Introduction

Population growth is deemed to be continuous over time and there is an increase in growth rate as time passes. This scenario depicts the exponential function. Population growth of organisms, increase of money in banks, as well as decay of a substance, are some of the real-life occurrences where exponential functions are involved.

Exponential Expression

An **exponential expression** is an expression of the form $a \cdot b^{x-c} + d$, where $b > 0$ but $b \neq 1$.

The definitions of exponential function, exponential equation, and exponential inequality are shown below.

	Exponential Equation	**Exponential Inequality**	**Exponential Function**
Definition	An equation involving exponential expressions	An inequality involving exponential expressions	Function of the form $f(x)=b^x$ $f(x) = b^x$, where $b>0$ $b > 0$, $b \neq 1$ $b \neq 1$
Example	$7^{2x-x^2} = 1/343$	$5^{2x} - 5^{x+1} \leq 0$	$f(x)=(1.8)^x$ or $y=(1.8)^x$

Note that an exponential equation or an exponential inequality can be solved for all *x* values that satisfy the equation or inequality.

On the other hand, an exponential function is not 'solved.' Rather, it expresses a relationship between two variables (such as *x* and *y*) and can be represented by a table of values or a graph.

Solving Exponential Equation

Solving Exponential Equations

One-to-One Property of Exponential Functions

If **b>0** such that **b≠1**, then **b^x=b^y** if and only if **x=y**, for any real numbers *x* and *y*.

Note:

1. This property tells us that if the two sides of equality have the same bases then we can “bring down” the exponent.

2. Apply Laws on Exponents if both sides have different bases.

Example 1: Solve for *x* in the exponential equation **7^x=49.**

Solution:

Write 49 as a power of 7.

7^x=49

7^x=72

Since the bases are already equal, we can now apply the One-to-One Property of Exponential Equations. Hence, **x=2.**

Example 2: **Solve 16^x−3 = 8^2x+1.**.

Solution:

Although 16 is a multiple of 8, there is no whole number exponent to express 16 as a power of 8. So we need to find another positive base such that we can express 16 and 8 as the power of that number with the whole number exponent. We choose 2 as the new base since 24 = 16 and 23 = 8.

16^x−3 = 8^2x+1

(2^4)^x−3 = (2^3)^2x+1

(apply Power of a Product Law on Exponents)

2^4x−12 = 2^6x+3

(apply the One-to-One Property of Exponential Equations)

4x−12 = 6x+3

−12−3 = 6x−4x

x=−15/2 or x=−7.5

***Example 3*: Solve $(25/9)^{x-2} = (3/5)^{x+1}$.**
Solution:
Note that **$9 = 3^2$ and $25 = 5^2$ so**

$$25/9 = 5^2/3^2 = (5/3)^2$$

Recalling that a negative exponent reciprocates the fraction, we can make the two bases equal by using −2 instead of 2 so that

$$(5/3)^2 = (3/5)^{-2}.$$

Hence,

$$(25/9)^{x-2} = (3/5)^{x+1}$$

$$[(3/5)^{-2}]^{x-2} = (3/5)^{x+1}$$

(apply Power of Products Law on Exponents)

$$(3/5)^{x-2x+4} = (3/5)^{x+1}$$

(apply the One-to-One Property of Exponential Equations)

$$-2x+4 = x+1$$

$$-2x-x = 1-4$$

$$9x = 1$$

Exponential Inequality

<u>Solving Exponential Inequalities0</u>

<u>Property of Exponential Inequalities</u>

For any real numbers *x* and *y*:

(i) If **$b>1$**, then **$b^x>b^y$** if and only **if $x>y$**. Likewise, i**f $b>1$**, then **$b^x<b^y$** if and only if **$x<y$.** [Hint: exponents and base with same direction of inequality]
(ii) If **$0<b<1$**, then **$b^x>b^y$** if and only **if $x<y$**. Likewise, **if $0<b<1$**, then **$b^x<b^y$** if and only if **$x>y$**. [Hint: exponents and base with opposite direction of inequality]

Note:
1. The property tells us that if the two sides of equality have the same bases then we can “bring down” the exponent.
2. Apply the Laws on Exponent if both sides have different bases.
3. The property is also true for **≥ and ≤.**

Example 4: **Solve for *x* in the exponential inequality $2^{3x+1} \geq 4^{x-3}$**

Solution:

$2^{3x+1} \geq 4^{x-3}$

$2^{3x+1} \geq (2^2)^{x-3}$

$2^{3x+1} \geq 2^{2x-6}$

[apply the Property of Exponential Inequalities (i) since the base 2 is in *b > 1*]

$3x+1 \geq 2x-6$

$x \geq -7$ or $[-7,\infty)$

Example 5: **Solve $(0.027)^{2x-3} < (0.3)^{3x}$.**
Solution:

$[(0.3)3]^{2x-3} < (0.3)^{3x}$

$(0.3)^{6x-9} < (0.3)^{3x}$

[apply the Property of Exponential Inequalities (ii) since the base 0.3 is in *0 < b < 1*]

$6x-9 > 3x$

$x>3$ or $(3,\infty)$

Example 6**: Solve $(1/4)^{x-1} \geq 2^{3x+2}$**

Solution:

$(1/4)^{x-1} \geq 2^{3x+2}$

$[(1/2)^2]^{x-1} \geq 2^{3x+2}$

$[(2)^{-2}]^{x-1} \geq 2^{3x+2}$

$2^{-2x+2} \geq 2^{3x+2}$

[apply the Property of Exponential Inequalities (i) since the base 2 is in *b > 1*]

$-2x+2 \geq 3x+2$

$0 \geq x$ or $x \leq 0$ or $(-\infty,0]$

Exponential Function

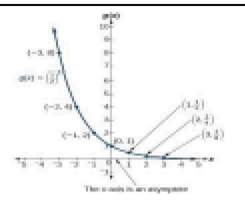
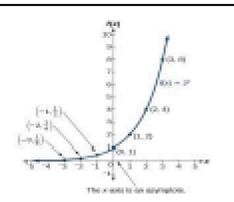
Exponential Function – an exponential function with base *b* is a function of the form ***f(x) = bx* or *y = bx*** where *b* is a positive number not equal to 1. Some examples of exponential functions are **f(x)=2^x, y=(1/5)6x, g(x)=3^2x+1, and *h*(x)=−4^x.**

Note that the base in ***h*(x)=−4^x is 4 not −4**. The negative sign acts as multiplier to 4x.

On the other hand, the function **y=(−4)^x** is **not** an exponential function because the base is −4, which is not a positive number .

<u>Properties of Exponential Functions</u>

Properties of Exponential Functions

<table>
<tr><td></td><td>When the base is between 0 and 1</td><td>When the base is greater than 1.</td></tr>
<tr><td>Graph:</td><td></td><td></td></tr>
<tr><td>y-intercept</td><td colspan="2">$y = 1$ or the point $(0,1)$</td></tr>
<tr><td>x-intercept</td><td colspan="2">NONE</td></tr>
<tr><td>Horizontal asymptote</td><td>x −axis

The graph approaches the right side of x −axis. It implies that as the exponent becomes larger, the exponential becomes closer to 0.</td><td>x −axis

The graph approaches the left side of x −axis. It implies that as the exponent becomes smaller, the exponential becomes closer to 0.</td></tr>
<tr><td>Domain</td><td colspan="2">The set of all real numbers.</td></tr>
<tr><td>Vertical Asymptote</td><td colspan="2">NONE</td></tr>
<tr><td>Range</td><td colspan="2">All positive real numbers.

Since the graphs do not touch the x −axis then the exponential cannot be zero. Also, the graphs stay above x −axis so the exponential function cannot have negative values.</td></tr>
</table>

Example 7: Complete a table of values of *x* = 3,2, 1, 0, 1, 2, and 3 for the exponential functions
y=(1/3)^x, y=10^x, and y=(0.8)^x.

Answers:

x	-3	-2	-1	0	1	2	3
$y=\left(\frac{1}{3}\right)^x$	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$
$y=10^x$	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
$y=(0.8)^x$	1.953125	1.5625	1.25	1	0.8	0.64	0.512

Example 8: **If f(x)=3^x, evaluate f(2), f(−2), f(1/2), f(0.4), and f(π).**

Solution and Answers:

$$f(2)=3^2=9$$

$$f(-2)=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$$

$$f\left(\frac{1}{2}\right)=3^{1/2}=\sqrt{3}$$

$$f(0.4)=3^{0.4}=3^{2/5}=\sqrt[5]{3^2}=\sqrt[5]{9}$$

Since **π≈3.14159** is irrational, the rules for rational exponents are not applicable. We can evaluate **f(π)=3^π.**
Many real-life applications involve transformations of exponential functions.

Transformation of an Exponential Function

Definition: Let *b* be a positive number not equal to 1. A transformation of an exponential function with base *b* is a function of the form **g(x)=a⋅b^x−c+d** where *a*, *c*, and *d* are real numbers.

Example 9: Let *t* = time in days. At *t* = 0, there were initially 20 bacteria. Suppose that the bacteria doubles every 100 hours. Give an exponential model for the bacteria as a function of *t*.

Solution and Answers:

$$\begin{array}{lll} \text{Initially,} & \text{at } t = 0 & \text{Number of bacteria} = 20 \\ & \text{at } t = 100 & \text{Number of bacteria} = 20(2) \\ & \text{at } t = 200 & \text{Number of bacteria} = 20(2)^2 \\ & \text{at } t = 300 & \text{Number of bacteria} = 20(2)^3 \\ & \text{at } t = 400 & \text{Number of bacteria} = 20(2)^4 \end{array}$$

Solution. An exponential model for this situation is $y = 20(2)^{t/100}$.

While an exponential function may have various bases, a frequently-used base is the irrational number **e≈2.71828**. This is called the Euler's number.

The Natural Exponential Function

The **natural exponential function** is the function **f(x)=e^x.**

*Note that when substituting values of this function in the scientific calculator, you do not need to substitute 2.71828. You should find *e* in your sci. cal. instead.

Example 10: A large slab of meat is taken from the refrigerator and placed in a preheated oven. The temperature *T* of the slab *t* minutes after being placed in the oven is given by **T=17.01e^0.005t** degrees Celsius. Construct a table of values for the following values of *t:* 0, 10, 20, 30, 40, 50, and interpret your results. Round off values to the nearest integer.

Solution and Answers:

t	0	10	20	30	40	50
T	17	18	19	20	21	22

Thus, we can see that the slab of meat is increasing in temperature roughly at the same rate.

