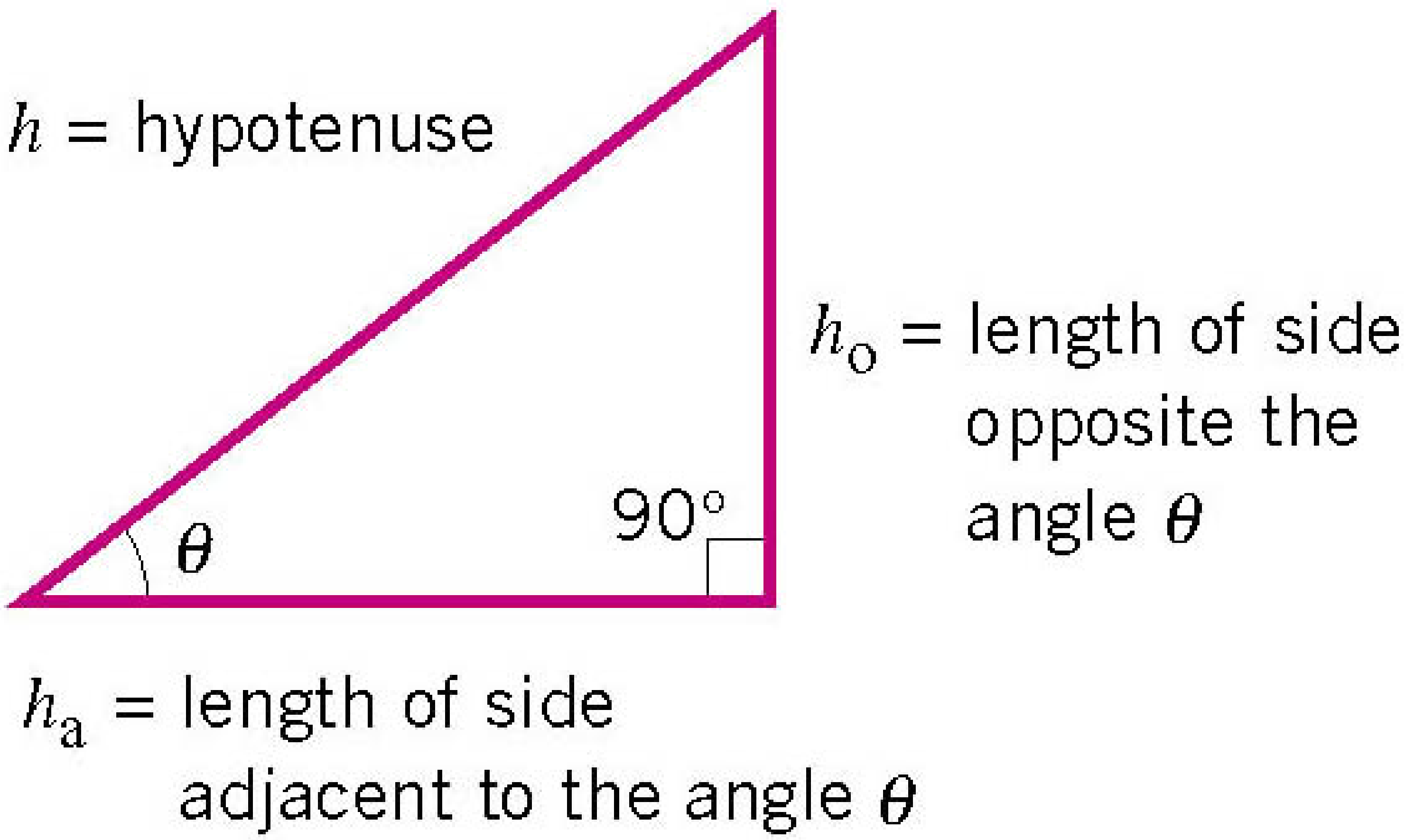
$$[\mathrm{L}] = \left[\frac{\mathrm{L}}{\mathrm{T}}\right][\mathrm{T}]^2 = [\mathrm{L}][\mathrm{T}]$$

Is the following equation dimensionally correct?

