

Health Risk and the Value of Life^{*}

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Abstract

We develop a stochastic life-cycle framework for valuing health and longevity improvements and apply it to data on mortality, quality of life, and medical spending for adults with different comorbidities. We find that sick adults are willing to pay over two times more per quality-adjusted life-year (QALY) to reduce mortality risk than healthy adults, and that prevention of serious illness risk is worth more per QALY than prevention of mild illness risk. Our results provide a rational explanation for why people oppose a single threshold value for rationing care and why they invest less in prevention than in treatment.

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1 Introduction

The economic analysis of risks to life and health has made enormous contributions to academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity ([Murphy and Topel, 2006](#)). Economic concepts such as the value of statistical life (VSL) play central roles in discussions surrounding public and private investments in medical care, public safety, environmental hazards, and countless other arenas.

However, the conventional life-cycle framework used to study the value of life does not accommodate unforeseen changes in health. As a result, it is ill-equipped to investigate how VSL varies with underlying health, and it cannot meaningfully distinguish between preventive care and medical treatment or between illness and death. By contrast, survey research suggests that the value people place on health gains varies considerably with health state, which has led to controversial debates in numerous countries about whether reimbursements for a fixed health gain should vary by disease severity ([Nord et al., 1995](#); [Buxton and Chambers, 2011](#); [Linley and Hughes, 2013](#)). And, an array of evidence suggests that society invests less in prevention than treatment, even when both have the same health benefits ([Weisbrod, 1991](#); [Dranove, 1998](#); [Pryor and Volpp, 2018](#)). The conventional framework’s failure to explain this apparent underinvestment in prevention has led researchers to posit alternative behavioral or market failure explanations, although the evidence remains inconclusive ([Fang and Wang, 2015](#); [Newhouse, 2021](#)).

This paper provides a rational explanation for why and how the value of health varies systematically with health state and disease risk, and quantifies what this insight means for the value of different types of health risk reductions. We develop a new stochastic life-cycle framework that introduces heterogeneity along two connected dimensions. First, individuals can fall ill, which alters their life-cycle consumption patterns and generates empirically meaningful heterogeneity in their ex post willingness to pay for health improvements. Second, in addition to the usual risk of death, individuals face multiple illness risks with heterogeneous effects on health and longevity. When individuals are risk averse, the ex ante willingness to pay to reduce illness risk rises with the severity of the illness.

We apply our model to individual-level data from a representative cohort of US adults ages 50–80. We quantify each individual’s marginal willingness to pay for the prevention and treatment of twenty different health conditions with varying mortality, quality of life, and financial risk profiles. We measure health improvements using quality-adjusted

life-years (QALYs), which combine longevity and morbidity into a single metric. We find that VSL rises on average by \$63,000 (21%) per QALY in the year following an adverse health shock, and by over \$177,000 (51%) per QALY following the worst five percent of shocks. Among 70-year-olds, those in the sickest health state are willing to pay 2.4 times more per QALY to reduce mortality risk than healthy people. Similar patterns prevail for preventive investments, although the differences are smaller than for treatment: reducing extreme risks such as serious cancer or death is worth up to \$36,000 (16%) more per QALY than reducing mild risks such as developing hypertension. These results indicate that the willingness to pay for a fixed health improvement is substantially larger when treating sicker individuals and when preventing more serious illnesses, and contrast with conventional valuation methods that assume independence between disease severity and the value of health when pricing health care reimbursements ([Garber and Phelps, 1997](#)).

The first half of this paper presents our theoretical framework. Our key theoretical innovation is the introduction of multiple health states. In addition to facing mortality risk, individuals in our model face different illness risks with heterogeneous health consequences. We derive the value of statistical illness (VSI), which measures the willingness to pay to reduce illness risk and includes VSL as a special case where that risk is death. We focus initially on a setting without financial markets, and then later extend our results to a more realistic setting with incomplete financial markets. In both settings, we find that the value of reducing a health risk varies with baseline health and with risk severity. We formalize this insight by establishing three theoretical results.

First, we provide a sufficient condition under which VSL rises following an adverse shock to longevity. This condition depends on consumer risk preferences and holds under a wide range of common parameterizations, including isoelastic utility. Intuitively, longevity shocks have two countervailing effects on VSL, and our condition clarifies when one dominates the other. On the one hand, a shorter lifespan reduces the lifetime utility of life-extension. On the other hand, a shorter lifespan reduces future consumption opportunities, which increases willingness to pay for health and longevity.

Second, we show that the value of reducing illness risk will be higher for those in worse health, if people are risk averse over illness severity. Heuristically, risk aversion over illness means consumers prefer living with mild illness for certain to living in good health with the threat of severe illness looming. Third, just as a risk-averse individual is willing to pay more per dollar to insure larger losses, an individual who is risk averse over illness severity is willing to pay more per unit of health improvement to reduce the risk of more serious illnesses.

While our theoretical framework is fully general, quantifying the value of health im-

provements requires making assumptions about preferences and consulting data. The second half of this paper parameterizes our model using standard assumptions and applies it to microsimulation data produced by the Future Elderly Model (FEM). FEM builds on nationally representative, individual-level data to provide detailed information on how mortality, medical spending, and quality of life evolve over the life cycle for people over age 50 with different comorbidities. The underlying data include more comprehensive information than any single national survey and have been widely used to study elderly health and medical spending (e.g., [Goldman et al., 2010, 2013](#); [Leaf et al., 2020](#)).

We first assign individuals from the FEM to one of twenty possible health states based on their number and type of comorbidities. We then conduct a simulation with a cohort of 50,000 adults who are representative of the US population. Each person's health path evolves at random over the life cycle according to the FEM's estimated transition probabilities, and each health state transition (health shock) is accompanied by a change in mortality risk, quality of life, and medical spending as estimated by the FEM. At age 70, the inter-vigintile range for VSL spans \$0.4 million to \$2.9 million. This heterogeneity in values is caused by differences in initial health and wealth at age 50, as well as by differences in the evolution of health over time.

To isolate the effect of health shocks, we focus the rest of our analysis on the 22,214 individuals who are initially healthy at age 50 (no comorbidities) and have the same initial wealth. This cohort experiences about 58,000 health shocks between the ages of 50 and 80. Because the effect of a shock depends on an individual's health path, which varies markedly across individuals over time, our exercise results in significant heterogeneity even for this initially identical group of individuals. On average, VSL per QALY rises by \$72,000 (21%) following a shock, and by over \$177,000 (51%) following the worst five percent of shocks. By age 70, VSL per QALY ranges from \$260,000 for healthy individuals to \$660,000 for people in the worst health state.¹ For a healthy 70-year-old who never fell ill, the marginal value of reducing the risk of illness (VSI) ranges from \$224,000 per QALY for mild illness risk to \$260,000 per QALY for the most extreme possible risk, death. While the absolute values of our estimates are moderately sensitive to alternative assumptions about consumer risk preferences or the presence of a bequest motive, our qualitative conclusions—that the value of reducing a health risk increases with baseline health risk and with the severity of the risk—hold up across a number of alternative parameterizations. Finally, we show that these patterns are driven by mortality risk, not quality of life or financial risk.

Our primary contribution is the development and application of a new, more general

¹These values span the typical range produced by a life-cycle model ([Murphy and Topel, 2006](#)).

life-cycle model for estimating the value of life. Our results help explain puzzles such as why consumers invest less in prevention than treatment, why end of life spending is high (Zeltzer et al., 2021), and why preventive care interventions frequently fail to deliver results (Jones et al., 2019), although we do not rule out alternative explanations such as market inefficiencies or hyperbolic discounting that may reinforce these effects. Our finding that the value of health improvements depends on baseline health and ex ante risk severity explains consumer opposition to the use of a single threshold value when making decisions about health resource allocation (Lucarelli et al., 2021). More generally, our model provides a framework for exploring whether and how value-based reimbursement of medical technology should reflect concerns about health disparities. For example, if insurer reimbursements reflect a patient’s willingness to pay for health improvements, then our model outlines circumstances under which those reimbursements should vary with disease severity. With its (health) states appropriately redefined, our stochastic framework can also be applied to a number of other distinct questions, such as why societies appear to invest less in preventing pandemics than in mitigating them and how to value insurance in a setting with shocks to health, longevity, and spending (Kowalski, 2015; Ericson and Sydnor, 2018; Fang and Shephard, 2019; Atal et al., 2020).

The economic literature on the value of life includes seminal studies by Arthur (1981), Rosen (1988), Murphy and Topel (2006), and Hall and Jones (2007). Shepard and Zeckhauser (1984) and Ehrlich (2000) note the important role played by financial markets. Aldy and Smyth (2014) use microsimulation to assess heterogeneity in VSL by race and sex. Córdoba and Ripoll (2016) use Epstein-Zin-Weil preferences to study the implications of state non-separable utility on the value of life. The models used in these prior studies include only a single health state for alive individuals. Our stochastic framework allows for an arbitrary number of health states, accommodates general additively separable preferences and incomplete financial markets, and to our knowledge is the first to provide a life-cycle analysis of the value of preventing illness.² In addition, we derive results concerning the value of a statistical health improvement, which is more relevant than the value of a statistical life to assessing policies such as optimal safety investments or health care reimbursements.³ Our framework reveals that the value of a health improvement varies not just across health states; it also varies across different illness risks, even when valued from the perspective of a single person in a single health state. These results demonstrate that the rational model of consumer behavior strongly rejects the

²The value of preventing illness has already found application in the empirical literature (Cameron and DeShazo, 2013; Hummels et al., 2016).

³Researchers and policymakers frequently measure health benefits in units of life-years or QALYs rather than lives. Our theoretical analysis employs a general measure that encompasses both life-years and QALYs.

notion that an individual has a single value for health.

Our model also reconciles the standard life-cycle framework with results from a distinct literature that uses one-period models to study the value of mortality risk-reduction (Raiffa, 1969; Weinstein et al., 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). These static models predict that an increase in baseline health risk must raise VSL when financial markets are incomplete, a result often referred to as the “dead-anyway” effect. We contribute to this literature by showing that this result does not hold—in theory or in practice—in a dynamic life-cycle setting. Adverse longevity shocks can raise *or* lower VSL in our model, depending on consumer risk preferences. In our empirical exercises, we find that most health shocks reduce VSL, although the effect goes in both directions.

The remainder of this paper is organized as follows. Section 2 presents the model, derives key results, and discusses welfare. Section 3 applies the model to data and shows how VSL varies across people with different health histories and how the value of preventing illness varies with the degree of illness risk. Section 4 concludes.

2 Model

Consider an individual who faces a health risk such as illness or death. We are interested in analyzing the value of a marginal reduction in that risk. We begin with a “Robinson Crusoe” model where the consumer cannot incur debt or purchase annuities to insure against her uncertain longevity. This simple setting allows us to transparently communicate our main insights; we then later show how these insights extend to a more realistic setting with incomplete financial markets. Section 2.1 derives the value of statistical life (VSL) and the value of statistical illness (VSI) in the Robinson Crusoe model. Section 2.2 provides a sufficient condition under which VSL rises following an adverse shock to longevity. Section 2.3 describes how the value of a statistical health improvement varies across health states and how the ex ante value of prevention varies with illness severity. Section 2.4 extends our results to an incomplete markets setting where the consumer earns income over the life-cycle, has access to health care insurance, and can optimally invest her wealth in a constant annuity. Section 2.5 discusses welfare. Because a complete markets setting lacks realism, we relegate its analysis to Appendix D.

Like prior studies on the value of life, we focus throughout this paper on the demand for health and longevity. Quantifying optimal health spending requires additionally modeling the supply of health care (Hall and Jones, 2007). In light of all the institutional differences across health care delivery systems, a wide variety of plausible approaches can be taken to this modeling problem, which we leave to future research.

2.1 The value of health and longevity

Let Y_t denote the consumer's health state at time t . We assume Y_t is a continuous-time Markov chain with finite state space $Y = \{1, 2, \dots, n, n+1\}$, where state $i \in \{1, \dots, n\}$ represents different possible health states while alive, and state $i = n+1$ represents death. Denote the transition rates by:

$$\begin{aligned}\lambda_{ij}(t) &= \lim_{h \rightarrow 0} \frac{1}{h} \mathbb{P}[Y_{t+h} = j | Y_t = i], \quad j \neq i, \\ \lambda_{ii}(t) &= - \sum_{j \neq i} \lambda_{ij}(t)\end{aligned}$$

For analytical convenience and without meaningful loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0 \quad \forall j < i$.⁴ The probability that a consumer in state i at time 0 remains in state i at time t is then equal to:

$$\tilde{S}(i, t) = \exp \left[- \int_0^t \sum_{j > i} \lambda_{ij}(s) ds \right]$$

For expositional purposes we shall refer to transitions as either “falling ill” or “dying,” but our model also accommodates transitions from sick states to healthy states. We denote the stochastic mortality rate at time t as:

$$\mu(t) = \sum_{i=1}^n \lambda_{i,n+1}(t) \mathbf{1}\{Y_t = i\}$$

where $\mathbf{1}\{Y_t = i\}$ is an indicator variable equal to 1 if the individual is in state i at time t and 0 otherwise. When the number of states is equal to $n = 1$, we obtain the setting with deterministic health risk studied in prior literature (e.g., [Shepard and Zeckhauser, 1984](#); [Rosen, 1988](#); [Murphy and Topel, 2006](#)). The maximum lifespan of an individual is T , and we denote her stochastic probability of surviving until $t \leq T$ as:

$$S(t) = \exp \left[- \int_0^t \mu(s) ds \right]$$

Let $c(t)$ be consumption at time t , W_0 be baseline wealth, ρ be the rate of time preference, and r be the rate of interest. Quality of life at time t , $q_{Y_t}(t)$, is exogenous and

⁴That is, a person can transition from state i to j , $i < j$, but not vice versa. This restriction does not meaningfully limit the generality of our model because one can always define a new state $k > j$ with properties similar to state i .

depends on the health state, Y_t . Let the state variable $W(t)$ represent current wealth at time t . Normalizing the utility of death to zero, the consumer's maximization problem for $Y_0 \in \{1, \dots, n\}$ is:

$$V(0, W_0, Y_0) = \max_{c(t)} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0, W_0 \right] \quad (1)$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) - c(t) \end{aligned}$$

The no-debt constraint, $W(t) \geq 0$, means the consumer cannot borrow. The utility function, $u(c, q)$, is time-separable and depends on both consumption and quality of life. We assume throughout that $u(\cdot)$ is strictly increasing and concave in its first argument, and twice continuously differentiable. Hence, we must have $W(T) = 0$, since it cannot be optimal to have wealth remaining at the maximum possible age. We denote the marginal utility of consumption as $u_c(\cdot)$ and assume that this function diverges to positive infinity as consumption approaches zero, so that optimal consumption is always positive.

Define the consumer's objective function at time t as:

$$J(t, W(t), i) = \mathbb{E} \left[\int_0^{T-t} e^{-\rho u} \exp \left\{ - \int_0^u \mu(t+s) ds \right\} u(c(t+u), q_{Y_{t+u}}(t+u)) du \middle| Y_t = i, W(t) \right]$$

Define the optimal value function as:

$$V(t, W(t), i) = \max_{c(s), s \geq t} \{J(t, W(t), i)\}$$

subject to the wealth dynamics above and $V(t, W(t), n+1) = 0$. Under conventional regularity conditions, if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} \rho V(t, W(t), i) = \max_{c(t)} \left\{ u(c(t), q_i(t)) + \frac{\partial V(t, W(t), i)}{\partial W(t)} [rW(t) - c(t)] + \frac{\partial V(t, W(t), i)}{\partial t} \right. \\ \left. + \sum_{j>i} \lambda_{ij}(t) [V(t, W(t), j) - V(t, W(t), i)] \right\}, \quad i = 1, \dots, n \quad (2) \end{aligned}$$

where $c(t) = c(t, W(t), i)$ is the optimal rate of consumption.

In order to apply our value of life analysis, we exploit recent advances in the systems and control literature. [Parpas and Webster \(2013\)](#) show that one can reformulate a stochastic finite-horizon optimization problem as a deterministic problem that takes $V(t, W(t), j)$, $j \neq i$, as exogenous. More precisely, we focus on the path of Y that begins in state i and remains in state i until time T . We denote optimal consumption and wealth in that path by $c_i(t)$ and $W_i(t)$, respectively.⁵ A key advantage of this method is that it allows us to apply the standard deterministic Pontryagin maximum principle and derive analytic expressions.

Lemma 1. *Consider the following deterministic optimization problem for $Y_0 = i$ and $W(0) = W_0$:*

$$V(0, W_0, i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right] \quad (3)$$

subject to:

$$\begin{aligned} W_i(0) &= W_0, \\ W_i(t) &\geq 0, \\ \frac{\partial W_i(t)}{\partial t} &= rW_i(t) - c_i(t) \end{aligned}$$

where $V(t, W_i(t), j)$, $j \neq i$, are taken as exogenous. Then the optimal value function, $V(t, W_i(t), i)$, satisfies the HJB equation given by (2), for all $i \in \{1, \dots, n\}$.

Proof. See Appendix A ■

Because the value function $V(t, W_i(t), i)$ corresponding to (3) satisfies the HJB equation given by (2), it must also be equal to the consumer's optimal value function ([Bertsekas, 2005](#), Proposition 3.2.1). The present value Hamiltonian corresponding to (3) is:

$$H(W_i(t), c_i(t), p_t^{(i)}) = e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) + p_t^{(i)} [rW_i(t) - c_i(t)]$$

⁵Consumption, $c(t)$, is a stochastic process. We occasionally denote it as $c(t, W(t), Y_t)$ to emphasize that it depends on the states $(t, W(t), Y_t)$. When we reformulate our stochastic problem as a deterministic problem and focus on a single path $Y_t = i$, consumption is no longer stochastic because there is no uncertainty in the development of health states. We emphasize this point in our notation here by writing consumption as $c_i(t)$, and wealth as $W_i(t)$.

where $p_t^{(i)}$ is the costate variable for state i . The necessary costate equation is:

$$\dot{p}_t^{(i)} = -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)}r - e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \quad (4)$$

The solution to the costate equation can be obtained using the variation of the constant method:

$$p_t^{(i)} = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant. The necessary first-order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) \quad (5)$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0, W_0, i)}{\partial W_0} = p_0^{(i)} = u_c(c_i(0), q_i(0))$. Since the Hamiltonian is concave in $c_i(t)$ and $W_i(t)$, the necessary conditions for optimality are also sufficient (Seierstad and Sydsaeter, 1977).

To analyze the value of health and longevity, we follow Rosen (1988). Let $\delta_{ij}(t)$ be a perturbation on the transition rate, $\lambda_{ij}(t)$, $0 \leq t \leq T$, where $\sum_{j>i} \int_0^T \delta_{ij}(t) dt = 1$. The impact of a small (ε) perturbation on the likelihood of exiting state i is:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t \sum_{j>i} (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right], \text{ where } \varepsilon > 0 \quad (6)$$

The marginal value of preventing illness or death is equal to $\frac{\partial V / \partial \varepsilon}{\partial V / \partial W} \Big|_{\varepsilon=0}$, the marginal rate of substitution between longer life and wealth. The next two lemmas provide the two components of this marginal value expression.

Lemma 2. *The marginal utility of preventing illness or death in state i is given by:*

$$\begin{aligned} \frac{\partial V(0, W_0, i)}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) \right. \\ &\quad \left. - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \end{aligned}$$

Proof. See Appendix A ■

Lemma 3. *The marginal utility of wealth in state i is equal to:*

$$\begin{aligned} \frac{\partial V(0, W_0, i)}{\partial W_0} &= u_c(c_i(0), q_i(0)) \\ &= \mathbb{E} \left[e^{(r-\rho)t} \exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t, W(t), Y_t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right], \forall t > 0 \end{aligned}$$

Proof. See Appendix A ■

The first equality in Lemma 3 follows immediately from the first-order condition in state i in the HJB (2). Our proof derives the second equality, which shows that the consumer sets the expected discounted marginal utility of consumption at time t equal to the current marginal utility of wealth. This result is the stochastic analogue of the first-order condition from a conventional (deterministic health risk) model.

Lemma 2 pertains to a marginal reduction in transition rates for all states and times. Consider as a special case perturbing only $\lambda_{i,n+1}(t)$, the mortality rate in state i , and set the perturbation $\delta(\cdot)$ in equation (6) equal to the Dirac delta function, so that the mortality rate is perturbed at $t = 0$ and remains unaffected otherwise (Rosen, 1988). This then yields an expression that is commonly known as the value of statistical life (VSL).

Proposition 4. *Set $\delta_{ij}(t) = 0 \forall j < n + 1$ in the marginal utility expression given in Lemma 2 and let $\delta_{i,n+1}(t)$ equal the Dirac delta function. Dividing by the marginal utility of wealth given in Lemma 3 yields:*

$$VSL(i) = \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u_c(c(0), q_{Y_0}(0))} dt \middle| Y_0 = i, W(0) = W_0 \right] = \frac{V(0, W_0, i)}{u_c(c_i(0), q_i(0))} \quad (7)$$

Applying the second equality given in Lemma 3 and rearranging yields the following, equivalent expression for VSL in state i :

$$VSL(i) = \int_0^T e^{-rt} v(i, t) dt$$

where $v(i, t)$ represents the value of a one-period change in survival from the perspective of current time:

$$v(i, t) = \frac{\mathbb{E} \left[S(t) u(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]}{\mathbb{E} \left[S(t) u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]}$$

Proof. See Appendix A ■

VSL is the value of a marginal reduction in the risk of death in the current period. Put differently, it is the amount that a large group of individuals are collectively willing to pay to eliminate a current risk that is expected to kill one of them. Proposition 4 shows that VSL is proportional to expected lifetime utility, and inversely proportional to the marginal utility of consumption.

