HS CALC 001: Precalculus
Trigonometry
Lesson 7: Unit Circle

Most Essential Learning Competencies

At the end of the lesson, the learners shall be able to:

1. illustrate the unit circle and the relationship between the linear and angular measures of a central angle in a unit circle;
2. convert degree measure to radian measure and vice versa;
3. Illustrate angles in standard position and coterminal angles.
4. solve situational problems involving circular functions

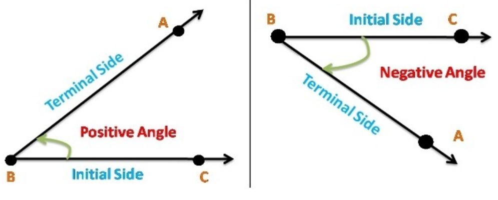
Discussion Points

There are many problems involving angles in several fields like engineering, medical imaging, electronics, astronomy, geography and many more. Surveyors, pilots, landscapers, designers, soldiers, and people in many other professions heavily use angles and trigonometry to accomplish a variety of practical tasks. In this lesson, we will deal with the basics of angle measures together with arc length and sectors.

Angle Measure

Angles in trigonometry differ from angles in Euclidean geometry in the sense of motion. An angle in geometry is defined as a union of rays (that is, static) and has measure between 0 and 180. An angle in trigonometry is a rotation of a ray, and, therefore, has no limit. It has positive and negative directions and measures.

Two Movements of Angles

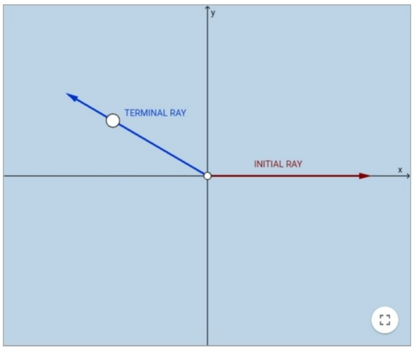


In the figure above, the initial side of ∠ABC is BC while its terminal side is BA. If the rotation of the terminal side is counterclockwise, the angle is *positive* and *negative* if the rotation of terminal side is clockwise.

Angle in Standard Position

An angle is in standard position if it is drawn in the cartesian plane with its vertex at the origin and its initial side is on the positive x - axis.

The applet below, shows the illustration of angle whose initial side is at positive x-axis and the vertex is at the origin. The terminal side moves counterclockwise and the angles are positive.



Illustrations of angles in standard position.

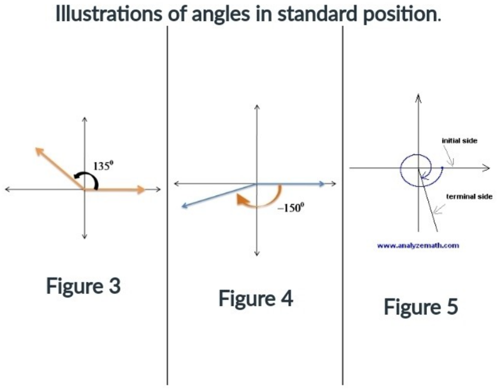


Figure 3 Figure 4 Figure 5

Converting Degrees to Radian and Vice Versa

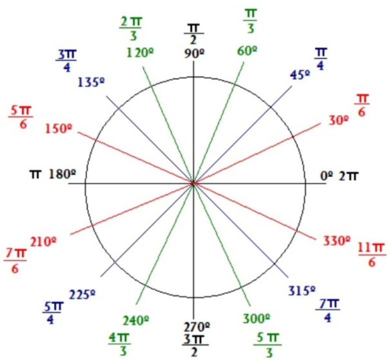
To convert,

- Degrees to Radian
  Multiply it by π/180
- Radian to Degrees
  Multiply by 180/π

Example 1: Convert the following:
a. 30°
b. 225°
c. 360°
d. π/3
e. 3π/2
f. 4π/3

Solutions:

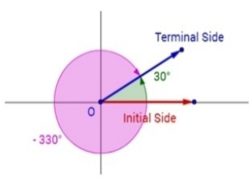
a. 30° • π/180 = π/6 rad
b. 225° • π/180 = 5π/4 rad
c. 360° • π/180 = 2π rad
d. π/3 • 180/π = 60°
e. 3π/2 • 180/π = 270°
f. 4π/3 • 180/π = 240°



Coterminal Angles

Two angles in standard position that have a common terminal side are called coterminal angles. Observe that the degree measures of coterminal angles differ by multiples of 360.

Angles in standard position



In **figure 6,** the coterminal angle of 30 degrees is -330 degrees. The other coterminal angle of 30 degrees is 390 degrees.

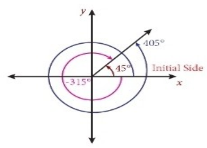


Figure 7

In **figure 7**, the coterminal angle of 45 degrees are - 315 degrees and 405 degrees.

