An important and impactful paper

Abstract. Stochastic control in multi-class queueing networks has been extensively studied primarily focusing on minimizing operational costs (e.g., waiting, abandonment). However, in many real-world applications, the system operator must balance the trade-off between waiting costs and maximizing immediate rewards when assigning customers to service units. This necessitates the integration of learning algorithms that focus on maximizing immediate rewards through matching, and stochastic control methods that aim to minimize waiting costs by managing queue dynamics. Specifically, we consider a multi-class queueing system where each customer type is associated with a waiting cost and a feature vector. Each service unit consist of multiple idential servers each with a fixed service rate and a parameter. The inner product of the customer feature vector and the server parameter determines the reward when a customer is assigned to a server. When the server parameters are known, we design a priority rule based on fluid approximation and demonstrate its asymptotic optimality, effectively balancing the trade-offs between reward maximization and waiting cost minimization. When the server parameters are unknown, we incorporate an online learning algorithm into the system control, achieving a regret bound of order $O(\ln T)$. Numerical experiments demonstrate the effectiveness of our proposed algorithms in balancing the trade-offs between immediate rewards and waiting costs.

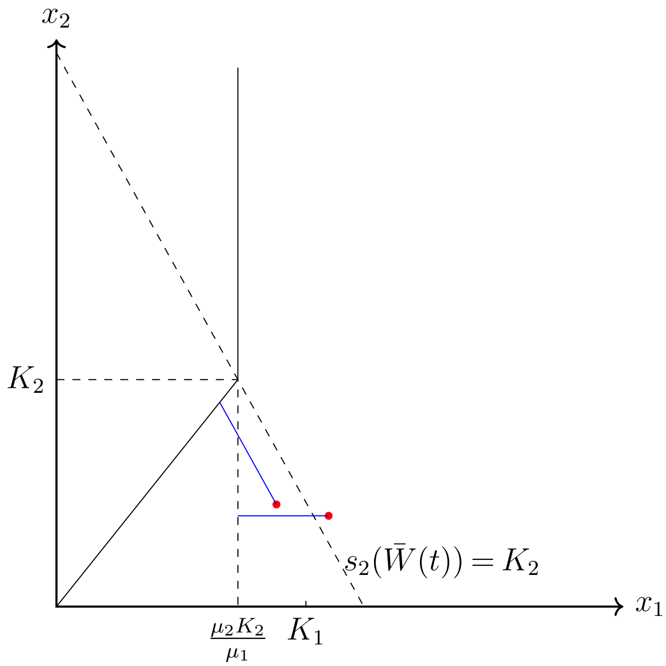
Key words: Stochastic control, Queueing network,Uncertainty, Online learning, Optimization

1. Introduction

Queueing control problems are widely studied in stochastic control, with applications in telecommunications, emergency services, project management, and more. In a traditional multi-class queueing control problem, the decision maker must design a priority rule—such as the $c\mu$ rule—to determine which type of customer to serve based on certain exogenous properties of the system. The primary objective is to minimize operational costs, including waiting costs and abandonment.

Erlang (1948), Dantzig (1955), Dynkin (1956), Bellman (1957), Little (1961), Skorokhod (1961), McKean (1965), Iglehart (1965)

2. Model



3. Conclusion

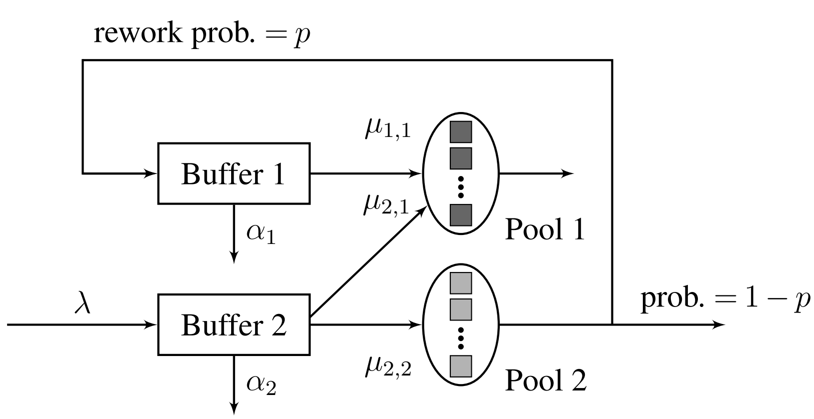


Figure 1 A schematic Model of Outsourcing with rework

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Proofs

EC.1. Proof of Results

EC.1.1. Proof of Lemma

LEMMA EC.1. *As long as* $t > 8\frac{d\log 9+\log(T/\alpha)}{p_*^2}$, *the following lower bound*

Proof of Lemma EC.1 $\square$