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CS4442 Assignment 3

1.) Showing the Gaussian kernel is spatially ~~equival~~ seperable convolution.

(Two 1D kernels applied to the image row-wise and column-wise in sequence)

$$h(i,j) = g[u,v] \circ f[u,v] = \sum_{i,j} g(u,v)\, f(u-i, v-j)$$

Because of commutative property

$$(1) \quad h(i,j) = f[u,v] \circ g[u,v] = \sum_{i,j} f(i,j) \cdot g(u-i, v-j)$$

and $f[u,v]$ is seperable to 1D kernels)

$$f[m,n] = f_1[m] \circ f_2[n]$$

subbing this in to (1)

$$h(i,j) = \sum_{i,j} f_1[i] \circ f_2[j] \cdot g(u-i, v-j)$$

$$= \sum_j f_2[j] \left(\sum_i f_1[i] \cdot g(u-i, v-j) \right)$$

This is a convolution of the input and f_1 then another convolution with h_2. This is a convolution of a row-wise and column wise vector in sequence. And due to associativity, then can be done in either order. Therefore the 2D gaussian kernel is spatially seperable.

the Sobel kernel is a 3x3 matrix giving horizontal detection and vertical detection ~~in~~ the case of images.

This is a spatially seperable convolution as a 3x3 matrix can be seperated into a 1x3 and a 3x1.

Example
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Seperable convolutions are preffered as they reduce the number of parameters, and ~~makes each~~ making each convolution operation cheaper reducing memory limitations.

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