Problem Set #5

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Exercise 8.1

Optimizer: $(\frac{14}{5}, \frac{16}{5})$

Exercise 8.2

Exercise 8.3

$$\begin{aligned} \text{maximize} \quad & 4b + 3j \\ \text{subject to} \quad & 15b + 10j \leq 1800 \\ & 2b + 2j \leq 300 \\ & j \leq 200 \\ & b, j \geq 0 \end{aligned}$$

Exercise 8.4

$$\begin{aligned} \text{maximize} \quad & 2x_{AB} + 5x_{AD} + 5x_{BC} + 2x_{BD} + 7x_{BE} + 9x_{BF} + 2x_{CF} + 4x_{DE} + 3x_{EF} \\ \text{subject to} \quad & x_{AB} + x_{AD} = 10 \\ & x_{BC} + x_{BD} + x_{BE} + x_{BF} - x_{AB} = 1 \\ & x_{CF} - x_{BC} = -2 \\ & x_{DE} - x_{AD} - x_{BD} = -3 \\ & x_{EF} - x_{BE} - x_{DE} = 4 \\ & -x_{BF} - x_{CF} - x_{EF} = -10 \\ & 0 \leq x_{AB}, x_{AD}, x_{BC}, x_{BD}, x_{BE}, x_{BF}, x_{CF}, x_{DE}, x_{EF} \leq 6 \end{aligned}$$

Exercise 8.5

(i)

$$\begin{aligned} \text{maximize} \quad & 3x_1 + x_2 \\ \text{subject to} \quad & x_1 + 3x_2 + w_1 = 15 \\ & 2x_1 + 3x_2 + w_2 = 18 \\ & x_1 - x_2 + w_3 = 4 \\ & x_1, x_2, w_1, w_2, w_3 \geq 0 \end{aligned}$$

$$
\begin{array}{rcrcrcr}
\zeta & = & & & 3x_1 & + & x_2 \\
\hline
w_1 & = & 15 & - & x_1 & - & 3x_2 \\
w_2 & = & 18 & - & 2x_1 & - & 3x_2 \\
w_3 & = & 4 & - & x_1 & + & x_2 \\
\hline
\zeta & = & 12 & + & 4x_2 & - & 3w_3 \\
\hline
w_1 & = & 11 & - & 4x_2 & + & w_3 \\
w_2 & = & 10 & - & 5x_2 & + & 2w_3 \\
x_1 & = & 4 & + & x_2 & - & w_3 \\
\hline
\zeta & = & 20 & - & \frac{4}{5}w_2 & - & \frac{7}{5}w_3 \\
\hline
w_1 & = & 3 & + & \frac{4}{5}w_2 & - & \frac{3}{5}w_3 \\
x_2 & = & 2 & - & \frac{1}{5}w_2 & + & \frac{2}{5}w_3 \\
x_1 & = & 6 & - & \frac{1}{5}w_2 & - & \frac{3}{5}w_3
\end{array}
$$

Optimizer: $(6, 2)$
Optimum value: 20

(ii)

$$
\begin{aligned}
\text{maximize} \quad & 4x + 6y \\
\text{subject to} \quad & -x + 3x_2 + w_1 = 11 \\
& x + y + w_2 = 27 \\
& 2x + 5y + w_3 = 90 \\
& x, y, w_1, w_2, w_3 \geq 0
\end{aligned}
$$

ζ	$=$			$4x$	$+$	$6y$
w_1	$=$	11	$+$	x	$-$	y
w_2	$=$	27	$-$	x	$-$	y
w_3	$=$	90	$-$	$2x$	$-$	$5y$
ζ	$=$	66	$+$	$10x$	$-$	$6w_1$
y	$=$	11	$+$	x	$-$	w_1
w_2	$=$	16	$-$	$2x$	$+$	w_1
w_3	$=$	35	$-$	$7x$	$+$	$5w_1$
ζ	$=$	116	$+$	$\frac{8}{7}w_1$	$-$	$\frac{10}{7}w_3$
y	$=$	16	$-$	$\frac{2}{7}w_1$	$-$	$\frac{1}{7}w_3$
w_2	$=$	6	$-$	$\frac{3}{7}w_1$	$+$	$\frac{2}{7}w_3$
x	$=$	5	$+$	$\frac{5}{7}w_1$	$-$	$\frac{1}{7}w_3$
ζ	$=$	132	$-$	$\frac{8}{3}w_2$	$-$	$\frac{2}{7}w_3$
y	$=$	12	$+$	$\frac{2}{3}w_2$	$-$	$\frac{1}{3}w_3$
w_1	$=$	14	$-$	$\frac{7}{3}w_2$	$+$	$\frac{2}{3}w_3$
x	$=$	15	$-$	$\frac{5}{3}w_2$	$+$	$\frac{1}{3}w_3$

