

Mortality Risk, Insurance, and the Value of Life*

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Abstract. We develop a new framework for valuing health and longevity improvements that departs from conventional but unrealistic assumptions of full annuitization and deterministic health. Our framework can value the prevention of mortality *and* of illness, and it can quantify the effects of retirement policies on the value of life. We apply the framework to life-cycle data and generate new insights absent from the conventional approach. First, treatment is up to five times more valuable than prevention, even when both extend life equally. This asymmetry helps explain low observed investment in preventive care. Second, severe illness can significantly increase the value of statistical life, helping to reconcile theory with empirical findings that consumers value life-extension more in bleaker health states. Third, retirement annuities boost aggregate demand for life-extension. We calculate that Social Security adds \$10.6 trillion (11 percent) to the value of post-1940 longevity gains and would add \$127 billion to the value of a one percent decline in future mortality.

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I. INTRODUCTION

The economic analysis of risks to life and health has made enormous contributions to both academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity (Rosen 1988; Murphy and Topel 2006). Economic concepts such as the value of statistical life play central roles in discussions surrounding public and private investments in medical care, public safety, environmental hazards, and countless other arenas.

The standard life-cycle framework employed by the value of life literature assumes full annuitization and deterministic health risk. While analytically convenient and useful for illustrating some of the underlying economics, these assumptions are not realistic: it is well known that most people are far from fully annuitized (Brown et al. 2008), and that health risk depends on one's health state. Moreover, these assumptions hamper explanatory power in several ways: the standard framework sheds no light on what happens to the value of life upon falling ill, cannot meaningfully distinguish between preventive care and medical treatment, and glosses over policy-relevant relationships between demand for health care and the structure of annuity markets. These issues are empirically relevant. An array of evidence suggests that society invests less in prevention than treatment, even when both have the same consequences for health and longevity (Weisbrod 1991; Dranove 1998; Pryor and Volpp 2018). And, prior research suggests the value consumers place on improvements in quality of life and longevity varies considerably with health state (Nord et al. 1995; Shah 2009; Shah, Tsuchiya, and Wailoo 2018).

We develop and apply a framework for valuing health improvements that relaxes the unrealistic assumptions of full annuitization and deterministic health risk. We establish three main results. First, we derive the value of statistical illness (VSI), which captures the willingness to pay to avoid sickness and includes VSL as a special case. We calculate that—holding wealth constant—a sick individual's initial willingness to pay for medical treatment is several times greater than a healthy individual's willingness to pay for preventive care that improves longevity by the same amount. Second, we derive conditions under which the value of life can rise following a negative health shock, and we demonstrate that this effect is economically significant under reasonable parameterizations. For example, we calculate that the value of statistical life (VSL) for a 70-year-old rises by \$600 thousand (25 percent) following the development of chronic conditions that impair her everyday living. Third, we calculate that the US Social Security program adds \$10.6 trillion (11 percent) to the value of post-1940 longevity gains.

Incomplete annuitization drives all three of these results. A simple example illustrates the intuition. Imagine a 60-year-old retiree with no bequest motive and a flat optimal consumption profile. If she fully

annuitizes her savings, her consumption remains flat at, say, \$30,000 annually. Now suppose annuities are unavailable. In this case, it is well known that the optimal consumption profile shifts forward to earlier ages (Yaari 1965), in response to the risk of dying with money still left in the bank (see Figure 1). Because VSL depends greatly on consumption, the life-cycle profile of VSL will also shift forward. Thus, reductions in annuitization lower VSL at older ages, and increase VSL at younger ages. Conversely, retirement savings programs such as Social Security that increase annuitization levels will raise VSL at older ages and lower it at younger ages.

The incorporation of stochastic mortality risk yields our other results. It is optimal for an incompletely annuitized individual to shift her consumption forward, i.e., to spend down her wealth, following an adverse shock to life expectancy. Although a negative shock to longevity reduces lifetime utility, the accompanying reduction in the contemporaneous marginal utility of consumption can be large enough to increase her VSL. This result contrasts with the conventional (fully annuitized) life-cycle model, where a reduction in longevity always reduces VSL (Murphy and Topel 2006). Similarly, in our framework a sick individual's willingness to pay for treatment can be higher than a healthy individual's willingness to pay for preventive care, even when both interventions add the same number of life-years.

The first half of this paper provides a formal framework that yields these insights. We show that optimal consumption increases following an adverse shock to longevity and derive a sufficient condition under which VSL also increases. This condition holds under a wide range of typical parameterizations.¹ We focus on mortality shocks, but allow for shocks to quality of life and income as well. We also derive VSI, a generalization of VSL that can be interpreted as a person's willingness to pay for a marginal decrease in the risk of acquiring an illness. VSI allows us to compare the value of prevention to the value of treatment. We show that prevention and treatment are valued equally only when consumers are fully annuitized. The value of treatment can exceed the value of prevention when annuity markets are incomplete. This result sheds new light on why consumers, firms, and health insurers appear reluctant to invest in prevention, even when there are considerable private life expectancy benefits (Weisbrod 1991; Dranove 1998; Pryor and Volpp 2018).

¹ Intuitively, the condition holds when the loss in lifetime utility is more than offset by a corresponding decrease in marginal utility. Specifically, an adverse mortality shock increases VSL when demand for current consumption is sufficiently inelastic, or when the marginal utility of demand is sufficiently linear (as measured by relative prudence).

The second half of the paper applies our model to data. Our first empirical exercise incorporates detailed real-world data from the Future Elderly Model into a stochastic life-cycle model that allows mortality, medical spending, and quality of life to vary across 20 different health states. Under typical utility parameterizations, we calculate that the value of treating lethal conditions like cancer is worth up to 5 times more to individuals than equivalent preventive care that adds the same number of years to life expectancy. This asymmetry arises because the value of life rises substantially following a health shock. For instance, VSL rises from \$2.4 million to \$3.0 million for a 70-year-old who suffers a debilitating health shock that reduces her life expectancy by nearly 7 years and also worsens her quality of life. This dynamic relationship between health shocks and VSL generates substantial variability in the aggregate: Monte Carlo simulations performed on a population of initially healthy 50-year-olds yield an inter-vigintile (middle 90 percent) VSL range of \$1.7 to \$2.5 million by age 70.

Our second exercise illustrates the connections between public annuity programs and the aggregate value of increases in longevity. We calculate that Social Security adds \$10.6 trillion (11 percent) to the value of post-1940 longevity gains, relative to a setting with no annuity markets, by raising the value of life at older ages. This gain is worth over \$30,000 per person to the current population, or more than half the longevity insurance value of Social Security. Moreover, Social Security increases the aggregate value of potential future increases in longevity by over 10 percent, so that a 1 percent reduction in population-wide mortality is \$127 billion more valuable than it would have been without the program. Increasing the generosity of Social Security by 50 percent would add a further \$64 billion of value to this mortality decline. This result also implies that Medicare is more valuable than previously recognized, by revealing that the value of old-age health insurance increases when coupled with annuity programs like Social Security. Finally, we show that a strong bequest motive reduces the effect of Social Security on the value of longevity improvements by 20 percent. This result suggests the effect of annuitization on the value of life matters more for low-income individuals, who are less likely to have a significant bequest motive.

Our stochastic model helps explain puzzles such as why consumers invest less in prevention than treatment or why preventive care interventions frequently fail to deliver results (e.g., Jones, Molitor, and Reif (2019)), although we do not rule out alternative behavioral explanations that may reinforce these effects, such as inattention or hyperbolic discounting (Lawless, Drichoutis, and Nayga 2013). We caution that our model does not necessarily imply that underinvestment in preventive care is socially optimal. As we discuss in the main text, a full accounting of the normative implications requires taking a stance on unsettled questions regarding the welfare economics of risk (Fleurbaey 2010). For this reason, we employ a deterministic model when quantifying the effect of public annuity programs on the aggregate value of life at older ages.

Our primary contribution is the development and application of a new and more general life-cycle model of the value of life. The economic literature on the value of life reaches back to Schelling (1968) and includes seminal studies by Arthur (1981), Rosen (1988), Murphy and Topel (2006), and Hall and Jones (2007). A few studies have considered departures from the assumption of full annuitization, but only under specialized preferences or alternative contexts (Shepard and Zeckhauser 1984; Ehrlich 2000; Ehrlich and Yin 2005). Córdoba and Ripoll (2016) use Epstein-Zin-Weil preferences to study the implications of state non-separable utility on the value of life when mortality is deterministic. Our stochastic framework accommodates general additively separable preferences, allows for incomplete annuity markets, and to our knowledge is the first to provide a life-cycle analysis of the value of preventing illness.² We establish the important result that, under standard assumptions about risk preferences, consumers value treatment more than prevention even when both extend life equally.

Our model also reconciles the standard life-cycle framework with results from a distinct literature that uses one-period models to study the value of mortality risk-reduction (Raiffa 1969; Weinstein, Shepard, and Pliskin 1980; Pratt and Zeckhauser 1996; Hammitt 2000). These static models predict that an increase in baseline risk must raise the value of statistical life when insurance markets are incomplete, a result often referred to as the “dead-anyway” effect. In contrast, we show in our more general setting that mortality shocks in life-cycle models can raise *or* lower the value of statistical life, depending on risk attitudes and other utility parameters.

The remainder of this paper is organized as follows. Section II reviews the predictions of the conventional theory on the value of life and demonstrates how relaxing its assumption of full annuitization alters these predictions. Section III generalizes the framework further by allowing health and income to be stochastic and provides a discussion of welfare. Section IV presents empirical analyses that quantify: (1) the value of preventing different kinds of illness; (2) the effect of health shocks on the value of statistical life; and (3) the effect of Social Security on the aggregate value of life-extension. Section V concludes.

II. DETERMINISTIC MODEL

Consider an individual who faces mortality risk. We are interested in analyzing the value of a marginal reduction in this risk. Section II.A quantifies this value in the conventional setting where annuity markets are complete (Rosen 1988; Murphy and Topel 2006). Section II.B then repeats this exercise in a

² The value of preventing illness has already found application in the empirical literature on mortality risk-reduction (Cameron and DeShazo 2013; Hummels, Munch, and Xiang 2016).

“Robinson Crusoe” economy where the consumer cannot purchase annuities to insure against her uncertain lifetime (Shepard and Zeckhauser 1984; Johansson 2002). We compare these two polar cases to illustrate the basic insights of the paper. Finally, Section II.C considers a more realistic situation where the consumer optimally invests part of her wealth in a constant annuity. We focus on the value of longevity improvements, but we allow for improvements in quality of life as well.³

Except for certain special cases, it will not be optimal for the consumer to fully annuitize when annuity markets are incomplete (Davidoff, Brown, and Diamond 2005). Section II.C demonstrates this point in the context of deterministic health, and Section III.C extends it to a setting that allows for stochastic health and correlated income shocks (Reichling and Smetters 2015). Section IV uses a numerical model to probe the sensitivity of our results to different assumptions about consumer preferences, such as the presence of a bequest motive, which prior studies have argued might also rationalize low observed rates of annuitization. There continues to be debate over why real-world consumption trajectories and annuity purchase decisions look the way they do. However, as we show, the implications for life-extension depend primarily on the consumption trajectory itself, not the reasons that lie beneath.

Like prior studies on the value of life, we focus throughout this paper on the demand for health and longevity. Quantifying optimal health spending requires additionally modeling the supply of health care (Hall and Jones 2007). In light of all the variation in health care delivery systems, a wide variety of plausible approaches can be taken to this modeling problem, which we leave to future research.

II.A. The fully annuitized value of life

Let $c(t)$ be consumption at time t , W_0 be baseline wealth, $m(t)$ be exogenous income, ρ be the rate of time preference, and r be the rate of interest. Let W be the net present value of wealth and future earnings at baseline. Finally, define $q(t)$ as health-related quality of life at time t . Since it sacrifices little generality in our application, we take $q(t)$ as exogenous.⁴ As needed, one can consider any relevant quality of life profile in concert with a given profile of mortality, and we investigate this issue in our

³ In keeping with the vast majority of prior literature, we abstract from indemnity health insurance and treat non-fatal health risks as exogenous and uninsurable. Since indemnity health insurance does not exist outside a few specialized disease areas, and since health care insurance is imperfect, this does not sacrifice substantial generality. Our empirical exercises in Section IV.B consider scenarios with and without out-of-pocket medical spending.

⁴ It is straightforward to incorporate endogenous labor supply (Murphy and Topel 2006). In the stochastic model presented in Section III.C, we allow income and quality of life to depend on the health state.

empirical analysis later. The maximum lifespan of a consumer is T , and her mortality (hazard) rate at any point in time is given by $\mu(t)$, where $0 \leq t \leq T$. The probability that a consumer will be alive at time t is:

$$S(t) = \exp \left[- \int_0^t \mu(s) ds \right]$$

We assume that annuity markets are complete and actuarially fair. The consumer's maximization problem is:

$$V(0) = \max_{c(t)} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$

subject to the budget constraint:

$$\int_0^T e^{-rt} S(t) c(t) dt = W = W_0 + \int_0^T e^{-rt} S(t) m(t) dt$$

The consumer's utility function, $u(c(t), q(t))$, depends on both consumption and health-related quality of life. We assume throughout this paper that $u(\cdot)$ is strictly increasing and concave in its first argument, and twice continuously differentiable. Let $u_c(\cdot)$ denote the marginal utility of consumption, and assume that this function diverges to positive infinity as consumption approaches zero, so that optimal consumption is always positive. Associating the multiplier θ with the wealth constraint, optimal consumption is characterized by the first-order condition:

$$\frac{\partial V(0)}{\partial W} = \theta = e^{(r-\rho)t} u_c(c(t), q(t))$$

To analyze the value of life, let $\delta(t)$ be a perturbation on the mortality rate with $\int_0^T \delta(t) dt = 1$, and consider:

$$S^\varepsilon(t) = \exp \left[- \int_0^t (\mu(s) - \varepsilon \delta(s)) ds \right], \varepsilon > 0$$

Let $c^\varepsilon(t)$ represent the equilibrium variation in $c(t)$ caused by this perturbation. As shown in Rosen (1988), the marginal utility of this life-extension is given by:

$$\begin{aligned} \frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} S^\varepsilon(t) u(c^\varepsilon(t), q(t)) dt \Big|_{\varepsilon=0} \\ &= \int_0^T \left[e^{-\rho t} u(c(t), q(t)) + e^{-rt} \theta (m(t) - c(t)) \right] \left[\int_0^t \delta(s) ds \right] S(t) dt \end{aligned}$$

The marginal value of life-extension is equal to the marginal rate of substitution between longer life and wealth:

$$\left. \frac{\partial V / \partial \varepsilon}{\partial V / \partial W} \right|_{\varepsilon=0} = \int_0^T e^{-rt} S(t) \left(\frac{u(c(t), q(t))}{u_c(c(t), q(t))} + m(t) - c(t) \right) \left[\int_0^t \delta(s) ds \right] dt \quad (1)$$

The value of a life-year is the value of a one-period change in survival from the perspective of current time:

$$v(t) = \frac{u(c(t), q(t))}{u_c(c(t), q(t))} + m(t) - c(t) \quad (2)$$

The value of a life-year, $v(t)$, is equal to the value of consumption in that year plus net savings, $m(t) - c(t)$, which can be used to finance consumption in other periods.

A canonical choice for $\delta(\cdot)$ in equation (1) is the Dirac delta function, so that the mortality rate is perturbed at $t = 0$ and remains unaffected otherwise. This then yields an expression that is commonly called the value of statistical life (VSL):

$$VSL = \int_0^T e^{-rt} S(t) v(t) dt = \int_0^T e^{-rt} S(t) \frac{u(c(t), q(t))}{u_c(c(t), q(t))} dt - W_0 \quad (3)$$

VSL corresponds to the value that the individual places on a marginal reduction in the risk of death in the current period. For example, it is the amount that 1,000 people are collectively willing to pay to eliminate a current risk that is expected to kill one of them. It is equal to the present discounted value of lifetime utility (the marginal benefit to the annuity pool from saving a life), net of baseline wealth at time zero (the marginal cost to the annuity pool from saving a life). Here and elsewhere, W_0 can be interpreted as baseline wealth or expected net dissaving over the individual's lifetime. Holding wealth constant, VSL increases with survival, which implies increasing returns in health improvements (Murphy and Topel 2006). Conversely, this leads to the conventional result that VSL falls when mortality rises.

VSL depends on how substitutable consumption is at different ages, i.e., on how easily an individual can reallocate consumption over time. Intuitively, if present consumption is a good substitute for future consumption, then living longer is less valuable. Define the elasticity of intertemporal substitution, σ , as:

$$\frac{1}{\sigma} \equiv - \frac{u_{cc} c}{u_c}$$

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq} q}{u_c}$$

When η is positive, the marginal utility of consumption is higher in healthier states, and vice-versa. Taking logarithms of the first-order condition for consumption and differentiating with respect to time yields the rate of change for consumption over the life cycle:

$$\frac{\dot{c}}{c} = \sigma(r - \rho) + \sigma\eta \frac{\dot{q}}{q} \quad (4)$$

A crucial feature of the conventional model is that consumption growth over the life-cycle is independent of the mortality rate, because the individual is fully insured against longevity risk. This feature in turn implies that the rate of change in the value of a life-year is also not a function of the mortality rate:

$$\frac{\dot{v}}{v} = \left(\frac{1}{\sigma v} \frac{u}{u_c} \right) \frac{\dot{c}}{c} + \left(\frac{-\eta}{v} \frac{u}{u_c} + \frac{q}{v} \frac{u_q}{u_c} \right) \frac{\dot{q}}{q} + \frac{\dot{m}}{v}$$

In sum, we have identified two major features of the theory on the value of life under the conventional assumptions of full annuitization and deterministic health risk:

- The relative value of a life-year within a lifetime is independent of the mortality rate.
- The value of statistical life falls when mortality rises.

II.B. The uninsured value of life

Next, we consider a setting where the consumer lacks access to annuity markets. We employ the classical Yaari (1965) model of consumption behavior under survival uncertainty.⁵ Let the state variable $W(t)$ represent current wealth at time t . The consumer's maximization problem is:

$$V(0, W_0) = \max_{c(t)} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, W(T) = 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) - c(t) \end{aligned}$$

Optimal consumption is again characterized by the first-order condition:

⁵ We do not allow for income in this model, so that we can focus on interior solutions (Leung 1994). We relax this assumption in Section II.C.

$$\frac{\partial V(0, W_0)}{\partial W_0} = \theta = e^{(r-\rho)t} S(t) u_c(c(t), q(t))$$

Unlike in the case of perfect markets, the survival function enters the consumer's first-order condition for consumption. Instead of setting the discounted marginal utility of consumption equal to the marginal utility of wealth, the consumer sets the *expected* discounted marginal utility of consumption at time t equal to the marginal utility of wealth. This shifts consumption to earlier ages, which is rational because consumption allocated to later time periods will not be enjoyed in the event of an early death.

The expression for the marginal utility of life-extension is:

$$\begin{aligned} \left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} S^\varepsilon(t) u(c^\varepsilon(t), q(t)) dt \Big|_{\varepsilon=0} \\ &= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds \right] S(t) u(c(t), q(t)) dt + \int_0^T e^{-\rho t} S(t) u_c(c(t), q(t)) \left. \frac{\partial c^\varepsilon(t)}{\partial \varepsilon} \right|_{\varepsilon=0} dt \\ &= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds \right] S(t) u(c(t), q(t)) dt + \theta \frac{\partial}{\partial \varepsilon} \int_0^T e^{-rt} c^\varepsilon(t) dt \\ &= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds \right] S(t) u(c(t), q(t)) dt, \end{aligned}$$

where the last equality follows from the budget constraint.⁶

Dividing this result by the marginal utility of wealth, θ , then yields the marginal value of life-extension:

$$\begin{aligned} \left. \frac{\partial V / \partial \varepsilon}{\partial V / \partial W} \right|_{\varepsilon=0} &= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds \right] S(t) \frac{u(c(t), q(t))}{u_c(c(0), q(0))} dt \\ &= \int_0^T e^{-rt} \left[\int_0^t \delta(s) ds \right] \frac{u(c(t), q(t))}{u_c(c(t), q(t))} dt \end{aligned}$$

In this setting, the value of a life-year from the perspective of current time is:

$$v(t) = \frac{u(c(t), q(t))}{u_c(c(t), q(t))} \quad (5)$$

When the consumer is uninsured, the value of a life-year depends only on the value of consumption.

Recall that the VSL expression in equation (3) also included a term reflecting the marginal cost of saving a life, equal to baseline wealth. This term is absent in equation (5), because the consumer's wealth has not

⁶ The budget constraint $W(T) = 0$ implies $0 = W_0 - \int_0^T e^{-rt} c^\varepsilon(t) dt$, so that differentiation yields zero.

been pooled into annuity markets. Therefore, saving a life does not deprive others in an annuity pool of any consumption.

Choosing again the Dirac delta function for $\delta(\cdot)$ yields an expression for VSL that differs from the complete annuity markets case:

$$VSL = \int_0^T e^{-rt} v(t) dt = \int_0^T e^{-\rho t} S(t) \frac{u(c(t), q(t))}{u_c(c(0), q(0))} dt \quad (6)$$

The value of statistical life is proportional to (expected) lifetime utility, and inversely proportional to the marginal utility of consumption. It is well known that removing annuity markets lowers lifetime utility (Yaari 1965). As we show more formally below, removing these markets also shifts consumption to earlier ages, thereby lowering the marginal utility of consumption at earlier ages. When consumers shift consumption forward, near-term life-years rise in value but distant life-years fall in value. Thus, the net effect of annuity markets on VSL is in general ambiguous. Put differently, exposure to longevity risk does not necessarily lower VSL. In Section III, we will show that this basic insight extends to exposing a consumer to a longevity “shock.” We emphasize that in both cases the ambiguity in the relationship between mortality shocks and VSL depends critically on the absence of complete annuity markets.

Unlike in the complete markets case, the rate of change for consumption over the life-cycle now depends explicitly on the mortality rate. Taking logarithms of the first-order condition for consumption and differentiating with respect to time yields:

$$\frac{\dot{c}}{c} = \sigma(r - \rho) + \sigma\eta \frac{\dot{q}}{q} - \sigma\mu(t) \quad (7)$$

Comparing this result to the standard case, given by equation (4), reveals both similarities and differences. As in the standard, fully annuitized model, the non-annuitized consumption profile described by equation (7) changes shape when the rate of time preference is above or below the rate of interest and when the quality of life changes. Unlike in the standard model, the consumption profile here depends explicitly on the mortality rate, $\mu(t)$. Higher rates of mortality depress the rate of consumption growth over the life-cycle. This rate of growth is always higher in the fully annuitized case, in which the last term drops out of the consumption growth equation (7). Put another way, eliminating annuity markets “pulls consumption earlier” in the life-cycle.

The rate of change in the value of a life-year is:

$$\frac{\dot{v}}{v} = \left(\frac{1}{\sigma} + \frac{c}{v}\right) \frac{\dot{c}}{c} + \left(\frac{qu_q}{u} - \eta\right) \frac{\dot{q}}{q} \quad (8)$$

Equation (8) shows that the rate of change in the value of a life-year depends on the rate of change in consumption, \dot{c}/c , and thus on mortality. Holding quality of life constant, it is evident from equation (5) that increases in the mortality rate—which shift consumption forward—will raise v , the current value of a life-year. Thus, mortality also shifts forward the value of life. All else equal, individuals who face poor survival prospects will pay more for a marginal (near-term) life-year, but less for a distant life-year, than healthy peers who face good survival prospects. This differs from the implications of the conventional model, in which higher mortality reduces the values of life-years but has no impact on their relative values.

To summarize, we have identified the following two properties of the uninsured model that contrast with those of the fully annuitized model:

- When mortality rises, near-term life-years rise in value, but distant life-years fall in value.
- The value of statistical life may rise or fall when mortality rises.

II.C. The incompletely annuitized value of life

Finally, we consider a more realistic setting that introduces incomplete annuity markets and life-cycle income, $m(t)$. These features can generate non-interior solutions, as shown in Appendix Figure A1. For convenience of exposition, we consider a consumer near retirement age who has a one-time opportunity to purchase a constant annuity, \bar{m} , and focus on the case where there is a single set of non-interior solutions. For example, this will occur if life-cycle income is constant and the mortality rate profile satisfies the condition $\mu(t) \geq r - \rho + \eta \frac{\dot{q}}{q}$. In this case, consumption will decrease with age and eventually converge to a constant level (e.g., the left panel in Appendix Figure A1). In addition to matching observed elderly consumption profiles, this case allows us to communicate our results without having to sequentially consider the multiple corner solutions that may occur when income or mortality profiles are allowed to vary arbitrarily (e.g., see the right panel in Appendix Figure A1). Our stochastic model, which we present later, includes these deterministic results as a special case. Thus, we keep our presentation brief and refer the reader to Section III.C for more formal derivations.

Our model is based on the Leung (1994) model of consumption behavior under survival uncertainty. We assume the consumer has an option at time zero to purchase a flat lifetime annuity at a level \bar{m} with a price markup of $\xi \geq 0$. The consumer cannot finance the purchase of the annuity using future income, and she cannot purchase or sell annuities after time zero. The consumer's maximization problem is:

$$V(0, W_0) = \max_{c(t), \bar{m}} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$

subject to:

$$W(0) = W_0 - (1 + \xi)\bar{m} \int_0^T e^{-rt} S(t) dt,$$

$$W(t) \geq 0, W(T) = 0,$$

$$\frac{\partial W(t)}{\partial t} = rW(t) + m(t) + \bar{m} - c(t)$$

The Hamiltonian for this problem is:

$$H(t) = e^{-\rho t} S(t) u(c(t), q(t)) + p(t)(rW(t) + m(t) + \bar{m} - c(t)) + \psi(t)W(t)$$

where $p(t)$ and $\psi(t)$ are the costate variables for the law of motion of wealth and the non-negative wealth constraint, respectively. If the non-negative wealth constraint binds, then the solution to the consumer's problem is to set $c(t) = m(t) + \bar{m}$. Under our maintained assumptions, this will occur during old ages only. When the wealth constraint does not bind, optimal consumption is characterized by the first-order condition:

$$\frac{\partial V(0, W_0)}{\partial W_0} = \theta = e^{(r-\rho)t} S(t) u_c(c(t), q(t))$$

The first-order condition for the optimal flat annuity is:

$$\frac{\partial V(0, W_0)}{\partial \bar{m}} = \frac{\partial V(0, W_0)}{\partial W_0} (1 + \xi) \int_0^T e^{-rt} S(t) dt$$

At the optimum, the marginal benefit of an increase in annuitization is equal to the marginal cost of the annuity. An increase in the price, ξ , weakly decreases the optimal annuitization level. Because the consumer may prefer a non-flat consumption profile—for example, because of life-cycle changes in the quality of life—the optimal level of annuitization may be partial even if the markup ξ is equal to zero (Davidoff, Brown, and Diamond 2005). However, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and income are constant, so this model nests the full annuitization scenario in some special cases.

The rest of the analysis proceeds analogously to the uninsured case presented in Section II.B. The marginal utility of life-extension is:

$$\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} = \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds \right] S(t) u(c(t), q(t)) dt - \theta (1 + \xi) \bar{m} \int_0^T e^{-rt} \left[\int_0^t \delta(s) ds \right] S(t) dt \quad (9)$$

Choosing the Dirac delta function for $\delta(\cdot)$ and dividing by the marginal utility of wealth yields:

$$VSL = \int_0^T e^{-\rho t} S(t) \frac{u(c(t), q(t))}{u_c(c(0), q(0))} dt - (1 + \xi) \bar{m} \int_0^T e^{-rt} S(t) dt \quad (10)$$

In the special case where the consumer has a flat optimal consumption profile and the markup $\xi = 0$, this VSL expression simplifies to the one given by (3).⁷ Otherwise, the expression will be equivalent to the uninsured VSL expression given by (6), net of the marginal cost to the annuity pool of saving a life. In the latter case, the willingness to pay for longevity will again depend on the life-cycle mortality profile.

To summarize, introducing incomplete annuity markets has the following effects:

- Except for certain special cases, the optimal level of annuitization is partial.
- When annuitization is partial, the qualitative conclusions from the uninsured model in Section II.B continue to hold. In particular, the value of statistical life may rise or fall when mortality rises.

Our results imply that public programs that increase annuitization rates, such as Social Security, will affect society's willingness to pay for longevity, thereby creating a feedback loop that could dampen or increase program expenditures.⁸ In our empirical exercises, we will quantify how the degree of annuitization influences the value of statistical life.

In the next section, we allow mortality to be stochastic so that we can investigate the effect of disease and other health shocks on the value of life. Before turning to that analysis, we pause to note that suffering an adverse shock to longevity is similar to removing access to annuity markets: both expose an individual to longevity risk. Not surprisingly, we shall see that longevity shocks also shift the value of life-years forward, with an ambiguous net effect on VSL.

III. STOCHASTIC MODEL

The previous section illustrated how relaxing the conventional assumption of full annuitization affects the relationship between mortality risk and the value of life. The conventional framework is ill-equipped to study the influence of mortality risk for another reason as well. Just like our deterministic model above, it treats the mortality rate as a nonrandom parameter, i.e., shifts in the mortality rate are preordained and

⁷ Wealth at time $t = 0$, $W(0)$, is zero upon full annuitization. This implies $W_0 = (1 + \xi) \bar{m} \int_0^T e^{-rt} S(t) dt$.

⁸ Philipson and Becker (1998) make the important, but distinct, point that the moral hazard effects of public annuity programs also increase an individual's willingness to pay for longevity gains.

fully anticipated. In the real world, however, neither the timing nor the size of shifts in the mortality rate is known. As a related matter, the conventional framework does not allow for different health states. This omission precludes a meaningful analysis of the value of preventing health deterioration.