We can also value a marginal reduction in the risk of falling ill. As before, it is helpful to choose the Dirac delta function for $\delta(t)$, so that the transition rates are perturbed at $t = 0$ only. Consider a reduction in the transition rate for a single alternative state, $j \leq n+1$, so that $\delta_{ik}(t) = 0 \ \forall k \neq j$. Applying these two conditions in Lemma 3 then yields what we term the value of statistical illness, $VSI(i, j)$:

$$VSI(i, j) = \frac{V(0, W_0, i) - V(0, W_0, j)}{u_c(c_i(0), q_i(0))} = VSL(i) - VSL(j) \frac{u_c(c_j(0), q_j(0))}{u_c(c_i(0), q_i(0))} \quad (8)$$

The interpretation of VSI is analogous to VSL: it is the amount that a large group of individuals are collectively willing to pay in order to eliminate a current disease risk that is expected to befall one of them. Note that if health state j corresponds to death, so that $VSL(j) = VSL(n+1) = 0$, then $VSI(i, j) = VSL(i)$. Thus, VSI is a generalization of VSL.

The values of statistical life and illness depend on how substitutable consumption is at different ages and states. Intuitively, if present consumption is a good substitute for future consumption, then living a longer life is less valuable. Define the elasticity of intertemporal substitution, σ , as:

$$\frac{1}{\sigma} \equiv -\frac{u_{cc}c}{u_c}$$

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq}q}{u_c}$$

When η is positive, the marginal utility of consumption is higher in healthier states, and vice-versa. Taking logarithms of equation (5), differentiating with respect to t , plugging in the result for the costate equation and its solution, and rearranging yields an expression for the life-cycle profile of consumption:

$$\frac{\dot{c}_i}{c_i} = \sigma(r - \rho) + \sigma\eta \frac{\dot{q}_i}{q_i} - \sigma\lambda_{i,n+1}(t) - \sigma \sum_{j=i+1}^n \lambda_{ij}(t) \left[1 - \frac{u_c(c(t, W_i(t), j), q_j(t))}{u_c(c(t, W_i(t), i), q_i(t))} \right] \quad (9)$$

The first two terms in equation (9) relate the growth rate of consumption to the consumer rate of time preference and to life-cycle changes in the quality of life. The third term

shows that consumption growth is a declining function of the individual’s current mortality rate, $\lambda_{i,n+1}(t)$. Because the consumer cannot purchase annuities to insure against her uncertain lifetime, higher rates of mortality depress the rate of consumption growth over the life-cycle. Put another way, removing annuity markets “pulls consumption earlier” in the life-cycle (Yaari, 1965). The fourth term in equation (9)—which is absent in a deterministic setting—accounts for the possibility that the consumer might transition to a different health state in the future. The possibility of transitioning to a state with low marginal utility of consumption shifts life-cycle consumption earlier still.

Equation (9) describes consumption dynamics conditional on the individual’s health state i . It is not readily apparent from (9) whether modeling health as stochastic causes consumption to shift forward, *on average* across all states, relative to modeling health as deterministic. We confirmed in numerical exercises that modeling health as stochastic has an ambiguous effect on consumption (and VSL), even when holding quality of life constant across states and time.⁶

2.2 The effect of longevity shocks on VSL

This section considers the effect of stochastic changes in expected longevity on VSL. The effect of accompanying changes in quality of life depends crucially on the relationship between quality of life and the marginal utility of consumption, a phenomenon often referred to as “health state dependence.” Because there is no consensus regarding the sign or magnitude of health state dependence, we hold quality of life constant for the time being and return to this issue in Section 2.3 and in our empirical analysis.⁷

When quality of life is constant, VSL can increase or decrease following a health state transition, depending on consumer preferences and expectations of future mortality. We isolate the role played by preferences by analyzing a two-state model, where mortality in state 2 is uniformly higher than mortality in state 1. We focus on the case where consumption is declining. Prior empirical work suggests this case is a reasonable description for the typical consumer nearing retirement.⁸ In our model, constant quality of life and

⁶Modeling health as stochastic has a positive effect on lifetime utility because a stochastic environment allows the consumer to adjust consumption after a health shock. Put differently, a deterministic model is equivalent to a stochastic model where the consumer is forced to keep consumption constant across states.

⁷Viscusi and Evans (1990), Sloan et al. (1998), and Finkelstein et al. (2013) find evidence of negative state dependence. Lillard and Weiss (1997) and Edwards (2008) find evidence of positive state dependence. Evans and Viscusi (1991) find no evidence of state dependence. Murphy and Topel (2006) assume negative state dependence when performing their empirical exercises, while Hall and Jones (2007) assume state independence.

⁸A typical consumption profile is constrained by low income at early ages, increasing during middle ages when income is high, and then declines during retirement until consumption equals the consumer’s

$r \leq \rho$ are sufficient conditions for declining consumption.⁹ The next proposition states that consumption increases when transitioning to a state where current and future expected mortality are high.

Proposition 5. *Let there be $n = 2$ states with constant quality of life, so that $q_1(s) = q_2(s) = q \forall s$. Assume that $r \leq \rho$, that the transition rates $\lambda_{12}(s)$ are uniformly bounded (finite), and that the mortality rate is uniformly higher in state 2: $\lambda_{13}(s) < \lambda_{23}(s) \forall s$. Suppose the consumer transitions from state 1 to state 2 at time t . Then $c_1(t, W(t), 1) < c_2(t, W(t), 2)$.*

Proof. See Appendix A ■

An adverse shock to longevity that increases current consumption has an ambiguous effect on VSL. Decreased longevity lowers lifetime utility, which all else equal reduces VSL, but the simultaneous decrease in the marginal utility of consumption produces an offsetting increase in VSL. The net effect depends on the curvature of the utility function relative to the curvature of the marginal utility function.

We formally demonstrate this tradeoff by comparing a persistently healthy individual to someone who suffers an adverse shock to life expectancy but is otherwise identical. To make headway we must introduce the notion of prudence. The elasticity of intertemporal substitution, σ , measures utility curvature. Prudence, π , is the analogous measure for the curvature of marginal utility (Kimball, 1990):

$$\pi \equiv -\frac{c u_{ccc}(\cdot)}{u_{cc}(\cdot)}$$

It will also be convenient to define the elasticity of the flow utility function:

$$\epsilon \equiv \frac{c u_c(\cdot)}{u(\cdot)}$$

The utility elasticity, ϵ , is positive when utility is positive. Positive utility ensures well-behaved preferences, and is often enforced by adding a constant to the utility function. Although adding a constant to the utility function does not affect the solution to the consumer's maximization problem, this constant matters for the value of life.¹⁰

pension. This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers, 1991; Banks et al., 1998; Fernandez-Villaverde and Krueger, 2007).

⁹From equation (9), $\frac{\dot{c}_i}{c_i} \leq 0$ when $\lambda_{i,n+1} \geq r - \rho + \eta \frac{\dot{q}_i}{q_i} - \sum_{j=i+1}^n \lambda_{ij}(t) \left[1 - \frac{u_c(c(t, W_i(t), j), q_j(t))}{u_c(c(t, W_i(t), i), q_i(t))} \right]$. This condition is satisfied when $r \leq \rho$, quality of life is constant, and the consumer can transition only to states with higher mortality.

¹⁰Rosen (1988) was the first to point out that the level of utility is an important determinant of the value of life. See also additional discussion on this point in Hall and Jones (2007) and Córdoba and Ripoll (2016).

The following proposition provides sufficient conditions for VSL to rise following an adverse shock to longevity.

Proposition 6. *Consider a two-state setting with assumptions set out in Proposition 5. Assume that utility is positive and satisfies the condition:*

$$\pi < \frac{2}{\sigma} + \epsilon \quad (10)$$

Suppose that the consumer transitions from state 1 to state 2 at time t , and that $\lambda_{12}(\tau) = 0 \forall \tau > t$. Then, $VSL(1, t) < VSL(2, t)$.

Proof. See Appendix A ■

Proposition 6 shows that the effect of longevity shocks on VSL depends on both prudence and the elasticity of intertemporal substitution. Consumers with inelastic demand for current consumption (low σ) prefer to smooth consumption over time because consumption expenditures at different ages are poor substitutes. They therefore have a high willingness to pay for life-extension and, all else equal, are more likely to exhibit a rise in VSL following an adverse longevity shock than consumers with more elastic demand. Likewise, consumers with low levels of prudence, π , have near-linear marginal utility that decreases rapidly with consumption. This generates a high willingness to pay for life-extension following a shock that increases consumption.

Condition (10) is satisfied by hyperbolic absolute risk aversion (HARA) utility functions, a class that includes isoelastic and quadratic utility, provided that utility is positive. The condition is not innocuous: one can easily find linear combinations of isoelastic and polynomial utility functions where VSL declines following an illness. Nonetheless, prior studies on the value of life generally assume that 0.5 to 0.8 is a reasonable range for the value of σ (Murphy and Topel, 2006; Hall and Jones, 2007), and recent empirical studies suggest that π is about 2 (Noussair et al., 2013; Christelis et al., 2020). Under these parameterizations, condition (10) will hold whenever utility is positive.

In our dynamic model, VSL can rise or fall following an increase in mortality risk. In static models commonly used in prior studies, however, VSL always rises with baseline mortality risk (Weinstein et al., 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). This discrepancy arises because these prior studies focus on a one-period setting with two states, alive and dead. In that context, if the marginal utility of consumption is lower in the dead state, then an increase in baseline mortality risk must lower the expected marginal utility of consumption and thus raise the willingness to pay for survival (the

“dead-anyway” effect).¹¹ Proposition 5 confirms that an increase in the risk of death also reduces marginal utility in our dynamic context. However, unlike in a static setting, the resulting effect on VSL is ambiguous because of an offsetting decrease in lifetime utility.

2.3 The value of a statistical health improvement

Let D_i denote some measure of health for an individual in state i at time 0, such as quality-adjusted life expectancy. We assume this measure is exogenous, non-negative, and equals 0 only when dead, but otherwise impose no restrictions on its form. When VSL rises following a transition from state i to some state j with lower health, the value per unit of health must rise as well (i.e., $VSL(i)/D_i < VSL(j)/D_j$). This section considers a more general case: How does the value of illness risk reduction per unit of health improvement, $VSI(i, j)/(D_i - D_j)$, vary across different health states i and across different potential illnesses j ? Unlike in Section 2.2, our analysis here will allow for an arbitrary number of health states and will not require quality of life to be constant. Instead, our main results will rely on the concavity of the value function.

For simplicity, it is helpful to assume that health states are ordered in terms of severity. Define the optimal value function in (3) to be concave in health states i , j , and k with respect to changes in our health measure D if the following inequality holds:

$$V(0, W_0, j) > D \times V(0, W_0, i) + (1 - D) \times V(0, W_0, k), \text{ where } D = \frac{D_j - D_k}{D_i - D_k}, D_i > D_j > D_k \quad (11)$$

Let states i , j , and k correspond to “healthy,” “mildly ill,” and “severely ill.” The “value function concavity” condition (11) requires that lifetime utility when mildly ill be larger than the weighted average of the lifetime utilities when healthy or severely ill. In other words, the individual is risk averse over illness severity, preferring mild illness with certainty to good health with a risk of severe illness. This condition will generally be satisfied when health severity is clearly ordered across states and well-measured by D .¹²

The following proposition states that value function concavity is necessary and sufficient for the ex ante value of a statistical health improvement to rise with the severity of illness risk. In addition, the proposition states that if the value function is concave and the marginal utility of consumption decreases weakly with the severity of one’s current

¹¹Let expected utility be equal to $EU = p u(0, c) + (1 - p) u(1, c)$, where $p \in (0, 1)$ is the probability of death and the states $\{0, 1\}$ represent death and life, respectively. The willingness to pay for a marginal reduction in the probability of dying is given by $VSL = \frac{u(1, c) - u(0, c)}{p u_c(0, c) + (1 - p) u_c(1, c)}$, which increases with p if $u_c(1, c) > u_c(0, c)$.

¹²When using QALYs, the condition can be violated if state i has higher life expectancy but lower quality of life than state j . See the discussion of Figure 7 in Section 3.3 for a numerical example.

health state, then the ex post value of a health improvement will rise with the severity of the current health state.¹³

Proposition 7. *The optimal value function is concave in health states i , j , and k with respect to changes in the health measure D , as described by (11), if and only if the ex ante value of a statistical health improvement increases with illness severity:*

$$\frac{VSI(i,j)}{D_i - D_j} < \frac{VSI(i,k)}{D_i - D_k} \text{ where } D_i > D_j > D_k$$

In addition, if the value function is concave in health states i , j , and k , and $u_c(c_i(0), q_i(0)) \geq u_c(c_j(0), q_j(0))$, then the value of a statistical health improvement is larger in sicker states:

$$\frac{VSI(i,k)}{D_i - D_k} < \frac{VSI(j,k)}{D_j - D_k} \text{ where } D_i > D_j > D_k$$

Proof. See Appendix A ■

Propositions 6 and 7 both provide conditions under which VSL per unit of health is higher for those in worse health. However, Proposition 7 can be applied to both VSI and VSL. For example, consider three different ways to improve one's health: a healthy individual quits smoking to reduce her risk of developing lung cancer (ex ante illness risk reduction); a healthy individual reduces her risk of dying by wearing a seat belt (ex ante mortality risk reduction); and a metastatic lung cancer patient reduces her risk of dying by undergoing chemotherapy (ex post mortality risk reduction). Proposition 7 implies that, under value function concavity, the health benefits of smoking cessation are worth less per unit of health improvement than the life-extension benefits of wearing a seat belt, which in turn is worth less than chemotherapy.

Our results contrast with traditional cost-effectiveness analysis, which assumes that a unit of health is equally valuable regardless of baseline health risk or illness severity (Drummond et al., 2015, Chapter 5). In fact, a constant value arises only when the utility of consumption is constant (Bleichrodt and Quiggin, 1999). Analogously, recent theoretical work in cost-effectiveness shows that risk aversion over quality of life states leads to consumers valuing quality of life improvements more when they are in worse health (Lakdawalla and Phelps, 2020). In Appendix D, we show that constant utility of consumption occurs in our model in the special case where markets are complete, the rate of time preference equals the interest rate, and quality of life is constant.

¹³Proposition 5 provides an example where marginal utility of consumption will be lower for the ill than the healthy. All else equal, this condition is likely to arise when expected survival is lower in the sick state than in the healthy state.

2.4 Incomplete markets

This section extends our analysis to a setting with incomplete insurance markets and life-cycle income fluctuations. Let income, m_{Y_t} , be exogenous and equal to:

$$m_{Y_t} = \delta_{Y_t} - \omega_{Y_t} + \pi_{Y_t}$$

Income is equal to labor earnings in health state Y_t , δ_{Y_t} , minus health care spending, ω_{Y_t} , plus health insurance reimbursements, π_{Y_t} . Borrowing an approach from [Reichling and Smetters \(2015\)](#), we assume the consumer has an option at time zero to purchase a flat lifetime annuity that pays out $\bar{a}_{Y_0} \geq 0$ in all health states and has a price markup of $\xi \geq 0$. The consumer cannot finance the purchase of the annuity using future earnings or sell her annuity after the purchase. Because the market is incomplete, it will not be optimal to fully annuitize except in certain special cases ([Davidoff et al., 2005](#)).¹⁴

The consumer's maximization problem is:

$$V(0, W_0, Y_0) = \max_{c(t), \bar{a}_{Y_0}} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0 - (1 + \xi) \bar{a}_{Y_0} \mathbb{E} \left[\int_0^T e^{-rt} S(t) dt \middle| Y_0 \right], \\ W(t) &\geq 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) + \bar{a}_{Y_0} - c(t) \end{aligned}$$

The optimal annuity amount is chosen in the consumer's initial state, Y_0 , and the net present value of the annuity may change following a transition to a new health state because a fixed payout is worth more to a person with higher life expectancy. We emphasize this relationship in our notation below by writing the value function V as a function of the optimally chosen annuity and remaining wealth. In addition, it is helpful to define the value of a one-dollar annuity at time t in state i as:

$$a(t, i) = \mathbb{E} \left[\int_t^T e^{-r(s-t)} \exp \left\{ - \int_t^s \mu(u) du \right\} ds \middle| Y_t = i \right]$$

¹⁴Section 3 uses a numerical model to probe the sensitivity of our results to different assumptions about consumer preferences, such as the presence of a bequest motive, which prior studies have argued might also rationalize low observed rates of annuitization.

Incomplete annuity markets and life-cycle income complicate our analysis by introducing the possibility of multiple sets of non-interior solutions within and across states. (See the right panel in Figure 1 for an example.) For convenience of exposition, we focus on the case where future income is nondecreasing over time and the growth rate of consumption is weakly declining, as illustrated by the left panel in Figure 1. As discussed in Section 2.2, this case is a reasonable description for the typical consumer nearing retirement. We do not take a stance on the reason underlying the (weakly) negative growth rate in consumption, but we note that it arises in our model under a wide variety of typical parameterizations. Under these conditions, one can derive a simple expression for VSL.

Proposition 8. *Suppose that annuity markets are incomplete as described above, consumption growth is weakly declining ($\frac{\dot{c}_i}{c_i} \leq 0 \forall i$), and that income, $m_i(t)$, is nondecreasing in t . Then VSL in state i at time 0 is equal to:*

$$VSL(i) = \frac{V(0, W_i(0), \bar{a}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi) \bar{a}_i a(0, i) \quad (12)$$

Proof. See Appendix A ■

The second term in equation (12)—sometimes referred to as “net savings”—represents the marginal cost to the annuity pool from saving a life and arises because the price of an annuity is linked to survival (Murphy and Topel, 2006). VSL under incomplete markets captures elements of both the uninsured and fully insured cases. When annuities are absent ($\bar{a}_i = 0$), equation (12) simplifies to the uninsured case given by equation (7). Similarly, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case equation (12) simplifies to the complete markets case given by equation (D.7) in Appendix D.¹⁵

The following corollary shows that the value of statistical illness also takes an intermediate form when markets are incomplete.

Corollary 9. *Consider a setting with assumptions set out in Proposition 8. Then the value of a marginal reduction in the risk of transitioning from state i to state j at time 0 is equal to:*

$$\begin{aligned} VSI(i, j) &= \left(\frac{V(0, W_i(0), \bar{a}_i, i) - V(0, W_i(0), \bar{a}_i, j)}{u_c(c_i(0), q_i(0))} \right) - ((1 + \xi) \bar{a}_i a(0, i) - (1 + \xi) \bar{a}_i a(0, j)) \\ &= VSL(i) - \left(\frac{V(0, W_i(0), \bar{a}_i, j)}{u_c(c_i(0), q_i(0))} - (1 + \xi) \bar{a}_i a(0, j) \right) \end{aligned}$$

Proof. See Appendix A ■

¹⁵Remaining wealth at time 0, $W_i(0)$, is zero under full annuitization, which implies $W_0 = (1 + \xi) \bar{a}_i a(0, i)$.

The expression for VSI in Corollary 9 is similar to the expression for VSI in the Robinson Crusoe case (see equation 8), except here there is again an extra term that reflects the effect on net savings of a change in survival.

The net savings term in the VSL and VSI expressions presented above arises only because those expressions are evaluated at time $t = 0$, when the annuity is purchased. The term disappears when evaluating VSI and VSL at $t > 0$ —or, equivalently, in a setting with life-cycle income but no opportunity to purchase an annuity—because survival changes occurring after the purchase of the annuity do not affect its price.¹⁶ Because the effect of health transitions on the value of life will generally occur at time $t > 0$, we will assume in what follows that life-extension does not affect the annuity’s price.¹⁷

We first consider the special case of full annuitization. Because the marginal utility of consumption is constant across states under full annuitization, an adverse shock to longevity must reduce VSL, as shown by the following proposition.

Proposition 10. *Consider a two-state setting with assumptions set out in Proposition 5. Assume further that $\xi = 0$, $r = \rho$, and that future income and quality of life are constant across both time and states, so that it is optimal for the consumer to fully annuitize. Suppose the consumer transitions from state 1 (healthy) to state 2 (sick) at time t . Then $VSL(1, t) > VSL(2, t)$.*

Proof. See Appendix A ■

From Propositions 6 and 10, it immediately follows that VSL may in general rise or fall following an adverse health shock when markets are incomplete. Unlike in the Robinson Crusoe case, here the direction of the effect can also depend on the degree of annuitization. For example, full annuitization is optimal in our incomplete markets setting when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case Proposition 10 shows that VSL can fall following the health shock. However, when the load, ξ , is sufficiently large then the incomplete markets setting is well-approximated by the Robinson Crusoe case and Proposition 6 will hold, indicating that VSL can rise following the shock.

Finally, we show that our results from Section 2.3 regarding the willingness to pay per unit of health improvement continue to hold in this incomplete markets setting. Let D_i be a measure of health in state i at time t , where $D_i = 0$ indexes death. Define the value function of a consumer who purchased an optimal annuity in state i to be concave in health states at time t if for health states i, j, k with $D_i > D_j > D_k$, the following inequality

¹⁶Philipson and Becker (1998) argue that this “moral hazard” effect induces excessive longevity because individuals do not internalize the costs to annuity programs of their increased lifespan.

¹⁷We derive VSL and VSI for this case in part (i) of the proof of Proposition 8 and Corollary 9.

holds:

$$V(t, W_i(t), \bar{a}_i, j) > D \times V(t, W_i(t), \bar{a}_i, i) + (1 - D) \times V(t, W_i(t), \bar{a}_i, k) \text{ where } D = \frac{D_j - D_k}{D_i - D_k} \quad (13)$$

Corollary 11. *Suppose the optimal value function is concave in health states at time $t > 0$, as described by (13). Then the two conclusions of Proposition 7 hold at time t in a setting with incomplete insurance markets and life-cycle income fluctuations.*

Proof. See Appendix A ■

2.5 Welfare

This paper studies the willingness to pay for health and longevity. Our framework is useful for understanding puzzles such as why individuals invest less in prevention than treatment. Often, however, policymakers must decide how to allocate resources across different people. Who should receive limited supplies of a vaccine against a pandemic? Should a payer with a fixed budget focus resources on the elderly or the young, on the sick or the poor?

