Axiom 2.1.6 $(xy)\,z = x\,(yz)\,,$ *(associative law for multiplication).*

Axiom 2.1.7 $1x = x,$ *(multiplicative identity).*

Axiom 2.1.8 *For each* $x \neq 0$*, there exists* x^{-1} *such that* $xx^{-1} = 1$.*(existence of multiplicative inverse).*

Axiom 2.1.9 $x\,(y + z) = xy + xz$.*(distributive law).*

These axioms are known as the field axioms and any set (there are many others besides $\mathbb{R}$) which has two such operations satisfying the above axioms is called a field. Division and subtraction are defined in the usual way by $x - y \equiv x + (-y)$ and $x/y \equiv x\left(y^{-1}\right)$. It is assumed that the reader is completely familiar with these axioms in the sense that he or she can do the usual algebraic manipulations taught in high school and junior high algebra courses. The axioms listed above are just a careful statement of exactly what is necessary to make the usual algebraic manipulations valid. A word of advice regarding division and subtraction is in order here. Whenever you feel a little confused about an algebraic expression which involves division or subtraction, think of division as multiplication by the multiplicative inverse as just indicated and think of subtraction as addition of the additive inverse. Thus, when you see x/y, think $x\left(y^{-1}\right)$ and when you see $x - y$, think $x + (-y)$. In many cases the source of confusion will disappear almost magically. The reason for this is that subtraction and division do not satisfy the associative law. This means there is a natural ambiguity in an expression like $6 - 3 - 4$. Do you mean $(6 - 3) - 4 = -1$ or $6 - (3 - 4) = 6 - (-1) = 7$? It makes a difference doesn't it? However, the so called binary operations of addition and multiplication are associative and so no such confusion will occur. It is conventional to simply do the operations in order of appearance reading from left to right. Thus, if you see $6-3-4$, you would normally interpret it as the first of the above alternatives. This is no problem for English speakers, but what if you grew up speaking Hebrew or Arabic in which you read from right to left?

In the first part of the following theorem, the claim is made that the additive inverse and the multiplicative inverse are unique. This means that for a given number, only one number has the property that it is an additive inverse and that, given a nonzero number, only one number has the property that it is a multiplicative inverse. The significance of this is that if you are wondering if a given number is the additive inverse of a given number, all you have to do is to check and see if it acts like one.

Theorem 2.1.10 *The above axioms imply the following.*

1. *The multiplicative inverse and additive inverses are unique.*
2. $0x = 0,\; -(-x) = x,$
3. $(-1)\,(-1) = 1,\; (-1)\,x = -x$
4. *If* $xy = 0$ *then either* $x = 0$ *or* $y = 0$.

Proof: Suppose then that x is a real number and that $x + y = 0 = x + z$. It is necessary to verify $y = z$. From the above axioms, there exists an additive inverse, $-x$ for x. Therefore,

$$-x + 0 = (-x) + (x + y) = (-x) + (x + z)$$

and so by the associative law for addition,

$$((-x) + x) + y = ((-x) + x) + z$$

which implies $0 + y = 0 + z$. Now by the definition of the additive identity, this implies $y = z$. You should prove the multiplicative inverse is unique.