$$x = vt$$

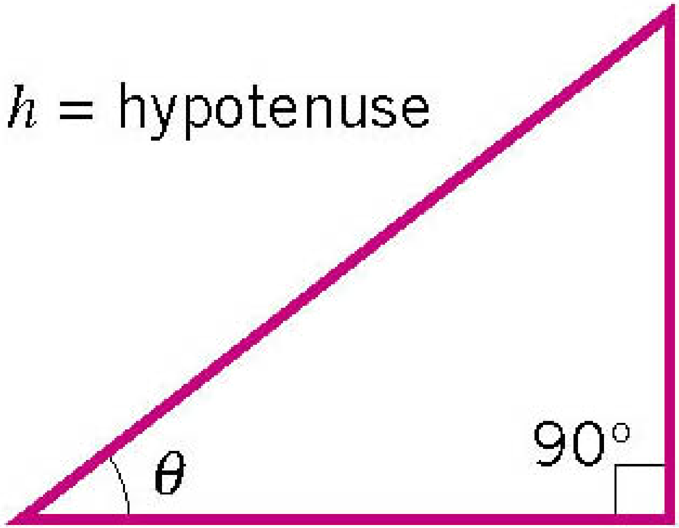
$$[\mathrm{L}] = \left[\frac{\mathrm{L}}{\mathrm{T}}\right][\mathrm{T}] = [\mathrm{L}]$$

Trigonometry



Trigonometry

h = hypotenuse



h_o = length of side opposite the angle θ

h_a = length of side adjacent to the angle θ

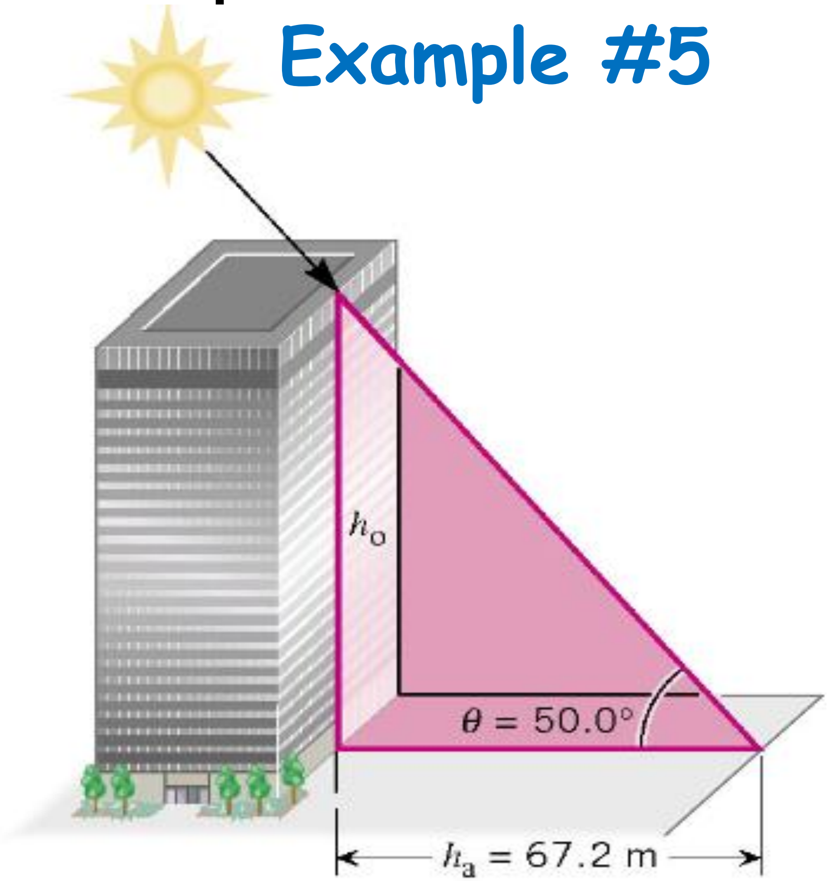
$$\sin\theta = \frac{h_o}{h}$$

$$\cos\theta = \frac{h_a}{h}$$

$$\tan\theta = \frac{h_o}{h_a}$$

Trigonometry

Example #5



$$\tan\theta = \frac{h_o}{h_a}$$

$$\tan 50^\circ = \frac{h_o}{67.2\text{m}}$$

$$h_o = \tan 50^\circ (67.2\text{m}) = 80.0\text{m}$$

Scalars and Vectors

A *scalar* quantity is one that can be described by a single number:

temperature, speed, mass

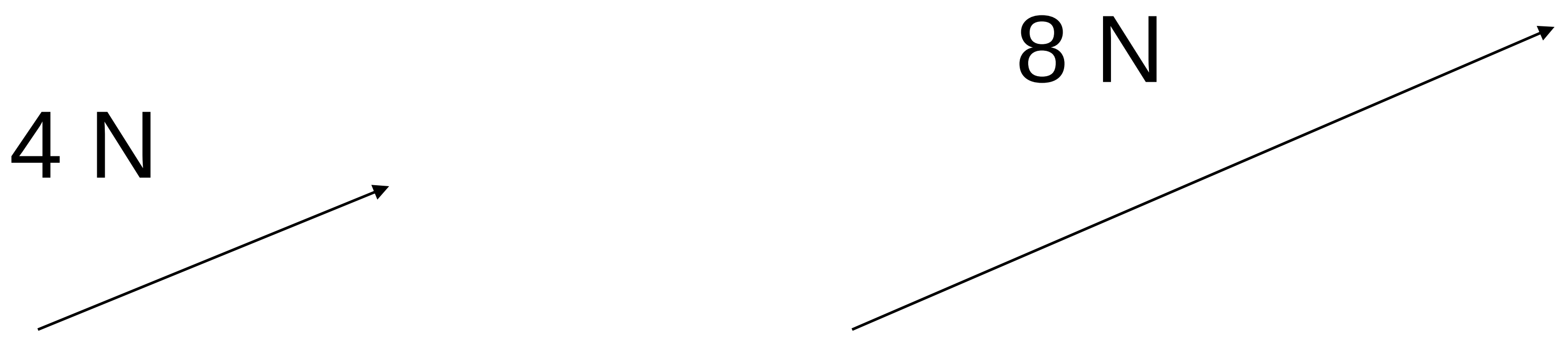
A *vector* quantity deals inherently with both magnitude and direction:

velocity, force, displacement

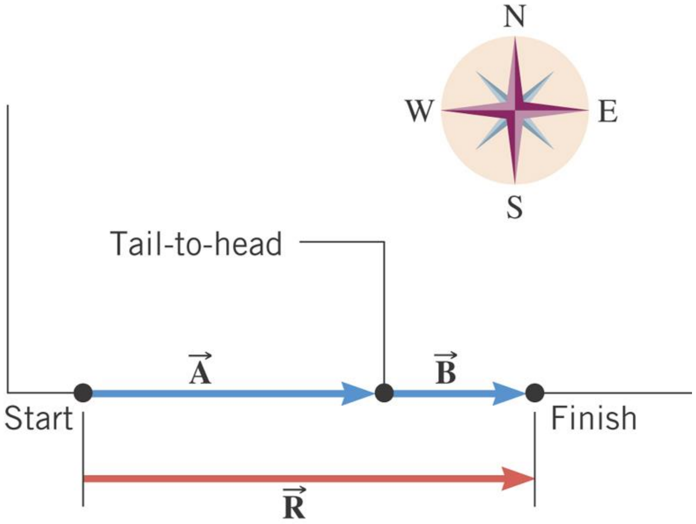
Scalars and Vectors

Arrows are used to represent vectors. The direction of the arrow gives the direction of the vector.

By convention, the length of a vector arrow is proportional to the magnitude of the vector.