In such contexts, economists frequently rely on comparisons of aggregate social surplus, that is, the aggregate sum of willingness to pay. For example, [Murphy and Topel \(2006\)](#) employ this approach in the framework of the standard life-cycle VSL model. [Garber and Phelps \(1997\)](#) rely on it to develop the theory of cost-effectiveness for health interventions. [Einav et al. \(2010\)](#) use it to study the welfare effects of health insurance. Industrial organization economists use it, in the form of deadweight loss comparisons, to evaluate the welfare consequences of market power ([Martin, 2019](#)).

While popular among applied economists and policymakers, the aggregate surplus approach has been criticized by welfare theorists for several reasons ([Boadway, 1974](#); [Blackorby and Donaldson, 1990](#)). Equity concerns arise because each dollar of surplus is weighted equally, regardless of differences in wealth or income across people. Aggregation can also produce intransitive rankings of alternative allocations. Heterogeneity in marginal utility across consumers can break the necessary link between growth in aggregate surplus and increases in utility ([Martin, 2019](#)). This last point matters little when valuing the prevention of different illnesses, which can be accomplished from the perspective of a single healthy individual, but it does suggest a need for caution when making welfare inferences across individuals residing in different health states.

One could address these limitations by aggregating utilities rather than monetized surplus, but debate persists about how to aggregate utilities in situations involving risk

(Fleurbaey, 2010). In a foundational study, Harsanyi (1955) shows that a social welfare function satisfying both rationality and the Pareto principle must be a weighted sum of ex ante individual utilities. However, this utilitarian approach ignores distributional concerns (Diamond, 1967). As a result, one cannot simultaneously satisfy both rationality and the Pareto principle while still pursuing equity. Theorists have argued for abandoning one or the other of these principles. Diamond (1967) advocates minimizing ex ante inequality, but this violates rationality. Adler and Sanichirico (2006) advocate minimizing ex post inequality, but this violates the Pareto principle. In the specific context of VSL, Pratt and Zeckhauser (1996) advocate maximizing ex ante utility, but this ignores equity concerns in light of Diamond’s result. We do not aim to resolve this longstanding debate in welfare economics, but instead note that our stochastic model can be incorporated into these different welfare frameworks as desired. In our own empirical analysis, we maintain agnosticism about the correct welfare framework and instead focus on the implications of our results for demand.

3 Quantitative Analysis

This section quantifies the value of health improvements achieved through prevention or treatment. While our theoretical model provided useful insights, some of our theoretical results required either imposing restrictions on the consumer’s setting, such as limiting it to two health states, or assuming value function concavity, which cannot be assessed without data. Our quantitative analysis calculates the value of health improvements for a consumer with standard preferences and whose mortality, medical spending, and quality of life can vary across 20 different health states.¹⁸ We calculate both VSI and VSL but focus on their normalized values, VSI per QALY and VSL per QALY, which are more easily compared. All of our data and code are publicly available online.¹⁹

3.1 Framework

We employ a discrete time analogue of the model from Section 2. There are n health states (excluding death). Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}[Y_{t+1} = j | Y_t = i]$$

¹⁸Our empirical framework is related to a number of papers that study the savings behavior of the elderly (Kotlikoff, 1988; Palumbo, 1999; De Nardi et al., 2010). These prior studies allow health to affect wealth accumulation by including two or three different health states in the model.

¹⁹They are available at: <https://julianreif.com/research/reif.wp.healthrisk.replication.zip>.

The mortality rate at time t , $d(t)$, depends on the individual's health state:

$$d(t) = \sum_{j=1}^n \bar{d}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\{\bar{d}_j(t)\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator equal to 1 if the individual is in state j at time t and 0 otherwise.²⁰ The maximum lifespan of a consumer is T , so $d(T) = 1$. We denote the stochastic probability of surviving from time t to time $s \leq T$ as $S_t(s)$, where:

$$\begin{aligned} S_t(t) &= 1, \\ S_t(s) &= S_t(s-1)(1 - d(s-1)), \quad s > t \end{aligned}$$

Let $c(t)$ and $W(t)$ denote consumption and wealth in period t , respectively. Quality of life at time t , $q_{Y_t}(t)$, depends on the individual's health state, Y_t . Let ρ denote the rate of time preference, and r the interest rate. We measure a health improvement in state i and time t using quality-adjusted life expectancy, defined as:

$$D_i(t) = \mathbb{E} \left[\sum_{j=t}^T e^{-\rho(j-t)} q_{Y_j}(j) S_t(j) \middle| Y_t = i \right] \quad (14)$$

We assume annuity markets are absent. This simplification allows us to calculate the value of life using an analytical solution to the consumer's problem. It is possible to incorporate partial annuitization in this setting along the lines discussed in Section 2.4. However, generalization requires numerical optimization, which may necessitate limiting the number of health states included in the model. In our sensitivity analysis, we model the effects of a bequest motive and of decreasing the substitutability of consumption over time, both of which—similar to annuitization—reduce consumption at earlier ages.

The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E} \left[\sum_{t=0}^T e^{-\rho t} S_0(t) u(c(t), q_{Y_t}(t)) \middle| Y_0, W_0 \right]$$

²⁰Because our mortality data are distinct from our health state transition data, we denote the probability of dying in state i as $\bar{d}_i(t)$ rather than $p_{i,n+1}(t)$, which differs slightly from the notation used in Section 2.

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t)) e^{r(t, Y_t)} \end{aligned}$$

The individual's effective interest rate, $r(t, Y_t)$, depends on her health state, Y_t . This dependence allows us to model health shocks that affect income, spending, and wealth. Our baseline model sets $r(t, Y_t) = r + \ln[1 - s(t, Y_t)]$, where r is the rate of interest and $s(t, Y_t)$ is the average share of an individual's wealth spent on medical and nursing home care in state Y_t at time t .²¹ Instead of deducting medical costs from wealth directly, we treat them as modifying the interest rate. Although unconventional, this approach achieves our desired change in the life-cycle income profile while preserving the closed-form solution that facilitates our quantitative analysis. In addition, the approach allows us to model health shocks that have proportional effects on wealth. We assume throughout that $r = \rho = 0.03$ (Siegel, 1992; Moore and Viscusi, 1990).

Finally, we assume that utility takes the following form:

$$u(c, q) = q \left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \quad (15)$$

where $q \leq 1$ and $q = 1$ indexes perfect health. This isoelastic utility function satisfies condition (10) from Proposition (6). Our main specification sets $\gamma = 1.25$ and $\underline{c} = \$5,000$, consistent with the parameterization employed in Murphy and Topel (2006). As discussed previously, there is no consensus regarding the sign or magnitude of health state dependence, $u_{cq}(\cdot)$. Here, we assume a multiplicative relationship where the marginal utility of consumption is higher when quality of life is high, and vice versa.

The value function for the consumer's maximization problem is defined as:

$$V(t, w, i) = \max_{c(t)} \mathbb{E} \left[\sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q_i(s)) \middle| Y_t = i, W(t) = w \right]$$

²¹We calculate $s(t, Y_t)$ by dividing out-of-pocket spending in health state Y_t at time t by average wealth at time t , as estimated by our model for a healthy individual with no medical spending. Our results are similar if we instead use age-specific wealth estimates from the Health and Retirement Study.

We reformulate this optimization problem as a recursive Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left[u(c(t), q_i(t)) + \frac{1 - \bar{d}_i(t)}{e^{\rho}} \sum_{j=1}^N p_{ij}(t) V(t+1, (w - c(t))e^{r(t, Y_t)}, j) \right]$$

We solve for consumption analytically and then use the formulas derived in Section 2 to calculate the value of life (see Appendix C). We calibrate initial wealth by assuming that average VSL at age 50 is \$6 million, which matches the value from [Murphy and Topel \(2006\)](#) and is within the range estimated by empirical studies of VSL for working-age individuals ([O'Brien, 2018](#)).

There is significant uncertainty among economists regarding the proper values of many of the parameters in our model. The goal of the subsequent analyses is to quantify the economic significance of our insights by applying our model to real-world data using reasonable parameterizations. To investigate the sensitivity of our results to the parameterization of our utility function, we estimate specifications with alternative assumptions regarding the elasticity of intertemporal substitution, $1/\gamma$. We also estimate an alternative specification that includes a bequest motive. Rather than setting the utility of death to zero, our bequest motive specification follows [Fischer \(1973\)](#) and sets it equal to $u(W(t+1), b(t))$, where $u(\cdot)$ takes the form given in (15), $W(t+1)$ is wealth at death, and the parameter $b(t)$ governs the strength of the bequest motive. We conservatively set $b(t) = 1.2$, the largest value considered in [Fischer \(1973\)](#), for all t .

3.2 Data

We obtain individual-level data on mortality, disease incidence, quality of life, and medical spending from the Future Elderly Model (FEM), a widely published microsimulation model that combines nationally representative information from the Health and Retirement Study (HRS), the Medical Expenditure Panel Survey (MEPS), the Panel Study of Income Dynamics, and the National Health Interview Survey (see Appendix B). The FEM provides a uniquely rich set of information about the US elderly. For instance, while the HRS provides detailed data on health and wealth, it lacks survey questions that would allow us to calculate quality of life using standard survey instruments. To solve this problem, the FEM weaves together validated quality of life estimates from the MEPS and maps them to the HRS using variables common to both databases.

The FEM, which has been released into the public domain, produces estimates for individuals ages 50–100 with different comorbid conditions. It accounts for six different chronic conditions (cancer, diabetes, heart disease, hypertension, chronic lung disease,

and stroke) and six different impaired activities of daily living (bathing, eating, dressing, walking, getting into or out of bed, and using the toilet). We divide the health space within the FEM into $n = 20$ states. Each state corresponds to the number (0, 1, 2, 3 or more) of impaired activities of daily living (ADL) and the number (0, 1, 2, 3, 4 or more) of chronic conditions, for a total of $4 \times 5 = 20$ health states. Health states are ordered first by number of ADLs and then by number of chronic diseases, so that state 1 corresponds to 0 ADLs and 0 chronic conditions, state 2 corresponds to 0 ADLs and 1 chronic condition, and so on. This aggregation provides a parsimonious way of incorporating information about functional status and several major diseases.²²

The FEM provides estimates of average annual medical spending, quality of life, mortality, and probabilities of transition to other health states, for all pairwise combinations of health state (1–20) and age (50–100). We then use those estimates as inputs into our life-cycle model. As in the theoretical model, individuals can transition only to higher-numbered states: $p_{ij}(t) = 0 \forall j < i$. In other words, all ADLs and chronic conditions are permanent. Quality of life is measured by the EuroQol five dimensions questionnaire (EQ-5D). These five dimensions are based on five survey questions that elicit the extent of a respondent’s problems with mobility, self-care, daily activities, pain, and anxiety/depression. These questions are then combined using weights derived from stated preference data.²³ The result is a single quality of life index, the EQ-5D, which is anchored at 0 (equivalent to death) and 1 (perfect health).

Table 1 reports means from the FEM for ages 50 and 70, by health state. At age 50, life expectancy ranges from 30.9 years to 9.1 years, quality of life ranges from 0.88 to 0.54, and average out-of-pocket medical spending ranges from \$686 to \$2,759 per year. Columns (10) and (11) report the probability that an individual exits her health state but remains alive, i.e., acquires at least one new ADL or chronic condition within the following year. Health states are relatively persistent, with exit rates never exceeding 15 percent at ages 50 or 70. State 20 is an absorbing state with an exit rate of 0 percent.

Figure 2 plots average out-of-pocket medical spending for the healthiest and the sickest health states, by age. These data include all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Spending is higher in sicker health states, and increases greatly at older ages, when long-term care expenses arise

²²While fully interacting all these variables would provide a more granular state space, it would also result in a very large number of possible states and correspondingly small cell sizes within many of them.

²³The five dimensions of the EQ-5D are weighted using estimates from Shaw et al. (2005). The specific process for estimating the quality of life score is explained in the FEM technical documentation, which can be found in the supplemental appendix of Agus et al. (2016). The methods used to measure the quality of life are consistent with our assumed utility specification, given in (15).

(De Nardi et al., 2010). The effect of sickness on out-of-pocket spending is modest in comparison to long-term care costs, causing the overall gap in spending across states to shrink with age.²⁴

We estimate our life-cycle model using FEM data for ages 50–100 but focus our discussion below on ages 50–80, where the FEM estimates are more precise and consumption decisions are less affected by our assumption that annuity markets are absent.

3.3 Elderly value of life

We begin with a simple example. The solid red and dashed blue lines in Figure 3 report VSL and consumption for a healthy individual who experiences a mild health shock at age 60, suffers a severe health shock at age 70, and then dies at age 75. Each shock produces sudden changes to expected survival, quality of life, and medical spending, as estimated by the FEM.

Consumption increases sharply following the two health shocks depicted in Figure 3. There is little change in VSL at age 60. By contrast, VSL rises from \$2.5 million to \$2.7 million following the severe health shock at age 70, even though life expectancy fell. The dotted black line in Figure 3 demonstrates directly the role played by financial shocks: VSL would fall at age 70, rather than rise, if the severe health shock reduced wealth by 20 percent instead of reducing it by the more modest amount estimated by the FEM. Overall, this simple example suggests that our results from Propositions 5 and 6 are relevant to the more general setting we study here.

As illustrated in Figure 3, the effect of a health shock on VSL depends on the nature of the shock as well as the individual’s health history and wealth. To characterize these effects among the US elderly population more generally, we conduct a Monte Carlo exercise that begins with 50,000 nationally representative individuals at age 50.²⁵ Each person’s health path then evolves at random according to the nationally representative health transition probabilities estimated by the FEM. Figure 4a illustrates how the mean, 5th percentile, and 95th percentile of VSL vary over the life cycle for these 50,000 individuals. At age 50, the inter-vigintile range spans \$1.8 million to \$6.8 million. The distribution is skewed towards zero, but the dispersion in VSL narrows over time as VSL decreases towards zero.

²⁴FEM medical spending estimates have been validated by comparing them to estimates from the National Health Expenditure Accounts (see Section 8.2, Appendix B of [National Academies of Sciences, Engineering, and Medicine, 2015](#)).

²⁵The FEM provides prevalence estimates for our 20 health states as well as average labor earnings by health state and age. We assume that the distribution of initial wealth across health states is proportional to labor earnings at age 50. We then calibrate wealth so that average VSL at age 50 is equal to \$6 million.

While much of the dispersion in VSL shown in Figure 4a is due to differences in initial health and wealth at age 50, individual-level health shocks also generate substantial variability in VSL over time. Figure 4b illustrates VSL over the life cycle for the subset (44%) of individuals who were in health state 1 at age 50. Although initially identical, these 22,214 individuals follow different health paths as they age. By age 70, the VSL inter-vigintile range spans \$1.9 to \$2.9 million.

The remainder of this section focuses on the 22,214 individuals who were initially healthy at age 50. This cohort experiences about 58,000 health shocks between the ages of 50 and 80. Figure 5a displays the distribution of the change in VSL in the year following each of those shocks. On average, a health shock reduces VSL by \$90,000 (2.3%), but there is much heterogeneity in this effect. About 5 percent of health shocks increase VSL, with some increases exceeding \$100,000. An even larger number of shocks reduce VSL by over \$100,000.

Although most health shocks reduce VSL, Figure 5b shows that on average they increase VSL per QALY by \$72,000 (21%) because of the accompanying reductions in health and longevity. This effect is consistently positive: fewer than 0.1% of health shocks cause a decline in the value of a QALY. The distribution is skewed to the right, with the value of a QALY rising by over \$177,000 (51%) in 5% of cases.

The dashed blue line in Figure 6 illustrates how VSL at age 70 varies with quality-adjusted life expectancy across the twenty health states in our model. The positive slope indicates that, on average, VSL rises with life expectancy, consistent with recent work finding that VSL is higher for people in better health states (Ketcham et al., 2020).²⁶ Nevertheless, the solid red line in Figure 6 indicates that, on average, VSL per QALY still falls rapidly with life expectancy. Individuals in the worst health state have an average VSL per QALY of \$660,000, over 2.4 times higher than individuals in the healthiest state, where VSL per QALY is \$260,000.

Finally, we consider the value of prevention. The dashed blue line in Figure 7 reports VSI's for different illnesses, including death, from the perspective of a healthy 70-year-old. Each value represents the healthy individual's willingness to pay for a marginal, contemporaneous reduction in the probability of dying or of transitioning to one of the 19 other health states in our model. The values are inversely related to life expectancy in the sick state because it is more valuable in absolute terms to prevent a severe illness

²⁶Because health states are persistent (see Columns (10)–(11) of Table 1), the averages shown in Figure 6 describe individuals who mostly have not experienced a recent health shock. (By contrast, Figure 5a described changes in VSL for individuals who had just experienced a shock.) Over a long enough time horizon, an adverse shock to longevity must eventually reduce VSL, relative to no shock, because it causes the individual to spend down her wealth more quickly.

than a mild one. A marginal reduction in the probability of transitioning to the worst health state (2.6 QALYs) is worth about \$2.1 million. This value is the amount that a large number of healthy individuals would collectively be willing to pay to reduce a risk that is expected to cause the onset of this health state for one of them. VSL, which is a special case of VSI where life expectancy is 0 years in the sick state, is \$2.8 million.

The solid red line in Figure 7 reports VSI per QALY. The negative slope indicates that these values increase with the severity of the disease being prevented. Reducing the risk of death (\$260,000 per QALY) is worth 16% more on a per QALY basis than reducing the risk of transitioning to health state 2 (\$224,000 per QALY), the state corresponding to the mildest possible illness (9.7 QALYs). Some sections of the red line occasionally have positive slopes, which can occur if there is no clear ordering in the severity of different health states. For example, at age 70 life expectancy in state 9 (5.4 QALYs) is lower than in state 17 (5.6 QALYs), but quality of life in state 9 is higher (see Table 1).²⁷ Nevertheless, the general concordance between the estimates shown in Figure 7 and the first inequality stated in Proposition 7 provides evidence that value function concavity holds for most elderly health risks when consumer preferences take the form (15).²⁸

Figure 8 shows how different utility function parameterizations and the presence of a bequest motive affect our estimates. Setting $\gamma = 1.5$, which makes demand for current consumption more inelastic, flattens the life-cycle consumption profile and increases the value of a QALY. Setting $\gamma = 0.8$, by contrast, pulls consumption forward in time and reduces the value of life-extension because consumption at early ages provide a good substitute for consumption at later ages. A bequest motive encourages individuals to delay consumption, because money saved for consumption in old age has the added benefit of increasing bequests in the event of death (Figure 8a). Likewise, it reduces the value of life-extension because death is less costly (Figure 8b).

Figure 9 shows that our results are driven by changes in mortality, not quality of life or medical spending. Setting quality of life equal to 1 (perfect health) and medical spending to 0 for all ages in all health states shifts consumption to later ages and raises VSL at later ages, but the effect is small when compared to our main estimates (Figures 9a and 9b).²⁹

Overall, while these alternative specifications produce meaningful shifts in the absolute values of VSL and VSI, they do not affect our qualitative conclusions. In all cases, the

²⁷Individuals in these two states also face different illness risks. QALYs measure the effect of risk on expected survival, but do not capture uncertainty in survival.

²⁸It is also worth noting that value function concavity implies the positive values shown in Figure 5b and the negative slope shown in Figure 6 (see the second inequality in Proposition 7).

²⁹The effect of quality of life and the rate of interest on consumption growth is given by Equation 9. Recall that we model shocks to medical spending as modifications to the interest rate.

value of a QALY is larger when treating sicker individuals (Figures 8c and 9c) or when preventing more serious illnesses (Figures 8d and 9d).

4 Conclusion

The economic theory surrounding the value of life has many important applications. Yet, a number of limitations have surfaced over time. The traditional model does not distinguish between prevention and treatment, and fails to explain several empirical facts, such as the apparent preferences of consumers to pay more for life-extension when survival prospects are bleaker.

Our model overcomes these limitations by relaxing the standard assumption that health risk is deterministic. Our framework provides a practical tool for policymakers and health agencies, since many health investments involve preventing the deterioration of health rather than reducing an immediate mortality risk. Using nationally representative data, we estimate that the willingness to pay to prevent an illness is less than the willingness to pay to treat the illness, holding fixed the health gains. We also find that people are willing to pay more per QALY to prevent more serious health risks.

Our findings provide a rational explanation for why many people state preferences for prioritizing the severely ill over other patients (Nord et al., 1995). They also help explain why it has proven so difficult for policymakers and public health advocates to encourage investments in the prevention of disease. Kremer and Snyder (2015) show that heterogeneity in consumer valuations distorts R&D incentives by allowing firms to extract more consumer surplus from treatments than preventives. Our results suggest that differences in private VSL may reinforce this result and further disadvantage incentives to develop preventives.

Our analysis raises a number of important questions for further research. First, what are our model’s implications for the value of health insurance and how that interacts with medical technology? Technology that improves quality of life can act as insurance by compressing the difference in utility between the sick and healthy states (Lakdawalla et al., 2017). It is less clear how these effects operate in a stochastic life-cycle setting with incomplete markets. Second, what are the most practical strategies for incorporating our insights into the literature on cost-effectiveness of alternative medical technologies? Traditional cost-effectiveness models imply that quality-adjusted life-years possess a constant value (Garber and Phelps, 1997). While flawed, this approach is simpler to implement than allowing the value to depend on health histories and illness severity. Future research should focus on practical strategies for aligning cost-effectiveness analyses

with the generalized theory of the value of life. Finally, what are the implications for the empirical literature on VSL? Prior studies have assumed that health histories can be ignored when estimating VSL ([Hirth et al., 2000](#); [Mrozek and Taylor, 2002](#); [Viscusi and Aldy, 2003](#)), but more recent research suggests otherwise ([Ketcham et al., 2020](#)). This missing insight may be one reason for the widely disparate empirical estimates of the value of statistical life.