Graphing Exponential Functions

An exponential function can be represented by its table of values, graph, and equation. For example, the exponential function **f(x)=2^** is also an equation whose table of values and graph are shown below.

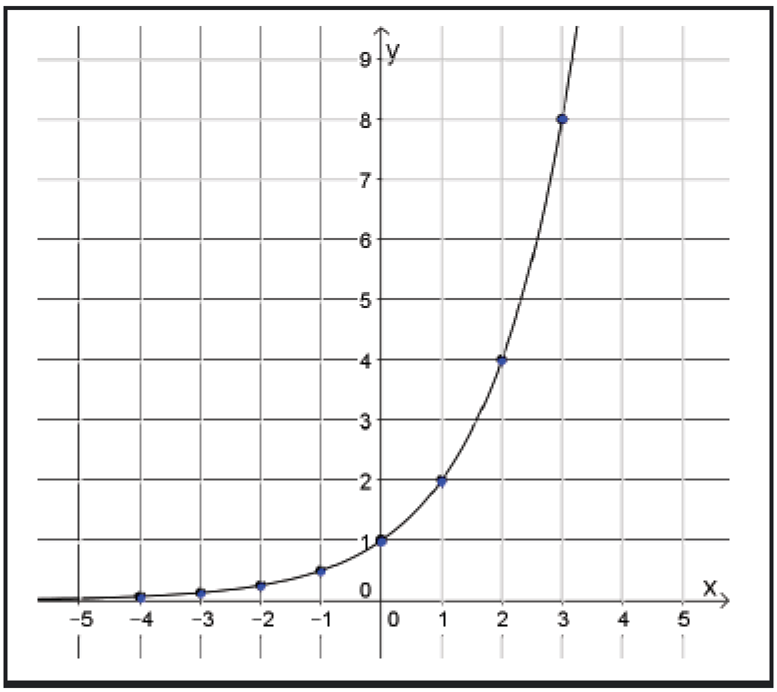
Table of Values

If the following values of *x* are substituted, *f(x)* will have the corresponding values.

x	−4	−3	−2	−1	0	1	2	3
f(x)	116	18	14	12	1	2	4	8

Graph

On a graphing paper, plot the values from the table above as ordered pairs where *x* is the first coordinate and *f(x)* is the second coordinate. Then connect the points with a smooth curve. The graph will look like this:



Observe that the function is defined for all values of *x*, is strictly increasing, and attains only positive y-values. As *x* decreases without bound, the function approaches 0. That is, the line *y = 0* is a horizontal asymptote.

Graphing Transformations of Exponential Functions Adding different parameters to a function affects its graph. The graph of the exponential function can be shrunk, stretched, shifted vertically or horizontally, and reflected about the *x*- and y- axes. For this part of this lesson, we will look at the different transformations of the graph of the exponential function **f(x)=2^x** by adding different parameters to it.

1. Reflection:

a. The graph of **y=−b^x** is the reflection about the *x*-axis of the graph of **y=b^x**

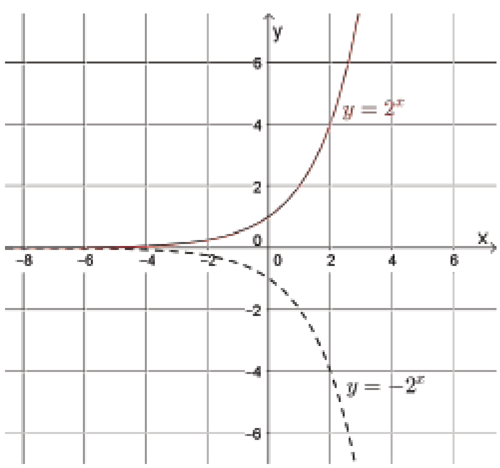
.

b. The graph of **y=b^−x** is the reflection about the *y*-axis of the graph of **y=b^x.**

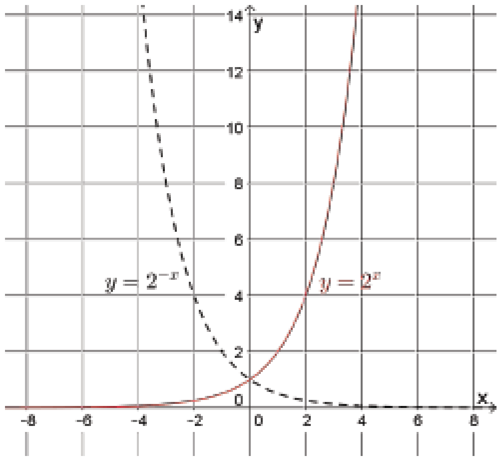
Example 11: Use the graph of **y=2^x** to graph the functions **y=−2^x and y=2^−x**.
Solution and Answers:
Substitute values of *x* in each function to form a table of values:

x	-3	-2	-1	0	1	2	3
$y=2^x$	0.125	0.25	0.5	1	2	4	8
$y=-2^x$	-0.125	-0.25	-0.5	-1	-2	-4	-8
$y=2^{-x}$	8	4	2	1	0.5	0.25	0.125

The *y*-coordinate of each point on the graph of **y=−2^x** is the negative of the corresponding *y*-coordinate of the graph of **y=2^x**. Thus, the graph of **y=−2^x** is the reflection of the graph of **y=2^x** about the *x*-axis.



