

Problem Set #5

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Exercise 8.1

Optimizer: $(\frac{14}{5}, \frac{16}{5})$

Exercise 8.2

Exercise 8.3

$$\begin{aligned} &\text{maximize} && 4b + 3j \\ &\text{subject to} && 15b + 10j \leq 1800 \\ &&& 2b + 2j \leq 300 \\ &&& j \leq 200 \\ &&& b, j \geq 0 \end{aligned}$$

Exercise 8.4

$$\begin{aligned} &\text{maximize} && 2x_{AB} + 5x_{AD} + 5x_{BC} + 2x_{BD} + 7x_{BE} + 9x_{BF} + 2x_{CF} + 4x_{DE} + 3x_{EF} \\ &\text{subject to} && x_{AB} + x_{AD} = 10 \\ &&& x_{BC} + x_{BD} + x_{BE} + x_{BF} - x_{AB} = 1 \\ &&& x_{CF} - x_{BC} = -2 \\ &&& x_{DE} - x_{AD} - x_{BD} = -3 \\ &&& x_{EF} - x_{BE} - x_{DE} = 4 \\ &&& -x_{BF} - x_{CF} - x_{EF} = -10 \\ &&& 0 \leq x_{AB}, x_{AD}, x_{BC}, x_{BD}, x_{BE}, x_{BF}, x_{CF}, x_{DE}, x_{EF} \leq 6 \end{aligned}$$

Exercise 8.5

(i)

$$\begin{aligned} &\text{maximize} && 3x_1 + x_2 \\ &\text{subject to} && x_1 + 3x_2 + w_1 = 15 \\ &&& 2x_1 + 3x_2 + w_2 = 18 \\ &&& x_1 - x_2 + w_3 = 4 \\ &&& x_1, x_2, w_1, w_2, w_3 \geq 0 \end{aligned}$$

ζ	=			$3x_1$	+	x_2
w_1	=	15	-	x_1	-	$3x_2$
w_2	=	18	-	$2x_1$	-	$3x_2$
w_3	=	4	-	x_1	+	x_2
ζ	=	12	+	$4x_2$	-	$3w_3$
w_1	=	11	-	$4x_2$	+	w_3
w_2	=	10	-	$5x_2$	+	$2w_3$
x_1	=	4	+	x_2	-	w_3
ζ	=	20	-	$\frac{4}{5}w_2$	-	$\frac{7}{5}w_3$
w_1	=	3	+	$\frac{4}{5}w_2$	-	$\frac{3}{5}w_3$
x_2	=	2	-	$\frac{1}{5}w_2$	+	$\frac{2}{5}w_3$
x_1	=	6	-	$\frac{1}{5}w_2$	-	$\frac{3}{5}w_3$

Optimizer: (6, 2)

Optimum value: 20

(ii)

$$\begin{aligned}
 &\text{maximize } 4x + 6y \\
 &\text{subject to } -x + 3x_2 + w_1 = 11 \\
 &\quad \quad \quad x + y + w_2 = 27 \\
 &\quad \quad \quad 2x + 5y + w_3 = 90 \\
 &\quad \quad \quad x, y, w_1, w_2, w_3 \geq 0
 \end{aligned}$$

ζ	=		$4x$	+	$6y$
w_1	=	11	+	x	- y
w_2	=	27	-	x	- y
w_3	=	90	-	$2x$	- $5y$
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ζ	=	66	+	$10x$	- $6w_1$
y	=	11	+	x	- w_1
w_2	=	16	-	$2x$	+ w_1
w_3	=	35	-	$7x$	+ $5w_1$
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ζ	=	116	+	$\frac{8}{7}w_1$	- $\frac{10}{7}w_3$
y	=	16	-	$\frac{2}{7}w_1$	- $\frac{1}{7}w_3$
w_2	=	6	-	$\frac{3}{7}w_1$	+ $\frac{2}{7}w_3$
x	=	5	+	$\frac{5}{7}w_1$	- $\frac{1}{7}w_3$
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ζ	=	132	-	$\frac{8}{3}w_2$	- $\frac{2}{7}w_3$
y	=	12	+	$\frac{2}{3}w_2$	- $\frac{1}{3}w_3$
w_1	=	14	-	$\frac{7}{3}w_2$	+ $\frac{2}{3}w_3$
x	=	15	-	$\frac{5}{3}w_2$	+ $\frac{1}{3}w_3$