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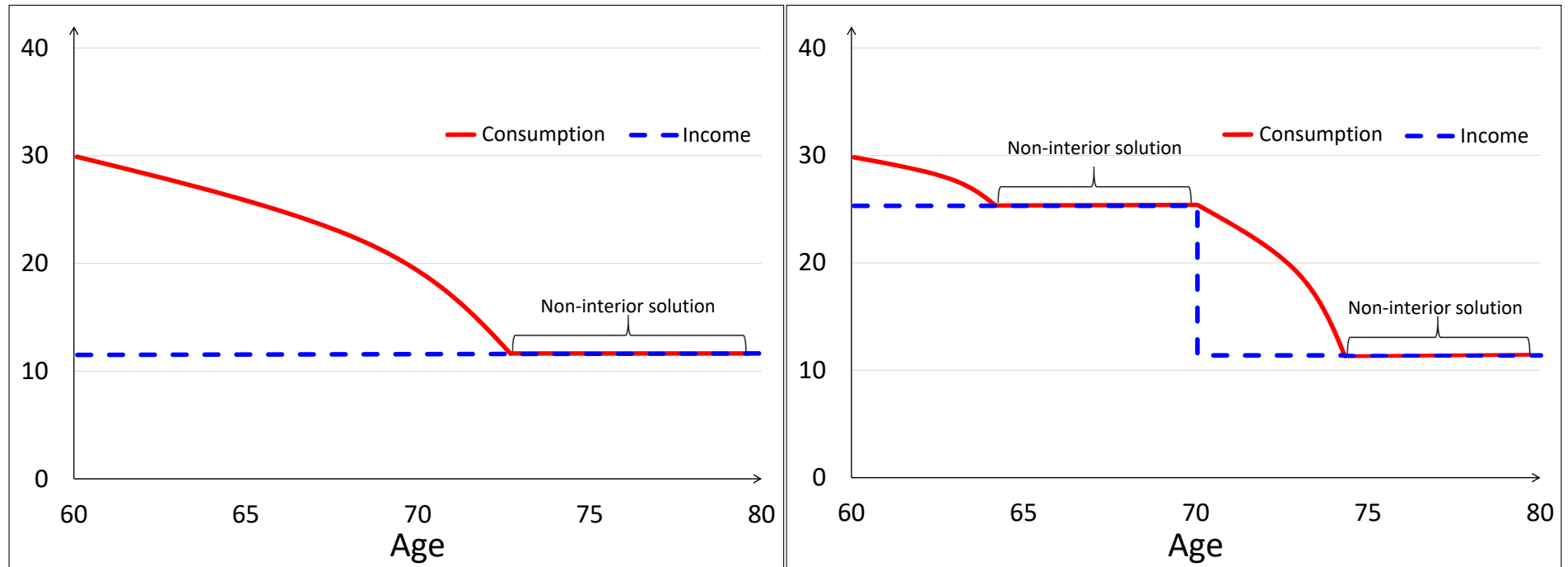
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Figure 1: Illustrative example: survival-contingent income can generate non-interior solutions

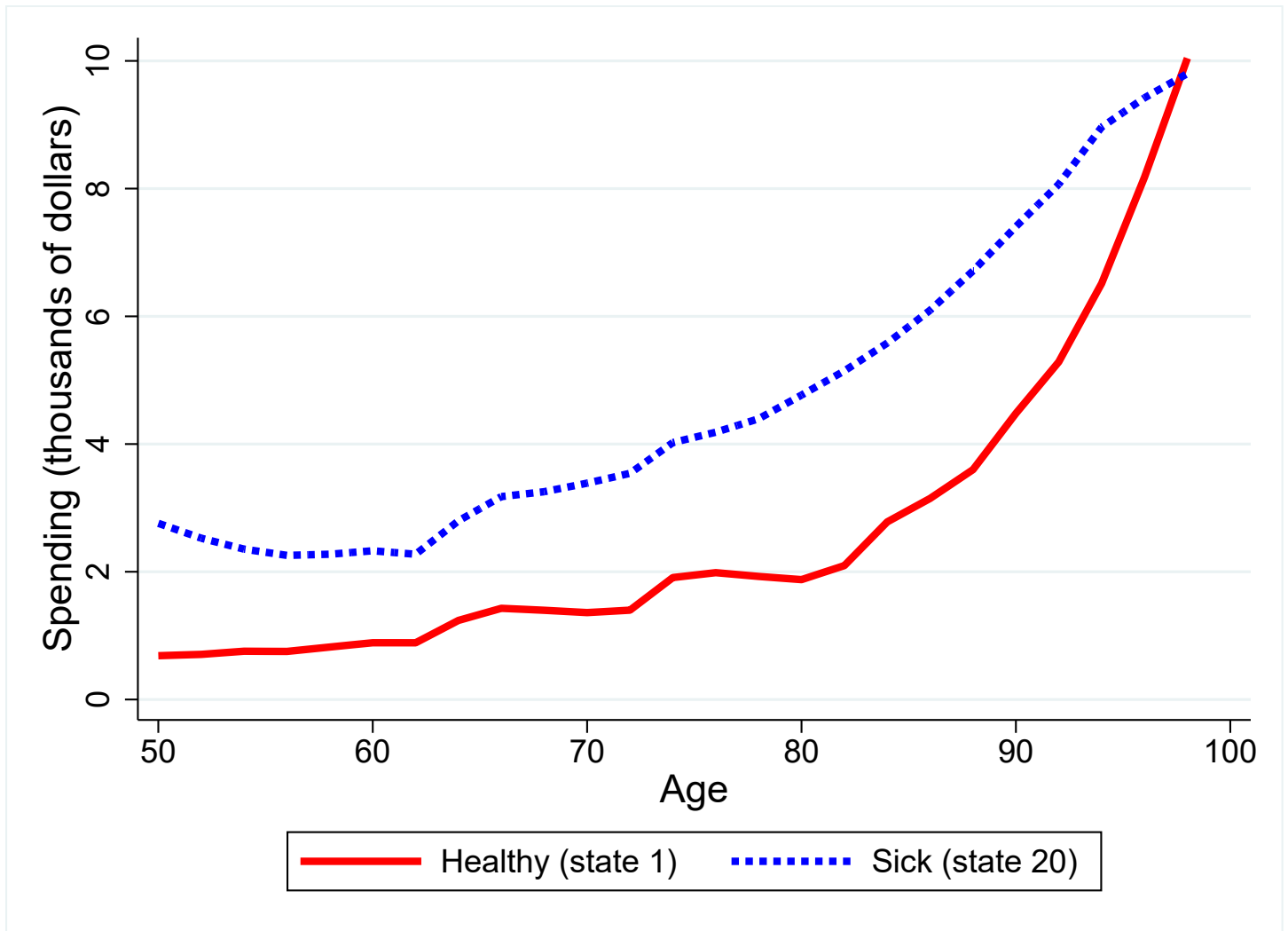
(a) One set of non-interior solutions

(b) Two sets of non-interior solutions



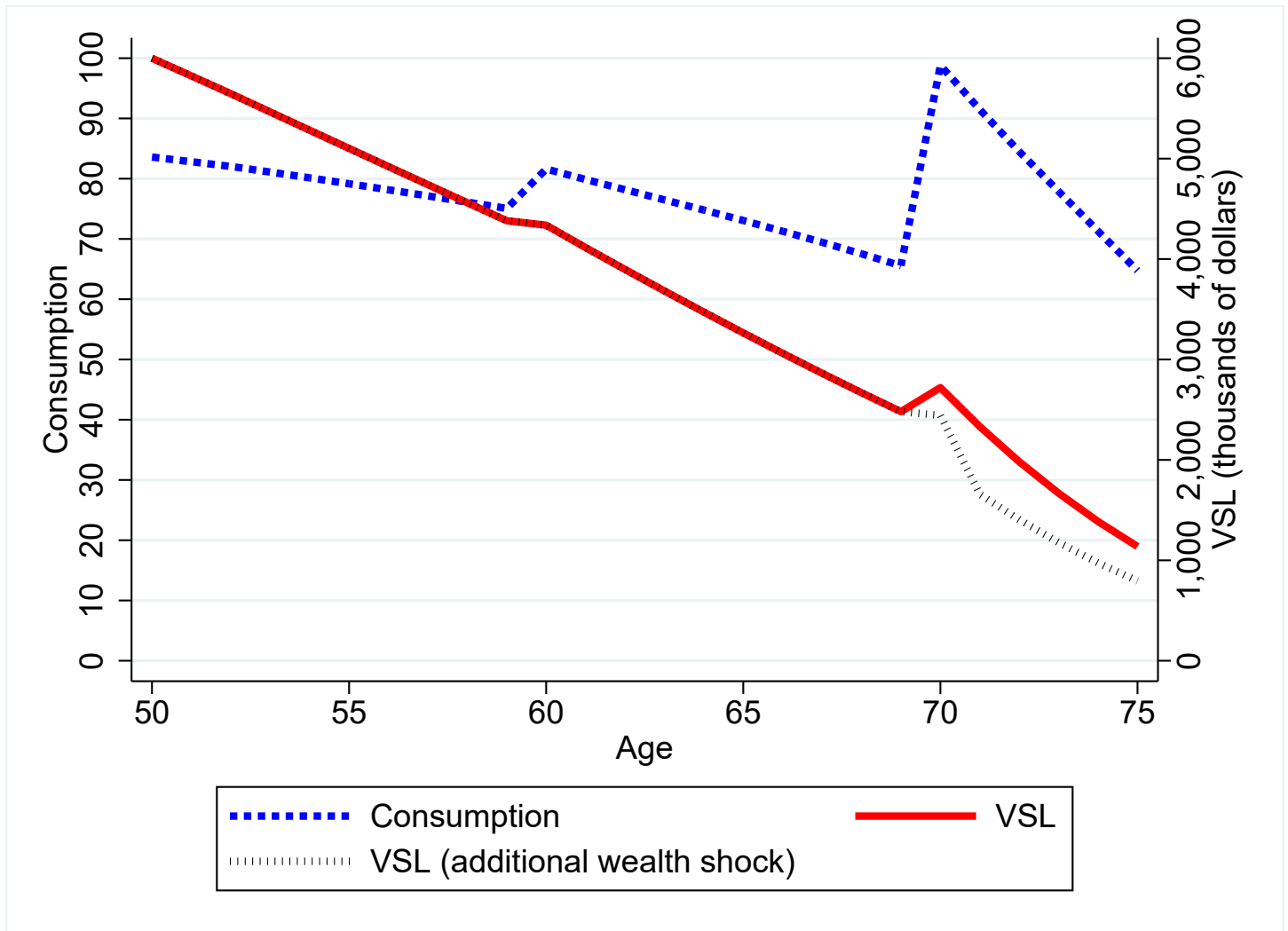
Notes: The solution to the consumer's maximization problem may be non-interior in the presence of survival-contingent income. Panel (a) gives an example where there is one set of non-interior solutions. Panel (b) gives an example where there are two sets of non-interior solutions. Income, illustrated by the dashed blue line, includes both labor income and annuity income.

Figure 2: Average annual out-of-pocket medical spending, by age



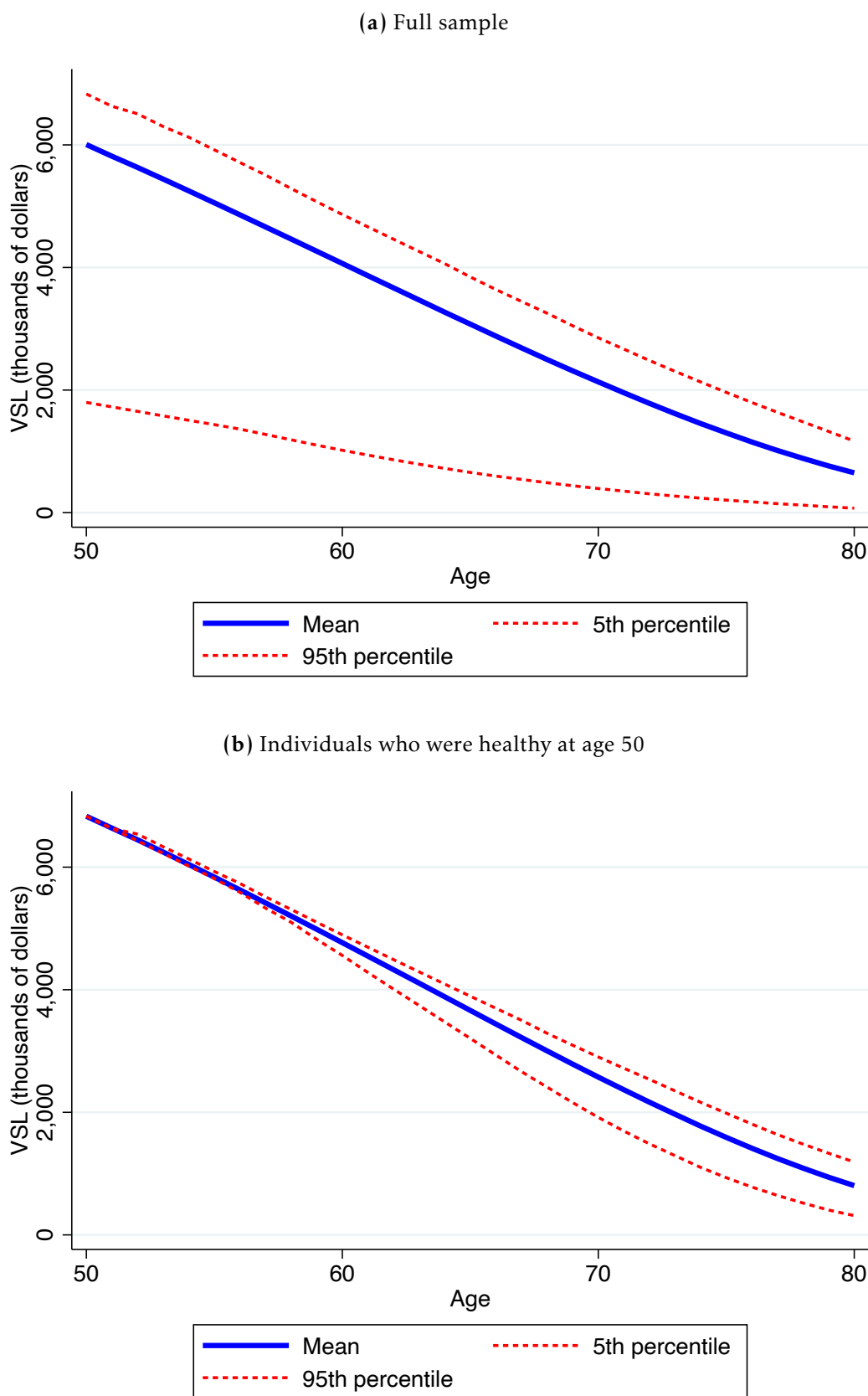
Notes: These medical spending estimates include all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Health state 1 describes healthy individuals with no impaired activities of daily living (ADL) and no chronic conditions. Health state 20 describes very ill individuals with three or more impaired ADLs and four or more chronic conditions. Additional characteristics for these health states are provided in Table 1. These estimates are produced by the Future Elderly Model (FEM).

Figure 3: Consumption and the value of statistical life for an individual who suffers two health shocks



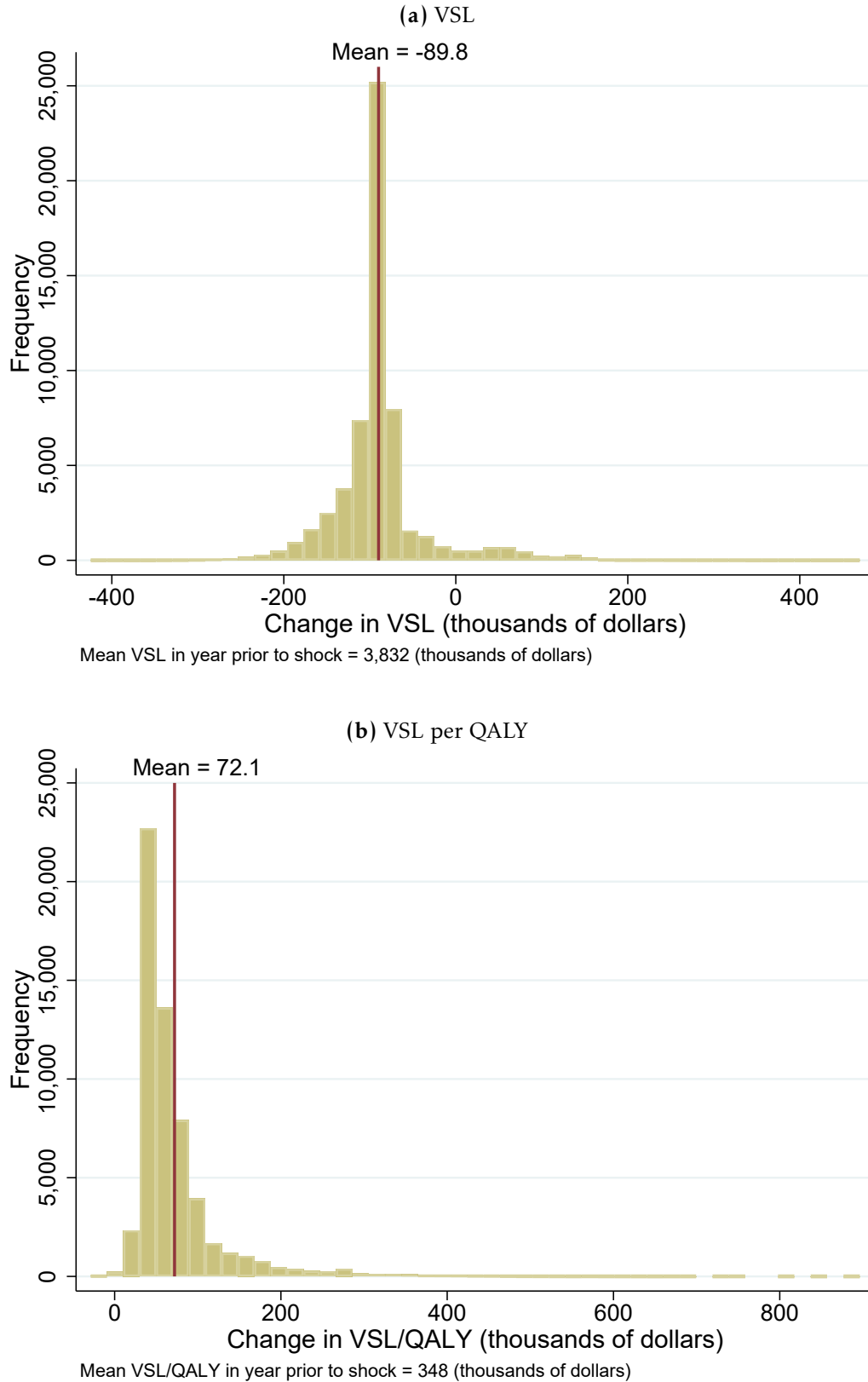
Notes: This figure plots an individual's life-cycle consumption and value of statistical life in a setting where mortality, quality of life, and medical spending are stochastic. The individual is healthy at age 50, but then falls ill twice, once at age 60 and then again at age 70. At age 60, the illness impairs one activity of daily living (ADL). At age 70, she is diagnosed with three chronic conditions and one additional impaired ADL. Equivalently, she transitions from state 1 to state 6 at age 60, and then from state 6 to state 14 at age 70 (see Table 1). The individual dies at age 75. The dashed blue line (consumption) and the solid red line (VSL) assume the individual's medical spending is equal to the average for her age and health state. The dotted black line shows how VSL would change if, in addition to the medical spending shock, there was also another financial shock that reduces the individual's wealth by 20 percent at age 70.

Figure 4: The value of statistical life among the US elderly population



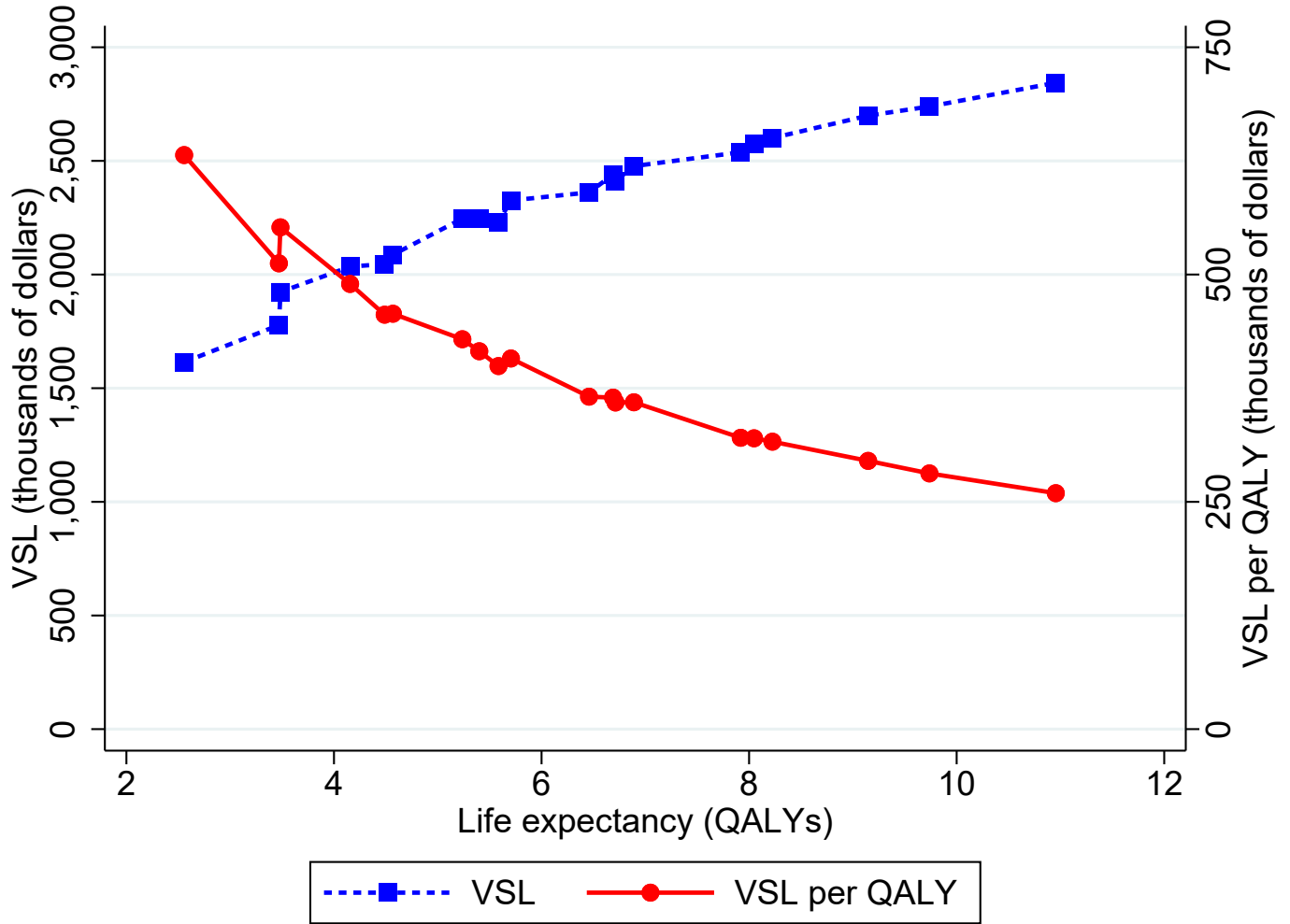
Notes: This figure reports the mean, 5th percentile, and 95th percentile of the value of statistical life (VSL) for 50,000 adults who are nationally representative of the US health and wealth distribution, as estimated by the Future Elderly Model (FEM). Panel (a) reports statistics for the full sample of individuals. Panel (b) reports statistics for the 22,214 individuals who were in state 1 (“healthy”) at age 50. The health and spending data employed in this exercise are summarized in Table 1.

