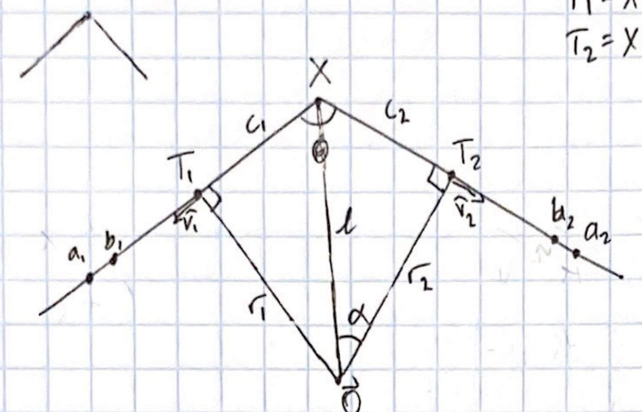

$$T_1 = X + c_1\hat{v}_1 \qquad V_1 = b_1 - a_1$$
$$T_2 = X + c_2\hat{v}_2 \qquad V_2 = b_2 - a_2$$

find X

$$b_1 - t_1V_1 = b_2 - t_2V_2$$
$$t_2V_2 - t_1V_1 = b_2 - b_1$$
$$V = [V_1 \ V_2] \quad V \in \mathbb{R}^{2\times 2}$$
$$t = [-t_1 \ t_2]^T \quad t \in \mathbb{R}^2$$
$$Vt = b_2 - b_1$$

solve with LUsolve() or similar
consistent as long as $\theta \neq 0, \pi$

$$X = b_1 - t_1V_1 = b_2 - t_2V_2$$

$c = |c_1| = |c_2|$, $|r_1| = |r_2| = r$ = given

find c

$$\tan(\theta/2) = \frac{r}{c}$$
$$c = \frac{r}{\tan(\frac{\theta}{2})}$$

$$\hat{v}_1^T\hat{v}_2 = \cos\theta$$
$$\theta = \cos^{-1}(\hat{v}_1^T\hat{v}_2)$$
$$\alpha = \frac{\pi}{2} - \frac{\theta}{2}$$

now find $\vec{O}$. Could do funny equation with sliding r_1 and r_2 until they match, but there's an easier way:

$$l = \left(\tfrac{1}{2}(v_1+v_2)\right)\left(\tfrac{1}{2}|v_1+v_2|\right)^{-1}\sqrt{r^2+c^2}$$
$$\text{let } \bar{V} = \tfrac{1}{2}(v_1+v_2)$$
$$\vec{O} = X + \hat{\bar{V}}\left(\sqrt{r^2+c^2}\right)$$

to interpolate with distance Δx, (approximately)

$$n = \frac{2\alpha r}{\Delta x}$$
$$\Delta\varphi = \frac{2\alpha}{n} = \frac{\Delta x}{r} \quad \text{but no! rounding } n$$
$$\Delta\varphi = \frac{2\alpha}{\text{round}(n)} = 2\alpha\left(\text{round}\left(\frac{2\alpha r}{\Delta x}\right)\right)^{-1}$$

so the arc is defined by

$\vec{O} + r(-\hat{\bar{V}})$ with $-r\hat{\bar{V}}$ rotated from $-\alpha$ to α

$$= \vec{O} + \text{rot}(\varphi)(-r\hat{\bar{V}}), \ \varphi \in [-\alpha, \alpha] \qquad \varphi = \varphi\,\text{sign}(\det(V))$$

since we flip arc direction for L/R turns

$$= \vec{O} + (-r)\begin{bmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{bmatrix}\hat{\bar{V}}, \ \varphi \in [-\alpha, \alpha]$$

now we do the same thing for the straight bits:

$$\begin{cases} t_1b_1 + (1-t_1)(T_1) & t_1 \in [0,1] \\ t_2b_2 + (1-t_2)(T_2) & t_2 \in [0,1] \end{cases}$$

$$\Delta t_1 = \left(|b_1 - T_1|/\Delta x\right)^{-1} = \frac{1}{\text{round}\left(\frac{|b_1-T_1|}{\Delta x}\right)}$$

$$\begin{cases} t_1b_1 + (1-t_1)(X + c_1\hat{v}_1) & t_1 \in [0,1] \\ t_2b_2 + (1-t_2)(X + c\hat{v}_2) & t_2 \in [0,1] \end{cases}$$

$$\Delta t_2 = \left(|b_2 - T_2|/\Delta x\right)^{-1} = \frac{1}{\text{round}\left(\frac{|b_2-T_2|}{\Delta \alpha}\right)}$$