You add these just like they were numbers. Write the first expression as $\frac{x(x-1)}{(x^2+y)(x-1)}$ and the second as $\frac{y(x^2+y)}{(x-1)(x^2+y)}$. Then since these have the same common denominator, you add them as follows.

$$\frac{x}{x^2+y}+\frac{y}{x-1}=\frac{x(x-1)}{(x^2+y)(x-1)}+\frac{y\left(x^2+y\right)}{(x-1)(x^2+y)}=\frac{x^2-x+yx^2+y^2}{(x^2+y)(x-1)}.$$

2.2 Exercises

1. Consider the expression $x+y(x+y)-x(y-x)\equiv f(x,y)$. Find $f(-1,2)$.

2. Show $-(ab)=(-a)\,b$.

3. Show on the number line the effect of multiplying a number by -1.

4. Add the fractions $\frac{x}{x^2-1}+\frac{x-1}{x+1}$.

5. Find a formula for $(x+y)^2, (x+y)^3$, and $(x+y)^4$. Based on what you observe for these, give a formula for $(x+y)^8$.

6. When is it true that $(x+y)^n=x^n+y^n$?

7. Find the error in the following argument. Let $x=y=1$. Then $xy=y^2$ and so $xy-x^2=y^2-x^2$. Therefore, $x(y-x)=(y-x)(y+x)$. Dividing both sides by $(y-x)$ yields $x=x+y$. Now substituting in what these variables equal yields $1=1+1$.

8. Find the error in the following argument. $\sqrt{x^2+1}=x+1$ and so letting $x=2$, $\sqrt{5}=3$. Therefore, $5=9$.

9. Find the error in the following. Let $x=1$ and $y=2$. Then $\frac{1}{3}=\frac{1}{x+y}=\frac{1}{x}+\frac{1}{y}=1+\frac{1}{2}=\frac{3}{2}$. Then cross multiplying, yields $2=9$.

10. Find the error in the following argument. Let $x=3$ and $y=1$. Then $1=3-2=3-(3-1)=x-y(x-y)=(x-y)(x-y)=2^2=4$.

11. Find the error in the following. $\frac{xy+y}{x}=y+y=2y$. Now let $x=2$ and $y=2$ to obtain $3=4$.

12. Show the rational numbers satisfy the field axioms. You may assume the associative, commutative, and distributive laws hold for the integers.

13. Show that for n a positive integer, $\sum_{k=0}^{n}(a+bk)=\sum_{k=0}^{n}(a+b(n-k))$. Explain why

$$2\sum_{k=0}^{n}(a+bk)=\sum_{k=0}^{n}2a+bn=(n+1)(2a+bn)$$

and so $\sum_{k=0}^{n}(a+bk)=(n+1)\frac{a+(a+bn)}{2}$.

2.3 Set Notation

A set is just a collection of things called elements. Often these are also referred to as points in calculus. For example $\{1,2,3,8\}$ would be a set consisting of the elements 1,2,3, and 8. To indicate that 3 is an element of $\{1,2,3,8\}$, it is customary to write $3\in\{1,2,3,8\}$. $9\notin\{1,2,3,8\}$ means 9 is not an element of $\{1,2,3,8\}$. Sometimes a rule specifies a set. For example you could specify a set as all integers larger than 2. This