Figure 5: Distribution of changes in VSL following a health shock



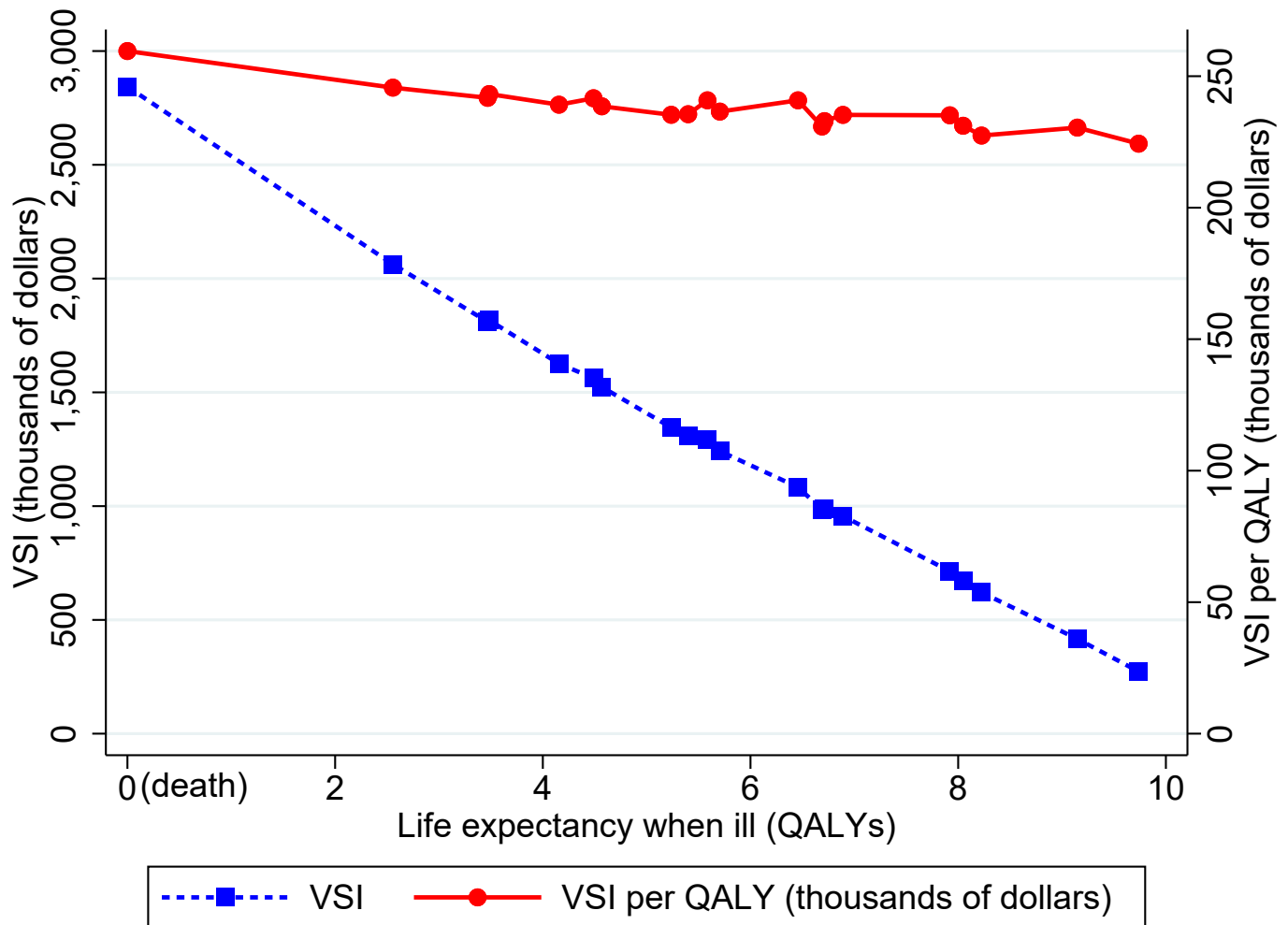
Notes: This figure illustrates how VSL changes following a health shock among the sample of 22,214 initially healthy adults from the Future Elderly Model. These individuals experience 57,981 shocks between the ages of 50–80. Panel (a) plots the distribution of the change in VSL in the year following a health shock. Panel (b) plots the distribution of the change in VSL per quality-adjusted life year (QALY). The vertical red lines report the means of the distributions. Quality-adjusted life-years are discounted at a rate of 3 percent. Figure 4b reports how average VSL evolves over the life cycle for this cohort of individuals.

Figure 6: Average VSL at age 70, by health state



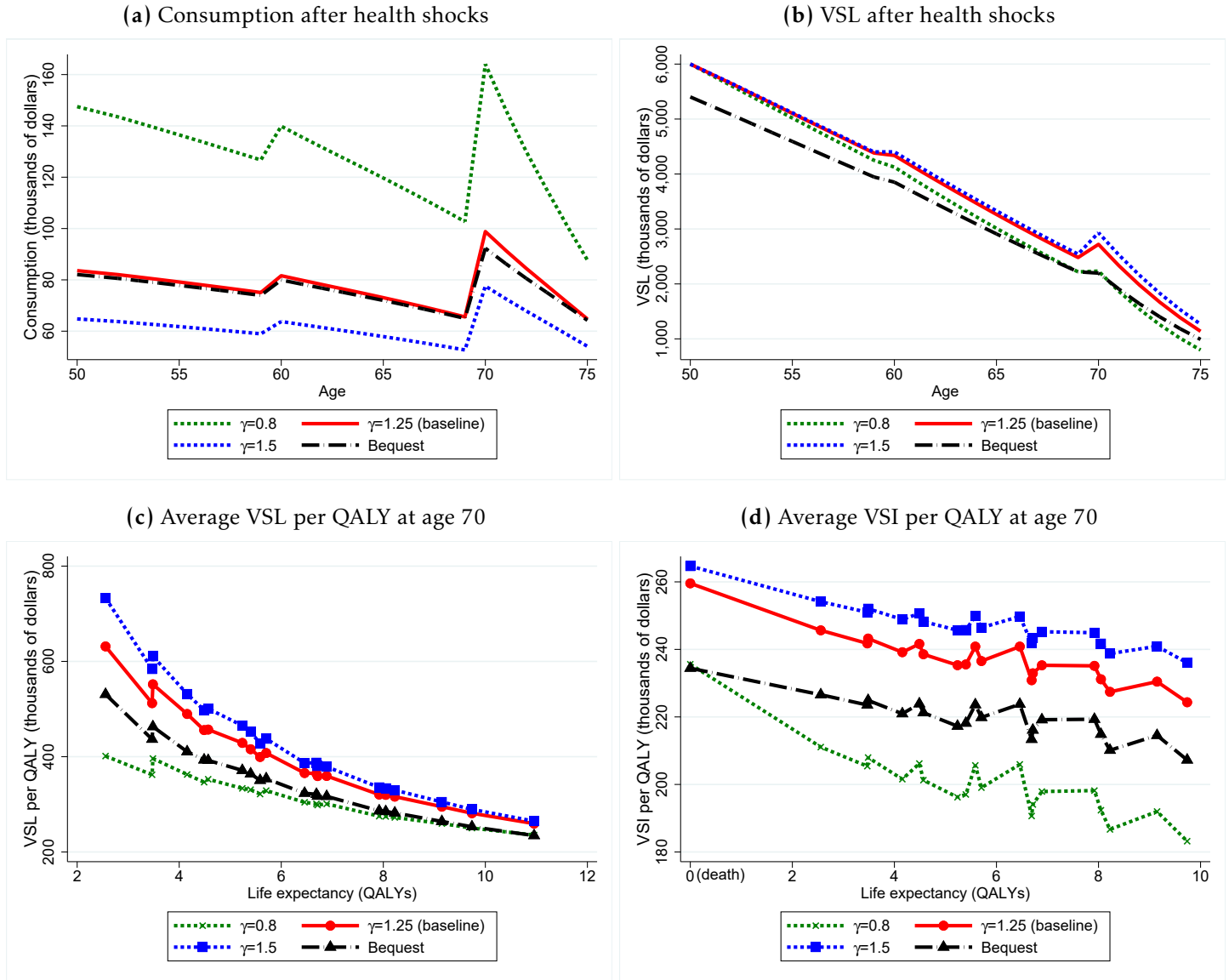
Notes: This figure presents VSL calculations for a sample of US adults from the Future Elderly Model. The dashed blue line reports average VSL at age 70 for each of the 20 health states described in Table 1. The solid red normalizes that value by the life expectancy for a person in that health state. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent.

Figure 7: Value to a healthy person of preventing different illnesses and death, at age 70



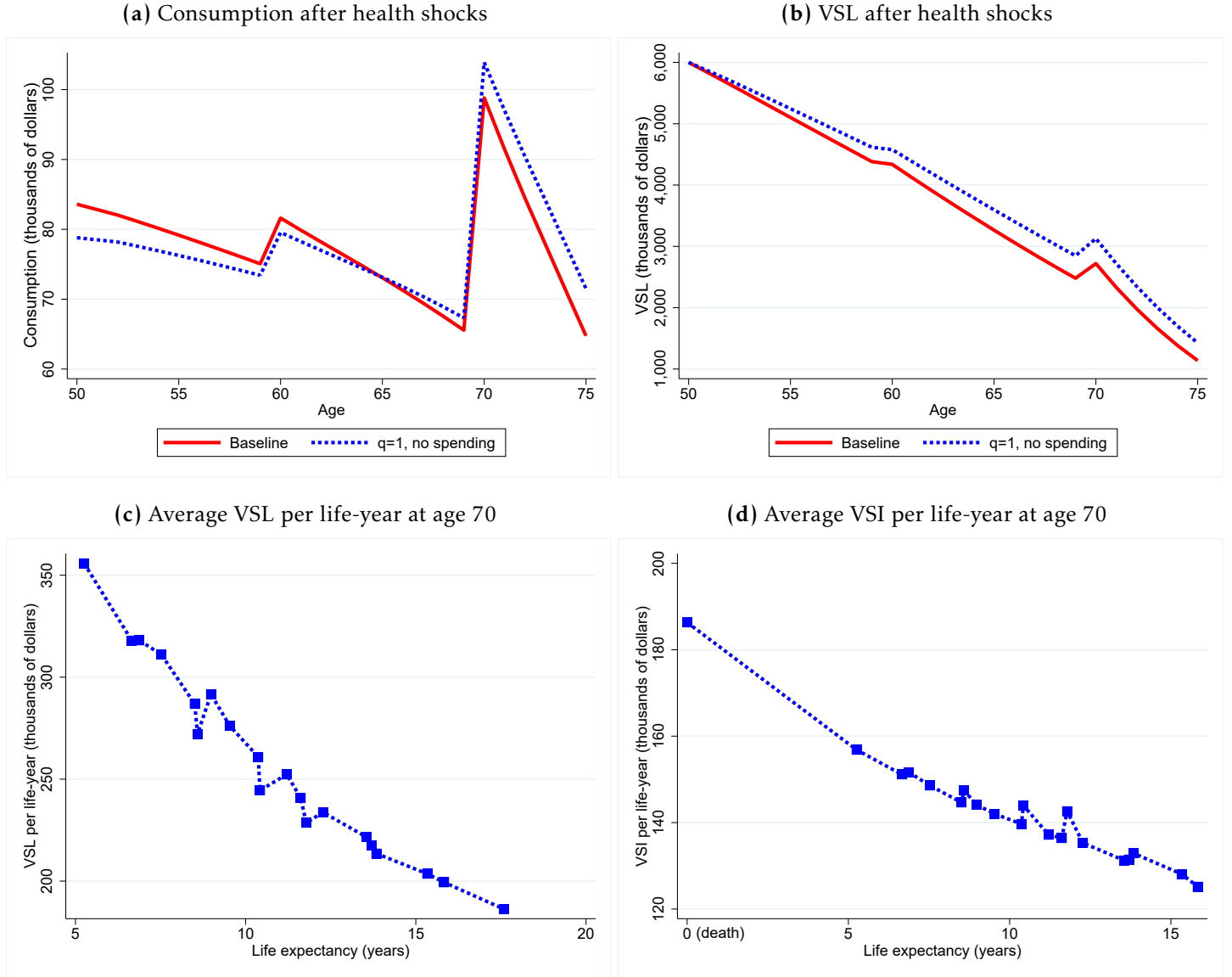
Notes: This figure presents VSI calculations for a sample of US adults from the Future Elderly Model. The dashed blue line reports a healthy (health state 1) 70-year-old's value of statistical illness (VSI) for different illnesses, which includes her marginal willingness to pay to avoid death (value 0 on the x-axis) and to avoid transitioning to one of the 19 other, sicker health states described in Table 1. The solid red line normalizes that value by the change in life expectancy caused by the illness. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent. Life expectancy for a 70-year-old in health state 1 is equal to 11.0 QALYs (see Table 1).

Figure 8: Sensitivity of results to different parameterizations of utility and to presence of bequest motive



Notes: The solid red lines in panels (a), (b), (c), and (d) replicate the baseline results from Figure 3 (consumption and VSL), Figure 6, and Figure 7. The dashed green and dashed blue lines present results under the alternative parameter assumptions $\gamma = 0.8$ and $\gamma = 1.5$, respectively, for the utility function (15). The bequest motive specification, depicted by the black dashed line, is based on Fischer (1973) and sets the bequest motive parameter $b(t) = 1.2$ (see Appendix C). Life expectancy is measured in quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent.

Figure 9: Sensitivity of results to quality of life and medical spending



Notes: The solid red lines in panels (a) and (b) replicate the baseline results from Figure 3 (consumption and VSL). The dashed blue lines present results when setting quality of life equal to 1 and out-of-pocket medical spending equal to 0 for all ages in all health states, i.e., $q_{Y_t} = q = 1$ and $r(t, Y_t) = r = .03$. (Medical spending is modeled as a modification to the interest rate in our framework.) Life expectancy is measured in life-years and is undiscounted. Panels (c) and (d) omit the baseline results from Figure 6 and Figure 7 because those baseline results were measured in quality-adjusted life-years and discounted at a rate of 3 percent.

Table 1: Summary means for the Future Elderly Model data, by health state

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		Life expectancy (years)		Life expectancy (QALYs)		Quality of life (EQ-5D)		Medical spending (\$)		Exit probability (%)	
Health state	ADLs / CCs	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70
1 (healthy)	0 / 0	30.9	17.6	16.3	11.0	0.88	0.87	686	1,361	4.2	12.6
2	0 / 1	28.2	15.8	14.8	9.7	0.85	0.84	866	1,578	3.6	10.8
3	0 / 2	24.6	13.6	12.8	8.2	0.81	0.80	1,145	1,925	3.6	10.2
4	0 / 3	20.5	11.2	10.7	6.7	0.77	0.76	1,487	2,366	3.9	10.2
5	0 / 4+	16.1	9.0	8.3	5.2	0.73	0.72	2,318	3,193	3.9	7.9
6	1 / 0	26.6	15.3	13.5	9.1	0.83	0.82	598	1,378	6.3	14.7
7	1 / 1	24.0	13.7	12.1	8.0	0.80	0.78	812	1,573	5.7	12.7
8	1 / 2	20.5	11.6	10.2	6.7	0.75	0.75	1,129	1,940	6.1	12.2
9	1 / 3	16.8	9.5	8.3	5.4	0.72	0.71	1,394	2,439	6.4	11.7
10	1 / 4+	13.2	7.5	6.5	4.2	0.67	0.66	2,098	3,287	6.1	8.6
11	2 / 0	24.3	13.8	11.9	7.9	0.78	0.77	585	1,314	7.3	14.3
12	2 / 1	21.5	12.3	10.4	6.9	0.75	0.73	797	1,600	7.5	14.3
13	2 / 2	18.1	10.4	8.7	5.7	0.71	0.69	1,043	1,934	7.5	13.8
14	2 / 3	15.0	8.5	7.1	4.6	0.67	0.66	1,348	2,412	7.5	13.1
15	2 / 4+	11.5	6.7	5.4	3.5	0.63	0.61	1,997	3,322	7.3	10.6
16	3+ / 0	21.9	11.8	10.3	6.5	0.70	0.69	693	1,358	3.4	11.1
17	3+ / 1	19.0	10.4	8.9	5.6	0.66	0.66	948	1,567	2.8	8.5
18	3+ / 2	15.7	8.6	7.3	4.5	0.62	0.62	1,105	1,965	2.3	7.1
19	3+ / 3	12.7	6.9	5.8	3.5	0.58	0.58	1,671	2,472	1.4	5.3
20	3+ / 4+	9.1	5.3	4.1	2.6	0.54	0.54	2,759	3,388	0.0	0.0

Notes: This table reports selected means for the health data obtained from the Future Elderly Model (FEM). Column (1) reports the number of impaired activities of daily living (ADLs) and the number of chronic conditions (CCs), which together define a health state. Columns (2)–(3) report life expectancy in years. Columns (4)–(5) reports life expectancy in QALYs, which is calculated using equation (14) with a 3% discount rate. Columns (6)–(7) report average quality of life as measured by the EQ-5D index, where 0 indexes death and 1 indexes perfect health. Columns (8)–(9) report average annual out-of-pocket medical spending, which includes all inpatient, outpatient, prescription, and long-term care spending not covered by insurance. Columns (10)–(11) report the percentage probability that an individual transitions to a different health state in the following year (excluding death). All impaired ADLs and chronic conditions are permanent, i.e., individuals can transition only to higher-numbered health states. Additional details about the FEM are available in Appendix B.

Online Appendix

“Health Risk and the Value of Life”

Daniel Bauer, University of Wisconsin-Madison

Darius Lakdawalla, University of Southern California and NBER

Julian Reif, University of Illinois and NBER

Appendix **A**: Mathematical Proofs

Appendix **B**: Future Elderly Model

Appendix **C**: Supporting Calculations for Quantitative Analysis

Appendix **D**: Complete Markets Model

A Mathematical Proofs

Proof of Lemma 1. Recall that the transition rates $\lambda_{ij}(t) = 0 \forall j < i$. The optimization problem in state n is therefore the standard problem with a single health state. We can contemplate a successive solution strategy by starting in state n and then moving sequentially to state $n - 1$, $n - 2$, etc. Thus, we can consider the deterministic optimization problem for an arbitrary state i by taking $V(t, w, j)$, $j > i$, as given (exogenous):

$$V(0, W_0, i) = \max_{c_i(t)} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

subject to:

$$\begin{aligned} \frac{\partial W_i(t)}{\partial t} &= r W_i(t) - c_i(t), \\ W_i(0) &= W_0 \end{aligned}$$

Optimal consumption and wealth in state i are denoted by $c_i(t)$ and $W_i(t)$, respectively. Denote the optimal value-to-go function as:

$$\tilde{V}(u, W_i(u), i) = \max_{c_i(t)} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

Setting $\tilde{V}(t, W_i(t), i) = e^{-\rho t} \tilde{S}(i, t) V(t, W_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (2) for i . See Theorem 1 and the proof of Theorem 2 in [Parpas and Webster \(2013\)](#) for additional details and intuition behind this result. ■

Proof of Lemma 2. From (3), the marginal utility of preventing an illness or death is:

$$\begin{aligned} \left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \left. \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp \left\{ - \int_0^t \sum_{j>i} (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right\} \left(u(c_i^\varepsilon(t), q_i(t)) + \sum_{j>i} (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, W_i^\varepsilon(t), j) \right) dt \right|_{\varepsilon=0} \\ &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \\ &\quad + \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i^\varepsilon(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \right) dt \Big|_{\varepsilon=0} \end{aligned}$$

where $c_i^\varepsilon(t)$ and $W_i^\varepsilon(t)$ represent the equilibrium variations in $c_i(t)$ and $W_i(t)$ caused by this perturbation.