Vector Addition and Subtraction

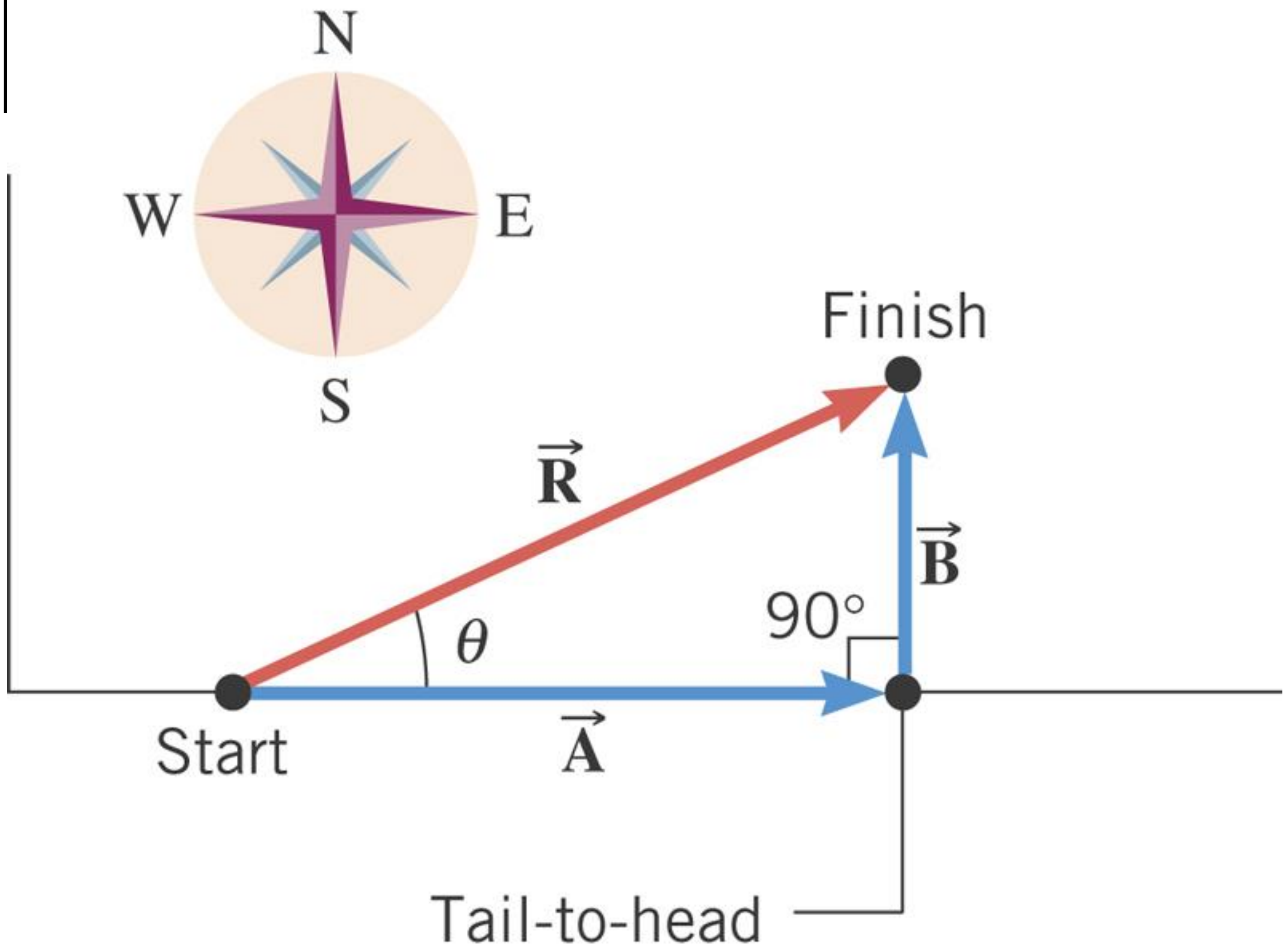


5 m

3 m

8 m

Vector Addition and Subtraction



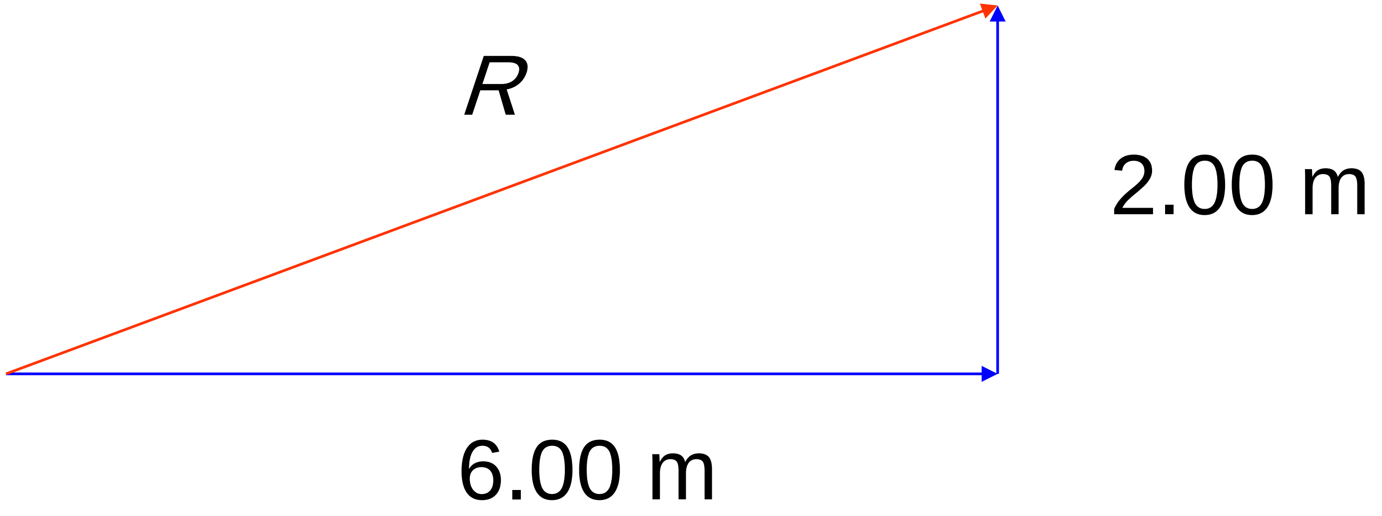
Vector Addition and Subtraction

$$R^2 = (2.00\,\mathrm{m})^2 + (6.00\,\mathrm{m})^2$$

$$R = \sqrt{(2.00\,\mathrm{m})^2 + (6.00\,\mathrm{m})^2} = 6.32\mathrm{m}$$

