Assignment 1

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1. Theoretical Questions

1.1. Problem #1

$$pixelSize = \frac{1024pixels}{7mm} = 6.836\mu m$$

$$imageSizeResolution(ISR) = \frac{1000\mu m/mm}{2 * 6.836mm} 73.14\ linepairs/mm$$

$$FOV = \frac{7mm * 500m}{35mm} = 100mm$$

$$PMAG = \frac{sensorSize}{FOV}$$

$$OSR = ISR * PMAG = \frac{73.14\ linepairs/mm * 7mm}{100mm} 5.12\ linepairs/mm$$

1.2. Problem #2

$$P_r(r) = -2r + 2$$
$$P_z(z) = 2z$$

Using the histogram equalization transformation,

$$T(r) = \int_0^r p_r(w)dw = \int_0^r (-2w + 2)dw = -r^2 + 2r$$
$$G(z) = \int_0^z p_z(w)dw = \int_0^z (2w)dw = z^2$$

Setting the two transformations equal to each other creates the composite transform

$$z = \sqrt{-r^2 + 2r}$$

which is the transformation function between $P_r(r)$ and $P_z(z)$

1.3. Problem #3

Assuming all operations are point-wise

1.3.1 Addition

For $f(x, y) + g(x, y)$, the histogram of the sum is distributed more towards the higher gray levels (ie 255), when compared to the original histogram of $f(x, y)$. The overall distribution would be relatively similar, just shifted to the right. The gray levels of both inputs are positive, so all of the summations are either increasing, or constant (for the case of $g(x, y) = 0$).

1.3.2 Subtraction

For $f(x,y) - g(x,y)$, the histogram of the difference is distributed more towards the lower gray levels (ie 0), when compared to the original histogram of $f(x,y)$. The overall distribution would be relatively similar, just shifted to the left. The gray levels of both inputs are positive, so the differences will either be less than, or equal to (for the case of $g(x,y) = 0$) of the original image.

1.3.3 Multiplication

For $f(x,y) \times g(x,y)$, the histogram of the product is generally distributed more towards the higher gray-levels, but the spread of the distribution will be larger because multiplication is not a linear operation.

1.3.4 Division

For $f(x,y) \div g(x,y)$, the histogram of the quotient is generally distributed more towards the lower gray-levels, but the spread of the distribution will be smaller because division is not a linear operation.

1.4. Problem #4

1.4.1 Part A

The resulting image from filter A would be noticeably sharper than the result from filter B. This is because the pixels that affect the result are farther away from the central pixel, so more detail is covered by the filter. The result from filter C would be even sharper than the result from the previous two, because all of the neighboring pixels are included in the result. This would make the filter detect more details, but the filter would also more susceptible to noise.

1.4.2 Part B

$$D = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -24 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The result from the 5x5 laplacian filter above would be even sharper than the other three results. The result would also be noisier because the larger filter size makes the result even more susceptible to noise. The larger size means that more detail is detected and emphasized, but the noise affects the resulting image more as well.

1.4.3 Part C

This type of filter emphasizes less detail the smaller the mask size, but the smaller size also reduces the affect of noise on the resulting image. As the mask size increases, the detail increases, but so does the noise.

1.5. Problem #5

1.5.1 Linearity

$$af_1(x,y) + bf_2(x,y) \Leftrightarrow aF_1(u,v) + bF_2(u,v)$$

$$aF_1(u,v) + bF_2(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [af_1(x,y) + bf_2(x,y)] e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}}$$

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} af_1(x,y) e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}} + \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} bf_2(x,y) e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}}$$

$$a \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f_1(x,y) e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}} + b \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f_2(x,y) e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}}$$

$$aF_1(u,v) + bF_2(u,v)$$

1.5.2 Translation - General

$$f(x,y) e^{j2\pi(\frac{u_o x}{M} + \frac{v_o y}{N})} \Leftrightarrow F(u-u_o, v-v_o)$$

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u-u_o, v-v_o) e^{\frac{j2\pi xu}{M}} e^{\frac{j2\pi yv}{N}} e^{\frac{j2\pi xu_o}{M}} e^{\frac{-j2\pi xu_o}{M}} e^{\frac{j2\pi yv_o}{N}} e^{\frac{-j2\pi yv_o}{N}}$$

$$e^{\frac{j2\pi xu_o}{M}} e^{\frac{j2\pi yv_o}{N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u-u_o, v-v_o) e^{\frac{j2\pi x(u-u_o)}{M}} e^{\frac{j2\pi y(v-v_o)}{N}}$$

$$\text{Let} \quad p = u-u_o, q = v-v_o$$

$$e^{j2\pi(\frac{u_o x}{M} + \frac{v_o y}{N})} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(p,q) e^{\frac{j2\pi x(p)}{M}} e^{\frac{j2\pi y(q)}{N}}$$

$$e^{j2\pi(\frac{u_o x}{M} + \frac{v_o y}{N})} f(x,y)$$

$$f(x-x_o, y-y_o) \Leftrightarrow F(u,v) e^{-j2\pi(\frac{ux_o}{M} + \frac{vy_o}{N})}$$