Optimizer: (15, 12)
Optimum value: 132

Exercise 8.6

maximize $4b + 3j$
subject to $15b + 10j + w_1 = 1800$
 $2b + 2j + w_2 = 300$
 $j + w_3 = 200$
 $b, j, w_1, w_2, w_3 \geq 0$

$$\begin{array}{rclclcl}
\zeta & = & & 4b & + & 3j \\
\hline
w_1 & = & 1800 & - & 15b & - & 10j \\
w_2 & = & 300 & - & 2b & - & 2j \\
w_3 & = & 200 & - & j & & \\
\hline
\zeta & = & 450 & + & b & - & \frac{3}{2}w_2 \\
\hline
w_1 & = & 300 & - & 5b & + & 5w_2 \\
j & = & 150 & - & b & - & \frac{1}{2}w_2 \\
w_3 & = & 50 & + & b & + & \frac{1}{2}w_2 \\
\hline
\zeta & = & 510 & - & \frac{1}{5}w_1 & - & \frac{1}{2}w_2 \\
b & = & 60 & - & \frac{1}{5}w_1 & + & w_2 \\
j & = & 90 & + & \frac{1}{5}w_1 & - & \frac{3}{2}w_2 \\
w_3 & = & 110 & - & \frac{1}{5}w_1 & + & \frac{3}{2}w_2 \\
\hline
\end{array}$$

Optimal choice: 60 GI Barb soldiers, 90 Joey dolls

Maximal profit: \$510

Exercise 8.7

(i)

$$\begin{array}{ll}
\text{maximize} & x_1 + 2x_2 \\
\text{subject to} & -4x_1 - 2x_2 + x_3 = -8 \\
& -2x_1 + 3x_2 + x_4 = 6 \\
& x_1 + x_5 = 3 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{array}$$

Auxiliary problem:

$$\begin{array}{ll}
\text{maximize} & -x_0 \\
\text{subject to} & -4x_1 - 2x_2 + x_3 - x_0 = -8 \\
& -2x_1 + 3x_2 + x_4 - x_0 = 6 \\
& x_1 + x_5 - x_0 = 3 \\
& x_0, x_1, x_2, x_3, x_4, x_5 \geq 0
\end{array}$$

ζ	=					-	x_0	
x_3	=	-8	+	$4x_1$	+	$2x_2$	+	x_0
x_4	=	6	+	$2x_1$	-	$3x_2$	+	x_0
x_5	=	3	-	x_1			+	x_0
ζ	=	-8	+	$4x_1$	+	$2x_2$	-	x_3
x_0	=	8	-	$4x_1$	-	$2x_2$	+	x_3
x_4	=	14	-	$2x_1$	-	$5x_2$	+	x_3
x_5	=	11	-	$5x_1$	-	$2x_2$	+	x_3
ζ	=						-	x_0
x_1	=	2	-	$\frac{1}{2}x_2$	+	$\frac{1}{4}x_3$	-	$\frac{1}{4}x_0$
x_4	=	10	-	$4x_2$	+	$\frac{1}{2}x_3$	+	$\frac{1}{2}x_0$
x_5	=	1	+	$\frac{1}{2}x_2$	-	$\frac{1}{4}x_3$	+	$\frac{5}{4}x_0$
ζ	=	2	+	$\frac{3}{2}x_2$	+	$\frac{1}{4}x_3$		
x_1	=	2	-	$\frac{1}{2}x_2$	+	$\frac{1}{4}x_3$		
x_4	=	10	-	$4x_2$	+	$\frac{1}{2}x_3$		
x_5	=	1	+	$\frac{1}{2}x_2$	-	$\frac{1}{4}x_3$		
ζ	=	3	+	$2x_2$	-	x_5		
x_1	=	3			-	x_5		
x_4	=	12	-	$3x_2$	-	$2x_5$		
x_3	=	4	+	$2x_2$	-	$4x_5$		
ζ	=	11	-	$\frac{2}{3}x_4$	-	$\frac{7}{3}x_5$		
x_1	=	3			-	x_5		
x_2	=	4	-	$\frac{1}{3}x_4$	-	$\frac{2}{3}x_5$		
x_3	=	4	-	$\frac{2}{3}x_4$	-	$\frac{16}{3}x_5$		

Optimal point: (3, 4)

Optimal value: 11

(ii)

$$\begin{aligned}
& \text{maximize} && 5x_1 + 2x_2 \\
& \text{subject to} && 5x_1 + 3x_2 + x_3 = 15 \\
& && 3x_1 + 5x_2 + x_4 = 15 \\
& && 4x_1 - 3x_2 + x_5 = -12 \\
& && x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