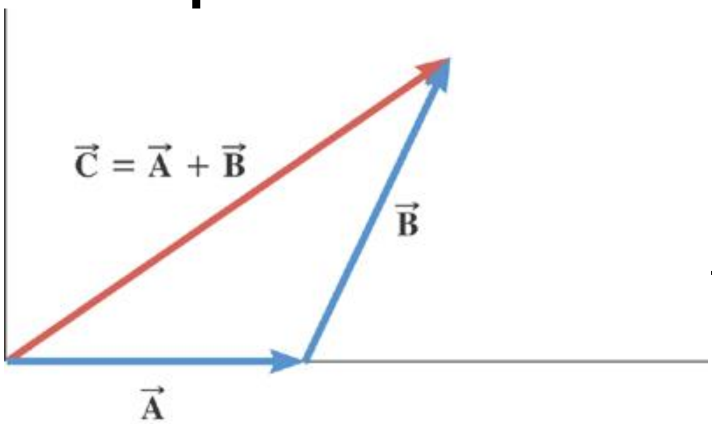
$$\tan\theta = 2.00/6.00$$

$$\theta = \tan^{-1}(2.00/6.00) = 18.4^\circ$$

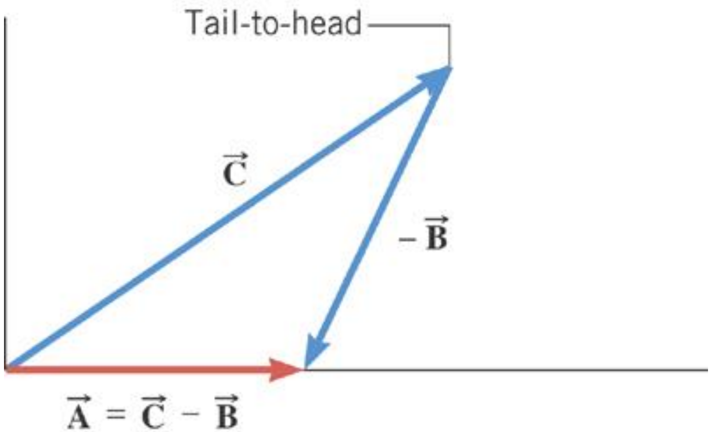


Vector Addition and Subtraction

When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed.