This section extends our analysis to allow for stochastic health shocks. Specifically, we assume that the individual's mortality rate and quality of life now depend on her health state. Let Y_t be a continuous-time Markov chain with finite state space $Y = \{1, 2, \dots, n\}$. Denote the transition intensities by:

$$\lambda_{ij}(t) = \lim_{h \rightarrow 0} \frac{1}{h} \mathbb{P}[Y_{t+h} = j | Y_t = i], j \neq i,$$

$$\lambda_{ii}(t) = - \sum_{j \neq i} \lambda_{ij}(t)$$

The mortality rate at time t is defined as:

$$\mu(t) = \sum_{j=1}^n \bar{\mu}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\{\bar{\mu}_j(t)\}$ is exogenous and $\mathbf{1}\{Y_t = j\}$ is an indicator variable equal to 1 if the individual is in state j at time t and 0 otherwise. Quality of life at time t , $q(t)$, is defined similarly. For analytical convenience and without meaningful loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0 \forall j < i$, so that the probability that a consumer in state i at time 0 remains in state i at time t is equal to:⁹

$$\tilde{S}(i, t) = \exp \left[- \int_0^t \left(\bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) \right) ds \right]$$

Introducing stochastic mortality does not alter the theoretical predictions of the conventional (deterministic) model when annuity markets are complete because the consumer can still fully insure against all longevity risks. We therefore relegate the fully annuitized case to Appendix D, and in Section III.A focus instead on the “Robinson Crusoe” (uninsured) case. We explain how the value of statistical life can rise or fall following an adverse shock to longevity, and in Section III.B we derive an expression for the value of statistical illness that allows us to compare the value of prevention to the value of

⁹ That is, an individual can transition from state i to j , $i < j$, but not vice versa. This does not meaningfully limit the generality of our model because one can always define a new state $k > j$ with properties identical to state i .

treatment. Section III.C incorporates incomplete annuity markets and life-cycle income, and Section III.D discusses welfare implications.

III.A. The uninsured value of life

The consumer's maximization problem is:

$$V(0, W_0, Y_0) = \max_{c(t)} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \mid Y_0, W_0 \right] \quad (11)$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, W(T) = 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) - c(t) \end{aligned}$$

Define the consumer's objective function at time t as:

$$J(t, W(t), i) = \mathbb{E} \left[\int_0^{T-t} e^{-\rho u} \exp \left\{ - \int_0^u \mu(t+s) ds \right\} u(c(t+u), q_{Y_{t+u}}(t+u)) du \mid Y_t = i, W(t) \right]$$

Define the optimal value function as:

$$V(t, W(t), i) = \max_{c(s), s \geq t} \{J(t, W(t), i)\}$$

subject to the wealth dynamics above. Under conventional regularity conditions, if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} (\rho + \bar{\mu}_i(t)) V(t, W(t), i) &= \max_{c(t)} \left\{ u(c(t), q_i(t)) + \frac{\partial V(t, W(t), i)}{\partial W(t)} [rW(t) - c(t)] + \frac{\partial V(t, W(t), i)}{\partial t} \right. \\ &\quad \left. + \sum_{j>i} \lambda_{ij}(t) [V(t, W(t), j) - V(t, W(t), i)] \right\}, i = 1, \dots, n \end{aligned} \quad (12)$$

where $c(t) = c(t, W(t), i)$ is the (optimal) rate of consumption. In order to apply our value of life analysis, we exploit recent advances in the systems and control literature. Parpas and Webster (2013) show that one can reformulate a stochastic finite-horizon optimization problem as a deterministic problem that takes $V(t, W(t), j), j \neq i$, as exogenous. More precisely, we focus on the path of Y that begins in state i and remains in state i until time T . We denote optimal consumption and wealth in that path by

$c_i(t)$ and $W_i(t)$, respectively.¹⁰ A key advantage of this method is that it allows us to apply the standard deterministic Pontryagin maximum principle and derive analytic expressions.

Lemma 1:

Consider the following *deterministic* optimization problem for $Y_0 = i$ and $W(0) = W_0$:

$$V(0, W_0, i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right] \quad (13)$$

subject to:

$$\begin{aligned} W_i(0) &= W_0, \\ W_i(t) &\geq 0, W_i(T) = 0, \\ \frac{\partial W_i(t)}{\partial t} &= rW_i(t) - c_i(t) \end{aligned}$$

where $V(t, W_i(t), j)$ are taken as exogenous. The optimal value function, $V(t, W_i(t), i)$, satisfies the HJB equation given by (12), for all $i \in \{1, \dots, n\}$.

Proof of Lemma 1: see Appendix A

Because the value function V corresponding to (13) satisfies the HJB equation given by (12), it must also be equal to the consumer's optimal value function (see Proposition 3.2.1, Bertsekas (2005)). The present value Hamiltonian corresponding to (13) is:

$$H(W_i(t), c_i(t), p_t^{(i)}) = e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) + p_t^{(i)} [rW_i(t) - c_i(t)]$$

where $p_t^{(i)}$ is the costate variable for state i . The necessary costate equation is:

$$\dot{p}_t^{(i)} = -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)} r - e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \quad (14)$$

¹⁰ Consumption, $c(t)$, is a stochastic process. We occasionally denote it as $c(t, W(t), Y_t)$ to emphasize that it depends on the states $(t, W(t), Y_t)$. When we reformulate our stochastic problem as a deterministic problem and focus on a single path $Y_t = i$, consumption is no longer stochastic because there is no uncertainty in the development of health states. We emphasize this point in our notation here by writing consumption as $c_i(t)$, and wealth as $W_i(t)$.

The solution to the costate equation can be obtained using the variation of the constant method:

$$p_t^{(i)} = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant. The necessary first-order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) \quad (15)$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0, W_0, i)}{\partial W_0} = p_0^{(i)} = u_c(c_i(0), q_i(0))$. Since the Hamiltonian is concave in $c_i(t)$ and $W_i(t)$, the necessary conditions for optimality are also sufficient (Seierstad and Sydsaeter 1977).

To analyze the value of life, we let $\delta(t)$ be a perturbation on the mortality rate in state i with $\int_0^T \delta(t) dt = 1$ and consider:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t (\bar{\mu}_i(s) - \varepsilon \delta(s)) + \sum_{j>i} \lambda_{ij}(s) ds \right], \text{ where } \varepsilon > 0$$

We first derive an expression for the effect of this perturbation on expected lifetime utility.

Lemma 2:

The marginal utility of life extension in state i is equal to:

$$\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} = \int_0^T \left[e^{-\rho t} \left(\int_0^t \delta(s) ds \right) \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) \right] dt$$

Proof of Lemma 2: see Appendix A

In order to facilitate comparison to the deterministic case, it is useful to derive an expression for the marginal utility of wealth at time t .

Lemma 3:

The expected marginal utility of wealth in state i at time t is equal to:

$$\frac{\partial V(t, W_i(t), i)}{\partial W_i(t)} = u_c(c_i(t), q_i(t))$$

$$= \mathbb{E} \left[e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \middle| Y_t = i, W(t) = W_i(t) \right], \forall \tau > t$$

Proof of Lemma 3: see Appendix A

This is the stochastic analogue of the consumer's first-order condition from Section II.B, and it shows that the consumer sets the expected discounted marginal utility of consumption at time $\tau > t$ equal to the current marginal utility of wealth. Our next result demonstrates that the value of statistical life also takes the same basic form as in the deterministic case.

Proposition 4:

Set $\delta(\cdot)$ in the expression for the marginal utility of life-extension given in **Lemma 2** equal to the Dirac delta function. Dividing the result by the marginal utility of wealth at time $t = 0$ yields:

$$VSL(i) = \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u_c(c(0), q_{Y_0}(0))} dt \middle| Y_0 = i, W(0) = W_0 \right] = \frac{V(0, W_0, i)}{u_c(c_i(0), q_i(0))} \quad (16)$$

Applying **Lemma 3** and rearranging yields the following, equivalent expression for VSL in state i :

$$VSL(i) = \int_0^T e^{-rt} v(i, t) dt$$

where the value of a life-year, $v(i, t)$, is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i, t) = \frac{\mathbb{E} [S(t) u(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0]}{\mathbb{E} [S(t) u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = W_0]}$$

Proof of Proposition 4: see Appendix A

As in the earlier setting with deterministic health (see equation 6), the value of statistical life here is proportional to expected lifetime utility, and inversely proportional to the marginal utility of consumption. As we shall show later, an adverse shock to mortality increases current consumption, causing the net effect on VSL to be ambiguous.

We can derive an expression for the life-cycle profile of consumption from (15), the first-order condition for consumption. Differentiating with respect to t , plugging in the result for the costate equation and its solution, and rearranging yields:

$$\frac{\dot{c}_i}{c_i} = \sigma(r - \rho) + \sigma\eta \frac{\dot{q}_i}{q_i} - \sigma\bar{\mu}_i(t) - \sigma \sum_{j>i} \lambda_{ij}(t) \left[1 - \frac{u_c(c(t, W_i(t), j), q_j(t))}{u_c(c(t, W_i(t), i), q_i(t))} \right] \quad (17)$$

As in the deterministic case, the rate of change in consumption is a declining function of the individual's current mortality rate, $\bar{\mu}_i(t)$: removing annuity markets “pulls consumption earlier” in the life-cycle.

There is also now an additional source of risk, captured by the fourth term in equation (17). This term represents the possibility that the consumer might transition to a different health state in the future. This transition would shift life-cycle consumption earlier still if the marginal utility of consumption in those future states is likely to be low.

Equation (17) describes consumption dynamics conditional on the individual's health state i . It is not readily apparent from (17) whether modeling health as stochastic causes consumption to shift forward, *on average* across all states, relative to modeling health as deterministic. We confirmed in numerical exercises that modeling health as stochastic has an ambiguous effect on consumption (and VSL), even when holding quality of life constant across states and time.¹¹

Consumption will jump when an uninsured consumer transitions between health states. The sign of that jump depends on how the accompanying changes in mortality risk and quality of life jointly affect the marginal utility of consumption. Because there is no consensus regarding the sign or magnitude of health state dependence, $u_{cq}(\cdot)$, we hold quality of life constant for the time being and return to this issue in our empirical analysis.¹² Under this assumption, the model predicts that transitioning to a state where current

¹¹ Counterintuitively, modeling health as stochastic has a positive effect on lifetime utility. This positive effect arises because a stochastic environment allows the consumer to react to health shocks by adjusting consumption. Put differently, a deterministic model is equivalent to a stochastic model where the consumer must keep consumption constant across states. Consumers prefer the ability to adjust consumption across states.

¹² Viscusi and Evans (1990), Sloan et al. (1998), and Finkelstein, Luttmer, and Notowidigdo (2013) find evidence of negative state dependence. Lillard and Weiss (1997) and Edwards (2008) find evidence of positive state dependence. Evans and Viscusi (1991) find no evidence of state dependence. Murphy and Topel (2006) assume negative state dependence when performing their empirical exercises, while Hall and Jones (2007) assume state independence.

and future expected mortality are high will shift consumption forward, and vice versa (see Figure 2). Our next result proves this formally for a two-state case.¹³

Proposition 5:

Let there be $n = 2$ states with identical quality of life profiles, so that $q_1(s) = q_2(s) \forall s$. Assume that the transition intensities $\lambda_{12}(s)$ are uniformly bounded (finite), and that $\bar{\mu}_1(s) < \bar{\mu}_2(s) \forall s$, so that state 1 is “healthy” and state 2 is “sick.” Suppose that the consumer transitions from state 1 to state 2 at time t . Then $c_1(t, w, 1) \leq c_2(t, w, 2), w > 0$.

Proof of Proposition 5: see Appendix A

It follows immediately from **Proposition 5** that the value of near-term life-years will increase, and the value of distant life-years will decrease, when transitioning from a healthy state with low mortality to a sick state with higher mortality. Whether VSL rises or falls is ambiguous, however. A rise in mortality risk lowers lifetime utility, which reduces VSL, but it also reduces the marginal utility of consumption, which increases VSL. Thus, the net effect depends on the curvature of the utility function relative to the curvature of the marginal utility function.

We formally demonstrate this tradeoff by again comparing a (persistently) healthy individual to someone who suffers an adverse shock to life expectancy but is otherwise identical. We know from **Proposition 5** that the sick person’s optimal consumption is initially higher. Under what conditions is the sick person’s VSL also higher? To make headway we must introduce the notion of prudence. The elasticity of intertemporal substitution, $1/\sigma$, measures utility curvature. Prudence is the analogous measure for the curvature of marginal utility (Kimball 1990). Define relative prudence as:

$$\pi \equiv -\frac{cu_{ccc}(\cdot)}{u_{cc}(\cdot)}$$

It will also be convenient to define the elasticity of the flow utility function:

$$\epsilon = \frac{cu_c(\cdot)}{u(\cdot)}$$

¹³ The proof can be extended to allow for a larger number of states, but the conditions required to sign the jump in consumption then become a complicated function of the matrix of transition probabilities and state-specific mortality rates. The two-state case conveys the basic result without a meaningful loss of generality.

The utility elasticity, ϵ , is positive when utility is positive. Positive utility ensures well-behaved preferences, and is often enforced by adding a constant to the utility function. (See Section IV.A. for a related discussion on this point.)

Our next result provides sufficient conditions for VSL to rise following an adverse shock to longevity.

Proposition 6:

Consider a two-state setting with assumptions set out in **Proposition 5**. Assume further that $r \leq \rho$, and that utility is positive and satisfies the condition:

$$\pi \leq \frac{2}{\sigma} + \epsilon \quad (18)$$

Suppose that the consumer transitions from state 1 to state 2 at time t , and that $\lambda_{12}(\tau) = 0 \forall \tau > t$. Then $VSL(1, t) \leq VSL(2, t)$.

Proof of Proposition 6: see Appendix A

The assumption $r \leq \rho$ is consistent with prior studies on discount and interest rates (Moore and Viscusi 1990).¹⁴ VSL will rise following a longevity shock if prudence, π , is low relative to the elasticity of intertemporal substitution, $1/\sigma$. Consumers with inelastic demand (low σ) prefer to smooth consumption over time. They therefore have a high willingness to pay for life-extension and are more likely to exhibit a rise in VSL following an adverse mortality shock. Likewise, consumers with low levels of prudence, π , have near-linear marginal utility that decreases rapidly with consumption. This generates a high willingness to pay for life-extension following a shock that increases consumption.

The condition (18) specified in **Proposition 6** is satisfied by hyperbolic absolute risk aversion (HARA) utility functions, a class that includes CRRA and quadratic utility, provided that utility is positive. However, this condition is not innocuous: for example, one can easily find linear combinations of CRRA and polynomial utility functions where VSL *declines* following an illness. Prior studies on the value of life generally assume that 0.5 to 0.8 is a reasonable range for the value of σ (Murphy and Topel 2006; Hall and Jones 2007), and recent empirical studies suggest that π is about 2 (Noussair, Trautmann, and Van de Kuilen 2013; Christelis et al. forthcoming). Under these parameterizations, condition (18) will hold whenever utility is positive.

¹⁴ The assumption $r \leq \rho$ is made for technical convenience: it ensures that the growth rate of consumption is negative, which facilitates the comparison of VSL across the two states.

We emphasize that this result differs from the findings of prior studies that examine the effect of baseline risk on VSL in a static environment (Weinstein, Shepard, and Pliskin 1980; Pratt and Zeckhauser 1996; Hammitt 2000). Those studies consider a one-period setting with two states, dead and alive. In this static setting, if the marginal utility of consumption is lower in the dead state, then an increase in the risk of death necessarily lowers the expected marginal utility of consumption and thus raises the willingness to pay for survival (the “dead-anyway” effect).¹⁵ **Proposition 5** describes how the effect of mortality risk on marginal utility plays out in a life-cycle model. In this dynamic context, an increase in the risk of death shifts consumption forward, which does reduce marginal utility. However, unlike in the conventional one-period setting, the resulting effect on VSL is ambiguous because of an offsetting effect on lifetime utility.

III.B. The value of statistical illness

The stochastic model permits us to investigate the value of avoiding transitions to other health states. This requires only a slight modification to the analysis presented above, and will result in a more general concept we term the *value of statistical illness*. With a slight abuse of notation, let state $N + 1$ correspond to death, so that $V(t, W(t), N + 1) = 0$. Let $\delta_{ij}(t)$, $j \leq N$, be a perturbation on the transition intensity, $\lambda_{ij}(t)$, and let $\delta_{i,N+1}(t)$ be a perturbation on the mortality rate, $\bar{\mu}_i(t)$, $i \leq N$, where $\sum_{j=i+1}^{N+1} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t (\bar{\mu}_i(s) - \varepsilon \delta_{i,N+1}(s)) + \sum_{j=i+1}^N (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right], \text{ where } \varepsilon > 0$$

Proposition 7:

The marginal utility of preventing an illness or death is given by:

$$\left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} = \int_0^T e^{-\rho t} \tilde{S}^\varepsilon(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt$$

¹⁵ Let expected utility be equal to $EU = pu(0, c) + (1 - p)u(1, c)$, where $p \in (0, 1)$ is the probability of death and the states $\{0, 1\}$ represent death and life, respectively. The willingness to pay for a marginal reduction in the probability of dying is given by $VSL = \frac{u(1, c) - u(0, c)}{pu_c(0, c) + (1 - p)u_c(1, c)}$, which increases with p if $u_c(1, c) > u_c(0, c)$. Pratt and Zeckhauser (1996) also discuss an offsetting “high-payment” effect, which arises when the at-risk individual increases spending on risk reduction. This reduction in wealth raises her marginal utility and thus lowers VSL.

Proof of Proposition 7: see Appendix A

The value of preventing an illness or death is equal to the marginal rate of substitution between the transition perturbation and wealth:

$$\left. \frac{\partial V / \partial \varepsilon}{\partial V / \partial W} \right|_{\varepsilon=0} = \int_0^T \frac{e^{-\rho t} \tilde{S}(i, t)}{u_c(c_i(0), q_i(0))} \left[\left(\int_0^t \sum_{j \geq i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j \geq i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j \geq i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt$$

As before, it is helpful to choose the Dirac delta function for $\delta(\cdot)$, so that the intensities are perturbed at $t = 0$ and remain unaffected otherwise. It is also helpful to consider a reduction in the transition probability for only one alternative state, j , so that $\delta_{ik}(t) = 0 \forall k \neq j$. Applying these two conditions then yields what we term the value of statistical illness, $VSI(i, j)$:

$$\begin{aligned} VSI(i, j) &= \frac{V(0, W_0, i) - V(0, W_0, j)}{u_c(c_i(0), q_i(0))} \\ &= VSL(i) - VSL(j) \frac{u_c(c_j(0), q_j(0))}{u_c(c_i(0), q_i(0))} \end{aligned} \tag{19}$$

The interpretation of VSI is analogous to VSL: it is the amount that 1,000 individuals are collectively willing to pay in order to eliminate a current disease risk that is expected to befall one of them. Note that if health state j corresponds to death, so that $VSL(j) = VSL(N + 1) = 0$, then $VSI(i, j) = VSL(i)$. Thus, VSI is a generalization of VSL.

It is instructive to compare VSI for the uninsured consumer, given in (19), to VSI for a fully annuitized consumer, which we denote as VSI^* :¹⁶

$$VSI^*(i, j) = VSL^*(i) - VSL^*(j) \tag{20}$$

Under full annuitization, the value of a life-year is equal across health states (holding quality of life constant). As shown by equation (20), this implies that prevention and treatment are equally valuable, as

¹⁶ The value of the consumer's annuity depends on the health state. If she purchases an annuity in state i and then later transitions to a worse health state j , causing her life expectancy to fall, then the value of her annuity will also fall, an effect not reflected in the notation used in equation (20). See equation (D8) and accompanying discussion in Appendix D.

long as they add the same number of expected life-years.¹⁷ In other words, full annuitization justifies the common cost-effectiveness practice of equating the values of prevention and treatment (Drummond et al. 2015).

In contrast, equation (19) shows that eliminating annuity markets breaks this equivalence between treatment and prevention. VSI in this case is not equal to the simple difference in VSL between the healthy and sick states, because VSL in the sick state is valued from the perspective of the sick, who are likely to have a lower marginal utility of consumption due to their shorter life span. This leads to the natural hypothesis that whenever VSL rises following an illness, the value of treatments (VSL per life-year) will be higher than equivalent preventive care consumed prior to the illness (VSI per life-year). It is simple to prove this for the case where the illness reduces life expectancy by one-half or more (see **Proposition 8** in Appendix A), and our numerical exercises suggest that the hypothesis is true under far more general conditions. Our empirical exercises find that, under reasonable parameterizations, the value of treatment is higher than the value of prevention for a number of different diseases.

To summarize, the uninsured stochastic model yields the following implications:

- All else equal, when an individual transitions to a higher mortality state, near-term life-years rise in value, and distant life-years fall in value.
- The value of statistical life may rise or fall when an individual transitions to a higher mortality state. If the individual's demand is sufficiently inelastic, or if the individual is insufficiently prudent, then VSL will rise.
- Therapies that increase survival by treating sick patients are not generally worth the same as preventives that add the same amount of life expectancy for healthy patients. If the disease causes VSL to rise, then we expect treatment to be worth more than equally effective preventive care.

III.C. The incompletely annuitized value of life

We now introduce a one-time opportunity to purchase a flat lifetime annuity, and also endow the consumer with state-dependent life-cycle income, $m_{Y_t}(t)$. Recall that we previously solved the consumer's problem for each state i by focusing on the path of Y that begins in state i and remains in state i until time T (Parpas and Webster 2013). Incomplete annuity markets and life-cycle income complicate

¹⁷ Consistent with our model, Rheinberger, Herrera-Araujo, and Hammitt (2016) point out that prevention can be more valuable (*ex ante*) than treatment for a highly lethal, but rare, disease, because a disease-specific mortality reduction in this case has a much smaller effect on *total* life-years gained than a reduction in disease incidence.

our analysis by introducing the possibility of multiple sets of non-interior solutions within and across different states. For convenience, we focus here on the case where there is a single set of non-interior solutions in each state. In particular, we follow Section II.C and consider the case where, conditional on remaining in the same state, consumption decreases with age and eventually converges to the consumer's annuity income (e.g., left panel in Appendix Figure A1). This occurs if, for example, the consumer has a constant income across and within states, has a mortality rate profile in that state that obeys the condition $\bar{\mu}_i(t) \geq r - \rho + \eta \frac{\dot{q}_i}{q_i}$, and cannot transition to a healthier state.¹⁸

Borrowing an approach from Reichling and Smetters (2015), we assume the consumer has an option at time zero to purchase a flat lifetime annuity that pays out $\bar{m}_{Y_0} \geq 0$ in all health states and that has a price markup of $\xi \geq 0$. The consumer cannot finance the purchase of the annuity using future income, and she cannot purchase or sell annuities after time zero. The consumer's maximization problem is:

$$V(0, W_0, Y_0) = \max_{c(t), \bar{m}_{Y_0}} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0 - (1 + \xi) \bar{m}_{Y_0} \mathbb{E} \left[\int_0^T e^{-rt} S(t) dt \middle| Y_0 = i \right], \\ W(t) &\geq 0, W(T) = 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) + \bar{m}_{Y_0} - c(t) \end{aligned}$$

The optimal annuity amount is chosen in the consumer's initial state, Y_0 , and its value may change following a transition to a new health state because a fixed payout is worth more to a person with higher life expectancy. We emphasize this in our notation below by writing V as a function of the optimally chosen annuity and remaining wealth. In addition, it is helpful to define the value of a one-dollar annuity at time t in state i as:

¹⁸ More precisely, we are interested in studying the case where $\frac{\dot{c}_i}{c_i} \leq 0 \forall i$. From equation (17), this occurs when

$\bar{\mu}_i(t) \geq r - \rho + \eta \frac{\dot{q}_i}{q_i} - \sum_{j>i} \lambda_{ij}(t) \left[1 - \frac{u_c(c(t, W_i(t), j), q_j(t))}{u_c(c(t, W_i(t), i), q_i(t))} \right]$. The fourth term is negative if, for example, quality of life is constant and the consumer can only transition to states with higher mortality, as in **Proposition 5**.

$$a(t, i) = \mathbb{E} \left[\int_t^T e^{-r(s-t)} \exp \left\{ - \int_t^s \mu(u) du \right\} ds \middle| Y_t = i \right]$$

Proposition 9:

The value of statistical life in state i is equal to:

$$VSL(i) = \frac{V(0, W_i(0), \bar{m}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi) \bar{m}_i a(0, i) \quad (21)$$

Proof of Proposition 9: see Appendix A

As in the deterministic case given by equation (10), the expression for the partially annuitized value of life here captures elements of both the uninsured and fully insured cases. When annuities are absent ($\bar{m}_i = 0$), equation (21) simplifies to the uninsured case given by equation (16). Similarly, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case equation (21) simplifies to the complete markets case given by equation (D7) in Appendix D.¹⁹

Finally, the value of statistical illness under partial annuitization again takes an intermediate form.

Corollary 10:

The value of a marginal reduction in the risk of transitioning from state i to state j is equal to:

$$\begin{aligned} VSI(i, j) &= \left(\frac{V(0, W_i(0), \bar{m}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi) a(0, i) \bar{m}_i \right) - \left(\frac{V(0, W_i(0), \bar{m}_i, j)}{u_c(c_i(0), q_i(0))} - (1 + \xi) a(0, j) \bar{m}_i \right) \\ &= VSL(i) - \left(\frac{V(0, W_i(0), \bar{m}_i, j)}{u_c(c_i(0), q_i(0))} - (1 + \xi) a(0, j) \bar{m}_i \right) \end{aligned}$$

Proof of Corollary 10: see Appendix A

VSI is again not exactly equal to the difference in VSL between the two health states. One reason why is because, as in the uninsured case, the utility of state j is valued from the perspective (marginal utility) of state i . A second reason is because the flat annuity was purchased in state i , and the size of this annuity may differ from the optimal flat annuity that would have been purchased in state j .

To summarize, combining stochastic health with incomplete annuity markets has the following effects:

¹⁹ Remaining wealth at time $t = 0$, $W(0)$, is zero upon full annuitization. This implies $W_0 = (1 + \xi) \bar{m}_i a(0, i)$.

- As in the deterministic model, the optimal level of annuitization is partial except for certain special cases.
- The insights from Sections III.A and III.B continue to hold when annuitization is partial. In particular, the value of statistical life may in general rise or fall following a mortality shock, and treatment and prevention are not valued equally.²⁰

III.D. Welfare

Our stochastic model describes a person's willingness to pay to extend life and shows how it changes *ex post* following a health shock. This model generates several positive predictions. It helps explain why the value of life-extension varies considerably with a person's health state and why people value prevention and treatment differently, even when both generate the same gain in life expectancy. The normative implications depend on how one resolves several longstanding controversies in the theoretical literature on welfare economics.

Aggregate social surplus remains the most widely used welfare criterion in applied microeconomics, including within the literature on life-extension. Murphy and Topel (2006) employ this principle in the framework of the standard life-cycle VSL model. Garber and Phelps (1997) rely on it to develop the theory of cost-effectiveness for health interventions. Einav, Finkelstein, and Cullen (2010) use it to study the welfare effects of health insurance. More generally, industrial organization economists use it, in the form of deadweight loss, to evaluate the welfare consequences of market power (Martin 2019).

However, the aggregate surplus approach has several shortcomings (Boadway 1974; Blackorby and Donaldson 1990). Each dollar of consumer or producer surplus is weighted equally, regardless of its owner, which raises equity concerns. Aggregation can produce intransitive rankings of alternative allocations. Heterogeneity in marginal utility can break the link between surplus and utility (Martin 2019). Since part of the *ex post* variation in VSL following a health shock is driven by changes in marginal utility, the surplus approach may result in misleading normative implications when applied to our setting.

A social welfare approach, by contrast, aggregates utilities rather than surplus. But debate persists about how to apply this approach under uncertainty, where the *ex ante* and *ex post* perspectives of a consumer

²⁰ Under the conditions outlined in **Proposition 5**, consumption will always increase following a mortality shock provided the consumer is not fully annuitized. Whether there is an accompanying rise in VSL will in general depend on the degree of annuitization in addition to the usual conditions outlined in **Proposition 6**. We show in Appendix D that VSL always declines following a mortality shock when the consumer is fully annuitized.

might differ (Fleurbaey 2010). In a foundational study, Harsanyi (1955) shows that a social welfare function satisfying both rationality and the Pareto principle must be a weighted sum of *ex ante* individual utilities. However, this utilitarian approach ignores distributional concerns (Diamond 1967). As a result, one cannot simultaneously satisfy both rationality and the Pareto principle while still pursuing equity. Theorists have argued for abandoning one or the other of these principles. Diamond (1967) advocates minimizing *ex ante* inequality, but this violates rationality. Adler and Sanichirico (2006) advocate minimizing *ex post* inequality, but this violates the Pareto principle. In the specific context of VSL, Pratt and Zeckhauser (1996) advocate maximizing *ex ante* utility, but this ignores equity concerns in light of Diamond’s result.

Given the inevitable trade-offs, we do not aim to defend one or another specific welfare perspective. When quantifying the value of statistical life under uncertainty (Section IV.B), we focus on positive implications only, e.g., explaining why there is less observed investment in prevention than treatment. When evaluating the effects of retiree programs on the value of life (Section IV.C), we employ a deterministic model and pursue an aggregate surplus approach. A deterministic setting avoids the need to model welfare under uncertainty and allows for a straightforward comparison to prior work (Murphy and Topel 2006).

As a practical matter, health policymakers and analysts frequently evaluate policy using cost-benefit analysis (CBA), which equates social welfare with aggregate social surplus (Viscusi 1992). It is therefore worth briefly mentioning the implications of our stochastic model for CBA, in spite of CBA’s well-documented shortcomings. CBA is equivalent to a utilitarian approach where all individuals’ utilities are weighted by the inverse of their marginal utilities of consumption. We showed previously that when annuity markets are incomplete, the marginal utility of consumption is lower for people with short life expectancies. Thus, the incomplete annuities framework causes CBA to place more weight on the sick, which some scholars have advocated (Adler, Hammitt, and Treich 2014).²¹

²¹ A special case of CBA, cost-effectiveness analysis, is widely used for studying the optimal allocation of healthcare resources. The traditional cost-effectiveness framework presumes that the value of a “quality-adjusted life-year” is independent of health state. Our model suggests that it should instead vary with the health state of the individuals responsible for financing health care.

IV. QUANTITATIVE ANALYSIS

This section demonstrates how the value of statistical life depends on an individual's health history and illustrates that, under typical consumer preferences, the willingness to pay for treatment exceeds the willingness to pay for prevention. We also measure the aggregate value of gains to health and longevity and how that value depends on the level of annuitization.

Our empirical framework, which incorporates survival and health status uncertainty into a life-cycle model, is related to a number of papers that study the savings behavior of the elderly (Kotlikoff 1988; Palumbo 1999; De Nardi, French, and Jones 2010). These prior studies allow health to affect wealth accumulation by including two or three different health states in the model. By contrast, we allow mortality, medical spending, and quality of life to vary across 20 different health states.

