would be written as $S = \{x \in \mathbb{Z} : x > 2\}$. This notation says: the set of all integers, x, such that $x > 2$.

If A and B are sets with the property that every element of A is an element of B, then A is a subset of B. For example, $\{1, 2, 3, 8\}$ is a subset of $\{1, 2, 3, 4, 5, 8\}$, in symbols, $\{1, 2, 3, 8\} \subseteq \{1, 2, 3, 4, 5, 8\}$. The same statement about the two sets may also be written as $\{1, 2, 3, 4, 5, 8\} \supseteq \{1, 2, 3, 8\}$.

The union of two sets is the set consisting of everything which is contained in at least one of the sets, A or B. As an example of the union of two sets, $\{1, 2, 3, 8\} \cup \{3, 4, 7, 8\} = \{1, 2, 3, 4, 7, 8\}$ because these numbers are those which are in at least one of the two sets. In general

$$A \cup B \equiv \{x : x \in A \text{ or } x \in B\}.$$

Be sure you understand that something which is in both A and B is in the union. It is not an exclusive or.

The intersection of two sets, A and B consists of everything which is in both of the sets. Thus $\{1, 2, 3, 8\} \cap \{3, 4, 7, 8\} = \{3, 8\}$ because 3 and 8 are those elements the two sets have in common. In general,

$$A \cap B \equiv \{x : x \in A \text{ and } x \in B\}.$$

When with real numbers, $[a, b]$ denotes the set of real numbers x, such that $a \leq x \leq b$ and $[a, b)$ denotes the set of real numbers such that $a \leq x < b$. (a, b) consists of the set of real numbers, x such that $a < x < b$ and $(a, b]$ indicates the set of numbers, x such that $a < x \leq b$. $[a, \infty)$ means the set of all numbers, x such that $x \geq a$ and $(-\infty, a]$ means the set of all real numbers which are less than or equal to a. These sorts of sets of real numbers are called intervals. The two points, a and b are called endpoints of the interval. Other intervals such as $(-\infty, b)$ are defined by analogy to what was just explained. In general, the curved parenthesis indicates the end point it sits next to is not included while the square parenthesis indicates this end point is included. The reason that there will always be a curved parenthesis next to ∞ or $-\infty$ is that these are not real numbers. Therefore, they cannot be included in any set of real numbers. It is assumed that the reader is already familiar with order which is discussed in the next section more carefully. The emphasis here is on the geometric significance of these intervals. That is $[a, b)$ consists of all points of the number line which are to the right of a possibly equaling a and to the left of b. In the above description, I have used the usual description of this set in terms of order.

A special set which needs to be given a name is the empty set also called the null set, denoted by $\emptyset$. Thus $\emptyset$ is defined as the set which has no elements in it. Mathematicians like to say the empty set is a subset of every set. The reason they say this is that if it were not so, there would have to exist a set A, such that $\emptyset$ has something in it which is not in A. However, $\emptyset$ has nothing in it and so the least intellectual discomfort is achieved by saying $\emptyset \subseteq A$.

If A and B are two sets, $A \setminus B$ denotes the set of things which are in A but not in B. Thus

$$A \setminus B \equiv \{x \in A : x \notin B\}.$$

Set notation is used whenever convenient.

2.4 Order

The real numbers also have an order defined on them. This order may be defined by reference to the positive real numbers, those to the right of 0 on the number line, denoted by $\mathbb{R}^+$ which is assumed to satisfy the following axioms.

Axiom 2.4.1 *The sum of two positive real numbers is positive.*