Econ 237, Winter 2022, Problem set 3, due Wednesday February 9

This problem set asks you to compute a model with labor income, borrowing constraint and two risky assets, equity and bonds. It will also make you familiar with the Survey of Consumer Finances.

We first adapt the problem from PS1 to a finite life cycle. We assume that agents live for 36 periods starting at age 25 ($t = 0$) and ending at age 60 ($t = T = 35$). Agents can save only in bonds. The budget constraint and borrowing constraint are

$$w_{t+1} = (w_t - c_t) R^f + y_{t+1} \qquad c_t \leq w_t, ....t = 0, .., T$$

where $w_t$ is initial cash on hand.

Preferences are represented by

$$\sum_{t=0}^{T-1} \beta^t u(c_t) + \beta^T \phi u(w_T)$$

We introduce the parameter $\phi$ to stand in for agents' need for funds at the end of working life.

1. *Wealth data.* Download the 2019 Survey of Consumer Finances data from the Fed website (the public extract file will do). Look for household net worth. Understand how weighting of households works to compute statistics for the US population (browse file to find weights; detailed description is on SCF website).

To make the income process from PS1 (which is for persons) comparable with the SCF data (where the unit of observations is a household), we use equivalence scales, that is, we divide household net worth by the "effective" number of household members. See explanation here

www.oecd.org/eco/growth/OECD-Note-EquivalenceScales.pdf

Select a scale you find reasonable, and use the number of adults in the household to derive net worth of the household head (viewed as a person).

Make a picture of average net worth by 10 year age cohorts to get an idea of savings over the life cycle.

2. *Numbers for income, returns and bequest motive.* Use an age profile and residual annual income process you estimated in PS1 to set up an age dependent income process. Choose one particular demographic profile that you find interesting. Set the interest rate on bonds to 2% (i.e. $R^f = 1.02$) and the discount factor to $\beta = .98$. We then need one more parameter, namely the parameter $\phi$ on old age utility. We will use the average level of net worth for an agent of your demographic profile at age 60 to calibrate $\phi$ (more below)..

3. *Separable expected utility.* Adapt your code from PS1 to the finite horizon case. Write it so it can compute optimal choice for any $\phi$ and simulate lifecycle income profiles as well as associated savings.

Fix $\gamma = 5$ as in PS1 and calibrate $\phi$ such that the code matches average $w_T$ to your average net worth number at age 60 (to do this, write an outer loop that experiments with $\phi$; it may pay off here to use your economic insight into the problem to pick starting values and adjust the values depending on what comes out in a given iteration).

Once the calibration is complete, use the simulated data to compute and plot average age profiles for savings. Compare to the cross sectional plot from part 1 and interpret the differences.

4. *Epstein-Zin utility* Assume now that agents have Epstein-Zin utility with IES $\sigma$ and CRRA $\gamma$. The utility process satisfies

$$V_t^{1-\frac{1}{\sigma}} = c_t^{1-\frac{1}{\sigma}} + \beta \left( E_t V_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\sigma}}{1-\gamma}}$$

for $t = 1,,.44$, with $V_T = \phi w_T$. Adapt your code to EZ utility. Test it with $\gamma = 1/\sigma = 5$, and then recompute using $\gamma = 5$ and $\sigma = 2$. Again plot age profiles.

5. *Portfolio choice.* Agents now choose between equity and bonds, both of which they cannot sell short. The budget constraint becomes

$$w_{t+1} = (w_t - c_t) \left( (1 - \alpha_t) R^f + \alpha_t R_{t+1} \right) + y_{t+1}$$

where $R_t^f$ is the gross interest rate on bonds, $R_{t+1}$ is the gross return on equity and $y_t$ is labor income. Since short sale constraints are prohibited, $\alpha$ must lie in the interval $[0, 1]$.

Adapt your code to allow choice of $\alpha$. Assume that the return on equity is iid and independent of labor income. Make a reasonable choice for the volatility of the return. (Remark: portfolio choice with low risk aversion & risk can be numerically unstable, since assets are almost perfect substitutes. If you run into trouble, experiment first with higher vol. Also corner solutions are a possibility!)

Again plot average savings as a function of age, but now also the split of savings into equity and bonds.