IV.A. Framework

We employ the discrete time analogue of our stochastic theoretical model. There are n health states. Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}[Y_{t+1} = j | Y_t = i]$$

The mortality rate at time t , $d(t)$, depends on the individual's health state:

$$d(t) = \sum_{j=1}^n \bar{d}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\{\bar{d}_j(t)\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator equal to 1 if the individual is in state j at time t and 0 otherwise. The probability of surviving from time period t to time period s is denoted as $S_t(s)$, where:

$$\begin{aligned} S_t(t) &= 1, \\ S_t(s) &= S_t(s-1)(1 - d(s-1)), s > t \end{aligned}$$

The survival probability is stochastic because it depends on the individual's health history. Let $c(t)$ and $W(t)$ denote consumption and wealth in period t , respectively. The individual's health state at time t , Y_t , determines her quality of life, $q_{Y_t}(t)$. Let ρ denote the rate of time preference, and r the interest rate.

Assume that in each period the consumer receives exogenous income, $y(t)$, and that the maximum lifespan of a consumer is T (i.e., $d(T) = 1$). Our baseline model assumes there is no bequest motive, although we relax this assumption in later exercises.

The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E} \left[\sum_{t=0}^T e^{-\rho t} S_0(t) u(c(t), q_{Y_t}(t)) \middle| Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) + y(t) - c(t))e^r \end{aligned}$$

The individual's period income is equal to $y(t) = (1 - \tau)m(t) + a(t)$, where $a(t)$ is nonwage defined-benefit income financed by an actuarially fair earnings tax, τ . We choose the individual's labor earnings, $m(t)$, to fit data on average life-cycle earnings as estimated by the Current Population Survey and the Health and Retirement Study (see Appendix B1 for details). Our retirement policy exercises, described in detail later, consider different levels of generosity for $a(t)$.

Unless stated otherwise, we assume that $r = \rho = 0.03$ (Siegel 1992; Moore and Viscusi 1990). Finally, we follow Murphy and Topel (2006) and assume that utility takes the following CRRA form:

$$u(c, q) = qu(c) = q \left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \quad (22)$$

As discussed in Section III, there is no consensus regarding the sign or magnitude of health state dependence ($u_{cq}(\cdot)$). Here, we assume a multiplicative relationship where the marginal utility of consumption is higher when quality of life is high, and vice versa.

Quality of life, q , is an index that ranges from 0 to 1, where $q = 1$ indexes perfect health. We have normalized the utility of death to zero in (22). A consumer receives positive utility if she consumes an amount greater than \underline{c} , which represents a subsistence level of consumption. Consuming an amount less than \underline{c} generates utility that is worse than death. Although adding a constant to the utility function does not affect the solution to the consumer's maximization problem, this constant matters for the value of life.²² We are unaware of any empirical evidence on the magnitude of \underline{c} , the subsistence level of consumption in the United States. We assume it is equal to \$5,000, which is in line with the parameterization employed in Murphy and Topel (2006).

²² Rosen (1988) was the first to point out that the level of utility is an important determinant of the value of life. See also additional discussion on this point in Hall and Jones (2007) and Córdoba and Ripoll (2016).

The parameter γ is the inverse of the elasticity of intertemporal substitution, an important determinant of both the value of life and the value of annuitization. We follow Hall and Jones (2007) and set $\gamma = 2$ in our main specification.

We employ dynamic programming techniques to solve for the optimal consumption path. The value function is defined as:

$$V(t, w, i) = \max_{c(t)} \mathbb{E} \left[\sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q_i(s)) \middle| Y_t = i, W(t) = w \right]$$

We then reformulate the optimization problem as a recursive Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left[u(c(t)) + \frac{1 - \bar{d}_i(t)}{e^\rho} \sum_{j=1}^N p_{ij}(t) V(t+1, (w + y(t) - c(t))e^r, j) \right]$$

After solving for the optimal consumption path, we use the analytical formulas derived in the previous sections to calculate the value of life. We provide complete details in Appendix C.

There is significant uncertainty among economists regarding the proper values of many of the parameters in our model. The goal of the subsequent analyses is to illustrate the economic significance of our insights when applying our model to data using reasonable parameterizations. In some analyses, we investigate the sensitivity of our results to alternative assumptions regarding the elasticity of intertemporal substitution, $1/\gamma$, and to the presence of a bequest motive. While the value of γ matters greatly for the magnitude of VSL, it does not have any qualitative effect on our findings regarding the relative determinants of VSL.

The remainder of this section reports results from two separate empirical exercises. The first exercise illustrates the novel implications of our framework by showing that the value of treatment generally exceeds the value of prevention and that the value of statistical life can increase following a health shock. The second exercise illustrates the effect of different annuitization schemes on the aggregate value of improvements in longevity. All code and data underlying these exercises are publicly available online.²³

IV.B. The value of life when health is stochastic

This section applies our stochastic life-cycle model to real-world data on mortality and quality of life that vary according to a person's health state. Later exercises further incorporate medical spending that varies

²³ See http://julianreif.com/research/mortality_risk_replication.zip.

by health state. We interpret our estimates here as the private value of statistical life, i.e., the individual's willingness to pay for life-extension and disease prevention.

The data for this set of exercises are provided by the Future Elderly Model (FEM), a widely published microsimulation model that employs comprehensive, nationally representative data from a wide array of sources (see Appendix B2). The model, which has been released into the public domain, produces estimates of mortality, disease incidence, quality of life, and medical spending at the individual level for people over the age of 50 with different comorbid conditions. The FEM accounts for six different chronic conditions (cancer, diabetes, heart disease, hypertension, chronic lung disease, and stroke) and six different impaired activities of daily living (bathing, eating, dressing, walking, getting into or out of bed, and using the toilet). The FEM provides us with a well-validated tool that combines information from the Health and Retirement Study (HRS), the Medical Expenditure Panel Survey (MEPS), the Panel Study of Income Dynamics, and the National Health Interview Survey. This combination provides a number of advantages. For instance, while the HRS possesses a uniquely rich set of covariates on health and wealth, it lacks survey questions that would allow us to calculate quality of life using validated survey instruments. To solve this problem, the FEM weaves together validated quality of life estimates from the MEPS and maps them to the HRS using variables common to both databases.

We divide the health space within the FEM into $n = 20$ states. Each state corresponds to the number (0, 1, 2, 3 or more) of impaired activities of daily living (ADL) and the number (0, 1, 2, 3, 4 or more) of chronic conditions, for a total of $4 \times 5 = 20$ health states. Health states are ordered first by number of ADLs and then by number of chronic diseases, so that state 1 corresponds to 0 ADLs and 0 chronic conditions, state 2 corresponds to 0 ADLs and 1 chronic condition, and so on. This aggregation provides a parsimonious way of incorporating information about functional status and several major diseases.²⁴ For each health state and age, the FEM estimates the probability of dying and the probability of transitioning to each of the other health states in the next year. As in the theoretical model, individuals can transition only to higher-numbered states, i.e., $p_{ij}(t) = 0 \forall j < i$. In other words, all ADLs and chronic conditions are permanent. The FEM also estimates quality of life for each health state and age, as measured by the EuroQol five dimensions questionnaire (EQ-5D). These five dimensions are based on five survey questions that elicit the extent of a respondent's problems with mobility, self-care, daily activities, pain,

²⁴ While fully interacting all these variables would provide a more granular state space, it would also result in a very large number of possible states and correspondingly small cell sizes within many of them.

and anxiety/depression. These questions are then weighted using stated preference data to compute the relative importance of each.²⁵ The result is a single quality of life measure, the EQ-5D, typically reported on a scale from zero to one.

Table 1 presents descriptive statistics for the data provided by the FEM. Initial life expectancy at age 50 ranges from 30.4 years for a healthy individual in state 1 to 8.6 years for an ill individual in state 20. Quality of life, as measured by the EQ-5D, ranges from 0.54 to 0.88 at age 50. Columns (7) and (8) of Table 1 report the probability that an individual exits her health state but remains alive, i.e., acquires at least one new ADL or chronic condition within the following year. Health states are relatively persistent, with exit rates never exceeding 15 percent. State 20 is an absorbing state with an exit rate of 0 percent.

We set wealth equal to \$862,947 and provide all of it to the individual at baseline (age 50). This value is approximately equal to the net present value of future earnings at age 50 plus savings at that age, as estimated by the retirement policy model we employ in Section IV.C (see Appendix C3). We also assume annuity markets are absent. These two simplifications allow us to calculate the effect of stochastic health shocks on the value of life using an analytical solution to the consumer's problem (see Appendix C2).²⁶ This analytical solution avoids numerical precision error and speeds up calculations, which is especially useful when performing the Monte Carlo simulations described below. We consider partial annuitization scenarios and allow for life-cycle income in the numerical model presented in the next section.

VSL in our baseline scenario for a healthy 50-year old in health state 1 is \$5.4 million, which is within the range estimated by empirical studies of VSL for working-age individuals (see O'Brien 2018 for a recent review).

IV.B.1 The value of prevention

We begin by calculating the value of statistical illness (VSI) for different diseases. Column (4) of Table 2 reports VSI at age 50 from the perspective of a healthy individual. Each value represents the healthy

²⁵ The five dimensions of the EQ-5D are weighted using estimates from Shaw, Johnson, and Coons (2005). The specific process for estimating the quality of life score is explained in the FEM technical documentation, which can be found in the supplemental information appendix of Agus et al. (2016). The methods used to measure the quality of life are consistent with our assumed utility specification, given in (22).

²⁶ It is possible (and available upon request) to incorporate partial annuitization in this setting along the lines discussed in Section III.C. Further generalization requires numerical optimization, which likely would necessitate significantly limiting the number of health states included in the model.

individual's willingness to pay for a marginal, contemporaneous reduction in the probability of developing an illness corresponding to one of the 19 other health states. The values are inversely related to life expectancy in the sick state because it is more valuable to prevent the onset of a lethal disease than a mild one. The highest VSI is \$4.0 million, which corresponds to the value of preventing the onset of a sick state with 3 ADLs and 4 chronic conditions (health state 20). The interpretation is analogous to VSL: it is the amount that 1,000 healthy individuals would collectively be willing to pay in order to reduce their risk of developing this illness by 1/1000. This value remains below the healthy individual's VSL, which represents the willingness to pay to avoid the extreme "illness" of dying.

How does the value of prevention compare to the value of treatment? We investigate this question by calculating VSL and VSI in a given health state, as discussed in Section IV.A, and normalizing both by the number of life-years saved. In contrast to the conventional (fully annuitized) framework, here the value of a life-year depends on whether the individual is sick or healthy. Intuitively, health gains are worth more after health shocks than before them, because those shocks accelerate consumption and increase the value of life.

Table 2 illustrates this point. For example, column (5) reports that a 50-year-old with two chronic conditions and no ADLs (health state 3) has a marginal willingness to pay of \$242,000 per life-year for a treatment that extends her life. Column (6), by contrast, reveals that a healthy individual is only willing to pay \$177,000 per life-year saved through preventing the onset of health state 3. In this case, treatment is 37 percent more valuable than prevention. Column (7) of Table 1 shows that the value of life-years saved by treating an illness always exceeds the corresponding value gained by preventing that illness – by as much as a factor of 5, for the sickest state in our model. These results help explain low observed investment in prevention (Dranove 1998; Pryor and Volpp 2018).

Figure 3 displays these results graphically. It depicts how VSL and VSI vary across our health states, which are arrayed along the x-axis from longest to shortest life expectancy. The solid blue bars depict VSL per life-year and demonstrate that the value gained through treatment is monotonically higher for states with lower remaining life expectancy. The dotted red bars show the value per life-year gained by preventing each health state, from the perspective of a perfectly healthy person. For instance, the left-most dotted red bar reports the value of each life-year saved when a perfectly healthy consumer reduces the risk of entering the health state with 27.7 years of life expectancy. VSI is relatively stable across health states. Recall that VSI is calculated from the fixed perspective of a perfectly healthy person; therefore, consumption profiles and the marginal utility of consumption remain stable. The minor variation in VSI per life-year across these health states is due primarily to differences in current and expected future quality of life.

While large initially, the gap between the value of treatment and prevention narrows in the years following an adverse health shock. For example, Figure 4 compares the value of treatment for a consumer who suffers a health shock at age 70 to an otherwise identical consumer who remains healthy. The value of treatment exceeds the value of prevention, but only for the first 7 years following the shock. After that point, the sick patient has spent down much of her wealth, which causes a significant reduction in her VSL, although we note that most patients will have died before reaching this point. (The FEM estimates that life expectancy for this sick patient is 8.0 years at age 70.)

IV.B.2 The effect of health shocks on the value of life

If an individual never suffers a health shock, then her consumption and VSL will decline smoothly with age. However, the arrival of a health shock can increase VSL, sometimes substantially. Figure 5 displays consumption and VSL for an initially healthy individual who develops one ADL (health state 6) at age 60, and then one more ADL plus three chronic conditions (health state 14) at age 70. The first shock reduces her life expectancy by 3.0 years and her quality of life by 0.06. The second one reduces her life expectancy by 6.8 years and her quality of life by 0.16. Both shocks increase consumption. The first shock has a mild effect on the declining trend in VSL, but the second increases VSL at age 70 by 24 percent, from \$2.4 million to \$3.0 million. This jump is driven by the reduction in life expectancy and would remain large even if quality of life were held constant.

Individual-level shocks generate substantial variability in VSL in the aggregate. Figure 6 reports results from a Monte Carlo simulation of 10,000 life-cycle modeling exercises. At age 50, all individuals are identical and have a VSL of \$5.4 million. As they age, some begin to suffer health shocks that, at least initially, increase their VSL. By age 70, the VSL inter-vigintile range spans \$1.7 to \$2.5 million. This dispersion is compressed towards the end of life, when mortality reaches 100 percent.

Next, we incorporate medical spending data from the FEM into our framework. Appendix Figure B3 reports average out-of-pocket medical spending for selected health states, by age. These data are comprehensive and include all inpatient, outpatient, prescription drug, and long-term care spending that is not paid for by insurance. Spending is higher in sicker health states, and—consistent with De Nardi, French, and Jones (2010)—increases greatly at older ages, when long-term care expenses arise. The effect of sickness on out-of-pocket spending is modest in comparison to long-term care costs, and the overall gap in spending across states shrinks with age. This shrinkage occurs because out-of-pocket medical

expenses are concentrated in the first year of incidence, and their effect on average spending is dampened in health states which include few newly diagnosed individuals.²⁷

Incorporating these spending data directly into our model would require numerical optimization methods. Instead, we reformulate these data as wealth shocks, which should yield qualitatively similar results while still allowing us to calculate an exact solution to the consumer's problem. This approach has the additional benefit of yielding insight into health shocks that reduce labor supply, since these would also ultimately reduce wealth. To implement, we modify the law of motion for wealth so that the individual's effective interest rate depends on her health state:

$$W(t + 1) = (W(t) - c(t))e^{r(t, Y_t)}$$

where $r(t, Y_t) = 0.03 + \ln[1 - s(t, Y_t)]$ and $s(t, Y_t)$ is the share of an individual's wealth spent on medical and nursing home care at time t in health state Y_t .²⁸ Instead of deducting medical costs from wealth directly, we treat them as modifying the interest rate. While this is unconventional, it achieves our desired change in the life-cycle consumption profile, while preserving the closed-form solution that facilitates our empirical analysis. In addition, treating medical costs as a percentage reduction in wealth sacrifices relatively little generality as compared to an approach that subtracts those costs from wealth.

Figure 7 illustrates that incorporating medical spending reduces VSL slightly but does not otherwise appreciably alter its life-cycle profile, even in the presence of significant health shocks. This remains true even if we employ total, rather than out-of-pocket, medical spending. The reason is that the difference in medical spending between healthy and sick individuals is small relative to the variation in spending by age (see Appendix Figure B3). A sufficiently large idiosyncratic spending shock will have a significant impact, however. This effect is illustrated by the dotted black line in Figure 7, which plots VSL for a hypothetical case where the individual's wealth falls by 20 percent following the health shock at age 70, rather than by the much smaller medical spending amount estimated by the FEM. Although VSL still

²⁷ For more details, see the appendix materials in National Academies of Sciences (2015).

²⁸ Specifically, we calculate $s(t, Y_t)$ by dividing out-of-pocket medical spending in health state Y_t at time t by $W(t)$, where $W(t)$ was estimated by our model for a healthy individual in a setting with no medical spending. Our results are similar if we instead use wealth estimates from the Health and Retirement Study.

increases slightly at age 70, the rise is far smaller than in the other two cases. Thus, while accounting for typical medical spending does not appear to alter our basic results, catastrophic expenditures do matter.

Our last exercise values the longevity gains experienced over the past 15 years. During this period, all-cause mortality for the US population ages 50 and over has fallen by 18%, with cancer and heart disease mortality both falling by 21%.²⁹ Panel A of Table 3 values these health gains from the perspective of a current 50-year-old. In a setting with no out-of-pocket medical spending, the private value of the reduction in all-cause mortality is worth \$56,000 to \$240,000, depending on the assumed value of relative risk version. Including out-of-pocket medical spending, which causes a small fall in consumption, reduces these values slightly. Panel B shows that these estimates are reduced by about 10 percent if we incorporate a bequest motive into the model.³⁰ The next section discusses the relationship between bequest motives and the value of life in more detail.

IV.C. Retirement policy and the aggregate value of life

This section explores the link between retirement policy and the value of life. We build up to these results by calculating how the value of statistical life varies over the life-cycle under alternative annuitization policies. We then quantify how these alternative policies influence the aggregate value of permanent reductions in mortality.

We initiate the model at age 20 and assume nobody survives past age 100. We obtain data on age-specific mortality rates from the Human Mortality Database. These mortality data are not available by health state, so this model includes only one health state (i.e., mortality is deterministic). We obtain nationally representative data on quality of life from the MEPS. These data are measured using the EQ-5D and are described further in Appendix B1. Since everyone is in the same (*ex ante*) health state, we interpret the estimates produced by this model as the long-run aggregate value of longevity improvements. (See Section III.D for additional discussion on this point.)

All our calculations account for the effect of life-extension on societal wealth in the same way, regardless of the degree of annuitization. A fully annuitized consumer who dies leaves behind wealth that is

²⁹ Source: authors' calculations using mortality data from the National Vital Statistics.

³⁰ We follow Fischer (1973) and assume the bequest motive takes a CRRA form, which again allows us to calculate an exact solution to the consumer's problem. See Appendix C2 for details.

distributed to the rest of the annuity pool. To net out this purely financial consequence of annuitization, all our calculations distribute remaining wealth in this same way, even when annuity markets are incomplete or absent.³¹ This facilitates comparison across different annuitization scenarios and makes it more appropriate to interpret our estimates as the aggregate value of increased longevity.

Unlike in the previous section, the individual receives a flow of income instead of a baseline endowment of wealth. This feature is important here because it allows us to model the effects of retirement and annuitization. Moreover, it is computationally simple to incorporate into this model because we only have to contend with a single health state. Recall that the individual's period income is equal to $y(t) = (1 - \tau)m(t) + a(t)$, where $a(t)$ is nonwage defined-benefit income financed by an earnings tax, τ . We consider three different policy scenarios in the main text. In the first, annuity markets are absent, and the consumer's income equals her labor earnings: $y^1(t) = m(t)$. Thus, her consumption is limited by current period income and savings from prior periods. The second scenario introduces an actuarially fair Social Security program that provides an annuity equal to \$16,195 beginning at age 65.³² In this second scenario, the consumer is partially annuitized, but she still lacks access to private annuity markets and cannot borrow against her future income. The third scenario increases the size of the Social Security pension by 50 percent. Finally, in the appendix we also present results for the case where the consumer fully annuitizes at age 20 and enjoys a constant annuity stream, $\bar{y} = \bar{a}$, provided by an actuarially fair annuity market. The income streams in all scenarios are related according to the following equation:

$$\sum_{t=0}^T \frac{y^1(t)S(t)}{e^{rt}} = \sum_{t=0}^T \frac{y^2(t)S(t)}{e^{rt}} = \sum_{t=0}^T \frac{y^3(t)S(t)}{e^{rt}} = \bar{y} \sum_{t=0}^T \frac{S(t)}{e^{rt}}$$

Our assumed interest rate of 3 percent and our data on mortality and earnings imply a full annuity value of $\bar{y} = \$37,897$.

The life-cycle profiles of consumption for the first two policy scenarios are displayed in Figure 8. Consumption is constrained by the consumer's low income in early life. She saves during middle age

³¹ For example, we set VSL at time $t = 0$ equal to $\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q(t))}{u(c(0), q(0))} dt - W_0$ when annuity markets are absent. Unlike equation (6), this VSL formula follows the fully annuitized equation (3) and subtracts wealth, W_0 .

³² This corresponds to the average retirement benefit paid by Social Security to retired workers in 2016 (www.ssa.gov/policy/docs/quickfacts/stat_snapshot/2016-07.pdf).

when income is high, and then consumes her savings during retirement until eventually her consumption equals her pension (if available).³³ Consumption for an individual with no annuity is “shifted forward” relative to an individual with a Social Security pension. This effect is particularly dramatic in the final 10 years of life, when old consumers outlive their wealth. This is not surprising: a primary benefit of an annuity is its ability to provide income to consumers in their oldest ages.

Appendix Figure A2 shows that this difference in consumption generates a corresponding difference in the value of a life-year. Individuals place a low value on life-years at very young and very old ages, because consumption is low. The slight drop at age 65 reflects the effect of retirement on the net savings component of the value of life.

Figure 9 displays the corresponding value of statistical life (VSL) for these two scenarios. At age 40, VSL is equal to \$6.8 million for an individual with no annuity, and \$7.8 million for an individual who will be eligible for Social Security at age 65. Figure 9 also shows that VSL is greater at older ages for a person with a Social Security pension than for a person with no annuity. This difference suggests that annuity programs are complementary with retiree health care programs and other investments in life-extension for the elderly population.

Next, we calculate the value of historical reductions in mortality for these different annuitization scenarios, as well as the prospective value of permanent reductions in future mortality for selected diseases. Let δ denote a vector of mortality reductions for different ages. As in Murphy and Topel (2006), we calculate the total value of a mortality reduction by aggregating over the age distribution of the 2015 US population:

$$Aggregate\ Value = \sum_{a=0}^{110} VLE(a, \delta) f(a)$$

³³ This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers 1991; Banks, Blundell, and Tanner 1998; Fernandez-Villaverde and Krueger 2007).

where $VLE(a, \delta)$ is based on equation (9), and $f(a)$ is the count of people alive in 2015 at age a .³⁴

We report our results in Table 4. Life expectancy at birth increased by over 10 years between 1940 and 2010. Like Murphy and Topel (2006), we find that the aggregate value of these past longevity gains is substantial: the post-1940 gains are worth about \$100 trillion today, and the post-1970 gains are worth about \$50 trillion. Comparing results for different annuitization scenarios informs our understanding of the interaction between retirement policies and the value of longevity. For example, consider the introduction of Social Security over the last century. Comparing column (1) to column (2) of Table 4 suggests that Social Security increased the value of post-1940 longevity gains by \$10.6 trillion (11.0 percent) and increased the value of post-1970 gains by \$5.7 trillion (12.2 percent). One way to interpret these values is to compare them to the longevity insurance value of Social Security, which is approximately \$17 trillion.³⁵ Thus, the interaction between post-1940 longevity gains and Social Security is worth half as much as the longevity insurance value of the entire Social Security program itself.

Table 4 also reveals that Social Security has raised the value of a 10 percent cancer mortality reduction by \$394 billion, or 14 percent. Alternatively, Social Security has raised the value of a 10 percent reduction in all-cause mortality by \$1.27 trillion (13 percent). Column (3) reports that increasing the size of Social Security pensions by 50 percent would add \$639 billion more to that value.

A bequest motive encourages individuals to delay consumption, because money saved for consumption in old age also has the added benefit of increasing bequests in the event of death. The effect of a bequest motive on consumption and the value of longevity is therefore similar to that of increased annuitization. Prior work suggests a bequest motive is most relevant to the rich (Hurd and Smith 2002; De Nardi, French, and Jones 2010). However, for illustrative purposes we repeat our main exercise under the assumption of a strong bequest motive.³⁶ Those results, illustrated in Figure 10, demonstrate that a

³⁴ Specifically, $VLE(a, \delta) = \int_a^{100} e^{-\rho(t-a)} \left[\int_a^t \delta(s) ds \right] S(t) \frac{u(c(t))}{u(c(a))} dt - \omega(a)$. The effect of life-extension on societal wealth is $\omega(a) = \int_a^{100} e^{-r(t-a)} \left[\int_a^t \delta(s) ds \right] S(t) (m(t) - c(t)) dt$. We assume $VLE(a, \delta) = VLE(20, \delta)$ for $a < 20$, and equal to $VLE(100, \delta)$ for $a > 100$. Unlike Murphy and Topel (2006), our calculation does not account for the value that mortality reductions generate for future (unborn) populations.

³⁵ The longevity insurance value is calculated using the methodology of Mitchell et al. (1999) and does not account for other potential benefits of Social Security such as protection against inflation risk. See Appendix C1 for details.

³⁶ When accounting for a bequest motive in this exercise, we follow Kopczuk and Lupton (2007) and assume the utility from leaving a bequest is linear in wealth. See Appendix C1 for details.

bequest motive lowers the value of statistical life prior to age 65, and increases it at older ages. Intuitively, bequest motives increase the value of saving at younger ages. Appendix Table A3 further shows that in this case, the effect of Social Security on the value of post-1940 longevity gains is \$8.6 trillion (9.3 percent), which is about 20 percent smaller than in the setting with no bequest motive. This result suggests that the effect of retirement policy on the value of life matters more for non-wealthy individuals, who are less likely to have a significant bequest motive.

To summarize, our model predicts that annuitization raises the value of life for the elderly. This increase in VSL should cause them to spend more on health care and invest more in healthy behaviors, which in turn should ultimately manifest in increased life expectancy. This dovetails with the point, made by Philipson and Becker (1998), that the moral hazard effects of retirement programs also increase the willingness to pay for longevity. Philipson and Becker (1998) analyze data from Virga (1996) and find that people with more generous annuities live longer than those with less generous annuities. They interpret this increase in lifespan as the effect of endogenous longevity investments, which are encouraged among highly annuitized individuals who do not bear the full cost of an increase in their longevity. In our model, by contrast, annuitization increases the value of life because it protects against the risk of outliving one's wealth. Given that these effects reinforce each other, it is not surprising that increases in the generosity of public pensions in developed countries have been accompanied by large increases in public spending on retiree health care.

V. CONCLUSION

The economic theory surrounding the life-cycle value of life has many important applications. Yet, a number of limitations have surfaced over time. The traditional model does not distinguish between prevention and treatment. It also suffers from several anomalies that appear at odds with intuition or empirical facts, e.g., the apparent preferences of consumers to pay more for life-extension when survival prospects are bleaker. We overcome these limitations without abandoning the standard life-cycle framework, simply by relaxing its assumptions about full annuitization and deterministic health.

Our model offers a single unified framework for valuing both treatment and prevention. This framework provides a more practical tool for policymakers and decision makers, since many health investments involve preventing the deterioration of health rather than reducing an immediate mortality risk. Our result that treatment can be more valuable to individuals than equally effective preventive care also provides one explanation for why it has proven so difficult for policymakers and public health advocates to encourage investments in the prevention of disease. Kremer and Snyder (2015) show that heterogeneity in consumer valuations distorts R&D incentives by allowing firms to extract more consumer surplus from

treatments than preventives. Our results suggest that differences in private VSL may reinforce this result and further disadvantage incentives to develop preventives.

We also show that a given gain in longevity can be more valuable to a consumer with a lower life expectancy. Under conventional parameterizations, we calculate that differences in baseline risk cause a sick person's VSL to exceed a healthy person's VSL by over \$1 million at age 50. This inverse relationship between health and VSL may explain public preferences for giving priority to patients with severe diseases (Nord et al. 1995; Shah 2009; Shah, Tsuchiya, and Wailoo 2018).

Finally, our framework suggests that expanding public annuity programs boosts the demand for life-extending technologies. Intuitively, annuities calm consumer fears about outliving their wealth and thus enable more aggressive investments in life-extension. Viewed differently, our results also show that market failures in annuities affect the value of statistical life, and thus the socially optimal level of health care spending. This relationship suggests that researchers and policymakers should pay more attention to the public finance interactions between pension and health care systems.

Our analysis raises a number of important questions for further research. First, how does the theory change if we endogenize the demand for health and longevity and introduce incomplete health insurance markets? In this setting, medical technology that improves quality of life can act as insurance by compressing the difference in utility between the sick and healthy states (Lakdawalla, Malani, and Reif 2017). Less clear is how demands for the quantity and quality of life interact with financial market incompleteness of various kinds. Second, what are the most practical strategies for incorporating these insights into the literature on cost-effectiveness of alternative medical technologies? This literature typically assumes that quality-adjusted life-years possess a constant value. While flawed, this approach is simpler to implement than allowing the value to depend on health histories. Future research should focus on practical strategies for aligning cost-effectiveness analyses with the generalized theory of the value of life. Finally, what are the implications for the empirical literature on VSL? Empirical analysis of the economic theory of life-extension has typically proceeded under the assumption that different kinds of mortality risk are all valued the same way, as long as they imply similar changes in the probability of dying (Hirth et al. 2000; Mrozek and Taylor 2002; Viscusi and Aldy 2003). Our framework suggests the need for a more nuanced empirical approach. This missing insight may be one reason for the widely disparate empirical estimates of the value of statistical life.

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VII. TABLES AND FIGURES

Table 1. Summary statistics for the Future Elderly Model data

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Life expectancy		Quality of life		Exit probability	
Health state	ADLs	Chronic conditions	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70
1 (healthy)	0	0	30.4	14.0	0.884	0.873	4.2%	12.6%
2	0	1	27.7	12.4	0.850	0.840	3.6%	10.8%
3	0	2	24.1	10.4	0.812	0.804	3.6%	10.2%
4	0	3	20.0	8.4	0.773	0.765	3.9%	10.2%
5	0	4+	15.6	6.6	0.730	0.720	3.9%	7.9%
6	1	0	26.1	12.0	0.830	0.816	6.3%	14.7%
7	1	1	23.5	10.6	0.795	0.783	5.7%	12.7%
8	1	2	20.0	8.8	0.754	0.745	6.1%	12.2%
9	1	3	16.3	7.1	0.716	0.707	6.4%	11.7%
10	1	4+	12.7	5.5	0.669	0.662	6.1%	8.6%
11	2	0	23.8	10.8	0.781	0.765	7.3%	14.3%
12	2	1	21.0	9.4	0.746	0.731	7.5%	14.3%
13	2	2	17.6	7.8	0.706	0.693	7.5%	13.8%
14	2	3	14.5	6.3	0.669	0.655	7.5%	13.1%
15	2	4+	11.0	4.8	0.630	0.610	7.3%	10.6%
16	3+	0	21.4	8.9	0.700	0.692	3.4%	11.1%
17	3+	1	18.5	7.9	0.664	0.660	2.8%	8.5%
18	3+	2	15.2	6.4	0.622	0.622	2.3%	7.1%
19	3+	3	12.2	5.0	0.584	0.584	1.4%	5.3%
20	3+	4+	8.6	3.8	0.536	0.540	0.0%	0.0%

Notes: This table reports selected summary statistics for the Future Elderly Model (FEM) data employed by the stochastic life-cycle modeling exercise presented in Section IV.B. Columns (1) and (2) report the number of impaired activities of daily living (ADL) and the number of chronic conditions, which together define a health state. Column (3)-(6) report life expectancy (in years) and quality of life for an individual in one of these health states. Quality of life is measured using the EQ-5D index, which ranges from 0 (death) to 1 (perfectly healthy). Columns (7) and (8) report the probability that an individual transitions to a different health state in the following year. All ADLs and chronic conditions are permanent, so individuals can transition only to higher-numbered health states. Additional details about the Future Elderly Model are available in Appendix B2.

