Chapter 2

The Real and Complex Numbers

2.1 Real and Rational Numbers

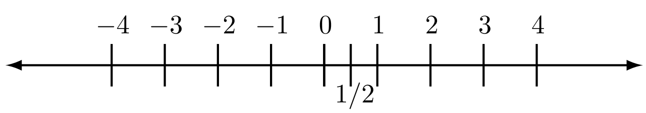
To begin with, consider the real numbers, denoted by $\mathbb{R}$, as a line extending infinitely far in both directions. In this book, the notation, $\equiv$ indicates something is being defined. Thus the integers are defined as

$$\mathbb{Z} \equiv \{\cdots - 1, 0, 1, \cdots\},$$

the natural numbers, $\mathbb{N} \equiv \{1, 2, \cdots\}$ and the rational numbers, defined as the numbers which are the quotient of two integers.

$$\mathbb{Q} \equiv \left\{ \frac{m}{n} \text{ such that } m, n \in \mathbb{Z}, n \neq 0 \right\}$$

are each subsets of $\mathbb{R}$ as indicated in the following picture.



As shown in the picture, $\frac{1}{2}$ is half way between the number 0 and the number, 1. By analogy, you can see where to place all the other rational numbers. It is assumed that $\mathbb{R}$ has the following algebra properties, listed here as a collection of assertions called axioms. These properties will not be proved which is why they are called axioms rather than theorems. In general, axioms are statements which are regarded as true. Often these are things which are "self evident" either from experience or from some sort of intuition but this does not have to be the case. We always assume $0 \neq 1$ because if not, you would end up with $x = x1 = x0 = 0$ for all x and we are not interested in such a stupid thing.

Axiom 2.1.1 $x + y = y + x$, *(commutative law for addition)*

Axiom 2.1.2 $x + 0 = x$, *(additive identity).*

Axiom 2.1.3 *For each* $x \in \mathbb{R}$, *there exists* $-x \in \mathbb{R}$ *such that* $x + (-x) = 0$, *(existence of additive inverse).*

Axiom 2.1.4 $(x + y) + z = x + (y + z)$, *(associative law for addition).*

Axiom 2.1.5 $xy = yx$, *(commutative law for multiplication).*