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4442 Assignment 3

March 19th 2020

250914839

1)

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- 1.) Showing the Gaussian kernel is spatially separable convolution.

(Two 1D kernels applied to the image row-wise and column-wise in sequence)

$$h(i, j) = g(u, v) \cdot f(u, v) = \sum_{i,j} g(u, v) f(u-i, v-j)$$

Because of commutative property

$$\textcircled{1} \quad h(i, j) = f(u, v) \cdot g(u, v) = \sum_{i,j} f(i, j) \cdot g(u-i, v-j)$$

and $F(u, v)$ is separable to 1D kernels)
 $f[m, n] = f_1[m] \cdot f_2[n]$

subbing this in to $\textcircled{1}$

$$\begin{aligned} h(i, j) &= \sum_{i,j} f_1[i] \cdot f_2[j] \cdot g(u-i, v-j) \\ &= \sum_j f_2[j] \left(\sum_i f_1[i] \cdot g(u-i, v-j) \right) \end{aligned}$$

This is a convolution of the input and f_1 . Then another convolution with h_2 . This is a convolution of a row-wise and column-wise vector in sequence. And due to associativity, there can be done in either order. Therefore the 2D gaussian kernel is spatially separable.

the Sobel kernel is a 3×3 matrix giving horizontal detection and vertical detection in the case of images.

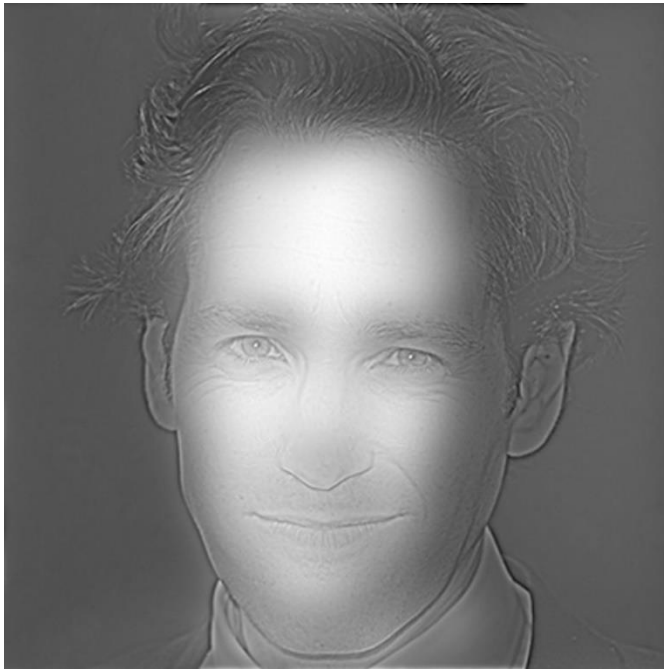
This is a spatially separable convolution as a 3×3 matrix can be separated into a 1×3 and a 3×1 .

Example
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

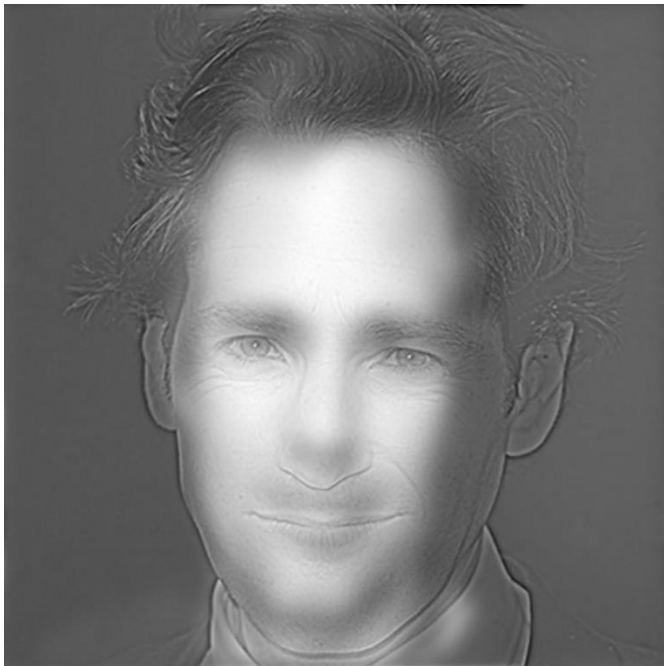
Separable convolutions are preferred as they reduce the number of parameters, and makes each making each convolution operation cheaper reducing memory limitations.

