Health Risk and the Value of Life*

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Abstract

We develop a stochastic life-cycle framework for valuing health and longevity improvements and apply it to data on mortality, quality of life, and medical spending for adults with different comorbidities. We estimate that the average value of a quality-adjusted life-year (QALY) rises by $64,000 (22%) following a typical health shock. Among the healthy, prevention of serious health risks is worth up to 17% more per QALY than prevention of mild risks. In contrast, conventional practice assumes that QALYs are equal in value regardless of context. Our results help explain why people value health improvements more in bleaker health states and why they invest less in prevention than in treatment.

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1 Introduction

The economic analysis of risks to life and health has made enormous contributions to academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity (Murphy and Topel, 2006). Economic concepts such as the value of statistical life (VSL) play central roles in discussions surrounding public and private investments in medical care, public safety, environmental hazards, and countless other arenas.

The conventional life-cycle framework assumes deterministic health risk that does not vary with one's health state, which hampers explanatory power in several ways. For example, this framework is ill-equipped to investigate how VSL varies with underlying health and cannot meaningfully distinguish between preventive care and medical treatment. These issues are empirically relevant. Prior research suggests that the value people place on health gains varies considerably with health state (Nord et al., 1995; Ketcham, Kuminoff and Saha, 2020). And, an array of evidence suggests that society invests less in prevention than treatment, even when both have the same consequences for health and longevity (Weisbrod, 1991; Dranove, 1998; Pryor and Volpp, 2018).

We develop a general framework for valuing health improvements that accommodates stochastic health risks and apply it to individual-level data on health and medical spending for the US population over age 50. We estimate that the average value of a quality-adjusted life-year (QALY) rises by \$63,000 (21%) in the year following an adverse health shock. Among healthy 70-year-olds, the value of reducing health risks ranges from \$194,000 per QALY for mild illness to \$227,000 per QALY for the most extreme health risk, death. Our results imply that the willingness to pay for a fixed health improvement is larger when treating sicker individuals and when preventing more serious illnesses.

The first half of this paper presents our theoretical framework. We derive the value of statistical illness (VSI), which captures the willingness to pay to reduce illness risk and includes VSL as a special case where that risk is death. When financial markets are incomplete, the value of reducing a health risk varies with baseline health and risk severity. This novel feature of our model allows us to establish two important results. First, we provide a sufficient condition under which both VSL and VSL per QALY rise following an adverse shock to longevity. This condition, which depends on consumer prudence and the elasticity of intertemporal substitution, holds under a wide range of typical preferences, including isoelastic utility. Intuitively, VSL can rise following an adverse health shock because the decrease in the individual's expected lifespan causes her to spend down her wealth, which increases her willingness to pay for life extension. This result generalizes to all health risks, i.e., VSI per QALY also rises following a health shock, when the consumer's value function is concave in health states. Value function concavity produces aversion to the risk of illness severity, meaning that a consumer prefers to live with mild illness than to live in good health but under the threat of severe illness. Our second theoretical result is that the value of reducing a health risk increases with the severity of the risk if and only if value function concavity holds. In this case, the value of investing in risk-reduction for a serious illness such as cancer is larger per QALY than for a mild illness such as the flu.

While our theoretical framework is fully general, quantifying the value of health improvements requires making assumptions about preferences and consulting data. For example, a quality of life shock can cause VSL to rise or fall, depending on one's assumptions about the effect of health on the marginal utility of consumption, and a large financial shock can offset an increase in VSL that would otherwise have occurred.

The second half of this paper parameterizes our model using standard assumptions and applies it to microsimulation data produced by the Future Elderly Model (FEM). These nationally representative, individual-level data provide detailed information on how mortality, medical spending, and quality of life evolve over the life cycle for people over age 50 with different comorbidities. These data include more comprehensive information than any single national survey and have been extensively validated in prior research (e.g., Leaf et al., 2020).

We first assign individuals from the FEM to one of twenty possible health states based on their number and type of comorbidities, and compute how average health, medical spending, and health state transitions vary by health state and age. We then run a Monte Carlo simulation with 10,000 individuals who are representative of the US population over age 50. Each person's health path evolves at random over the life cycle according to the FEM's estimated transition probabilities, and each health state transition (health shock) is accompanied by the average change in mortality, quality of life, and medical spending as estimated by the FEM.

These 10,000 individuals experience over 25,000 health shocks between the ages of 50 and 80. The average value of a QALY rises by $63,000 following a shock, although there is substantial heterogeneity: about 2% of shocks cause the value of a QALY to increase by over $100,000. Among healthy 70-year-olds, the value of reducing the risk of illness ranges from $194,000 per QALY for a single chronic condition to $227,000 per QALY for the most extreme risk, death. These numerical results imply that value function concavity does indeed hold for most elderly health risks. While the absolute values of our estimates are sensitive to alternative assumptions about the value of the elasticity of intertemporal substitution or the presence of a bequest motive, our qualitative findings—that the value per QALY of reducing a health risk increases with baseline health risk and with the severity of the risk in question—hold up across these alternative parameterizations.

Our primary contribution is the development and application of a new life-cycle model of the value of life. Our results help explain puzzles such as why consumers invest less in prevention than treatment, why end of life spending is high (Zeltzer et al., 2020), and why preventive care interventions frequently fail to deliver results (Jones, Molitor and Reif, 2019), although we do not rule out alternative explanations such as market inefficiencies or hyperbolic discounting that may reinforce these effects (Lawless, Drichoutis and Nayga, 2013). Our finding that the value of health improvements depends on baseline health and risk severity explains consumer opposition to the use of a single QALY value when making decisions about health resource allocation. More generally, our model provides a formal framework for exploring how and whether value-based pricing should reflect concerns about health equity (Linley and Hughes, 2013). Our framework can also be applied to a number of other distinct questions. For example, health researchers could use it to investigate why societies appear to invest less in preventing pandemics than in mitigating them and finance researchers could use it to assess the value of insurance in a stochastic setting (Kowalski, 2015; Lakdawalla, Malani and Reif, 2017; Ericson and Sydnor, 2018). As we discuss in the main text, while we focus on the positive implications of our model, it could also be used to investigate normative questions provided one is willing to take a stance on unsettled questions regarding the welfare economics of risk (Fleurbaey, 2010).

The economic literature on the value of life reaches back to Schelling (1968) and includes seminal studies by Arthur (1981), Rosen (1988), Murphy and Topel (2006), and Hall and Jones (2007). Shepard and Zeckhauser (1984) and Ehrlich (2000) note the important role played by financial markets. Aldy and Smyth (2014) use microsimulation to assess heterogeneity in VSL by race and sex. Córdoba and Ripoll (2016) use Epstein-Zin-Weil preferences to study the implications of state non-separable utility on the value of life when mortality is deterministic. Our stochastic framework accommodates general additively separable preferences, allows for incomplete financial markets, and to our knowledge is the first to provide a life-cycle analysis of the value of preventing illness.[1] Our model also reconciles the standard life-cycle framework with results from a distinct literature that uses one-period models to study the value of mortality risk-reduction

[1] The value of preventing illness has already found application in the empirical literature on mortality risk-reduction (Cameron and DeShazo, 2013; Hummels, Munch and Xiang, 2016).

(Raiffa, 1969; Weinstein, Shepard and Pliskin, 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). These static models predict that an increase in baseline health risk must raise VSL when financial markets are incomplete, a result often referred to as the "dead-anyway" effect. However, this result does not hold—in theory or in practice—in our dynamic setting. Adverse longevity shocks can raise *or* lower VSL in our model, depending on consumer risk preferences. In our empirical exercises, we find that most health shocks reduce VSL.

The remainder of this paper is organized as follows. Section 2 presents the model, derives key results, and provides a discussion of welfare. Section 3 applies the model to data and shows how the value of life-extension varies across consumers with different health histories and how the value of prevention varies within consumers across different potential illnesses. Section 4 concludes.

2 Model

Consider an individual who faces a stochastic health risk such as illness or death. We are interested in analyzing the value of a marginal reduction in that risk. We focus on the value of improvements in longevity, but allow for improvements in quality of life as well.

A simple and transparent setting with no financial markets yields insights substantially similar to a more complex and realistic incomplete markets model. Therefore, we begin with a simple "Robinson Crusoe" economy where the consumer cannot incur debt or purchase annuities to insure against her uncertain longevity. Section 2.1 derives the values of statistical life and illness. Section 2.2 provides conditions under which these values rise following an adverse health shock. Section 2.3 describes how the value of treatment varies with baseline risk and how the value of prevention varies with the severity of illnesses. Section 2.4 extends our Robinson Crusoe model to an incomplete markets setting where the consumer has access to health care insurance and can optimally invest her wealth in a constant annuity. Section 2.5 discusses welfare. Because a complete markets setting lacks both realism and meaningfully new implications, we relegate its analysis to Appendix D.

Like prior studies on the value of life, we focus throughout this paper on the demand for health and longevity. Quantifying optimal health spending requires additionally modeling the supply of health care (Hall and Jones, 2007). In light of all the institutional differences across health care delivery systems, a wide variety of plausible approaches can be taken to this modeling problem, which we leave to future research.

2.1 The value of health and longevity

Let Y_t denote the consumer's health state at time t. We assume Y_t is a continuous-time Markov chain with finite state space $Y = \{1, 2, \ldots, n, n+1\}$, where state $i \in \{1, \ldots, n\}$ represents different possible health states while alive, and state $i = n+1$ represents death. Denote the transition intensities by:

$$\lambda_{ij}(t) = \lim_{h \to 0} \frac{1}{h} \mathbb{P}[Y_{t+h} = j | Y_t = i], \; j \neq i,$$
$$\lambda_{ii}(t) = -\sum_{j \neq i} \lambda_{ij}(t)$$

For analytical convenience and without meaningful loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0 \; \forall j < i$.[2] The probability that a consumer in state i at time 0 remains in state i at time t is then equal to:

$$\tilde{S}(i, t) = \exp\left[-\int_0^t \sum_{j > i} \lambda_{ij}(s)\, ds\right]$$

For expositional purposes we shall refer to transitions as either "falling ill" or "dying," but our model accommodates transitions from sick states to healthy states. We denote the stochastic mortality rate at time t as:

$$\mu(t) = \sum_{i=1}^{n} \lambda_{i,n+1}(t)\, \mathbf{1}\{Y_t = i\}$$

where $\mathbf{1}\{Y_t = i\}$ is an indicator variable equal to 1 if the individual is in state i at time t and 0 otherwise. The maximum lifespan of an individual is T, and we denote her stochastic probability of surviving until $t \leq T$ as:

$$S(t) = \exp\left[-\int_0^t \mu(s)\, ds\right]$$

Let $c(t)$ be consumption at time t, W_0 be baseline wealth, ρ be the rate of time preference, and r be the rate of interest. Quality of life at time t, $q_{Y_t}(t)$, is exogenous and depends on the health state, Y_t. Let the state variable $W(t)$ represent current wealth at time t. Normalizing the utility of death to zero, the consumer's maximization problem for $Y_0 \in$

[2] That is, a person can transition from state i to j, $i < j$, but not vice versa. This restriction does not meaningfully limit the generality of our model because one can always define a new state $k > j$ with properties similar to state i.

$\{1,\ldots,n\}$ is:

$$V(0, W_0, Y_0) = \max_{c(t)} \mathbb{E}\left[\int_0^T e^{-\rho t}\, S(t)\, u(c(t), q_{Y_t}(t))\, dt \,\middle|\, Y_0, W_0 \right] \tag{1}$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) - c(t) \end{aligned}$$

The no-debt constraint, $W(t) \geq 0$, means the consumer cannot borrow against future earnings. The utility function, $u(c,q)$, is time-separable and depends on both consumption and quality of life. We assume throughout that $u(\cdot)$ is strictly increasing and concave in its first argument, and twice continuously differentiable. Hence, we must have $W(T) = 0$, since it cannot be optimal to have wealth remaining at the maximum possible age. We denote the marginal utility of consumption as $u_c(\cdot)$ and assume that this function diverges to positive infinity as consumption approaches zero, so that optimal consumption is always positive.

Define the consumer's objective function at time t as:

$$J(t, W(t), i) = \mathbb{E}\left[\int_0^{T-t} e^{-\rho u} \exp\left\{ -\int_0^u \mu(t+s) ds \right\} u\left(c(t+u), q_{Y_{t+u}}(t+u) \right) du \,\middle|\, Y_t = i, W(t) \right]$$

Define the optimal value function as:

$$V(t, W(t), i) = \max_{c(s),\ s \geq t} \{ J(t, W(t), i) \}$$

subject to the wealth dynamics above and $V(t, W(t), n+1) = 0$. Under conventional regularity conditions, if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} \rho V(t, W(t), i) = \max_{c(t)} \Bigg\{ & u(c(t), q_i(t)) + \frac{\partial V(t, W(t), i)}{\partial W(t)} [rW(t) - c(t)] + \frac{\partial V(t, W(t), i)}{\partial t} \\ & + \sum_{j > i} \lambda_{ij}(t) [V(t, W(t), j) - V(t, W(t), i)] \Bigg\},\ i = 1, \ldots, n \end{aligned} \tag{2}$$

where $c(t) = c(t, W(t), i)$ is the optimal rate of consumption. In order to apply our value of life analysis, we exploit recent advances in the systems and control literature. Parpas and

Webster (2013) show that one can reformulate a stochastic finite-horizon optimization problem as a deterministic problem that takes $V(t, W(t), j)$, $j \neq i$, as exogenous. More precisely, we focus on the path of Y that begins in state i and remains in state i until time T. We denote optimal consumption and wealth in that path by $c_i(t)$ and $W_i(t)$, respectively.[3] A key advantage of this method is that it allows us to apply the standard deterministic Pontryagin maximum principle and derive analytic expressions.

Lemma 1. *Consider the following deterministic optimization problem for $Y_0 = i$ and $W(0) = W_0$:*

$$V(0, W_0, i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right] \tag{3}$$

subject to:

$$\begin{aligned} W_i(0) &= W_0, \\ W_i(t) &\geq 0, \\ \frac{\partial W_i(t)}{\partial t} &= r W_i(t) - c_i(t) \end{aligned}$$

where $V(t, W_i(t), j)$, $j \neq i$, are taken as exogenous. Then the optimal value function, $V(t, W_i(t), i)$, satisfies the HJB equation given by (2), for all $i \in \{1, \ldots, n\}$.

Proof. See Appendix A ■

Because the value function $V(t, W_i(t), i)$ corresponding to (3) satisfies the HJB equation given by (2), it must also be equal to the consumer's optimal value function (Bertsekas, 2005, Proposition 3.2.1). The present value Hamiltonian corresponding to (3) is:

$$H\left(W_i(t), c_i(t), p_t^{(i)}\right) = e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) + p_t^{(i)} \left[r W_i(t) - c_i(t) \right]$$

where $p_t^{(i)}$ is the costate variable for state i. The necessary costate equation is:

$$\dot{p}_t^{(i)} = -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)} r - e^{-\rho t} \tilde{S}(i,t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \tag{4}$$

[3] Consumption, $c(t)$, is a stochastic process. We occasionally denote it as $c(t, W(t), Y_t)$ to emphasize that it depends on the states $(t, W(t), Y_t)$. When we reformulate our stochastic problem as a deterministic problem and focus on a single path $Y_t = i$, consumption is no longer stochastic because there is no uncertainty in the development of health states. We emphasize this point in our notation here by writing consumption as $c_i(t)$, and wealth as $W_i(t)$.

The solution to the costate equation can be obtained using the variation of the constant method:

$$p_t^{(i)} = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant. The necessary first-order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t} \tilde{S}(i,t) u_c(c_i(t), q_i(t)) \tag{5}$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0,W_0,i)}{\partial W_0} = p_0^{(i)} = u_c(c_i(0), q_i(0))$. Since the Hamiltonian is concave in $c_i(t)$ and $W_i(t)$, the necessary conditions for optimality are also sufficient (Seierstad and Sydsaeter, 1977).

To analyze the value of health and longevity, we follow Rosen (1988). Let $\delta_{ij}(t)$ be a perturbation on the transition intensity, $\lambda_{ij}(t)$, $0 \le t \le T$, where $\sum_{j>i} \int_0^T \delta_{ij}(t)\,dt = 1$. The impact on survival is:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[-\int_0^t \sum_{j>i} \left(\lambda_{ij}(s) - \varepsilon \delta_{ij}(s) \right) ds \right], \text{ where } \varepsilon > 0 \tag{6}$$

The marginal value of preventing illness or death in state i is equal to $\frac{\partial V/\partial \varepsilon}{\partial V/\partial W}\Big|_{\varepsilon=0}$, the marginal rate of substitution between longer life in that state and wealth. The next two lemmas provide the two components of this marginal value expression.

Lemma 2. *The marginal utility of preventing illness or death in state i is given by:*

$$\begin{aligned}\frac{\partial V}{\partial \varepsilon}\Big|_{\varepsilon=0} \quad = \quad & \int_0^T e^{-\rho t} \tilde{S}(i,t) \Bigg[\left(\int_0^t \sum_{j>i} \delta_{ij}(s)\,ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) \\ & \qquad - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \Bigg] dt\end{aligned}$$

Proof. See Appendix A ■

Lemma 3. *The marginal utility of wealth in state i at time t is equal to:*

$$\begin{aligned}\frac{\partial V(t, W_i(t), i)}{\partial W_i(t)} &= u_c(c_i(t), q_i(t)) \\ &= \mathbb{E}\left[e^{(r-\rho)(\tau-t)} \exp\left\{ -\int_t^\tau \mu(s) ds \right\} u_c\Big(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)\Big) \Big| Y_t = i, W(t) = W_i(t) \right], \forall \tau > t\end{aligned}$$

Proof. See Appendix A ■

The first equality in Lemma 3 follows immediately from the first-order condition in state i in the HJB (2). Our proof derives the second equality, which shows that the consumer sets the expected discounted marginal utility of consumption at time $\tau > t$ equal to the current marginal utility of wealth. This result is the stochastic analogue of the first-order condition from a conventional (deterministic) model.

Lemma 2 pertains to a marginal reduction in transition intensities for all states and times. Consider as a special case perturbing only the mortality rate in state i, $\lambda_{i,n+1}(t)$. Next, set the perturbation $\delta(\cdot)$ in equation (6) equal to the Dirac delta function, so that the mortality rate is perturbed at $t = 0$ and remains unaffected otherwise (Rosen, 1988). These two simplifications then yield an expression that is commonly called the value of statistical life (VSL).

Proposition 4. *Set $\delta_{ij}(t) = 0\ \forall j < n+1$ in the marginal utility expression given in Lemma 2 and let $\delta_{i,n+1}(t)$ equal the Dirac delta function. Dividing by the current marginal utility of wealth given in Lemma 3 yields:*

$$VSL(i) = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u_c\big(c(0), q_{Y_0}(0)\big)} dt \,\middle|\, Y_0 = i, W(0) = W_0\right] = \frac{V(0, W_0, i)}{u_c(c_i(0), q_i(0))} \tag{7}$$

Applying the second equality given in Lemma 3 and rearranging yields the following, equivalent expression for VSL in state i:

$$VSL(i) = \int_0^T e^{-rt} v(i,t)\, dt$$

where $v(i,t)$ represents the value of a one-period change in survival from the perspective of current time:

$$v(i,t) = \frac{\mathbb{E}\Big[S(t)\ u(c(t), q_{Y_t}(t))\Big| Y_0 = i, W(0) = W_0\Big]}{\mathbb{E}\Big[S(t)\ u_c\big(c(t), q_{Y_t}(t)\big)\Big| Y_0 = i, W(0) = W_0\Big]}$$

Proof. See Appendix A ∎

VSL is the value of a marginal reduction in the risk of death in the current period. It is the amount that a large group of individuals are collectively willing to pay to eliminate a current risk that is expected to kill one of them. Proposition 4 shows that VSL is proportional to expected lifetime utility, and inversely proportional to the marginal utility of consumption.

We can also value a marginal reduction in the risk of falling ill. As before, it is helpful to choose the Dirac delta function for $\delta(t)$, so that the intensities are perturbed at $t = 0$ only. Consider a reduction in the transition intensity for a single alternative state, $j \leq n+1$, so that $\delta_{ik}(t) = 0\ \forall k \neq j$. Applying these two conditions in Lemma 3 then yields what we term the value of statistical illness, $VSI(i,j)$:

$$VSI(i,j) = \frac{V(0,W_0,i) - V(0,W_0,j)}{u_c(c_i(0),q_i(0))} = VSL(i) - VSL(j)\frac{u_c\big(c_j(0),q_j(0)\big)}{u_c(c_i(0),q_i(0))} \tag{8}$$

The interpretation of VSI is analogous to VSL: it is the amount that a large group of individuals are collectively willing to pay in order to eliminate a current disease risk that is expected to befall one of them. Note that if health state j corresponds to death, so that $VSL(j) = VSL(n+1) = 0$, then $VSI(i,j) = VSL(i)$. Thus, VSI is a generalization of VSL.

The values of statistical life and illness depend on how substitutable consumption is at different ages and states. Intuitively, if present consumption is a good substitute for future consumption, then living a longer life is less valuable. Define the elasticity of intertemporal substitution, σ, as:

$$\frac{1}{\sigma} \equiv -\frac{u_{cc}\,c}{u_c}$$

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq}\,q}{u_c}$$

When η is positive, the marginal utility of consumption is higher in healthier states, and vice-versa. Taking logarithms of equation (5), differentiating with respect to t, plugging in the result for the costate equation and its solution, and rearranging yields an expression for the life-cycle profile of consumption:

$$\frac{\dot{c}_i}{c_i} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}_i}{q_i} - \sigma\lambda_{i,n+1}(t) - \sigma\sum_{j=i+1}^{n}\lambda_{ij}(t)\left[1 - \frac{u_c\big(c(t,W_i(t),j),q_j(t)\big)}{u_c(c(t,W_i(t),i),q_i(t))}\right] \tag{9}$$

The first two terms in equation (9) relate the growth rate of consumption to the consumer rate of time preference and life-cycle changes in the quality of life. The third term shows that the rate of change in consumption is a declining function of the individual's current mortality rate, $\lambda_{i,n+1}(t)$. Because the consumer cannot purchase annuities to insure against her uncertain lifetime, higher rates of mortality depress the rate of consumption growth over the life-cycle. Put another way, removing annuity markets "pulls consumption earlier" in the life-cycle (Yaari, 1965).

The fourth term in equation (9)—which is absent in a deterministic setting—accounts for the possibility that the consumer might transition to a different health state in the future. This transition would shift life-cycle consumption earlier still if it pulls the individual to a state with low marginal utility of consumption.

Equation (9) describes consumption dynamics conditional on the individual's health state i. It is not readily apparent from (9) whether modeling health as stochastic causes consumption to shift forward, *on average* across all states, relative to modeling health as deterministic. We confirmed in numerical exercises that modeling health as stochastic has an ambiguous effect on consumption (and VSL), even when holding quality of life constant across states and time.[4]

2.2 Longevity shocks

This section considers the effect of stochastic changes in longevity on VSL. The effect of any accompanying changes in quality of life on VSL depends crucially on the relationship between quality of life and the marginal utility of consumption, a phenomenon often referred to as "health state dependence." Because there is no consensus regarding the sign or magnitude of health state dependence, we hold quality of life constant for the time being and return to this issue in Section 2.3 and in our empirical analysis.[5]

When quality of life is constant, the value of life can increase or decrease following a health state transition, depending on consumer preferences and expectations of future mortality. In this section we isolate the role played by preferences by analyzing a two-state model, where mortality in state 2 is uniformly higher than mortality in state 1. We focus on the case where consumption is declining, as illustrated in Figure 1. Prior empirical work suggests this case is a reasonable description for the typical consumer nearing retirement.[6] In our model, constant quality of life and $r \leq \rho$ are sufficient conditions for

[4]Counterintuitively, modeling health as stochastic has a positive effect on lifetime utility. This positive effect arises because a stochastic environment allows the consumer to react to health shocks by adjusting consumption. Put differently, a deterministic model is equivalent to a stochastic model where the consumer must keep consumption constant across states. Consumers prefer the ability to adjust consumption across states.