(*a*)



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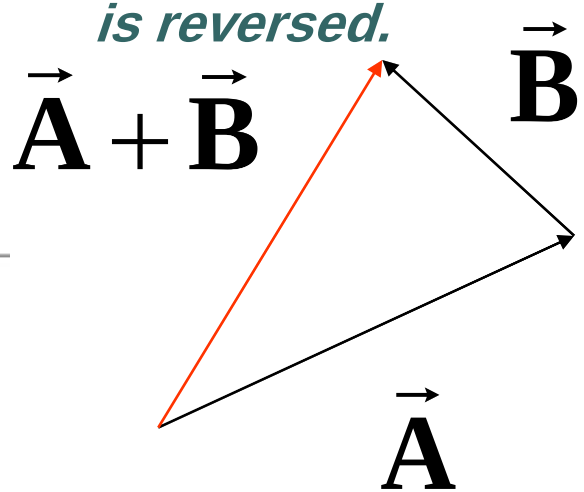
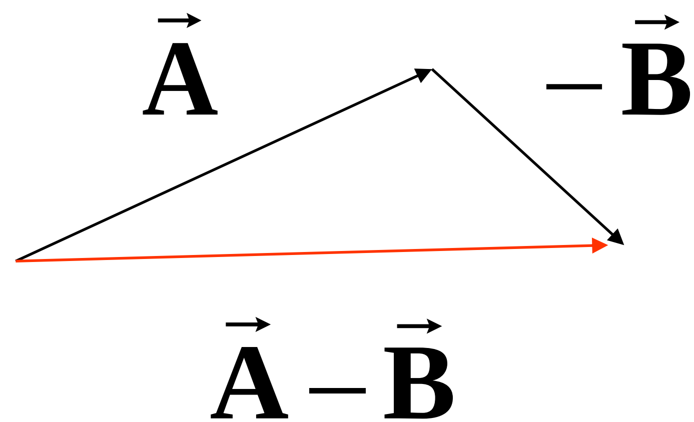
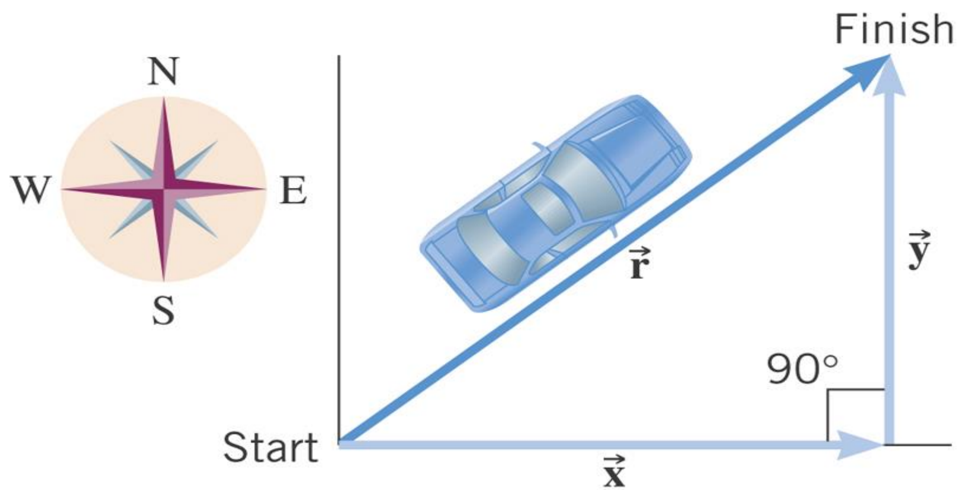


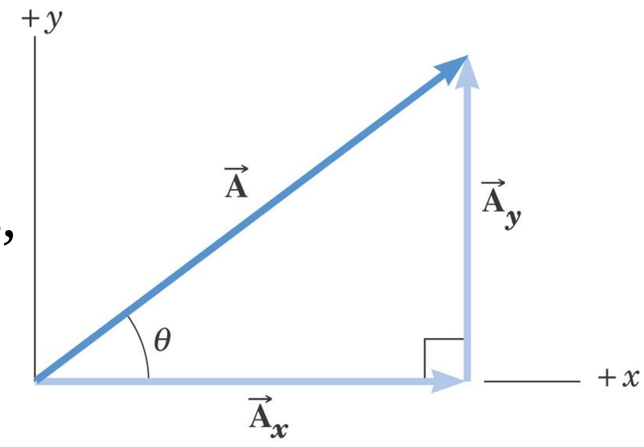


The Components of a Vector



$\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are called the *x* vector component and the *y* vector component of $\vec{\mathbf{r}}$.

The vector components of $\vec{\mathbf{A}}$ are two perpendicular vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ that are parallel to the *x* and *y* axes, and add together vectorially so that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$.