Auxiliary problem:

$$\begin{aligned}
& \text{maximize} && -x_0 \\
& \text{subject to} && 5x_1 + 3x_2 + x_3 - x_0 = 15 \\
& && 3x_1 + 5x_2 + x_4 - x_0 = 15 \\
& && 4x_1 - 3x_2 + x_5 - x_0 = -12 \\
& && x_0, x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

ζ	=							-	x_0
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x_3	=	15	-	$5x_1$	-	$3x_2$	+	x_0	
x_4	=	15	-	$3x_1$	-	$5x_2$	+	x_0	
x_5	=	-12	-	$4x_1$	+	$3x_2$	+	x_0	
<hr/>									
ζ	=	-12	-	$4x_1$	+	$3x_2$	-	x_5	
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x_3	=	27	-	$9x_1$			+	x_5	
x_4	=	27	-	$7x_1$	-	$2x_2$	+	x_5	
x_0	=	12	+	$4x_1$	-	$3x_2$	+	x_5	
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ζ	=							-	x_0
<hr/>									
x_3	=	27	-	$9x_1$	+	x_5			
x_4	=	19	-	$\frac{29}{3}x_1$	+	$\frac{1}{3}x_5$	+	$\frac{2}{3}x_0$	
x_2	=	4	+	$\frac{4}{3}x_1$	+	$\frac{1}{3}x_5$	-	$\frac{1}{3}x_0$	
<hr/>									
ζ	=	8	+	$\frac{23}{3}x_1$	+	$\frac{2}{3}x_5$			
<hr/>									
x_3	=	27	-	$9x_1$	+	x_5			
x_4	=	19	-	$\frac{29}{3}x_1$	+	$\frac{1}{3}x_5$			
x_2	=	4	+	$\frac{4}{3}x_1$	+	$\frac{1}{3}x_5$			
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The problem is unbounded.

(iii)

$$\begin{aligned}
& \text{maximize} && -3x_1 + x_2 \\
& \text{subject to} && x_2 + x_3 = 4 \\
& && -2x_1 + 3x_2 + x_4 = 6 \\
& && x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

ζ	=		-	$3x_1$	+	x_2
<hr/>						
x_3	=	4			-	x_2
x_4	=	6	+	$2x_1$	-	$3x_2$
<hr/>						
ζ	=	2	-	$\frac{7}{3}x_1$	-	$\frac{1}{3}x_4$
<hr/>						
x_3	=	2	-	$\frac{2}{3}x_1$	+	$\frac{1}{3}x_4$
x_2	=	2	+	$\frac{2}{3}x_1$	-	$\frac{1}{3}x_4$
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Optimal point: $(0, 2)$

Optimal value: 2

Exercise 8.8

$$\begin{aligned}
 &\text{maximize} && 10x_1 - 57x_2 - 9x_3 - 24x_4 \\
 &\text{subject to} && 0.5x_1 - 1.5x_2 - 0.5x_3 + x_4 + x_5 = 0 \\
 &&& 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 + x_6 = 0 \\
 &&& x_1 + x_7 = 0 \\
 &&& x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0
 \end{aligned}$$

ζ	=			$10x_1$	−	$57x_2$	−	$9x_3$	−	$24x_4$
x_5	=		−	$0.5x_1$	+	$1.5x_2$	+	$0.5x_3$	−	x_4
x_6	=		−	$0.5x_1$	+	$5.5x_2$	+	$2.5x_3$	−	$9x_4$
x_7	=	1	−	x_1						
ζ	=		−	$27x_2$	+	x_3	−	$44x_4$	−	$20x_5$
x_1	=			$3x_2$	+	x_3	−	$2x_4$	−	$2x_5$
x_6	=			$4x_2$	+	$2x_3$	−	$8x_4$	+	x_5
x_7	=	1	−	$3x_2$	−	x_3	+	$2x_4$	+	$2x_5$
ζ	=	−	$30x_2$	−	$42x_4$	−	$18x_5$	− x_7	−	x_2
x_1	=	1							−	x_7
x_6	=	2	−	$2x_2$	−	$4x_4$	+	$5x_5$	−	$2x_7$
x_3	=	1	−	$3x_2$	+	$2x_4$	+	$2x_4$	−	x_7

Exercise 8.9

Exercise 8.10

Exercise 8.11

Exercise 8.12

Exercise 8.15

Exercise 8.17

Exercise 8.18