[5]Viscusi and Evans (1990), Sloan et al. (1998), and Finkelstein, Luttmer and Notowidigdo (2013) find evidence of negative state dependence. Lillard and Weiss (1997) and Edwards (2008) find evidence of positive state dependence. Evans and Viscusi (1991) find no evidence of state dependence. Murphy and Topel (2006) assume negative state dependence when performing their empirical exercises, while Hall and Jones (2007) assume state independence.

[6]A typical consumption profile is constrained by low income at early ages, increasing during middle ages when income is high, and then declines during retirement until consumption equals the consumer's pension. This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers, 1991; Banks, Blundell and Tanner, 1998; Fernandez-Villaverde and Krueger, 2007).

declining consumption.[7] The next proposition states that consumption increases when transitioning to a state where current and future expected mortality are high.

Proposition 5. *Let there be $n = 2$ states with constant quality of life, so that $q_1(s) = q_2(s) = q\ \forall s$. Assume that $r \leq \rho$, that the transition intensities $\lambda_{12}(s)$ are uniformly bounded (finite), and that the mortality rate is uniformly higher in state 2: $\lambda_{13}(s) < \lambda_{23}(s)\ \forall s$. Suppose the consumer transitions from state 1 to state 2 at time t. Then $c_1(t, W(t), 1) < c_2(t, W(t), 2)$.*

Proof. See Appendix A ■

While the health shock described in Proposition 5 produces a definite rise in current consumption, the shock's effect on VSL is ambiguous. A rise in mortality risk lowers lifetime utility, which all else equal reduces VSL, but it also reduces the marginal utility of consumption, which produces an offsetting increase in VSL. The net effect depends on the curvature of the utility function relative to the curvature of the marginal utility function.

We formally demonstrate this tradeoff by comparing a persistently healthy individual to someone who suffers an adverse shock to life expectancy but is otherwise identical. To make headway we must introduce the notion of prudence. The elasticity of intertemporal substitution, σ, measures utility curvature. Prudence, π, is the analogous measure for the curvature of marginal utility (Kimball, 1990):

$$\pi \equiv -\frac{c\, u_{ccc}(\cdot)}{u_{cc}(\cdot)}$$

It will also be convenient to define the elasticity of the flow utility function:

$$\epsilon \equiv \frac{c\, u_c(\cdot)}{u(\cdot)}$$

The utility elasticity, ϵ, is positive when utility is positive. Positive utility ensures well-behaved preferences, and is often enforced by adding a constant to the utility function. Although adding a constant to the utility function does not affect the solution to the consumer's maximization problem, this constant matters for the value of life.[8]

The following proposition provides sufficient conditions for VSL to rise following an adverse shock to longevity.

[7] From equation (9), $\frac{\dot{c}_i}{c_i} \leq 0$ when $\lambda_{i,n+1} \geq r - \rho + \eta \frac{\dot{q}_i}{q_i} - \sum_{j=i+1}^{n} \lambda_{ij}(t) \left[1 - \frac{u_c(c(t,W_i(t),j),q_j(t))}{u_c(c(t,W_i(t),i),q_i(t))}\right]$. This condition is satisfied when $r \leq \rho$, quality of life is constant, and the consumer can only transition to states with higher mortality.

[8] Rosen (1988) was the first to point out that the level of utility is an important determinant of the value of life. See also additional discussion on this point in Hall and Jones (2007) and Córdoba and Ripoll (2016).

Proposition 6. *Consider a two-state setting with assumptions set out in Proposition 5. Assume that utility is positive and satisfies the condition:*

$$\pi < \frac{2}{\sigma} + \epsilon \tag{10}$$

Suppose that the consumer transitions from state 1 to state 2 at time t, and that $\lambda_{12}(\tau) = 0 \; \forall \tau > t$. Then, $VSL(1,t) < VSL(2,t)$.

Proof. See Appendix A ∎

Proposition 6 shows that the effect of longevity shocks on VSL depends on both prudence and the elasticity of intertemporal substitution. Consumers with inelastic demand for current consumption (low σ) prefer to smooth consumption over time because consumption expenditures at different ages are poor substitutes. They therefore have a high willingness to pay for life-extension and, all else equal, are more likely to exhibit a rise in VSL following an adverse longevity shock than consumers with more elastic demand. Likewise, consumers with low levels of prudence, π, have near-linear marginal utility that decreases rapidly with consumption. This generates a high willingness to pay for life-extension following a shock that increases consumption.

The condition (10) specified in Proposition 6 is satisfied by hyperbolic absolute risk aversion (HARA) utility functions, a class that includes isoelastic and quadratic utility, provided that utility is positive. However, the condition is not innocuous: for example, one can easily find linear combinations of isoelastic and polynomial utility functions where VSL declines following an illness. Nonetheless, prior studies on the value of life generally assume that 0.5 to 0.8 is a reasonable range for the value of σ (Murphy and Topel, 2006; Hall and Jones, 2007), and recent empirical studies suggest that π is about 2 (Noussair, Trautmann and Van de Kuilen, 2013; Christelis et al., 2020). Under these parameterizations, condition (10) will hold whenever utility is positive.

Our finding that VSL could rise or fall following a health shock differs from prior findings, based on static VSL models, that higher baseline mortality risk always increase VSL (Weinstein, Shepard and Pliskin, 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). This discrepancy arises because these prior studies focused on a one-period setting with two states, dead and alive. In that context, if the marginal utility of consumption is lower in the dead state, then an increase in the risk of death must lower the expected marginal utility of consumption and thus raise the willingness to pay for survival (the "dead-anyway" effect).[9] By contrast, Proposition 5 describes how the effect of mortality risk on marginal

[9] Let expected utility be equal to $EU = p\,u(0,c) + (1-p)\,u(1,c)$, where $p \in (0,1)$ is the probability of death

utility plays out in a life-cycle model. In this dynamic context, an increase in the risk of death shifts consumption forward, which does reduce marginal utility. However, unlike in a static setting, the resulting effect on VSL is ambiguous because of an offsetting decrease in lifetime utility.

2.3 The value of a quality-adjusted life-year

The benefits of investments in medical care and safety are frequently measured in units of quality-adjusted life-years (QALY). This widely available metric is helpful when comparing different health improvements because the number of QALYs accounts for differences in survival, reflects the notion that one year of poor health is worth less than one year of good health, and discounts future benefits by the rate of time preference. Let D_i denote the expected number of quality-adjusted life-years in state i:

$$D_i = \mathbb{E}\left[\int_0^T e^{-\rho t} q_{Y_t}(t) S(t) dt \middle| Y_0 = i\right]$$

where $q_{Y_t}(t) \leq 1$ and $q = 1$ indexes perfect health (Drummond et al., 2015). We are interested in analyzing how VSI per QALY, $VSI(i,j)/D_i$, varies across people in different health states i and across different potential diseases j, including death.

We shall assume for the remainder of this Section that utility takes a form consistent with how researchers measure QALYs and quantify VSL (Murphy and Topel, 2006). Specifically, we assume that utility is isoelastic in consumption and multiplicative in the quality of life, and that the consumer receives positive utility only if she consumes an amount above some subsistence level, $\underline{c}$.

Lemma 7. *Let the utility function take the following form:*

$$u(c,q) = q\left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right)$$

where $\underline{c} > 0$ is the level of subsistence consumption and $\gamma = 1/\sigma$ is the inverse of the elasticity of intertemporal substitution. Then the optimal value function is equal to:

$$V(0, W_0, Y_0) = K_{0,Y_0} \frac{W_0^{1-\gamma}}{1-\gamma} - \frac{\underline{c}^{1-\gamma}}{1-\gamma} D_{Y_0}$$

---

and the states $\{0,1\}$ represent death and life, respectively. The willingness to pay for a marginal reduction in the probability of dying is given by $VSL = \frac{u(1,c)-u(0,c)}{p\, u_c(0,c)+(1-p)\, u_c(1,c)}$, which increases with p if $u_c(1,c) > u_c(0,c)$.

where K_{0,Y_0} is the solution (initial value) to a system of ordinary differential equations.

Proof. See Appendix A ■

Consider three different ways to improve one's health: a healthy individual quits smoking to reduce her risk of developing lung cancer (prevention); a healthy individual reduces her risk of dying by wearing a seat belt (ex ante mortality risk reduction); and a lung cancer patient reduces her risk of dying by undergoing chemotherapy (ex post mortality risk reduction). The following proposition states that prevention is worth less per QALY gained than both ex ante and ex post mortality risk reduction, for any illness that increases VSL.

Proposition 8. *Let utility take the form given in Lemma 7. Suppose that transitioning from healthy state i to sick state j reduces quality-adjusted life expectancy and increases VSL, so that $D_i > D_j$ and $VSL(i) < VSL(j)$. Then:*

$$\frac{VSI(i,j)}{D_i - D_j} < \frac{VSL(i)}{D_i} < \frac{VSL(j)}{D_j}$$

Proof. See Appendix A ■

In the context of the motivating example above, Proposition 8 suggests that the health benefits of smoking cessation are worth less per QALY than the life-extension benefits of wearing a seatbelt or of chemotherapy. The rise in VSL following illness is the key assumption required for this result. Because isoleastic utility satisfies condition (10), this assumption is automatically met in any two-state setting with constant quality of life as described in Proposition 5.

We can derive more general results by imposing additional structure on consumer preferences. Define a value function as concave in health states with respect to changes in life expectancy ("value function concavity") if the following inequality holds:

$$V(0, W_0, j) > D \times V(0, W_0, i) + (1 - D) \times V(0, W_0, k), \text{ where } D = \frac{D_j - D_k}{D_i - D_k} \tag{11}$$

Let states i, j, and k correspond to "healthy," "mildly ill," and "severely ill." Value function concavity requires that lifetime utility when mildly ill be larger than the weighted average of the lifetime utilities when healthy or severely ill. This condition will generally be satisfied when there is a clear ordering in the severity of health states, a point which we discuss further in Section 3.3.

The following proposition states that value function concavity is necessary and sufficient for the value of a QALY to rise with the severity of the illness being prevented, and sufficient for the value of a QALY to rise with the severity of one's current health state.

Proposition 9. *The value function given in Lemma 7 is concave in health states, as described by (11), if and only if the value of prevention per QALY increases with illness severity:*

$$\frac{VSI(i,j)}{D_i - D_j} < \frac{VSI(i,k)}{D_i - D_k} \; \forall i,j,k \text{ where } D_i > D_j > D_k$$

In addition, if the value function given in Lemma 7 is concave in health states then the value of a QALY increases with baseline health risk:

$$\frac{VSI(i,k)}{D_i - D_k} < \frac{VSI(j,k)}{D_j - D_k} \; \forall i,j,k \text{ where } D_i > D_j > D_k$$

Proof. See Appendix A ■

Setting state $k = n+1$ (death) in Proposition 9 yields the two inequalities from Proposition 8. While Proposition 9 requires value function concavity, it does not require quality of life to be constant and it allows for an arbitrary number of health states.

Our results contrast with traditional cost-effectiveness analysis, which assumes that QALYs are equally valuable regardless of baseline health risk or illness severity (Drummond et al., 2015, Chapter 5). A constant value for QALYs arises only when the utility of consumption is constant (Bleichrodt and Quiggin, 1999). In Appendix D, we show that constant utility of consumption occurs in our model in the special case where markets are complete, the rate of time preference equals the interest rate, and quality of life is constant.

2.4 Incomplete markets

Our analysis above illustrated our main insights in a setting with no financial markets. This section extends our analysis to a setting with incomplete insurance markets and life-cycle income fluctuations. Except for certain special cases, it is not optimal for the consumer to fully annuitize when annuity markets are incomplete (Davidoff, Brown and Diamond, 2005; Reichling and Smetters, 2015). There continues to be debate over why real-world consumption trajectories and annuity purchase decisions look the way they do. However, the implications for life-extension depend primarily on the consumption trajectory itself, not the reasons that lie beneath.[10]

Let income, m_{Y_t}, be exogenous and equal to:

$$m_{Y_t} = \delta_{Y_t} - \omega_{Y_t} + \pi_{Y_t}$$

Income is equal to labor earnings in health state Y_t, δ_{Y_t}, minus health care spending, ω_{Y_t}, plus health insurance reimbursements, π_{Y_t}. Borrowing an approach from Reichling and Smetters (2015), we assume the consumer has an option at time zero to purchase a flat lifetime annuity that pays out $\overline{a}_{Y_0} \geq 0$ in all health states and that has a price markup of $\xi \geq 0$. The consumer cannot finance the purchase of the annuity using future earnings, and she cannot purchase or sell annuities after time zero. The consumer's maximization problem is:

$$V(0, W_0, Y_0) = \max_{c(t), \overline{a}_{Y_0}} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) \, dt \middle| Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0 - (1+\xi)\overline{a}_{Y_0} \mathbb{E}\left[\int_0^T e^{-rt} S(t) \, dt \middle| Y_0 \right], \\ W(t) &\geq 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) + \overline{a}_{Y_0} - c(t) \end{aligned}$$

The optimal annuity amount is chosen in the consumer's initial state, Y_0, and its net present value may change following a transition to a new health state, because a fixed payout is worth more to a person with higher life expectancy. We emphasize this relationship in our notation below by writing the value function V as a function of the optimally chosen annuity and remaining wealth. In addition, it is helpful to define the value of a one-dollar annuity at time t in state i as:

$$a(T, i) = \mathbb{E}\left[\int_t^T e^{-r(s-t)} \exp\left\{ -\int_t^s \mu(u) \, du \right\} \, ds \middle| Y_t = i \right]$$

Incomplete annuity markets and life-cycle income complicate our analysis by introducing the possibility of multiple sets of non-interior solutions within and across states. (See the

[10] Section 3 uses a numerical model to probe the sensitivity of our results to different assumptions about consumer preferences, such as the presence of a bequest motive, which prior studies have argued might also rationalize low observed rates of annuitization.

right panel in Figure 2 for an example.) For convenience of exposition, we focus on the case where future income is nondecreasing over time and the growth rate of consumption is weakly declining, as illustrated by the left panel in Figure 2. As discussed in Section 2.2, this case is a reasonable description for the typical consumer nearing retirement. We do not take a stance on the reason underlying the negative growth rate in consumption, but we note that it arises in our model under a wide variety of typical parameterizations. Under these conditions, one can derive a simple expression for VSL.

Proposition 10. *Suppose that consumption growth is weakly declining ($\frac{\dot{c}_i}{c_i} \leq 0 \; \forall i$) and that income, $m_i(t)$, is nondecreasing in t. Then VSL in state i when annuity markets are incomplete is equal to:*

$$VSL(i) = \frac{V\left(0, W_i(0), \overline{a}_i, i\right)}{u_c(c_i(0), q_i(0))} - (1+\xi)\overline{a}_i \; a(0,i) \tag{12}$$

Proof. See Appendix A ■

The second term in equation (12)—sometimes referred to as "net savings"—represents the marginal cost to the annuity pool from saving a life (Murphy and Topel, 2006). VSL under incomplete markets captures elements of both the uninsured and fully insured cases. When annuities are absent ($\overline{a}_i = 0$), equation (12) simplifies to the uninsured case given by equation (7). Similarly, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case equation (12) simplifies to the complete markets case given by equation (D.7) in Appendix D.[11]

The value of statistical illness also takes an intermediate form when markets are incomplete.

Corollary 11. *Consider a setting with assumptions set out in Proposition 10. Then the value of a marginal reduction in the risk of transitioning from state i to state j when annuity markets are incomplete is equal to:*

$$\begin{aligned} VSI(i,j) &= \left(\frac{V\left(0, W_i(0), \overline{a}_i, i\right) - V\left(0, W_i(0), \overline{a}_i, j\right)}{u_c(c_i(0), q_i(0))}\right) - \left((1+\xi)\,\overline{a}_i \; a(0,i) - (1+\xi)\,\overline{a}_i \; a(0,j)\right) \\ &= VSL(i) - \left(\frac{V\left(0, W_i(0), \overline{a}_i, j\right)}{u_c(c_i(0), q_i(0))} - (1+\xi)\,\overline{a}_i \; a(0,j)\right) \end{aligned}$$

Proof. See Appendix A ■

The expression for VSI in Corollary 11 is similar to equation (8), but also includes a term that reflects the effect of prevention on the value of the annuity, $\overline{a}_i$. The size of this

[11] Remaining wealth at time $t = 0$, $W(0)$, is zero upon full annuitization. This implies $W_0 = (1+\xi)\overline{a}_i \, a(0,i)$.

annuity may differ from the optimal flat annuity that would have been purchased in state j, and its value depends on life expectancy.

When markets are incomplete, the marginal utility of consumption will generally jump when transitioning from one state to another. One exception is the full annuitization case mentioned above. In this special case, the marginal utility of consumption is constant across states, so falling ill affects VSL only by reducing lifetime utility, with no offsetting effect on marginal utility.

Proposition 12. *Consider a two-state setting with assumptions set out in Proposition 5. Assume further that $\xi = 0$, $r = \rho$, and that future income is constant across both time and states, so that it is optimal for the consumer to fully annuitize. Suppose the consumer transitions from state 1 (healthy) to state 2 (sick) at time t. Then $VSL(1,t) > VSL(2,t)$.*

Proof. See Appendix A ■

From Propositions 6 and 12, it immediately follows that VSL may in general rise or fall following a health shock when markets are incomplete. The sign will depend on the degree of annuitization. For example, full annuitization is optimal in our incomplete markets setting when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case VSL will fall following a health shock. However, when the load, ξ, is sufficiently large then the incomplete markets setting is well-approximated by the uninsured case and Proposition 6 will hold.

2.5 Welfare

This paper studies the willingness to pay for health and longevity. Our framework models how demand for life-extension varies with an individual's health condition and helps explain why individuals invest less in prevention than treatment. Often, however, policymakers must decide how to allocate resources across different people. Who should receive limited supplies of a vaccine against a pandemic? Should a payer with a fixed budget focus resources on the elderly or the young, on the sick or the poor?

In such contexts, economists frequently rely on comparisons of aggregate social surplus, that is, the aggregate sum of willingness to pay. For example, Murphy and Topel (2006) employ this approach in the framework of the standard life-cycle VSL model. Garber and Phelps (1997) rely on it to develop the theory of cost-effectiveness for health interventions. Einav, Finkelstein and Cullen (2010) use it to study the welfare effects of health insurance. Industrial organization economists use it, in the form of deadweight loss comparisons, to evaluate the welfare consequences of market power (Martin, 2019).

While popular among applied economists and policymakers, the aggregate surplus approach has been criticized by welfare theorists for several reasons (Boadway, 1974; Blackorby and Donaldson, 1990). Equity concerns arise because each dollar of surplus is weighted equally, regardless of differences in wealth or income across people. Aggregation can also produce intransitive rankings of alternative allocations. Heterogeneity in marginal utility can break the correspondence between growth in surplus and increases in utility (Martin, 2019). This last point matters little when using willingness to pay to value the prevention of different illnesses, since that can be accomplished from the perspective of a single healthy individual (e.g., Proposition 9), but it suggests caution when making welfare inferences across individuals residing in different health states.

To address these limitations, one may prefer a social welfare approach that aggregates utilities rather than monetized surplus. But debate persists about how to apply this approach under uncertainty, where the ex ante and ex post utilities of a consumer can differ (Fleurbaey, 2010). In a foundational study, Harsanyi (1955) shows that a social welfare function satisfying both rationality and the Pareto principle must be a weighted sum of ex ante individual utilities. However, this utilitarian approach ignores distributional concerns (Diamond, 1967). As a result, one cannot simultaneously satisfy both rationality and the Pareto principle while still pursuing equity. Theorists have argued for abandoning one or the other of these principles. Diamond (1967) advocates minimizing ex ante inequality, but this violates rationality. Adler and Sanchirico (2006) advocate minimizing ex post inequality, but this violates the Pareto principle. In the specific context of VSL, Pratt and Zeckhauser (1996) advocate maximizing ex ante utility, but this ignores equity concerns in light of Diamond's result. We do not aim to resolve this longstanding debate in welfare economics, but instead note that our stochastic model can be incorporated into these different welfare frameworks as desired.

3 Quantitative Analysis

This section quantifies the value of health improvements achieved through prevention or treatment. While our model provided useful insights, some of our theoretical results required either imposing restrictions on the consumer's setting, such as limiting it to two health states, or assuming value function concavity, which cannot be assessed without data. Our quantitative analysis calculates the value of health improvements for a consumer with standard preferences and whose mortality, medical spending, and quality of life can vary across 20 different health states.[12] We calculate both VSI and VSL but focus on their normalized values, VSI per QALY and VSL per QALY, which are more easily compared.

3.1 Framework

We employ a discrete time analogue of the model presented in Section 2. There are n health states (excluding death). Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}[\,Y_{t+1} = j \,|\, Y_t = i\,]$$

The mortality rate at time t, $d(t)$, depends on the individual's health state:

$$d(t) = \sum_{j=1}^{n} \overline{d}_j(t)\,\mathbf{1}\{Y_t = j\}$$

where $\left\{\overline{d}_j(t)\right\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator equal to 1 if the individual is in state j at time t and 0 otherwise.[13] The maximum lifespan of a consumer is T, i.e., $d(T) = 1$. The probability of surviving from time period t to time period $s \le T$ is denoted as $S_t(s)$, where:

$$\begin{aligned} &S_t(t) = 1, \\ &S_t(s) = S_t(s-1)(1 - d(s-1)),\ s > t \end{aligned}$$

The survival probability is stochastic because it depends on the individual's health history. Let $c(t)$ and $W(t)$ denote consumption and wealth in period t, respectively. Quality of life at time t, $q_{Y_t}(t)$, depends on the individual's health state, Y_t. Let ρ denote the rate of time preference, and r the interest rate. Quality-adjusted life expectancy in state i at time t is:

$$D_i(t) = \mathbb{E}\left[\sum_{j=t}^{T} e^{-\rho(j-t)} q_{Y_j}(j)\, S_t(j) \,\middle|\, Y_t = i\right] \tag{13}$$

We assume annuity markets are absent. This simplification allows us to calculate the

[12] Our empirical framework is related to a number of papers that study the savings behavior of the elderly (Kotlikoff, 1988; Palumbo, 1999; De Nardi, French and Jones, 2010). These prior studies allow health to affect wealth accumulation by including two or three different health states in the model.

[13] Our notation here differs slightly from Section 2. Because our mortality data is distinct from our health state transition data, we denote the probability of dying in state i as $\overline{d}_i(t)$ rather than $p_{i,n+1}(t)$.

value of life using an analytical solution to the consumer's problem. It is possible to incorporate partial annuitization in this setting along the lines discussed in Section 2.4. However, generalization requires numerical optimization, which may necessitate limiting the number of health states included in the model. In our sensitivity analysis, we model the effects of a bequest motive and of decreasing the substitutability of consumption over time, both of which—similar to annuitization—reduce consumption at earlier ages.

The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E}\left[\sum_{t=0}^{T} e^{-\rho t} S_0(t)\, u(c(t), q_{Y_t}(t)) \middle| Y_0, W_0\right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t))\, e^{r(t, Y_t)} \end{aligned}$$

The individual's effective interest rate, $r(t, Y_t)$, depends on her health state, Y_t. This dependence allows us to model health shocks that affect income, spending, and wealth. Our baseline model sets $r(t, Y_t) = r + \ln[1 - s(t, Y_t)]$, where r is the rate of interest and $s(t, Y_t)$ is the average share of an individual's wealth spent on medical and nursing home care in state Y_t at time t.[14] Instead of deducting medical costs from wealth directly, we treat them as modifying the interest rate. Although unconventional, this approach achieves our desired change in the life-cycle income profile while preserving the closed-form solution that facilitates our quantitative analysis. In addition, the approach allows us to model health shocks that have proportional effects on wealth. We assume throughout that $r = \rho = 0.03$ (Siegel, 1992; Moore and Viscusi, 1990).