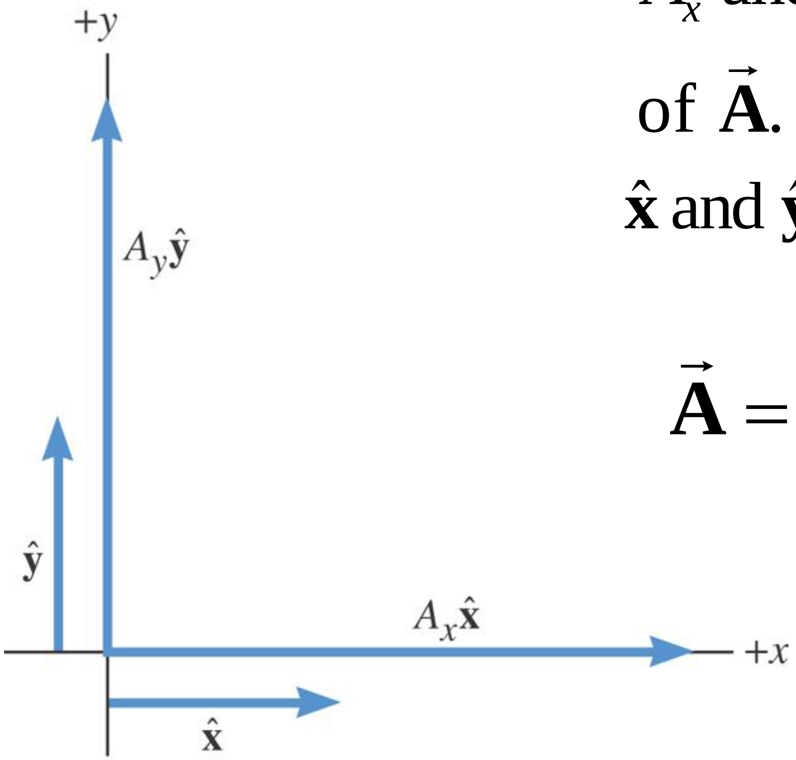
The Components of a Vector

It is often easier to work with the scalar components rather than the vector components.

A_x and A_y are the scalar components of $\vec{\mathbf{A}}$.

$\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors with magnitude 1.

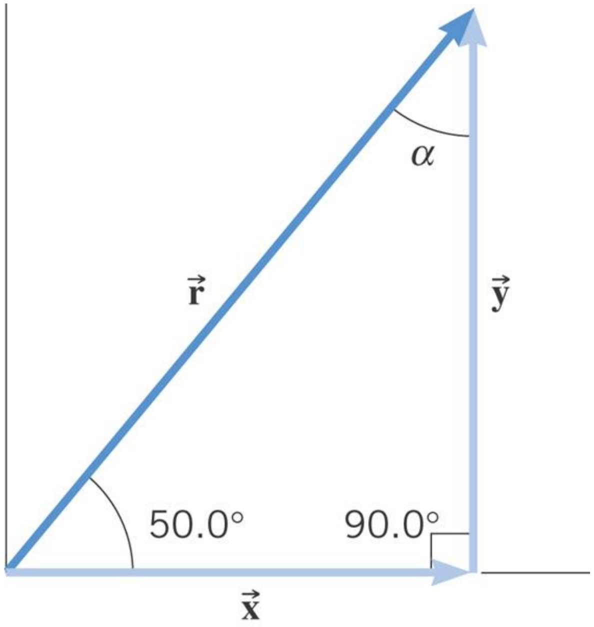
$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$



The Components of a Vector

Example #6

A displacement vector has a magnitude of 175 m and points at an angle of 50.0 degrees relative to the *x* axis. Find the *x* and *y* components of this vector.



$$\sin\theta = y/r$$

$$y = r\sin\theta = (175\,\mathrm{m})(\sin 50.0^\circ) = 134\,\mathrm{m}$$

$$\cos\theta = x/r$$

$$x = r\cos\theta = (175\,\mathrm{m})(\cos 50.0^\circ) = 112\,\mathrm{m}$$

$$\vec{\mathbf{r}} = (112\,\mathrm{m})\hat{\mathbf{x}} + (134\,\mathrm{m})\hat{\mathbf{y}}$$