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x-x_o, y-y_o) e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}}$$

$$\text{Let} \quad m = x-x_o, n = y-y_o$$

$$\sum_{x=-x_o}^{M-1-x_o} \sum_{y=-y_o}^{N-1-y_o} f(m,n) e^{\frac{-j2\pi(m+x_o)u}{M}} e^{\frac{-j2\pi(n+y_o)v}{N}}$$

$$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) e^{\frac{-j2\pi mu}{M}} e^{\frac{-j2\pi x_o u}{M}} e^{\frac{-j2\pi nv}{N}} e^{\frac{-j2\pi y_o v}{N}}$$

$$e^{-j2\pi(\frac{x_o u}{M} + \frac{y_o v}{N})} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) e^{\frac{-j2\pi mu}{M}} e^{\frac{-j2\pi nv}{N}}$$

$$F(u,v) e^{-j2\pi(\frac{x_o u}{M} + \frac{y_o v}{N})}$$

1.5.3 Translation - Centered

$$f(x,y)(-1)^{x+y} \Leftrightarrow F(u-\frac{M}{2}, v-\frac{N}{2})$$

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u-\frac{M}{2}, v-\frac{N}{2}) e^{\frac{j2\pi xu}{M}} e^{\frac{j2\pi yv}{N}} e^{\frac{j\pi x(\frac{M}{2})}{M}} e^{\frac{-j\pi x(\frac{M}{2})}{M}} e^{\frac{j2\pi y(\frac{N}{2})}{N}} e^{\frac{-j2\pi y(\frac{N}{2})}{N}}$$

$$e^{j\pi x} e^{j\pi y} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u-\frac{M}{2}, v-\frac{N}{2}) e^{\frac{j\pi x(u-\frac{M}{2})}{M}} e^{\frac{j\pi y(v-\frac{N}{2})}{N}}$$

Let $p = u - \frac{M}{2}, q = v - \frac{N}{2}$

$$e^{j\pi(x+y)} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(p,q) e^{\frac{j2\pi x(p)}{M}} e^{\frac{j2\pi y(q)}{N}}$$

$$(-1)^{x+y} f(x,y)$$

$$f(x-\frac{M}{2}, y-\frac{N}{2}) \Leftrightarrow F(u,v)(-1)^{u+v}$$

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x-\frac{M}{2}, y-\frac{N}{2}) e^{\frac{-j2\pi xu}{M}} e^{\frac{-j2\pi yv}{N}}$$

Let $m = x - \frac{M}{2}, n = y - \frac{N}{2}$

$$\sum_{x=-\frac{M}{2}}^{M-1-\frac{M}{2}} \sum_{y=-\frac{N}{2}}^{N-1-\frac{N}{2}} f(m,n) e^{\frac{-j2\pi(m+\frac{M}{2})u}{M}} e^{\frac{-j2\pi(n+\frac{M}{2})v}{N}}$$

$$\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n) e^{\frac{-j2\pi mu}{M}} e^{-j\pi u} e^{\frac{-j2\pi nv}{N}} e^{-j\pi v}$$

$$e^{-j\pi(u+v)} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n) e^{\frac{-j2\pi mu}{M}} e^{\frac{-j2\pi nv}{N}}$$

Note: $(-1)^x = (-1)^{-x}$ when x is a whole number

$$F(u,v)(-1)^{u+v}$$

2. Programming Questions

2.1. Problem #6

Our histogram must have been implemented fairly well by OpenCV's standard, because our histogram came out looking roughly identical to the result of OpenCV's calcHist() function (see figures 2 and 3).
In addition, our equalized histogram came out looking quite similar (if not identical) to the result of OpenCV's equlizeHist() function (see figures 4 and 5). This tells us that our implementation of the equalization is on par with OpenCV's implementation.

2.2. Problem #7

2.2.1 Part C

The Mean Squared Errors are shown below (See section 2.2.3). We found that we produced a higher mean squared error in Part B than in Part A. Both of the error comparisons in Part B were larger than the comparison found in Part A. Unsurprisingly, the subjective quality of the noisy quantized image is much better than that of the original quantized image, as the original quantized appears very blocky and not smooth. However, it is slightly strange that the quantized image had the least error with the original image.

2.2.2 Part D

Again, the Mean Squared Errors are shown in the figure below (Section 2.2.3). As one can see, the errors are significantly lower for all three images after they were subjected to the lowpass filter. This shows us that the lowpass filter will certainly improve the quality of a quantized or noisy image, since it is essentially more similar to the original image (which holds the highest quality). Corresponding to that, the subjective quality of all three filtered images is noticeably better than each of their respective unfiltered images, which proves the usefulness of smoothing. All of these images are shown in the figures below as well.