The value of **y=2^−x** at **x** is the same as that of **y=2^x** at **−x**. Thus, the graph of **y=2^−x** is the reflection of the graph of **y=2^x** about the *y*-axis.



2. Vertical Stretch/Shrink:

Let **a>0**, meaning a is positive.The *y*-coordinate of each point in the graph of **y=a(b^x)** is equal to the *y*-coordinate of each point multiplied by a.

2.a. When **a>1**, the graph of **y=b^x** is stretched vertically.

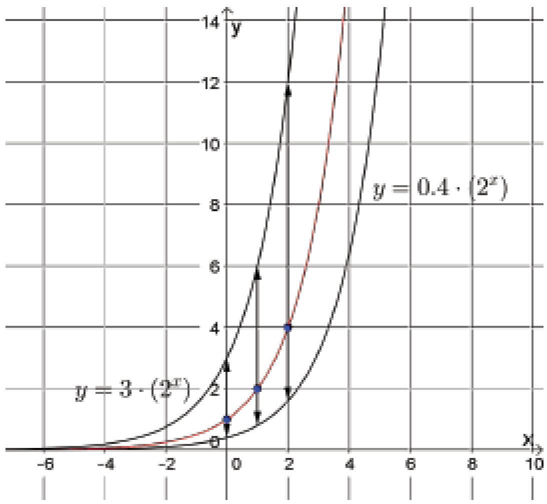
2.b. When **0<a<1**, the graph of **y=b^x** is shrunk vertically.

Note: The *y*-intercept of **y=a(b^x)** is **(0,a).**

Example 12: Use the graph of **y=2^x** to graph the functions **y=3(2^x)** and **y=0.4(2^x).**
Solution and Answers:
Substitute values of *x* in each function to form a table of values:

x	-3	-2	-1	0	1	2	3
$y = 2^x$	0.125	0.25	0.5	1	2	4	8
$y = 3(2^x)$	0.375	0.75	1.5	3	6	12	24
$y = 0.4(2^x)$	0.05	0.1	0.2	0.4	0.8	1.6	3.2

The *y*-coordinate of each point on the graph of **y=3(2^x)** is 3 times the *y*-coordinate of the corresponding point on **y=2^x**. Similarly, the *y*-coordinate of each point on the graph of **y=0.4(2^x)** is 0.4 times the *y*-coordinate of the corresponding point on **y=2^x**. The graphs of these functions are shown below.



3. Vertical Shifts: Let **d** be a real number.The *y*-coordinate of each point in **y=b^x+d** is equal to the *y*-coordinate of each point in **y=b^x** increased by **d.**

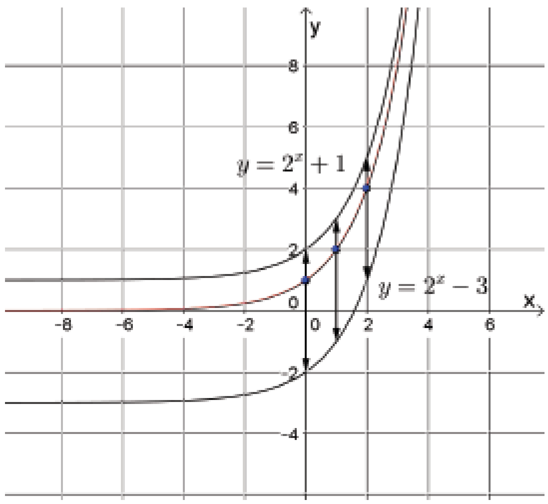
3a. If **d>0**, meaning d is positive, the graph of **y=b^x** shifts **d** units upward.
3b. If **d<0**, meaning d is negative, the graph of **y=b^x** shifts **d** units downward.

Note: The *y*-intercept and the horizontal asymptote of **y=b^x** will also shift accordingly when **d** is added.

Example 13: Use the graph of y=2^x to graph the functions y=2^x −3 and x=2^x +1.
Solution and Answers:
Substitute values of *x* in each function to form a table of values:

x	-3	-2	-1	0	1	2	3
$y = 2^x$	0.125	0.25	0.5	1	2	4	8
$y = 2^x - 3$	-2.875	-2.75	-2.5	-2	-1	1	5
$y = 2^x + 1$	1.125	1.25	1.5	2	3	5	9

Plotting the ordered pairs for each function, we see that the graph of **y=2^x −3** is **y=2^x** shifted 3 units downward while the graph of **y=2^x +1** is **y=2^x** is shifted 1 unit upward.



4. Horizontal Shifts:

Let **c** be a real number.The *x*-coordinate of each point in **y=b^x−c** is equal to the *x*-coordinate of each point in **y=b^x** decreased by 2.

4.a. If **c>0**, the graph of **y=b^x** shifts **c** units to the right.

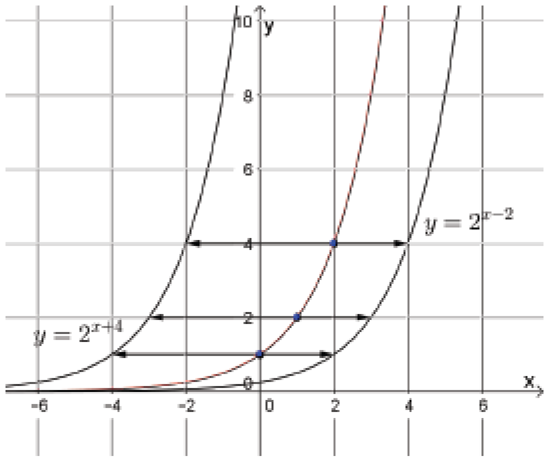
4b. If **c<0**, the graph of **y=b^x** shifts **c** units to the left.