Table 2. Per capita private value of medical treatment and preventive care at age 50, by health state (thousands of dollars)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Health state	Life expectancy	VSL	VSI	Willingness to pay per life-year		
				Treatment	Prevention	Treatment/Prevention
1 (healthy)	30.4	\$5,413	N/A	\$178	N/A	N/A
2	27.7	\$5,576	\$488	\$201	\$181	1.11
6	26.1	\$5,672	\$904	\$217	\$212	1.02
3	24.1	\$5,834	\$1,116	\$242	\$177	1.37
11	23.8	\$5,809	\$1,425	\$244	\$217	1.13
7	23.5	\$5,873	\$1,366	\$250	\$198	1.26
16	21.4	\$5,902	\$1,944	\$276	\$215	1.28
12	21.0	\$6,055	\$1,912	\$289	\$203	1.43
4	20.0	\$6,202	\$1,817	\$310	\$175	1.77
8	20.0	\$6,208	\$1,968	\$311	\$189	1.65
17	18.5	\$6,175	\$2,420	\$335	\$203	1.65
13	17.6	\$6,438	\$2,458	\$365	\$193	1.90
9	16.3	\$6,676	\$2,571	\$408	\$183	2.23
5	15.6	\$6,726	\$2,580	\$431	\$174	2.47
18	15.2	\$6,615	\$2,944	\$435	\$194	2.25
14	14.5	\$6,964	\$2,968	\$481	\$186	2.58
10	12.7	\$7,322	\$3,181	\$575	\$180	3.19
19	12.2	\$7,212	\$3,428	\$593	\$188	3.15
15	11.0	\$7,732	\$3,536	\$705	\$182	3.88
20	8.6	\$8,197	\$3,992	\$950	\$183	5.18

Notes: This table displays values (in thousands of dollars) produced by the stochastic life-cycle modeling exercise presented in Section IV.B. Values are sorted by life expectancy at age 50, as reported in column (2). Column (3) reports the value of statistical life (VSL) for a 50-year-old in each health state. Column (4) reports the values of statistical illness (VSI) from the perspective of a healthy individual in state 1, and can be interpreted as a healthy individual's willingness to pay (WTP) to prevent a marginal increase in the probability of transitioning to the health state specified in column (1). Column (5) reports a sick individual's WTP per life-year for a therapeutic treatment, which is equal to the value in column (3) divided by the value in column (2). Column (6) reports the healthy individual's corresponding WTP for preventive care, which is equal to the value in column (4) divided by the difference between 30.4 (life expectancy when healthy) and the value in column (2). Column (7) reports the ratio of the values reported in columns (5) and (6). The twenty health states listed in column (1) are defined in Table 1.

Table 3. Per capita private value of historical 2001-2015 health gains, at age 50 (thousands of dollars)

Disease	Increase in life expectancy at age 50 (years)	(1)	(2)	(3)	(4)	(5)	(6)
<u>A. No bequest motive</u>							
All causes	1.43	\$56	\$117	\$240	\$49	\$101	\$205
Cancer	0.39	\$12	\$28	\$59	\$10	\$23	\$49
Heart disease	1.21	\$32	\$73	\$160	\$22	\$55	\$125
<u>B. Bequest motive</u>							
All causes	1.43	\$52	\$109	\$228	\$43	\$91	\$188
Cancer	0.39	\$12	\$26	\$56	\$9	\$20	\$44
Heart disease	1.21	\$29	\$68	\$150	\$19	\$47	\$109
Relative risk aversion		1.5	2	2.5	1.5	2	2.5
OOP medical spending		No	No	No	Yes	Yes	Yes

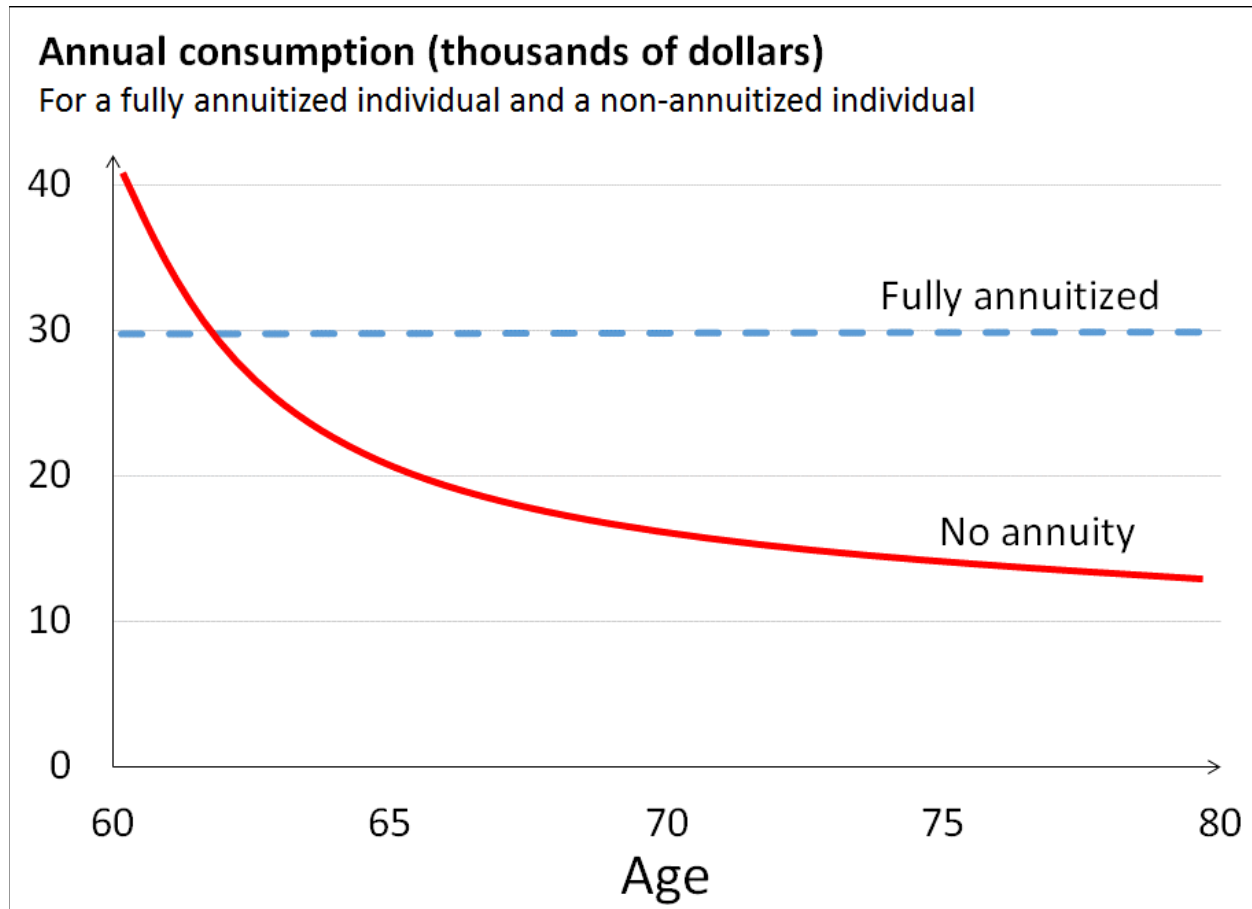
Notes: This table reports the value of the reduction in mortality experienced in the United States between 2001 and 2015, from the perspective of a 50-year-old alive in 2015. The cancer and heart disease calculations do not account for competing risks, and thus should be interpreted as holding mortality from all other causes constant. Columns (1)-(3) report results under the assumption that the individual has no out-of-pocket health care or nursing home costs. Columns (4)-(6) report results under the assumption that the health shocks are accompanied by an increase in health care and nursing home costs. The values in Panel A are calculated under the assumption that individuals do not have a bequest motive, while those in Panel B assume the bequest motive specification described in Appendix C2. The table also shows that these values increase with the size of the coefficient of relative risk aversion, which in our utility specification is equal to the inverse of the elasticity of intertemporal substitution.

Table 4. Aggregate value of historical and prospective reductions in mortality (billions of dollars)

	(1)	(2)	(3)
	No annuity	Social Security	Social Security + 50%
<u>A. Historical reduction</u>			
1940-2010	\$96,116	\$106,695	\$111,619
1970-2010	\$46,695	\$52,414	\$55,143
<u>B. 10% reduction, all ages</u>			
All causes	\$9,894	\$11,150	\$11,789
Cancer	\$2,885	\$3,279	\$3,477
Diabetes	\$316	\$359	\$380
Heart disease	\$2,060	\$2,351	\$2,505
Homicide	\$96	\$93	\$90
Infectious diseases	\$139	\$159	\$170

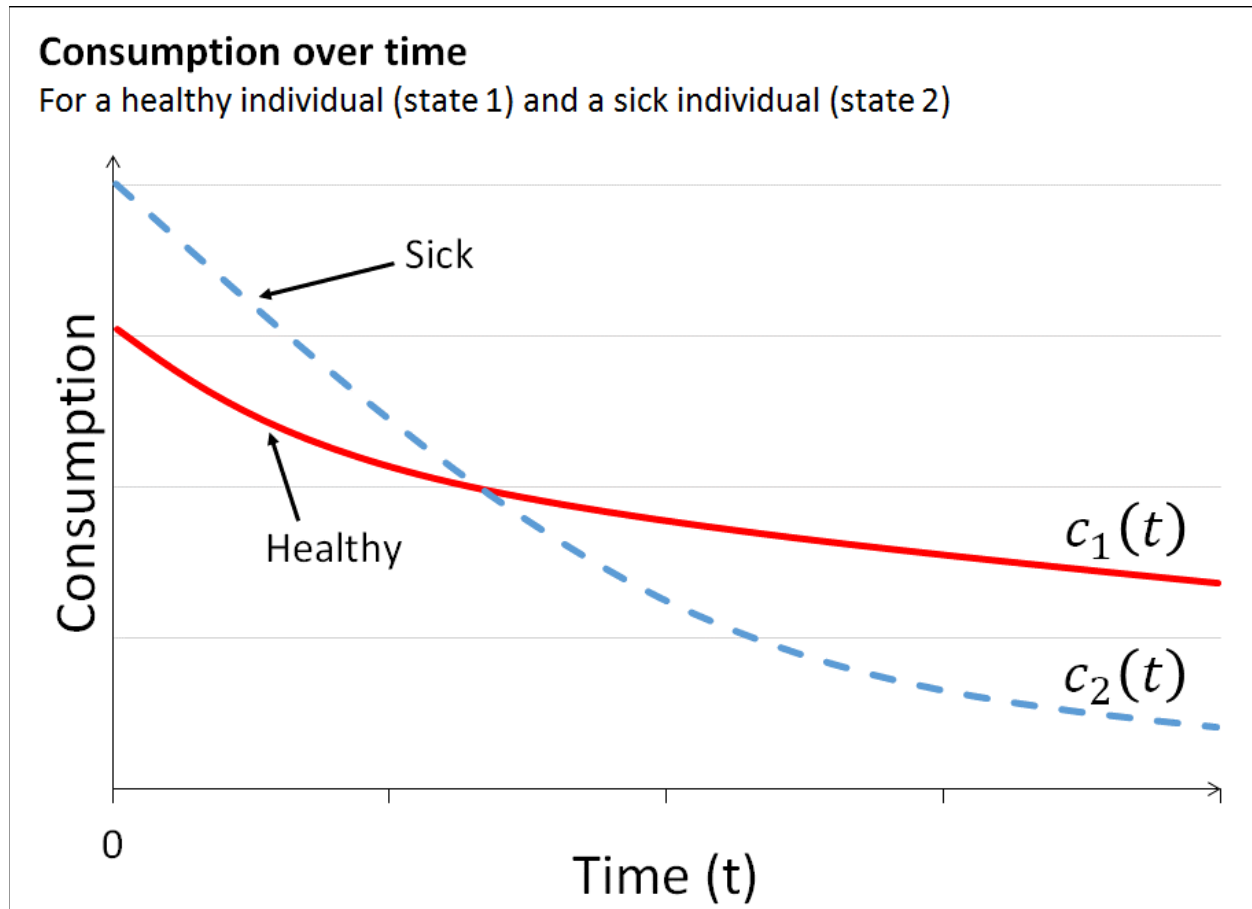
Notes: These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns.

Figure 1. Illustrative example: annual consumption for fully annuitized and non-annuitized consumers



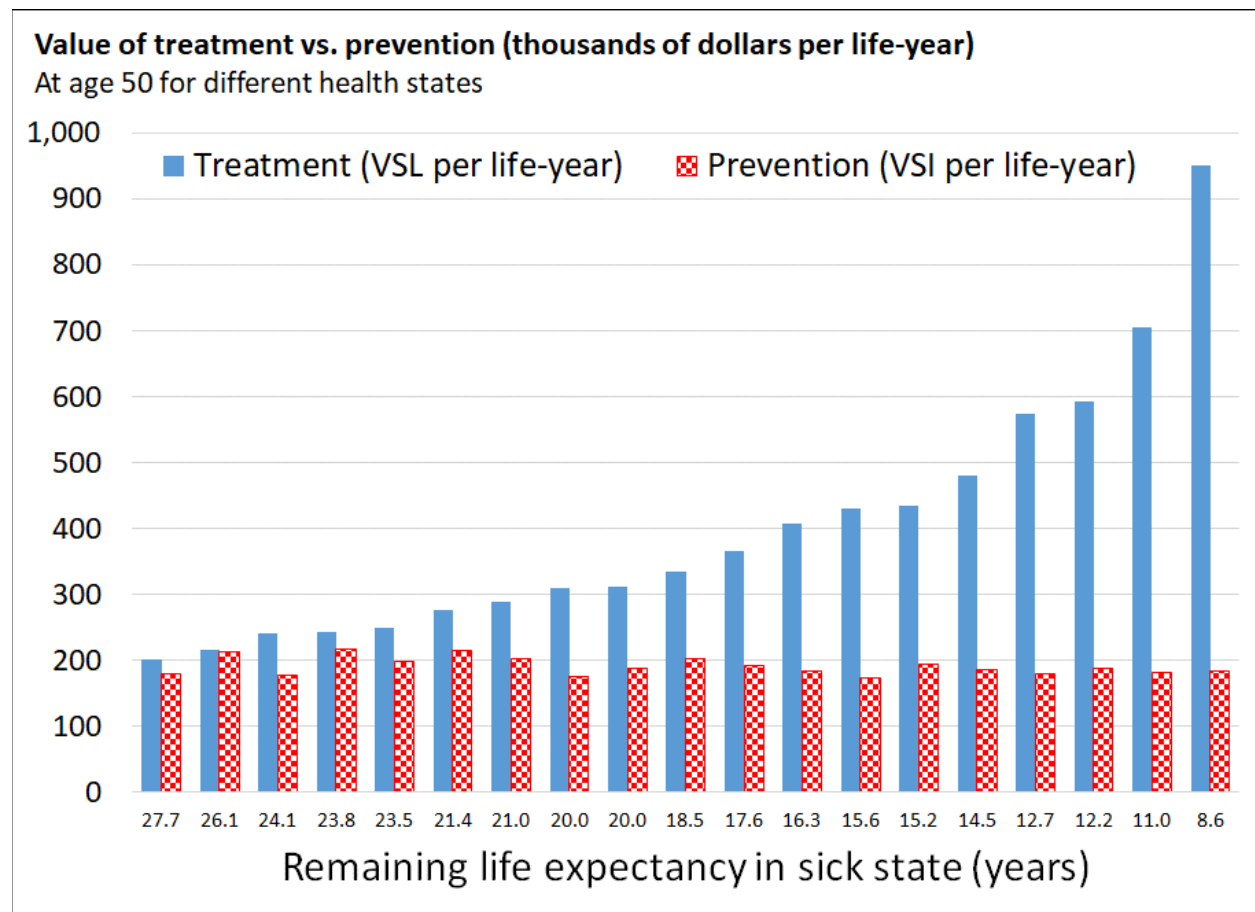
Notes: This figure illustrates the well-known result that it is optimal for a non-annuitized consumer who is exposed to longevity risk to shift her consumption forward in time, relative to a fully annuitized consumer. For simplicity, this example assumes that the optimal consumption profile of the fully annuitized consumer is flat.

Figure 2. Illustrative example: upon falling ill, consumption initially increases



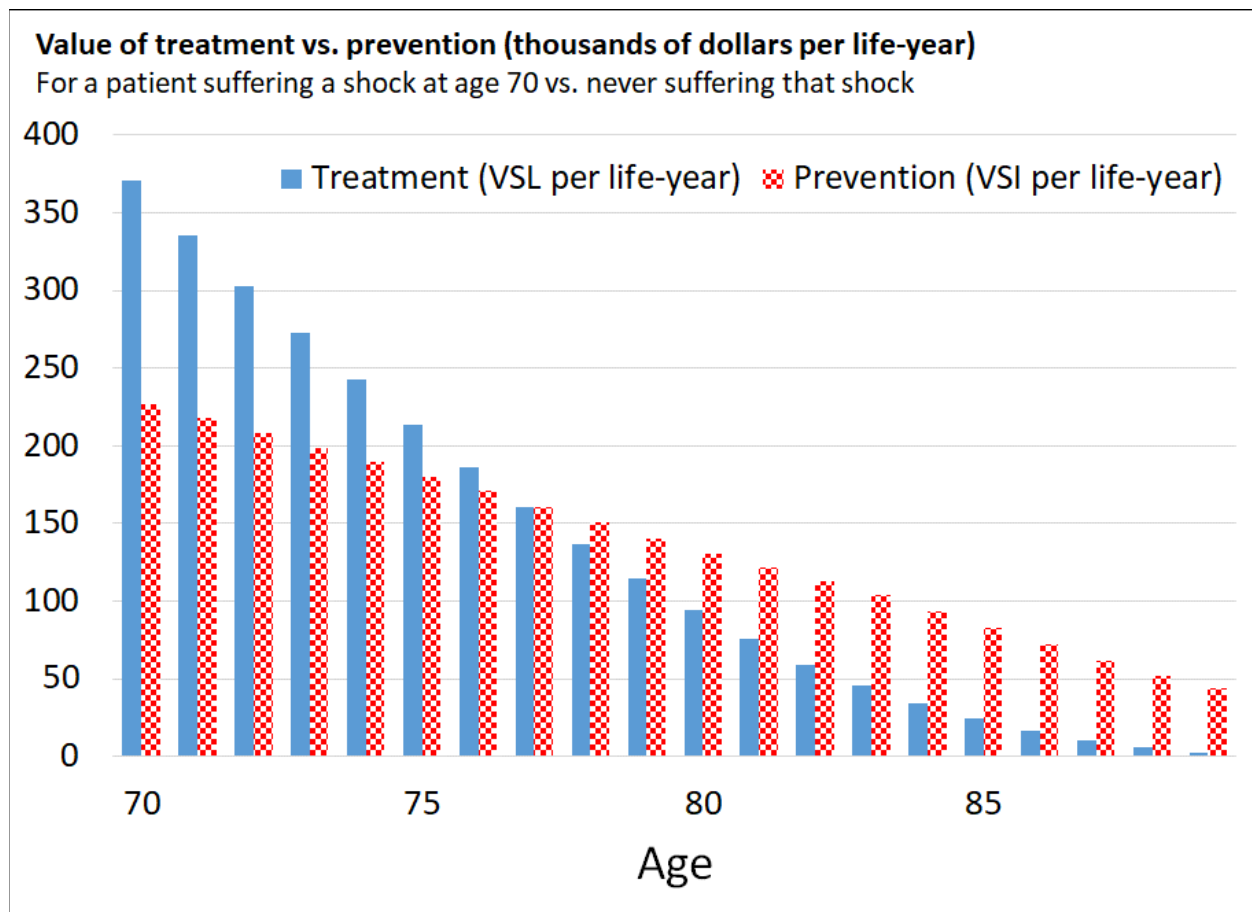
Notes: Both individuals have identical initial wealth at time $t = 0$. It is optimal for the sick individual (state 2) to consume at a higher rate than the healthy individual (state 1) because she has lower life expectancy. Thus, initial consumption at time $t = 0$ in the sick state is higher than in the healthy state, i.e., $c_2(0) > c_1(0)$. **Proposition 6** provides conditions under which VSL at time $t = 0$ in the sick state is also higher.

Figure 3. Treatments for an ill patient are worth more than preventive care for a healthy individual



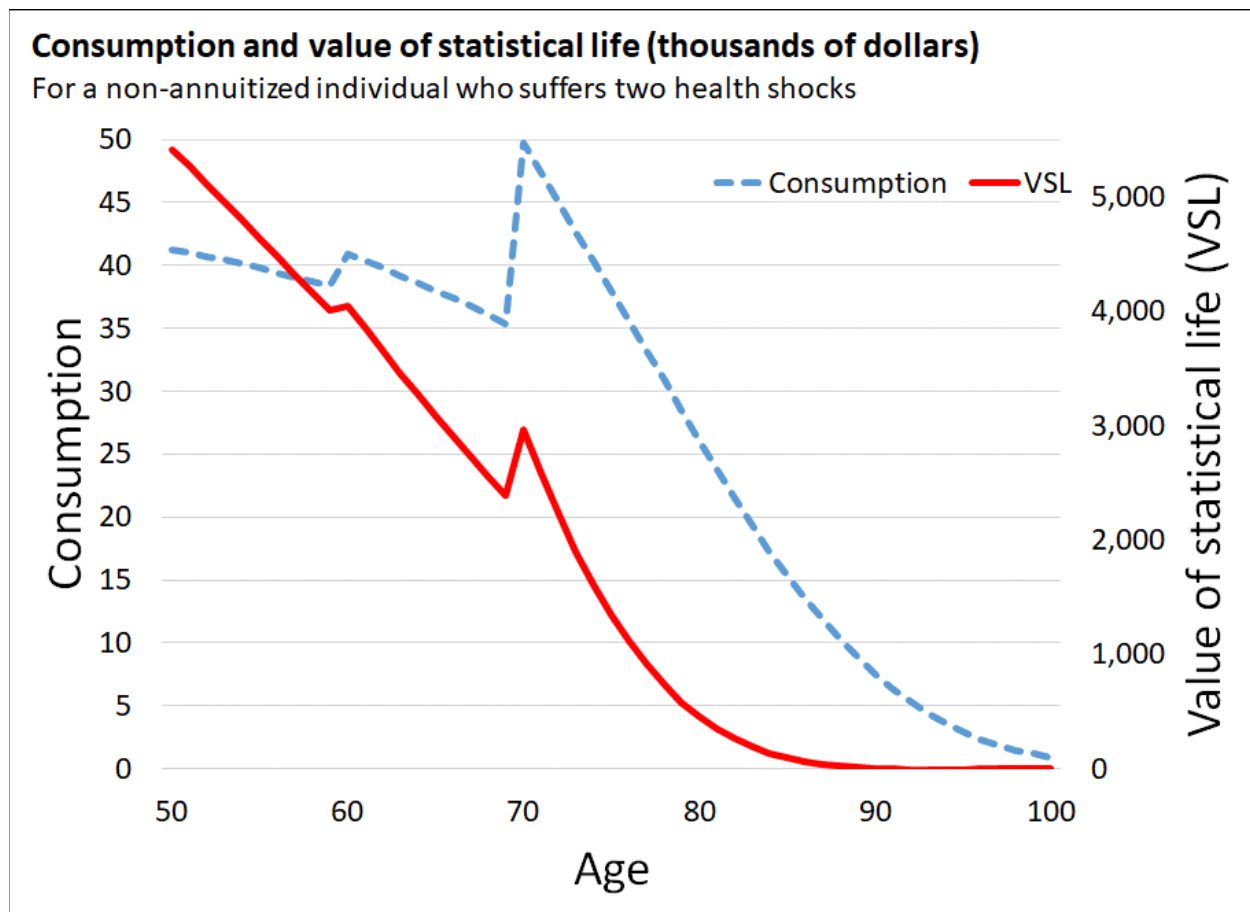
Notes: The solid blue bars report the value of statistical life (VSL) for an individual in one of 19 different sick states, divided by life expectancy in that state. The dotted red bars report the value of statistical illness (VSI) for a healthy individual (life expectancy: 30.4 years) divided by the reduction in life expectancy she would experience if she fell ill. The data plotted in this figure are also reported in columns (5) and (6) of Table 2.

Figure 4. The value of treatment relative to prevention declines with time since illness



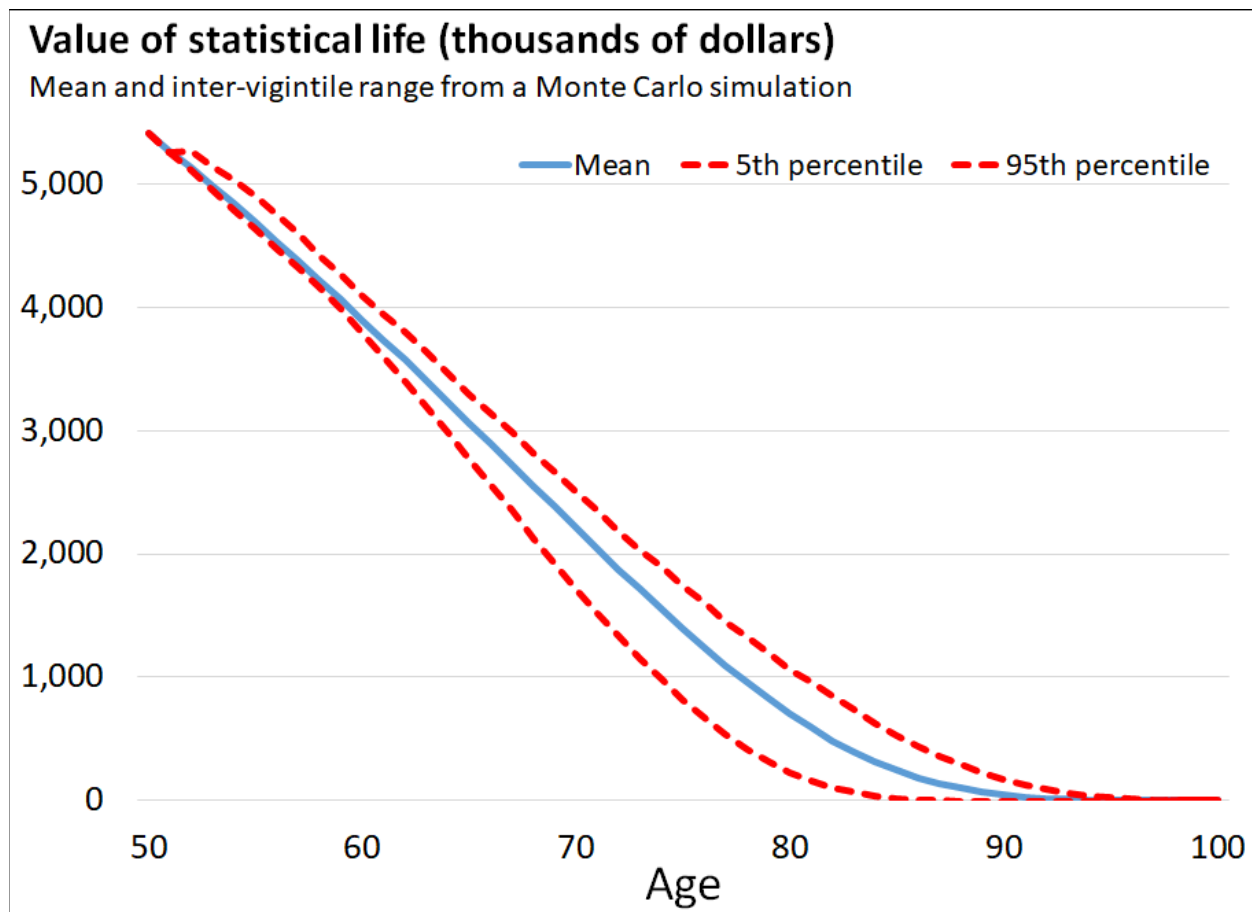
Notes: The solid blue bars report the value of statistical life (VSL) divided by life expectancy for an individual who suffers a health shock at age 70 that reduces her life expectancy by 6.8 years and her quality of life by 0.16. (This same health shock is also depicted in Figure 5 at age 70.) The dotted red bars report the value of statistical illness (VSI) for an otherwise identical individual divided by the *reduction* in life expectancy she would experience if she fell ill with the same disease.

