Example 2.4.9 *Solve the inequality* $2x + 4 \leq x - 8$

Subtract $2x$ from both sides to yield $4 \leq -x - 8$. Next add 8 to both sides to get $12 \leq -x$. Then multiply both sides by (-1) to obtain $x \leq -12$. Alternatively, subtract x from both sides to get $x+4 \leq -8$. Then subtract 4 from both sides to obtain $x \leq -12$.

Example 2.4.10 *Solve the inequality* $(x + 1)(2x - 3) \geq 0$.

If this is to hold, either both of the factors, $x+1$ and $2x-3$ are nonnegative or they are both non-positive. The first case yields $x + 1 \geq 0$ and $2x - 3 \geq 0$ so $x \geq -1$ and $x \geq \frac{3}{2}$ yielding $x \geq \frac{3}{2}$. The second case yields $x + 1 \leq 0$ and $2x - 3 \leq 0$ which implies $x \leq -1$ and $x \leq \frac{3}{2}$. Therefore, the solution to this inequality is $x \leq -1$ or $x \geq \frac{3}{2}$.

Example 2.4.11 *Solve the inequality* $(x)(x + 2) \geq -4$

Here the problem is to find x such that $x^2 + 2x + 4 \geq 0$. However, $x^2 + 2x + 4 = (x + 1)^2 + 3 \geq 0$ for all x. Therefore, the solution to this problem is all $x \in \mathbb{R}$.

Example 2.4.12 *Solve the inequality* $2x + 4 \leq x - 8$

This is written as $(-\infty, -12]$.

Example 2.4.13 *Solve the inequality* $(x + 1)(2x - 3) \geq 0$.

This was worked earlier and $x \leq -1$ or $x \geq \frac{3}{2}$ was the answer. In terms of set notation this is denoted by $(-\infty, -1] \cup [\frac{3}{2}, \infty)$.

Example 2.4.14 *Solve the equation* $|x - 1| = 2$

This will be true when $x - 1 = 2$ or when $x - 1 = -2$. Therefore, there are two solutions to this problem, $x = 3$ or $x = -1$.

Example 2.4.15 *Solve the inequality* $|2x - 1| < 2$

From the number line, it is necessary to have $2x - 1$ between -2 and 2 because the inequality says that the distance from $2x - 1$ to 0 is less than 2. Therefore, $-2 < 2x - 1 < 2$ and so $-1/2 < x < 3/2$. In other words, $-1/2 < x$ and $x < 3/2$.

Example 2.4.16 *Solve the inequality* $|2x - 1| > 2$.

This happens if $2x - 1 > 2$ or if $2x - 1 < -2$. Thus the solution is $x > 3/2$ or $x < -1/2$. Written in terms of intervals this is $\left(\frac{3}{2}, \infty\right) \cup \left(-\infty, -\frac{1}{2}\right)$.

Example 2.4.17 *Solve* $|x + 1| = |2x - 2|$

There are two ways this can happen. It could be the case that $x + 1 = 2x - 2$ in which case $x = 3$ or alternatively, $x + 1 = 2 - 2x$ in which case $x = 1/3$.

Example 2.4.18 *Solve* $|x + 1| \leq |2x - 2|$

In order to keep track of what is happening, it is a very good idea to graph the two relations, $y = |x + 1|$ and $y = |2x - 2|$ on the same set of coordinate axes. This is not a hard job. $|x + 1| = x + 1$ when $x > -1$ and $|x + 1| = -1 - x$ when $x \leq -1$. Therefore, it is not hard to draw its graph. Similar considerations apply to the other relation. Functions and their graphs are discussed formally later but I assume the reader has seen these things. The result is