We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to:

$$W_i(t) = W_0 e^{rt} - \int_0^t e^{r(t-s)} c_i(s) ds$$

which implies $\frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} = - \int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds$. From the solution to the costate equation, we know that:

$$e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

Thus, we can rewrite the second term in the expression for $\frac{\partial V}{\partial \varepsilon}|_{\varepsilon=0}$ above as:

$$\begin{aligned}
& \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds + \theta^{(i)} \right] e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt - \int_0^T e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds dt \Bigg|_{\varepsilon=0} \\
&= \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt - \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt + \int_0^T \theta^{(i)} e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt \Bigg|_{\varepsilon=0} \\
&= \theta^{(i)} \frac{\partial}{\partial \varepsilon} \underbrace{\int_0^T e^{-rt} c_i^\varepsilon(t) dt}_{W_0} \Bigg|_{\varepsilon=0} \\
&= 0
\end{aligned}$$

where the last equality follows from application of the budget constraint. ■

Proof of Lemma 3. We show the result at an arbitrary time t and a future time $\tau > t$:

$$\frac{\partial V(t, W_i(t), i)}{\partial W_i(t)} = u_c(c_i(t), q_i(t)) = \mathbb{E} \left[e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \middle| Y_t = i, W(t) = W_i(t) \right], \forall \tau > t$$

The proof proceeds by induction on $i \leq n$. For the base case $i = n$, in which no state transitions are possible, the solution to the costate equation (4) simplifies to:

$$\begin{aligned}
p_\tau^{(n)} &= \theta^{(n)} e^{-r\tau} \\
&= \exp \left\{ - \int_0^\tau \rho + \lambda_{n,n+1}(s) ds \right\} u_c(c_n(\tau), q_n(\tau)) \\
&= \theta^{(n)} e^{-rt} e^{-r(\tau-t)} \\
&= p_t^{(n)} e^{-r(\tau-t)} \\
&= \exp \left\{ - \int_0^t \rho + \lambda_{n,n+1}(s) ds \right\} u_c(c_n(t), q_n(t)) e^{-r(\tau-t)}
\end{aligned}$$

where the second equality makes use of the first-order condition (5). Using the expressions in the second and the last lines then gives:

$$u_c(c_n(t), q_n(t)) = e^{r(\tau-t)} e^{-\rho(\tau-t)} \exp \left\{ - \int_t^\tau \lambda_{n,n+1}(s) ds \right\} u_c(c_n(\tau), q_n(\tau))$$

which shows that the lemma holds for $i = n$.

For the induction step, suppose the lemma is true for $j > i$, $1 \leq i \leq n-1$. For any subinterval $[0, \tau]$, the solution of the costate equation can be written as:

$$p_t^{(i)} = \left[\int_t^\tau e^{(r-\rho)s} \exp \left\{ - \int_0^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta(\tau, i) e^{-rt} \quad (\text{A.1})$$

where $\theta(\tau, i)$ is a constant that depends on the choice of τ and i . (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating equation (A.1) at $t = \tau$ and combining with equation (5) from the main text yields:

$$p_\tau^{(i)} = \theta(\tau, i) e^{-r\tau} = \exp \left\{ - \int_0^\tau \rho + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau))$$

which implies:

$$\theta(\tau, i) = e^{(r-\rho)\tau} \exp \left\{ - \int_0^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \quad (\text{A.2})$$

Plugging equations (5) and (A.2) into equation (A.1) yields:

$$\begin{aligned} u_c(c_i(t), q_i(t)) \exp \left\{ - \int_0^t \rho + \sum_{j>i} \lambda_{ij}(s) ds \right\} &= \left[\int_t^\tau e^{(r-\rho)s} \exp \left\{ - \int_0^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \\ &\quad + e^{-rt} e^{(r-\rho)\tau} \exp \left\{ - \int_0^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \end{aligned}$$

Since $\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} = u_c(c(s, W_i(s), j), q_j(s))$ from the first-order condition in the HJB for state j , we obtain:

$$\begin{aligned} u_c(c_i(t), q_i(t)) &= \int_t^\tau e^{(r-\rho)(s-t)} \exp \left\{ - \int_t^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) u_c(c(s, W_i(s), j), q_j(s)) ds + e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \\ &= \int_t^\tau e^{(r-\rho)(s-t)} \exp \left\{ - \int_t^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \mathbb{E} \left[e^{(r-\rho)(\tau-s)} \exp \left\{ - \int_s^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \middle| Y_s = j, W(s) = W_i(s) \right] ds \\ &\quad + e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \\ &= \mathbb{E} \left[e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \middle| Y_t = i, W(t) = W_i(t) \right] \end{aligned}$$

where the second equality follows from the induction hypothesis. ■

Proof of Proposition 4. Choosing the Dirac delta function for $\delta(t)$ in Lemma 2 yields:

$$\begin{aligned} \frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T \left[e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) \right] dt \\ &= \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0 = i, W(0) = W_0 \right] \end{aligned}$$

Dividing the result by the marginal utility of wealth at time $t = 0$ then yields the value of statistical life given by equation (7):

$$VSL(i) = \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u(c(0), q_{Y_0}(0))} dt \middle| Y_0 = i, W(0) = W_0 \right]$$

Applying Lemma 3 for $t = 0$ allows us to rewrite VSL as:

$$\begin{aligned} VSL(i) &= \mathbb{E} \left[\int_0^T e^{-\rho t} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E} \left[e^{(r-\rho)t} \exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]} dt \middle| Y_0 = i, W(0) = W_0 \right] \\ &= \mathbb{E} \left[\int_0^T e^{-rt} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]} dt \middle| Y_0 = i, W(0) = W_0 \right] \end{aligned}$$

Exchanging expectation and integration then yields:

$$VSL(i) = \int_0^T e^{-rt} v(i, t) dt$$

where $v(i, t)$ is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i, t) = \frac{\mathbb{E} \left[S(t) u(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]}{\mathbb{E} \left[S(t) u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0 \right]}$$

■

Proof of Proposition 5. Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Because we hold quality of life constant, we omit $q_i(t)$ in the notation below in order to keep the presentation concise.

We want to prove that $c_2(0) \geq c_1(0)$. Assume by way of contradiction that $c_2(0) < c_1(0)$. We will show that this assumption implies $c_2(t) < c_1(t)$ for all $t > 0$, which is a contradiction since the feasible consumption plan $c_1(\cdot)$ dominates $c_2(\cdot)$.

We proceed by inductively constructing a sequence $0 < t_1 < t_2 \dots$ where for each element in the sequence:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p_{t_i}^{(1)} &< \exp \left\{ - \int_0^{t_i} \lambda_{12}(s) ds \right\} p_{t_i}^{(2)} \end{aligned}$$

To construct the sequence, for the base case $i = 1$, we first note that from the first-order condition (5), we obtain:

$$p_0^{(1)} = u_c(c_1(0)) < u_c(c_2(0)) = p_0^{(2)}$$

The costate equation (4) then implies:

$$\begin{aligned} \dot{p}_0^{(1)} &= -p_0^{(1)} r - \lambda_{12}(0) u_c(c_2(0)) \\ &= -p_0^{(1)} \left[r + \lambda_{12}(0) \underbrace{\frac{u_c(c_2(0))}{u_c(c_1(0))}}_{>1} \right] \\ &< -p_0^{(1)} [r + \lambda_{12}(0)] = \frac{\partial g(t)}{\partial t} \Big|_{t=0} \end{aligned}$$

where $g(t) = p_0^{(1)} \exp \left\{ - \int_0^t r + \lambda_{12}(s) ds \right\}$. Hence, there exists a $t_1 > t_0 = 0$ such that:

$$p_t^{(1)} \leq g(t) < p_0^{(2)} \exp \left\{ - \int_0^t (r + \lambda_{12}(s)) ds \right\} = p_t^{(2)} \exp \left\{ - \int_0^t \lambda_{12}(s) ds \right\}, \quad 0 \leq t \leq t_1$$

which together with the first-order condition (5) implies:

$$e^{-\rho t} \exp \left\{ - \int_0^t (\lambda_{12}(s) + \lambda_{13}(s)) ds \right\} u_c(c_1(t)) < e^{-\rho t} \exp \left\{ - \int_0^t (\lambda_{12}(s) + \lambda_{23}(s)) ds \right\} u_c(c_2(t)), \quad 0 \leq t \leq t_1$$

so that $c_1(t) > c_2(t)$, $0 \leq t \leq t_1$. This inequality in turn implies $W_1(t_1) \leq W_2(t_1)$.

For the induction step, suppose that the following properties also hold for $i \geq 1$:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p_{t_i}^{(1)} &< \exp \left\{ - \int_0^{t_i} \lambda_{12}(s) ds \right\} p_{t_i}^{(2)} \end{aligned}$$

The induction hypothesis implies:

$$c(t_i, W_1(t_i), 2) \leq c(t_i, W_2(t_i), 2) = c_2(t_i) < c_1(t_i)$$

so that:

$$\begin{aligned} \dot{p}_{t_i}^{(1)} &= -p_{t_i}^{(1)} r - e^{-\rho t_i} \tilde{S}(1, t_i) \lambda_{12}(t_i) u(c(t_i, W_1(t_i), 2)) \\ &= -p_{t_i}^{(1)} \left[r + \lambda_{12}(t_i) \underbrace{\frac{u_c(c(t_i, W_1(t_i), 2))}{u_c(c_1(t_i))}}_{>1} \right] \\ &< -p_{t_i}^{(1)} [r + \lambda_{12}(t_i)] = \frac{\partial \tilde{g}(t)}{\partial t} \Big|_{t=0} \end{aligned}$$

with $\tilde{g}(t) = p_{t_i}^{(1)} \exp \left\{ - \int_{t_i}^t (r + \lambda_{12}(s)) ds \right\}$. Hence, there exists a $t_{i+1} > t_i$ such that:

$$\begin{aligned} p_t^{(1)} &\leq \tilde{g}(t) \\ &< \exp \left\{ - \int_0^{t_i} \lambda_{12}(s) ds \right\} p_{t_i}^{(2)} \exp \left\{ - \int_{t_i}^t (r + \lambda_{12}(s)) ds \right\} \\ &= p_t^{(2)} \exp \left\{ - \int_0^t \lambda_{12}(s) ds \right\}, \quad t_i \leq t \leq t_{i+1} \end{aligned}$$

Applying again the first-order condition (5) for all $t_i \leq t \leq t_{i+1}$ yields:

$$\exp \left\{ - \int_0^t (\lambda_{12}(s) + \lambda_{13}(s)) ds \right\} u_c(c_1(t)) < \exp \left\{ - \int_0^t (\lambda_{12}(s) + \lambda_{23}(s)) ds \right\} u_c(c_2(t))$$

which in turn implies $u_c(c_1(t)) < u_c(c_2(t))$ and $c_2(t) < c_1(t)$. Once again, this inequality implies $W_1(t_{i+1}) \leq W_2(t_{i+1})$.

Thus, we have proven the existence of the sequence. We then obtain $c_2(t) < c_1(t) \forall t$ by noting that $\{t_i\}_{i \geq 0}$ strictly increases due to the uniformly boundedness condition on $\lambda_{12}(t)$, which is the desired contradiction.

We note that this proof implies that the consumption paths $c_1(t)$ and $c_2(t)$ cross (at most) once. As soon as $c_1(t)$ exceeds $c_2(t)$ for some time t_0 , $c_1(t)$ will exceed $c_2(t)$ for $t > t_0$. However, we have that $c_2(t)$ exceeds $c_1(t)$ prior to t_0 . In particular, consumption jumps up at the transition point. ■

Proof of Proposition 6. Without loss of generality, consider the case $t = 0$. Under our assumptions, from equation (9) and Proposition 5 it is clear that $c_1(t)$ and $c_2(t)$ are decreasing, $c_2(0) > c_1(0)$, $c_2(t) > c_1(t)$ for $t \leq t_0$, and $c_2(t) < c_1(t)$ for $t > t_0$. Making use of the assumption that no state transitions occur for $t > 0$, we have that:

$$VSL(2, 0) = \int_0^T e^{-rt} \frac{S_2(t) u(c_2(t))}{S_2(t) u_c(c_2(t))} dt = \int_0^T e^{-rt} \frac{u(c_2(t))}{u_c(c_2(t))} dt$$

and:

$$VSL(1, 0) = \int_0^T e^{-rt} \frac{u(c_1(t))}{u_c(c_1(t))} dt$$

Let $Y(x) \equiv \frac{u(x)}{u_c(x)}$. Under the stated assumptions on preferences, we have that:

$$Y'(x) = 1 - \frac{u(x)u_{cc}(x)}{(u_c(x))^2} > 0,$$

$$Y''(x) = \frac{2(u_{cc}(x))^2 u(x) - u_c^2(x)u_{cc}(x) - u_c(x)u(x)u_{ccc}(x)}{(u_c(x))^3} > 0$$

Employing Taylor's theorem then implies that for some $\xi(t)$ that lies in-between $c_1(t)$ and $c_2(t)$:

$$\begin{aligned} VSL(2,0) &= \int_0^T e^{-rt} Y(c_2(t)) dt \\ &= \int_0^T e^{-rt} \left[Y(c_1(t)) + [c_2(t) - c_1(t)] Y'(c_1(t)) + \underbrace{\frac{1}{2}[c_2(t) - c_1(t)]^2 Y''(\xi(t))}_{>0} \right] dt \\ &> \int_0^T e^{-rt} Y(c_1(t)) dt + \int_0^{t_0} e^{-rt} Y'(c_1(t)) \underbrace{[c_2(t) - c_1(t)]}_{\geq 0} dt + \int_{t_0}^T e^{-rt} Y'(c_1(t)) \underbrace{[c_2(t) - c_1(t)]}_{\leq 0} dt \\ &\geq \int_0^T e^{-rt} Y(c_1(t)) dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0)) [c_2(t) - c_1(t)] dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0)) [c_2(t) - c_1(t)] dt \\ &= \int_0^T e^{-rt} Y(c_1(t)) dt + Y'(c_1(t_0)) \underbrace{\left[\int_0^T e^{-rt} c_2(t) dt - \int_0^T e^{-rt} c_1(t) dt \right]}_{=0} \\ &= \int_0^T e^{-rt} Y(c_1(t)) dt \\ &= VSL(1,0) \end{aligned}$$

where the final step follows from the budget constraint. ■

Proof of Proposition 7. The proposition assumes concavity in health states i, j , and k :

$$V(0, W_0, j) > D \times V(0, W_0, i) + (1 - D) \times V(0, W_0, k), \text{ where } D = \frac{D_j - D_k}{D_i - D_k}$$

This condition is equivalent to:

$$\begin{aligned} V(0, W_0, i) - V(0, W_0, j) &< (1 - D) \times [V(0, W_0, i) - V(0, W_0, k)] \\ \iff \frac{V(0, W_0, i) - V(0, W_0, j)}{D_i - D_j} &< \frac{V(0, W_0, i) - V(0, W_0, k)}{D_i - D_k} \end{aligned}$$

since $(1 - D) = (D_i - D_j)/(D_i - D_k)$. Dividing both sides of the final expression by $u_c(c_i(0), q_i(0))$ and applying equation (8) yields the first part of the proposition:

$$\frac{VSI(i, j)}{D_i - D_j} < \frac{VSI(i, k)}{D_i - D_k}$$

For the second part, note that concavity in health states i, j , and k implies:

$$\begin{aligned}
V(0, W_0, j) - V(0, W_0, k) &> D \times [V(0, W_0, i) - V(0, W_0, k)] \\
\iff \frac{V(0, W_0, j) - V(0, W_0, k)}{D_j - D_k} &> \frac{V(0, W_0, i) - V(0, W_0, k)}{D_i - D_k} \\
\iff \frac{V(0, W_0, j) - V(0, W_0, k)}{u_c(c_j(0), q_j(0))} \frac{1}{D_j - D_k} &> \frac{u_c(c_i(0), q_i(0))}{u_c(c_j(0), q_j(0))} \frac{V(0, W_0, i) - V(0, W_0, k)}{u_c(c_i(0), q_i(0))} \frac{1}{D_i - D_k} \\
&\iff \frac{VSI(j, k)}{D_j - D_k} > \frac{u_c(c_i(0), q_i(0))}{u_c(c_j(0), q_j(0))} \frac{VSI(i, k)}{D_i - D_k}
\end{aligned}$$

where the second equivalence follows from dividing by $u_c(c_j(0), q_j(0))$. Finally, the assumption that $u_c(c_i(0), q_i(0)) \geq u_c(c_j(0), q_j(0))$ yields the second part of the proposition:

$$VSI(j, k)/(D_j - D_k) > VSI(i, k)/(D_i - D_k)$$

■

Proof of Proposition 8 and Corollary 9. Our goal is to derive expressions for VSL and VSI when annuity markets are incomplete and the consumer is endowed with state-dependent life-cycle income. We first consider in part (i) the case with life-cycle earnings only. This part also provides expressions for the incomplete markets case at time $t > 0$, because after a flat annuity has been purchased it is equivalent to adding a constant to life-cycle earnings. Part (ii) considers the optimal purchase of the annuity and provides expressions for VSL and VSI at time $t = 0$.

(i) No annuity markets

Denote the consumer's earnings in state i at time t as $m_i(t)$. The consumer's maximization problem is again equation (1), but the law of motion for wealth now includes earnings:

$$\begin{aligned}
W(0) &= W_0, \\
W(t) &\geq 0, \\
\frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) - c(t)
\end{aligned}$$

Once again, we solve this stochastic finite-horizon optimization problem by reformulating it as a deterministic optimization problem. Specifically, we consider equation (3), subject to:

$$\begin{aligned}
W_i(0) &= W_0, \\
W_i(t) &\geq 0, \\
\frac{\partial W_i(t)}{\partial t} &= rW_i(t) + m_i(t) - c_i(t)
\end{aligned}$$

The present-value Hamiltonian corresponding to this deterministic problem is:

$$H(W_i(t), c_i(t), p_t^{(i)}, \Psi_t^{(i)}) = e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) + p_t^{(i)} [rW_i(t) + m_i(t) - c_i(t)] + \Psi_t^{(i)} W_i(t)$$

where $p_t^{(i)}$ is the costate variable for the wealth dynamics in state i and $\Psi_t^{(i)}$ is the multiplier for the wealth constraint. The

first-order conditions are:

$$\begin{aligned}\dot{p}_t^{(i)} &= -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)} r - e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} - \Psi_t^{(i)} \\ p_t^{(i)} &= e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) \\ \Psi_t^{(i)} &\geq 0, \\ \Psi_t^{(i)} W_i(t) &= 0\end{aligned}$$

Following Proposition 1 in [Leung \(1994\)](#), one can show the following: the Hamiltonian is regular on $[0, T)$, so optimal consumption $c_i(t)$ is everywhere continuous; the state-variable inequality constraint is of first-order, so $p_t^{(i)}$ is everywhere continuous; and optimal consumption $c_i(t)$ is continuously differentiable when $W_i(t) > 0$ (i.e., when the wealth constraint is not binding).

First, consider the case when $W_i(t) > 0$. Differentiating the first-order condition for consumption with respect to t , plugging in the result for the costate equation and its solution, and then rearranging yields the rate of change in life-cycle consumption. This rate of change, $\frac{\dot{c}_i}{c_i}$, is identical to the one described by equation (9), and is weakly declining by assumption.

The presence of life-cycle earnings introduces the possibility of multiple sets of non-interior solutions (e.g., right panel of Figure 1). Modeling these scenarios is possible, but cumbersome. As discussed in the main text, we therefore restrict ourselves to considering the case with a single set of non-interior solutions (i.e., a single “kink point”, see left panel of Figure 1). A sufficient (but not necessary) assumption is that consumption growth is weakly declining. We employ that assumption in the following Lemma, which establishes the existence of a single kink point, T_i , where the consumer runs out of wealth.

Lemma A.1. *Assume $m_i(t)$ is non-decreasing. Then there must exist a T_i such that (1) $W_i(t) = 0$ and $c_i(t) = m_i(t)$ for $t \geq T_i$; and (2) $c_i(t) > m_i(t)$ for $t < T_i$. The solution to the costate equation on $[0, T_i]$ is thus:*

$$p_t^{(i)} = \left[\int_t^{T_i} e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant.