2) Here are a variety of images of varying Sigmas used. The Gaussian kernel was used with Discrete Fourier Transform to obtain the results.

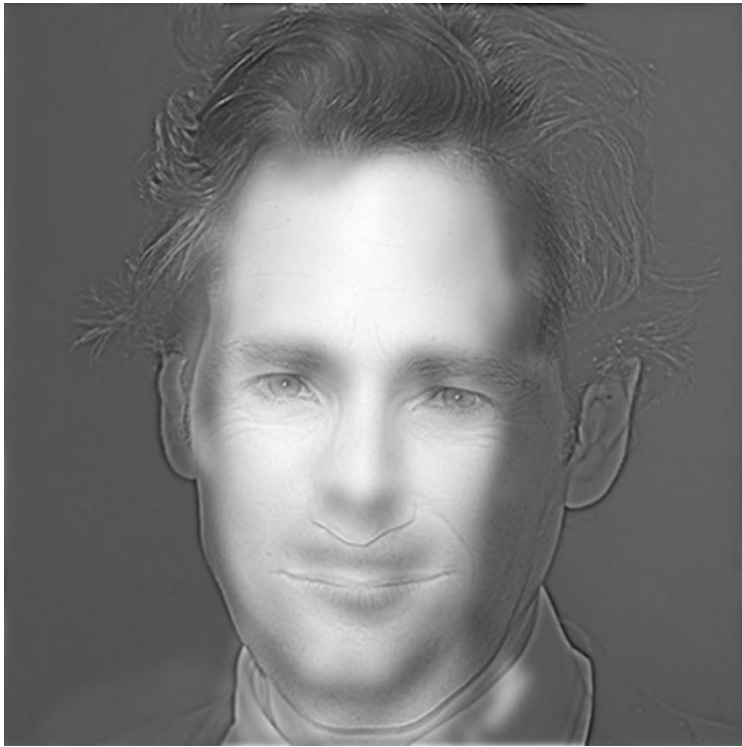
Low Sigma Cut-off: 5 High Sigma Cut-off: 35



