Chapter 3

Set Theory

3.1 Basic Definitions

This chapter has more on set theory. Recall a set is a collection of things called elements of the set. For example, the set of integers, the collection of signed whole numbers such as 1,2,-4, etc. This set whose existence will be assumed is denoted by $\mathbb{Z}$. Other sets could be the set of people in a family or the set of donuts in a display case at the store. Sometimes parentheses, $\{\ \}$ specify a set by listing the things which are in the set between the parentheses. For example the set of integers between -1 and 2, including these numbers could be denoted as $\{-1, 0, 1, 2\}$. The notation signifying x is an element of a set S, is written as $x \in S$. Thus, $1 \in \{-1, 0, 1, 2, 3\}$. Here are some axioms about sets.

Axiom 3.1.1 *Two sets are equal if and only if they have the same elements.*

Axiom 3.1.2 *To every set, A, and to every condition $S(x)$ there corresponds a set B, whose elements are exactly those elements x of A for which $S(x)$ holds.*

Axiom 3.1.3 *For every collection of sets there exists a set that contains all the elements that belong to at least one set of the given collection.*

Axiom 3.1.4 *The Cartesian product of a nonempty family of nonempty sets is nonempty.*

Axiom 3.1.5 *If A is a set there exists a set $\mathcal{P}(A)$, such that $\mathcal{P}(A)$ is the set of all subsets of A. This is called the power set.*

These axioms are referred to as the axiom of extension, axiom of specification, axiom of unions, axiom of choice, and axiom of powers respectively.

It seems fairly clear you should want to believe in the axiom of extension. It is merely saying, for example, that $\{1, 2, 3\} = \{2, 3, 1\}$ since these two sets have the same elements in them. Similarly, it would seem you should be able to specify a new set from a given set using some "condition" which can be used as a test to determine whether the element in question is in the set. For example, the set of all integers which are multiples of 2. This set could be specified as follows.

$$\{x \in \mathbb{Z} : x = 2y \text{ for some } y \in \mathbb{Z}\}.$$

In this notation, the colon is read as "such that" and in this case the condition is being a multiple of 2.

Another example of political interest, could be the set of all judges who are not judicial activists. I think you can see this last is not a very precise condition since there is no way to determine to everyone's satisfaction whether a given judge is an