Finally, we follow Section 2.3 and assume that utility takes the following form:

$$u(c, q) = q\left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right) \tag{14}$$

where $q \leq 1$ and $q = 1$ indexes perfect health. Our main specification sets $\gamma = 1.25$ and $\underline{c} = \$5,000$, consistent with the parameterization employed in Murphy and Topel (2006). As discussed previously, there is no consensus regarding the sign or magnitude

[14] We calculate $s(t, Y_t)$ by dividing out-of-pocket spending in health state Y_t at time t by average wealth at time t, as estimated by our model for a healthy individual in a setting with no medical spending. Our results are similar if we instead use wealth estimates from the Health and Retirement Study.

of health state dependence, $u_{cq}(\cdot)$. Here, we assume a multiplicative relationship where the marginal utility of consumption is higher when quality of life is high, and vice versa.

The value function for the consumer's maximization problem is defined as:

$$V(t,w,i) = \max_{c(t)} \mathbb{E}\left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s)\, u(c(s), q_i(s)) \middle| Y_t = i, W(t) = w \right]$$

We reformulate this optimization problem as a recursive Bellman equation:

$$V(t,w,i) = \max_{c(t)} \left[u(c(t), q_i(t)) + \frac{1-\overline{d}_i(t)}{e^{\rho}} \sum_{j=1}^{N} p_{ij}(t)\, V(t+1, (w-c(t))\, e^{r(t,Y_t)}, j) \right]$$

We solve for consumption analytically and then use the formulas derived in Section 2 to calculate the value of life (see Appendix C). We calibrate initial wealth by assuming that VSL for a healthy 50-year-old in health state 1 is \$6 million, which matches the value from Murphy and Topel (2006) and is within the range estimated by empirical studies of VSL for working-age individuals (O'Brien, 2018).

There is significant uncertainty among economists regarding the proper values of many of the parameters in our model. The goal of the subsequent analyses is to illustrate the economic significance of our insights when applying our model to data using reasonable parameterizations. To investigate the sensitivity of our results to the parameterization of our utility function, we estimate specifications with alternative assumptions regarding the elasticity of intertemporal substitution, $1/\gamma$. We also estimate an alternative specification that includes a bequest motive. Rather than setting the utility of death to zero, our bequest motive specification follows Fischer (1973) and sets it equal to $u(W(t+1), b(t))$, where $u(\cdot)$ takes the form given in (14), $W(t+1)$ is wealth at death, and the parameter $b(t)$ governs the strength of the bequest motive. We conservatively set $b(t) = 1.2$, the largest value considered in Fischer (1973), for all t.

3.2 Data

We obtain individual-level data on mortality, disease incidence, quality of life, and medical spending from the Future Elderly Model (FEM), a widely published microsimulation model that combines nationally representative information from the Health and Retirement Study (HRS), the Medical Expenditure Panel Survey (MEPS), the Panel Study of Income Dynamics, and the National Health Interview Survey (see Appendix B). The FEM provides a uniquely rich set of information about the US elderly. For instance, while the

HRS provides detailed data on health and wealth, it lacks survey questions that would allow us to calculate quality of life using standard survey instruments. To solve this problem, the FEM weaves together validated quality of life estimates from the MEPS and maps them to the HRS using variables common to both databases.

The FEM, which has been released into the public domain, produces estimates for individuals ages 50–100 with different comorbid conditions. It accounts for six different chronic conditions (cancer, diabetes, heart disease, hypertension, chronic lung disease, and stroke) and six different impaired activities of daily living (bathing, eating, dressing, walking, getting into or out of bed, and using the toilet). We divide the health space within the FEM into $n = 20$ states. Each state corresponds to the number (0, 1, 2, 3 or more) of impaired activities of daily living (ADL) and the number (0, 1, 2, 3, 4 or more) of chronic conditions, for a total of $4 \times 5 = 20$ health states. Health states are ordered first by number of ADLs and then by number of chronic diseases, so that state 1 corresponds to 0 ADLs and 0 chronic conditions, state 2 corresponds to 0 ADLs and 1 chronic condition, and so on. This aggregation provides a parsimonious way of incorporating information about functional status and several major diseases.[15]

For each health state and age, we calculate average medical spending, the average quality of life, and the annual probabilities of dying and of transitioning to each of the other health states in the next year. We then use those averages as inputs into our life-cycle model. As in the theoretical model, individuals can transition only to higher-numbered states, i.e., $p_{ij}(t) = 0 \ \forall j < i$. In other words, all ADLs and chronic conditions are permanent. Quality of life is measured by the EuroQol five dimensions questionnaire (EQ-5D). These five dimensions are based on five survey questions that elicit the extent of a respondent's problems with mobility, self-care, daily activities, pain, and anxiety/depression. These questions are then combined using weights derived from stated preference data.[16] The result is a single quality of life index, the EQ-5D, which is anchored at 0 (equivalent to death) and 1 (perfect health).

Table 1 reports means from the FEM for ages 50 and 70, by health state. At age 50, life expectancy ranges from 30.9 years to 9.1 years, the average EQ-5D quality of life index ranges from 0.88 to 0.54, and average out-of-pocket medical spending ranges from $686 to $2,759 per year. Columns (10) and (11) report the probability that an individual

[15]While fully interacting all these variables would provide a more granular state space, it would also result in a very large number of possible states and correspondingly small cell sizes within many of them.

[16]The five dimensions of the EQ-5D are weighted using estimates from Shaw, Johnson and Coons (2005). The specific process for estimating the quality of life score is explained in the FEM technical documentation, which can be found in the supplemental appendix of Agus et al. (2016). The methods used to measure the quality of life are consistent with our assumed utility specification, given in (14).

exits her health state but remains alive, i.e., acquires at least one new ADL or chronic condition within the following year. Health states are relatively persistent, with exit rates never exceeding 15 percent. State 20 is an absorbing state with an exit rate of 0 percent.

Figure 3 plots average out-of-pocket medical spending for the healthiest and the sickest health states, by age. These data include all spending on inpatient, outpatient, prescription drug, and long-term care that is not paid for by insurance. Spending is higher in sicker health states, and increases greatly at older ages, when long-term care expenses arise (De Nardi, French and Jones, 2010). The effect of sickness on out-of-pocket spending is modest in comparison to long-term care costs, and the overall gap in spending across states shrinks with age. This shrinkage occurs because out-of-pocket medical expenses are concentrated in the first year of incidence, and their effect on average spending is dampened in health states that include few newly diagnosed individuals.[17]

We estimate our life-cycle model using FEM data for ages 50–100 but focus our discussion below on ages 50–80, where the FEM estimates are more precise and consumption decisions are less affected by our assumption that annuity markets are absent.

3.3 Elderly value of life

We begin with a simple example. The solid red and dashed blue lines in Figure 4 report VSL and consumption for a healthy individual who experiences a mild health shock at age 60, suffers an uncommonly severe health shock at age 70, and then dies at age 75. Each shock produces sudden changes to expected survival, quality of life, and medical spending, as measured in our FEM data.

Consumption increases sharply following both health shocks. These increases are consistent with the result stated in Proposition 5, which was restricted to a setting with two health states, constant quality of life, and no financial shocks. Figure 4 also shows that VSL rises from \$2.5 million to \$2.7 million following the severe health shock at age 70. This rise in VSL is consistent with Proposition 6, which also imposed restrictions on the consumer's setting.[18] However, VSL does not increase following the mild health shock at age 60, in part because the higher willingness to pay for life-extension associated with the consumption increase is offset by the changes in current and expected changes in quality of life and medical spending. The dotted black line in Figure 4 demonstrates directly the role played by financial shocks: VSL would fall at age 70, rather than rise, if

[17] For more details, see the appendix materials in National Academies of Sciences, Engineering, and Medicine (2015).

[18] In addition, Proposition 6 imposed a condition on consumer preferences, which is satisfied by the utility function (14).

the severe health shock reduced wealth by 20 percent instead of reducing it by the more modest amount measured in the FEM data.

As shown in Figure 4, the effect of a health shock on VSL depends on the nature of the shock as well as the individual's health history and wealth. To characterize these effects among the US elderly population more generally, we turn to Monte Carlo simulation. Each simulation begins with a 50-year-old person in perfect health. That person's health path then evolves at random according to the nationally representative health transition probabilities estimated by the FEM. Figure 5 illustrates how the mean, 5th percentile, and 95th percentile of VSL for 10,000 of these individuals vary over the life cycle. Individual-level health shocks generate substantial variability in VSL. At age 50, all individuals are identical and have a VSL of \$6 million. As they age, these individuals follow different health paths. By age 70, the VSL inter-vigintile range spans \$1.5 to \$2.5 million.

These 10,000 people experience over 25,000 health shocks between the ages of 50 and 80. Figure 6a displays the distribution of the change in VSL in the year following each of those shocks. On average, a health shock reduces VSL by \$77,000, but there is much heterogeneity in this effect. About 5 percent of health shocks increase VSL, with some increases exceeding \$100,000. An even larger number of shocks reduce VSL by over \$100,000.

While health shocks reduce average VSL by \$77,000 in absolute terms, Figure 6b shows that VSL per QALY actually increases by \$63,000 because of the accompanying reductions in health and longevity. This effect is consistently positive: fewer than 0.1% of health shocks cause a decline in the value of a QALY. The distribution is skewed to the right, with the value of a QALY rising by over \$100,000 in 1.7% of cases.

The dashed blue line in Figure 7 illustrates how VSL at age 70 varies with quality-adjusted life expectancy across the twenty health states in our model. The positive slope indicates that, on average, VSL rises with life expectancy, consistent with recent work finding that VSL is higher for people in better health states (Ketcham, Kuminoff and Saha, 2020). Because health states are relatively persistent (see Column (9) of Table 1), the averages shown in Figure 7 describe individuals who mostly have not experienced a recent health shock. By contrast, the distribution in Figure 6a described changes in VSL for individuals who had just transitioned to a new health state.[19] Nevertheless, the solid red line in Figure 7 indicates that, on average, VSL per QALY still falls with life

[19]Over a long enough time horizon, an adverse shock to longevity always reduces VSL (relative to no shock) because it causes the individual to spend down her wealth more quickly. For example, in Figure 4 the slopes of both the consumption and VSL trajectories increase in magnitude following the shock at age 70. The dashed blue line in Figure 7 reports estimates for the typical elderly person, who has not experienced a recent health shock.

expectancy.

Finally, we consider the value of prevention. The dashed blue line in Figure 8 reports VSI's for different illnesses, including death, from the perspective of a healthy 70-year-old. Each value represents the healthy individual's willingness to pay for a marginal, contemporaneous reduction in the probability of dying or of transitioning to one of the 19 other health states in our model. The values are inversely related to life expectancy in the sick state because it is more valuable in absolute terms to prevent a severe illness than a mild one. A marginal reduction in the probability of transitioning to the worst health state (2.6 QALYs) is worth about \$1.8 million. This value is the amount that a large number of healthy individuals would collectively be willing to pay to reduce a risk that is expected to cause the onset of this health state for one of them. VSL, which is a special case of VSI where life expectancy is 0 years in the sick state, is \$2.5 million.

The solid red line in Figure 8 reports VSI per QALY. The negative slope indicates that these values increase with the severity of the disease being prevented, consistent with the first inequality stated in Proposition 9. Reducing the risk of death (\$227,000 per QALY) is worth 17% more than reducing the risk of transitioning to health state 2 (\$194,000 per QALY). Some sections of the red line occasionally have positive slopes, which can occur if there is no clear ordering in the severity of different health states. For example, at age 70 life expectancy in state 9 (5.4 QALYs) is lower than in state 17 (5.6 QALYs), but quality of life in state 9 is higher (see Table 1). Nevertheless, the general concordance between the estimates shown in Figure 8 and the first inequality stated in Proposition 9 provides evidence that value function concavity holds for most elderly health risks when consumer preferences take the form (14).[20]

Figure 9 shows how different utility function parameterizations and the presence of a bequest motive affect our estimates. Setting $\gamma = 1.5$, which makes demand for current consumption more inelastic, flattens the life-cycle consumption profile and increases the value of a QALY. Setting $\gamma = 0.8$, by contrast, pulls consumption forward in time and reduces the value of life-extension because consumption at early ages provide a good substitute for consumption at later ages. A bequest motive encourages individuals to delay consumption, because money saved for consumption in old age has the added benefit of increasing bequests in the event of death (Figure 9a). Likewise, it reduces the value of life-extension because death is less costly (Figure 9b).

Overall, while these alternative specifications produce meaningful shifts in the absolute values of VSL/VSI, they do not affect our qualitative conclusions. In all cases, the

[20] It is also worth noting that value function concavity implies the positive values shown in Figure 6b and the negative slope shown in Figure 7 (see the second inequality in Proposition 9).

value of a QALY is larger when treating sicker individuals (Figure 9c) or when preventing more serious illnesses (Figure 9d).

4 Conclusion

The economic theory surrounding the value of life has many important applications. Yet, a number of limitations have surfaced over time. The traditional model does not distinguish between prevention and treatment. It also suffers from several anomalies that appear at odds with intuition or empirical facts, such as the apparent preferences of consumers to pay more for life-extension when survival prospects are bleaker. We overcome these limitations by relaxing the standard assumption about deterministic health.

Our model offers a unified framework for valuing both treatment and prevention. This framework provides a practical tool for policymakers and health agencies, since many health investments involve preventing the deterioration of health rather than reducing an immediate mortality risk. Using nationally representative data, we estimate that the willingness to pay to prevent an illness is less than the willingness to pay to treat the illness, holding fixed the health gains. We also estimate that healthy individuals are willing to pay more per QALY to prevent more lethal illnesses.

These findings provide one explanation for why many people state preferences for giving priority to patients with severe diseases (Nord et al., 1995). The findings also help explain why it has proven so difficult for policymakers and public health advocates to encourage investments in the prevention of disease. Kremer and Snyder (2015) show that heterogeneity in consumer valuations distorts R&D incentives by allowing firms to extract more consumer surplus from treatments than preventives. Our results suggest that differences in private VSL may reinforce this result and further disadvantage incentives to develop preventives.

Our analysis raises a number of important questions for further research. First, how does the theory change if we endogenize the demand for health and longevity (Ehrlich, 2000)? In this setting, medical technology that improves quality of life can act as insurance by compressing the difference in utility between the sick and healthy states (Lakdawalla, Malani and Reif, 2017). Less clear is how demands for the quantity and quality of life interact with financial market incompleteness of various kinds. Second, what are the most practical strategies for incorporating our insights into the literature on cost-effectiveness of alternative medical technologies? This literature typically assumes that quality-adjusted life-years possess a constant value. While flawed, this approach is simpler to implement than allowing the value to depend on health histories. Future research should focus on practical strategies for aligning cost-effectiveness analyses with the generalized theory of the value of life. Finally, what are the implications for the empirical literature on VSL? Prior studies have assumed that health histories can be ignored when estimating VSL (Hirth et al., 2000; Mrozek and Taylor, 2002; Viscusi and Aldy, 2003). Our framework suggests the need for a more nuanced empirical approach. This missing insight may be one reason for the widely disparate empirical estimates of the value of statistical life.

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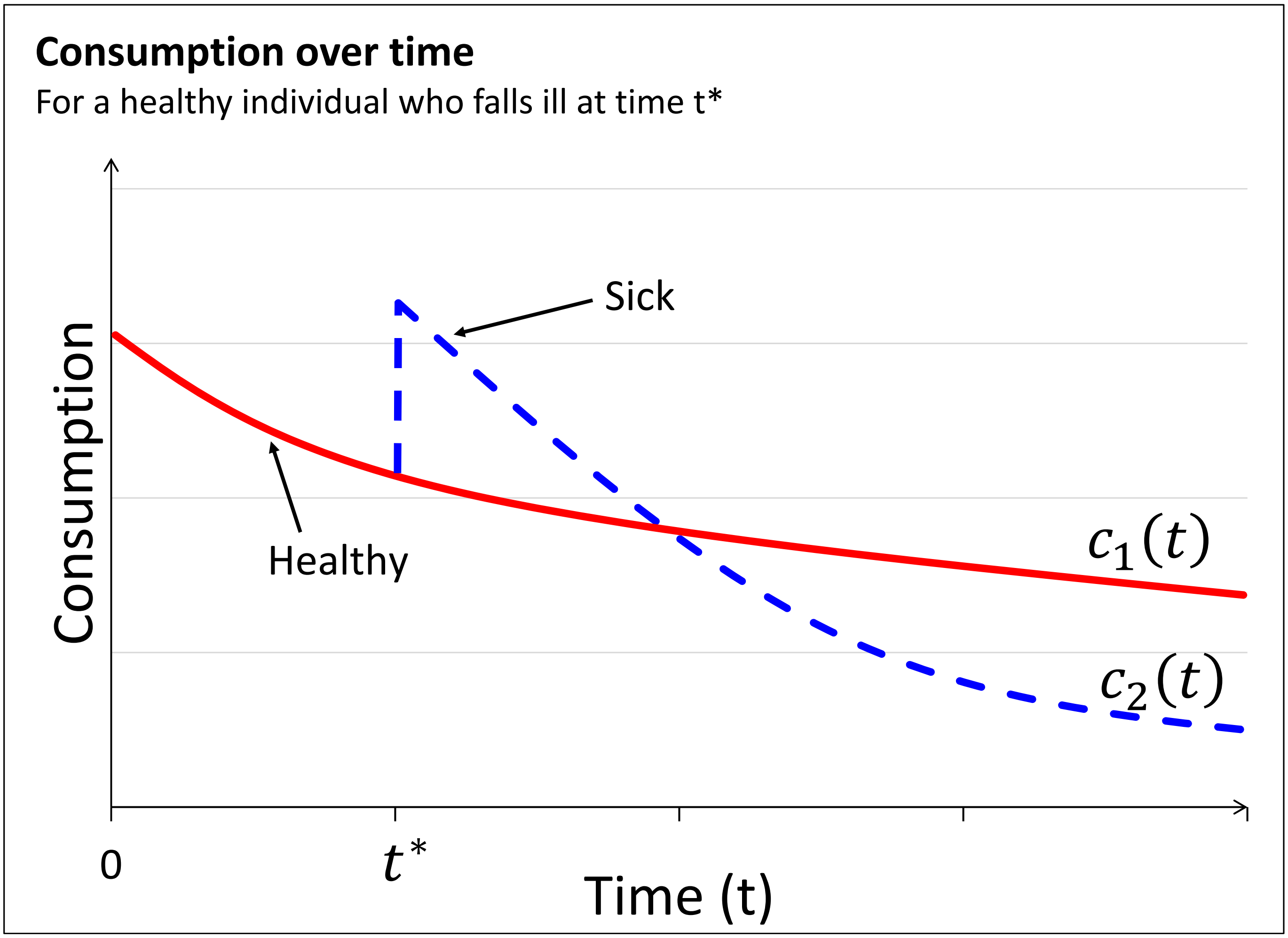
Figure 1: Consumption increases after an adverse shock to life expectancy

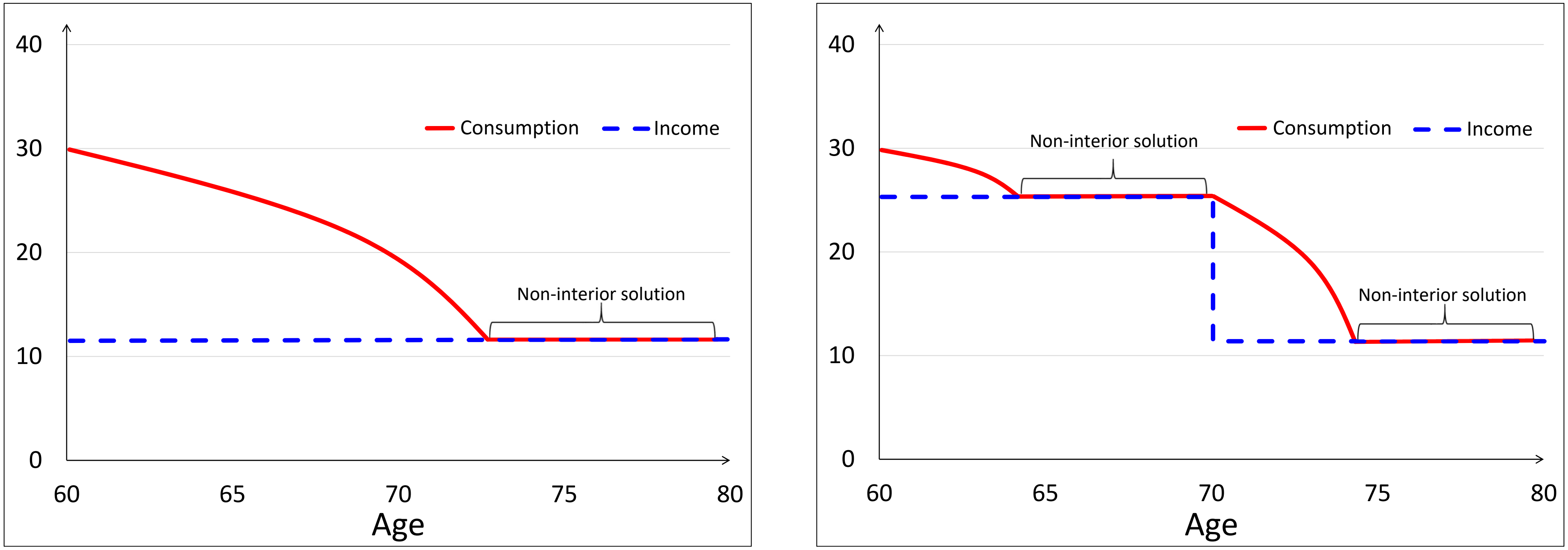


Notes: The solid red line displays a consumption path for a healthy individual who never falls ill. The dashed blue line displays a path for a healthy individual who experiences an adverse shock to life expectancy at time t^*. Proposition 5 provides sufficient conditions under which consumption rises at the transition, i.e., $c_2(t^*) > c_1(t^*)$. Under those conditions, it is optimal for the sick individual (state 2) to consume at a higher rate than the healthy individual (state 1) because she has lower life expectancy. Proposition 6 provides conditions under which VSL in the sick state is higher at time t^*.