Note: the *y*-intercept of **y=b^x** will also shift accordingly when **c** is added .

Example 14: Use the graph of **y=2^x** to graph the functions **y=2^x−2** and **y=2^x+4.**
Solution and Answers:
Substitute values of *x* in each function to form a table of values:

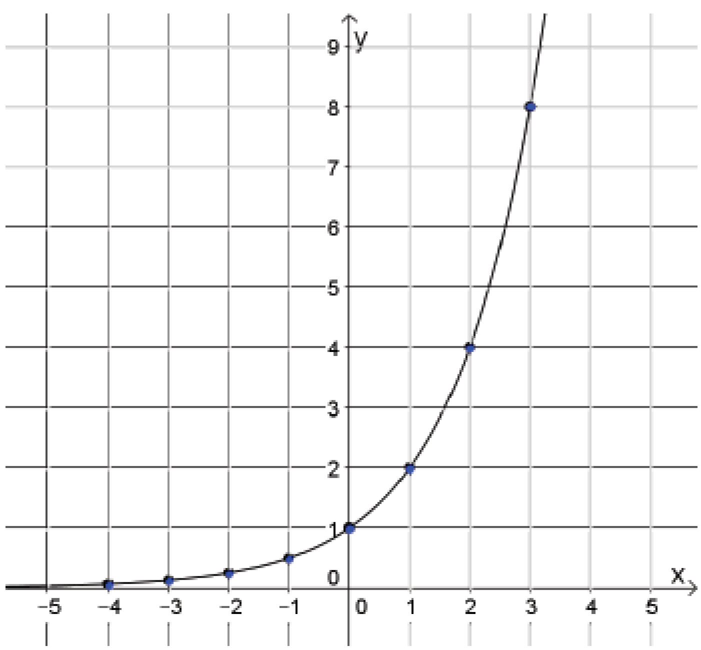
x	-3	-2	-1	0	1	2	3
$y = 2^x$	0.125	0.25	0.5	1	2	4	8
$y = 2^{x-2}$	0.031	0.063	0.125	0.25	0.5	1	2
$y = 2^{x+4}$	2	4	8	16	32	64	128

Plotting the ordered pairs for each function, we see that the graph of **y=2^x−2** is **y=2^x** shifted 2 units to the right while the graph of **y=2^x+4** is **y=2^x** is shifted 4 units to the left.



* The <u>domain of all exponential functions of the form f(x)=b^x</u> is always the <u>set of all real numbers</u>.
* The <u>range of all exponential functions of the form f(x)=b^x</u> is always the <u>set of all positive real numbers</u>.

For example, the exponential function **f(x)=2^x**, which has a base that is greater than 1, has the following graph and properties:



Property	**Answer**	**Explanation**
y-intercept:	1	The curve intersects with the *y*-axis at the point *(0,1).*
x-intercept:	None	The graph does not intersect with the *x*-axis.
Horizontal Asymptote:	*x*-axis	The graph approaches the left side of axis. As *x* decreases without bound, the function approaches 0.
Domain:	The set of all real numbers. In symbol forms: R or $(-\infty,\infty)$	The function is defined for all values of *x*.
Vertical Asymptote:	None	There is no value of *x* that will make the function undefined.
Range:	All positive real numbers. In symbol forms: $\{y \mid y>0\}$ or $(0,\infty)$	Since the graph does not touch the axis then the exponential function cannot be zero. Also, the graph stays above axis so the function cannot have non-positive values.

Also note that this exponential function is increasing. We can tell if a graph of a function is increasing if it goes higher as we move from left to right.

Now, how about the properties of the transformations of exponential functions?

Recall that transformations of exponential functions are of the form

a · b^x−c +d, where b>0, b≠1.

Their properties are:

Properties of Transformation of Exponential Functions

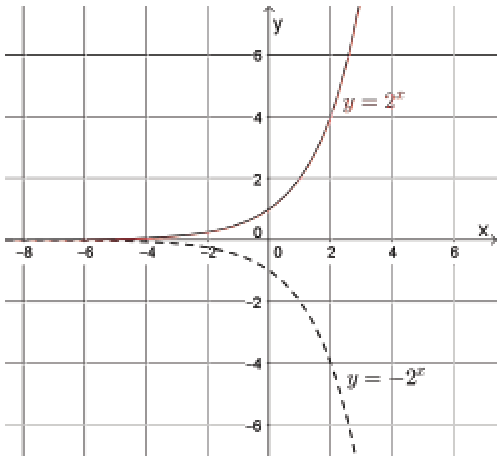
Properties	$y = b^x$	$y = a(b^{x-c}) + d$
y-intercept	$(0,1)$	$(0, a(b^{-c}) + d)$
x-intercept	None	May or may not have
Horizontal Asymptote	x −axis or $y = 0$	$y = d$
Vertical Asymptote	None	
Domain	All real numbers	
Range	$(0, \infty)$	(d, ∞) when $a > 0$ $(-\infty, d)$ when $a < 0$

Let us recall the different transformations of the graph of the exponential function

�(�)=2� and observe their properties.