Figure 5. Consumption and the value of statistical life can increase when an individual falls ill



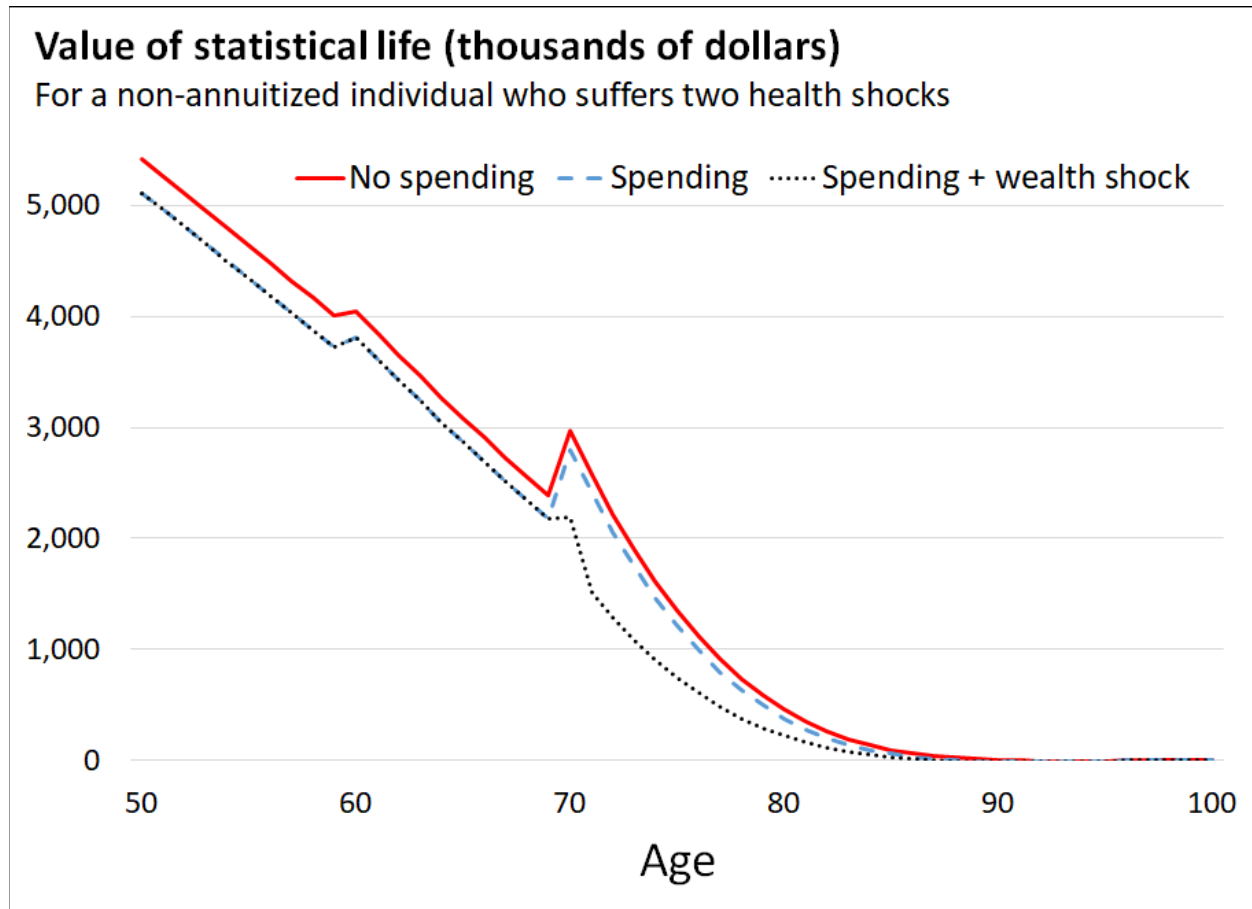
Notes: This figure plots an individual's consumption (left axis) and value of statistical life (right axis), as calculated by a life-cycle modeling exercise where mortality and quality of life are stochastic. The individual is healthy at age 50, but then falls ill twice, once at age 60 and then again at age 70. At age 60, the illness causes permanent difficulties with one routine activity of daily living (ADL). At age 70, she is diagnosed with three chronic conditions and one additional ADL. In our data, this corresponds to transitioning from state 1 to state 6 at age 60, and then from state 6 to state 14 at age 70. Summary statistics for these health states are available in Table 1.

Figure 6. The value of statistical life depends on an individual's health history



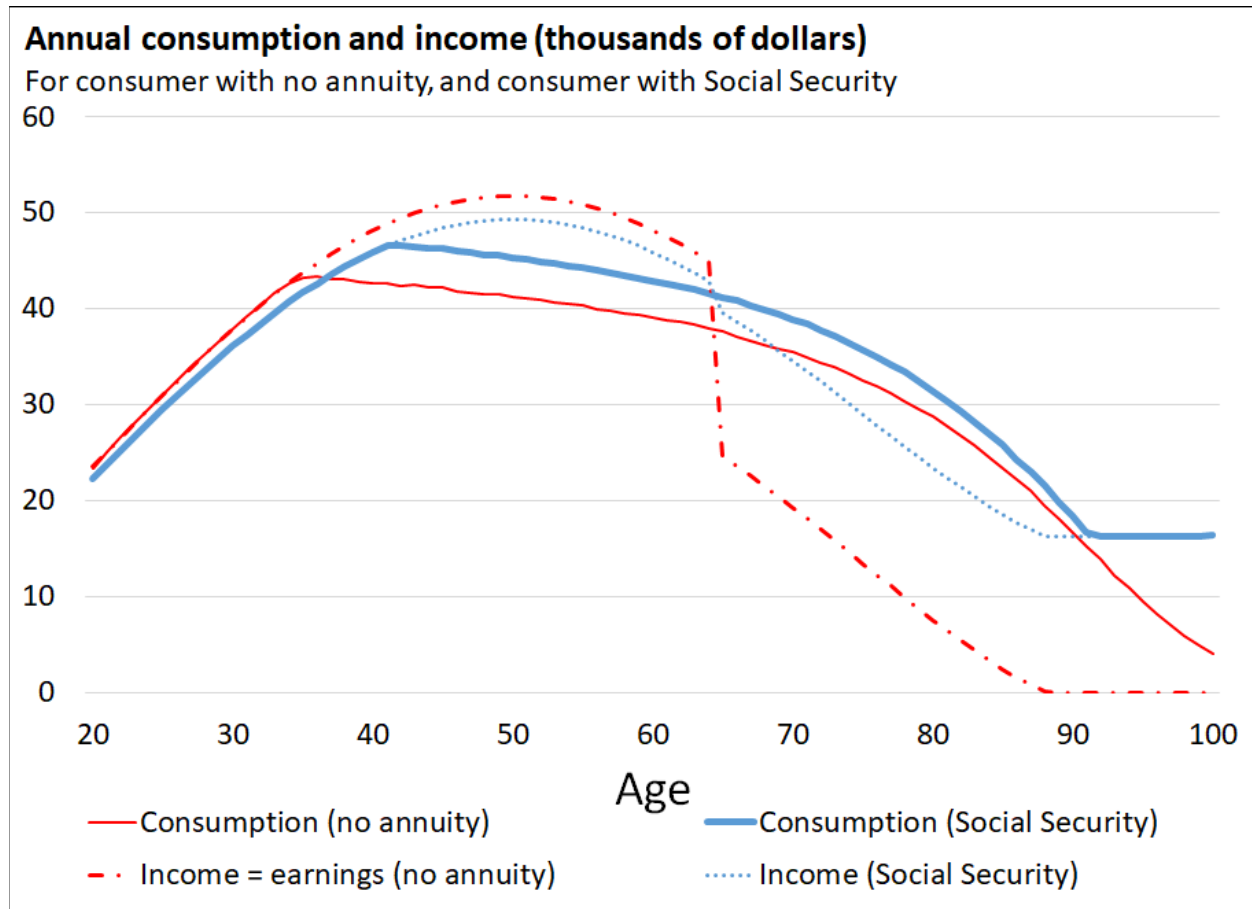
Notes: This figure reports the mean, 5th percentile, and 95th percentile of the value of statistical life (VSL) from a Monte Carlo simulation that is repeated 10,000 times. Each simulation begins at age 50 with a consumer in health state 1 (“healthy”). We then generate a health state path $\{Y_{51}, Y_{52}, \dots, Y_{100}\}$ using the transition probabilities estimated by the Future Elderly Model and solve for optimal consumption and VSL using the methods described in Appendix C2. Differences in VSL at older ages are caused by differences in the evolution of people’s health states.

Figure 7. Correlated spending shocks attenuate the rise in the value of statistical life following a health shock



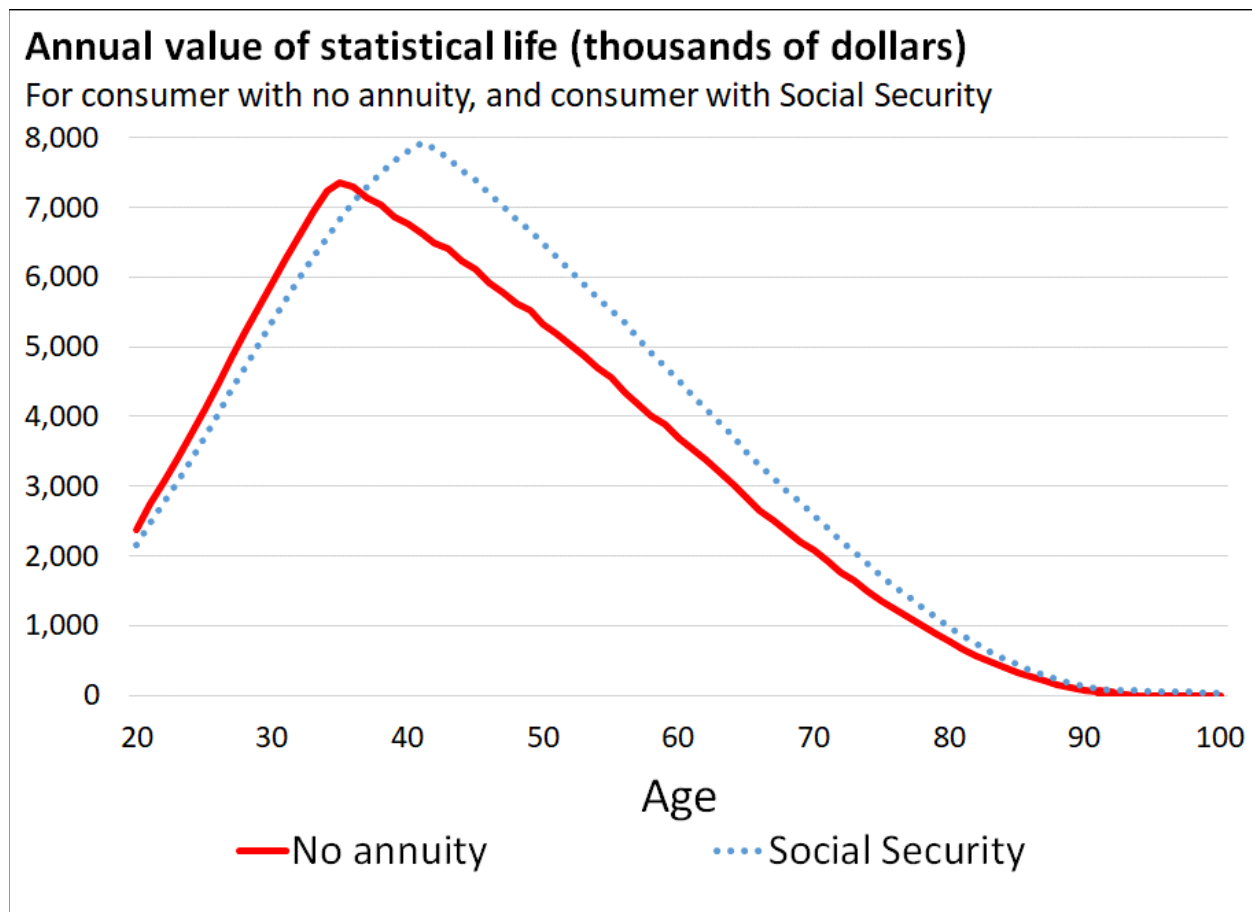
Notes: The solid red line, which reproduces the value of statistical life (VSL) estimates displayed in Figure 5, assumes that health shocks are not accompanied by medical spending shocks. The dashed blue line shows that VSL drops slightly following a health shock when we incorporate out-of-pocket medical spending shocks into the life-cycle model. The dotted black line additionally incorporates a wealth shock at age 70 that reduces the individual's wealth by 20 percent. Medical spending includes the expected effect of illness on both out-of-pocket health care costs and nursing home costs.

Figure 8. Life-cycle profiles of consumption and income when mortality is deterministic



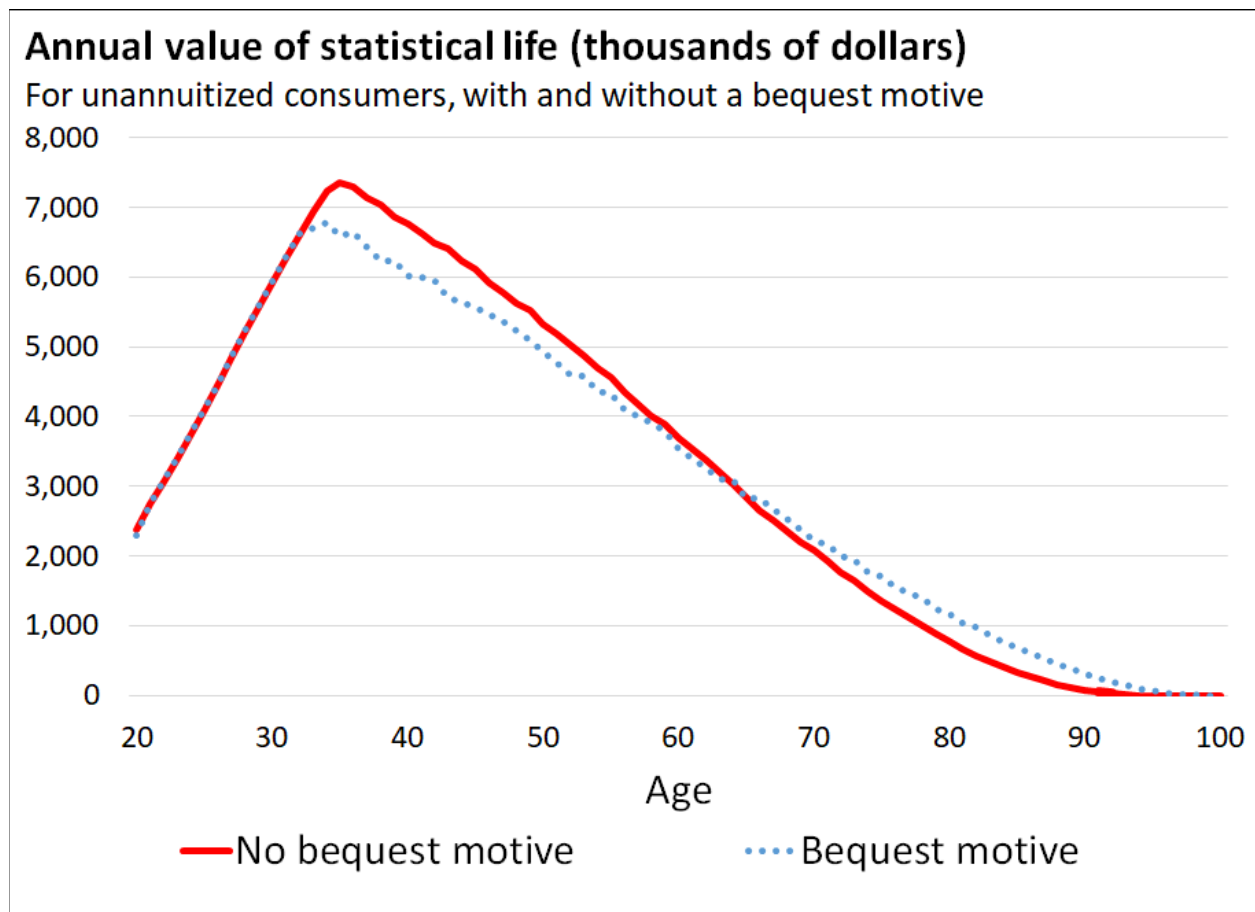
Notes: This figure plots consumption results from a life-cycle modeling exercise where mortality is deterministic. “Consumption (no annuity)” displays consumption for a consumer whose income equals her earnings. “Consumption (Social Security)” displays consumption for a consumer receiving typical Social Security benefits that are financed by an earnings tax. The net present value at age 20 of all future income is the same across both scenarios.

Figure 9. Life-cycle profile of the value of statistical life when mortality is deterministic



Notes: This figure plots the value of statistical life for the two scenarios displayed in Figure 8. The “No annuity” scenario assumes the consumer’s income equals her labor earnings. The “Social Security” scenario assumes the consumer receives typical Social Security benefits that are financed by an earnings tax. The net present value at age 20 of all future income is identical in both scenarios.

Figure 10. Similar to annuitization, a bequest motive shifts the value of statistical life towards older ages



Notes: This figure plots the value of statistical life in a setting with deterministic health and no annuity markets. The “No bequest motive” scenario is identical to the “No annuity” scenario depicted in Figure 9. The bequest motive specification is described at the end of Appendix C1.

ONLINE APPENDIX

Mortality Risk, Insurance, and the Value of Life

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Additional Tables and Figures

Appendix A: Mathematical Proofs

Appendix B: Data

Appendix C: Supporting Calculations for Quantitative Analysis

Appendix D: The Fully Annuitized Value of Life When Health Is Stochastic

Additional Tables and Figures

The value of life depends greatly on the elasticity of intertemporal substitution, which under CRRA is equal to the inverse of γ , the coefficient of relative risk aversion. The specification in the main text, which sets $\gamma = 2$, calculated that Social Security raised the aggregate value of post-1940 reductions by \$11.5 trillion (10.5 percent). Appendix Table A1 and Appendix Table A2, which both replicate Table 4 from the main text, show that varying γ over the range [1.5, 2.5] yields analogous increases that range from 8.3 percent to 14.2 percent.

The first two columns of Appendix Table A3 show that when a strong bequest motive is present, the increase in the aggregate value of life attributable to Social Security is equal to \$5.5 trillion (5.4 percent). Finally, the third column of Appendix Table A3 shows that fully annuitizing all wealth and future earnings at age 20 increases the aggregate value of life by \$17.7 trillion (16 percent), relative to a setting with no annuity markets.

Appendix Figure A1 illustrates how non-interior solutions can arise when income is survival-contingent. For convenience of exposition, the theoretical analysis presented in the main text considers the case where there is a single set of non-interior solutions, as illustrated by the left panel of Appendix Figure A1.

Appendix Table A1. Aggregate value of historical and prospective reductions in mortality when the degree of relative risk aversion is set equal to $\gamma = 2.5$ (billions of dollars)

	(1)	(2)	(3)
	No annuity	Social Security	Social Security + 50%
<u>Historical reduction:</u>			
1940-2010	\$196,619	\$225,781	\$240,641
1970-2010	\$96,449	\$111,964	\$120,030
<u>10% reduction, all ages:</u>			
All causes	\$20,646	\$24,007	\$25,804
Cancer	\$6,041	\$7,093	\$7,656
Diabetes	\$662	\$775	\$835
Heart disease	\$4,350	\$5,116	\$5,535
Homicide	\$174	\$171	\$167
Infectious diseases	\$295	\$349	\$379

Notes: These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities in retirement. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns. The degree of relative risk aversion, γ , is equal to the inverse of the elasticity of intertemporal substitution. In the main text, we assume that $\gamma = 2$.

Appendix Table A2. Aggregate value of historical and prospective reductions in mortality when the degree of relative risk aversion is set equal to $\gamma = 1.5$ (billions of dollars)

	(1)	(2)	(3)
	No annuity	Social Security	Social Security + 50%
<u>Historical reduction:</u>			
1940-2010	\$23,388	\$25,474	\$26,238
1970-2010	\$4,858	\$5,341	\$5,559
<u>10% reduction, all ages:</u>			
All causes	\$1,410	\$1,559	\$1,621
Cancer	\$155	\$171	\$178
Diabetes	\$991	\$1,106	\$1,162
Heart disease	\$57	\$55	\$53
Homicide	\$65	\$74	\$78
Infectious diseases	\$0	\$0	\$0

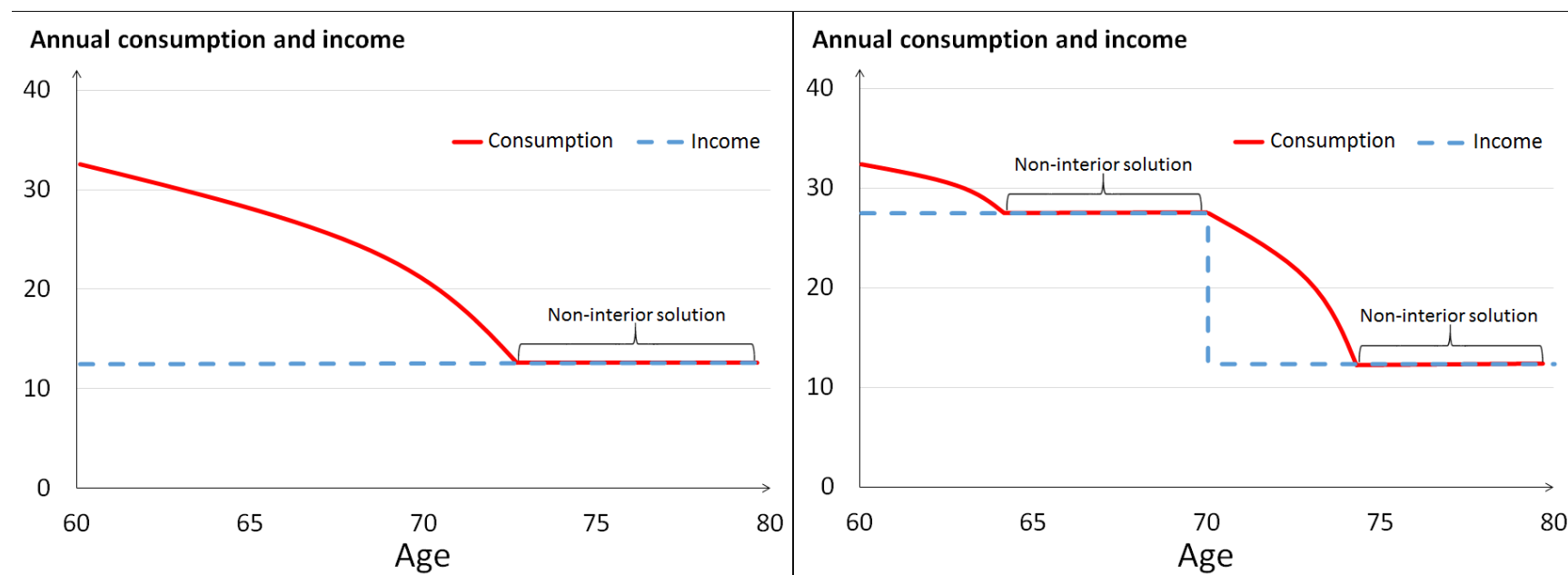
Notes: These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities in retirement. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns. The degree of relative risk aversion, γ , is equal to the inverse of the elasticity of intertemporal substitution. In the main text, we assume that $\gamma = 2$.

Appendix Table A3. Aggregate value of historical and prospective reductions in mortality when a bequest motive is present or when consumer annuitizes all future earnings (billions of dollars)

	(1)	(2)	(3)	(4)
	Bequest motive		No bequest motive	
	No annuity	Social Security	No annuity	Full annuitization
<u>Historical reduction:</u>				
1940-2010	\$92,519	\$101,081	\$96,116	\$127,030
1970-2010	\$44,877	\$49,453	\$46,695	\$60,571
<u>10% reduction, all ages:</u>				
All causes	\$9,644	\$10,683	\$9,894	\$13,403
Cancer	\$2,783	\$3,100	\$2,885	\$3,708
Diabetes	\$306	\$341	\$316	\$412
Heart disease	\$2,020	\$2,264	\$2,060	\$2,755
Homicide	\$92	\$89	\$96	\$173
Infectious diseases	\$138	\$155	\$139	\$192

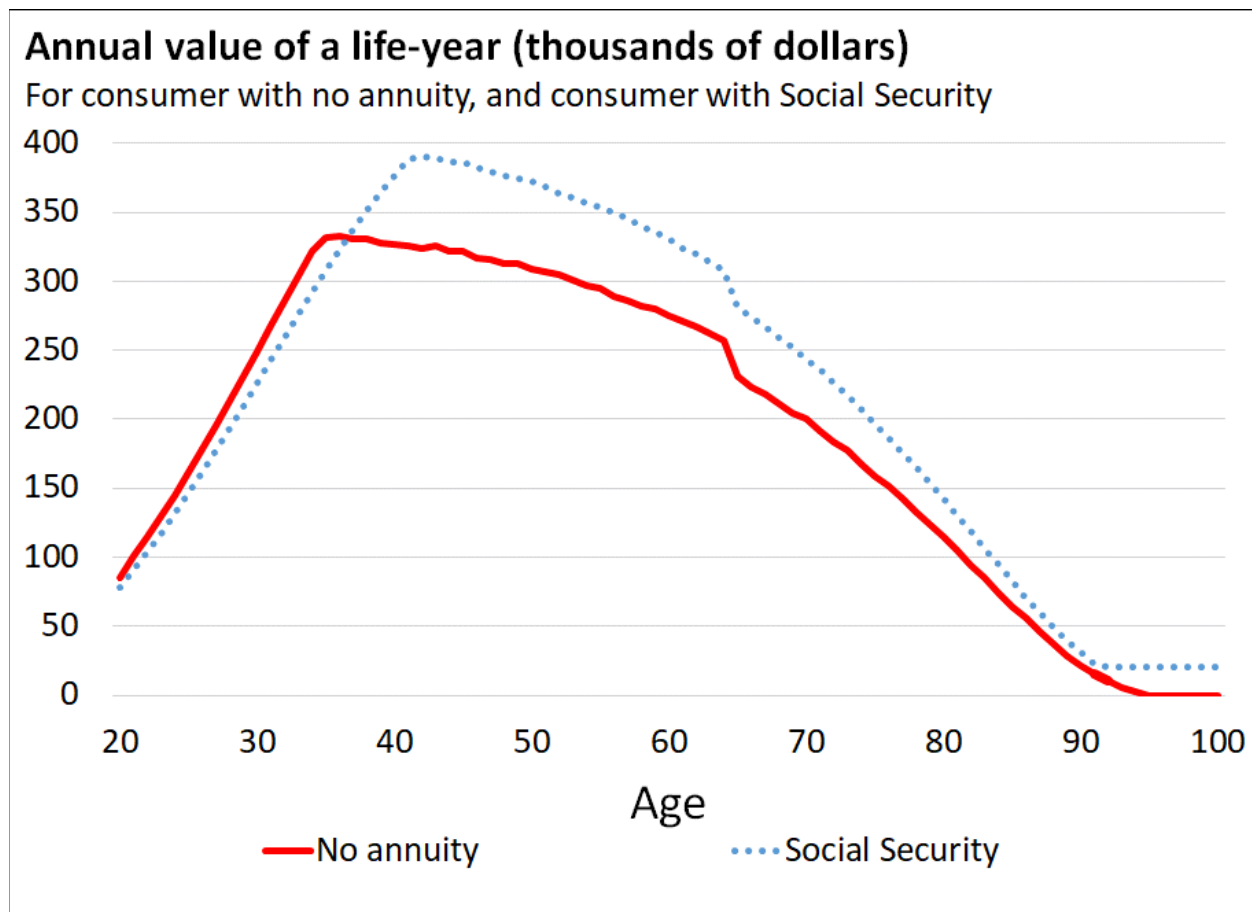
Notes: The bequest motive specification is described at the end of Appendix C1. These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Columns (1) and (3) presents estimates under the assumption that individuals have no annuities. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) replicates Column (1) from Table 4. Column (4) presents estimates under the assumption that the consumer annuitizes all future earnings in exchange for a flat annuity beginning at age 20.

Appendix Figure A1. Illustrative example: survival-contingent income can generate non-interior solutions.



Notes: The solution to the consumer's maximization problem may be non-interior in the presence of survival-contingent income. The left panel gives an example where there is one set of non-interior solutions. The right panel gives an example where there are two sets of non-interior solutions. Income, illustrated by the dashed blue line, includes both labor income and annuity income.

Appendix Figure A2. Life-cycle profile of the value of a life-year when mortality is deterministic



Notes: This figure plots the value of a life-year for the two scenarios displayed in Figure 8. The “No annuity” scenario assumes the consumer’s income equals her earnings. The “Social Security” scenario assumes the consumer receives typical Social Security benefits that are financed by an earnings tax. The net present value at age 20 of all future income is identical in both scenarios.

Appendix A: Mathematical Proofs

Proof of Lemma 1:

Recall that the transition intensities $\lambda_{ij}(t) = 0 \forall j < i$. The optimization problem in the absorbing state n is therefore a standard deterministic problem. We can contemplate a successive solution strategy by starting in state n and then moving sequentially to state $n - 1$, $n - 2$, etc. Thus, we can consider the deterministic optimization problem for an arbitrary state i by taking $V(t, w, j), j > i$, as given (exogenous):

$$V(0, W_0, i) = \max_{c_i(t)} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

subject to:

$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) - c_i(t), W_i(0) = W_0$$

Optimal consumption and wealth in state i are denoted by $c_i(t)$ and $W_i(t)$, respectively. Denote the optimal value-to-go function as:

$$\tilde{V}(u, W_i(u), i) = \max_{c_i(t)} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

Setting $\tilde{V}(t, W_i(t), i) = e^{-\rho t} \tilde{S}(i, t) V(t, W_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (12) for i . See Theorem 1 and the proof of Theorem 2 in Parpas and Webster (2013) for additional details and intuition behind this result.

QED

Proof of Lemma 2:

From (13), the marginal utility of life-extension is:

$$\begin{aligned} \left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp \left\{ - \int_0^t (\mu(s) - \varepsilon \delta(s)) + \sum_{j>i} \lambda_{ij}(s) ds \right\} \left(u(c_i^\varepsilon(t), q_i(t)) \right. \\ &\quad \left. + \sum_{j>i} \lambda_{ij}(t) V(t, W_i^\varepsilon(t), j) \right) dt \Big|_{\varepsilon=0} \\ &= \int_0^T e^{-\rho t} \left(\int_0^t \delta(s) ds \right) \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \\ &\quad + \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \right) dt \Big|_{\varepsilon=0} \end{aligned}$$

where $c_i^\varepsilon(t)$ and $W_i^\varepsilon(t)$ represent the equilibrium variations in $c_i(t)$ and $W_i(t)$ caused by this perturbation. We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to:

$$W_i(t) = W_0 e^{rt} - \int_0^t e^{r(t-s)} c_i(s) ds,$$

which implies $\frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} = - \int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds$. From the solution to the costate equation, we know that:

$$e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

Thus, we can rewrite the second term in the expression for $\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0}$ above as:

$$\begin{aligned} & \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds + \theta^{(i)} \right] e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt \\ & \quad - \int_0^T e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} ds dt \Big|_{\varepsilon=0} \\ &= \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt \\ & \quad - \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt + \int_0^T \theta^{(i)} e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} dt \Big|_{\varepsilon=0} \\ &= \theta^{(i)} \frac{\partial}{\partial \varepsilon} \underbrace{\int_0^T e^{-rt} c_i^\varepsilon(t) dt}_{W_0} \Big|_{\varepsilon=0} \\ &= 0 \end{aligned}$$

where, as in the deterministic case, the last equality follows from application of the budget constraint.

