Consider 2. It is desired to verify $0x = 0$. From the definition of the additive identity and the distributive law it follows that

$$0x = (0+0)\,x = 0x + 0x.$$

From the existence of the additive inverse and the associative law it follows

$$\begin{aligned} 0 &= (-0x) + 0x = (-0x) + (0x + 0x) \\ &= ((-0x) + 0x) + 0x = 0 + 0x = 0x \end{aligned}$$

To verify the second claim in 2., it suffices to show x acts like the additive inverse of $-x$ in order to conclude that $-(-x) = x$. This is because it has just been shown that additive inverses are unique. By the definition of additive inverse, $x + (-x) = 0$ and so $x = -(-x)$ as claimed.

To demonstrate 3., $(-1)(1 + (-1)) = (-1)\,0 = 0$ and so using the definition of the multiplicative identity, and the distributive law, $(-1) + (-1)(-1) = 0$. It follows from 1. and 2. that $1 = -(-1) = (-1)(-1)$. To verify $(-1)\,x = -x$, use 2. and the distributive law to write

$$x + (-1)\,x = x\,(1 + (-1)) = x0 = 0.$$

Therefore, by the uniqueness of the additive inverse proved in 1., it follows $(-1)\,x = -x$ as claimed.

To verify 4., suppose $x \neq 0$. Then x^{-1} exists by the axiom about the existence of multiplicative inverses. Therefore, by 2. and the associative law for multiplication,

$$y = \left(x^{-1}x\right) y = x^{-1}\,(xy) = x^{-1}0 = 0.$$

This proves 4. ■

Recall the notion of something raised to an integer power. Thus $y^2 = y \times y$ and $b^{-3} = \frac{1}{b^3}$ etc.

Also, there are a few **conventions** related to the order in which operations are performed. Exponents are always done before multiplication. Thus $xy^2 = x\left(y^2\right)$ and is not equal to $(xy)^2$. Division or multiplication is always done before addition or subtraction. Thus $x - y\,(z + w) = x - [y\,(z + w)]$ and is not equal to $(x - y)(z + w)$. Parentheses are done before anything else. Be very careful of such things since they are a source of mistakes. When you have doubts, insert parentheses to resolve the ambiguities.

Also recall summation notation.

Definition 2.1.11 *Let $x_1, x_2, \cdots, x_m$ be numbers. Then $\sum_{j=1}^{m} x_j \equiv x_1 + x_2 + \cdots + x_m$. Thus this symbol, $\sum_{j=1}^{m} x_j$ means to take all the numbers, $x_1, x_2, \cdots, x_m$ and add them all together. Note the use of the j as a generic variable which takes values from 1 up to m. This notation will be used whenever there are things which can be added, not just numbers.*

As an example of the use of this notation, you should verify the following.

Example 2.1.12 $\sum_{k=1}^{6} (2k + 1) = 48$.

Be sure you understand why $\sum_{k=1}^{m+1} x_k = \sum_{k=1}^{m} x_k + x_{m+1}$. As a slight generalization of this notation, $\sum_{j=k}^{m} x_j \equiv x_k + \cdots + x_m$. It is also possible to change the variable of summation. $\sum_{j=1}^{m} x_j = x_1 + x_2 + \cdots + x_m$ while if r is an integer, the notation requires $\sum_{j=1+r}^{m+r} x_{j-r} = x_1 + x_2 + \cdots + x_m$ and so $\sum_{j=1}^{m} x_j = \sum_{j=1+r}^{m+r} x_{j-r}$.

Summation notation will be used throughout the book whenever it is convenient to do so.

Example 2.1.13 *Add the fractions, $\frac{x}{x^2+y} + \frac{y}{x-1}$.*