Optimizer: $(15, 12)$
Optimum value: 132

Exercise 8.6

$$\begin{aligned} \text{maximize} \quad & 4b + 3j \\ \text{subject to} \quad & 15b + 10j + w_1 = 1800 \\ & 2b + 2j + w_2 = 300 \\ & j + w_3 = 200 \\ & b, j, w_1, w_2, w_3 \geq 0 \end{aligned}$$

$$
\begin{array}{rcrcrcr}
\zeta & = & & & 4b & + & 3j \\
\hline
w_1 & = & 1800 & - & 15b & - & 10j \\
w_2 & = & 300 & - & 2b & - & 2j \\
w_3 & = & 200 & - & j & & \\
\hline
\zeta & = & 450 & + & b & - & \frac{3}{2}w_2 \\
\hline
w_1 & = & 300 & - & 5b & + & 5w_2 \\
j & = & 150 & - & b & - & \frac{1}{2}w_2 \\
w_3 & = & 50 & + & b & + & \frac{1}{2}w_2 \\
\hline
\zeta & = & 510 & - & \frac{1}{5}w_1 & - & \frac{1}{2}w_2 \\
\hline
b & = & 60 & - & \frac{1}{5}w_1 & + & w_2 \\
j & = & 90 & + & \frac{1}{5}w_1 & - & \frac{3}{2}w_2 \\
w_3 & = & 110 & - & \frac{1}{5}w_1 & + & \frac{3}{2}w_2
\end{array}
$$

Optimal choice: 60 GI Barb soldiers, 90 Joey dolls
Maximal profit: \$510

Exercise 8.7
(i)

$$
\begin{aligned}
\text{maximize} \quad & x_1 + 2x_2 \\
\text{subject to} \quad & -4x_1 - 2x_2 + x_3 = -8 \\
& -2x_1 + 3x_2 + x_4 = 6 \\
& x_1 + x_5 = 3 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}
$$

Auxiliary problem:

$$
\begin{aligned}
\text{maximize} \quad & -x_0 \\
\text{subject to} \quad & -4x_1 - 2x_2 + x_3 - x_0 = -8 \\
& -2x_1 + 3x_2 + x_4 - x_0 = 6 \\
& x_1 + x_5 - x_0 = 3 \\
& x_0, x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}
$$

ζ	$=$						$-$	x_0
x_3	$=$	-8	$+$	$4x_1$	$+$	$2x_2$	$+$	x_0
x_4	$=$	6	$+$	$2x_1$	$-$	$3x_2$	$+$	x_0
x_5	$=$	3	$-$	x_1			$+$	x_0
ζ	$=$	-8	$+$	$4x_1$	$+$	$2x_2$	$-$	x_3
x_0	$=$	8	$-$	$4x_1$	$-$	$2x_2$	$+$	x_3
x_4	$=$	14	$-$	$2x_1$	$-$	$5x_2$	$+$	x_3
x_5	$=$	11	$-$	$5x_1$	$-$	$2x_2$	$+$	x_3
ζ	$=$						$-$	x_0
x_1	$=$	2	$-$	$\frac{1}{2}x_2$	$+$	$\frac{1}{4}x_3$	$-$	$\frac{1}{4}x_0$
x_4	$=$	10	$-$	$4x_2$	$+$	$\frac{1}{2}x_3$	$+$	$\frac{1}{2}x_0$
x_5	$=$	1	$+$	$\frac{1}{2}x_2$	$-$	$\frac{1}{4}x_3$	$+$	$\frac{5}{4}x_0$
ζ	$=$	2	$+$	$\frac{3}{2}x_2$	$+$	$\frac{1}{4}x_3$		
x_1	$=$	2	$-$	$\frac{1}{2}x_2$	$+$	$\frac{1}{4}x_3$		
x_4	$=$	10	$-$	$4x_2$	$+$	$\frac{1}{2}x_3$		
x_5	$=$	1	$+$	$\frac{1}{2}x_2$	$-$	$\frac{1}{4}x_3$		
ζ	$=$	3	$+$	$2x_2$	$-$	x_5		
x_1	$=$	3			$-$	x_5		
x_4	$=$	12	$-$	$3x_2$	$-$	$2x_5$		
x_3	$=$	4	$+$	$2x_2$	$-$	$4x_5$		
ζ	$=$	11	$-$	$\frac{2}{3}x_4$	$-$	$\frac{7}{3}x_5$		
x_1	$=$	3			$-$	x_5		
x_2	$=$	4	$-$	$\frac{1}{3}x_4$	$-$	$\frac{2}{3}x_5$		
x_3	$=$	4	$-$	$\frac{2}{3}2x_4$	$-$	$\frac{16}{3}x_5$		

