Chapter 1

Introduction

The difference between advanced calculus and calculus is that all the theorems are proved completely and the role of plane geometry is minimized. Instead, the notion of completeness is of preeminent importance. Formal manipulations are of no significance at all unless they aid in showing something significant. Routine skills involving elementary functions and integration techniques are supposed to be mastered and have no place in advanced calculus which deals with the fundamental issues related to existence and meaning. This is a subject which places calculus as part of mathematics and involves proofs and definitions, not algorithms and busy work. Roughly speaking, it is nineteenth century calculus rather than eighteenth century calculus.

An orderly development of the elementary functions is included but I assume the reader is familiar enough with these functions to use them in problems which illustrate some of the ideas presented. I have placed the construction of the real numbers at the end in an appendix to conform with the historical development of analysis. Completeness of the real line was used as an axiom and all the classical major theorems proved long before Dedekind and Cantor showed how to construct the real numbers from the rational numbers. However, this could be presented earlier if desired.