Figure 2: Illustrative example: survival-contingent income can generate non-interior solutions

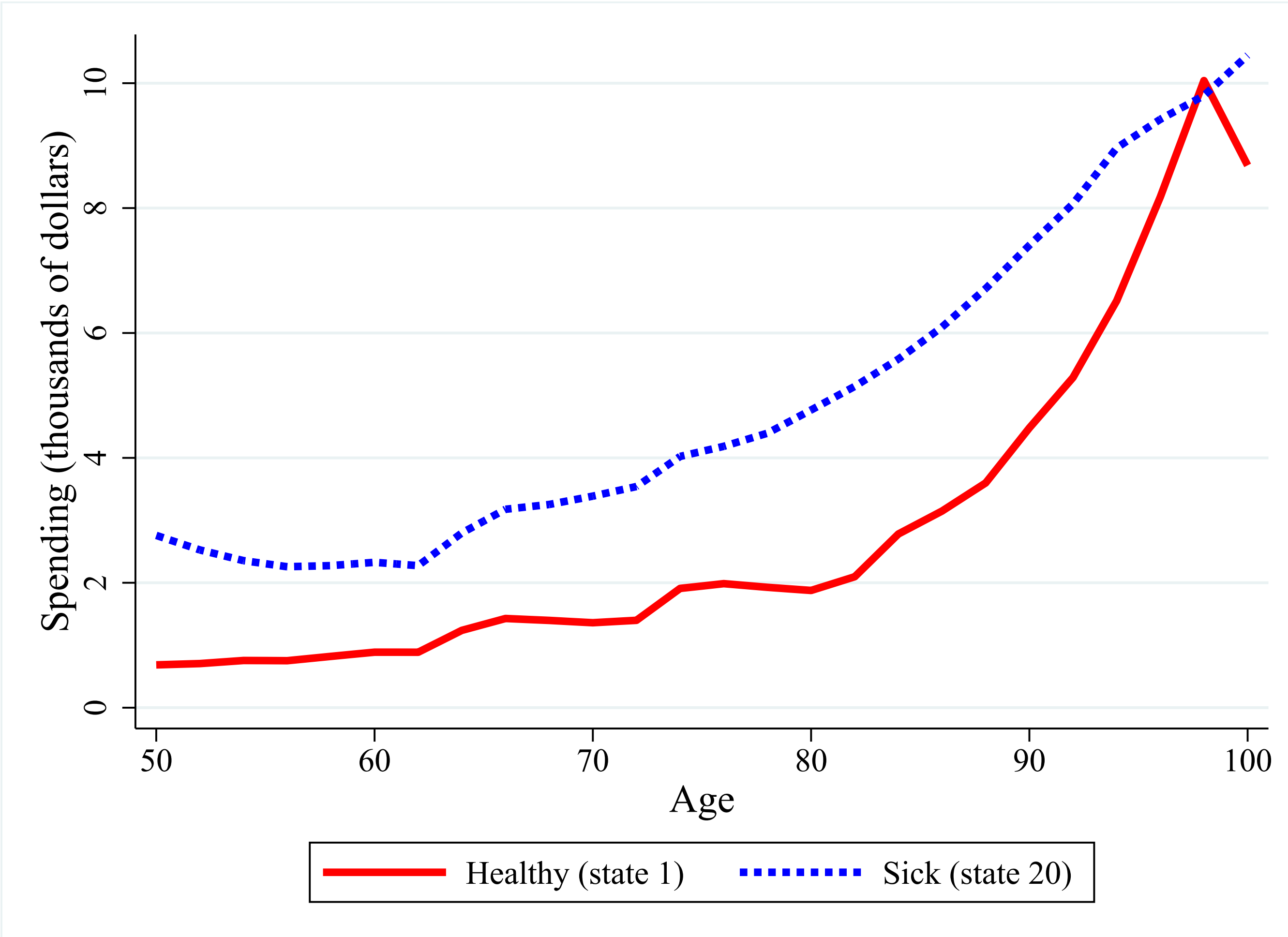
(a) One set of non-interior solutions

(b) Two sets of non-interior solutions



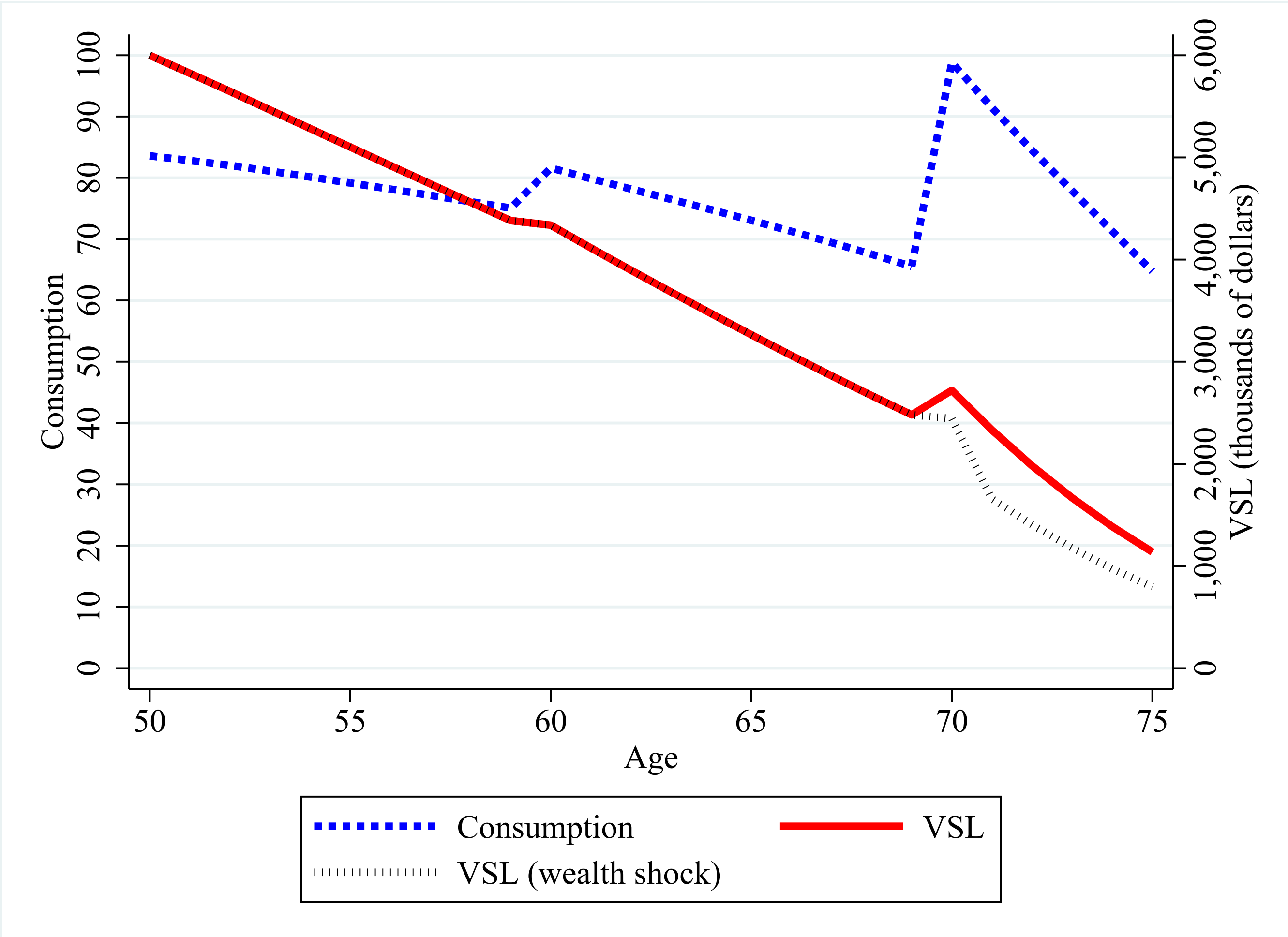
Notes: The solution to the consumer's maximization problem may be non-interior in the presence of survival-contingent income. Panel (a) gives an example where there is one set of non-interior solutions. Panel (b) gives an example where there are two sets of non-interior solutions. Income, illustrated by the dashed blue line, includes both labor income and annuity income.

Figure 3: Average annual out-of-pocket medical spending, by age



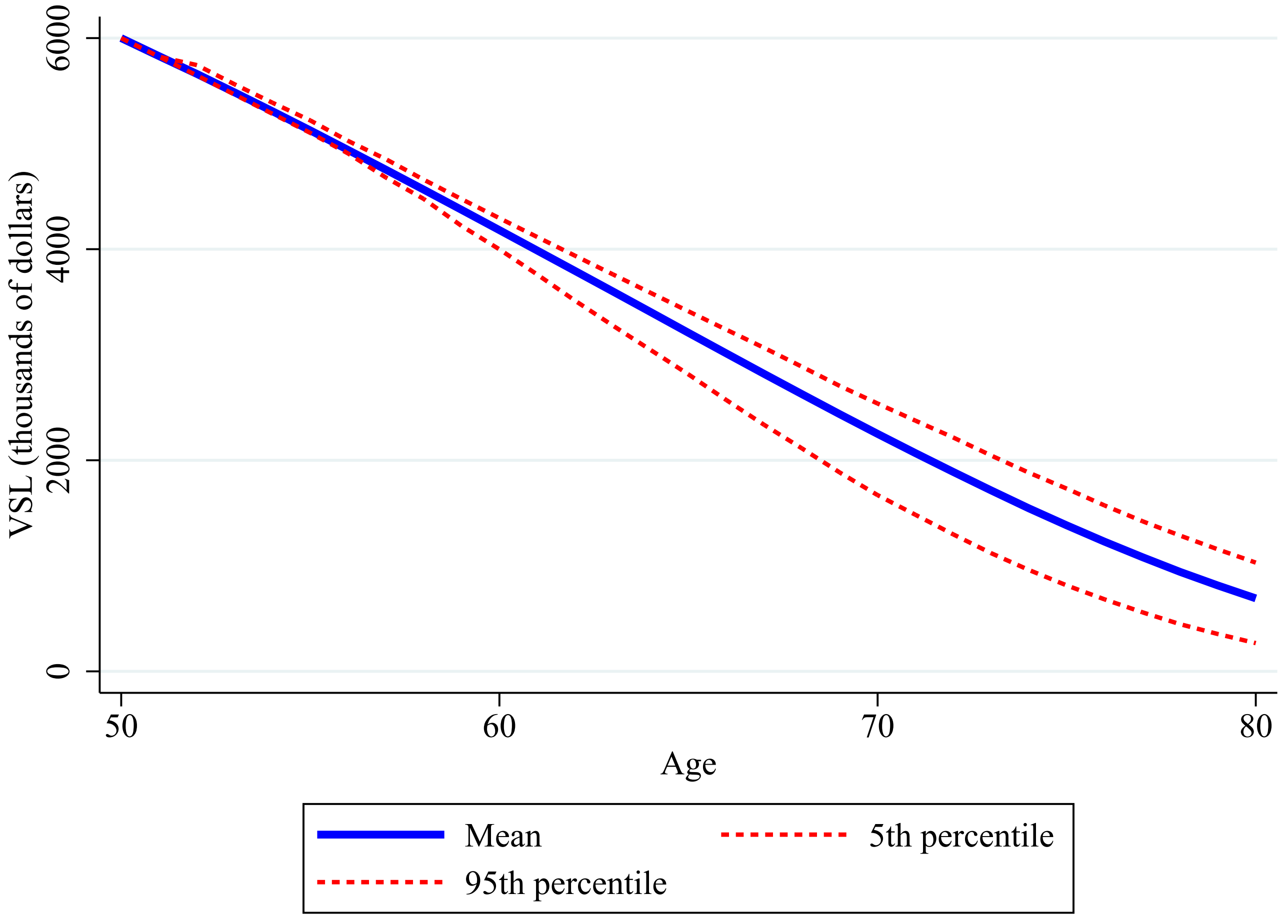
Notes: These medical spending estimates include all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Health state 1 describes healthy individuals with no impaired activities of daily living (ADL) and no chronic conditions. Health state 20 describes very ill individuals with three or more ADLs and four or more chronic conditions. Additional characteristics for these health states are provided in Table 1. These data are obtained from the Future Elderly Model (FEM).

Figure 4: Consumption and the value of statistical life for an individual who suffers two health shocks



Notes: This figure plots an individual's consumption and value of statistical life, as calculated by a life-cycle modeling exercise where mortality, quality of life, and medical spending are stochastic. The individual is healthy at age 50, but then falls ill twice, once at age 60 and then again at age 70. At age 60, the illness causes permanent difficulties with one routine activity of daily living (ADL). At age 70, she is diagnosed with three chronic conditions and one additional ADL. In our data, this corresponds to transitioning from state 1 to state 6 at age 60, and then from state 6 to state 14 at age 70. The individual dies at age 75. The dashed blue line (consumption) and the solid red line (VSL) assume that the individual's wealth falls following a health shock by the average medical spending associated with the new health state. The dotted black line (VSL) alternately assumes a financial shock at age 70 that reduces the individual's wealth by 20 percent.

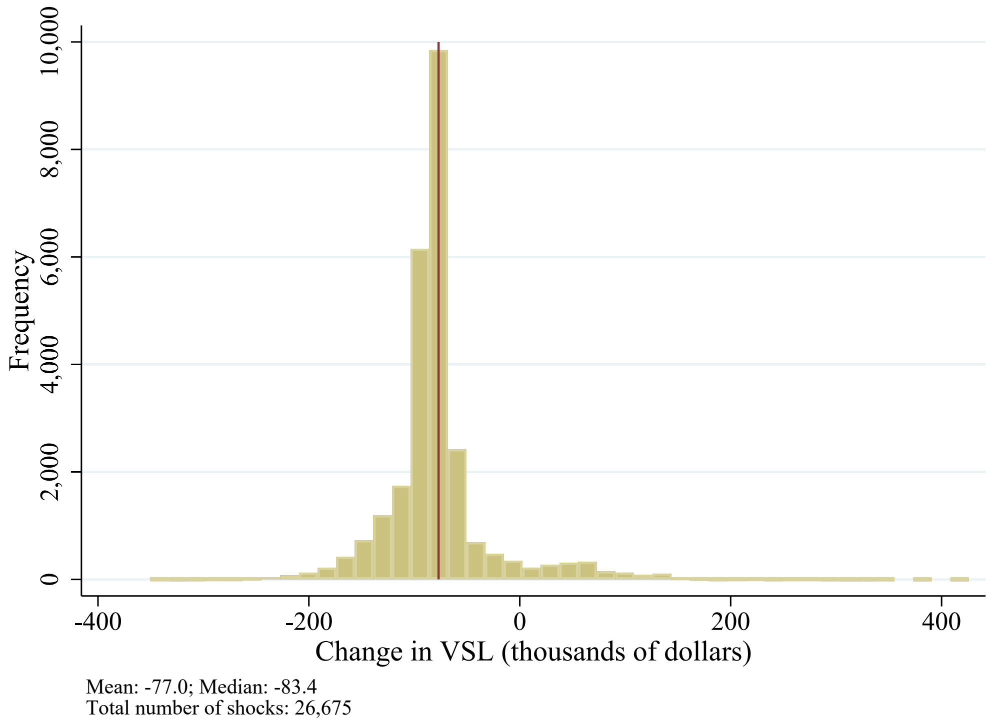
Figure 5: The value of statistical life among the US elderly population



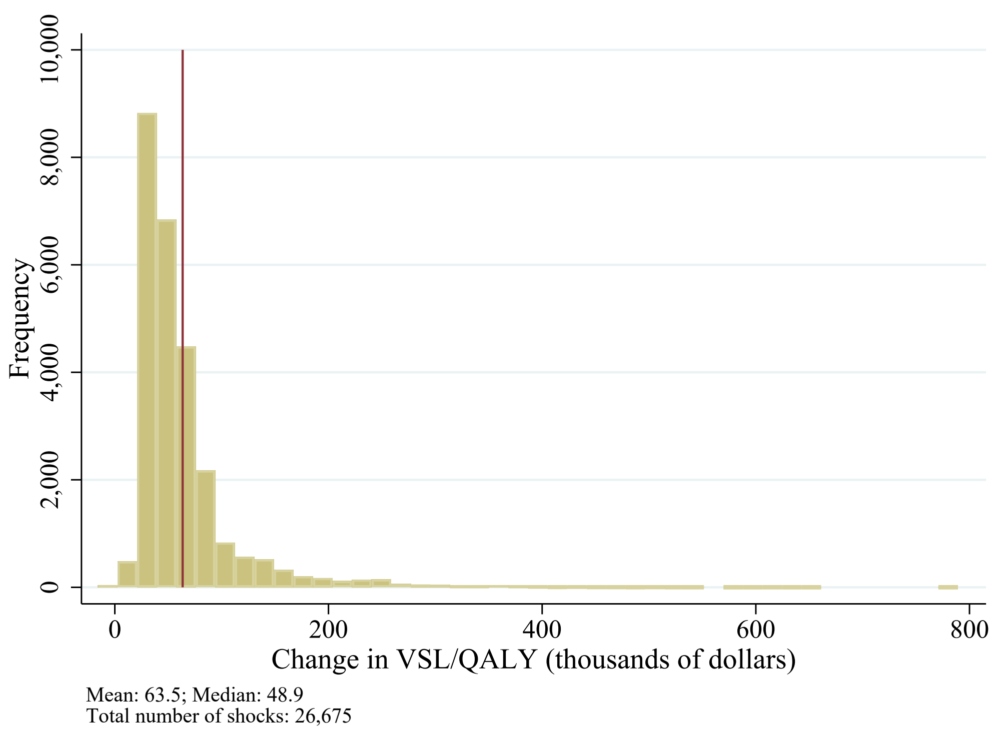
Notes: This figure reports the mean, 5th percentile, and 95th percentile of the value of statistical life (VSL) from a Monte Carlo simulation that is repeated 10,000 times. Each simulation begins at age 50 with a consumer in health state 1 ("healthy"). We then randomly generate a health state path $\{Y_{51}, Y_{52}, \ldots Y_{100}\}$ using the transition probabilities estimated by the Future Elderly Model and solve for optimal consumption and VSL using the methods described in Appendix C. The simulations employ data on medical spending, quality of life, and mortality obtained from the Future Elderly Model. Those data are summarized in Table 1.

Figure 6: Distribution of changes in VSL following a health shock

(a) VSL

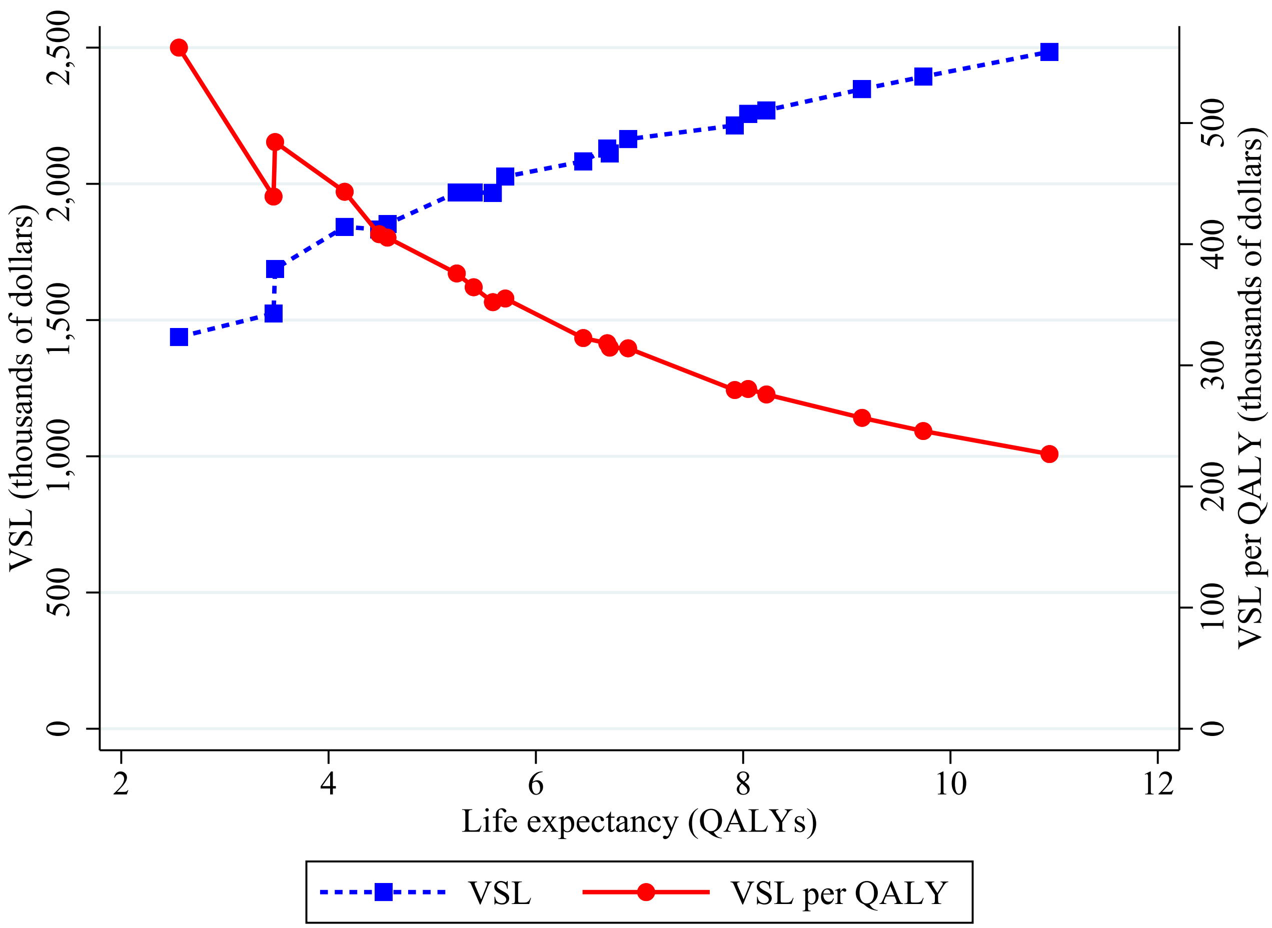


(b) VSL per QALY



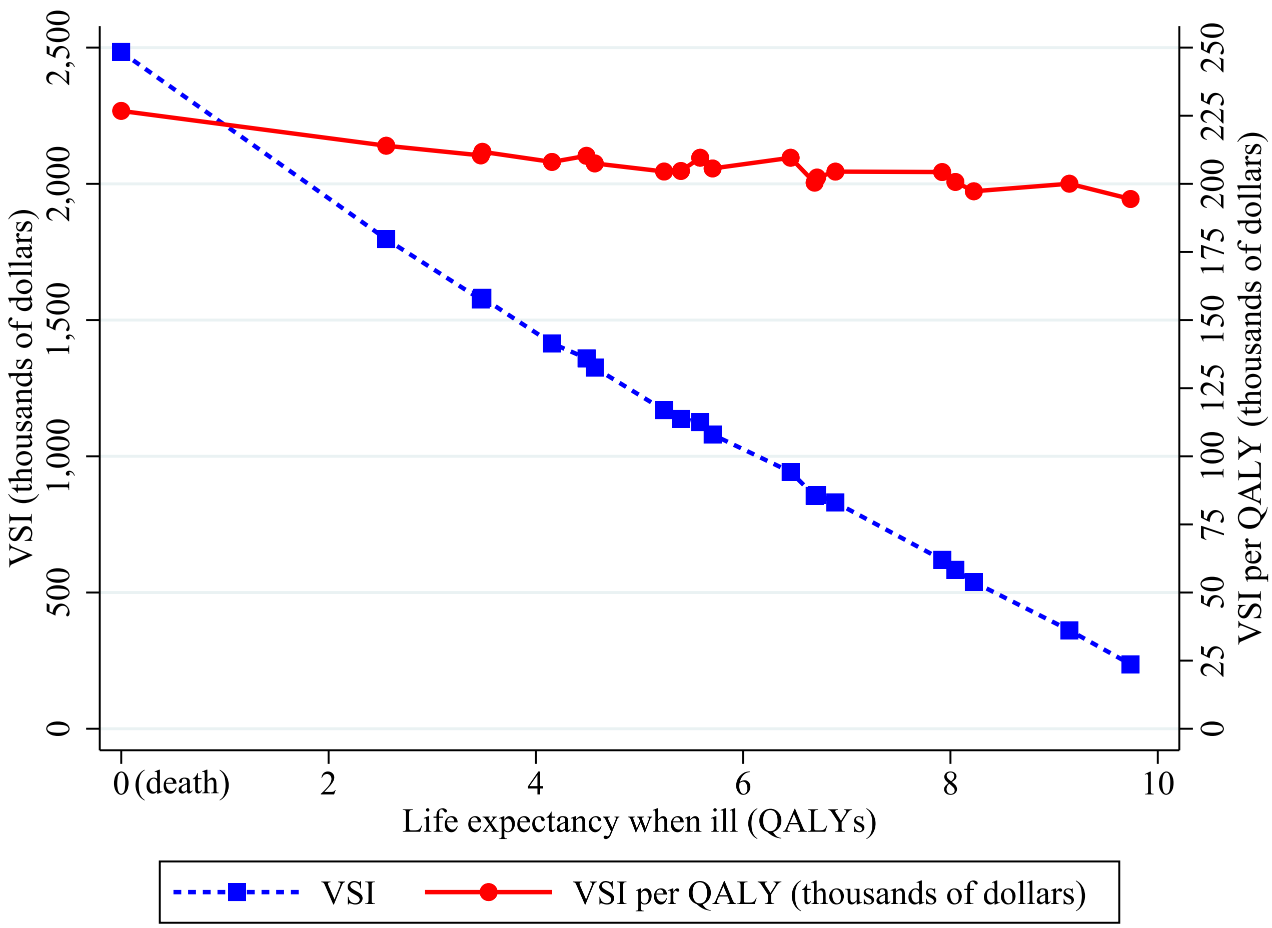
Notes: These figures are based on data from the Monte Carlo simulation described in Figure 5. Panel (a) plots the distribution of the change in VSL in the year following a health shock. Panel (b) plots the distribution of the change in VSL per quality-adjusted life year (QALY). The vertical red lines report the means of the distributions. Quality-adjusted life-years are discounted at a rate of 3 percent.