QED

Proof of Lemma 3:

The proof proceeds by induction on $i \leq n$. For the base case $i = n$, in which no state transitions are possible, the solution to the costate equation (14) simplifies to:³⁷

$$\begin{aligned} p_\tau^{(n)} &= \theta^{(n)} e^{-r\tau} = \exp \left\{ - \int_0^\tau \rho + \bar{\mu}_n(s) ds \right\} u_c(c_n(\tau), q_n(\tau)) \\ &= \theta^{(n)} e^{-r\tau} e^{-r(\tau-t)} \\ &= p_t^{(n)} e^{-r(\tau-t)} \\ &= \exp \left\{ - \int_0^t \rho + \bar{\mu}_n(s) ds \right\} u_c(c_n(t), q_n(t)) e^{-r(\tau-t)} \end{aligned}$$

³⁷ When no transitions are possible, the solution reduces to the first-order condition for consumption presented in Section II.B.

This then implies:

$$u_c(c_n(t), q_n(t)) = e^{r(\tau-t)} e^{-\rho(\tau-t)} \exp \left\{ - \int_t^\tau \bar{\mu}_n(s) ds \right\} u_c(c_n(\tau), q_n(\tau))$$

which shows that the lemma holds for $i = n$.

For the induction step, suppose the lemma is true for $j > i$, $1 \leq i \leq n-1$. For any subinterval $[0, \tau]$, the solution of the costate equation can be written as:

$$p_t^{(i)} = \left[\int_t^\tau e^{(r-\rho)s} \exp \left\{ - \int_0^s \bar{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta(\tau, i) e^{-r\tau} \quad (\text{A1})$$

where $\theta(\tau, i)$ is a constant that depends on the choice of τ and i . (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating equation (A1) at $t = \tau$ and combining with equation (15) from the main text yields:

$$p_\tau^{(i)} = \theta(\tau, i) e^{-r\tau} = \exp \left\{ - \int_0^\tau \rho + \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau))$$

which implies:

$$\theta(\tau, i) = e^{(r-\rho)\tau} \exp \left\{ - \int_0^\tau \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \quad (\text{A2})$$

Plugging equations (15) and (A2) into equation (A1) yields:

$$\begin{aligned} & u_c(c_i(t), q_i(t)) \exp \left\{ - \int_0^t \rho + \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} \\ &= \left[\int_t^\tau e^{(r-\rho)s} \exp \left\{ - \int_0^s \bar{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \\ &+ e^{-rt} e^{(r-\rho)\tau} \exp \left\{ - \int_0^\tau \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \end{aligned}$$

Since $\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} = u_c(c(s, W_i(s), j), q_j(s))$ from the first-order condition in the HJB for state j , we obtain:

$$\begin{aligned} u_c(c_i(t), q_i(t)) &= \int_t^\tau e^{(r-\rho)(s-t)} \exp \left\{ - \int_t^s \bar{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) u_c(c(s, W_i(s), j), q_j(s)) ds \\ &+ e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \end{aligned}$$

$$\begin{aligned}
&= \int_t^\tau e^{(r-\rho)(s-t)} \exp \left\{ - \int_t^s \bar{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \mathbb{E} \left[e^{(r-\rho)(\tau-s)} \exp \left\{ - \int_s^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \right] Y_s \\
&= j, W(s) = W_i(s) \Big] ds + e^{(r-\rho)(\tau-t)} \exp \left\{ - \int_t^\tau \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau)) \\
&= \mathbb{E} \left[e^{(r-\rho)(\tau-s)} \exp \left\{ - \int_t^\tau \mu(s) ds \right\} u_c(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)) \right] Y_t = i, W(t) = W_i(t) \Big]
\end{aligned}$$

where the second equality follows from the induction hypothesis.

QED

Proof of Proposition 4:

Choosing once again the Dirac delta function for $\delta(\cdot)$ in **Lemma 2** yields:

$$\begin{aligned}
\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T \left[e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) \right] dt \\
&= \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \right] Y_0 = i, W(0) = W_0
\end{aligned}$$

Dividing the result by the marginal utility of wealth at time $t = 0$ then yields the value of statistical life given by equation (16):

$$VSL(i) = \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u(c(0), q_{Y_0}(0))} dt \right] Y_0 = i, W(0) = W_0$$

Applying Lemma 3 for $t = 0$ allows us to rewrite VSL as:

$$\begin{aligned}
VSL(i) &= \mathbb{E} \left[\int_0^T e^{-\rho t} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E} \left[e^{(r-\rho)t} \exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \right] Y_0 = i, W(0) = W_0} dt \right] Y_0 = i, W(0) = W_0 \\
&= \mathbb{E} \left[\int_0^T e^{-rt} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \right] Y_0 = i, W(0) = W_0} dt \right] Y_0 = i, W(0) = W_0
\end{aligned}$$

Exchanging expectation and integration then yields:

$$VSL(i) = \int_0^T e^{-rt} v(i, t) dt$$

where the value of a life-year, $v(i, t)$, is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i, t) = \frac{\mathbb{E} \left[S(t) u(c(t), q_{Y_t}(t)) \right] Y_0 = i, W(0) = W_0}{\mathbb{E} \left[S(t) u_c(c(t), q_{Y_t}(t)) \right] Y_0 = i, W(0) = W_0}$$

QED

Proof of Proposition 5:

Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Because we hold quality of life constant, we omit $q_i(t)$ in the notation below in order to keep the presentation concise.

We want to prove that $c_2(0) \geq c_1(0)$. Assume by way of contradiction that $c_2(0) < c_1(0)$. We will show that this implies $c_2(t) < c_1(t)$ for all $t > 0$, which is a contradiction since the feasible consumption plan $c_1(\cdot)$ dominates $c_2(\cdot)$.

We proceed by inductively constructing a sequence $0 < t_1 < t_2 \dots$ where for each element in the sequence:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p_{t_i}^{(1)} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s) ds\right\} p_{t_i}^{(2)} \end{aligned}$$

To construct the sequence, for the base case $i = 1$, we first note that from the first-order condition (15), we obtain:

$$p_0^{(1)} = u_c(c_1(0)) < u_c(c_2(0)) = p_0^{(2)}$$

The costate equation (14) then implies:

$$\begin{aligned} \dot{p}_0^{(1)} &= -p_0^{(1)}r - \lambda_{12}(0)u_c(c_2(0)) \\ &= -p_0^{(1)}\left[r + \lambda_{12}(0)\frac{u_c(c_2(0))}{u_c(c_1(0))}\right] \\ &< -p_0^{(1)}[r + \lambda_{12}(0)] = \frac{\partial g(t)}{\partial t}\Big|_{t=0} \end{aligned}$$

where $g(t) = p_0^{(1)} \exp\left\{-\int_0^t r + \lambda_{12}(s) ds\right\}$. Hence, there exists a $t_1 > t_0 = 0$ such that:

$$p_t^{(1)} \leq g(t) < p_0^{(2)} \exp\left\{-\int_0^t (r + \lambda_{12}(s)) ds\right\} = p_t^{(2)} \exp\left\{-\int_0^t \lambda_{12}(s) ds\right\}, 0 \leq t \leq t_1$$

which together with the first-order condition (15) implies:

$$e^{-\rho t} \exp\left\{-\int_0^t (\bar{\mu}_1(s) + \lambda_{12}(s)) ds\right\} u_c(c_1(t)) < e^{-\rho t} \exp\left\{-\int_0^t (\bar{\mu}_2(s) + \lambda_{12}(s)) ds\right\} u_c(c_2(t)), 0 \leq t \leq t_1$$

so that $c_1(t) > c_2(t)$, $0 \leq t \leq t_1$. This in turn implies $W_1(t_1) \leq W_2(t_1)$.

For the induction step, suppose that the following properties also hold for $i \geq 1$:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p_{t_i}^{(1)} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s) ds\right\} p_{t_i}^{(2)} \end{aligned}$$

The induction hypothesis implies:

$$c(t_i, W_1(t_i), 2) \leq c(t_i, W_2(t_i), 2) = c_2(t_i) < c_1(t_i)$$

so that:

$$\begin{aligned}
\dot{p}_{t_i}^{(1)} &= -p_{t_i}^{(1)} r - e^{-\rho t_i} \tilde{S}(1, t_i) \lambda_{12}(t_i) u(c(t_i, W_1(t_i), 2)) \\
&= -p_{t_i}^{(1)} \left[r + \lambda_{12}(t_i) \underbrace{\frac{u_c(c(t_i, W_1(t_i), 2))}{u_c(c_1(t_i))}}_{>1} \right] \\
&< -p_{t_i}^{(1)} [r + \lambda_{12}(t_i)] = \frac{\partial \tilde{g}(t)}{\partial t} \Big|_{t=0}
\end{aligned}$$

with $\tilde{g}(t) = p_{t_i}^{(1)} \exp \left\{ - \int_{t_i}^t (r + \lambda_{12}(s)) ds \right\}$. Hence, there exists a $t_{i+1} > t_i$ such that:

$$\begin{aligned}
p_t^{(1)} &\leq \tilde{g}(t) \\
&< \exp \left\{ - \int_0^{t_i} \lambda_{12}(s) ds \right\} p_{t_i}^{(2)} \exp \left\{ - \int_{t_i}^t (r + \lambda_{12}(s)) ds \right\} = p_t^{(2)} \exp \left\{ - \int_0^t \lambda_{12}(s) ds \right\}, t_i \leq t \leq t_{i+1}
\end{aligned}$$

In particular, again with the first-order condition (15) for all $t_i \leq t \leq t_{i+1}$:

$$\exp \left\{ - \int_0^t (\bar{\mu}_1(s) + \lambda_{12}(s)) ds \right\} u_c(c_1(t)) < \exp \left\{ - \int_0^t (\bar{\mu}_2(s) + \lambda_{12}(s)) ds \right\} u_c(c_2(t))$$

which in turn implies $u_c(c_1(t)) < u_c(c_2(t))$ and $c_2(t) < c_1(t)$. Once again, this implies $W_1(t_{i+1}) \leq W_2(t_{i+1})$.

Thus, we have proven the existence of the sequence. We then obtain $c_2(t) < c_1(t) \forall t$ by noting that $\{t_i\}_{i \geq 0}$ strictly increases due to the uniformly boundedness condition on $\lambda_{12}(t)$, which is the desired contradiction.

We note that this proof implies that the consumption paths $c_1(t)$ and $c_2(t)$ cross (at most) once. As soon as $c_1(t)$ exceeds $c_2(t)$ for some time t_0 , $c_1(t)$ will exceed $c_2(t)$ for $t > t_0$. However, we have that $c_2(t)$ exceeds $c_1(t)$ prior to t_0 . In particular, consumption jumps up at the transition point. See Figure 2 for an illustration.

QED

Proof of Proposition 6:

Without loss of generality, consider the case $t = 0$, as depicted in Figure 2. Under our assumptions, from equation (17) and **Proposition 5** it is clear that $c_1(t)$ and $c_2(t)$ are decreasing, $c_2(0) \geq c_1(0)$, $c_2(t) \geq c_1(t)$ for $t \leq t_0$, and $c_2(t) \leq c_1(t)$ for $t > t_0$. Making use of the assumption that no state transitions occur for $t > 0$, we have that:

$$VSL(2,0) = \int_0^T e^{-rt} \frac{S_2(t) u(c_2(t))}{S_2(t) u_c(c_2(t))} dt = \int_0^T e^{-rt} \frac{u(c_2(t))}{u_c(c_2(t))} dt$$

and:

$$VSL(1,0) = \int_0^T e^{-rt} \frac{u(c_1(t))}{u_c(c_1(t))} dt$$

Let $Y(x) \equiv \frac{u(x)}{u_c(x)}$. Under the stated assumptions on preferences, we have that:

$$Y'(x) = 1 - \frac{u(x)u_{cc}(x)}{(u_c(x))^2} \geq 0,$$

$$Y''(x) = \frac{2(u_{cc}(x))^2 u(x) - u_c^2(x)u_{cc}(x) - u_c(x)u(x)u_{ccc}(x)}{(u_c(x))^3} \geq 0$$

Employing Taylor's theorem then implies that for some $\xi(t)$ that lies in-between $c_1(t)$ and $c_2(t)$:

$$\begin{aligned}
VSL(2,0) &= \int_0^T e^{-rt} Y(c_2(t)) dt \\
&= \int_0^T e^{-rt} \left[Y(c_1(t)) + [c_2(t) - c_1(t)] Y'(c_1(t)) + \underbrace{\frac{1}{2} [c_2(t) - c_1(t)]^2 Y''(\xi(t))}_{\geq 0} \right] dt \\
&\geq \int_0^T e^{-rt} Y(c_1(t)) dt + \int_0^{t_0} e^{-rt} Y'(c_1(t)) \underbrace{[c_2(t) - c_1(t)]}_{\geq 0} dt \\
&\quad + \int_{t_0}^T e^{-rt} Y'(c_1(t)) \underbrace{[c_2(t) - c_1(t)]}_{\leq 0} dt \\
&\geq \int_0^T e^{-rt} Y(c_1(t)) dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0)) [c_2(t) - c_1(t)] dt \\
&\quad + \int_0^{t_0} e^{-rt} Y'(c_1(t_0)) [c_2(t) - c_1(t)] dt \\
&= \int_0^T e^{-rt} Y(c_1(t)) dt + Y'(c_1(t_0)) \underbrace{\left[\int_0^T e^{-rt} c_2(t) dt - \int_0^T e^{-rt} c_1(t) dt \right]}_{=0} \\
&= \int_0^T e^{-rt} Y(c_1(t)) dt \\
&= VSL(1,0)
\end{aligned}$$

where the final step follows from the budget constraint.

QED

Proof of Proposition 7:

From (13), the marginal utility of preventing an illness or death is:

$$\begin{aligned}
\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp \left\{ - \int_0^t (\bar{\mu}_i(s) - \varepsilon \delta_{i,N+1}(t)) + \sum_{j>i} (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right\} \left(u(c_i^\varepsilon(t), q_i(t)) \right. \\
&\quad \left. + \sum_{j>i} (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, W_i^\varepsilon(t), j) \right) dt \Big|_{\varepsilon=0}
\end{aligned}$$

$$\begin{aligned}
&= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \\
&\quad + \int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i^\varepsilon(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \right) dt
\end{aligned}$$

Following the same argument as in the VSL case, the second term in the last equality is equal to 0.

QED

In Section III.B, we claimed that the value of treatment will be higher than the value of prevention if VSL rises following an illness that reduces life expectancy by one-half or more. We provide a formal proposition and proof below.

Proposition 8:

Consider a two-state setting with assumptions set out in **Proposition 6**. Denote life expectancy in healthy state 1 as L_1 , and life expectancy in sick state 2 as $L_2 < L_1$. Define the per-year value of treatment in the sick state as $\frac{VSL(2)}{L_2}$, and the per-year value of preventing a transition from the healthy state to the sick state as $\frac{VSI(1,2)}{L_1 - L_2}$. If $L_2 < \frac{L_1}{2}$, then the per-year value of treatment is greater than the per-year value of prevention.

Proof of Proposition 8:

Straightforward algebra yields:

$$\begin{aligned}
\frac{VSI(1,2)}{L_1 - L_2} &= \frac{VSL(1)}{L_1 - L_2} - \frac{VSL(2) \frac{u_c(c_2(0), q_2(0))}{u_c(c_1(0), q_1(0))}}{L_1 - L_2} \\
&< \frac{VSL(2)}{L_1 - L_2} - \frac{VSL(2) \frac{u_c(c_2(0), q_2(0))}{u_c(c_1(0), q_1(0))}}{L_1 - L_2} \\
&= \frac{VSL(2)}{L_1 - L_2} \left(1 - \underbrace{\frac{u_c(c_2(0), q_2(0))}{u_c(c_1(0), q_1(0))}}_{<1} \right) < \frac{VSL(2)}{L_1 - L_2} < \frac{VSL(2)}{L_2}
\end{aligned}$$

The first inequality following from application of **Proposition 6**. Application of **Proposition 5** implies that $\frac{u_c(c_2(0), q_2(0))}{u_c(c_1(0), q_1(0))} < 1$. The final inequality follows if $L_2 < \frac{L_1}{2}$.

QED

Proof of Proposition 9 and Corollary 10:

Our goal is to derive expressions for VSL and VSI when annuity markets are incomplete and the consumer is endowed with state-dependent life-cycle income. We first consider in part (i) the case with life-cycle income only. We then introduce incomplete annuity markets later in part (ii) of the proof.

(i) No annuity markets

Denote the consumer's income in state i at time t as $m_i(t)$. The consumer's maximization problem is again (11), but it is now subject to different constraints:

$$\begin{aligned} W(0) &= W_0, W(t) \geq 0, W(T) = 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) - c(t) \end{aligned}$$

Once again, we solve this stochastic finite-horizon optimization problem by reformulating it as a deterministic optimization problem. Specifically, we consider equation (13), subject to:

$$\begin{aligned} W_i(0) &= W_0, \\ \frac{\partial W_i(t)}{\partial t} &= rW_i(t) + m_i(t) - c_i(t) \end{aligned}$$

The present-value Hamiltonian corresponding to this deterministic problem is:

$$H(W_i(t), c_i(t), p_t^{(i)}, \psi_t^{(i)}) = e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) + p_t^{(i)} [rW_i(t) + m_i(t) - c_i(t)] + \psi_t^{(i)} W_i(t)$$

where $p_t^{(i)}$ is the costate variable for the wealth dynamics in state i and $\psi_t^{(i)}$ is the multiplier for the wealth constraint. The necessary first-order conditions for the costate variable p , consumption, and the wealth constraint are:

$$\begin{aligned} \dot{p}_t^{(i)} &= -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)} r - e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} - \psi_t^{(i)} \\ p_t^{(i)} &= e^{-\rho t} \tilde{S}(i, t) u_c(c_i(t), q_i(t)) \\ \psi_t^{(i)} &\geq 0, \psi_t^{(i)} W_i(t) = 0 \end{aligned}$$

Following Proposition 1 in Leung (1994), one can show the following: the Hamiltonian is regular on $[0, T)$, so optimal consumption $c_i(t)$ is everywhere continuous; the state-variable inequality constraint is of first-order, so $p_t^{(i)}$ is everywhere continuous; and optimal consumption $c_i(t)$ is continuously differentiable when $W_i(t) > 0$ (i.e., when the wealth constraint is not binding).

First, consider the case when $W_i(t) > 0$. Differentiating the first-order condition for consumption with respect to t , plugging in the result for the costate equation and its solution, and then rearranging yields the rate of change in life-cycle consumption. This rate of change, $\frac{\dot{c}_i}{c_i}$, is identical to the one for the uninsured case (see equation 17), and will be negative under the conditions outlined in footnote 18 of the main text.

The presence of life-cycle income introduces the possibility of multiple sets of non-interior solutions (e.g., right panel of Appendix Figure A1). Modeling these scenarios is possible, but cumbersome. As discussed in the main text, we therefore restrict ourselves to considering the case with a single set of non-interior solutions (i.e., a single “kink point”, see left panel of Appendix Figure A1). A sufficient (but not necessary) set of assumptions that delivers this case is given in footnote 18 of the main text. We employ those assumptions in the following Lemma, which establishes the existence of a single kink point, T_i , where the consumer runs out of wealth.

Lemma:

Assume $m_i(t)$ is constant. Then there must exist a T_i such that (1) $W_i(t) = 0$ and $c_i(t) = m_i(t)$ for $t \geq T_i$; and (2) $c_i(t) > m_i(t)$ for $t < T_i$. The solution to the costate equation on $[0, T_i]$ is thus:

$$p_t^{(i)} = \left[\int_t^{T_i} e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant.

Proof:

Under the maintained assumptions in footnote 18, $\frac{c_i}{c_i} < 0$ whenever $W_i(t) > 0$. Following the same argument as in Proposition 2 of Leung (1994), there is a smallest T_i such that $W_i(t) = 0$ on $[T_i, T]$ and, thus, $c_i(t) = m_i(t)$ on $[T_i, T]$. Since this is the smallest such T_i , there exists an interval (\underline{T}_i, T_i) such that $W_i(t) > 0$ and $c_i(t_0) > m_i(t_0)$ for a t_0 in the vicinity of T_i . Now assume $W_i(\underline{T}_i) = 0$. Then there exists a t_1 in the vicinity of \underline{T}_i such that $c_i(t_1) < m_i(t_1)$. This is a contradiction, since $m_i(t)$ is constant and $c_i(t)$ is decreasing whenever $W_i(t) > 0$. Hence $W_i(t) > 0$ on $[0, T_i)$ and $c_i(t) > m_i(t)$ for $t \in [0, T_i)$. As in the main text, the solution to the costate equation can be obtained using the variation of the constant method.

QED

Because the value of statistical illness (VSI) is a generalization of the value of statistical life (VSL), we will focus on deriving an expression for VSI. As in Section III.B of the main text, let $\delta_{ij}(t)$, $j \leq N$, be a perturbation on the transition intensity, $\lambda_{ij}(t)$, and let $\delta_{i,N+1}(t)$ be a perturbation on the mortality rate, $\bar{\mu}_i(t)$, $i \leq N$, where $\sum_{j=i+1}^{N+1} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t (\bar{\mu}_i(s) - \varepsilon \delta_{i,N+1}(s)) + \sum_{j=i+1}^N (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right], \text{ where } \varepsilon > 0$$

From equation (13), we obtain:

$$\begin{aligned} \frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \left[\int_0^{T_i(\varepsilon)} e^{-\rho t} \tilde{S}^\varepsilon(i, t) \left(u(c_i^\varepsilon(t), q_i(t)) + \sum_{j=i+1}^N (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, W_i^\varepsilon(t), j) \right) dt \right. \\ &\quad \left. + \int_{T_i(\varepsilon)}^T e^{-\rho t} \tilde{S}^\varepsilon(i, t) \left(u(m_i(t), q_i(t)) + \sum_{j=i+1}^N (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) V(t, 0, j) \right) dt \right] \Big|_{\varepsilon=0} \\ &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j=i+1}^N \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j=i+1}^N \delta_{ij}(t) V(t, W_i(t), j) \right] dt \\ &\quad + \underbrace{\int_0^{T_i} e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} \right) dt}_0 \end{aligned}$$

where the second term in the last equality is equal to 0:

$$\int_0^{T_i} e^{-\rho t} \tilde{S}(i, t) \left(u_c(c_i(t), q_i(t)) \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} \right) dt$$

$$\begin{aligned}
&= \int_0^{T_i} p_t^{(i)} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + e^{-\rho t} \tilde{S}(i, t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \left[- \int_0^t e^{r(t-s)} \frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon} \Big|_{\varepsilon=0} ds \right] dt \\
&= \int_0^{T_i} \theta^{(i)} e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} dt + \int_0^{T_i} \int_t^{T_i} e^{(r-\rho)s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} dt \\
&\quad - \int_0^{T_i} \int_t^{T_i} e^{-\rho s} \tilde{S}(i, s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds e^{rs} e^{-rt} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} dt \\
&= \theta^{(i)} \frac{\partial}{\partial \varepsilon} \int_0^{T_i} e^{-rt} c_i^\varepsilon(t) dt \Big|_{\varepsilon=0} \\
&= 0
\end{aligned}$$

The final equality follows because $W_i(T_i) = 0$ (by definition), which in turn implies $0 = W_0 + \int_0^{T_i} e^{-rt} m_i(t) dt - \int_0^{T_i} e^{-rt} c_i^\varepsilon(t) dt$, so that differentiation yields zero. Thus we obtain:

$$\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} = \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j=i+1}^N \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j=i+1}^N \delta_{ij}(t) V(t, W_i(t), j) \right] dt \quad (A3)$$

Dividing by the marginal utility of wealth yields the value of life-extension. Choosing the Dirac delta function for $\delta_{i,N+1}(\cdot)$ yields VSL, and choosing the Dirac delta function for $\delta_{ij}(\cdot), j < N + 1$, yields VSI:

$$\begin{aligned}
VSL(i) &= \frac{V(0, W(0), i)}{u_c(c_i(0), q_i(0))} \\
VSI(i, j) &= \frac{V(0, W(0), i) - V(0, W(0), j)}{u_c(c_i(0), q_i(0))}
\end{aligned}$$

Unlike the uninsured case, we cannot express VSI or VSL as functions of the value of a life-year, $v(i, t)$.

(ii) Incomplete annuity markets

Now, we introduce a one-time opportunity at time $t = 0$ to purchase a flat lifetime annuity at a level $\bar{m}_{y_0} \geq 0$ with a price markup $\xi \geq 0$. Let $a(t, i) = \mathbb{E} \left[\int_t^T e^{-r(s-t)} \exp\{-\int_t^s \mu(u) du\} ds \mid Y_t = i \right]$ be the expected value of a one-dollar annuity purchased at time t in state i . Note that for any given annuity, \bar{m}_i , the consumer's problem can be mapped to the no-annuity case in part (i) above by setting the constraints equal to:

$$\begin{aligned}
W_i(0) &= W_0 - (1 + \xi) \bar{m}_i a(0, i), \\
\frac{\partial W_i(t)}{\partial t} &= rW_i(t) + \bar{m}_i + m_i(t) - c_i(t)
\end{aligned}$$

Solving for the optimal fixed annuity then becomes a straightforward static optimization problem: $\bar{m}_i^* = \arg \max_{\bar{m}_i} V(0, W_i(0), \bar{m}_i, i)$.

The optimal annuity must satisfy the necessary first-order condition:

$$\frac{\partial V(0, W_i(0), \bar{m}_i, i)}{\partial \bar{m}_i} = \frac{\partial V(0, W_i(0), \bar{m}_i, i)}{\partial W(0)} (1 + \xi) a(0, i) \quad (A4)$$

Because the consumer may favor a non-flat optimal consumption profile, the optimal level of annuitization is likely to be partial even if the markup ξ is equal to zero. However, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and income are constant.³⁸

The value of an annuity depends on a consumer's expected future survival. Life-extension affects the value and cost of a given annuity, and may also affect the level of the optimal annuity. Thus, the effect of the mortality rate perturbation on the marginal utility of life-extension is:

$$\left. \frac{\partial V(0, W_i^\varepsilon(0), \bar{m}_i^\varepsilon, i)}{\partial \varepsilon} \right|_{\varepsilon=0} = (A3) + \left. \frac{\partial V}{\partial \bar{m}_i} \frac{\partial \bar{m}_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} + \left. \frac{\partial V}{\partial W_i(0)} \frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0}$$

where the first term on the right-hand side is equal to equation (A3) derived in part (i) above for the case with life-cycle income but no annuity. Note that:

$$\begin{aligned} \left. \frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \left(-(1+\xi) \bar{m}_i^\varepsilon \int_0^T \tilde{S}^\varepsilon(i, t) e^{-rt} \left[1 + \sum_{j=i+1}^N (\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)) a(t, j) \right] dt \right) \\ &= -(1+\xi) \left. \frac{\partial \bar{m}_i^\varepsilon}{\partial \varepsilon} \right|_{\varepsilon=0} a(0, i) - (1+\xi) \bar{m}_i \int_0^T e^{-rt} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(1 + \sum_{j=i+1}^N \lambda_{ij}(t) a(t, j) \right) - \sum_{j=i+1}^N \delta_{ij}(t) a(t, j) \right] dt \end{aligned}$$

Combining this with the first-order condition (A4) implies that:

$$\begin{aligned} \left. \frac{\partial V}{\partial \bar{m}_i} \frac{\partial \bar{m}_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} + \left. \frac{\partial V}{\partial W_i(0)} \frac{\partial W_i^\varepsilon(0)}{\partial \varepsilon} \right|_{\varepsilon=0} &= - \frac{\partial V}{\partial W_i(0)} (1+\xi) \bar{m}_i \int_0^T e^{-rt} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(1 + \sum_{j=i+1}^N \lambda_{ij}(t) a(t, j) \right) - \sum_{j=i+1}^N \delta_{ij}(t) a(t, j) \right] dt \end{aligned}$$

Thus the marginal utility of life-extension is equal to:

$$\begin{aligned} \left. \frac{\partial V}{\partial \varepsilon} \right|_{\varepsilon=0} &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j=i+1}^N \lambda_{ij}(t) V(t, W_i(t), \bar{m}_i, j) \right) - \sum_{j=i+1}^N \delta_{ij}(t) V(t, W_i(t), \bar{m}_i, j) \right] dt \\ &\quad - \frac{\partial V}{\partial W_i(0)} (1+\xi) \bar{m}_i \int_0^T e^{-rt} \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(1 + \sum_{j=i+1}^N \lambda_{ij}(t) a(t, j) \right) - \sum_{j=i+1}^N \delta_{ij}(t) a(t, j) \right] dt \end{aligned}$$

The marginal utility of wealth, $\partial V / \partial W_i(0)$, is equal to $u_c(c_i(0), q_i(0))$ when the solution is interior. Dividing by the marginal utility of wealth and rearranging yields the marginal value of life-extension:

$$\begin{aligned} \left. \frac{\partial V / \partial \varepsilon}{\partial V / \partial W} \right|_{\varepsilon=0} &= \int_0^T \tilde{S}(i, t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left[\frac{e^{-\rho t} u(c_i(t), q_i(t)) + \sum_{j=i+1}^N \lambda_{ij}(t) V(t, W_i(t), \bar{m}_i, j)}{u_c(c_i(0), q_i(0))} \right] \right. \\ &\quad \left. - (1+\xi) \bar{m}_i e^{-rt} \left(1 + \sum_{j=i+1}^N \lambda_{ij}(t) a(t, j) \right) \right] - \sum_{j=i+1}^N \delta_{ij}(t) \left(\frac{V(t, W_i(t), \bar{m}_i, j)}{u_c(c_i(0), q_i(0))} - (1+\xi) \bar{m}_i e^{-rt} a(t, j) \right) \right] dt \end{aligned}$$

Choosing the Dirac delta function for $\delta_{i, N+1}(\cdot)$ yields:

³⁸ Even in the case of full annuitization, the first-order condition (A4) holds with strict equality since the consumer is indifferent between an increase in the annuity level or a proportionate increase in baseline wealth.

$$\begin{aligned}
VSL(i) &= \frac{V(0, W_i(0), \overline{m}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi)\overline{m}_i \int_0^T \tilde{S}(i, t) e^{-rt} \left(1 + \sum_{j>i} \lambda_{ij}(s) a(t, j) \right) dt \\
&= \frac{V(0, W_i(0), \overline{m}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi)\overline{m}_i a(0, i)
\end{aligned}$$

Likewise, choosing the Dirac delta function for $\delta_{ij}(\cdot), j < N + 1$, yields:

$$VSI(i, j) = \left(\frac{V(0, W_i(0), \overline{m}_i, i)}{u_c(c_i(0), q_i(0))} - (1 + \xi)a(0, i)\overline{m}_i \right) - \left(\frac{V(0, W_i(0), \overline{m}_i, j)}{u_c(c_i(0), q_i(0))} - (1 + \xi)a(0, j)\overline{m}_i \right)$$

QED

Appendix B: Data

B1. Mortality, earnings, and quality of life by age

The empirical exercise presented in Section IV.C applies a deterministic life-cycle model to data on mortality, earnings, and quality of life.

