Predicting Flow in Pipes

A Practical Guide for Engineers

Abstract

This paper documents the development of an Excel-based tool for pipe flow calculations, implemented in three excel worksheet tabs: **Darcy Incompressible**, **Darcy Compressible**, and **Fanno Flow**. The tool provides engineers with practical methods for estimating flow rates and pressure drops in piping systems under varying conditions of fluid compressibility and frictional effects. The **Darcy Incompressible** tab applies the Bernoulli equation with the Darcy–Weisbach formulation. The **Darcy Compressible** tab modifies this approach with the net expansion factor from Crane Technical Paper No. 410. The **Fanno Flow** tab applies an explicit formulation of the adiabatic compressible flow model with friction in constant-area ducts, based on the polytropic approximation described by Kirkland (2019). Guidance is provided on assumptions, limitations, and solution methods (including Excel's Goal Seek function for iteration).

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1. Introduction

Pipe flow modeling is a central problem in chemical, mechanical, and nuclear engineering. Engineers require practical tools to determine flow rates, pressure losses, and design margins in process systems. Depending on the fluid and operating regime, three levels of model fidelity are typically considered:

1. **Incompressible Darcy–Weisbach** – Adequate for liquids and low-Mach-number gases.

2. **Compressible Darcy–Weisbach with Net Expansion Factor** – Used widely in industrial design for moderate pressure drop gas flows.

3. **Fanno Flow (Compressible Flow with Friction)** – The rigorous gas dynamics approach, necessary when compressibility and friction dominate.

The Excel file *Flow Rate in Pipes.xlsm* contains each of these methods implemented in separate tabs, with user inputs, calculated parameters, and comparison outputs. This paper describes each tab, underlying theory, assumptions, and instructions for use.

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2. Darcy Incompressible Model

2.1 Theory

The incompressible tab applies the **Darcy–Weisbach equation** to calculate head loss due to friction:

$$\Delta P = f \frac{L}{D} \cdot \frac{\rho v^2}{2} + \sum K_i \frac{\rho v^2}{2}$$

where ff is the friction factor, L/DL/D the relative pipe length, and ∑Ki\sum K_i the sum of minor losses (valves, elbows, tees, entrances). Flow velocity vv is obtained from continuity:

$$v = \frac{\dot{m}}{\rho A}$$

This approach assumes constant density (valid for liquids or gases at ΔP/P<0.1\Delta P/P < 0.1).

2.2 Assumptions

- Fluid density constant (incompressible).
- Pipe diameter and cross-sectional area constant.
- Single-phase flow, fully developed.
- K-factors from Crane TP-410 are additive.

2.3 Usage

Users input:

- Pipe size and length

- Fluid properties (density, viscosity)
- Pressure drop or flow rate
- Fitting types and counts (K-factors)

Outputs:

- Flow velocity
- Reynolds number
- Frictional and minor losses
- Mass and volumetric flow rate

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3. Darcy Compressible Model

3.1 Theory

Crane TP-410 extends the Darcy–Weisbach equation to compressible gases by introducing the **net expansion factor Y:**

$$\dot{m} = AY\sqrt{\frac{2\Delta P \rho_1}{K}}$$

where Y accounts for density reduction due to expansion across the system. Y was determined from reference 1.

3.2 Assumptions

- Ideal gas behavior.
- Flow remains subsonic.
- Polytropic expansion approximated via Y.
- Same assumptions for frictional loss coefficients as incompressible model.

3.3 Usage

Users input the same parameters as the incompressible tab plus:

- Gas compressibility factor (if available).
- Ratio of specific heats (γ\gamma).

Outputs:

- Mass flow rate corrected for compressibility.
- Net expansion factor Y.

- Comparison to incompressible estimate.

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4. Fanno Flow Model

4.1 Theory

Fanno flow describes **one-dimensional compressible flow with friction in constant-area ducts**. Kirkland (2019) demonstrated that the Fanno equations can be reformulated using a polytropic approximation, leading to explicit equations for mass flow rate and the net expansion factor.

The governing momentum/continuity equations yield an explicit mass flow rate expression:

$$\dot{m} = \left[\frac{2c^2 P_1 A^2}{(c+1)\, v_1} \times \frac{\left(\frac{P_2}{P_1}\right)^{\frac{c+1}{c}} - 1}{2 \ln\left(\frac{P_2}{P_1}\right) - cK} \right]^{\frac{1}{2}}$$

where c is the polytropic index (between 1 and γ), estimated via energy transfer ratio approximations.

4.2 Assumptions

- Adiabatic flow, no heat exchange.
- Perfect gas with constant γ\gamma.
- Constant pipe diameter.
- Friction modeled via Darcy factor or equivalent K-values.
- Flow remains subsonic unless choked.

4.3 Usage

Inputs:

- Upstream pressure and temperature.
- Downstream pressure (or desired flow rate).
- Pipe/fitting K factors (from Crane TP-410).
- Gas properties (γ, molecular weight).

Outputs:

- Polytropic index c.
- Net expansion factor Y.
- Mass flow rate (kg/s, lbm/hr, etc.).

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5. Iteration with Goal Seek

Some problems require iteration because pressure drop, flow rate, and velocity are interdependent. In Excel, **Goal Seek** can be used:

1. Go to **Data** → **What-If Analysis** → **Goal Seek**.
2. Select the output cell (e.g., calculated pressure drop).
3. Set it equal to the desired target.
4. Select the input cell to vary (e.g., guessed flow rate).
5. Run Goal Seek; Excel iterates until the target condition is met.

This procedure is essential in the **Darcy Compressible** and **Fanno Flow** tabs when solving for flow rate given upstream/downstream pressures.

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6. Comparison of Models

- **Darcy Incompressible** – Best for liquids or gases with negligible density change.

- **Darcy Compressible** – A practical engineering method; widely used due to simplicity and availability of Y factors.

- **Fanno Flow** – Highest fidelity; necessary when pressure ratios are large, flow approaches choked conditions, or regulatory/design standards require rigorous gas dynamics.

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7. References

1. Kirkland, W. M., *A Polytropic Approximation of Compressible Flow in Pipes with Friction*. Oak Ridge National Laboratory, 2019.

2. Crane Co., *Flow of Fluids Through Valves, Fittings, and Pipe*. Technical Paper No. 410, 2013.

3. Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. 1, John Wiley & Sons, 1953.

4. Anderson, J. D., *Modern Compressible Flow with Historical Perspective*, 3rd ed., McGraw-Hill, 2003.