Figure 7: Average VSL at age 70, by health state



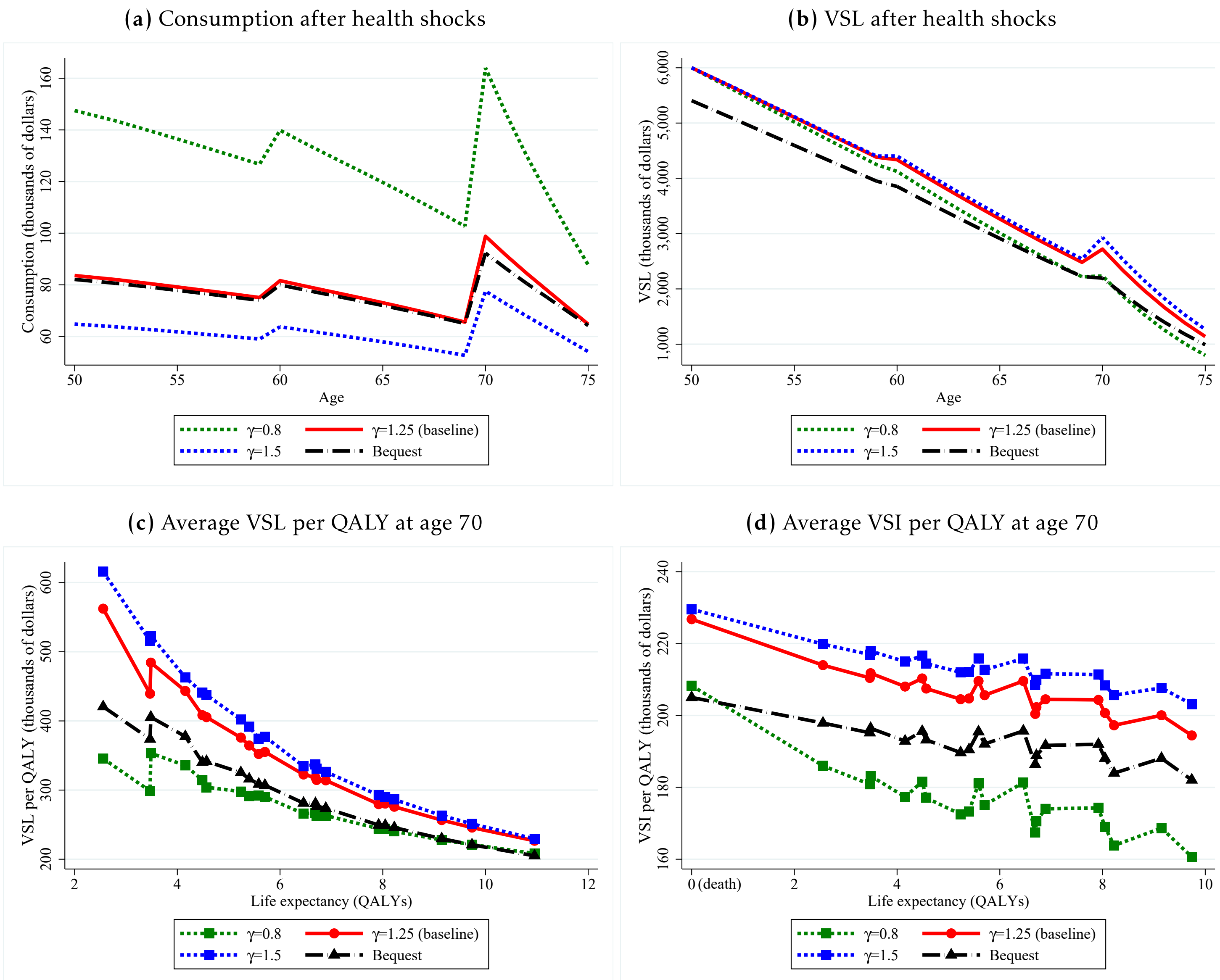
Notes: This figure is based on data from the Monte Carlo simulation described in Figure 5. The solid red line reports average VSL at age 70 for each of the 20 health states described in Table 1. The dashed blue line normalizes that value by the life expectancy for a person in that health state. Life expectancy is measured in quality-adjusted life-years (QALYs), which are discounted at a rate of 3 percent.

Figure 8: Value to a healthy person of preventing different illnesses and death, at age 70



Notes: This figure is based on data from the Monte Carlo simulation described in Figure 5. The solid red line reports a healthy (health state 1) 70-year-old's value of statistical illness (VSI) for different illnesses, i.e., their marginal willingness to pay to avoid death (value 0 on the x-axis) or to avoid transitioning to one of the 19 other, sicker health states described in Table 1. The dashed blue line normalizes that value by the change in life expectancy caused by the illness. Life expectancy is measured in quality-adjusted life-years (QALYs), which are discounted at a rate of 3 percent. Life expectancy for a 70-year-old in health state 1 is equal to 11.0 QALYs.

Figure 9: Sensitivity of results to different parameterizations of utility and to presence of bequest motive



Notes: The solid red lines in panels (a), (b), (c), and (d) replicate the baseline results from Figure 4 (consumption), Figure 4 (VSL), Figure 7, and Figure 8 . The dashed green and dashed blue lines present results under the alternative parameter assumptions $\gamma = 0.8$ and $\gamma = 1.5$, respectively, for the utility function (14). The bequest motive specification, depicted by the black dashed line, is based on Fischer (1973) and sets the bequest motive parameter $b_t = 2$ (see Appendix C). Life expectancy is measured in quality-adjusted life-years (QALYs), which are discounted at a rate of 3 percent.

Table 1: Summary means for the Future Elderly Model data, by health state

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		Life expectancy (years)		Life expectancy (QALYs)		Quality of life		Out-of-pocket spending ($)		Exit probability (%)	
Health state	ADLs / CCs	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70
1 (healthy)	0 / 0	30.9	17.6	16.3	11.0	0.88	0.87	686	1,361	4.2	12.6
2	0 / 1	28.2	15.8	14.8	9.7	0.85	0.84	866	1,578	3.6	10.8
3	0 / 2	24.6	13.6	12.8	8.2	0.81	0.80	1,145	1,925	3.6	10.2
4	0 / 3	20.5	11.2	10.7	6.7	0.77	0.76	1,487	2,366	3.9	10.2
5	0 / 4+	16.1	9.0	8.3	5.2	0.73	0.72	2,318	3,193	3.9	7.9
6	1 / 0	26.6	15.3	13.5	9.1	0.83	0.82	598	1,378	6.3	14.7
7	1 / 1	24.0	13.7	12.1	8.0	0.80	0.78	812	1,573	5.7	12.7
8	1 / 2	20.5	11.6	10.2	6.7	0.75	0.75	1,129	1,940	6.1	12.2
9	1 / 3	16.8	9.5	8.3	5.4	0.72	0.71	1,394	2,439	6.4	11.7
10	1 / 4+	13.2	7.5	6.5	4.2	0.67	0.66	2,098	3,287	6.1	8.6
11	2 / 0	24.3	13.8	11.9	7.9	0.78	0.77	585	1,314	7.3	14.3
12	2 / 1	21.5	12.3	10.4	6.9	0.75	0.73	797	1,600	7.5	14.3
13	2 / 2	18.1	10.4	8.7	5.7	0.71	0.69	1,043	1,934	7.5	13.8
14	2 / 3	15.0	8.5	7.1	4.6	0.67	0.66	1,348	2,412	7.5	13.1
15	2 / 4+	11.5	6.7	5.4	3.5	0.63	0.61	1,997	3,322	7.3	10.6
16	3+ / 0	21.9	11.8	10.3	6.5	0.70	0.69	693	1,358	3.4	11.1
17	3+ / 1	19.0	10.4	8.9	5.6	0.66	0.66	948	1,567	2.8	8.5
18	3+ / 2	15.7	8.6	7.3	4.5	0.62	0.62	1,105	1,965	2.3	7.1
19	3+ / 3	12.7	6.9	5.8	3.5	0.58	0.58	1,671	2,472	1.4	5.3
20	3+ / 4+	9.1	5.3	4.1	2.6	0.54	0.54	2,759	3,388	0.0	0.0

Notes: This table reports selected means for the health data obtained from the Future Elderly Model (FEM). Column (1) reports the number of impaired activities of daily living (ADLs) and the number of chronic conditions (CCs), which together define a health state. Columns (2)–(3) report life expectancy in years. Columns (4)–(5) reports life expectancy in QALYs, which is calculated using equation (13) with a 3% interest rate. Columns (6)–(7) report average quality of life as measured by the EQ-5D index, where 0 indexes death and 1 indexes perfect health. Columns (8)–(9) report average medical spending, which includes all inpatient, outpatient, prescription, and long-term care spending not covered by insurance. Columns (10)–(11) report the percentage probability that an individual transitions to a different health state in the following year (excluding death). All ADLs and chronic conditions are permanent, so individuals can transition only to higher-numbered health states. Additional details about the FEM are available in Appendix B.

Online Appendix

"Mortality Risk, Insurance, and the Value of Life"

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A Mathematical Proofs

Proof of Lemma 1. Recall that the transition intensities $\lambda_{ij}(t) = 0\ \forall j < i$. The optimization problem in state n is therefore the standard problem with a single health state. We can contemplate a successive solution strategy by starting in state n and then moving sequentially to state $n-1$, $n-2$, etc. Thus, we can consider the deterministic optimization problem for an arbitrary state i by taking $V(t,w,j),\ j>i$, as given (exogenous):

$$V(0, W_0, i) = \max_{c_i(t)} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

subject to:

$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) - c_i(t),$$
$$W_i(0) = W_0$$

Optimal consumption and wealth in state i are denoted by $c_i(t)$ and $W_i(t)$, respectively. Denote the optimal value-to-go function as:

$$\tilde{V}(u, W_i(u), i) = \max_{c_i(t)} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

Setting $\tilde{V}(t, W_i(t), i) = e^{-\rho t}\tilde{S}(i,t)V(t, W_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (2) for i. See Theorem 1 and the proof of Theorem 2 in Parpas and Webster (2013) for additional details and intuition behind this result.

■

Proof of Lemma 2. From (3), the marginal utility of preventing an illness or death is:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp\left\{ -\int_0^t \sum_{j>i} \left(\lambda_{ij}(s) - \varepsilon \delta_{ij}(s) \right) ds \right\} \left(u\left(c_i^\varepsilon(t), q_i(t)\right) + \sum_{j>i} \left(\lambda_{ij}(t) - \varepsilon \delta_{ij}(t) \right) V(t, W_i^\varepsilon(t), j) \right) dt \Bigg|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t} \tilde{S}(i,t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \\
&+ \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u_c\left(c_i^\varepsilon(t), q_i(t)\right) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \right) dt \Bigg|_{\varepsilon=0}
\end{aligned}$$

where $c_i^\varepsilon(t)$ and $W_i^\varepsilon(t)$ represent the equilibrium variations in $c_i(t)$ and $W_i(t)$ caused by this perturbation.

We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to:

$$W_i(t) = W_0 e^{rt} - \int_0^t e^{r(t-s)} c_i(s) ds,$$

which implies $\frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} = -\int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds$. From the solution to the costate equation, we know that:

$$e^{-\rho t}\tilde{S}(i,t) u_c(c_i(t), q_i(t)) = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

Thus, we can rewrite the second term in the expression for $\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0}$ above as:

$$
\begin{aligned}
&\left.\int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds+\theta^{(i)}\right]e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial\varepsilon}dt-\int_0^T e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\int_0^t e^{r(t-s)}\frac{\partial c_i^{\varepsilon}(s)}{\partial\varepsilon}ds\,dt\right|_{\varepsilon=0}\\
&=\left.\int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds\right]e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial\varepsilon}dt-\int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds\right]e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial\varepsilon}dt+\int_0^T\theta^{(i)}e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial\varepsilon}dt\right|_{\varepsilon=0}\\
&=\left.\theta^{(i)}\frac{\partial}{\partial\varepsilon}\underbrace{\int_0^T e^{-rt}c_i^{\varepsilon}(t)dt}_{W_0}\right|_{\varepsilon=0}\\
&=0
\end{aligned}
$$

where the last equality follows from application of the budget constraint.

■

Proof of Lemma 3. The proof proceeds by induction on $i \le n$. For the base case $i = n$, in which no state transitions are possible, the solution to the costate equation (4) simplifies to:

$$
\begin{aligned}
p_\tau^{(n)} &= \theta^{(n)}e^{-r\tau}\\
&= \exp\left\{-\int_0^\tau \rho+\lambda_{n,n+1}(s)\,ds\right\}u_c\left(c_n(\tau),q_n(\tau)\right)\\
&= \theta^{(n)}e^{-rt}e^{-r(\tau-t)}\\
&= p_t^{(n)}e^{-r(\tau-t)}\\
&= \exp\left\{-\int_0^t \rho+\lambda_{n,n+1}(s)\,ds\right\}u_c(c_n(t),q_n(t))\,e^{-r(\tau-t)}
\end{aligned}
$$

where the second equality makes use of the first-order condition (5). Using the expressions in the second and the last lines then implies:

$$
u_c(c_n(t),q_n(t)) = e^{r(\tau-t)}\,e^{-\rho(\tau-t)}\exp\left\{-\int_t^\tau \lambda_{n,n+1}(s)\,ds\right\}u_c\left(c_n(\tau),q_n(\tau)\right)
$$

which shows that the lemma holds for $i = n$.

For the induction step, suppose the lemma is true for $j > i$, $1 \le i \le n-1$. For any subinterval $[0,\tau]$, the solution of the costate equation can be written as:

$$
p_t^{(i)} = \left[\int_t^\tau e^{(r-\rho)s}\exp\left\{-\int_0^s\sum_{j>i}\lambda_{ij}(u)\,du\right\}\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}\,ds\right]e^{-rt}+\theta(\tau,i)\,e^{-rt} \tag{A.1}
$$

where $\theta(\tau,i)$ is a constant that depends on the choice of τ and i. (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating equation (A.1) at $t=\tau$ and combining with equation (5) from the main text yields:

$$
p_\tau^{(i)} = \theta(\tau,i)\,e^{-r\tau} = \exp\left\{-\int_0^\tau \rho+\sum_{j>i}\lambda_{ij}(s)\,ds\right\}u_c(c_i(\tau),q_i(\tau))
$$

which implies:

$$
\theta(\tau,i) = e^{(r-\rho)\tau}\exp\left\{-\int_0^\tau\sum_{j>i}\lambda_{ij}(s)ds\right\}u_c(c_i(\tau),q_i(\tau)) \tag{A.2}
$$

Plugging equations (5) and (A.2) into equation (A.1) yields:

$$u_c(c_i(t),q_i(t))\exp\left\{-\int_0^t \rho+\sum_{j>i}\lambda_{ij}(s)\,ds\right\} \quad = \quad \left[\int_t^\tau e^{(r-\rho)s}\exp\left\{-\int_0^s\sum_{j>i}\lambda_{ij}(u)\,du\right\}\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}\,ds\right]e^{-rt}$$
$$+e^{-rt}\,e^{(r-\rho)\tau}\exp\left\{-\int_0^\tau\sum_{j>i}\lambda_{ij}(s)ds\right\}\,u_c(c_i(\tau),q_i(\tau))$$

Since $\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}=u_c(c(s,W_i(s),j),q_j(s))$ from the first-order condition in the HJB for state j, we obtain:

$$\begin{aligned}
u_c(c_i(t),q_i(t)) &= \int_t^\tau e^{(r-\rho)(s-t)}\exp\left\{-\int_t^s\sum_{j>i}\lambda_{ij}(u)\,du\right\}\sum_{j>i}\lambda_{ij}(s)\,u_c(c(s,W_i(s),j),q_j(s))\,ds+e^{(r-\rho)(\tau-t)}\exp\left\{-\int_t^\tau\sum_{j>i}\lambda_{ij}(s)\,ds\right\}\,u_c(c_i(\tau),q_i(\tau))\\
&=\int_t^\tau e^{(r-\rho)(s-t)}\exp\left\{-\int_t^s\sum_{j>i}\lambda_{ij}(u)\,du\right\}\sum_{j>i}\lambda_{ij}(s)\mathbb{E}\left[e^{(r-\rho)(\tau-s)}\exp\left\{-\int_s^\tau\mu(s)ds\right\}\,u_c\left(c(\tau,W(\tau),Y_\tau),q_{Y_\tau}(\tau)\right)\middle|\,Y_s=j,W(s)=W_i(s)\right]ds\\
&\quad+e^{(r-\rho)(\tau-t)}\exp\left\{-\int_t^\tau\sum_{j>i}\lambda_{ij}(s)\,ds\right\}\,u_c(c_i(\tau),q_i(\tau))\\
&=\mathbb{E}\left[e^{(r-\rho)(\tau-s)}\exp\left\{-\int_t^\tau\mu(s)\,ds\right\}\,u_c\left(c(\tau,W(\tau),Y_\tau),q_{y_\tau}(\tau)\right)\middle|\,Y_t=i,\;W(t)=W_i(t)\right]
\end{aligned}$$

where the second equality follows from the induction hypothesis.

■

Proof of Proposition 4. Choosing the Dirac delta function for $\delta(t)$ in Lemma 2 yields:

$$\begin{aligned}
\left.\frac{\partial V}{\partial\varepsilon}\right|_{\varepsilon=0} &= \int_0^T\left[e^{-\rho t}\tilde{S}(i,t)\left(u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right)\right]dt\\
&=\mathbb{E}\left[\int_0^T e^{-\rho t}S(t)u(c(t),q_{Y_t}(t))dt\,\middle|\,Y_0=i,W(0)=W_0\right]
\end{aligned}$$

Dividing the result by the marginal utility of wealth at time $t=0$ then yields the value of statistical life given by equation (7):

$$VSL(i)=\mathbb{E}\left[\int_0^T e^{-\rho t}S(t)\frac{u(c(t),q_{Y_t}(t))}{u\left(c(0),q_{Y_0}(0)\right)}dt\,\middle|\,Y_0=i,W(0)=W_0\right]$$

Applying Lemma 3 for $t=0$ allows us to rewrite VSL as:

$$\begin{aligned}
VSL(i)&=\mathbb{E}\left[\int_0^T e^{-\rho t}\frac{S(t)\,u(c(t),q_{Y_t}(t))}{\mathbb{E}\left[e^{(r-\rho)t}\exp\left\{-\int_0^t\mu(s)ds\right\}\,u_c\left(c(t),q_{Y_t}(t)\right)\middle|\,Y_0=i,W(0)=W_0\right]}dt\,\middle|\,Y_0=i,W(0)=W_0\right]\\
&=\mathbb{E}\left[\int_0^T e^{-rt}\frac{S(t)\,u(c(t),q_{Y_t}(t))}{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}\,u_c\left(c(t),q_{Y_t}(t)\right)\middle|\,Y_0=i,W(0)=W_0\right]}dt\,\middle|\,Y_0=i,W(0)=W_0\right]
\end{aligned}$$

Exchanging expectation and integration then yields:

$$VSL(i)=\int_0^T e^{-rt}v(i,t)\,dt$$

where $v(i,t)$ is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i,t) = \frac{\mathbb{E}\left[S(t)\,u(c(t), q_{Y_t}(t))\middle| Y_0 = i, W(0) = W_0\right]}{\mathbb{E}\left[S(t)\,u_c\left(c(t), q_{Y_t}(t)\right)\middle| Y_0 = i, W(0) = W_0\right]}$$

■

Proof of Proposition 5. Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Because we hold quality of life constant, we omit $q_i(t)$ in the notation below in order to keep the presentation concise.

We want to prove that $c_2(0) \geq c_1(0)$. Assume by way of contradiction that $c_2(0) < c_1(0)$. We will show that this assumption implies $c_2(t) < c_1(t)$ for all $t > 0$, which is a contradiction since the feasible consumption plan $c_1(\cdot)$ dominates $c_2(\cdot)$.

We proceed by inductively constructing a sequence $0 < t_1 < t_2 \ldots$ where for each element in the sequence:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p^{(1)}_{t_i} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p^{(2)}_{t_i} \end{aligned}$$

To construct the sequence, for the base case $i = 1$, we first note that from the first-order condition (5), we obtain:

$$p^{(1)}_0 = u_c(c_1(0)) < u_c(c_2(0)) = p^{(2)}_0$$

The costate equation (4) then implies:

$$\begin{aligned} \dot{p}^{(1)}_0 &= -p^{(1)}_0 r - \lambda_{12}(0) u_c(c_2(0)) \\ &= -p^{(1)}_0 \left[r + \lambda_{12}(0) \underbrace{\frac{u_c(c_2(0))}{u_c(c_1(0))}}_{>1} \right] \\ &< -p^{(1)}_0 [r + \lambda_{12}(0)] = \left.\frac{\partial g(t)}{\partial t}\right|_{t=0} \end{aligned}$$

where $g(t) = p^{(1)}_0 \exp\left\{-\int_0^t r + \lambda_{12}(s)\,ds\right\}$. Hence, there exists a $t_1 > t_0 = 0$ such that:

$$p^{(1)}_t \leq g(t) < p^{(2)}_0 \exp\left\{-\int_0^t (r + \lambda_{12}(s))\,ds\right\} = p^{(2)}_t \exp\left\{-\int_0^t \lambda_{12}(s)ds\right\},\ 0 \leq t \leq t_1$$

which together with the first-order condition (5) implies:

$$e^{-\rho t}\exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{13}(s))\,ds\right\} u_c(c_1(t)) < e^{-\rho t}\exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{23}(s))\,ds\right\} u_c(c_2(t)),\ 0 \leq t \leq t_1$$

so that $c_1(t) > c_2(t)$, $0 \leq t \leq t_1$. This inequality in turn implies $W_1(t_1) \leq W_2(t_1)$.

For the induction step, suppose that the following properties also hold for $i \geq 1$:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p_{t_i}^{(1)} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)} \end{aligned}$$

The induction hypothesis implies:

$$c(t_i, W_1(t_i), 2) \leq c(t_i, W_2(t_i), 2) = c_2(t_i) < c_1(t_i)$$

so that:

$$\begin{aligned} \dot{p}_{t_i}^{(1)} &= -p_{t_i}^{(1)} r - e^{-\rho t_i} \tilde{S}(1, t_i)\, \lambda_{12}(t_i)\, u(c(t_i, W_1(t_i), 2)) \\ &= -p_{t_i}^{(1)} \left[r + \lambda_{12}(t_i) \underbrace{\frac{u_c(c(t_i, W_1(t_i), 2))}{u_c(c_1(t_i))}}_{>1} \right] \\ &< -p_{t_i}^{(1)} [r + \lambda_{12}(t_i)] = \left.\frac{\partial \tilde{g}(t)}{\partial t}\right|_{t=0} \end{aligned}$$

with $\tilde{g}(t) = p_{t_i}^{(1)} \exp\left\{-\int_{t_i}^{t} (r + \lambda_{12}(s))\, ds\right\}$. Hence, there exists a $t_{i+1} > t_i$ such that:

$$\begin{aligned} p_t^{(1)} &\leq \tilde{g}(t) \\ &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)\, ds\right\} p_{t_i}^{(2)} \exp\left\{-\int_{t_i}^{t} (r + \lambda_{12}(s))\, ds\right\} \\ &= p_t^{(2)} \exp\left\{-\int_0^{t} \lambda_{12}(s)\, ds\right\}, \; t_i \leq t \leq t_{i+1} \end{aligned}$$

Applying again the first-order condition (5) for all $t_i \leq t \leq t_{i+1}$ yields:

$$\exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{13}(s))\, ds\right\} u_c(c_1(t)) < \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{23}(s))\, ds\right\} u_c(c_2(t))$$

which in turn implies $u_c(c_1(t)) < u_c(c_2(t))$ and $c_2(t) < c_1(t)$. Once again, this inequality implies $W_1(t_{i+1}) \leq W_2(t_{i+1})$.