1. Reflection:

Example 15: Find the properties of the graphs of **y=−2^x** and **y=2^−x**.
Solution and Answers:
The original function is **y=2^x**, which is then reflected about the *x-axis* as **y=−2^x**.
The graph of **y=−2^x** is shown below.



Properties:

The domain of y=−2^x is the set of all real numbers R or (−∞,∞).
The range of y=−2^ is the set of all negative real numbers {y∣y<0} or (−∞,0) because now the graph is *below the* x-*axis*.
The graph of y=−2^x does not have a vertical asymptote.
The graph of y=−2^x still has the same horizontal asymptote, which is the line y=0 or the *x*-axis.
The *y*-intercept of each point on the graph of y=−2x is the negative of the corresponding *y*-coordinate of the graph of y=2^x.
The original function y=2x has a *y*-intercept of 1. The negative of this is −1, which would then be the *y*-intercept of y=−2^x.
The graph does not have an *x*-intercept.

Aside from observing the graphs, we could also find the properties of these exponential functions by calculation using the formulae in the table of properties above.

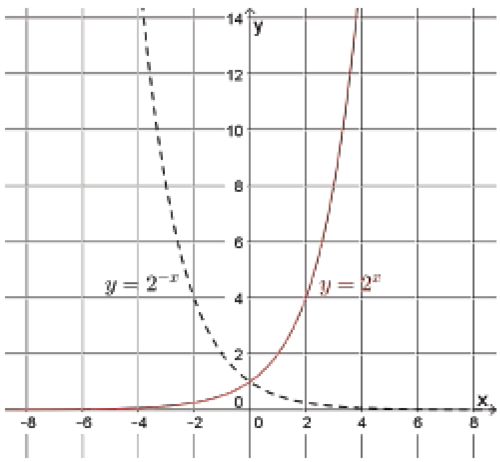
For y=−2^x, a=−1,b=2,c=0,d=0 because y=−1(2^x)=−1(2^x−0)+0=y=a(b^x−c)+d.

Thus, its:

- *y*-intercept is in the point
- (0,a(b^−c)+d)=(0,−1(2^−0+0))=(0,−1(1+0))=(0,−1(1))=(0,−1)
- The *y*-intercept of y=−2^x is −1;

- *x*-intercept of $y=-2^x$ is the value of *x* when *y = 0*. We arrive with $0=-2^x$ which will give us a non-real number because exponential functions cannot be non-positive. Similar to observing the graph, we see that there is no *x*-intercept;
- horizontal asymptote is $y=d \Rightarrow y=0$;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is $(-\infty,d)=(-\infty,0)$because $a=-1$ which is less than 0, i.e. negative.

Next, the original function is $y=2^x$, which is then reflected about the *y-axis* as $y=2^{-x}$. The graph of $y=2^{-x}$ is shown below.



Properties:

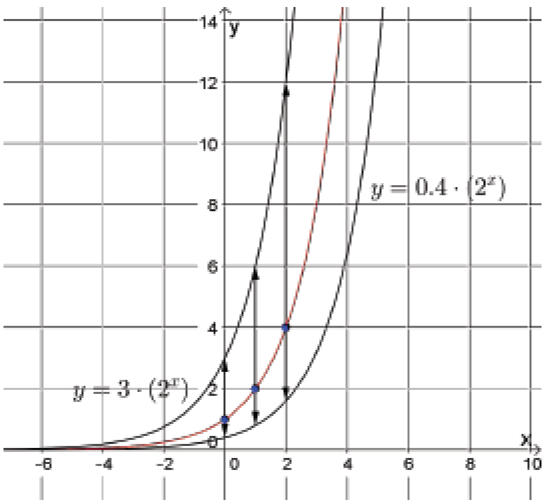
- The domain of $y=2^{-x}$ is the set of all real numbers R or $(-\infty,\infty)$.
- The range of $y=2^{-x}$is the set of all positive real numbers $\{y \mid y>0\}$ or $(0,\infty)$ because now the graph is *above the* x-*axis*.
- The graph of $y=2^{-x}$does not have a vertical asymptote.
- The graph of $y=2^{-x}$ still has the same horizontal asymptote, which is the line $y=0$ or the *x*-axis.
- The *y*-intercept of each point on the graph of $y=2^{-x}$ at x is the same as that of $y=2^x$ at $-x$. The original function $y=2^x$ has a *y*-intercept of 1. The reflection of this is about the *y*-axis is still 1, as seen in the graph. Thus, 1 would then be the *y*-intercept of $y=2^{-x}$.
- To find the *y*-intercept of $y=2^{-x}$, we could also set *x = 0*, because the *y*-intercept is the value of *y* when *x = 0*. Doing this, $y=2^{-x}=2^{-0}=2^0=1$. Thus, the
- *y*-intercept of $y=2^{-x}$ is indeed 1.
- The graph does not have an *x*-intercept.

2. Vertical Stretch/Shrink:

Example 16: Find the properties of the graphs of **y=3(2^x)** and **y=0.4(2^x)**

Solution and Answers:

The original function is **y=2^x**, which is then vertically stretched as **y=3(2^x)** and vertically shrunk as **y=0.4(2^x).**
The graphs of these functions are shown below.