We obtain age-specific mortality rates from the Human Mortality Database (www.mortality.org).

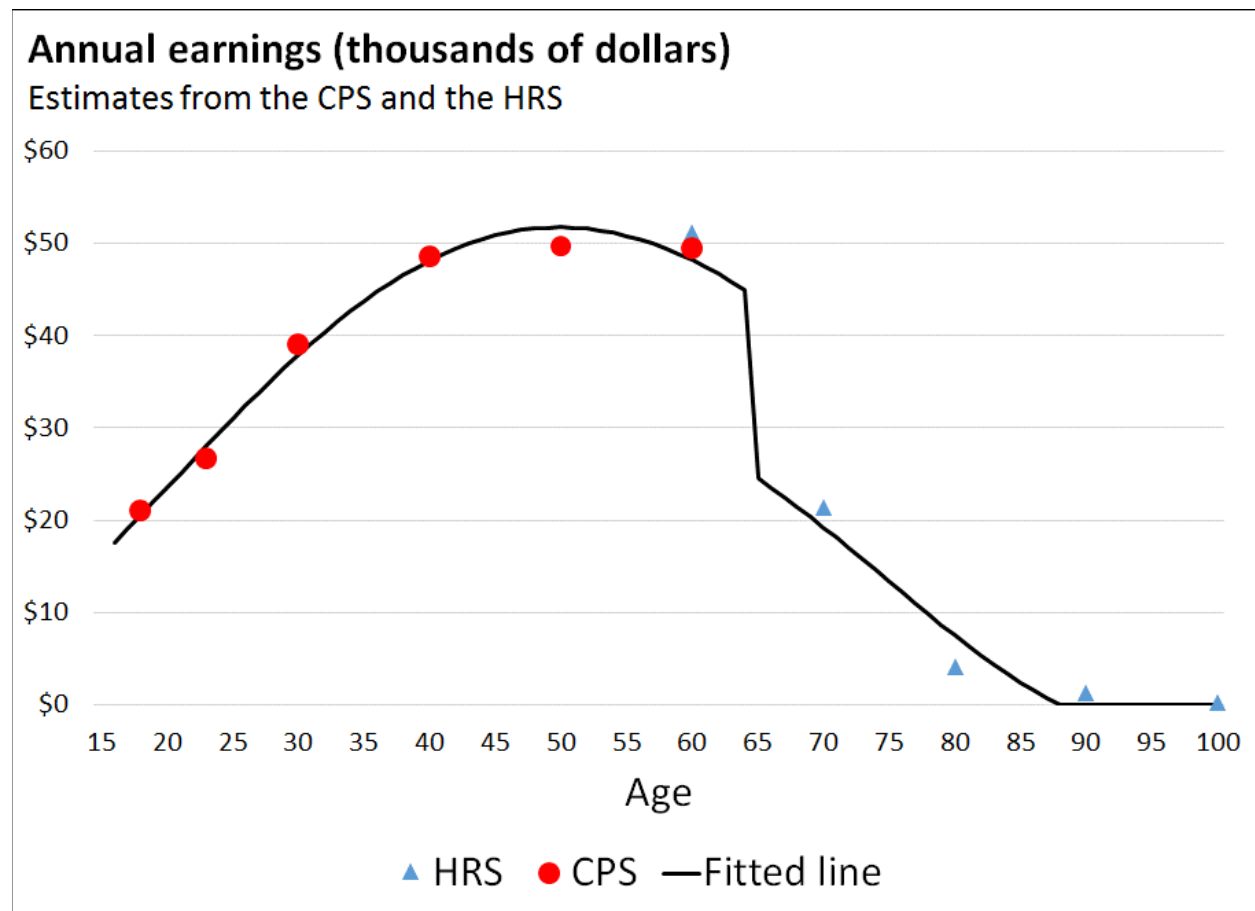
We obtain earnings data for employed individuals under the age of 65 from the 2016 Current Population Survey (CPS).³⁹ We also obtain earnings data for respondents over the age of 55 from the 2014 Health and Retirement Study (HRS). For both surveys, the data represent earnings before taxes and other deductions, and include wages, salaries, and tips. The HRS earnings data also include self-employment income. (The CPS data exclude self-employed individuals.)

The CPS earnings data are binned into the following age groups: 16-19, 20-24, 25-34, 35-44, 45-54, and 55-64. We collapse the HRS earnings data into the following age groups: 55-64, 65-74, 75-84, 85-94, and 95-104. The resulting estimates are plotted in Appendix Figure B1. We smooth the data by fitting it to a quartic polynomial in age, and include an indicator variable for ages over 65. The dependent variable in the regression is the CPS earnings estimate for ages under 65, and the HRS estimate for ages over 65. Finally, we constrain the fitted prediction to be non-negative.

We obtain nationally representative data on quality of life from the 2000-2003 Medical Expenditure Panel Surveys. These surveys measure quality of life using the EQ-5D, an index typically reported on a scale from 0 to 1, where 0 indexes death and 1 indexes perfect health. We smooth the data by fitting it to a quartic polynomial in age. The resulting estimates are plotted in Appendix Figure B2.

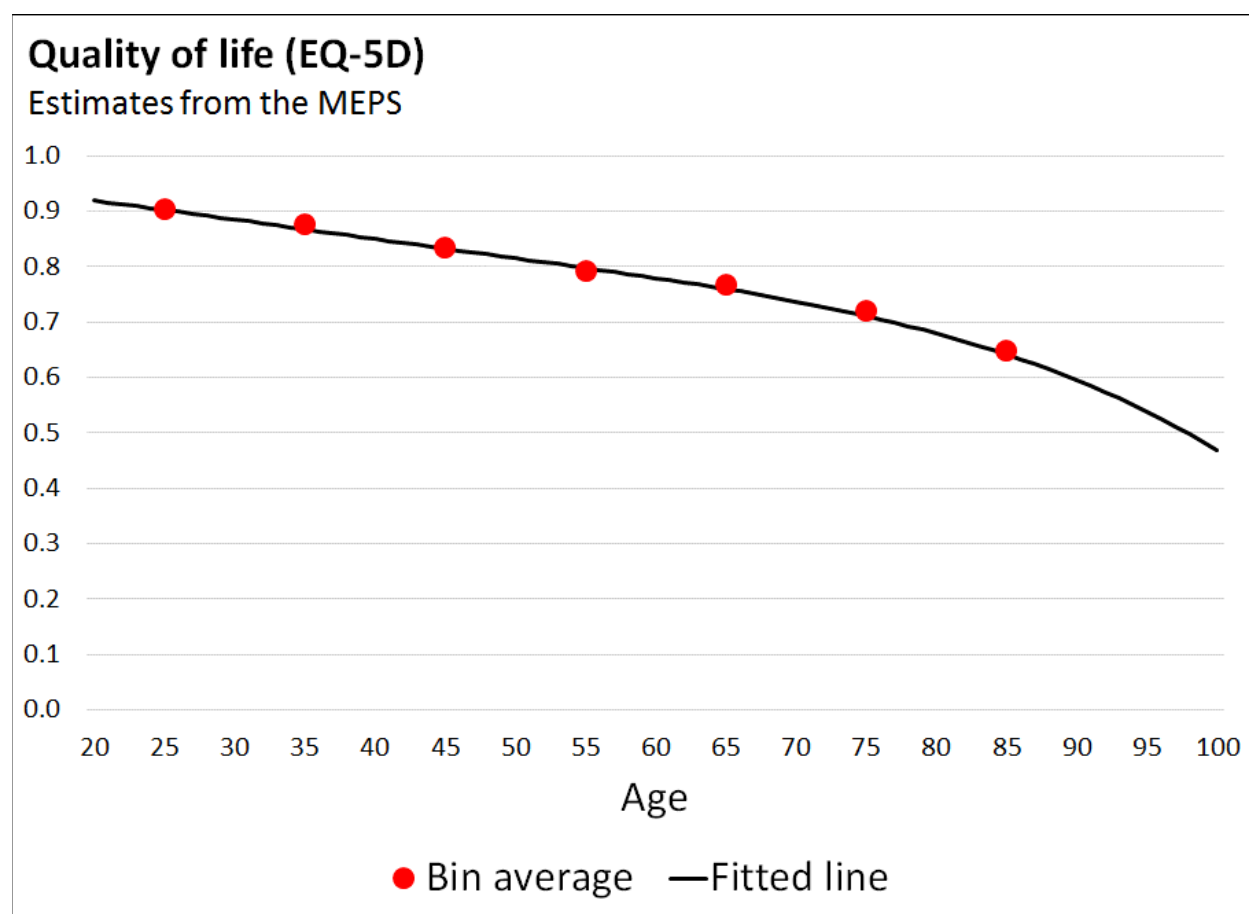
³⁹ These data are available at <http://data.bls.gov/pdq/querytool.jsp?survey=le>.

Appendix Figure B1. Annual earnings estimates from CPS and HRS



Notes: This figure plots annual earnings by midpoint of age group as estimated by the 2016 Current Population Survey (CPS) for respondents under age 65, and as estimated by the 2014 Health and Retirement Study (HRS) for respondents over age 55. The fitted line corresponds to a regression of annual earnings on a quartic polynomial in age and an indicator equal to 1 for ages 65 and over. The dependent variable in that regression, annual earnings, corresponds to CPS estimates for ages under 65 and HRS estimates for ages over 65.

Appendix Figure B2. Quality of life estimates from MEPS



Notes: This figure plots quality of life estimates from the 2000-2003 Medical Expenditure Panel Surveys (MEPS). Quality of life is measured using the EuroQol five dimensions questionnaire (EQ-5D). The fitted line corresponds to predicted values from an individual-level regression of quality of life on a quartic polynomial in age. Estimates are weighted using the MEPS sampling weights.

B2. Mortality, quality of life, and medical spending by health state and age

The empirical exercise presented in Section IV.B applies a stochastic life-cycle model to data on mortality, quality of life, and medical spending that varies across 20 different health states. We obtain these data from the Future Elderly Model (FEM).

The FEM follows Americans aged 50 years and older and projects their health and medical spending over time. It has been used by a variety of researchers and policy analysts to understand the future implications of population aging, health trends, new medical technologies, and possible health policy interventions in the US, Europe, and Asia (Goldman et al. 2005; Lakdawalla, Goldman, and Shang 2005; Lakdawalla et al. 2008; Goldman et al. 2009; Goldman et al. 2010; Michaud et al. 2011; Michaud et al. 2012; Goldman et al. 2013; Goldman and Orszag 2014; National Academies of Sciences 2015; Chen et al. 2016; Gonzalez-Gonzalez et al. 2017). A complete technical document detailing the FEM is available online.⁴⁰

⁴⁰ See roybalhealthpolicy.usc.edu/fem/technical-specifications/.

The FEM is a microsimulation that follows the evolution of individual-level health trajectories and economic outcomes, rather than the average or aggregate characteristics of a cohort.

The FEM has three core modules. The first is the Replenishing Cohorts module, which predicts economic and health outcomes of new cohorts of 50-year-olds with data from the Panel Study of Income Dynamics (PSID), and incorporates trends in disease and other outcomes based on data from external sources, such as the National Health Interview Survey and the American Community Survey. This module generates cohorts as the simulation proceeds, so that we can measure outcomes for the age 50+ population in any given year.

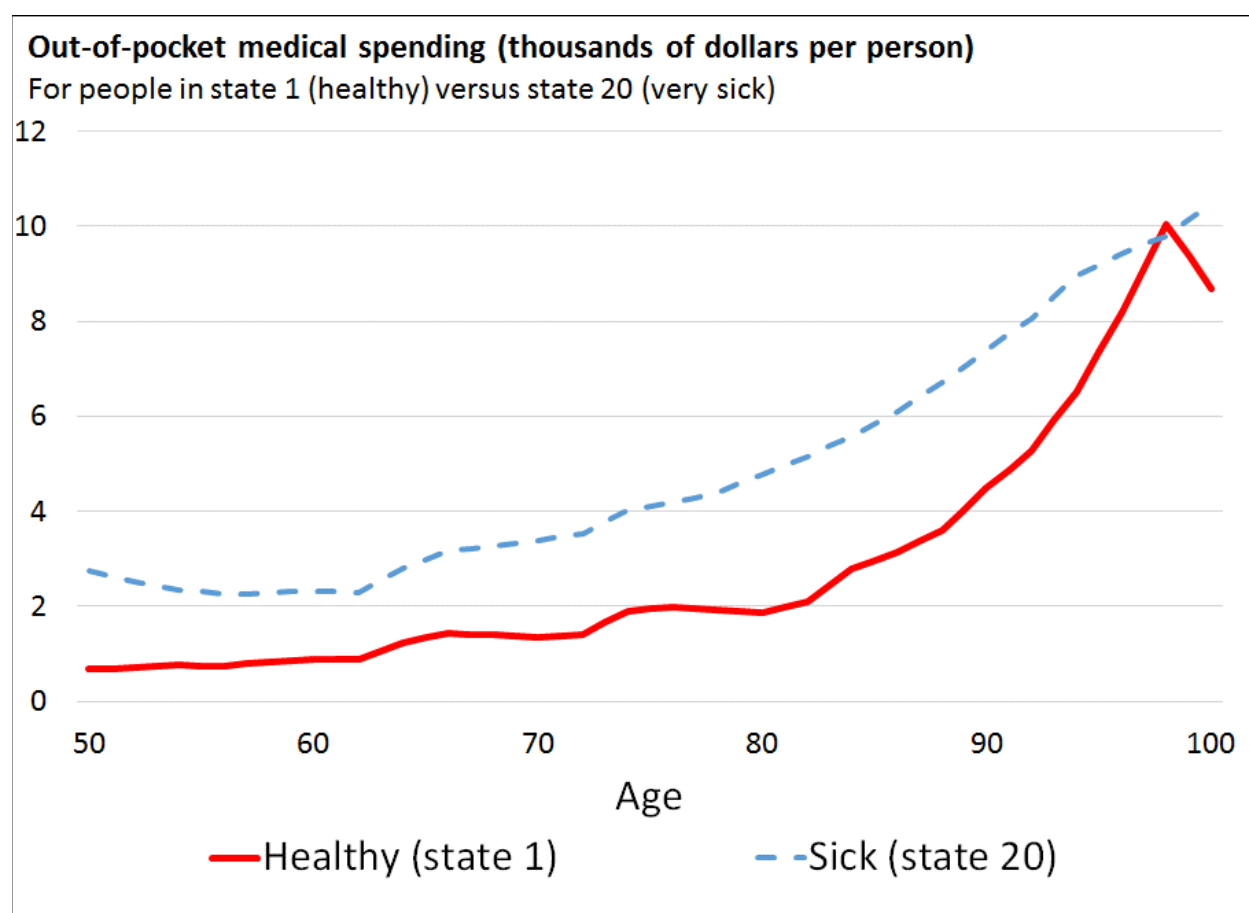
The second component is the Health Transition module, which uses the longitudinal structure of the Health and Retirement Study (HRS) to calculate transition probabilities across various health states, including chronic conditions, functional status, body-mass index, and mortality, using linear and nonlinear multivariate regression models. These transition probabilities depend on a battery of predictors: age, sex, education, race, ethnicity, smoking behavior, marital status, employment and health conditions. Baseline factors are also controlled for using a series of initial health variables measured at age 50. FEM transitions produce a large set of simulated outcomes, including diabetes, high-blood pressure, heart disease, cancer (except skin cancer), stroke or transient ischemic attack, and lung disease (either or both chronic bronchitis and emphysema), disability, and body-mass index. Disability is measured by limitations in instrumental activities of daily living, activities of daily living, and residence in a nursing home. This dynamic simulation method has undergone extensive benchmarking and validation.

Finally, the Policy Outcomes module combines individual-level outcomes into aggregate outcomes, such as medical care costs (Medicare, Medicaid and Private), federal, state and property taxes, and Social Security expenditures and contributions. Individual health spending is predicted with regard to health status (chronic conditions and functional status), demographics (age, sex, race, ethnicity and education), nursing home status, and mortality. Estimates are based on spending data from the Medical Expenditure Panel Survey for individuals aged 64 and younger and the Medicare Current Beneficiary Survey for individuals aged 65 and older, who constitute the bulk of the Medicare population. This module has been comprehensively tested against national aggregates.

An example of how the three modules interact is as follows. For year 2014, the model begins with the population of Americans aged 50 and older based on nationally representative data from the HRS. Individual-level health and economic outcomes for the next two years are predicted using the Policy Outcomes module. The cohort is then aged two years using the Health Transition Module. Aggregate health and functional status outcomes for those years are then calculated. At that point, a new cohort of 50-year-olds is introduced into the 2016 population using the Replenishing Cohort module, and they join those who survived from 2014 to 2016. This forms the age 50+ population for 2016. The transition model is then applied to this population. The same process is repeated until reaching the last year of the simulation.

As described in the main text, for the purposes of our analysis we divide the health space within the FEM into $n = 20$ states. We summarize the resulting dataset in Table 1. Appendix Figure B3 reports average out-of-pocket medical spending, by age, for a healthy individual in health state 1 and for a very sick individual in health state 20. These spending data include all inpatient, outpatient, prescription drug, and long-term care payments made by the individual, as estimated by the Future Elderly Model. The large increase in spending that occurs after age 80 is due primarily to the large costs of long-term care.

Appendix Figure B3. Annual out-of-pocket medical spending estimates from the FEM



Notes: These medical spending estimates include out-of-pocket spending on both health care and nursing homes.

State 1 corresponds to a healthy individual with no impaired activities of daily living (ADL) and no chronic conditions. State 20 corresponds to an individual with three or more ADLs and four or more chronic conditions.

Additional characteristics for these health states are provided in Table 1. These estimates are provided by the Future Elderly Model (FEM).

Appendix C: Supporting Calculations for Quantitative Analysis

Appendix C1 provides details regarding the implementation of the deterministic model employed in Section IV.C, and explains how it is used to derive the aggregate insurance value of Social Security. This model is estimated numerically using standard dynamic programming methods.

Appendix C2 provides a derivation of the stochastic numerical model employed in Section IV.B. This model is solved analytically and thus provides exact solutions.

Appendix C3 describes how we calibrate initial wealth in the stochastic health model.

We confirmed that these two models, one solved numerically and the other analytically, produce the same estimates in a “Robinson Crusoe” economy (no annuity markets or life-cycle income) when applied to the mortality and quality of life data employed in Section IV.C.

C1. Deterministic health

In this model, there is only one health state. The optimal value function then simplifies to:

$$V(t, W(t)) = \max_{c(t)} \sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q(s))$$

We can use the value function to rewrite the optimization problem as a recursive Bellman equation:

$$V(t, W(t)) = \max_{c(t)} u(c(t), q(t)) + \frac{1 - d(t)}{e^\rho} V(t + 1, W(t + 1))$$

Because the problem is finite, we can work backwards from the final period. We discretize the state space into $N_w = 3,000$ points evenly distributed across the interval $[0, W_{max}]$. Let that set of values be $\{W_n\}$. Define $g_t(W(t)) = W(t + 1)$ as a mapping from the current wealth state, $W(t)$, to the optimal wealth state in the following period, $W(t + 1)$.

It is clear that the consumer should consume all her wealth in the final period, i.e., $g_T(W(T)) = 0$ for all $W(T) \in \{W_n\}$. This implies that $V(T, W(T)) = u(W(T) + y(T))$ for all $W(T) \in \{W_n\}$.

Next, we calculate $V(T - 1, W_{T-1}) = \max_{g(W_{T-1})=W_T} u(W_{T-1} + y(T - 1) - W_T/e^r) + \frac{1-d(T-1)}{e^\rho} V(T, W_T)$.

In other words, for each $W(T - 1) \in \{W_n\}$, we calculate the optimal $V(T - 1, W(T - 1))$ by determining which choice of $g_{T-1}(W(T - 1)) = W(T) \in \{W_n\}$ will maximize utility. This algorithm is then repeated for $t = T - 2, T - 3, \dots, 1$.

Given the initial condition, W_1 , we can then employ our results to calculate $W(2) = g_1(W(1))$, $W(3) = g_2(W(2))$, ..., $W(T) = g_{T-1}(W(T - 1))$. Period consumption, $c(t)$, is then calculated using the equation for the budget constraint. Finally, we use the analytical formulas derived in Section II to calculate the value of statistical life.

When accounting for a bequest motive, we follow Kopczuk and Lupton (2007) and assume the utility from leaving a bequest is linear in wealth:

$$V(t, W(t)) = \max_{c(t)} u(c(t), q(t)) + \frac{1}{e^\rho} [(1 - d(t))V(t + 1, W(t + 1)) + d(t)\alpha W(t + 1)]$$

Kopczuk and Lupton (2007) estimate that the constant $\alpha^{-\gamma}$ is approximately equal to \$50,000, where γ is the coefficient of relative risk aversion from a CRRA utility function. We adopt their estimate of \$50,000 when accounting for a bequest motive. This parameterization implies that the marginal utility of

consumption is less than the marginal utility of leaving a bequest when consumption in the last year of life is more than \$50,000.

Insurance value of Social Security

We calculate the insurance value of Social Security at all ages by estimating its wealth equivalence. That is, we follow Mitchell et al. (1999) and estimate the amount of wealth, W^* , required to equalize the utilities of a non-annuitized individual and an individual with Social Security. In other words, we solve for compensating wealth at age t , $W^*(t)$, such that $V(t, W(t) + W^*(t)) = V^{SS}(t, W^{SS}(t))$. Wealth for a non-annuitized individual, $W(t)$, and wealth for an individual with Social Security, $W^{SS}(t)$, are calculated by the deterministic model for the first two policy scenarios discussed in the main text.

We solve for $W^*(t)$ by applying a numerical search algorithm. We estimate that, at age 65, having access to Social Security is equivalent to an increase in wealth of 19.7 percent for a non-annuitized individual. By way of comparison, Mitchell et al. (1999) estimate the before-tax value of *full* (complete) annuitization at age 65 to be 37.4 percent of wealth, using the same parameters for risk aversion, interest rate, and the discount rate.

The aggregate insurance value of Social Security is then calculated by aggregating over the 2015 US population:

$$\text{Aggregate Value SS} = \sum_{a=0}^{110} W^*(a)f(a)$$

C2. Stochastic health

We focus on the case where the consumer does not have access to annuities. We assume that the consumer's lifetime wealth is available at time $t = 0$, so that we can abstract away from the income-generating process. This allows us to generate an analytic solution to the consumer's problem, given by:

$$\max_{c(t)} \mathbb{E} \left[\sum_{t=0}^T e^{-\rho t} S_0(t) u(c(t), q_{Y_t}(t)) + e^{-\rho(t+1)} ((S_0(t) - S_0(t+1))u(W(t+1), b_t)) \middle| Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, W(T) = 0 \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t))e^{r(t, Y_t)} \end{aligned}$$

Here, Y_t denotes the consumer's health state at time t , and we allow the interest rate to depend on it so as to model health-related wealth shocks, as described in the main text. Of course, a constant interest rate $r(t, i) = r$ is included as a special case. The parameter b_t measures the bequest motive. This parameter is set equal to 0 in our main specification, which assumes no bequest motive. When incorporating a bequest motive, we set $b_t = 1$. The utility function is given by (22):

$$u(c, q) = q \left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right)$$

where \underline{c} is the subsistence level of consumption for a healthy person. Because optimal consumption is unaffected by affine transformations of utility, we will assume $u(c, q) = qc^{1-\gamma}/(1-\gamma)$ when solving the model for consumption.

Define the value function as:

$$V(t, W(t), Y_t) = \max_{c(s)} \mathbb{E} \left[\sum_{s=t}^T e^{-\rho(s-t)} S_t(s) u(c(s), q_{Y_s}(s)) + e^{-\rho(s+1-t)} (S_t(s) - S_t(s+1)) u(W(s+1), b_s) \middle| Y_t, W(t) \right]$$

subject to:

$$W(s+1) = (W(s) - c(s)) e^{r(s, Y_s)}, s > t, W(s) \geq 0$$

Then we obtain the following Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left\{ u(c(t), q_i(t)) + e^{-\rho} \bar{d}_i(t) u((w - c(t)) e^{r(t, i)}, b_t) + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) V(t+1, (w - c(t)) e^{r(t, i)}, j) \right\}$$

Appendix Proposition C1:

The value function and the optimal consumption level satisfy:

$$V(t, w, i) = \frac{w^{1-\gamma}}{1-\gamma} K_{t,i},$$

$$c^*(t, w, i) = w \cdot c_{t,i}$$

where:

$$c_{t,i} = \left[1 + e^{-r(t, i)} \left(\frac{e^{r(t, i)} [\bar{d}_i(t) b_t + (1 - \bar{d}_i(t)) (\sum_{j=1}^n p_{ij}(t) K_{t+1, j})]}{e^{\rho} q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}, t < T,$$

$$c_{T,i} = \left[1 + e^{-r(T, i)} \left(\frac{e^{r(T, i)} b_T}{e^{\rho} q_i(T)} \right)^{\frac{1}{\gamma}} \right]^{-1}$$

and $K_{t,i}$ satisfies the recursion:

$$K_{t,i} = \left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t, i)} \left[e^{r(t, i) - \rho} \left(\bar{d}_i(t) b_t + (1 - \bar{d}_i(t)) \left(\sum_{j=1}^n p_{ij}(t) K_{t+1, j} \right) \right) \right]^{\frac{1}{\gamma}} \right]^{\gamma}, t < T,$$

$$K_{T,i} = \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T, i)} (e^{r(T, i) - \rho} b_T)^{\frac{1}{\gamma}} \right]^{\gamma}$$

Proof of Appendix Proposition C1: see end of appendix C

When calculating VSL, we incorporate subsistence consumption back into the utility function. In this case, the value function is:

$$V(0, w, i) = \sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) \frac{c(t)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right] \quad (C1)$$

$$+ e^{-\rho(t+1)} \mathbb{E} \left[\left(\exp \left\{ - \int_0^t \mu(s) ds \right\} - \exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} \right) \left(b_t \frac{W(t+1)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right]$$

Rearranging yields:

$$\begin{aligned}
V(0, w, i) &= \sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \frac{c(t)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\
&\quad + e^{-\rho(t+1)} b_t \mathbb{E} \left[\left(\exp \left\{ - \int_0^t \mu(s) ds \right\} - \exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} \right) \frac{W(t+1)^{1-\gamma}}{1-\gamma} \middle| Y_0 = i, W(0) = w \right] \\
&\quad - \frac{\underline{c}^{1-\gamma}}{1-\gamma} \left[q_{Y_0}(0) + e^{-\rho} b_0 + \sum_{t=1}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} (q_{Y_t}(t) + e^{-\rho} b_t - b_{t-1}) \middle| Y_0 = i \right] \right] \\
&= \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \left[q_{Y_0} + e^{-\rho} b_0 + \sum_{t=1}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} (q_{Y_t}(t) + e^{-\rho} b_t - b_{t-1}) \middle| Y_0 = i \right] \right] \right]
\end{aligned}$$

Note that in the absence of a bequest motive ($b_t \equiv 0$), we obtain:

$$V(0, w, i) = \frac{1}{1-\gamma} \left[w^{1-\gamma} K_{0,i} - \underline{c}^{1-\gamma} \underbrace{\sum_{t=0}^T e^{-\rho t} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \middle| Y_0 = i \right]}_{\text{discounted quality-adjusted life expectancy in state } i} \right]$$

We can then calculate VSL in state i using the following formula:

$$VSL(i) = \frac{V(0, w, i)}{u_c(w c_{0,i}, q_i(0))} = \frac{V(0, w, i)}{V_w(0, w, i)} \quad (C2)$$

When bequests are absent ($b_t \equiv 0$) and $r(t, i) = r$, the theory presented in the main text then yields the following expression for VSL:

$$\begin{aligned}
VSL(i) &= \mathbb{E} \left[\sum_{t=0}^T \exp \left\{ - \int_0^t \rho + \mu(s) ds \right\} \frac{u(c(t), q_{Y_t}(t))}{u_c(c(0), q_{Y_0}(0))} \middle| Y_0 = i, W(0) = w \right] \\
&= \sum_{t=0}^T e^{-rt} \frac{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} u(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = w \right]}{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} u_c(c(t), q_{Y_t}(t)) \middle| Y_0 = i, W(0) = w \right]} \\
&= \sum_{t=0}^T e^{-rt} \frac{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} \left(q_{Y_t}(t) \frac{c(t)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma} \right) \middle| Y_0 = i, W(0) = w \right]}{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^{-\gamma} \middle| Y_0 = i, W(0) = w \right]}
\end{aligned}$$

which can also be written as:

$$\begin{aligned}
VSL(i) &= \frac{1}{1-\gamma} \sum_{t=0}^T e^{-rt} \frac{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^{1-\gamma} \middle| Y_0 = i, W(0) = w \right] - \underline{c}^{1-\gamma} \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) \middle| Y_0 = i, W(0) = w \right]}{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^{-\gamma} \middle| Y_0 = i, W(0) = w \right]} \quad (C3) \\
&\quad \underbrace{\hspace{10em}}_{v(Y_0, t)}
\end{aligned}$$

To evaluate this expression for VSL, we will make use of the following lemma.

Appendix Lemma C2: Let $W_{t,j}(\Psi) = \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} W(t)^\Psi \mathbf{1}\{Y_t = j\} \middle| Y_0, W_0 \right]$ for $\Psi \in (1, \infty)$. Then $W_{t,j}(\Psi)$ satisfies the following recursion:

$$\begin{aligned}
W_{0,Y_0}(\Psi) &= W_0^\Psi, W_{0,i}(\Psi) = 0, i \neq Y_0, \\
W_{t+1,j}(\Psi) &= e^{r\Psi} \sum_{k=1}^n W_{t,k}(\Psi) (1 - c_{t,k})^\Psi (1 - \bar{d}_k(t)) p_{k,j}(t)
\end{aligned}$$

Proof of Appendix Lemma C2: see end of appendix C

Note that for $\Psi = 0$, the expression $\sum_{j=1}^n W_{t,j}(0) = \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} \middle| Y_0 \right]$ is simply the t -year survival probability. Applying **Appendix Lemma C2**, we obtain:

Appendix Proposition C3:

VSL in state Y_0 is equal to:

$$VSL(Y_0) = \frac{1}{1-\gamma} \sum_{t=0}^T e^{-rt} \frac{\sum_{j=1}^n q_j(t) c_{t,j}^{1-\gamma} W_{t,j}(1-\gamma) - \underline{c}^{1-\gamma} \sum_{j=1}^n W_{t,j}(0) q_j(t)}{\underbrace{\sum_{j=1}^n q_j(t) c_{t,j}^{-\gamma} W_{t,j}(-\gamma)}_{v(Y_0, t)}}$$

Proof of Appendix Proposition C3: see end of appendix C

We also immediately obtain the following corollary.