Thus, we have proven the existence of the sequence. We then obtain $c_2(t) < c_1(t)\ \forall t$ by noting that $\{t_i\}_{i \geq 0}$ strictly increases due to the uniformly boundedness condition on $\lambda_{12}(t)$, which is the desired contradiction.

We note that this proof implies that the consumption paths $c_1(t)$ and $c_2(t)$ cross (at most) once. As soon as $c_1(t)$ exceeds $c_2(t)$ for some time t_0, $c_1(t)$ will exceed $c_2(t)$ for $t > t_0$. However, we have that $c_2(t)$ exceeds $c_1(t)$ prior to t_0. In particular, consumption jumps up at the transition point. See Figure 1 for an illustration.

■

Proof of Proposition 6. Without loss of generality, consider the case $t = 0$. Under our assumptions, from equation (9) and Proposition 5 it is clear that $c_1(t)$ and $c_2(t)$ are decreasing, $c_2(0) > c_1(0)$, $c_2(t) > c_1(t)$ for $t \leq t_0$, and $c_2(t) < c_1(t)$ for $t > t_0$. Making use of the assumption that no state transitions occur for $t > 0$, we have that:

$$VSL(2,0) = \int_0^T e^{-rt} \frac{S_2(t)\, u(c_2(t))}{S_2(t)\, u_c(c_2(t))} dt = \int_0^T e^{-rt} \frac{u(c_2(t))}{u_c(c_2(t))} dt$$

and:

$$VSL(1,0) = \int_0^T e^{-rt} \frac{u(c_1(t))}{u_c(c_1(t))} dt$$

Let $Y(x) \equiv \frac{u(x)}{u_c(x)}$. Under the stated assumptions on preferences, we have that:

$$
\begin{aligned}
Y'(x) &= 1 - \frac{u(x)u_{cc}(x)}{(u_c(x))^2} > 0, \\
Y''(x) &= \frac{2(u_{cc}(x))^2 u(x) - u_c^2(x)u_{cc}(x) - u_c(x)u(x)u_{ccc}(x)}{(u_c(x))^3} > 0
\end{aligned}
$$

Employing Taylor's theorem then implies that for some $\xi(t)$ that lies in-between $c_1(t)$ and $c_2(t)$:

$$
\begin{aligned}
VSL(2,0) &= \int_0^T e^{-rt}\, Y(c_2(t))\, dt \\
&= \int_0^T e^{-rt} \left[Y(c_1(t)) + [c_2(t) - c_1(t)]\, Y'(c_1(t)) + \underbrace{\frac{1}{2}[c_2(t) - c_1(t)]^2 Y''(\xi(t))}_{>0} \right] dt \\
&> \int_0^T e^{-rt}\, Y(c_1(t))\, dt + \int_0^{t_0} e^{-rt} Y'(c_1(t))\, \underbrace{[c_2(t) - c_1(t)]}_{\geq 0}\, dt + \int_{t_0}^T e^{-rt} Y'(c_1(t))\, \underbrace{[c_2(t) - c_1(t)]}_{\leq 0}\, dt \\
&\geq \int_0^T e^{-rt} Y(c_1(t))\, dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0))\, [c_2(t) - c_1(t)]\, dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0))\, [c_2(t) - c_1(t)]\, dt \\
&= \int_0^T e^{-rt} Y(c_1(t))\, dt + Y'(c_1(t_0)) \underbrace{\left[\int_0^T e^{-rt} c_2(t)\, dt - \int_0^T e^{-rt} c_1(t)\, dt \right]}_{=0} \\
&= \int_0^T e^{-rt} Y(c_1(t))\, dt \\
&= VSL(1,0)
\end{aligned}
$$

where the final step follows from the budget constraint.

■

Proof of Lemma 7. We first assume that utility takes the following simplified form:

$$
u(c,q) = q \left(\frac{c^{1-\gamma}}{1-\gamma} \right)
$$

This specification assumes the subsistence level of consumption is equal to 0. This assumption has no effect on optimal consumption, which is unaffected by affine transformations of utility. At the end of the proof, we generalize the optimal value function to allow for a positive level of subsistence consumption.

We will show that the optimal value function under this utility specification is equal to:

$$
V(t, W(t), i) = K_{t,i} \frac{W(t)^{1-\gamma}}{1-\gamma}
$$

where $\{K_{t,i}\}$ satisfies the following system of ordinary differential equations:

$$
\frac{\partial K_{t,i}}{\partial t} = K_{t,i} \left(\rho + \sum_{j>i} \lambda_{ij}(t) - (1-\gamma) r \right) - \sum_{j>i} \lambda_{ij}(t) K_{t,j} - \gamma K_{t,i} \left(\frac{K_{t,i}}{q_i(t)} \right)^{-1/\gamma}, \; K_{T,i} = 0 \,\forall i, \; K_{t,n+1} = 0 \,\forall t
$$

Plugging the utility function and our proposed solution to the value function into the Hamilton-Jacobi-Bellman (HJB) system of equations (2), the first-order condition for consumption yields:

$$q_i(t)\,c^*(t,W(t),i)^{-\gamma} = K_{t,i}\,W(t)^{-\gamma}$$

which implies:

$$c^*(t,W(t),i) = \left(\frac{K_{t,i}}{q_i(t)}\right)^{-1/\gamma} W(t)$$

Thus, the HJB (2) takes the form:

$$\rho\,K_{t,i}\frac{W(t)^{1-\gamma}}{1-\gamma} = q_i(t)\left(\frac{K_{t,i}}{q_i(t)}\right)^{1-1/\gamma}\frac{W(t)^{1-\gamma}}{1-\gamma} + K_{t,i}\,r\,W(t)^{1-\gamma}$$
$$-(1-\gamma)\left(\frac{K_{t,i}}{q_i(t)}\right)^{-\frac{1}{\gamma}} K_{t,i}\frac{W(t)^{1-\gamma}}{1-\gamma} + \left(\frac{\partial K_{t,i}}{\partial t}\right)\frac{W(t)^{1-\gamma}}{1-\gamma} + \frac{W(t)^{1-\gamma}}{1-\gamma}\sum_{j>i}\lambda_{ij}(t)\left(K_{t,j}-K_{t,i}\right)$$

Dividing by $\frac{W(t)^{1-\gamma}}{1-\gamma}$ and rearranging shows that this HJB will hold if and only if $\{K_{t,i}\}$ satisfies the system of ordinary differential equations given above.

Finally, we generalize our utility function to allow for a positive level of subsistence consumption, $\underline{c}$:

$$u(c,q) = q\left(\frac{c^{1-\gamma}-\underline{c}^{1-\gamma}}{1-\gamma}\right)$$

Plugging this generalized utility function into equation (1) and then applying our solution to the value function that we derived above yields:

$$\begin{aligned}
V(0,W_0,Y_0) &= \mathbb{E}\left[\int_0^T e^{-\rho t}\,S(t)\,q_{Y_t}\left(\frac{c^*(t,W(t),Y_t)^{1-\gamma}-\underline{c}^{1-\gamma}}{1-\gamma}\right)dt\,\middle|\,Y_0,W_0\right]\\
&= K_{0,Y_0}\frac{W_0^{1-\gamma}}{1-\gamma} - \frac{\underline{c}^{1-\gamma}}{1-\gamma}\mathbb{E}\left[\int_0^T e^{-\rho t}\,S(t)\,q_{Y_t}\,dt\,\middle|\,Y_0\right]\\
&= K_{0,Y_0}\frac{W_0^{1-\gamma}}{1-\gamma} - \frac{\underline{c}^{1-\gamma}}{1-\gamma}D_{Y_0}
\end{aligned}$$

where $D_{Y_0} = \mathbb{E}\left[\int_0^T e^{-\rho t}\,q_{Y_t}(t)\,S(t)\,dt\,\middle|\,Y_0\right]$ is quality-adjusted, discounted life expectancy. ■

Proof of Proposition 8. From the stated assumptions, it follows immediately that $\frac{VSL(i)}{D_i} < \frac{VSL(j)}{D_j}$. The remainder of this proof shows that $\frac{VSI(i,j)}{D_i-D_j} < \frac{VSL(i)}{D_i}$. Our proof uses the result from Lemma 7, and we will omit the subscript 0 from $K_{0,i}$ and $K_{0,j}$ in our notation below to keep the presentation concise. We have that:

$$\begin{aligned}
\frac{VSI(i,j)}{D_i-D_j} &= \frac{V(0,W_0,i)-V(0,W_0,j)}{u_c(c_i(0))\,[D_i-D_j]}\\
&= \frac{1}{1-\gamma}\left[\frac{W_0^{1-\gamma}[K_i-K_j]}{W_0^{-\gamma}K_i[D_i-D_j]} - \frac{\underline{c}^{1-\gamma}[D_i-D_j]}{W_0^{-\gamma}K_i[D_i-D_j]}\right]\\
&= \frac{1}{1-\gamma}W_0\frac{K_i-K_j}{K_i[D_i-D_j]} - \frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}K_i}
\end{aligned}$$

and that:

$$\begin{aligned}\frac{VSL(i)}{D_i} &= \frac{V(0,W_0,i)}{u_c(c_i(0))D_i} \\ &= \frac{1}{1-\gamma}\left[\frac{W_0^{1-\gamma}K_i}{W_0^{-\gamma}K_iD_i} - \frac{\underline{c}^{1-\gamma}D_i}{W_0^{-\gamma}K_iD_i}\right] \\ &= \frac{1}{1-\gamma}W_0\frac{1}{D_i} - \frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}K_i}\end{aligned}$$

Applying algebra yields:

$$\begin{aligned} & \frac{VSL(i)}{D_i} > \frac{VSI(i,j)}{D_i - D_j} \\ \iff & \frac{1}{1-\gamma}\frac{1}{D_i} > \frac{1}{1-\gamma}\frac{K_i-K_j}{K_i[D_i-D_j]} \\ \iff & \frac{1}{1-\gamma}\left\{K_i[D_i-D_j] - D_i[K_i-K_j]\right\} > 0 \\ \iff & \frac{1}{1-\gamma}\left\{K_jD_i - K_iD_j\right\} > 0 \\ \iff & \frac{1}{1-\gamma}\left\{\frac{K_j}{D_j} - \frac{K_i}{D_i}\right\} > 0\end{aligned}$$

Finally, notice that if $VSL(j) > VSL(i)$ then $\frac{1}{\gamma-1}\frac{D_j}{K_j} > \frac{1}{\gamma-1}\frac{D_i}{K_i}$, which can be rearranged to yield the final inequality above. ■

Proof of Proposition 9. The proposition assumes concavity in health states:

$$V(0,W_0,j) > D \times V(0,W_0,i) + (1-D) \times V(0,W_0,k), \text{ where } D = \frac{D_j - D_k}{D_i - D_k}$$

From Lemma 7, this is equivalent to the condition:

$$\frac{K_{0,j}}{1-\gamma} > \frac{K_{0,i}}{1-\gamma}D + \frac{K_{0,k}}{1-\gamma}(1-D) \tag{A.3}$$

We will prove the two statements separately. To keep the presentation concise, we omit the subscript 0 from $K_{0,i}, K_{0,j}$, and $K_{0,k}$ in the notation below.

(i) Proof that (A.3) $\iff VSI(i,j)/(D_i - D_j) < VSI(i,k)/(D_i - D_k)$

$$\begin{aligned}\frac{VSI(i,j)}{D_i-D_j} = \frac{V(0,W_0,i)-V(0,W_0,j)}{K_iW_0^{-\gamma}[D_i-D_j]} &< \frac{V(0,W_0,i)-V(0,W_0,k)}{K_iW_0^{-\gamma}[D_i-D_k]} = \frac{VSI(i,k)}{D_i-D_k} \\ \iff \frac{1}{1-\gamma}W_0\frac{K_i-K_j}{K_i[D_i-D_j]} &< \frac{1}{1-\gamma}W_0\frac{K_i-K_k}{K_i[D_i-D_k]} \\ \iff 0 &< \frac{1}{1-\gamma}\Big([K_i-K_k][D_i-D_j] - [K_i-K_j][D_i-D_k]\Big)\end{aligned}$$

The latter expression is equivalent to:

$$\begin{aligned}
0 &< -\frac{1}{1-\gamma}K_k[D_i-D_j]-\frac{1}{1-\gamma}K_i[D_j-D_k]+\frac{1}{1-\gamma}K_j[D_i-D_k] \\
\Longleftrightarrow 0 &< -\frac{1}{1-\gamma}K_k\frac{D_i-D_j}{D_i-D_k}-\frac{1}{1-\gamma}K_i\frac{D_j-D_k}{D_i-D_k}+\frac{1}{1-\gamma}K_j \\
\Longleftrightarrow \frac{K_i}{1-\gamma}D+\frac{K_k}{1-\gamma}(1-D) &< \frac{K_j}{1-\gamma}
\end{aligned}$$

(ii) Proof that (A.3) $\Longrightarrow VSI(i,k)/(D_i-D_k) < VSI(j,k)/(D_j-D_k)$

$$\begin{aligned}
\frac{VSI(i,k)}{D_i-D_k} &< \frac{VSI(j,k)}{D_j-D_k} \\
\Longleftrightarrow \frac{1}{1-\gamma}W_0\frac{K_i-K_k}{K_i[D_i-D_j]}-\frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}K_i} &< \frac{1}{1-\gamma}W_0\frac{K_j-K_k}{K_j[D_j-D_k]}-\frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}K_j} \\
\Longleftrightarrow \frac{1}{1-\gamma}W_0\frac{K_i-K_k}{D_i-D_k}-\frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}} &< \underbrace{\frac{K_i}{K_j}}_{\geq 1}\underbrace{\left[\frac{1}{1-\gamma}W_0\frac{K_j-K_k}{D_j-D_k}-\frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}}\right]}_{\geq 0}
\end{aligned}$$

Hence, the inequality will hold when:

$$\begin{aligned}
\frac{1}{1-\gamma}W_0\frac{K_i-K_k}{D_i-D_k}-\frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}} &< \frac{1}{1-\gamma}W_0\frac{K_j-K_k}{D_j-D_k}-\frac{1}{1-\gamma}\frac{\underline{c}^{1-\gamma}}{W_0^{-\gamma}} \\
\Longleftrightarrow \frac{1}{1-\gamma}\frac{K_i-K_k}{D_i-D_k} &< \frac{1}{1-\gamma}\frac{K_j-K_k}{D_j-D_k} \\
\Longleftrightarrow \frac{K_i}{1-\gamma}D+\frac{K_k}{1-\gamma}(1-D) &< \frac{K_j}{1-\gamma}
\end{aligned}$$

■

Proof of Proposition 10 and Corollary 11. Our goal is to derive expressions for VSL and VSI when annuity markets are incomplete and the consumer is endowed with state-dependent life-cycle income. We first consider in part (i) the case with life-cycle earnings only. We then introduce incomplete annuity markets later in part (ii) of the proof.

(i) No annuity markets

Denote the consumer's earnings in state i at time t as $m_i(t)$. The consumer's maximization problem is again equation (1), but the law of motion for wealth now includes earnings:

$$\begin{aligned}
&W(0)=W_0, \\
&W(t)\geq 0, \\
&\frac{\partial W(t)}{\partial t}=rW(t)+m_{Y_t}(t)-c(t)
\end{aligned}$$

Once again, we solve this stochastic finite-horizon optimization problem by reformulating it as a deterministic optimization problem. Specifically, we consider equation (3), subject to:

$$\begin{aligned}
&W_i(0)=W_0, \\
&W_i(t)\geq 0, \\
&\frac{\partial W_i(t)}{\partial t}=rW_i(t)+m_i(t)-c_i(t)
\end{aligned}$$

The present-value Hamiltonian corresponding to this deterministic problem is:

$$H\left(W_i(t), c_i(t), p_t^{(i)}, \Psi_t^{(i)}\right) = e^{-\rho t}\tilde{S}(i,t)\left(u(c_i(t), q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V(t, W_i(t), j)\right) + p_t^{(i)}\left[rW_i(t) + m_i(t) - c_i(t)\right] + \Psi_t^{(i)}W_i(t)$$

where $p_t^{(i)}$ is the costate variable for the wealth dynamics in state i and $\Psi_t^{(i)}$ is the multiplier for the wealth constraint. The first-order conditions are:

$$\begin{aligned}
\dot{p}_t^{(i)} &= -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)}r - e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} - \Psi_t^{(i)} \\
p_t^{(i)} &= e^{-\rho t}\tilde{S}(i,t)u_c(c_i(t), q_i(t)) \\
\Psi_t^{(i)} &\geq 0, \\
\Psi_t^{(i)}W_i(t) &= 0
\end{aligned}$$

Following Proposition 1 in Leung (1994), one can show the following: the Hamiltonian is regular on $[0, T)$, so optimal consumption $c_i(t)$ is everywhere continuous; the state-variable inequality constraint is of first-order, so $p_t^{(i)}$ is everywhere continuous; and optimal consumption $c_i(t)$ is continuously differentiable when $W_i(t) > 0$ (i.e., when the wealth constraint is not binding).

First, consider the case when $W_i(t) > 0$. Differentiating the first-order condition for consumption with respect to t, plugging in the result for the costate equation and its solution, and then rearranging yields the rate of change in life-cycle consumption. This rate of change, $\frac{\dot{c}_i}{c_i}$, is identical to the one described by equation (9), and is weakly declining by assumption.

The presence of life-cycle earnings introduces the possibility of multiple sets of non-interior solutions (e.g., right panel of Figure 2). Modeling these scenarios is possible, but cumbersome. As discussed in the main text, we therefore restrict ourselves to considering the case with a single set of non-interior solutions (i.e., a single "kink point", see left panel of Figure 2). A sufficient (but not necessary) assumption is that consumption growth is weakly declining. We employ that assumption in the following Lemma, which establishes the existence of a single kink point, T_i, where the consumer runs out of wealth.

Lemma 13. *Assume $m_i(t)$ is non-decreasing. Then there must exist a T_i such that (1) $W_i(t) = 0$ and $c_i(t) = m_i(t)$ for $t \geq T_i$; and (2) $c_i(t) > m_i(t)$ for $t < T_i$. The solution to the costate equation on $[0, T_i]$ is thus:*

$$p_t^{(i)} = \left[\int_t^{T_i} e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)}\,ds\right]e^{-rt} + \theta^{(i)}e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant.

Proof. By assumption, $\frac{\dot{c}_i}{c_i} < 0$ whenever $W_i(t) > 0$. Following the same argument as in Proposition 2 of Leung (1994), there is a smallest T_i such that $W_i(t) = 0$ on $[T_i, T]$ and, thus, $c_i(t) = m_i(t)$ on $[T_i, T]$. Since this is the smallest such T_i, there exists an interval $(\underline{T}_i, T_i)$ such that $W_i(t) > 0$ and $c_i(t_0) > m_i(t_0)$ for a t_0 in the vicinity of T_i. Now assume $W_i(\underline{T}_i) = 0$. Then there exists a t_1 in the vicinity of $\underline{T}_i$ such that $c_i(t_1) < m_i(t_1)$. This is a contradiction, since $m_i(t)$ is non-decreasing and $c_i(t)$ is decreasing whenever $W_i(t) > 0$. Hence $W_i(t) > 0$ on $[0, T_i)$ and $c_i(t) > m_i(t)$ for $t \in [0, T_i)$. As in the main text, the solution to the costate equation can be obtained using the variation of the constant method. ■

Because the value of statistical illness (VSI) is a generalization of the value of statistical life (VSL), we again focus on deriving an expression for VSI. Let $\delta_{ij}(t)$ be a perturbation on the transition intensity, and consider the impact on survival as described by equation (6). From equation (3), we obtain:

$$\frac{\partial V}{\partial \varepsilon}\Big|_{\varepsilon=0} = \frac{\partial}{\partial \varepsilon}\left[\int_0^{T_i(\varepsilon)} e^{-\rho t}\tilde{S}^{\varepsilon}(i,t)\left(u\left(c_i^{\varepsilon}(t),q_i(t)\right)+\sum_{j>i}\left(\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\right)V(t,W_i^{\varepsilon}(t),j)\right)dt+\int_{T_i(\varepsilon)}^{T} e^{-\rho t}\tilde{S}^{\varepsilon}(i,t)\left(u\left(m_i(t),q_i(t)\right)+\sum_{j>i}\left(\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\right)V(t,0,j)\right)dt\right]\Bigg|_{\varepsilon=0}$$

$$= \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left(u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right)-\sum_{j>i}\delta_{ij}(t)V(t,W_i(t),j)\right]dt$$

$$+\underbrace{\int_0^{T_i} e^{-\rho t}\tilde{S}(i,t)\left(u_c(c_i(t),q_i(t))\,\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}+\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\,\frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}\right)dt}_{=0}$$

where the second term in the last equality is equal to 0:

$$\begin{aligned}
&\int_0^{T_i} e^{-\rho t}\tilde{S}(i,t)\left(u_c(c_i(t),q_i(t))\,\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}+\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\,\frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}\right)dt\\
&=\int_0^{T_i} p_t^{(i)}\,\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}+e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\left[-\int_0^t e^{r(t-s)}\,\frac{\partial c_i^{\varepsilon}(s)}{\partial \varepsilon}\Big|_{\varepsilon=0}ds\right]dt\\
&=\int_0^{T_i}\theta^{(i)}e^{-rt}\,\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}dt+\int_0^{T_i}\int_t^{T_i} e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}\,ds e^{-rt}\,\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}dt\\
&\quad-\int_0^{T_i}\int_t^{T_i} e^{-\rho s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}\,ds e^{rs}e^{-rt}\,\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\Big|_{\varepsilon=0}dt\\
&=\theta^{(i)}\frac{\partial}{\partial \varepsilon}\int_0^{T_i} e^{-rt}c_i^{\varepsilon}(t)\,dt\Big|_{\varepsilon=0}\\
&=0
\end{aligned}$$

The final equality follows because $W_i(T_i)=0$ (by definition), which in turn implies $0=W_0+\int_0^{T_i}e^{-rt}m_i(t)dt-\int_0^{T_i}e^{-rt}c_i^{\varepsilon}(t)dt$, so that differentiation yields zero. Thus we obtain:

$$\frac{\partial V}{\partial \varepsilon}\Big|_{\varepsilon=0} = \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left(u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right)-\sum_{j>i}\delta_{ij}(t)V(t,W_i(t),j)\right]dt \tag{A.4}$$

Dividing by the marginal utility of wealth yields the value of life-extension. Choosing the Dirac delta function for $\delta_{i,n+1}(t)$ yields VSL, and choosing the Dirac delta function for $\delta_{ij}(t)$, $j<n+1$, yields VSI:

$$VSL(i)=\frac{V(0,W(0),i)}{u_c(c_i(0),q_i(0))}$$
$$VSI(i,j)=\frac{V(0,W(0),i)-V(0,W(0),j)}{u_c(c_i(0),q_i(0))}$$

(ii) Incomplete annuity markets

Now, we introduce a one-time opportunity at time $t=0$ to purchase a flat lifetime annuity at a level $\bar{a}_{Y_0}\geq 0$ with a price markup $\xi\geq 0$. Let $a(T,i)=\mathbb{E}\left[\int_t^T e^{-r(s-t)}\exp\left\{-\int_t^s \mu(u)\,du\right\}\,ds\,\Big|\,Y_t=i\right]$ be the expected value of a one-dollar annuity purchased at time t in state i. Note that for any given annuity, $\bar{a}_i$, the consumer's problem can be mapped to the no-annuity case in part (i) above by setting the constraints equal to:

$$W_i(0) = W_0 - (1+\xi)\bar{a}_i\ a(0,i),$$
$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) + m_i(t) + \bar{a}_i - c_i(t)$$

Solving for the optimal fixed annuity then becomes a straightforward static optimization problem:

$$\bar{a}_i^* = \arg\max_{\bar{a}_i} V\left(0, W_i(0), \bar{a}_i, i\right)$$

The optimal annuity must satisfy the necessary first-order condition:

$$\frac{\partial V\left(0, W_i(0), \bar{a}_i, i\right)}{\partial \bar{a}_i} = \frac{\partial V\left(0, W_i(0), \bar{a}_i, i\right)}{\partial W(0)}(1+\xi)a(0,i) \tag{A.5}$$