Proof. By assumption, $\frac{\dot{c}_i}{c_i} < 0$ whenever $W_i(t) > 0$. Following the same argument as in Proposition 2 of [Leung \(1994\)](#), there is a smallest T_i such that $W_i(t) = 0$ on $[T_i, T]$ and, thus, $c_i(t) = m_i(t)$ on $[T_i, T]$. Since this is the smallest such T_i , there exists an interval (\underline{T}_i, T_i) such that $W_i(t) > 0$ and $c_i(t_0) > m_i(t_0)$ for a t_0 in the vicinity of T_i . Now assume $W_i(\underline{T}_i) = 0$. Then there exists a t_1 in the vicinity of \underline{T}_i such that $c_i(t_1) < m_i(t_1)$. This is a contradiction, since $m_i(t)$ is non-decreasing and $c_i(t)$ is decreasing whenever $W_i(t) > 0$. Hence $W_i(t) > 0$ on $[0, T_i)$ and $c_i(t) > m_i(t)$ for $t \in [0, T_i)$. As in the main text, the solution to the costate equation can be obtained using the variation of the constant method. ■

Because the value of statistical illness (VSI) is a generalization of the value of statistical life (VSL), we again focus on deriving an expression for VSI. Let $\delta_{ij}(t)$ be a perturbation on the transition rate, and consider the impact on survival as

described by equation (6). From equation (3), we obtain:

$$\begin{aligned}
\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \left[\int_0^{T_i(\varepsilon)} e^{-\rho t} \tilde{S}^\varepsilon(i, t) \left(u(c_i^\varepsilon(t), q_i(t)) + \sum_{j>i} (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, W_i^\varepsilon(t), j) \right) dt + \int_{T_i(\varepsilon)}^T e^{-\rho t} \tilde{S}^\varepsilon(i, t) \left(u(m_i(t), q_i(t)) + \sum_{j>i} (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, 0, j) \right) dt \right] \Big|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \\
&\quad + \underbrace{\int_0^{T_i} e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} \right) dt}_{=0}
\end{aligned}$$

where the second term in the last equality is equal to 0:

$$\begin{aligned}
&\int_0^{T_i} e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} \right) dt \\
&= \int_0^{T_i} p_t^{(i)} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \left[- \int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} \Big|_{\varepsilon=0} ds \right] dt \\
&= \int_0^{T_i} \theta^{(i)} e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} dt + \int_0^{T_i} \int_t^{T_i} e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} dt \\
&\quad - \int_0^{T_i} \int_t^{T_i} e^{-\rho s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds e^{rs} e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} dt \\
&= \theta^{(i)} \frac{\partial}{\partial \varepsilon} \int_0^{T_i} e^{-rt} c_i^\varepsilon(t) dt \Big|_{\varepsilon=0} \\
&= 0
\end{aligned}$$

The final equality follows because $W_i(T_i) = 0$ (by definition), which in turn implies $0 = W_0 + \int_0^{T_i} e^{-rt} m_i(t) dt - \int_0^{T_i} e^{-rt} c_i^\varepsilon(t) dt$, so that differentiation yields zero. Thus we obtain:

$$\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} = \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \quad (\text{A.3})$$

Dividing by the marginal utility of wealth yields the value of life-extension. Choosing the Dirac delta function for $\delta_{i,n+1}(t)$ yields VSL, and choosing the Dirac delta function for $\delta_{ij}(t)$, $j < n+1$, yields VSI:

$$VSL(i) = \frac{V(0, W(0), i)}{u_c(c_i(0), q_i(0))} \quad (\text{A.4})$$

$$VSI(i, j) = \frac{V(0, W(0), i) - V(0, W(0), j)}{u_c(c_i(0), q_i(0))} \quad (\text{A.5})$$

(ii) Incomplete annuity markets

Now, we introduce a one-time opportunity at time $t = 0$ to purchase a flat lifetime annuity at a level $\bar{a}_{Y_0} \geq 0$ with a price markup $\xi \geq 0$. Let $a(T, i) = \mathbb{E} \left[\int_t^T e^{-r(s-t)} \exp \left\{ - \int_t^s \mu(u) du \right\} ds \mid Y_t = i \right]$ be the expected value of a one-dollar annuity purchased at time t in state i . Note that for any given annuity, \bar{a}_i , the consumer's problem can be mapped to the no-annuity case in

part (i) above by setting the constraints equal to:

$$W_i(0) = W_0 - (1 + \xi)\bar{a}_i a(0, i),$$

$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) + m_i(t) + \bar{a}_i - c_i(t)$$

Solving for the optimal fixed annuity then becomes a straightforward static optimization problem:

$$\bar{a}_i^* = \arg \max_{\bar{a}_i} V(0, W_i(0), \bar{a}_i, i)$$

The optimal annuity must satisfy the necessary first-order condition:

$$\frac{\partial V(0, W_i(0), \bar{a}_i, i)}{\partial \bar{a}_i} = \frac{\partial V(0, W_i(0), \bar{a}_i, i)}{\partial W(0)} (1 + \xi) a(0, i) \quad (\text{A.6})$$

Because the consumer may favor a non-flat optimal consumption profile, the optimal level of annuitization is likely to be partial even if the markup ξ is equal to zero. However, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and income are constant.¹

The value of an annuity depends on a consumer's expected future survival. Life-extension affects the value and cost of a given annuity, and may also affect the level of the optimal annuity. Thus, the effect of the mortality rate perturbation on the marginal utility of life-extension is:

$$\left. \frac{\partial V(0, W_i^\varepsilon(0), \bar{a}_i^\varepsilon, i)}{\partial \varepsilon} \right|_{\varepsilon=0} = (\text{A.3}) + \left. \frac{\partial V}{\partial \bar{a}_i} \frac{\partial \bar{a}_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} + \left. \frac{\partial V}{\partial W_i(0)} \frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0}$$

where the first term on the right-hand side is equal to equation (A.3) derived in part (i) above for the case with life-cycle earnings but no annuity. Note that:

$$\begin{aligned} \left. \frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \left(-(1 + \xi)\bar{a}_i^\varepsilon \int_0^T \tilde{S}^\varepsilon(i, t) e^{-rt} \left[1 + \sum_{j>i} (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) a(t, j) \right] dt \right) \\ &= -(1 + \xi) \left. \frac{\partial \bar{a}_i^\varepsilon}{\partial \varepsilon} \right|_{\varepsilon=0} a(0, i) - (1 + \xi)\bar{a}_i \int_0^T e^{-rt} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(1 + \sum_{j>i} \lambda_{ij}(t) a(t, j) \right) - \sum_{j>i} \delta_{ij}(t) a(t, j) \right] dt \end{aligned}$$

Combining this with the first-order condition (A.6) implies that:

$$\left. \frac{\partial V}{\partial \bar{a}_i} \frac{\partial \bar{a}_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} + \left. \frac{\partial V}{\partial W_i(0)} \frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} = - \frac{\partial V}{\partial W_i(0)} (1 + \xi)\bar{a}_i \int_0^T e^{-rt} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(1 + \sum_{j>i} \lambda_{ij}(t) a(t, j) \right) - \sum_{j>i} \delta_{ij}(t) a(t, j) \right] dt$$

Thus the marginal utility of life-extension is equal to:

$$\begin{aligned} \left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), \bar{a}_i, j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), \bar{a}_i, j) \right] dt \\ &\quad - \frac{\partial V}{\partial W_i(0)} (1 + \xi)\bar{a}_i \int_0^T e^{-rt} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(1 + \sum_{j>i} \lambda_{ij}(t) a(t, j) \right) - \sum_{j>i} \delta_{ij}(t) a(t, j) \right] dt \end{aligned}$$

The marginal utility of wealth, $\partial V / \partial W_i(0)$, is equal to $u_c(c_i(0), q_i(0))$ when the solution is interior. Dividing by the marginal

¹Even in the case of full annuitization, the first-order condition (A.6) holds with strict equality since the consumer is indifferent between an increase in the annuity level or a proportionate increase in baseline wealth.

utility of wealth and rearranging yields the marginal value of life-extension:

$$\begin{aligned} & \left. \frac{\partial V / \partial \varepsilon}{\partial V / \partial W} \right|_{\varepsilon=0} \\ &= \int_0^T \tilde{S}(i, t) \left\{ \left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left[\left(\frac{e^{-\rho t} u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), \bar{a}_i, j)}{u_c(c_i(0), q_i(0))} \right) - (1 + \xi) \bar{a}_i e^{-rt} \left(1 + \sum_{j>i} \lambda_{ij}(t) a(t, j) \right) \right] \right. \\ & \quad \left. - \sum_{j>i} \delta_{ij}(t) \left(\frac{V(t, W_i(t), \bar{a}_i, j)}{u_c(c_i(0), q_i(0))} - (1 + \xi) \bar{a}_i e^{-rt} a(t, j) \right) \right\} dt \end{aligned}$$

Choosing the Dirac delta function for $\delta_{i,n+1}(t)$ yields:

$$\begin{aligned} VSL(i) &= \frac{V(0, W_i(0), \bar{a}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi) \bar{a}_i \int_0^T \tilde{S}(i, t) e^{-rt} \left(1 + \sum_{j>i} \lambda_{ij}(s) a(t, j) \right) dt \\ &= \frac{V(0, W_i(0), \bar{a}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi) \bar{a}_i a(0, i) \end{aligned}$$

Likewise, choosing the Dirac delta function for $\delta_{ij}(t)$, $j < n + 1$, yields:

$$VSI(i, j) = \left(\frac{V(0, W_i(0), \bar{a}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi) a(0, i) \bar{a}_i \right) - \left(\frac{V(0, W_i(0), \bar{a}_i, j)}{u_c(c_i(0), q_i(0))} - (1 + \xi) a(0, j) \bar{a}_i \right)$$

■

Proof of Proposition 10. If $\xi = 0$, $r = \rho$, and future income and quality of life are constant across both time and states, then it is optimal for the consumer to fully annuitize, in which case optimal consumption will be constant:

$$c(t) = m_i(t) + \bar{a}_1 = \bar{m}_i + \bar{a}_1 = \bar{c}$$

Without loss of generality, consider a transition from state 1 to state 2 at time $t = 0+$, the instant after the consumer has purchased her annuity. Hence, we rely on the VSL expression (A.4) from part (i) of the proof of Proposition 8 and Corollary 9. We have:

$$VSL(1, 0) = \mathbb{E} \left[\int_0^T e^{-rt} S(t) \frac{u(\bar{c}, q)}{u_c(\bar{c}, q)} dt \middle| Y_0 = 1 \right] = \frac{u(\bar{c}, q)}{u_c(\bar{c}, q)} a(0, 1)$$

where $a(0, 1)$ is the value of a one-dollar annuity at time $t = 0$ in state 1 as defined in the main text. Similarly,

$$VSL(2, 0) = \frac{u(\bar{c}, q)}{u_c(\bar{c}, q)} a(0, 2)$$

By assumption, survival in the healthy state is larger than survival in the sick state: $\mathbb{E}[S(t)|Y_0 = 1] > \mathbb{E}[S(t)|Y_0 = 2]$. This assumption implies $a(0, 2) < a(0, 1)$, which in turn implies $VSL(1, 0) > VSL(2, 0)$.

■

Proof of Corollary 11. Again, as in the proof of Proposition 10, we consider transitions at time $t = 0+$, the instant after the consumer has purchased her annuity. Using the VSI expression (A.5) from part (i) of the proof of Proposition 8 and Corollary 9, we have:

$$\frac{VSI(i, j)}{D_i - D_j} = \frac{V(0, W_i(0), \bar{a}_i, i) - V(0, W_i(0), \bar{a}_i, j)}{u_c(c_i(0), q_i(0)) (D_i - D_j)}.$$

With condition (13), the results then follow by employing the same arguments as in the proof of Proposition 7.

■

B Future Elderly Model

The empirical exercises presented in Section 3 employ data obtained from the Future Elderly Model (FEM). The FEM is a microsimulation model that projects future health and medical spending for Americans ages 50 and over. It has been used by a variety of researchers and policy analysts to understand the implications of population aging, health trends, new medical technologies, and possible health policy interventions in the US, Europe, and Asia (Goldman et al., 2005; Lakdawalla et al., 2005, 2008; Goldman et al., 2009, 2010; Michaud et al., 2011, 2012; Goldman et al., 2013; Goldman and Orszag, 2014; National Academies of Sciences, Engineering, and Medicine, 2015; Chen et al., 2016; Gonzalez-Gonzalez et al., 2017). Technical information about its data sources and methods is available online at:

<https://roybalhealthpolicy.usc.edu/fem/technical-specifications/>.

The FEM has three core modules. The first is the Replenishing Cohorts module, which predicts economic and health outcomes of new cohorts of 50-year-olds using data from the Panel Study of Income Dynamics, and incorporates trends in disease and other outcomes based on data from external sources, such as the National Health Interview Survey and the American Community Survey. This module generates new cohorts as the simulation proceeds, so that we can measure outcomes for the age 50+ population in any given year.

The second component is the Health Transition module, which uses the longitudinal structure of the Health and Retirement Study (HRS) to calculate transition probabilities across various health states, including chronic conditions, functional status, body-mass index, and mortality. These transition probabilities depend on a battery of predictors: age, sex, education, race, ethnicity, smoking behavior, marital status, employment and health conditions. FEM transitions produce a large set of simulated outcomes, including diabetes, high-blood pressure, heart disease, cancer (except skin cancer), stroke or transient ischemic attack, and lung disease (either or both chronic bronchitis and emphysema), disability, and body-mass index. Disability is measured by limitations in instrumental activities of daily living, activities of daily living, and residence in a nursing home.

Finally, the Policy Outcomes module estimates medical spending, including payments made by insurers (Medicare, Medicaid and Private) and out-of-pocket payments made by individuals. Medical spending for an individual is predicted as a function of health status (chronic conditions and functional status), demographics (age, sex, race, ethnicity and education), nursing home status, and mortality. Estimates are based on spending data from the Medical Expenditure Panel Survey for individuals ages 64 and younger and the Medicare Current Beneficiary Survey for individuals ages 65 and older.

The following example illustrates how the three modules interact. For year 2014, the model begins with the population of Americans ages 50 and over based on nationally representative data from the HRS. Individual-level health and economic outcomes for the next two years are predicted using the Policy Outcomes module. The cohort is then aged two years using the Health Transition Module. Aggregate health and functional status outcomes for those years are then calculated. At that point, a new cohort of 50-year-olds is introduced into the 2016 population using the Replenishing Cohort module, and they join those who survived from 2014 to 2016. This forms the age 50+ population for 2016. The transition model is then applied to this population. The same process is repeated until reaching the last year of the simulation. For our study, we ran the simulation until the year 2064, which gives us complete life-cycle data for ages 50–100 for all people who were ages 50 and over as of 2014.

The projections produced by the FEM have been extensively validated. Mortality forecasts line up closely with published death counts and achieve lower error rates than alternative forecasts used by the Social Security Administration (Leaf et al., 2020). Population, smoking behavior, cancer, diabetes, heart disease, hypertension, lung disease, and stroke forecasts perform well in cross-validation exercises. Medical spending data have been comprehensively tested against national aggregates.

C Supporting Calculations for Quantitative Analysis

This appendix provides the solution to the discrete-time dynamic programming problem described in Section 3.1. This model is solved analytically and provides exact solutions for optimal consumption.

The consumer's problem is:

$$\max_{c(t)} \mathbb{E} \left[\sum_{t=0}^T e^{-\rho t} S_0(t) u(c(t), q_{Y_t}(t)) + e^{-\rho(t+1)} ((S_0(t) - S_0(t+1)) u(W(t+1), b(t))) \right] \Big| Y_0, W_0$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t)) e^{r(t, Y_t)} \end{aligned}$$

where all variables are defined as in the main text. The strength of the bequest motive is governed by the parameter $b(t)$. We set $b(t) = 0$ in our baseline specification, which assumes no bequest motive (and normalizes utility of death to zero). The utility function is given by equation (15) from the main text:

$$u(c, q) = q \left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right)$$

where \underline{c} is the subsistence level of consumption for a healthy person with no bequest motive. Because optimal consumption is unaffected by affine transformations of utility, we shall initially assume $u(c, q) = q c^{1-\gamma} / (1-\gamma)$ when solving the model for consumption.

Define the value function as:

$$V(t, W(t), Y_t) = \max_{c(s)} \mathbb{E} \left[\sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q_{Y_s}(s)) + e^{-\rho(s+1-t)} (S_t(s) - S_t(s+1)) u(W(s+1), b(s)) \right] \Big| Y_t, W(t)$$

subject to:

$$W(s+1) = (W(s) - c(s)) e^{r(s, Y_s)}, \quad s > t, \quad W(s) \geq 0$$

Then we obtain the following Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left\{ u(c(t), q_i(t)) + e^{-\rho} \bar{d}_i(t) u((w - c(t)) e^{r(t, i)}, b(t)) + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) V(t+1, (w - c(t)) e^{r(t, i)}, j) \right\}$$

Proposition C.1. *The value function and the optimal consumption level satisfy:*

$$\begin{aligned} V(t, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} K_{t, i}, \\ c^*(t, w, i) &= w \times c_{t, i} \end{aligned}$$

where:

$$c_{t,i} = \left[1 + e^{-r(t,i)} \left(\frac{e^{r(t,i)} [\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) (\sum_{j=1}^n p_{ij}(t) K_{t+1,j})]}{e^\rho q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}, t < T,$$

$$c_{T,i} = \left[1 + e^{-r(T,i)} \left(\frac{e^{r(T,i)} b(T)}{e^\rho q_i(T)} \right)^{\frac{1}{\gamma}} \right]^{-1}$$

and $K_{t,i}$ satisfies the recursion:

$$K_{t,i} = \left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)} \left(e^{r(t,i)-\rho} \left(\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \left(\sum_{j=1}^n p_{ij}(t) K_{t+1,j} \right) \right) \right)^{\frac{1}{\gamma}} \right]^\gamma, t < T, K_{T,i} = \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)} (e^{r(T,i)-\rho} b(T))^{\frac{1}{\gamma}} \right]^\gamma$$

Proof. See Appendix C.1 ■

When calculating VSL, we incorporate subsistence consumption back into the utility function. In this case, the value function is:

$$V(0, w, i) = \sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) \frac{c(t)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \\ + e^{-\rho(t+1)} \mathbb{E} \left[\left(\exp \left\{ - \int_0^t \mu(s) ds \right\} - \exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} \right) \left(b(t) \frac{W(t+1)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \quad (C.1)$$

Rearranging yields:

$$V(0, w, i) = \sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \frac{c(t)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\ + e^{-\rho(t+1)} b(t) \mathbb{E} \left[\left(\exp \left\{ - \int_0^t \mu(s) ds \right\} - \exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} \right) \frac{W(t+1)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\ - \frac{\underline{c}^{1-\gamma}}{1-\gamma} \left[q_{Y_0}(0) + e^{-\rho} b(0) + \sum_{t=1}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} (q_{Y_t}(t) + e^{-\rho} b(t) - b(t-1)) \middle| Y_0 = i \right] \right] \\ = \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \left[q_{Y_0}(0) + e^{-\rho} b(0) + \sum_{t=1}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} (q_{Y_t}(t) + e^{-\rho} b(t) - b(t-1)) \middle| Y_0 = i \right] \right] \right]$$

We can then calculate VSL in state i using the following formula:

$$VSL(i) = \frac{V(0, w, i) - b(0) \left(\frac{w^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right)}{u_c(w c_{0,i}, q_i(0))} \quad (C.2)$$

The second term in the numerator of (C.2) is the utility at death (the bequest function). When the bequest motive is absent ($b(t) \equiv 0$), the value function simplifies to:

$$V(0, w, i) = \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \underbrace{\sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \middle| Y_0 = i \right]}_{\text{discounted quality-adjusted life expectancy in state } i} \right]$$

and the expression for VSL simplifies to equation (7) from the main text.

Once one has calculated VSL, it is straightforward to calculate VSI:

Corollary C.2. *The value of a marginal reduction in the probability of transitioning from state i to state j is equal to:*

$$VSI(i, j) = VSL(i) - VSL(j) \frac{q_j(0) c_{0,j}^{-\gamma}}{q_i(0) c_{0,i}^{-\gamma}} = VSL(i) - \left(\frac{q_j(0)}{q_i(0)} \right) \left(\frac{c_{0,i}}{c_{0,j}} \right)^{\gamma} VSL(j)$$

Proof. See Appendix C.1 ■

C.1 Proofs

Proof of Proposition C.1. The proof proceeds by induction on $t \leq T$. For the base case $t = T$, note that $\bar{d}_i(t) = 1$, so that the first-order condition from the Bellman equation gives:

$$q_i(T) c(T)^{-\gamma} = e^{r(T,i)-\rho} b(T) (w - c(T))^{-\gamma} e^{-r(T,i)\gamma}$$

Rearranging this first-order condition yields:

$$c(T) = \frac{w e^{r(T,i)} e^{\frac{(\rho-r(T,i))}{\gamma}} \left(\frac{q_i(T)}{b(T)} \right)^{\frac{1}{\gamma}}}{1 + e^{r(T,i)} e^{\frac{(\rho-r(T,i))}{\gamma}} \left(\frac{q_i(T)}{b(T)} \right)^{\frac{1}{\gamma}}} = w \underbrace{\left[1 + e^{-r(T,i)} \left(\frac{e^{r(T,i)} b(T)}{e^{\rho} q_i(T)} \right)^{\frac{1}{\gamma}} \right]^{-1}}_{c_{T,i}}$$

Hence, we obtain:

$$\begin{aligned} V(T, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} \left(q_i(T) c_{T,i}^{1-\gamma} + e^{-\rho} b(T) e^{r(T,i)(1-\gamma)} (1 - c_{T,i})^{1-\gamma} \right) \\ &= \frac{e^{-\rho} e^{r(T,i)(1-\gamma)}}{\left[b_T^{\frac{1}{\gamma}} + e^{r(T,i)} e^{\frac{(\rho-r(T,i))}{\gamma}} q_i(T)^{\frac{1}{\gamma}} \right]^{-\gamma}} = \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)} \left(e^{r(T,i)-\rho} b(T) \right)^{\frac{1}{\gamma}} \right]^{\gamma} \end{aligned}$$

For the induction step, suppose the proposition is true for case $t + 1$. We have:

$$V(t, w, i) = \max_c \left\{ q_i(t) \frac{c^{1-\gamma}}{1-\gamma} + b(t) e^{-\rho} \bar{d}_i(t) \frac{((w - c) e^{r(t,i)})^{1-\gamma}}{1-\gamma} + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) \frac{K_{t+1,j}}{1-\gamma} [(w - c) e^{r(t,i)}]^{1-\gamma} \right\}$$

From the first-order condition we obtain:

$$q_i(t) c^{-\gamma} = b(t) e^{r(t,i)-\rho} \bar{d}_i(t) e^{-r(t,i)\gamma} (w - c)^{-\gamma} + e^{r(t,i)-\rho} (1 - \bar{d}_i(t)) e^{-\gamma r(t,i)} (w - c)^{-\gamma} \sum_{j=1}^n p_{ij}(t) K_{t+1,j}$$

Rearranging yields:

$$q_i(t) c^{-\gamma} = (w - c)^{-\gamma} e^{r(t,i)-\rho} e^{-r(t,i)\gamma} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) K_{t+1,j} \right]$$

which implies:

$$q_i(t)^{-1/\gamma} c = (w - c) e^{(\rho-r(t,i))/\gamma} e^{r(t,i)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) K_{t+1,j} \right]^{-1/\gamma}$$

Rearranging further yields:

$$\begin{aligned}
c &= w \times \frac{e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right] \right]^{-1/\gamma}}{e^\rho q_i(t)^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right] \right]^{-1/\gamma}} \\
&= w \times \underbrace{\left[1 + e^{-r(t,i)} \left(\frac{e^{r(t,i)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right]}{e^\rho q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}}_{c_{t,i}}
\end{aligned}$$

Thus we obtain:

$$\begin{aligned}
V(t, w, i) &= q_i(t) c_{t,i}^{1-\gamma} \frac{w^{1-\gamma}}{1-\gamma} + b(t) e^{-\rho} \bar{d}_i(t) \frac{w^{1-\gamma}}{1-\gamma} (1 - c_{t,i})^{1-\gamma} e^{r(t,i)(1-\gamma)} + e^{-\rho} (1 - \bar{d}_i(t)) \frac{w^{1-\gamma}}{1-\gamma} (1 - c_{t,i})^{1-\gamma} e^{r(t,i)(1-\gamma)} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \\
&= \frac{w^{1-\gamma}}{1-\gamma} \left[q_i(t) c_{t,i}^{1-\gamma} + e^{-\rho} (1 - c_{t,i})^{1-\gamma} e^{r(t,i)(1-\gamma)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right] \right] \\
&= \frac{w^{1-\gamma}}{1-\gamma} \frac{q_i(t) e^{r(t,i)(1-\gamma)} \left[e^{r(t,i)} \left(\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right) \right]^{1-1/\gamma} + e^{-\rho} e^{r(t,i)(1-\gamma)} (e^\rho q_i(t))^{1-1/\gamma} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right]}{\left[(e^\rho q_i(t))^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right] \right]^{-\frac{1}{\gamma}} \right]^{1-\gamma}} \\
&= \frac{w^{1-\gamma}}{1-\gamma} \frac{e^{r(t,i)(1-\gamma)} q_i(t) \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right]}{\left[(e^\rho q_i(t))^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right] \right]^{-\frac{1}{\gamma}} \right]^{-\gamma}} \\
&= \frac{w^{1-\gamma}}{1-\gamma} \underbrace{\left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)-\rho} \left[e^{r(t,i)-\rho} \left(\bar{d}_i(t) b(t) + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \right) \right]^{\frac{1}{\gamma}} \right]^\gamma}_{K_{t,i}}
\end{aligned}$$

■

Proof of Corollary C.2. The proof follows immediately from the expression for VSI, given by equation (8), and from noting that $u_c(c_i(0), q_i(0)) = q_i(0) c_{i,0}^{-\gamma} w^{-\gamma}$. ■

D Complete Markets Model

We assume a full menu of actuarially fair annuities is available, where consumers can choose consumption streams, $c(t)$, that depend on the evolution of their health state. Thus, the consumer is able to fully insure against consumption risk. The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0 \right] \quad (\text{D.1})$$

subject to:

$$\mathbb{E} \left[\int_0^T e^{-rt} S(t) c(t) dt \middle| Y_0 \right] = W_0 + \mathbb{E} \left[\int_0^T e^{-rt} S(t) m_{Y_t}(t) dt \middle| Y_0 \right] \equiv \bar{W}(0, Y_0)$$

where $\bar{W}(0, Y_0)$ is the net present value of wealth and future earnings.