Low Sigma Cut-off: 10 High Sigma Cut-off: 35



Low Sigma Cut-off: 15 High Sigma Cut-off: 35



Low Sigma Cut-off: 20 High Sigma Cut-off: 35



Low Sigma Cut-off: 25 High Sigma Cut-off: 35

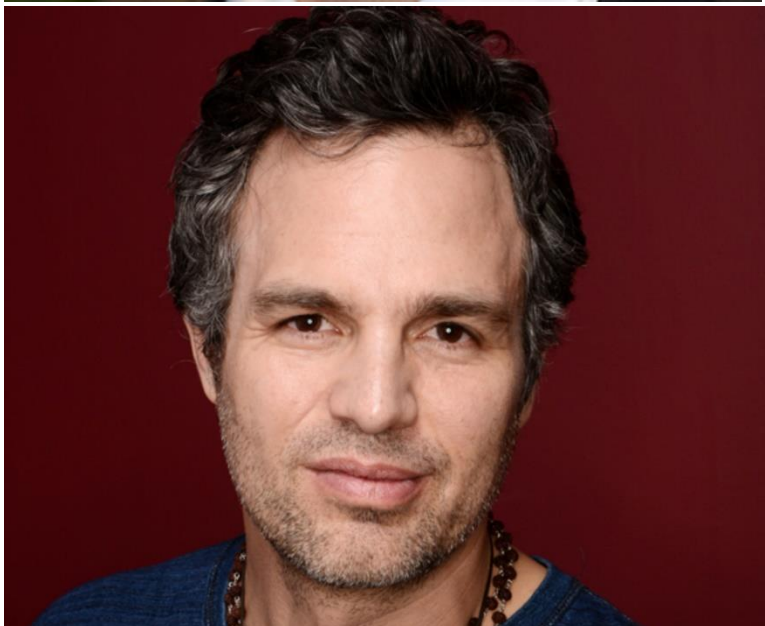
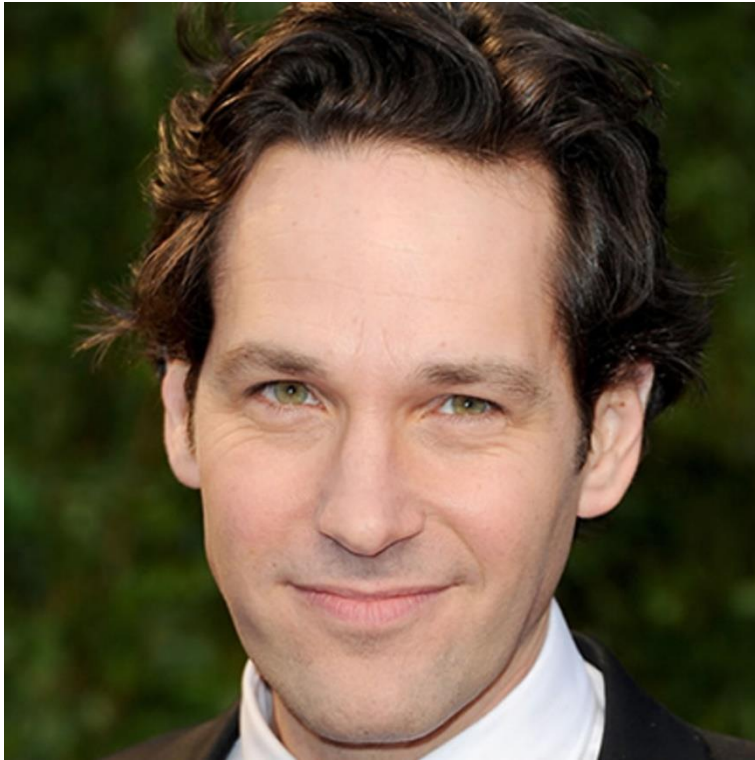


Low Sigma Cut-off: 30 High Sigma Cut-off: 35

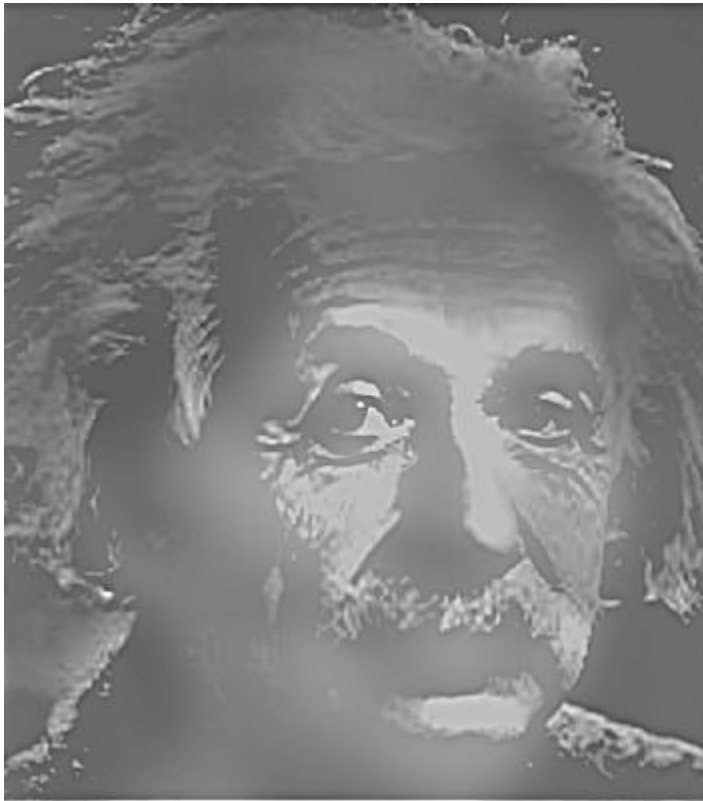


The different low sigma values blur the result differently, with higher low-cut offs making a sharper background image. This however makes the hybrid image transition more difficult, making it harder to see one image over another. The best balance from my values here was the Low Cut-off of 15 and a high of 35 as it was enough blur to see the close-up face of Paul Rudd, and the crisper face of Mark Ruffalo from farther away.

Starting images:



3) From taking the low and high passes of the einsteinandwho image, these are the results



From the high pass and



as the high pass image. The guess I have for the low-pass is Paul McCartney.