Optimal point: $(3, 4)$
Optimal value: 11

(ii)

$$\begin{aligned} \text{maximize} \quad & 5x_1 + 2x_2 \\ \text{subject to} \quad & 5x_1 + 3x_2 + x_3 = 15 \\ & 3x_1 + 5x_2 + x_4 = 15 \\ & 4x_1 - 3x_2 + x_5 = -12 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Auxiliary problem:

$$\begin{aligned}
\text{maximize} \quad & -x_0 \\
\text{subject to} \quad & 5x_1 + 3x_2 + x_3 - x_0 = 15 \\
& 3x_1 + 5x_2 + x_4 - x_0 = 15 \\
& 4x_1 - 3x_2 + x_5 - x_0 = -12 \\
& x_0, x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

$$\begin{array}{lcrcrcrcr}
\zeta & = & & & & & & - & x_0 \\
\hline
x_3 & = & 15 & - & 5x_1 & - & 3x_2 & + & x_0 \\
x_4 & = & 15 & - & 3x_1 & - & 5x_2 & + & x_0 \\
x_5 & = & -12 & - & 4x_1 & + & 3x_2 & + & x_0 \\
\hline\hline
\zeta & = & -12 & - & 4x_1 & + & 3x_2 & - & x_5 \\
\hline
x_3 & = & 27 & - & 9x_1 & & & + & x_5 \\
x_4 & = & 27 & - & 7x_1 & - & 2x_2 & + & x_5 \\
x_0 & = & 12 & + & 4x_1 & - & 3x_2 & + & x_5 \\
\hline\hline
\zeta & = & & & & & & - & x_0 \\
\hline
x_3 & = & 27 & - & 9x_1 & + & x_5 & & \\
x_4 & = & 19 & - & \frac{29}{3}x_1 & + & \frac{1}{3}x_5 & + & \frac{2}{3}x_0 \\
x_2 & = & 4 & + & \frac{4}{3}x_1 & + & \frac{1}{3}x_5 & - & \frac{1}{3}x_0 \\
\hline\hline
\zeta & = & 8 & + & \frac{23}{3}x_1 & + & \frac{2}{3}x_5 & & \\
\hline
x_3 & = & 27 & - & 9x_1 & + & x_5 & & \\
x_4 & = & 19 & - & \frac{29}{3}x_1 & + & \frac{1}{3}x_5 & & \\
x_2 & = & 4 & + & \frac{4}{3}x_1 & + & \frac{1}{3}x_5 & & \\
\hline
\end{array}$$

The problem is unbounded.

(iii)

$$\begin{aligned}
\text{maximize} \quad & -3x_1 + x_2 \\
\text{subject to} \quad & x_2 + x_3 = 4 \\
& -2x_1 + 3x_2 + x_4 = 6 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

$$\begin{array}{lcrcrcr}
\zeta & = & & - & 3x_1 & + & x_2 \\
\hline
x_3 & = & 4 & & & - & x_2 \\
x_4 & = & 6 & + & 2x_1 & - & 3x_2 \\
\hline\hline
\zeta & = & 2 & - & \frac{7}{3}x_1 & - & \frac{1}{3}x_4 \\
\hline
x_3 & = & 2 & - & \frac{2}{3}x_1 & + & \frac{1}{3}x_4 \\
x_2 & = & 2 & + & \frac{2}{3}x_1 & - & \frac{1}{3}x_4 \\
\hline
\end{array}$$

Optimal point: $(0, 2)$
Optimal value: 2

Exercise 8.8
Exercise 8.9

Exercise 8.10

Exercise 8.11

Exercise 8.12

$$
\begin{aligned}
\text{maximize} \quad & 10x_1 - 57x_2 - 9x_3 - 24x_4 \\
\text{subject to} \quad & 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 + x_5 = 0 \\
& 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 + x_6 = 0 \\
& x_1 + x_7 = 0 \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0
\end{aligned}
$$

ζ	$=$			$10x_1$	$-$	$57x_2$	$-$	$9x_3$	$-$	$24x_4$
x_5	$=$		$-$	$0.5x_1$	$+$	$1.5x_2$	$+$	$0.5x_3$	$-$	x_4
x_6	$=$		$-$	$0.5x_1$	$+$	$5.5x_2$	$+$	$2.5x_3$	$-$	$9x_4$
x_7	$=$	1	$-$	x_1						
ζ	$=$		$-$	$27x_2$	$+$	x_3	$-$	$44x_4$	$-$	$20x_5$
x_1	$=$			$3x_2$	$+$	x_3	$-$	$2x_4$	$-$	$2x_5$
x_6	$=$			$4x_2$	$+$	$2x_3$	$-$	$8x_4$	$+$	x_5
x_7	$=$	1	$-$	$3x_2$	$-$	x_3	$+$	$2x_4$	$+$	$2x_5$
ζ	$=$	1	$-$	$30x_2$	$-$	$42x_4$	$-$	$18x_5$	$-$	x_7
x_1	$=$	1							$-$	x_7
x_6	$=$	2	$-$	$2x_2$	$-$	$4x_4$	$+$	$5x_5$	$-$	$2x_7$
x_3	$=$	1	$-$	$3x_2$	$+$	$2x_4$	$+$	$2x_5$	$-$	x_7

Optimal point: $(1, 0, 1, 0)$ Optimum value: 1
Exercise 8.15

Exercise 8.17

Exercise 8.18