2.2.3 Mean Squared Errors

- MSE between image and quantized image: 83.8895
- MSE between image and noisy image: 90.7379
- MSE between noisy image and quantized noisy image: 85.3259
- MSE between original image and quantized filtered image: 59.1023
- MSE between original image and noisy filitered image: 26.8854
- MSE between original image and noisy quantized filtered image: 38.6416

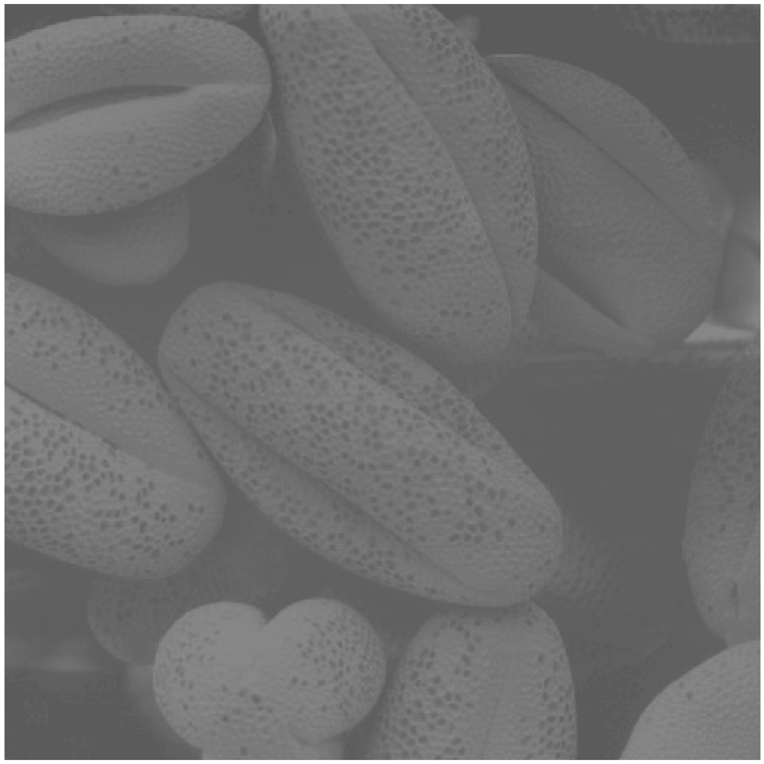

Figure 1. Original Image

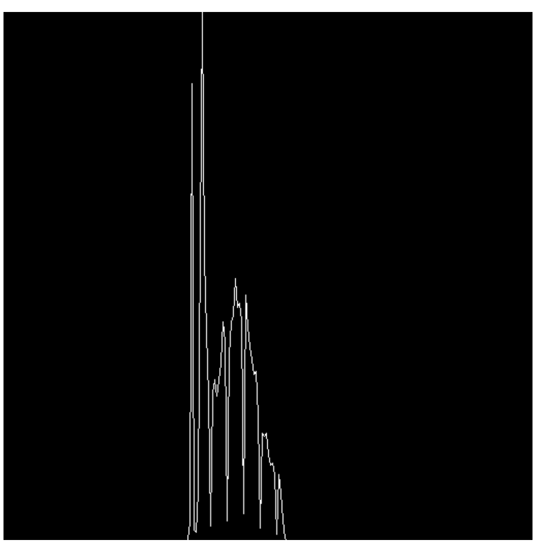

Figure 2. Our Histogram

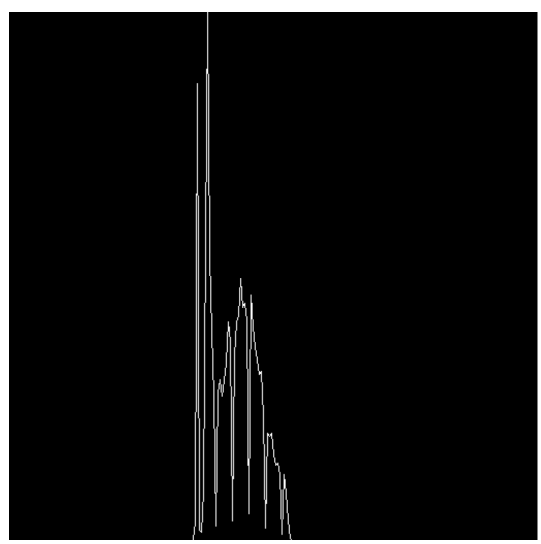

Figure 3. OpenCV's Histogram

Figure 4. Our Equalized Image

Figure 5. OpenCV's Equalized Image

Figure 6. Original Image

Figure 7. Quantized Image

Figure 8. Noisy Image

Figure 9. Noisy Quantized Image

Figure 10. Quantized Image - Filtered

Figure 11. Noisy Image - Filtered

Figure 12. Noisy Quantized Image - Filtered