Problem Set #1

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Exercise 1

1. The state variables are the barrels of oil B and the price p_t.
2. The control variables are q_t, the quantity of oil the owner choses to sell at time t.
3. The transition equation is $B_{t+1} = B_t - q_t$
4. The sequence problem of the owner is

$$V(B) = \max_{q_1, q_2, \cdots} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t p_t q_t.$$

The Bellman equation is

$$V(B_t) = \max_{q_t} p_t q_t + \left(\frac{1}{1+r}\right) V(B_{t+1}).$$

5. The owner's Euler equation is

$$p_t = \left(\frac{1}{1+r}\right) p_{t+1}.$$

We substitute the transition equation into the Bellman equation

$$V(B_t) = \max_{B_{t+1}} p_t(B_t - B_{t+1}) + \left(\frac{1}{1+r}\right) V(B_{t+1}).$$

The first order condition says

$$\frac{dV}{dB_{t+1}} = -p_t + \frac{1}{1+r} V'(B_{t+1}) = 0 \implies p_t = \frac{1}{1+r} V'(B_{t+1}) \tag{1}$$

Additionally, using the envelope condition,

$$\frac{dV}{dB_t} = p_t - p_t \frac{dB_{t+1}}{dB_t} + \frac{1}{1+r} V'(B_{t+1}) \frac{dB_{t+1}}{dB_t} \implies V'(B_t) = p_t \quad \text{and} \quad V'(B_{t+1}) = p_{t+1}. \tag{2}$$

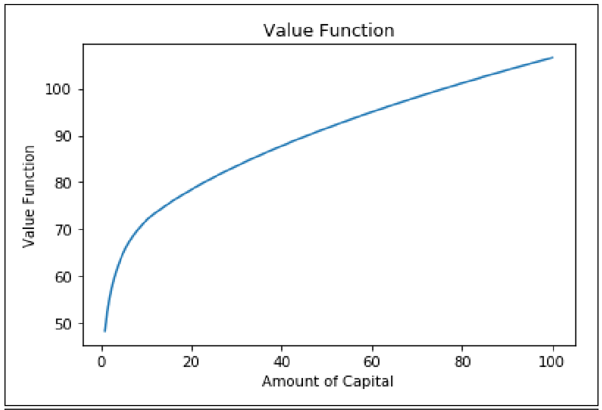
Substituting (2) into (1), gives us the Euler equation.

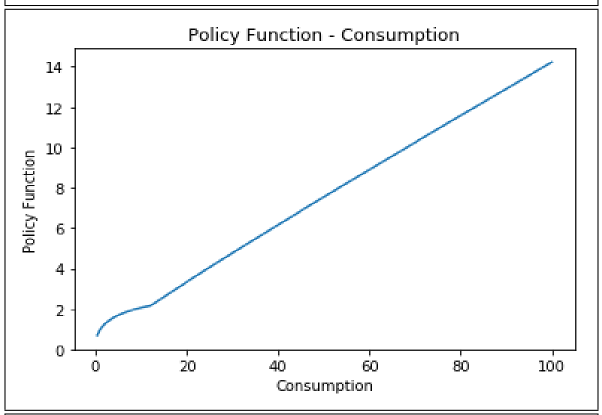
6. If $p_{t+1} = p_t$ for all t, the owner will sell all B barrels of oil in the first period. If $p_{t+1} > (1+r)p_t$ for all t, the owner will never sell any oil. The conditon on the path of prices necessary for an interior solution is $p_t < p_{t+1} < (1+r)p_t$.

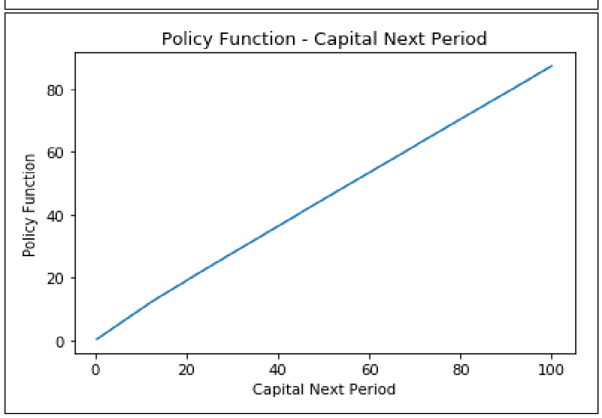
Exercise 2

1. The state variables are $k_t, z_t,$ and y_t.
2. The control variables are c_t and i_t. 3. The Bellman equation that represents this sequence problem is

$$V(k_t, z_t) = \max_{c} u(c_t) + \beta E[V(k_{t+1}, z_{z+1})].$$


Value Function
Value Function
100
90
80
70
60
50
0
20
40
60
80
100
Amount of Capital

