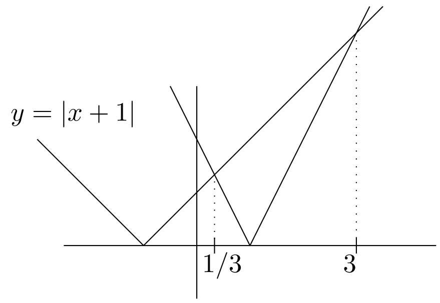

Equality holds exactly when $x = 3$ or $x = \frac{1}{3}$ as in the preceding example. Consider x between $\frac{1}{3}$ and 3. You can see these values of x do not solve the inequality. For example $x = 1$ does not work. Therefore, $\left(\frac{1}{3}, 3\right)$ must be excluded. The values of x larger than 3 do not produce equality so either $|x+1| < |2x-2|$ for these points or $|2x-2| < |x+1|$ for these points. Checking examples, you see the first of the two cases is the one which holds. Therefore, $[3, \infty)$ is included. Similar reasoning obtains $(-\infty, \frac{1}{3}]$. It follows the solution set to this inequality is $(-\infty, \frac{1}{3}] \cup [3, \infty)$.

Example 2.4.19 *Suppose $\varepsilon > 0$ is a given positive number. Obtain a number, $\delta > 0$, such that if $|x-1| < \delta$, then $\left|x^2 - 1\right| < \varepsilon$.*

First of all, note $\left|x^2 - 1\right| = |x-1| \, |x+1| \leq (|x| + 1) \, |x-1|$. Now if $|x-1| < 1$, it follows $|x| < 2$ and so for $|x-1| < 1$, $\left|x^2 - 1\right| < 3 \, |x-1|$.Now let $\delta = \min\left(1, \frac{\varepsilon}{3}\right)$. This notation means to take the minimum of the two numbers, 1 and $\frac{\varepsilon}{3}$. Then if $|x-1| < \delta$, $\left|x^2 - 1\right| < 3 \, |x-1| < 3\frac{\varepsilon}{3} = \varepsilon$.

2.5 Exercises

1. Solve $(3x+2)(x-3) \leq 0$.
2. Solve $(3x+2)(x-3) > 0$.
3. Solve $\frac{x+2}{3x-2} < 0$.
4. Solve$\frac{x+1}{x+3} < 1$.
5. Solve $(x-1)(2x+1) \leq 2$.
6. Solve $(x-1)(2x+1) > 2$.
7. Solve $x^2 - 2x \leq 0$.
8. Solve $(x+2)(x-2)^2 \leq 0$.
9. Solve $\frac{3x-4}{x^2+2x+2} \geq 0$.
10. Solve $\frac{3x+9}{x^2+2x+1} \geq 1$.
11. Solve $\frac{x^2+2x+1}{3x+7} < 1$.
12. Solve $|x+1| = |2x-3|$.
13. Solve $|3x+1| < 8$. Give your answer in terms of intervals on the real line.
14. Sketch on the number line the solution to the inequality $|x-3| > 2$.
15. Sketch on the number line the solution to the inequality $|x-3| < 2$.
16. Show $|x| = \sqrt{x^2}$.
17. Solve $|x+2| < |3x-3|$.
18. Tell when equality holds in the triangle inequality.
19. Solve $|x+2| \leq 8 + |2x-4|$.
20. Solve $(x+1)(2x-2)\,x \geq 0$.
21. Solve $\frac{x+3}{2x+1} > 1$.
22. Solve $\frac{x+2}{3x+1} > 2$.
23. Describe the set of numbers, a such that there is no solution to $|x+1| = 4 - |x+a|$.
24. Suppose $0 < a < b$. Show $a^{-1} > b^{-1}$.
25. Show that if $|x-6| < 1$, then $|x| < 7$.
26. Suppose $|x-8| < 2$. How large can $|x-5|$ be?