The consumer chooses the consumption profile at time t based on her health state, $Y_t = i$, and on her available wealth, $\bar{W}(t, i)$. We define the present value of future earnings as:

$$M(t, i) = \mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} m_{Y_u}(u) du \middle| Y_t = i \right]$$

Her available wealth finances future consumption such that:

$$\bar{W}(t, i) = \mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} c(u) du \middle| Y_t, \bar{W}(t, i) \right]$$

Lemma D.1. *The law of motion for wealth is:*

$$\frac{\partial \bar{W}(t, i)}{\partial t} = r \bar{W}(t, i) - c(t, \bar{W}(t, i), i) + \sum_{j>i} \lambda_{ij}(t) [\bar{W}(t, i) - \bar{W}(t, j)], \quad i = 1, \dots, n, \quad \bar{W}(t, n+1) = 0 \quad \forall t$$

Proof. See Appendix D.1 ■

Note that the dynamics for $\bar{W}(t, i)$ will depend on $\bar{W}(t, j)$, $j > i$, so that $(Y_t, \bar{W}(t, Y_t))$ is not Markov, but $(Y_t, \bar{W}(t))$, where we define the wealth vector $\bar{W}(t) \equiv (\bar{W}(t, 1), \dots, \bar{W}(t, n+1))$, is Markov.

Define the optimal value-to-go function as:

$$V(t, \bar{W}(t), Y_t) = \max_{c(u)} \mathbb{E} \left[\int_t^T e^{-\rho(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} u(c(u), q_{Y_u}(u)) du \middle| Y_t, \bar{W}(t) \right]$$

subject to the law of motion for wealth given above. As a stochastic dynamic programming problem, $V(\cdot)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} \rho V(t, \bar{W}(t), i) = & \frac{\partial V(t, \bar{W}(t), i)}{\partial t} + \max_{c(t)} \left\{ u(c(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) [V(t, \bar{W}(t), j) - V(t, \bar{W}(t), i)] \right. \\ & \left. + \sum_{k \geq i} \frac{\partial V(t, \bar{W}(t), i)}{\partial \bar{W}(t, k)} \left[r \bar{W}(t, k) - c(t) + \sum_{l>k} \lambda_{kl}(t) [\bar{W}(t, k) - \bar{W}(t, l)] \right] \right\}, \quad 1 \leq i \leq n \quad (\text{D.2}) \end{aligned}$$

where $V(t, \bar{W}(t), n+1) = 0$. Similarly to the uninsured case presented in the main text, we follow [Parpas and Webster \(2013\)](#) and focus on the path of Y that begins in state i and remains in i until time t , with $c_i(t)$ and $\bar{W}_i(t)$ denoting the corresponding optimal consumption and wealth paths. We take optimal consumption rules and value functions from other states as exogenous. As in the uninsured case, this approach will allow us to apply the standard Pontryagin maximum

principle and derive analytic expressions.

Lemma D.2. *The optimal value function for $Y_0 = i$, $V(0, \bar{W}(0, i), i)$, for the following deterministic optimization problem also satisfies the HJB given by (D.2), for each $i \in \{1, \dots, n\}$:*

$$V(0, \bar{W}(0, i), i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right) dt \right] \quad (D.3)$$

subject to:

$$\begin{aligned} \frac{\partial \bar{W}_i(t, j)}{\partial t} &= r \bar{W}_i(t, j) - c(t, \bar{W}_i(t), j) + \sum_{k>j} \lambda_{jk}(t) [\bar{W}_i(t, j) - \bar{W}_i(t, k)], \quad j > i \\ \frac{\partial \bar{W}_i(t, i)}{\partial t} &= r \bar{W}_i(t, i) - c_i(t) + \sum_{k>i} \lambda_{ik}(t) [\bar{W}_i(t, i) - \bar{W}_i(t, k)] \end{aligned}$$

where $V(t, \bar{W}_i(t), j)$ and $c(t, \bar{W}_i(t), j)$, $j > i$, are taken as exogenous.

Proof. See Appendix D.1 ■

Following Bertsekas (2005), the Hamiltonian for the (deterministic) maximization problem (D.3) is:

$$\begin{aligned} H(\bar{W}_i(t), c_i(t), p_i(t)) &= e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right) \\ &+ \sum_{k>i} p_i(t, k) \left[r \bar{W}_i(t, k) - c(t, \bar{W}_i(t), k) + \sum_{l>k} \lambda_{kl}(t) [\bar{W}_i(t, k) - \bar{W}_i(t, l)] \right] \\ &+ p_i(t, i) \left[r \bar{W}_i(t, i) - c_i(t) + \sum_{l>i} \lambda_{il}(t) [\bar{W}_i(t, i) - \bar{W}_i(t, l)] \right] \quad (D.4) \end{aligned}$$

where $p_i(t) = (p_i(t, 1), \dots, p_i(t, n))$ is the vector of costate variables corresponding to wealth $\bar{W}_i(t)$.

Lemma D.3. *We have that $p_i(t, i) = \theta e^{-\rho t} \tilde{S}(i, t)$ for θ independent of i , and $p_i(t, k) = 0, k \neq i$. The necessary first-order condition for consumption is:*

$$e^{(r-\rho)t} u_c(c_i(t), q_i(t)) = \theta \quad (D.5)$$

where $\theta = p_i(0, i) = \partial V(0, \bar{W}_i(0), i) / \partial \bar{W}_i(0, i)$ is the marginal utility of wealth.

Proof. See Appendix D.1 ■

Equation (D.5) shows that the discounted marginal utility of consumption is constant within the path that remains in state i . The following result extends this insight by showing that the same is true across states.

Lemma D.4. *The first-order condition (D.5) holds across different states. That is, if a consumer transitions from state i to state j , then $u_c(c(t, i, \bar{W}(t)), q_i(t)) = u_c(c(t, j, \bar{W}(t)), q_j(t)) \quad \forall j$.*

Proof. See Appendix D.1 ■

To analyze the values of life and illness, let $\delta_{ij}(t)$, $i < j, i \leq n, j \leq n+1$, be a perturbation on the transition rate $\lambda_{ij}(t)$, where $\sum_{j>i} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t \sum_{j>i} (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right], \text{ where } \varepsilon > 0$$

Proposition D.5. *The marginal utility of preventing an illness or death is given by:*

$$\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} = \int_0^T \left[\tilde{S}(i, t) \left\{ e^{-\rho t} \left[u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right] + \theta e^{-rt} \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} - \tilde{S}(i, t) \sum_{j>i} \delta_{ij}(t) \left\{ e^{-\rho t} V(t, \bar{W}_i(t), j) - \theta e^{-rt} [\bar{W}_i(t, j) - M(t, j)] \right\} \right] dt \quad (D.6)$$

Proof. See Appendix D.1 ■

To obtain the value of statistical life (VSL), we first set $\delta_{i,N+1}$ equal to the Dirac delta function, and set all other perturbations equal to 0. Dividing the result by the marginal utility of wealth, θ , then yields:

$$\begin{aligned} VSL &= \int_0^T \tilde{S}(i, t) e^{-rt} \left\{ \left[\frac{u(c_i(t), q_i(t))}{u_c(c_i(t), q_i(t))} + \sum_{j>i} \lambda_{ij}(t) \frac{V(t, \bar{W}_i(t), j)}{\partial V(t, \bar{W}_i(t), j) / \partial \bar{W}_i(t, j)} \right] + \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} dt \\ &= \frac{V(0, \bar{W}_i(0), i)}{u_c(c_i(0), q_i(0))} - W_0 \\ &= \mathbb{E} \left[\int_0^T e^{-rt} S(t) v(t) dt \mid Y_0 = i \right] \end{aligned} \quad (D.7)$$

where the value of a one-period change in survival from the perspective of current time is:

$$v(t) = \frac{u(c(t), q_{Y_t}(t))}{u_c(c(t), q_{Y_t}(t))} + m_{Y_t}(t) - c_{Y_t}(t)$$

Differentiating the first-order condition (D.5) with respect to t yields like the life-cycle profile of consumption:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \sigma(r - \rho) + \sigma \eta \frac{\dot{q}_i}{q_i} \quad (D.8)$$

Equation (D.8) matches the result one obtains in a setting with a single health state, such as [Murphy and Topel \(2006\)](#).

A novel feature of the stochastic model is that it permits an investigation into the value of prevention. Inspecting the expression for the marginal utility of life extension (D.6), the first term inside the integral represents the gain in marginal utility from a reduction in the probability of exiting state i . The second term represents the loss in marginal utility from the reduction in probability of transitioning to other possible states. The net effect depends on the consumer's marginal utility in the different states.

To analyze the value of prevention, consider a reduction in the transition probability for only one alternative state, j , so that $\delta_{ik}(t) = 0 \ \forall k \neq j$. The value of avoiding illness j is then equal to:

$$\begin{aligned} VSI(i, j) &= \int_0^T \tilde{S}(i, t) e^{-rt} \left\{ \left[\frac{u(c_i(t), q_i(t))}{u_c(c_i(t), q_i(t))} + \sum_{j>i} \lambda_{ij}(t) \frac{V(t, \bar{W}_i(t), j)}{\frac{\partial V(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, j)}} \right] + \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} dt \\ &\quad - \left[\frac{V(0, \bar{W}_i(0), j)}{\theta} - [\bar{W}_i(0, j) - M(0, j)] \right] \\ &= \frac{V(0, \bar{W}_i(0), i)}{u_c(c_i(0), q_i(0))} - W_0 - \left[\frac{V(0, \bar{W}_i(0), j)}{u_c(c_i(0), q_i(0))} - [\bar{W}_i(0, j) - M(0, j)] \right] \\ &= VSL(i) - VSL(j | W_0 = \bar{W}_i(0, j) - M(0, j)) \end{aligned} \quad (D.9)$$

Thus, equation (D.9) demonstrates that $VSI(i, j)$ is equal to the difference in VSL for states i and j , with the caveat that VSL in state j uses a measure of wealth evaluated from the perspective of a person in state i . This technicality arises because the value of the consumer's annuity depends on her expected survival. For example, an annuity is worth more to a healthy

65-year-old than it is to a 65-year-old who was just diagnosed with lung cancer.

A constant value per unit of health arises only when the utility of consumption is constant (Bleichrodt and Quiggin, 1999). Inspecting equation (D.8) shows that when markets are complete, consumption will be constant when the rate of time preference equals the interest rate and quality of life is constant.

D.1 Proofs

Proof of Lemma D.1. Available wealth can be written as:

$$\bar{W}(t, i) = \int_t^T \exp \left\{ - \int_t^u r + \sum_{j>i} \lambda_{ij}(s) ds \right\} \left[c_i(t, u) + \sum_{j>i} \lambda_{ij}(u) \bar{W}_i(u, t, j) \right] du$$

where with a slight abuse of notation, $c_i(t, u)$ and $\bar{W}_i(u, t, j)$ denote the consumption and wealth paths for an individual who is in state i at time t and remains in state i until time u —but jumps to state j at time u for the latter. The result then follows by taking the derivative with respect to t . ■

Proof of Lemma D.2. This proof follows the same logic as the proof of Lemma 1 in Appendix A. Consider the deterministic optimization problem (D.3). Denote the optimal value-to-go function as:

$$\bar{V}(t, \bar{W}_i(t), i) = \max_{c_i(t)} \left\{ \int_t^T e^{-\rho u} \tilde{S}(i, u) \left(u(c_i(u), q_i(u)) + \sum_{j>i} \lambda_{ij}(u) V(u, \bar{W}_i(u), j) \right) du \right\}$$

Setting $\bar{V}(t, \bar{W}_i(t), i) = e^{-\rho t} \tilde{S}(i, t) V(t, \bar{W}_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (D.2) for i . ■

Proof of Lemma D.3. The costate equations for the Hamiltonian (D.4) are:

$$\begin{aligned} \dot{p}_i(t, i) &= - \left[r + \sum_{j>i} \lambda_{ij}(t) \right] p_i(t, i), \\ \dot{p}_i(t, k) &= -e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, k)} + \sum_{k \geq j>i} p_i(t, j) \left(\frac{\partial c(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, k)} + \lambda_{jk}(t) \right) - p_i(t, k) \left[r + \sum_{l>k} \lambda_{kl}(t) \right] + p_i(T, i) \lambda_{ik}(t) \end{aligned}$$

for $k > i$. From the first costate equation, we obtain:

$$p_i(t, i) = e^{-rt} \tilde{S}(i, t) \theta$$

Taking first-order conditions in the Hamiltonian (D.4) and plugging this in then yields:

$$u_c(c_i(t), q_i(t)) = \frac{\partial V(t, \bar{W}_i(t), i)}{\partial \bar{W}_i(t, i)} = e^{(\rho-r)t} \theta$$

To see that this solution works, let θ be constant across states, and set $p_i(t, k) = 0 = \frac{\partial V(t, \bar{W}_i(t), i)}{\partial \bar{W}_i(t, k)}$. This expression then satisfies the costate equation system across i , k , and t . In particular, for the second equation we obtain:

$$\dot{p}_i(t, k) = -e^{-\rho t} \tilde{S}(i, t) \lambda_{ik}(t) \underbrace{\frac{\partial V(t, \bar{W}_i(t), k)}{\partial \bar{W}_i(t, k)}}_{e^{(\rho-r)t} \theta} + \lambda_{ik}(t) p_i(t, i) = 0$$

Proof of Lemma D.4. With Lemma D.3, the HJB (D.2) takes the form:

$$\rho V(t, \bar{W}(t, i), i) = \frac{\partial V(t, \bar{W}(t, i), i)}{\partial t} + \max_{c(t)} \left\{ u(c(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) [V(t, \bar{W}(t, j), j) - V(t, \bar{W}(t, i), i)] + \frac{\partial V(t, \bar{W}(t, i), i)}{\partial \bar{W}(t, i)} \left[r \bar{W}(t, i) - c(t) + \sum_{k>i} \lambda_{ik}(t) [\bar{W}(t, i) - \bar{W}(t, k)] \right] \right\}, \quad 1 \leq i \leq n$$

By taking the first-order condition, we get:

$$u_c(c(t), q_i(t)) = u_c(c(t, i, \bar{W}(t)), q_i(t)) = \frac{\partial V(t, \bar{W}(t, i), i)}{\partial \bar{W}(t, i)}$$

Furthermore, differentiating the HJB (D.2) with respect to $\bar{W}(t, j)$, j fixed, we get:

$$\frac{\partial V(t, \bar{W}(t, j), j)}{\partial \bar{W}(t, j)} = \frac{\partial V(t, \bar{W}(t, i), i)}{\partial \bar{W}(t, i)}$$

Combining these last two results completes the proof:

$$u_c(c(t, i, \bar{W}(t)), q_i(t)) = u_c(c(t, j, \bar{W}(t)), q_j(t))$$

Proof of Proposition D.5. Starting from equation (D.3), we have:

$$V^\varepsilon(0, \bar{W}_i(0, i), i) = \int_0^T e^{-\rho t} \exp \left\{ - \int_0^t \sum_{j>i} \lambda_{ij}(s) - \varepsilon \sum_{j>i} \delta_{ij}(s) ds \right\} \left[u(c_i^\varepsilon(t), q_i(t)) + \sum_{j>i} [\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)] V(t, \bar{W}_i^\varepsilon(t), j) \right] dt$$

where $c_i^\varepsilon(t)$ and $\bar{W}_i^\varepsilon(t)$ represent the equilibrium variations in $c_i(t)$ and $\bar{W}_i(t)$ caused by the perturbation, $\delta_{ij}(t)$. Differentiating then yields:

$$\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} = \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right] \left[\sum_{j>i} \int_0^t \delta_{ij}(s) ds \right] - e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \delta_{ij}(t) V(t, \bar{W}_i(t), j) + e^{-\rho t} \tilde{S}(i, t) \left[\underbrace{u_c(c_i(t), q_i(t))}_{e^{-(r-\rho)t}\theta}} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \right|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \underbrace{\frac{\partial V(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, j)}}_{e^{-(r-\rho)t}\theta} \frac{\partial \bar{W}_i(t, j)}{\partial \varepsilon} \right]_{\varepsilon=0} dt$$

We have:

$$\begin{aligned} W_0 &= \mathbb{E} \left[\int_0^T e^{-rt} S(t) [c(t) - m_{Y_t}(t)] dt \middle| Y_0 = i \right] \\ &= \int_0^T e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s) ds} (c_i(t) - m_i(t)) dt + \underbrace{\sum_{j>i} e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s) ds} \lambda_{ij}(t) \mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} c(u) du \middle| Y_t = j \right]}_{\bar{W}_i(t, j)} \\ &\quad - \underbrace{\sum_{j>i} e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s) ds} \lambda_{ij}(t) \mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} m_{Y_t}(u) du \middle| Y_t = j \right]}_{M(t, j)} \\ &= \int_0^T e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s) ds} \left[c_i(t) - m_i(t) + \sum_{j>i} \lambda_{ij}(t) (\bar{W}_i(t, j) - M(t, j)) \right] dt \end{aligned}$$

The budget constraint then implies:

$$\begin{aligned}
0 &= \left. \frac{\partial W_0}{\partial \varepsilon} \right|_{\varepsilon=0} \\
&= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-rt} \exp \left\{ - \int_0^t \sum_{j>i} \lambda_{ij}(s) - \varepsilon \sum_{j>i} \delta_{ij}(s) ds \right\} \left(c_i^\varepsilon(t) - m_i(t) + \sum_{j>i} [\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)] (\bar{W}_i^\varepsilon(t, j) - M(t, j)) \right) dt \Big|_{\varepsilon=0} \\
&= \int_0^T \left(e^{-rt} \tilde{S}(i, t) \left[c_i(t) - m_i(t) + \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \left[\sum_{j>i} \int_0^t \delta_{ij}(s) ds \right] \right. \\
&\quad \left. - e^{-rt} \tilde{S}(i, t) \sum_{j>i} \delta_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] + e^{-rt} \tilde{S}(i, t) \left[\left. \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \right|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \left. \frac{\partial \bar{W}_i^\varepsilon(t, j)}{\partial \varepsilon} \right|_{\varepsilon=0} \right] \right) dt
\end{aligned}$$

Plugging this last result into the expression for $\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0}$ then yields the desired result for marginal utility:

$$\begin{aligned}
\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \int_0^T \left(\tilde{S}(i, t) \left[\sum_{j>i} \int_0^t \delta_{ij}(s) ds \right] \left\{ e^{-\rho t} \left[u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t, j), j) \right] + \theta e^{-rt} \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} \right. \\
&\quad \left. - \tilde{S}(i, t) \left\{ e^{-\rho t} \sum_{j>i} \delta_{ij}(t) V(t, \bar{W}_i(t, j), j) - \theta e^{-rt} \sum_{j>i} \delta_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right\} \right) dt
\end{aligned}$$

■