Because the consumer may favor a non-flat optimal consumption profile, the optimal level of annuitization is likely to be partial even if the markup ξ is equal to zero. However, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and income are constant.[1]

The value of an annuity depends on a consumer's expected future survival. Life-extension affects the value and cost of a given annuity, and may also affect the level of the optimal annuity. Thus, the effect of the mortality rate perturbation on the marginal utility of life-extension is:

$$\left.\frac{\partial V\left(0, W_i^\varepsilon(0), \bar{a}_i^\varepsilon, i\right)}{\partial \varepsilon}\right|_{\varepsilon=0} = (\text{A.4}) + \left.\frac{\partial V}{\partial \bar{a}_i}\frac{\partial \bar{a}_i^\varepsilon(0)}{\partial \varepsilon}\right|_{\varepsilon=0} + \left.\frac{\partial V}{\partial W_i(0)}\frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon}\right|_{\varepsilon=0}$$

where the first term on the right-hand side is equal to equation (A.4) derived in part (i) above for the case with life-cycle earnings but no annuity. Note that:

$$\begin{aligned}
\left.\frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon}\left(-(1+\xi)\bar{a}_i^\varepsilon \int_0^T \tilde{S}^\varepsilon(i,t)e^{-rt}\left[1 + \sum_{j>i}\Big(\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\Big)a(t,j)\right]dt\right) \\
&= -(1+\xi)\left.\frac{\partial \bar{a}_i^\varepsilon}{\partial \varepsilon}\right|_{\varepsilon=0} a(0,i) - (1+\xi)\bar{a}_i \int_0^T e^{-rt}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left(1 + \sum_{j>i}\lambda_{ij}(t)a(t,j)\right) - \sum_{j>i}\delta_{ij}(t)a(t,j)\right]dt
\end{aligned}$$

Combining this with the first-order condition (A.5) implies that:

$$\left.\frac{\partial V}{\partial \bar{a}_i}\frac{\partial \bar{a}_i^\varepsilon(0)}{\partial \varepsilon}\right|_{\varepsilon=0} + \left.\frac{\partial V}{\partial W_i(0)}\frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon}\right|_{\varepsilon=0} = -\frac{\partial V}{\partial W_i(0)}(1+\xi)\bar{a}_i \int_0^T e^{-rt}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(1 + \sum_{j>i}\lambda_{ij}(t)a(t,j)\right) - \sum_{j>i}\delta_{ij}(t)a(t,j)\right]dt$$

Thus the marginal utility of life-extension is equal to:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = &\int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(u(c_i(t), q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V(t, W_i(t), \bar{a}_i, j)\right) - \sum_{j>i}\delta_{ij}(t)V(t, W_i(t), \bar{a}_i, j)\right]dt \\
&- \frac{\partial V}{\partial W_i(0)}(1+\xi)\bar{a}_i \int_0^T e^{-rt}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(1 + \sum_{j>i}\lambda_{ij}(t)a(t,j)\right) - \sum_{j>i}\delta_{ij}(t)a(t,j)\right]dt
\end{aligned}$$

The marginal utility of wealth, $\partial V/\partial W_i(0)$, is equal to $u_c(c_i(0), q_i(0))$ when the solution is interior. Dividing by the marginal

[1] Even in the case of full annuitization, the first-order condition (A.5) holds with strict equality since the consumer is indifferent between an increase in the annuity level or a proportionate increase in baseline wealth.

utility of wealth and rearranging yields the marginal value of life-extension:

$$\left.\frac{\partial V/\partial \varepsilon}{\partial V/\partial W}\right|_{\varepsilon=0}$$

$$= \int_0^T \tilde{S}(i,t)\left\{\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left[\left(\frac{e^{-\rho t}u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),\bar{a}_i,j)}{u_c(c_i(0),q_i(0))}\right)-(1+\xi)\bar{a}_i\, e^{-rt}\left(1+\sum_{j>i}\lambda_{ij}(t)a(t,j)\right)\right]\right.$$
$$\left. -\sum_{j>i}\delta_{ij}(t)\left(\frac{V(t,W_i(t),\bar{a}_i,j)}{u_c(c_i(0),q_i(0))}-(1+\xi)\bar{a}_i e^{-rt}a(t,j)\right)\right\}dt$$

Choosing the Dirac delta function for $\delta_{i,n+1}(t)$ yields:

$$\begin{aligned} VSL(i) &= \frac{V(0,W_i(0),\bar{a}_i,i)}{u_c(c_i(0),q_i(0))}-(1+\xi)\bar{a}_i\int_0^T \tilde{S}(i,t)e^{-rt}\left(1+\sum_{j>i}\lambda_{ij}(s)a(t,j)\right)dt \\ &= \frac{V(0,W_i(0),\bar{a}_i,i)}{u_c(c_i(0),q_i(0))}-(1+\xi)\bar{a}_i a(0,i) \end{aligned}$$

Likewise, choosing the Dirac delta function for $\delta_{ij}(t),\ j<n+1$, yields:

$$VSI(i,j)=\left(\frac{V(0,W_i(0),\bar{a}_i,i)}{u_c(c_i(0),q_i(0))}-(1+\xi)a(0,i)\bar{a}_i\right)-\left(\frac{V(0,W_i(0),\bar{a}_i,j)}{u_c(c_i(0),q_i(0))}-(1+\xi)a(0,j)\bar{a}_i\right)$$

■

Proof of Proposition 12. Without loss of generality, consider a transition from state 1 to state 2 at time $t=0$. If $\xi=0$, $r=\rho$, and future earnings and quality of life are constant across both time and states, then it is optimal for the consumer to fully annuitize, in which case optimal consumption will be constant:

$$c(t)=m_i(t)+\bar{a}_1=\overline{m}_i+\bar{a}_1=\bar{c}$$

Equation (12) then implies:

$$\begin{aligned} VSL(1,0) &= \mathbb{E}\left[\int_0^T e^{-rt}S(t)\left(\frac{u(\bar{c},q)}{u_c(\bar{c},q)}-\bar{a}_1\right)\middle| Y_0=1\right] \\ &= \left(\frac{u(\bar{c},q)}{u_c(\bar{c},q)}-\bar{a}_1\right)a(0,1) \end{aligned}$$

where $a(0,1)$ is the value of a one-dollar annuity at time $t=0$ in state 1 as defined in the main text. Similarly,

$$VSL(2,0)=\left(\frac{u(\bar{c},q)}{u_c(\bar{c},q)}-\bar{a}_1\right)a(0,2)$$

By assumption, survival in the healthy state is larger than survival in the sick state: $\mathbb{E}[S(t)|Y_0=1]>\mathbb{E}[S(t)|Y_0=2]$. This assumption implies $a(0,2)<a(0,1)$, which in turn implies $VSL(1,0)>VSL(2,0)$.

■

B Data

The empirical exercises presented in Section 3 apply a stochastic life-cycle model to data on mortality, quality of life, and medical spending that varies across 20 different health states. We obtain these data from the Future Elderly Model (FEM).

The FEM follows Americans aged 50 years and older and projects their health and medical spending over time. It has been used by a variety of researchers and policy analysts to understand the future implications of population aging, health trends, new medical technologies, and possible health policy interventions in the US, Europe, and Asia (Goldman et al., 2005; Lakdawalla, Goldman and Shang, 2005; Lakdawalla et al., 2008; Goldman et al., 2009, 2010; Michaud et al., 2011, 2012; Goldman et al., 2013; Goldman and Orszag, 2014; National Academies of Sciences, Engineering, and Medicine, 2015; Chen et al., 2016; Gonzalez-Gonzalez et al., 2017). A complete technical document detailing the FEM is available online.[2] The FEM is a microsimulation that follows the evolution of individual-level health trajectories and economic outcomes, rather than the average or aggregate characteristics of a cohort.

The FEM has three core modules. The first is the Replenishing Cohorts module, which predicts economic and health outcomes of new cohorts of 50-year-olds with data from the Panel Study of Income Dynamics (PSID), and incorporates trends in disease and other outcomes based on data from external sources, such as the National Health Interview Survey and the American Community Survey. This module generates cohorts as the simulation proceeds, so that we can measure outcomes for the age 50+ population in any given year.

The second component is the Health Transition module, which uses the longitudinal structure of the Health and Retirement Study (HRS) to calculate transition probabilities across various health states, including chronic conditions, functional status, body-mass index, and mortality, using linear and nonlinear multivariate regression models. These transition probabilities depend on a battery of predictors: age, sex, education, race, ethnicity, smoking behavior, marital status, employment and health conditions. Baseline factors are also controlled for using a series of initial health variables measured at age 50. FEM transitions produce a large set of simulated outcomes, including diabetes, high-blood pressure, heart disease, cancer (except skin cancer), stroke or transient ischemic attack, and lung disease (either or both chronic bronchitis and emphysema), disability, and body-mass index. Disability is measured by limitations in instrumental activities of daily living, activities of daily living, and residence in a nursing home. This dynamic simulation method has undergone extensive benchmarking and validation.

Finally, the Policy Outcomes module combines individual-level outcomes into aggregate outcomes, such as medical care costs (Medicare, Medicaid and Private), federal, state and property taxes, and Social Security expenditures and contributions. Individual health spending is predicted with regard to health status (chronic conditions and functional status), demographics (age, sex, race, ethnicity and education), nursing home status, and mortality. Estimates are based on spending data from the Medical Expenditure Panel Survey for individuals aged 64 and younger and the Medicare Current Beneficiary Survey for individuals aged 65 and older, who constitute the bulk of the Medicare population. This module has been comprehensively tested against national aggregates.

An example of how the three modules interact is as follows. For year 2014, the model begins with the population of Americans aged 50 and older based on nationally representative data from the HRS. Individual-level health and economic outcomes for the next two years are predicted using the Policy Outcomes module. The cohort is then aged two years using the Health Transition Module. Aggregate health and functional status outcomes for those years are then calculated. At that point, a new cohort of 50-year-olds is introduced into the 2016 population using the Replenishing Cohort module, and they join those who survived from 2014 to 2016. This forms the age 50+ population for 2016. The transition model is then applied to this population. The same process is repeated until reaching the last year of the simulation.

As described in the main text, for the purposes of our analysis we divide the health space within the FEM into $n = 20$ states. We summarize the resulting dataset in Table 1. Figure 3 reports average out-of-pocket medical spending, by age, for a healthy individual in health state 1 and for a very sick individual in health state 20. These spending data include all inpatient, outpatient, prescription drug, and long-term care payments made by the individual, as estimated by the Future Elderly Model. The large increase in spending that occurs after age 80 is due primarily to the large costs of long-term care.

[2]See roybalhealthpolicy.usc.edu/fem/technical-specifications/.

C Supporting Calculations for Quantitative Analysis

This appendix provides the solution to the discrete-time dynamic programming problem described in Section 3.1. This model is solved analytically and thus provides exact solutions for both optimal consumption and VSL.

The consumer's problem is:

$$\max_{c(t)} \mathbb{E}\left[\sum_{t=0}^{T} e^{-\rho t} S_0(t)\, u(c(t), q_{Y_t}(t)) + e^{-\rho(t+1)}\left((S_0(t) - S_0(t+1))\, u\left(W(t+1), b_t\right)\right) \middle| Y_0, W_0\right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t))\, e^{r(t, Y_t)} \end{aligned}$$

where all variables are defined as in the main text. Here, the consumer's problem also includes a bequest motive. The strength of the bequest motive is governed by the parameter b_t. This parameter is set equal to 0 in the baseline specification, which assumes no bequest motive (and normalizes utility of death to zero). The utility function is given by equation (14) from the main text:

$$u(c, q) = q\left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right)$$

where $\underline{c}$ is the subsistence level of consumption for a healthy person. Because optimal consumption is unaffected by affine transformations of utility, we shall initially assume $u(c, q) = q\, c^{1-\gamma}/(1-\gamma)$ when solving the model for consumption.

Define the value function as:

$$V\left(t, W(t), Y_t\right) = \max_{c(s)} \mathbb{E}\left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s)\, u\left(c(s), q_{Y_s}(s)\right) + e^{-\rho(s+1-t)}\left(S_t(s) - S_t\left(s+1\right)\right) u\left(W\left(s+1\right), b_s\right) \middle| Y_t, W(t)\right]$$

subject to:

$$W\left(s+1\right) = \left(W(s) - c(s)\right) e^{r(s, Y_s)},\ s > t, W(s) \geq 0$$

Then we obtain the following Bellman equation:

$$V(t, w, i) = \max_{c(t)}\left\{u(c(t), q_i(t)) + e^{-\rho}\overline{d}_i(t)\, u\left((w - c(t))\, e^{r(t,i)}, b_t\right) + e^{-\rho}\left(1 - \overline{d}_i(t)\right)\sum_{j=1}^{n} p_{ij}(t)\, V\left(t+1, (w - c(t))\, e^{r(t,i)}, j\right)\right\}$$

Proposition C.1. *The value function and the optimal consumption level satisfy:*

$$\begin{aligned} V(t, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} K_{t,i}, \\ c^*(t, w, i) &= w \times c_{t,i} \end{aligned}$$

where:

$$\begin{aligned} c_{t,i} &= \left[1 + e^{-r(t,i)}\left(\frac{e^{r(t,i)}\left[\overline{d}_i(t)\, b_t + \left(1 - \overline{d}_i(t)\right)\left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)\right]}{e^{\rho} q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1}, t < T, \\ c_{T,i} &= \left[1 + e^{-r(t,i)}\left(\frac{e^{r(t,i)}\, b_t}{e^{\rho}\, q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1} \end{aligned}$$

and $K_{t,i}$ satisfies the recursion:

$$K_{t,i} = \left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)} \left[e^{r(t,i)-\rho} \left(\overline{d}_i(t)\, b_t + \left(1-\overline{d}_i(t)\right) \left(\sum_{j=1}^{n} p_{ij}(t)\, K_{t+1,j} \right) \right) \right]^{\frac{1}{\gamma}} \right]^{\gamma},\ t<T, K_{T,i} = \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)} \left(e^{r(T,i)-\rho} b_T \right)^{\frac{1}{\gamma}} \right]^{\gamma}$$

Proof. See Appendix C.1 ∎

When calculating VSL, we incorporate subsistence consumption back into the utility function. In this case, the value function is:

$$\begin{aligned} V(0,w,i) = \sum_{t=0}^{T} e^{-\rho t}\, \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) \frac{c(t)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \\ + e^{-\rho(t+1)}\, \mathbb{E}\left[\left(\exp\left\{ -\int_0^t \mu(s) ds \right\} - \exp\left\{ -\int_0^{t+1} \mu(s) ds \right\} \right) \left(b_t \frac{W(t+1)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \end{aligned} \tag{C.1}$$

Rearranging yields:

$$\begin{aligned} V(0,w,i) &= \sum_{t=0}^{T} e^{-\rho t} \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} q_{Y_t}(t) \frac{c(t)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\ &\quad + e^{-\rho(t+1)}\, b_t\, \mathbb{E}\left[\left(\exp\left\{ -\int_0^t \mu(s) ds \right\} - \exp\left\{ -\int_0^{t+1} \mu(s) ds \right\} \right) \frac{W(t+1)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\ &\quad - \frac{\underline{c}^{1-\gamma}}{1-\gamma} \left[q_{Y_0}(0) + e^{-\rho} b_0 + \sum_{t=1}^{T} e^{-\rho t}\, \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) + e^{-\rho} b_t - b_{t-1} \right) \middle| Y_0 = i \right] \right] \\ &= \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \left[q_{Y_0}(0) + e^{-\rho} b_0 + \sum_{t=1}^{T} e^{-\rho t}\, \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) + e^{-\rho} b_t - b_{t-1} \right) \middle| Y_0 = i \right] \right] \right] \end{aligned}$$

We can then calculate VSL in state i using the following formula:

$$VSL(i) = \frac{V(0,w,i) - b_0 \left(\frac{w^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right)}{u_c(w\, c_{0,i}, q_i(0))} \tag{C.2}$$

The second term in the numerator of (C.2) is the utility at death (the bequest function). When the bequest motive is absent ($b_t \equiv 0$), the value function simplifies to:

$$V(0,w,i) = \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \underbrace{\sum_{t=0}^{T} e^{-\rho t} \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} q_{Y_t}(t) \middle| Y_0 = i \right]}_{\text{discounted quality–adjusted life expectancy in state } i} \right]$$

and the expression for VSL simplifies to equation (7) from the main text.

Once one has calculated VSL, it is straightforward to calculate VSI:

Corollary C.2. *The value of a marginal reduction in the probability of transitioning from state i to state j is equal to:*

$$VSI(i,j) = VSL(i) - VSL(j) \frac{q_j(0)\, c_{0,j}^{-\gamma}}{q_i(0)\, c_{0,i}^{-\gamma}} = VSL(i) - \left(\frac{q_j(0)}{q_i(0)} \right) \left(\frac{c_{0,i}}{c_{0,j}} \right)^{\gamma} VSL(j)$$

Proof. See Appendix C.1 ∎

C.1 Proofs

Proof of Proposition C.1. The proof proceeds by induction on $t \le T$. For the base case $t = T$, note that $\overline{d}_i(t) = 1$, so that the first-order condition from the Bellman equation gives:

$$q_i(T)\,c(T)^{-\gamma} = e^{r(T,i)-\rho}\,b_T(w - c(T))^{-\gamma}\,e^{-r(T,i)\gamma}$$

This implies that:

$$\begin{aligned} c(T) &= \frac{w\,e^{r(T,i)}e^{\frac{(\rho - r(T,i))}{\gamma}}\left(\frac{q_i(T)}{b_T}\right)^{\frac{1}{\gamma}}}{1 + e^{r(T,i)}e^{\frac{(\rho - r(T,i))}{\gamma}}\left(\frac{q_i(T)}{b_T}\right)^{\frac{1}{\gamma}}} \\ &= w\underbrace{\left[1 + e^{-r(T,i)}\left(\frac{e^{r(T,i)}b_T}{e^{\rho}q_i(T)}\right)^{\frac{1}{\gamma}}\right]^{-1}}_{c_{T,i}} \end{aligned}$$

Hence, we obtain:

$$\begin{aligned} V(T,w,i) &= \frac{w^{1-\gamma}}{1-\gamma}\left(q_i(T)\,c_{T,i}^{1-\gamma} + e^{-\rho}\,b_T\,e^{r(T,i)(1-\gamma)}(1 - c_{T,i})^{1-\gamma}\right) \\ &= \frac{e^{-\rho}e^{r(T,i)(1-\gamma)}}{\left[b_T^{\frac{1}{\gamma}} + e^{r(T,i)}e^{\frac{(\rho - r(T,i))}{\gamma}}\,q_i(T)^{\frac{1}{\gamma}}\right]^{-\gamma}} \\ &= \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)}\left(e^{(r(T,i)-\rho)}b_T\right)^{\frac{1}{\gamma}}\right]^{\gamma} \end{aligned}$$

For the induction step, suppose the proposition is true for case $t+1$. We have:

$$V(t,w,i) = \max_c\left\{q_i(t)\frac{c^{1-\gamma}}{1-\gamma} + b_t\,e^{-\rho}\,\overline{d}_i(t)\frac{\left((w-c)\,e^{r(t,i)}\right)^{1-\gamma}}{1-\gamma} + e^{-\rho}\left(1 - \overline{d}_i(t)\right)\sum_{j=1}^{n}p_{ij}(t)\,\frac{K_{t+1,j}}{1-\gamma}\left[(w-c)\,e^{r(t,i)}\right]^{1-\gamma}\right\}$$

From the first-order condition we obtain:

$$q_i(t)\,c^{-\gamma} = b_t e^{r(t,i)-\rho}\overline{d}_i(t)\,e^{-r(t,i)\gamma}(w-c)^{-\gamma} + e^{r(t,i)-\rho}\left(1 - \overline{d}_i(t)\right)e^{-\gamma r(t,i)}\,(w-c)^{-\gamma}\sum_{j=i}^{n}p_{ij}(t)\,K_{t+1,j}$$

Rearranging yields:

$$q_i(t)\,c^{-\gamma} = (w-c)^{-\gamma}\,e^{r(t,i)-\rho}e^{-r(t,i)\gamma}\left[\overline{d}_i(t)\,b_t + \left(1 - \overline{d}_i(t)\right)\sum_{j=i}^{n}p_{ij}(t)\,K_{t+1,j}\right]$$

which implies:

$$q_i(t)^{-1/\gamma}\,c = (w-c)\,e^{(\rho - r(t,i))/\gamma}e^{r(T,i)}\left[\overline{d}_i(t)\,b_t + \left(1 - \overline{d}_i(t)\right)\sum_{j=i}^{n}p_{ij}(t)\,K_{t+1,j}\right]^{-1/\gamma}$$

Rearranging further yields:

$$
\begin{aligned}
c &= w \times \frac{e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-1/\gamma}}{e^{\rho}\,q_i(t)^{-1/\gamma} + e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-1/\gamma}} \\
&= w \times \underbrace{\left[1 + e^{-r(t,i)}\left(\frac{e^{r(t,i)}\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]}{e^{\rho} q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1}}_{c_{t,i}}
\end{aligned}
$$

Thus we obtain:

$$
\begin{aligned}
V(t,w,i) &= q_i(t)\,c_{t,i}^{1-\gamma}\,\frac{w^{1-\gamma}}{1-\gamma} + b_t\,e^{-\rho}\,\overline{d}_i(t)\,\frac{w^{1-\gamma}}{1-\gamma}\Big(1-c_{t,i}\Big)^{1-\gamma} e^{r(t,i)(1-\gamma)} + e^{-\rho}\,\Big(1-\overline{d}_i(t)\Big)\,\frac{w^{1-\gamma}}{1-\gamma}\Big(1-c_{t,i}\Big)^{1-\gamma} e^{r(t,i)(1-\gamma)} \sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j} \\
&= \frac{w^{1-\gamma}}{1-\gamma}\left[q_i(t)\,c_{t,i}^{1-\gamma} + e^{-\rho}\Big(1-c_{t,i}\Big)^{1-\gamma} e^{r(t,i)(1-\gamma)}\left[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\right]\right] \\
&= \frac{w^{1-\gamma}}{1-\gamma}\,\frac{q_i(t)\,e^{r(t,i)(1-\gamma)}\Big[e^{r(T,i)}\Big(\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big)\Big]^{1-1/\gamma} + e^{-\rho}\,e^{r(t,i)(1-\gamma)}(e^{\rho}q_i(t))^{1-1/\gamma}\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]}{\left[(e^{\rho}q_i(t))^{-1/\gamma} + e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-\frac{1}{\gamma}}\right]^{1-\gamma}} \\
&= \frac{w^{1-\gamma}}{1-\gamma}\,\frac{e^{r(t,i)(1-\gamma)}q_i(t)\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]}{\left[(e^{\rho}q_i(t))^{-1/\gamma} + e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-\frac{1}{\gamma}}\right]^{-\gamma}} \\
&= \frac{w^{1-\gamma}}{1-\gamma}\underbrace{\left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)}\left[e^{r(t,i)-\rho}\left(\overline{d}_i(t)\,b_t + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\right)\right]^{\frac{1}{\gamma}}\right]^{\gamma}}_{K_{t,i}}
\end{aligned}
$$

■

Proof of Corollary C.2. The proof follows by the equation for VSI (8) and using that $u_c(c_i(0), q_i(0)) = q_i(0)\,c_{i,0}^{-\gamma}\,w^{-\gamma}$. ■

D Complete Markets Model

We assume a full menu of actuarially fair annuities is available, where consumers can choose consumption streams, $c(t)$, that depend on the evolution of their health state. Thus, the consumer is able to fully insure against consumption risk. The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t)\, u(c(t), q_{Y_t}(t))\, dt \,\middle|\, Y_0 \right] \tag{D.1}$$

subject to:

$$\mathbb{E}\left[\int_0^T e^{-rt} S(t) c(t)\, dt \,\middle|\, Y_0 \right] = W_0 + \mathbb{E}\left[\int_0^T e^{-rt} S(t) m_{Y_t}(t)\, dt \,\middle|\, Y_0 \right] \equiv \overline{W}(0, Y_0)$$

where $\overline{W}(0, Y_0)$ is the net present value of wealth and future earnings.