Properties:

- The domain for all three functions is the set of all real numbers Ror (−∞,∞).
- The range of all three functions is the set of all positive real numbers {y∣y>0}or (0,∞).
- All three graphs do not have vertical asymptotes.
- All three graphs have the same horizontal asymptote, which is the line y=0 or the *x*-axis.
- The *y*-intercepts were also multiplied according to the transformations. The *y*-intercept of y=3(2^x)is 3 while the *y*-intercept of y=0.4(2^x) is 0.4.
- All three graphs do not have *x*-intercepts.

Aside from observing the graphs, we could also find the properties of these exponential functions by calculation using the formulae in the table of properties above.

For y=3(2^x), a=3,b=2,c=0,d=0 because y=3(2^x)=3(2^x−0)+0=y=a(b^x−c)+d.

Thus, its:

- *y*-intercept is in the point (0,a(b^−c)+d)=(0,3(2^−0+0))=(0,3(1+0))=(0,3(1))=(0,3). The *y*-intercept of y=3(2^x) is 3;

- *x*-intercept of $y=3(2^x)$ is the value of *x* when *$y = 0$*. We arrive with $0=3(2^x)$
- , which will give us a non-real number because exponential functions cannot be non-positive. Similar to observing the graph, we see that there is no *x*-intercept;
- horizontal asymptote is $y=d \Rightarrow y=0$;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is $(d,\infty)=(0,\infty)$ because $a=1$ which is greater than 0.

For $y=0.4(2^x)$, $a=0.4,b=2,c=0,d=0$ because $y=0.4(2^x)=0.4(2^{x-0})+0=y=a(b^{x-c}+d$.

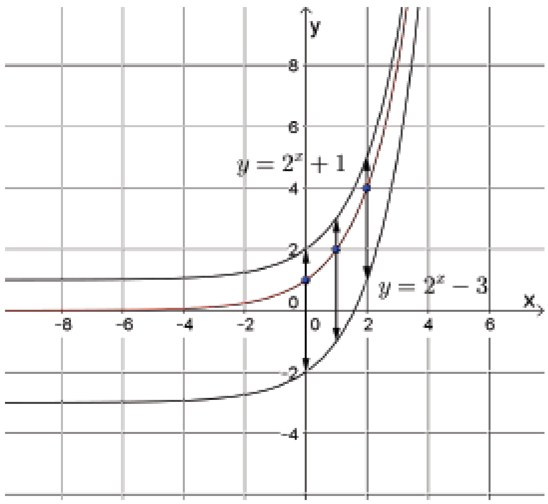
Thus, its:

- *y*-intercept is in the point $(0,a(b^{-c})+d)=(0,0.4(2^{-0}+0))=(0,0.4(1+0))=(0,0.4(1))=(0,0.4)$ The *y*-intercept of $y=0.4(2^x)$ is 0.4;
- *x*-intercept of $y=0.4(2^x)$ is the value of *x* when *$y = 0$*. We arrive with $0=0.4(2^x)$, which will give us a non-real number because exponential functions cannot be non-positive. Similar to observing the graph, we see that there is no *x*-intercept;
- horizontal asymptote is $y=d \Rightarrow y=0$;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is $(d,\infty)=(0,\infty)$ because $a=1$ which is greater than 0.

3. Vertical Shifts:

Example 17: Find the properties of the graphs of y=2^x−3 and y=2^x+1.
Solution and Answers:

The original function is y=2^x, which is then vertically shifted 3 units downward as y=2^x −3 and vertically shifted 1 unit upward as y=2^x+1.
The graphs of these functions are shown below.



Properties:

- The domain for all three graphs is the set of all real numbers R or (−∞,∞).
- Observing the graphs, we see that the range of y=2^x −3 is {y∣y>−3} or (−3,∞) while the range of y=2^x +1 is{y∣y>1} or (1,∞).
- All three graphs do not have vertical asymptotes.
- The *y*-intercepts and horizontal asymptotes were also vertically translated from the *y*-intercept and horizontal asymptote of y=2^x.
- For the *y*-intercept of y=2^x −3, shift the *y*-intercept of y=2^x 3 units downward, that is −2.
- For the *y*-intercept of y=2^x +1, shift the *y*-intercept of y=2^x 1 unit upward, that is 2.
- For the horizontal asymptote of y=2^x −3, shift the horizontal asymptote of y=2^x 3 units downward, that is y=−3.
- For the horizontal asymptote of y=2^x +1, shift the horizontal asymptote of y=2^x 1 unit upward, that is y=1.
- For the *x*-intercept of y=2^x −3, we set the value of *y = 0* then solve for *x*. We arrive with 3=2^x, where we should solve for *x* in the exponent. For now, we will not find the exact value of *x* here because that will be the discussion for

Logarithms on Week 8. We can just observe from the graph that the *x*-intercept of y=2^x −3 is between 1 and 2, estimatedly 1.6.

- For the *x*-intercept of x=2^x +1, we see from its graph that it does not have an *x*-intercept because the function does not touch the *x*-axis.

Aside from observing the graphs, we could also find the properties of these exponential functions by calculation using the formulae in the table of properties above.

For y=2^x −3, a=1,b=2,c=0,d=−3 because y=2^x −3=1(2^x−0)+(−3)=y=a(b^x−c)+d.

