Deterministic models and optimization 2018.

Piotr's Assignment 1: Continuous optimization. Due date: November 20, 2018

Problem 1 Let $x \in \mathbb{R}^n$ and consider the function

$$f(x) = \frac{1}{2}(x - m)^T A(x - m) - \sum_{i=1}^{n} \log(x_i^2),$$

where $m \in \mathbb{R}^n$ is a fixed vector and $A \in \mathbb{S}^n_{++}$ is a fixed positive definite matrix. Implement the gradient descent method for the problem of *maximizing* this function.

Now focus on the two-dimensional case ($n = 2$). Fix $m = (0.5, 0)$. Study convergence of this method depending on $\rho \in (-1, 1)$ in

$$A = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

Try to propose another line search method that overall performs better (e.g. in convergence rate or the number of iterations needed). Make a contour plot with sample paths of both algorithms.

Optimizing functions of this kind has recently become important in Bayesian model selection procedures involving high-dimensional regression problems (non-local priors). In that case n will be much larger, say between 100 and 1000. Choosing random values for m and A study (by simulations) how the performance of the gradient descent method and your proposed algorithm scales with n.