Appendix Corollary C4:

The value of a marginal reduction in the probability of transitioning from state i to state j is equal to:

$$\begin{aligned} VSI(i, j) &= VSL(i) - VSL(j) \frac{q_j(0) c_{0,j}^{-\gamma}}{q_i(0) c_{0,i}^{-\gamma}} \\ &= VSL(i) - \left(\frac{q_j(0)}{q_i(0)} \right) \left(\frac{c_{0,i}}{c_{0,j}} \right)^{\gamma} VSL(j) \end{aligned}$$

We have verified in our numerical calculations that **Appendix Proposition C3** and **Appendix Corollary C4** yield the same answer as the direct evaluation via equation (C2) above.

C3. Wealth calibration

As described in Section IV.B, the stochastic health model assumes annuity markets are absent and endows the consumer with wealth instead of life-cycle income. We choose initial wealth in that model by collapsing the FEM data into a single health state and then solving for the initial wealth value that yields a VSL at age 50 equal to \$5.35 million. That VSL was chosen because it corresponds to the estimated VSL (excluding net savings) at age 50 in the deterministic model for the “No annuity” scenario depicted in Figure 9. This procedure yields a calculated value of \$862,947 for initial wealth.

To evaluate whether this initial wealth value is reasonable, we compare it to wealth and earnings in the deterministic model. In the “No annuity” scenario depicted in Figure 9, estimated wealth plus the net present value of future earnings at age 50 is equal to \$871,833, which is similar to our chosen value of \$862,947 for initial wealth.

Appendix C Proofs

Proof of Appendix Proposition C1:

The proof proceeds by induction on $t \leq T$. For the base case $t = T$, note that $\bar{d}_i(t) = 1$, so that the first-order condition from the Bellman equation gives:

$$q_i(T)c(T)^{-\gamma} = e^{r(T,i)-\rho}b_T(w - c(T))^{-\gamma}e^{-r(T,i)\gamma}$$

This implies that:

$$\begin{aligned} c(T) &= \frac{we^{r(T,i)}e^{\frac{(\rho-r(T,i))}{\gamma}}\left(\frac{q_i(T)}{b_T}\right)^{\frac{1}{\gamma}}}{1 + e^{r(T,i)}e^{\frac{(\rho-r(T,i))}{\gamma}}\left(\frac{q_i(T)}{b_T}\right)^{\frac{1}{\gamma}}} \\ &= w \underbrace{\left[1 + e^{-r(T,i)}\left(\frac{e^{r(T,i)}b_T}{e^\rho q_i(T)}\right)^{\frac{1}{\gamma}}\right]^{-1}}_{c_{T,i}} \end{aligned}$$

Hence, we obtain:

$$\begin{aligned} V(T, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} \left(q_i(T)c_{T,i}^{1-\gamma} + e^{-\rho}b_T e^{r(T,i)(1-\gamma)}(1 - c_{T,i})^{1-\gamma} \right) \\ &= \frac{e^{-\rho}e^{r(T,i)(1-\gamma)}}{\left[b_T^{\frac{1}{\gamma}} + e^{r(T,i)}e^{\frac{(\rho-r(T,i))}{\gamma}}q_i(T)^{\frac{1}{\gamma}} \right]^{-\gamma}} \\ &= \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)}(e^{r(T,i)-\rho}b_T)^{\frac{1}{\gamma}} \right]^\gamma \end{aligned}$$

For the induction step, suppose the proposition is true for case $t + 1$. We have:

$$\begin{aligned} V(t, w, i) &= \max_c \left\{ q_i(t) \frac{c^{1-\gamma}}{1-\gamma} + b_t e^{-\rho} \bar{d}_i(t) \frac{((w - c)e^{r(t,i)})^{1-\gamma}}{1-\gamma} \right. \\ &\quad \left. + e^{-\rho} (1 - \bar{d}_i(t)) \sum_{j=1}^n p_{ij}(t) \frac{K_{t+1,j}}{1-\gamma} [(w - c)e^{r(t,i)}]^{1-\gamma} \right\} \end{aligned}$$

From the first-order condition we obtain:

$$\begin{aligned} q_i(t)c^{-\gamma} &= b_t e^{r(t,i)-\rho} \bar{d}_i(t) e^{-r(t,i)\gamma} (w - c)^{-\gamma} \\ &\quad + e^{r(t,i)-\rho} (1 - \bar{d}_i(t)) e^{-\gamma r(t,i)} (w - c)^{-\gamma} \sum_{j=i}^n p_{ij}(t) K_{t+1,j} \end{aligned}$$

Rearranging yields:

$$q_i(t)c^{-\gamma} = (w - c)^{-\gamma} e^{r(t,i)-\rho} e^{-r(t,i)\gamma} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right]$$

which implies:

$$q_i(t)^{-1/\gamma} c = (w - c) e^{(\rho - r(t,i))/\gamma} e^{r(t,i)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right]^{-1/\gamma}$$

Rearranging further yields:

$$\begin{aligned} c &= w \frac{e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right] \right]^{-1/\gamma}}{e^{\rho} q_i(t)^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right] \right]^{-1/\gamma}} \\ &= w \underbrace{\left[1 + e^{-r(t,i)} \left(\frac{e^{r(t,i)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right]}{e^{\rho} q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}}_{c_{t,i}} \end{aligned}$$

Thus we obtain:

$$\begin{aligned} V(t, w, i) &= q_i(t)c_{t,i}^{1-\gamma} \frac{w^{1-\gamma}}{1-\gamma} + b_t e^{-\rho} \bar{d}_i(t) \frac{w^{1-\gamma}}{1-\gamma} (1 - c_{t,i})^{1-\gamma} e^{r(t,i)(1-\gamma)} \\ &\quad + e^{-\rho} (1 - \bar{d}_i(t)) \frac{w^{1-\gamma}}{1-\gamma} (1 - c_{t,i})^{1-\gamma} e^{r(t,i)(1-\gamma)} \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \\ &= \frac{w^{1-\gamma}}{1-\gamma} \left[q_i(t)c_{t,i}^{1-\gamma} + e^{-\rho} (1 - c_{t,i})^{1-\gamma} e^{r(t,i)(1-\gamma)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right] \right] \\ &= \frac{w^{1-\gamma} q_i(t) e^{r(t,i)(1-\gamma)} \left[e^{r(t,i)} \left(\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right) \right]^{1-1/\gamma} + e^{-\rho} e^{r(t,i)(1-\gamma)} (e^{\rho} q_i(t))^{1-1/\gamma} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right]}{1-\gamma} \\ &\quad \left[(e^{\rho} q_i(t))^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right] \right]^{\frac{1}{\gamma}} \right]^{1-\gamma} \\ &= \frac{w^{1-\gamma}}{1-\gamma} \frac{e^{r(t,i)(1-\gamma)} q_i(t) \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right]}{\left[(e^{\rho} q_i(t))^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right] \right]^{\frac{1}{\gamma}} \right]^{1-\gamma}} \\ &= \frac{w^{1-\gamma}}{1-\gamma} \underbrace{\left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)} \left[e^{r(t,i)-\rho} \left(\bar{d}_i(t)b_t + (1 - \bar{d}_i(t)) \sum_{j=i}^n p_{ij}(t)K_{t+1,j} \right) \right]^{\frac{1}{\gamma}} \right]^{\gamma}}_{K_{t,i}} \end{aligned}$$

QED

Proof of Appendix Lemma C2:

$$\begin{aligned}
W_{t+1,j}(\Psi) &= \mathbb{E} \left[\exp \left\{ - \int_0^{t+1} \mu(s) ds \right\} (W(t+1))^\Psi \mathbf{1}_{\{Y_{t+1}=j\}} \middle| Y_0, W_0 \right] \\
&= \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} \left((W(t) - c(t)) e^r \right)^\Psi \mathbf{1}_{\{Y_{t+1}=j\}} \exp \left\{ - \int_t^{t+1} \mu(s) ds \right\} \middle| Y_0, W_0 \right] \\
&= \sum_{k=1}^n \mathbb{E} \left[\mathbf{1}_{\{Y_t=k\}} \exp \left\{ - \int_0^t \mu(s) ds \right\} e^{r\Psi} W(t)^\Psi (1 - c_{t,k})^\Psi \underbrace{\mathbb{E} \left[\mathbf{1}_{\{Y_{t+1}=j\}} \exp \left\{ - \int_t^{t+1} \mu(s) ds \right\} \middle| Y_t=k \right]}_{(1-\bar{d}_k(t))p_{kj}(t)} \middle| Y_0, W_0 \right] \\
&= e^{r\Psi} \sum_{k=1}^n W_{t,k}(\Psi) (1 - c_{t,k})^\Psi (1 - \bar{d}_k(t)) p_{kj}(t)
\end{aligned}$$

QED

Proof of Appendix Proposition C3:

Note that we can rewrite one of the terms in equation (C3) as follows:

$$\begin{aligned}
\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^\Psi \middle| Y_0, W_0 \right] &= \sum_{j=1}^n \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^\Psi \mathbf{1}_{\{Y_t=j\}} \middle| Y_0, W_0 \right] \\
&= \sum_{j=1}^n \mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} q_j(t) c_{t,j}^\Psi W(t)^\Psi \mathbf{1}_{\{Y_t=j\}} \middle| Y_0, W_0 \right] \\
&= \sum_{j=1}^n q_j(t) c_{t,j}^\Psi \underbrace{\mathbb{E} \left[\exp \left\{ - \int_0^t \mu(s) ds \right\} W(t)^\Psi \mathbf{1}_{\{Y_t=j\}} \middle| Y_0, W_0 \right]}_{W_{t,j}(\Psi)}
\end{aligned}$$

The proof follows by setting $\Psi = 1 - \gamma$, 0, and $-\gamma$ and then plugging those results into equation (C3) as appropriate.

QED

Appendix D: The Fully Annuitized Value of Life When Health Is Stochastic

We assume a full menu of actuarially fair annuities is available, where consumers can choose consumption streams, $c(t)$, that depend on the evolution of their health state. Thus, the consumer is able to fully insure against consumption risk. The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E} \left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0 \right] \quad (D1)$$

subject to:

$$\mathbb{E} \left[\int_0^T e^{-rt} S(t) c(t) dt \middle| Y_0 \right] = W_0 + \mathbb{E} \left[\int_0^T e^{-rt} S(t) m_{Y_t}(t) dt \middle| Y_0 \right] \equiv \bar{W}(0, Y_0)$$

where $\bar{W}(0, Y_0)$ is the net present value of wealth and future earnings.

The consumer chooses the consumption profile at time t based on her health state, $Y_t = i$, and on her available wealth, $\bar{W}(t, i)$. We define the present value of future earnings as:

$$M(t, i) = \mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} m_{Y_u}(u) du \middle| Y_t = i \right]$$

Her available wealth finances future consumption such that:

$$\bar{W}(t, i) = \mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} c(u) du \middle| Y_t, \bar{W}(t, i) \right]$$

Appendix Lemma D1:

The law of motion for wealth is:

$$\frac{\partial \bar{W}(t, i)}{\partial t} = (r + \bar{\mu}_i(t)) \bar{W}(t, i) - c(t, \bar{W}(t, i), i) + \sum_{j>i} \lambda_{ij}(t) [\bar{W}(t, i) - \bar{W}(t, j)], i = 1, \dots, n$$

Proof of Appendix Lemma D1: see end of Appendix D

Note that the dynamics for $\bar{W}(t, i)$ will depend on $\bar{W}(t, j)$, $j > i$, so that $(Y_t, \bar{W}(t, Y_t))$ is not Markov, but $(Y_t, \bar{W}(t))$, where we define the wealth vector $\bar{W}(t) \equiv (\bar{W}(t, 1), \dots, \bar{W}(t, n))$, is Markov.

Define the optimal value-to-go function as:

$$V(t, \bar{W}(t), Y_t) = \max_{c(u)} \mathbb{E} \left[\int_t^T e^{-\rho(u-t)} \exp \left\{ - \int_t^u \mu(s) ds \right\} u(c(u), q_{Y_u}(u)) du \middle| Y_t, \bar{W}(t) \right]$$

subject to the law of motion for wealth given above. As a stochastic dynamic programming problem, $V(\cdot)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned}
& (\rho + \bar{\mu}_i(t)) V(t, \bar{W}(t), i) \\
&= \frac{\partial V(t, \bar{W}(t), i)}{\partial t} + \max_{c(t)} \left\{ u(c(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) [V(t, \bar{W}(t), j) - V(t, \bar{W}(t), i)] \right. \\
&\quad + \sum_{k \geq i} \frac{\partial V(t, \bar{W}(t), i)}{\partial \bar{W}(t, k)} \left[(r + \bar{\mu}_k(t)) \bar{W}(t, k) - c(t, \bar{W}(t, k), k) \right. \\
&\quad \left. \left. + \sum_{l>k} \lambda_{kl}(t) [\bar{W}(t, k) - \bar{W}(t, l)] \right] \right\}, 1 \leq i \leq n
\end{aligned} \tag{D2}$$

Similarly to the uninsured case presented in the main text, we follow Pappas and Webster (2013) and focus on the path of Y that begins in i and remains in i until time t , with $c_i(t)$ and $\bar{W}_i(t)$ denoting the corresponding optimal consumption and wealth paths. We take optimal consumption rules and value functions from other states as exogenous. As in the uninsured case, this approach will allow us to apply the standard Pontryagin maximum principle and derive analytic expressions.

Appendix Lemma D2:

The optimal value function for $Y_0 = i$, $V(0, \bar{W}(0, i), i)$, for the following deterministic optimization problem also satisfies the HJB given by (D2), for each $i \in \{1, \dots, n\}$:

$$V(0, \bar{W}(0, i), i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right) dt \right] \tag{D3}$$

subject to:

$$\begin{aligned}
\frac{\partial \bar{W}_i(t, j)}{\partial t} &= (r + \bar{\mu}_j(t)) \bar{W}_i(t, j) - c(t, \bar{W}_i(t), j) + \sum_{k>j} \lambda_{jk}(t) [\bar{W}_i(t, j) - \bar{W}_i(t, k)], j > i \\
\frac{\partial \bar{W}_i(t, i)}{\partial t} &= (r + \bar{\mu}_i(t)) \bar{W}_i(t, i) - c_i(t) + \sum_{k>i} \lambda_{ik}(t) [\bar{W}_i(t, i) - \bar{W}_i(t, k)]
\end{aligned}$$

where $V(t, \bar{W}_i(t), j)$ and $c(t, \bar{W}_i(t), j), j > i$, are taken as exogenous.

Proof of Appendix Lemma D2: see end of Appendix D

Following Bertsekas (2005), the Hamiltonian for the (deterministic) maximization problem (D3) is:

$$\begin{aligned}
H(\bar{W}_i(t), c_i(t), p_i(t)) &= e^{-\rho t} \tilde{S}(i, t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right) \\
&\quad + \sum_{k>i} p_i(t, k) \left[(r + \bar{\mu}_k(t)) \bar{W}_i(t, k) - c(t, \bar{W}_i(t), k) \right. \\
&\quad \left. + \sum_{l>k} \lambda_{kl}(t) [\bar{W}_i(t, k) - \bar{W}_i(t, l)] \right] \\
&\quad + p_i(t, i) \left[(r + \bar{\mu}_i(t)) \bar{W}_i(t, i) - c_i(t) + \sum_{l>i} \lambda_{il}(t) [\bar{W}_i(t, i) - \bar{W}_i(t, l)] \right]
\end{aligned} \tag{D4}$$

where $p_i(t) = (p_i(t, 1), \dots, p_i(t, n))$ is the vector of costate variables corresponding to wealth $\bar{W}_i(t)$.

Appendix Lemma D3:

We have that $p_i(t, i) = \theta e^{-\rho t} \tilde{S}(i, t)$ for θ independent of i , and $p_i(t, k) = 0, k \neq i$. The necessary first-order condition for consumption is:

$$e^{(r-\rho)t} u_c(c_i(t), q_i(t)) = \theta \quad (\text{D5})$$

where $\theta = p_i(0, i) = \partial V(0, \bar{W}_i(0), i) / \partial \bar{W}(0, i)$ is the marginal utility of wealth.

Proof of Appendix Lemma D3: see end of Appendix D

To analyze the values of life and illness, let $\delta_{ij}(t), i, j \leq N$, be a perturbation on the transition intensity $\lambda_{ij}(t)$, and let $\delta_{i,N+1}(t)$ be a perturbation on the mortality rate, $\bar{\mu}_i(t)$, where $\sum_{j=i+1}^{N+1} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^\varepsilon(i, t) = \exp \left[- \int_0^t (\bar{\mu}_i(s) - \varepsilon \delta_{i,N+1}(s)) + \sum_{j=i+1}^N (\lambda_{ij}(s) - \varepsilon \delta_{ij}(s)) ds \right], \text{ where } \varepsilon > 0$$

Appendix Proposition D4:

The marginal utility of preventing an illness or death is given by:

$$\begin{aligned} \frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} = & \int_0^T \left(\tilde{S}(i, t) \left\{ e^{-\rho t} \left[u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right] \right. \right. \\ & \left. \left. + \theta e^{-rt} \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} \right. \\ & \left. - \tilde{S}(i, t) \sum_{j=i+1}^N \delta_{ij}(t) \{ e^{-\rho t} V(t, \bar{W}_i(t), j) - \theta e^{-rt} [\bar{W}_i(t, j) - M(t, j)] \} \right) dt \end{aligned} \quad (\text{D6})$$

Proof of Appendix Proposition D4: see end of Appendix D

To obtain the value of statistical life (VSL), we first set $\delta_{i,N+1}$ equal to the Dirac delta function, and set all other perturbations equal to 0. Dividing the result by the marginal utility of wealth, θ , then yields:

$$\begin{aligned} VSL = & \int_0^T \tilde{S}(i, t) e^{-rt} \left\{ \left[\frac{u(c_i(t), q_i(t))}{u_c(c_i(t), q_i(t))} + \sum_{j>i} \lambda_{ij}(t) \frac{V(t, \bar{W}_i(t), j)}{\partial V(t, \bar{W}_i(t), j) / \partial \bar{W}_i(t, j)} \right] \right. \\ & \left. + \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} dt \\ = & \frac{V(0, \bar{W}_i(0), i)}{u_c(c_i(0), q_i(0))} - W_0 \\ = & \mathbb{E} \left[\int_0^T e^{-rt} S(t) v(t) dt \mid Y_0 = i \right] \end{aligned} \quad (\text{D7})$$

where the value of a statistical life-year is:

$$v(t) = \frac{u(c(t), q_{Y_t}(t))}{u_c(c(t), q_{Y_t}(t))} + m_{Y_t}(t) - c_{Y_t}(t)$$

Comparing (D7) to (3) reveals that generalizing the standard model to account for stochastic health risk alone does not alter the basic expression for VSL. Consumers continue to discount future life-years by the rate of interest and by survival. We can obtain the life-cycle profile of consumption in state i by differentiating the first-order condition (D5) with respect to t . Doing so confirms that, as in the deterministic case, annuitization insulates consumption from mortality risk:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \sigma(r - \rho) + \sigma\eta \frac{\dot{q}}{q}$$

Our results demonstrate that stochastic health risk, by itself, does not alter the basic insights regarding VSL offered by the prior literature as long as one maintains the assumption of full annuitization. In particular, the conventional result that VSL falls when mortality rises continues to hold in this setting.

However, a novel feature of the stochastic model is that it permits an investigation into the value of prevention. Inspecting the expression for the marginal utility of life extension (D6), the first term inside the integral represents the gain in marginal utility from a reduction in the probability of exiting state i . The second term represents the loss in marginal utility from the reduction in probability of transitioning to other possible states. The net effect depends on the consumer's marginal utility in the different states.

To analyze the value of prevention, consider a reduction in the transition probability for only one alternative state, j , so that $\delta_{ik}(t) = 0 \forall k \neq j$. The value of avoiding illness j is then equal to:

$$\begin{aligned} VSI(i, j) &= \int_0^T \tilde{S}(i, t) e^{-rt} \left\{ \left[\frac{u(c_i(t), q_i(t))}{u_c(c_i(t), q_i(t))} + \sum_{j>i} \lambda_{ij}(t) \frac{V(t, \bar{W}_i(t), j)}{\frac{\partial V(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, j)}} \right] \right. \\ &\quad \left. + \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} dt - \left[\frac{V(0, \bar{W}_i(0), j)}{\theta} - [\bar{W}_i(0, j) - M(0, j)] \right] \\ &= \frac{V(0, \bar{W}_i(0), i)}{u_c(c_i(0), q_i(0))} - W_0 - \left(\frac{V(0, \bar{W}_i(0), j)}{u_c(c_i(0), q_i(0))} - [\bar{W}_i(0, j) - M(0, j)] \right) \\ &= VSL(i) - VSL(j|W_0 = \bar{W}_i(0, j) - M(0, j)) \end{aligned} \tag{D8}$$

Thus, equation (D8) demonstrates that $VSI(i, j)$ is equal to the difference in VSL for states i and j , with the caveat that VSL in state j uses a measure of wealth evaluated from the perspective of a person in state i . This technicality arises because the value of the consumer's annuity depends on her expected survival. For example, an annuity is worth more to a healthy 65-year-old than it is to a 65-year-old who was just diagnosed with lung cancer.

Appendix D Proofs

Proof of Appendix Lemma D1:

Available wealth can be written as:

$$\bar{W}(t, i) = \int_t^T \exp \left\{ - \int_t^u r + \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} \left[c_i(t, u) + \sum_{j>i} \lambda_{ij}(u) \bar{W}_i(u, t, j) \right] du$$

where with a slight abuse of notation, $c_i(t, u)$ and $\bar{W}_i(u, t, j)$ denote the consumption and wealth paths for an individual who is in state i at time t and remains in state i until time u . The result then follows by taking the derivative with respect to t .

Proof of Appendix Lemma D2:

This proof follows the same logic as the proof of **Lemma 1** in Appendix A. Consider the deterministic optimization problem (D3). Denote the optimal value-to-go function as:

$$\bar{V}(t, \bar{W}_i(t), i) = \max_{c_i(t)} \left\{ \int_t^T e^{-\rho u} \tilde{S}(i, u) \left(u(c_i(u), q_i(u)) + \sum_{j>i} \lambda_{ij}(u) V(u, \bar{W}_i(u), j) \right) du \right\}$$

Setting $\bar{V}(t, \bar{W}_i(t), i) = e^{-\rho t} \tilde{S}(i, t) V(t, \bar{W}_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (D2) for i .

QED

Proof of Appendix Lemma D3:

The costate equations for the Hamiltonian (D4) are:

$$\begin{aligned} \dot{p}_i(t, i) &= - \left[(r + \bar{\mu}_i(t)) + \sum_{l>i} \lambda_{il}(t) \right] p_i(t, i), \\ \dot{p}_i(t, k) &= - \sum_{j>k} \lambda_{ij}(t) \frac{\partial V(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, k)} + \sum_{k \geq j > i} p_i(t, j) \left(\frac{\partial c(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, k)} + \lambda_{jk}(t) \right) \\ &\quad - p_i(t, k) \left[(r + \bar{\mu}_k(t)) + \sum_{l>k} \lambda_{kl}(t) \right] + p_i(t, i) \lambda_{ik}(t), \text{ for } k > i \end{aligned}$$

From the first costate equation, we obtain:

$$p_i(t, i) = e^{-rt} \tilde{S}(i, t) \theta$$

Taking first-order conditions in the Hamiltonian (D4) and plugging this in then yields:

$$u_c(c_i(t), q_i(t)) = \frac{\partial V(t, \bar{W}_i(t), i)}{\partial \bar{W}_i(t, i)} = e^{(\rho-r)t} \theta$$

To see that this solution works, let θ be constant across states, and set $p_i(t, k) = 0 = \frac{\partial V(t, \bar{W}_i(t), i)}{\partial \bar{W}_i(t, k)}$. This then satisfies the costate equation system across i, k , and t . In particular, for the second equation we obtain

$$\begin{aligned}\dot{p}_i(t, k) &= -e^{-\rho t} \tilde{S}(i, t) \lambda_{ik}(t) \underbrace{\frac{\partial V(t, \bar{W}_i(t), k)}{\partial \bar{W}_i(t, k)}}_{e^{(\rho-r)t}\theta} + \lambda_{ik}(t) p_i(t, i) \\ &= 0\end{aligned}$$

QED

Proof of Appendix Proposition D4:

Starting from equation (D3), we have:

$$\begin{aligned}V^\varepsilon(0, \bar{W}_i(0, i), i) &= \int_0^T e^{-\rho t} \exp \left\{ -\int_0^t \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) - \varepsilon \sum_{j=i+1}^{N+1} \delta_{ij}(s) ds \right\} \left[u(c_i^\varepsilon(t), q_i(t)) \right. \\ &\quad \left. + \sum_{j=i+1}^N [\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)] V(t, \bar{W}_i^\varepsilon(t), j) \right] dt,\end{aligned}$$

where $c_i^\varepsilon(t)$ and $\bar{W}_i^\varepsilon(t)$ represent the equilibrium variations in $c_i(t)$ and $\bar{W}_i(t)$ caused by the perturbation, $\delta_{ij}(t)$. Differentiating then yields:

$$\begin{aligned}\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T e^{-\rho t} \tilde{S}(i, t) \left[u(c_i(t), q_i(t)) + \sum_{j=i+1}^N \lambda_{ij}(t) V(t, \bar{W}_i(t), j) \right] \left[\sum_{j=i+1}^{N+1} \int_0^t \delta_{ij}(s) ds \right] \\ &\quad - e^{-\rho t} \tilde{S}(i, t) \sum_{j=i+1}^N \delta_{ij}(t) V(t, \bar{W}_i(t), j) \\ &\quad + e^{-\rho t} \tilde{S}(i, t) \left[\underbrace{u_c(c_i(t), q_i(t))}_{e^{-(r-\rho)t}\theta}} \frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + \sum_{j=i+1}^N \underbrace{\frac{\partial V(t, \bar{W}_i(t), j)}{\partial \bar{W}_i(t, j)}}_{e^{-(r-\rho)t}\theta}} \frac{\partial \bar{W}_i(t, j)}{\partial \varepsilon} \Big|_{\varepsilon=0} \right] dt\end{aligned}$$

We have:

$$\begin{aligned}W_0 &= \mathbb{E} \left[\int_0^T e^{-rt} S(t) [c(t) - m_{Y_t}(t)] dt \mid Y_0 = i \right] \\ &= \int_0^T e^{-rt} \exp \left\{ -\int_0^t \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} (c_i(t) - m_i(t)) dt \\ &\quad + \sum_{j=i+1}^N e^{-rt} \exp \left\{ -\int_0^t \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} \lambda_{ij}(t) \underbrace{\mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ -\int_t^u \mu(s) ds \right\} c(u) du \mid Y_t = j \right]}_{\bar{W}_i(t, j)} \\ &\quad - \sum_{j=i+1}^N e^{-rt} \exp \left\{ -\int_0^t \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} \lambda_{ij}(t) \underbrace{\mathbb{E} \left[\int_t^T e^{-r(u-t)} \exp \left\{ -\int_t^u \mu(s) ds \right\} m_{Y_t}(u) du \mid Y_t = j \right]}_{M(t, j)} \\ &= \int_0^T e^{-rt} \exp \left\{ -\int_0^t \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) ds \right\} \left[c_i(t) - m_i(t) + \sum_{j=i+1}^N \lambda_{ij}(t) (\bar{W}_i(t, j) - M(t, j)) \right] dt\end{aligned}$$

The budget constraint then implies:

$$\begin{aligned}
0 &= \frac{\partial W_0}{\partial \varepsilon} \Big|_{\varepsilon=0} \\
&= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-rt} \exp \left\{ - \int_0^t \bar{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s) - \varepsilon \sum_{j=i+1}^{N+1} \delta_{ij}(s) ds \right\} \left(c_i^\varepsilon(t) - m_i(t) \right. \\
&\quad \left. + \sum_{j=i+1}^N [\lambda_{ij}(t) - \varepsilon \delta_{ij}(t)] (\bar{W}_i^\varepsilon(t, j) - M(t, j)) \right) dt \Big|_{\varepsilon=0} \\
&= \int_0^T \left(e^{-rt} \tilde{S}(i, t) \left[c_i(t) - m_i(t) + \sum_{j=i+1}^N \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \left[\sum_{j=i+1}^{N+1} \int_0^t \delta_{ij}(s) ds \right] \right. \\
&\quad \left. - e^{-rt} \tilde{S}(i, t) \sum_{j=i+1}^N \delta_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right. \\
&\quad \left. + e^{-rt} \tilde{S}(i, t) \left[\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} \Big|_{\varepsilon=0} + \sum_{j=i+1}^N \lambda_{ij}(t) \frac{\partial \bar{W}_i^\varepsilon(t, j)}{\partial \varepsilon} \Big|_{\varepsilon=0} \right] \right) dt
\end{aligned}$$

Plugging this last result into the expression for $\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0}$ then yields the desired result for marginal utility:

$$\begin{aligned}
\frac{\partial V}{\partial \varepsilon} \Big|_{\varepsilon=0} &= \int_0^T \left(\tilde{S}(i, t) \left[\sum_{j=i+1}^N \int_0^t \delta_{ij}(s) ds \right] \left\{ e^{-\rho t} \left[u(c_i(t), q_i(t)) + \sum_{j=i+1}^N \lambda_{ij}(t) V(t, \bar{W}_i(t, j), j) \right] \right. \right. \\
&\quad \left. \left. + \theta e^{-rt} \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right] \right\} \right. \\
&\quad \left. - \tilde{S}(i, t) \left\{ e^{-\rho t} \sum_{j=1}^N \delta_{ij}(t) V(t, \bar{W}_i(t, j), j) - \theta e^{-rt} \sum_{j=i+1}^N \delta_{ij}(t) [\bar{W}_i(t, j) - M(t, j)] \right\} \right) dt
\end{aligned}$$

QED