Example 2: Find the coterminal angle/s of the following with the given condition.

a. **60°** (closest positive and negative coterminal angle)
b. ***π/4 rad*** (closest positive and negative coterminal angle)
c. **840°** (between 0° and 360°)
d. **- 7π/6** (between 0° and 360°)

Solution:
a. 60°
= 60° - 360°**= - 300°**
= 60° + 360° **= 420°**

b. ***π/4 rad***
= π/4 - 2π = ***- 7π/4 rad***
= π/4 + 2π = ***9π/4 rad***

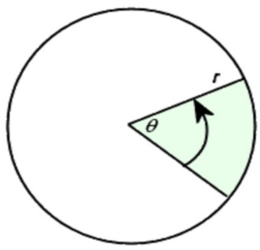
c. **840°**
= 840° - 360° = 480° - 360°
= **120°**

d. ***- 7π/6 rad***
= - 7π/6 + 2π
= ***5π/6 rad***

Arc Length and Area of a Sector

As you remember from geometry, the area A of a circle having a radius of length r is given by: A= πr^2

The circumference C (that is, the length around the outside) of the same circle is given by: C=2πr



In this circle, the green portion is the area of a sector. The central angle of a sector whose radian measure is θ subtends an arc that is the ratio θ/2π of the circumference of the circle. Thus in a circle of radius r, the length s of an arc that subtends the θ is,

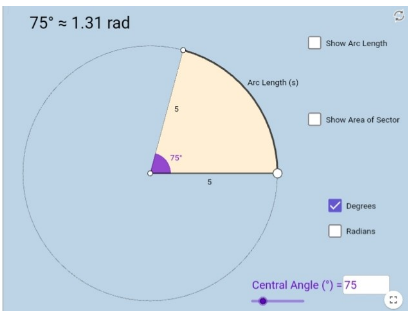
$$s = \theta/2\pi \times 2\pi r = r\theta$$

$$\mathbf{s = r\theta}$$

A sector of a circle is the portion of the interior of a circle bounded by the initial and terminal sides of a central angle and its intercepted arc. If a central angle of a sector has measure θ radians, then the sector makes up the ratio θ/2π of a complete circle.

$$\text{Area of a Sector} = \theta/2\pi\,(\pi r^2)$$

$$\mathbf{A = ½\,\pi r^2}$$



The applet above shows the relationship of Arc length, Area of a Sector and the Central Angle. 1 radian is approximately 57.3 degrees. As the central angle changing, the area of a sector and the arc length are also changing.

Example 3: Find the linear measure of central angle of a circle with radius 16 meters and subtends a central angle of measure 3π/4 rad.

Given: r = 16m
θ = 3π/4 *rad*
Find: s

__Solution:__
s = rθ
s = 16m (3π/4 rad)
s = 12π meters

Example 4: A central angle θ , in a circle of a radius 4 m is subtended by an arc of length 6 m. Find the measure of θ in radians.

Given: r = 4m
s = 6m
Find: θ

__Solution__: θ = s/r
θ = 6/4
θ = 3/2 *rad*

Example 5: Find the area of a sector of a circle with central angle 450 if the radius of the circle is 8 m.

__Solution__: Convert first the 450 into radians. Then, substitute the given to the equation.

$A = ½\,\theta r^2$
$A = ½\,(\pi/4)\,(8m)^2$
$A = 16\pi/8\ m^2$
$\mathbf{A = 2\pi\ m^2}$

Example 6: An adjustable-angle pop-up lawn sprinkler has been installed in an awkward corner of the neighbor's yard. This sprinkler, assuming full water pressure, can spray everything within 3 meters. Given that the angle has been set to 600, how much lawn will this sprinkler head water? (Round to two decimal places.)

Given: r = 3 m

$\theta = 60° = \pi/3$ rad

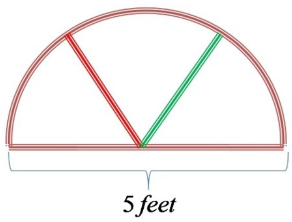
Find: Area of a Sector

<u>Solution:</u>

$A = ½ (\pi/3) (3m)^2$

$A = 3\pi/2 \approx 4.71\ m^2$

Example 7: A cathedral window is shown below. If the window is to contain three equal sections of glass, what is the area of each glass section?



Given:

diameter = 5 ft ; radius = 2.5 ft

$\theta = 60°$ or $\pi/3$ for each section of window glass

Find the area of each section, A.

Solution:

$A = ½\ \theta r^2$

$A = ½ (\pi/3) (2.5)^2$

$A = ½ (\pi/3) (6.25)$

$A = 3.27\ ft^2$

The area of each window glass section is $3.27\ ft^2$.