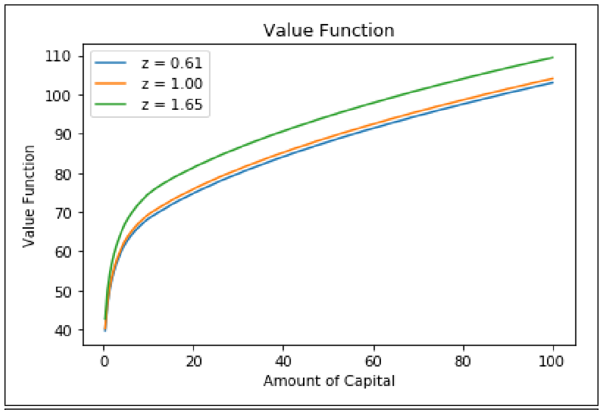

Policy Function - Consumption
Policy Function
14
12
10
8
6
4
2
0
0
20
40
60
80
100
Consumption

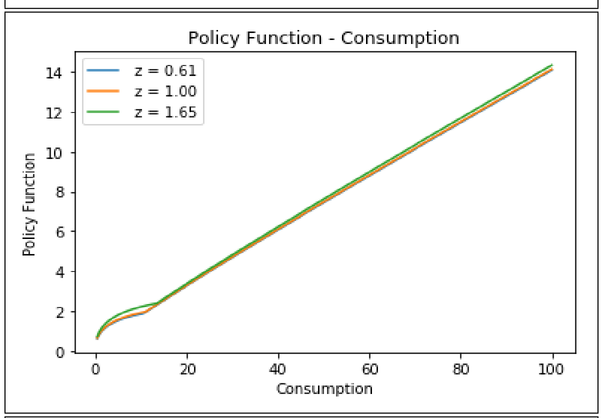

Policy Function - Capital Next Period
Policy Function
80
60
40
20
0
0
20
40
60
80
100
Capital Next Period

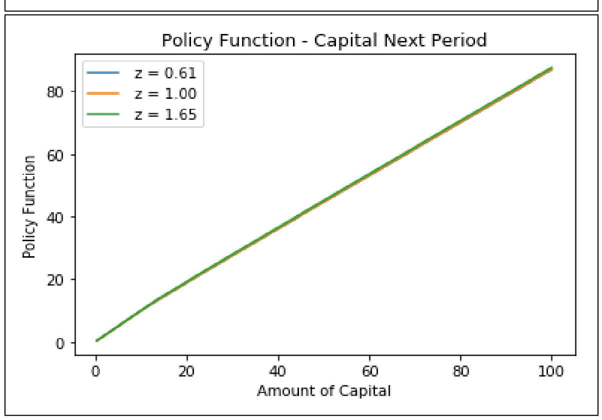
Exercise 3

1. The Bellman equation that represents this sequence problem is

$$V(k_t, z_t) = \max_{c} u(c_t) + \beta E_{z_{t+1}|z_t}[V(k_{t+1}, z_{z+1})].$$



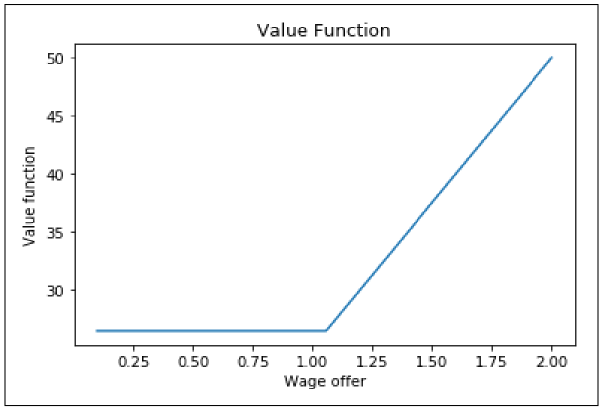




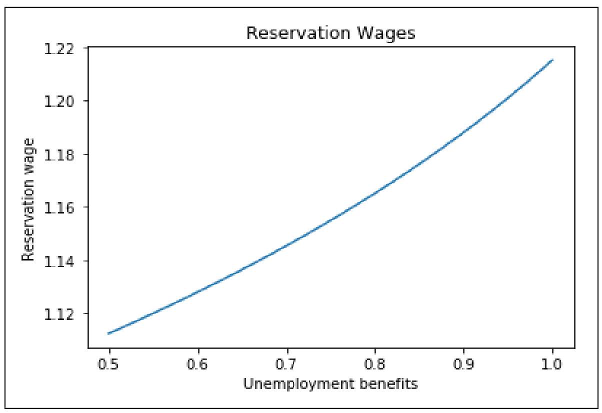
Exercise 4

The Bellman equation representing this optimal stopping problem is

$$V(w_t) = \max\{\frac{w_t}{1-\beta}, b + \beta E[V(w_{t+1})]\}.$$



The reservation wage for the unemployed worker that makes her indifferent between accepting the job offer and not for $b = 0.05$ is 1.057.



As unemployment benefits vary from 0.5 to 1.0, the reservation wage increases.