The consumer chooses the consumption profile at time t based on her health state, $Y_t = i$, and on her available wealth, $\overline{W}(t,i)$. We define the present value of future earnings as:

$$M(t,i) = \mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{ -\int_t^u \mu(s)\, ds \right\} m_{Y_u}(u) du \,\middle|\, Y_t = i \right]$$

Her available wealth finances future consumption such that:

$$\overline{W}(t,i) = \mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{ -\int_t^u \mu(s) ds \right\} c(u)\, du \,\middle|\, Y_t, \overline{W}(t,i) \right]$$

Lemma D.1. *The law of motion for wealth is:*

$$\frac{\partial \overline{W}(t,i)}{\partial t} = r\overline{W}(t,i) - c\left(t, \overline{W}(t,i), i\right) + \sum_{j>i} \lambda_{ij}(t) \left[\overline{W}(t,i) - \overline{W}(t,j) \right],\ i = 1, \dots, n,\ \overline{W}(t, n+1) = 0\, \forall t$$

Proof. See Appendix D.1 ∎

Note that the dynamics for $\overline{W}(t,i)$ will depend on $\overline{W}(t,j)$, $j > i$, so that $\left(Y_t, \overline{W}(t, Y_t)\right)$ is not Markov, but $\left(Y_t, \overline{W}(t)\right)$, where we define the wealth vector $\overline{W}(t) \equiv \left(\overline{W}(t,1), \dots, \overline{W}(t, n+1)\right)$, is Markov.

Define the optimal value-to-go function as:

$$V\left(t, \overline{W}(t), Y_t\right) = \max_{c(u)} \mathbb{E}\left[\int_t^T e^{-\rho(u-t)} \exp\left\{ -\int_t^u \mu(s)\, ds \right\} u\left(c(u), q_{Y_u}(u)\right) du \,\middle|\, Y_t, \overline{W}(t) \right]$$

subject to the law of motion for wealth given above. As a stochastic dynamic programming problem, $V(\cdot)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} \rho V(t, \overline{W}(t), i) = \frac{\partial V(t, \overline{W}(t), i)}{\partial t} + \max_{c(t)} \Bigg\{ & u(c(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) \left[V(t, \overline{W}(t), j) - V(t, \overline{W}(t), i) \right] \\ & + \sum_{k \geq i} \frac{\partial V(t, \overline{W}(t), i)}{\partial \overline{W}(t,k)} \left[r\overline{W}(t,k) - c(t) + \sum_{l>k} \lambda_{kl}(t) \left[\overline{W}(t,k) - \overline{W}(t,l) \right] \right] \Bigg\},\ 1 \leq i \leq n \end{aligned} \tag{D.2}$$

where $V(t, \overline{W}(t), n+1) = 0$. Similarly to the uninsured case presented in the main text, we follow Parpas and Webster (2013) and focus on the path of Y that begins in i and remains in i until time t, with $c_i(t)$ and $\overline{W}_i(t)$ denoting the corresponding optimal consumption and wealth paths. We take optimal consumption rules and value functions from other states as exogenous. As in the uninsured case, this approach will allow us to apply the standard Pontryagin maximum principle and derive analytic expressions.

Lemma D.2. *The optimal value function for $Y_0 = i$, $V\left(0, \overline{W}(0,i), i\right)$, for the following deterministic optimization problem also satisfies the HJB given by (D.2), for each $i \in \{1, \ldots, n\}$:*

$$V\left(0, \overline{W}(0,i), i\right) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right) dt \right] \tag{D.3}$$

subject to:

$$\frac{\partial \overline{W}_i(t,j)}{\partial t} = r\overline{W}_i(t,j) - c\left(t, \overline{W}_i(t), j\right) + \sum_{k>j} \lambda_{jk}(t) \left[\overline{W}_i(t,j) - \overline{W}_i(t,k)\right],\ j > i$$

$$\frac{\partial \overline{W}_i(t,i)}{\partial t} = r\overline{W}_i(t,i) - c_i(t) + \sum_{k>i} \lambda_{ik}(t) \left[\overline{W}_i(t,i) - \overline{W}_i(t,k)\right]$$

where $V\left(t, \overline{W}_i(t), j\right)$ and $c\left(t, \overline{W}_i(t), j\right)$, $j > i$, are taken as exogenous.

Proof. See Appendix D.1 ∎

Following Bertsekas (2005), the Hamiltonian for the (deterministic) maximization problem (D.3) is:

$$\begin{aligned} H\left(\overline{W}_i(t), c_i(t), p_i(t)\right) &= e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right) \\ &\quad + \sum_{k>i} p_i(t,k) \left[r\overline{W}_i(t,k) - c\left(t, \overline{W}_i(t), k\right) + \sum_{l>k} \lambda_{kl}(t) \left[\overline{W}_i(t,k) - \overline{W}_i(t,l)\right] \right] \\ &\quad + p_i(t,i) \left[r\overline{W}_i(t,k) - c_i(t) + \sum_{l>i} \lambda_{il}(t) \left[\overline{W}_i(t,i) - \overline{W}_i(t,l)\right] \right] \end{aligned} \tag{D.4}$$

where $p_i(t) = (p_i(t,1), \ldots, p_i(t,n))$ is the vector of costate variables corresponding to wealth $\overline{W}_i(t)$.

Lemma D.3. *We have that $p_i(t,i) = \theta e^{-\rho t} \tilde{S}(i,t)$ for θ independent of i, and $p_i(t,k) = 0, k \neq i$. The necessary first-order condition for consumption is:*

$$e^{(r-\rho)t} u_c(c_i(t), q_i(t)) = \theta \tag{D.5}$$

where $\theta = p_i(0,i) = \partial V\left(0, \overline{W}_i(0), i\right) / \partial \overline{W}(0,i)$ is the marginal utility of wealth.

Proof. See Appendix D.1 ∎

Equation (D.5) shows that the discounted marginal utility of consumption is constant within the path that remains in state i. The following result extends this insight by showing that the same is true across states.

Lemma D.4. *The first-order condition (D.5) holds across different states. That is, if a consumer transitions from state i to state j, then $u_c\left(c(t,i,\overline{W}(t)), q_i(t)\right) = u_c\left(c(t,j,\overline{W}(t)), q_j(t)\right)\ \forall j$.*

Proof. See Appendix D.1 ∎

To analyze the values of life and illness, let $\delta_{ij}(t)$, $i < j$, $i \leq n$, $j \leq n+1$, be a perturbation on the transition intensity $\lambda_{ij}(t)$, where $\sum_{j>i} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[- \int_0^t \sum_{j>i} \left(\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)\right) ds \right], \text{ where } \varepsilon > 0$$

Proposition D.5. *The marginal utility of preventing an illness or death is given by:*

$$\frac{\partial V}{\partial \varepsilon}\bigg|_{\varepsilon=0} = \int_0^T \left(\tilde{S}(i,t)\left\{ e^{-\rho t}\left[u(c_i(t),q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V\left(t,\overline{W}_i(t),j\right)\right] + \theta e^{-rt}\left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right]\right]\right\} - \tilde{S}(i,t)\sum_{j>i}\delta_{ij}(t)\left\{ e^{-\rho t}V\left(t,\overline{W}_i(t),j\right) - \theta e^{-rt}\left[\overline{W}_i(t,j) - M(t,j)\right]\right\}\right) dt \tag{D.6}$$

Proof. See Appendix D.1 ■

To obtain the value of statistical life (VSL), we first set $\delta_{i,N+1}$ equal to the Dirac delta function, and set all other perturbations equal to 0. Dividing the result by the marginal utility of wealth, θ, then yields:

$$VSL = \int_0^T \tilde{S}(i,t)e^{-rt}\left\{\left[\frac{u(c_i(t),q_i(t))}{u_c(c_i(t),q_i(t))} + \sum_{j>i}\lambda_{ij}(t)\frac{V\left(t,\overline{W}_i(t),j\right)}{\partial V\left(t,\overline{W}_i(t),j\right)/\partial \overline{W}_i(t,j)}\right] + \left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right]\right]\right\} dt \tag{D.7}$$

$$= \frac{V\left(0,\overline{W}_i(0),i\right)}{u_c(c_i(0),q_i(0))} - W_0$$

$$= \mathbb{E}\left[\int_0^T e^{-rt}S(t)v(t)dt \,\middle|\, Y_0 = i\right]$$

where the the value of a one-period change in survival from the perspective of current time is:

$$v(t) = \frac{u(c(t),q_{Y_t}(t))}{u_c\left(c(t),q_{Y_t}(t)\right)} + m_{Y_t}(t) - c_{Y_t}(t)$$

Equation (D.7) reveals that generalizing the standard (deterministic) model to account for stochastic health risk alone does not alter the basic expression for VSL (Murphy and Topel, 2006). We can obtain the life-cycle profile of consumption in state i by differentiating the first-order condition (D.5) with respect to t:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q} \tag{D.8}$$

As in the case where health is deterministic, full annuitization insulates consumption from mortality risk. Thus, our results demonstrate that stochastic health risk, by itself, does not alter the basic insights regarding VSL offered by the prior literature as long as one maintains the assumption of full annuitization.

However, a novel feature of the stochastic model is that it permits an investigation into the value of prevention. Inspecting the expression for the marginal utility of life extension (D.6), the first term inside the integral represents the gain in marginal utility from a reduction in the probability of exiting state i. The second term represents the loss in marginal utility from the reduction in probability of transitioning to other possible states. The net effect depends on the consumer's marginal utility in the different states.

To analyze the value of prevention, consider a reduction in the transition probability for only one alternative state, j, so that $\delta_{ik}(t) = 0\ \forall k \neq j$. The value of avoiding illness j is then equal to:

$$VSI(i,j) = \int_0^T \tilde{S}(i,t)e^{-rt}\left\{\left[\frac{u(c_i(t),q_i(t))}{u_c(c_i(t),q_i(t))} + \sum_{j>i}\lambda_{ij}(t)\frac{V\left(t,\overline{W}_i(t),j\right)}{\frac{\partial V(t,\overline{W}_i(t),j)}{\partial \overline{W}_i(t,j)}}\right] + \left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right]\right]\right\}dt \qquad \text{(D.9)}$$
$$- \left[\frac{V\left(0,\overline{W}_i(0),j\right)}{\theta} - \left[\overline{W}_i(0,j) - M(0,j)\right]\right]$$
$$= \frac{V\left(0,\overline{W}_i(0),i\right)}{u_c(c_i(0),q_i(0))} - W_0 - \left(\frac{V\left(0,\overline{W}_i(0),j\right)}{u_c(c_i(0),q_i(0))} - \left[\overline{W}_i(0,j) - M(0,j)\right]\right)$$
$$= VSL(i) - VSL\left(j|W_0 = \overline{W}_i(0,j) - M(0,j)\right)$$

Thus, equation (D.9) demonstrates that $VSI(i,j)$ is equal to the difference in VSL for states i and j, with the caveat that VSL in state j uses a measure of wealth evaluated from the perspective of a person in state i. This technicality arises because the value of the consumer's annuity depends on her expected survival. For example, an annuity is worth more to a healthy 65-year-old than it is to a 65-year-old who was just diagnosed with lung cancer.

A constant value for QALYs arises only when the utility of consumption is constant (Bleichrodt and Quiggin, 1999). Thus, the value of a QALY will not generally be constant in this setting because consumption is not constant, as shown by equation (D.8). The following proposition shows that when markets are complete, the value of a QALY is constant if the rate of time preference equals the interest rate, life-cycle earnings are constant, and quality of life is constant.

Proposition D.6. *Suppose that $r = \rho$, that life-cycle earnings are constant, and that quality of life is constant, so that consumption is constant across states and time. Let D_i be quality-adjusted life expectancy in state i, as defined in Section 2.3. Then the values per QALY of reducing illness risk, ex ante mortality risk, and ex post mortality risk are the same:*

$$\frac{VSI(i,j)}{D_i - D_j} = \frac{VSL(i)}{D_i} = \frac{VSL\left(j|W_0 = \overline{W}_i(0,j) - M(0,j)\right)}{D_j}\ \forall i < j$$

Proof. See Appendix D.1 ■

D.1 Proofs

Proof of Lemma D.1. Available wealth can be written as:

$$\overline{W}(t,i) = \int_t^T \exp\left\{-\int_t^u r + \sum_{j>i}\lambda_{ij}(s)\,ds\right\}\left[c_i(t,u) + \sum_{j>i}\lambda_{ij}(u)\overline{W}_i(u,t,j)\right]du$$

where with a slight abuse of notation, $c_i(t,u)$ and $\overline{W}_i(u,t,j)$ denote the consumption and wealth paths for an individual who is in state i at time t and remains in state i until time u—but jumps to state j at time u for the latter. The result then follows by taking the derivative with respect to t. ■

Proof of Lemma D.2. This proof follows the same logic as the proof of Lemma 1 in Appendix A. Consider the deterministic optimization problem (D.3). Denote the optimal value-to-go function as:

$$\overline{V}\left(t,\overline{W}_i(t),i\right) = \max_{c_i(t)}\left\{\int_t^T e^{-\rho u}\tilde{S}(i,u)\left(u(c_i(u),q_i(u)) + \sum_{j>i}\lambda_{ij}(u)V\left(u,\overline{W}_i(u),j\right)\right)du\right\}$$

Setting $\overline{V}\left(t,\overline{W}_i(t),i\right) = e^{-\rho t}\tilde{S}(i,t)V\left(t,\overline{W}_i(t),i\right)$ then demonstrates that $V(\cdot)$ satisfies the HJB (D.2) for i.

■

Proof of Lemma D.3. The costate equations for the Hamiltonian (D.4) are:

$$\dot{p}_i(t,i) = -\left[r + \sum_{j>i} \lambda_{ij}(t)\right] p_i(t,i),$$

$$\dot{p}_i(t,k) = -e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V\left(t,\overline{W}_i(t),j\right)}{\partial \overline{W}_i(t,k)} + \sum_{k\geq j>i} p_i(t,j)\left(\frac{\partial c\left(t,\overline{W}_i(t),j\right)}{\partial \overline{W}_i(t,k)} + \lambda_{jk}(t)\right) - p_i(t,k)\left[r + \sum_{l>k}\lambda_{kl}(t)\right] + p_i(T,i)\lambda_{ik}(t)$$

for $k > i$. From the first costate equation, we obtain:

$$p_i(t,i) = e^{-rt}\tilde{S}(i,t)\theta$$

Taking first-order conditions in the Hamiltonian (D.4) and plugging this in then yields:

$$u_c(c_i(t), q_i(t)) = \frac{\partial V\left(t,\overline{W}_i(t),i\right)}{\partial \overline{W}_i(t,i)} = e^{(\rho-r)t}\theta$$

To see that this solution works, let θ be constant across states, and set $p_i(t,k) = 0 = \frac{\partial V\left(t,\overline{W}_i(t),i\right)}{\partial \overline{W}_i(t,k)}$. This then satisfies the costate equation system across i, k, and t. In particular, for the second equation we obtain:

$$\dot{p}_i(t,k) = -e^{-\rho t}\tilde{S}(i,t)\lambda_{ik}(t)\underbrace{\frac{\partial V\left(t,\overline{W}_i(t),k\right)}{\partial \overline{W}_i(t,k)}}_{e^{(\rho-r)t}\theta} + \lambda_{ik}(t)p_i(t,i) = 0$$

■

Proof of Lemma D.4. With Lemma D.3, the HJB (D.2) takes the form:

$$\rho V\left(t,\overline{W}(T,i),i\right) = \frac{\partial V(t,\overline{W}(t,i),i)}{\partial t}$$
$$+\max_{c(t)}\left\{u(c(t),q_i(t)) + \sum_{j>i}\lambda_{ij}(t)\left[V(t,\overline{W}(t,j),j) - V\left(t,\overline{W}(t,i),i\right)\right] + \frac{\partial V\left(t,\overline{W}(t,i),i\right)}{\partial \overline{W}(t,i)}\left[r\overline{W}(t,i) - c(t) + \sum_{k>i}\lambda_{ik}(t)\left[\overline{W}(t,i) - \overline{W}(t,k)\right]\right]\right\},\ 1\leq i\leq n$$

By taking the first-order condition, we get:

$$u_c\left(c(t),q_i(t)\right) = u_c\left(c(t,i,\overline{W}(t)),q_i(t)\right) = \frac{\partial V(t,\overline{W}(t,i),i)}{\partial \overline{W}(t,i)}$$

Furthermore, differentiating the HJB (D.2) with respect to $\overline{W}(t,j)$, j fixed, we get:

$$\frac{\partial V(t,\overline{W}(t,j),j)}{\partial \overline{W}(t,j)} = \frac{\partial V(t,\overline{W}(t,i),i)}{\partial \overline{W}(t,i)}$$

Combining these last two results completes the proof:

$$u_c\left(c(t,i,\overline{W}(t)),q_i(t)\right) = u_c\left(c(t,j,\overline{W}(t)),q_j(t)\right)$$

■

Proof of Proposition D.5. Starting from equation (D.3), we have:

$$V^{\varepsilon}\left(0,\overline{W}_i(0,i),i\right)=\int_0^T e^{-\rho t}\exp\left\{-\int_0^t\sum_{j>i}\lambda_{ij}(s)-\varepsilon\sum_{j>i}\delta_{ij}(s)\,ds\right\}\left[u\left(c_i^{\varepsilon}(t),q_i(t)\right)+\sum_{j>i}\left[\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\right]V\left(t,\overline{W}_i^{\varepsilon}(t),j\right)\right]dt$$

where $c_i^{\varepsilon}(t)$ and $\overline{W}_i^{\varepsilon}(t)$ represent the equilibrium variations in $c_i(t)$ and $\overline{W}_i(t)$ caused by the perturbation, $\delta_{ij}(t)$. Differentiating then yields:

$$\frac{\partial V}{\partial\varepsilon}\bigg|_{\varepsilon=0}=\int_0^T e^{-\rho t}\tilde{S}(i,t)\left[u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V\left(t,\overline{W}_i(t),j\right)\right]\left[\sum_{j>i}\int_0^t\delta_{ij}(s)ds\right]-e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\delta_{ij}(t)V\left(t,\overline{W}_i(t),j\right)+e^{-\rho t}\tilde{S}(i,t)\left[\underbrace{u_c(c_i(t),q_i(t))}_{e^{-(r-\rho)t}\theta}\frac{\partial c_i^{\varepsilon}(t)}{\partial\varepsilon}\bigg|_{\varepsilon=0}+\sum_{j>i}\lambda_{ij}(t)\underbrace{\frac{\partial V\left(t,\overline{W}_i(t),j\right)}{\partial\overline{W}_i(t,j)}}_{e^{-(r-\rho)t}\theta}\frac{\partial\overline{W}_i(t,j)}{\partial\varepsilon}\bigg|_{\varepsilon=0}\right]dt$$

We have:

$$\begin{aligned}
W_0&=\mathbb{E}\left[\int_0^T e^{-rt}S(t)\left[c(t)-m_{Y_t}(t)\right]dt\,\middle|\,Y_0=i\right]\\
&=\int_0^T e^{-rt-\int_0^t\sum_{j>i}\lambda_{ij}(s)ds}\left(c_i(t)-m_i(t)\right)dt+\sum_{j>i}e^{-rt-\int_0^t\sum_{j>i}\lambda_{ij}(s)ds}\lambda_{ij}(t)\underbrace{\mathbb{E}\left[\int_t^T e^{-r(u-t)}\exp\left\{-\int_t^u\mu(s)\,ds\right\}c(u)\,du\,\middle|\,Y_t=j\right]}_{\overline{W}_i(t,j)}\\
&\quad-\sum_{j>i}e^{-rt-\int_0^t\sum_{j>i}\lambda_{ij}(s)ds}\lambda_{ij}(t)\underbrace{\mathbb{E}\left[\int_t^T e^{-r(u-t)}\exp\left\{-\int_t^u\mu(s)\,ds\right\}m_{Y_t}(u)\,du\,\middle|\,Y_t=j\right]}_{M(t,j)}\\
&=\int_0^T e^{-rt-\int_0^t\sum_{j>i}\lambda_{ij}(s)ds}\left[c_i(t)-m_i(t)+\sum_{j>i}\lambda_{ij}(t)\left(\overline{W}_i(t,j)-M(t,j)\right)\right]dt
\end{aligned}$$

The budget constraint then implies:

$$\begin{aligned}
0&=\frac{\partial W_0}{\partial\varepsilon}\bigg|_{\varepsilon=0}\\
&=\frac{\partial}{\partial\varepsilon}\int_0^T e^{-rt}\exp\left\{-\int_0^t\sum_{j>i}\lambda_{ij}(s)-\varepsilon\sum_{j>i}\delta_{ij}(s)\,ds\right\}\left(c_i^{\varepsilon}(t)-m_i(t)+\sum_{j>i}\left[\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\right]\left(\overline{W}_i^{\varepsilon}(t,j)-M(t,j)\right)\right)dt\,\Bigg|_{\varepsilon=0}\\
&=\int_0^T\Bigg(e^{-rt}\tilde{S}(i,t)\left[c_i(t)-m_i(t)+\sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j)-M(t,j)\right]\right]\left[\sum_{j>i}\int_0^t\delta_{ij}(s)ds\right]\\
&\qquad-e^{-rt}\tilde{S}(i,t)\sum_{j>i}\delta_{ij}(t)\left[\overline{W}_i(t,j)-M(t,j)\right]+e^{-rt}\tilde{S}(i,t)\left[\frac{\partial c_i^{\varepsilon}(t)}{\partial\varepsilon}\bigg|_{\varepsilon=0}+\sum_{j>i}\lambda_{ij}(t)\frac{\partial\overline{W}_i^{\varepsilon}(t,j)}{\partial\varepsilon}\bigg|_{\varepsilon=0}\right]\Bigg)dt
\end{aligned}$$

Plugging this last result into the expression for $\frac{\partial V}{\partial\varepsilon}\big|_{\varepsilon=0}$ then yields the desired result for marginal utility:

$$\begin{aligned}
\frac{\partial V}{\partial\varepsilon}\bigg|_{\varepsilon=0}=\int_0^T&\Bigg(\tilde{S}(i,t)\left[\sum_{j>i}\int_0^t\delta_{ij}(s)ds\right]\left\{e^{-\rho t}\left[u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V\left(t,\overline{W}_i(t,j),j\right)\right]+\theta e^{-rt}\left[m_i(t)-c_i(t)-\sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j)-M(t,j)\right]\right]\right\}\\
&-\tilde{S}(i,t)\left\{e^{-\rho t}\sum_{j>i}\delta_{ij}(t)V\left(t,\overline{W}_i(t),j\right)-\theta e^{-rt}\sum_{j>i}\delta_{ij}(t)\left[\overline{W}_i(t,j)-M(t,j)\right]\right\}\Bigg)dt
\end{aligned}$$

■

Proof of Proposition D.6. As in Proposition 8, define the value of prevention as $VSI(i,j)/(D_i - D_j)$ and the value of treatment as $VSL(j)/D_j$, where D_i is the quality-adjusted discounted life expectancy in state i. Note that the following equality:

$$\frac{VSI(i,j)}{D_i - D_j} = \frac{VSL(i) - VSL\left(j|W_0 = \overline{W}_i(0,j) - M(0,j)\right)}{D_i - D_j} = \frac{VSL\left(j|W_0 = \overline{W}_i(0,j) - M(0,j)\right))}{D_j}$$

will be true only under the following condition:

$$\frac{VSL(i)}{VSL\left(j|W_0 = \overline{W}_i(0,j) - M(0,j)\right)} = \frac{D_i}{D_j}$$

Suppose that $r = \rho$, quality of life is constant, and life-cycle earnings are constant. Then, $v(t)$ in the third equality in equation (D.7) is constant across states and time and the proposition must hold. ■