Thus, its:

- *y*-intercept is in the point
- (0,a(b^−c)+d)=(0,1(2^−0−3))=(0,1(1−3))=(0,1(−2))=(0,−2). The *y*-intercept of y=2^x −3 is −2;
- *x*-intercept of y=2^x −3 is the value of *x* when *y = 0*. We arrive with 3=2^x, where we should solve for *x* in the exponent. For now, we will not find the exact value of *x* here because that will be the discussion for Logarithms on Week 8;
- horizontal asymptote is y=d=y=−3;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is (d,∞)=(−3,∞) because a=1 which is greater than 0, meaning positive.

For y=2^x+1, a=1,b=2,c=0,d=1 because y=2^x +1=1(2^x−0)+1=y=a(B^x−c)+d.

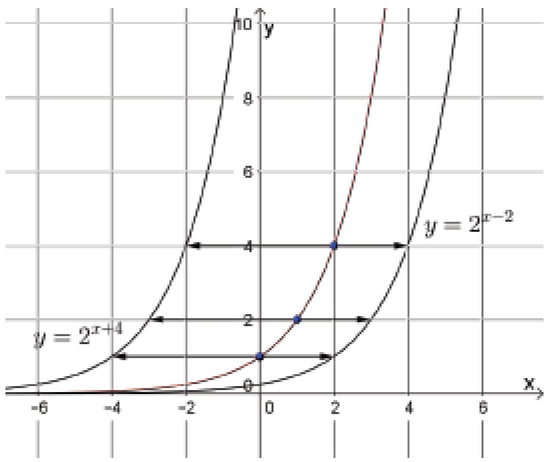
Thus, its:

- *y*-intercept is in the point
- (0,a(b^−c)+d)=(0,1(2^−0+1))=(0,1(1+1))=(0,1(2))=(0,2). The *y*-intercept of y=2^x +1 is 2;
- *x*-intercept of y=2^x +1 is the value of *x* when *y = 0*. We arrive with −1=2^x, which will give us a non-real number because exponential functions cannot be non-positive. Similar to observing the graph, we see that there is no *x*-intercept;
- horizontal asymptote is y=d=y=1;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is (d,∞)=(1,∞) because a=1 which is greater than 0, meaning positive.

4. Horizontal Shifts:

Example 18: Find the properties of the graphs of **y=2^x−2** and **y=2^x+4**
Solution and Answers:

The original function is **y=2^x**, which is then horizontally shifted 2 units to the right as **y=2^x−2** and horizontally shifted 4 units to the left as **x=2^x+4.**
The graphs of these functions are shown below.



Properties:

- The domain for all three functions is the set of all real numbers R or (−∞,∞).
- The range of all three functions is the set of all positive real numbers
- {y∣y>0} or 0,∞).
- All three graphs do not have vertical asymptotes.
- Translating a graph horizontally does not change the horizontal asymptote. Thus, the horizontal asymptotes of all three graphs is *y = 0*.
- We see in the graphs that the *y*-intercepts changed. To find them, substitute *x = 0* in each of the functions.
- For the *y*-intercept of y=2^x−2, y=2^0−2=2^−2=1/4=0.25. Thus, the *y*-intercept of y=2^x−2 is 0.25.
- For the *y*-intercept of y=2^x+4, y=2^0+4=24=16. Thus, the *y*-intercept of y=2^x+4 is 16.
- Since all three graphs do not touch the *x*-axis, they all do not have *x*-intercepts.

Aside from observing the graphs, we could also find the properties of these exponential functions by calculation using the formulae in the table of properties above.

For $y=2^{x-2}$, $a=1, b=2, c=2, d=0$ because $y=2^{x-2}=1(2^{x-2})+0=y=a(b^{x-c})+d$. Remember that c is the number being subtracted in the exponent of the base b; thus, $c=2$ not -2.

Thus, its:

- *y*-intercept is in the point $(0,a(b^{-c})+d)=(0,1(2^{-2}+0))=(0,1(1/4))=(0,1/4)=(0,0.25)$. The *y*-intercept of $y=2^{x-2}$ is 0.25;
- *x*-intercept of $y=2^{x-2}$ is the value of *x* when $y = 0$. We arrive with $0=2^{x-2}$, which will give us a non-real number because exponential functions cannot be non-positive. Similar to observing the graph, we see that there is no *x*-intercept;
- horizontal asymptote is $y=d \Rightarrow y=0$;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is $(d,\infty)=(0,\infty)$because $a=1$ which is greater than 0.

For $y=2^{x+4}$, $a=1, b=2, c=-4, d=0$ because $y=2^{x+4}=1(2^{x-(-4)})+0=y=a(b^{x-c})+d$. Remember that c is the number being subtracted in the exponent of the base b; thus, $c=-4$ not 4 because $x-(-4)=x+4$, which is the exponent.

Thus, its:

- *y*-intercept is in the point $(0,a(b^{-c})+d)=(0,1(2^{-(-4)}+0))=(0,1(2^4))=(0,1(16))=(0,16)$. The *y*-intercept of $y=2^{x+4}$ is 16;
- *x*-intercept of $y=2^{x+4}$ is the value of *x* when $y = 0$. We arrive with $0=2^{x+4}$, which will give us a non-real number because exponential functions cannot be non-positive. Similar to observing the graph, we see that there is no *x*-intercept;
- horizontal asymptote is $y=d \Rightarrow y=0$;
- vertical asymptote does not exist;
- domain is the set of all real numbers; and
- range is $(d,\infty)=(0,\infty)$ because $a=1$ which is greater than 0.

All these transformations could be combined or done to an exponential function. The properties observed and their formulae will still hold true.