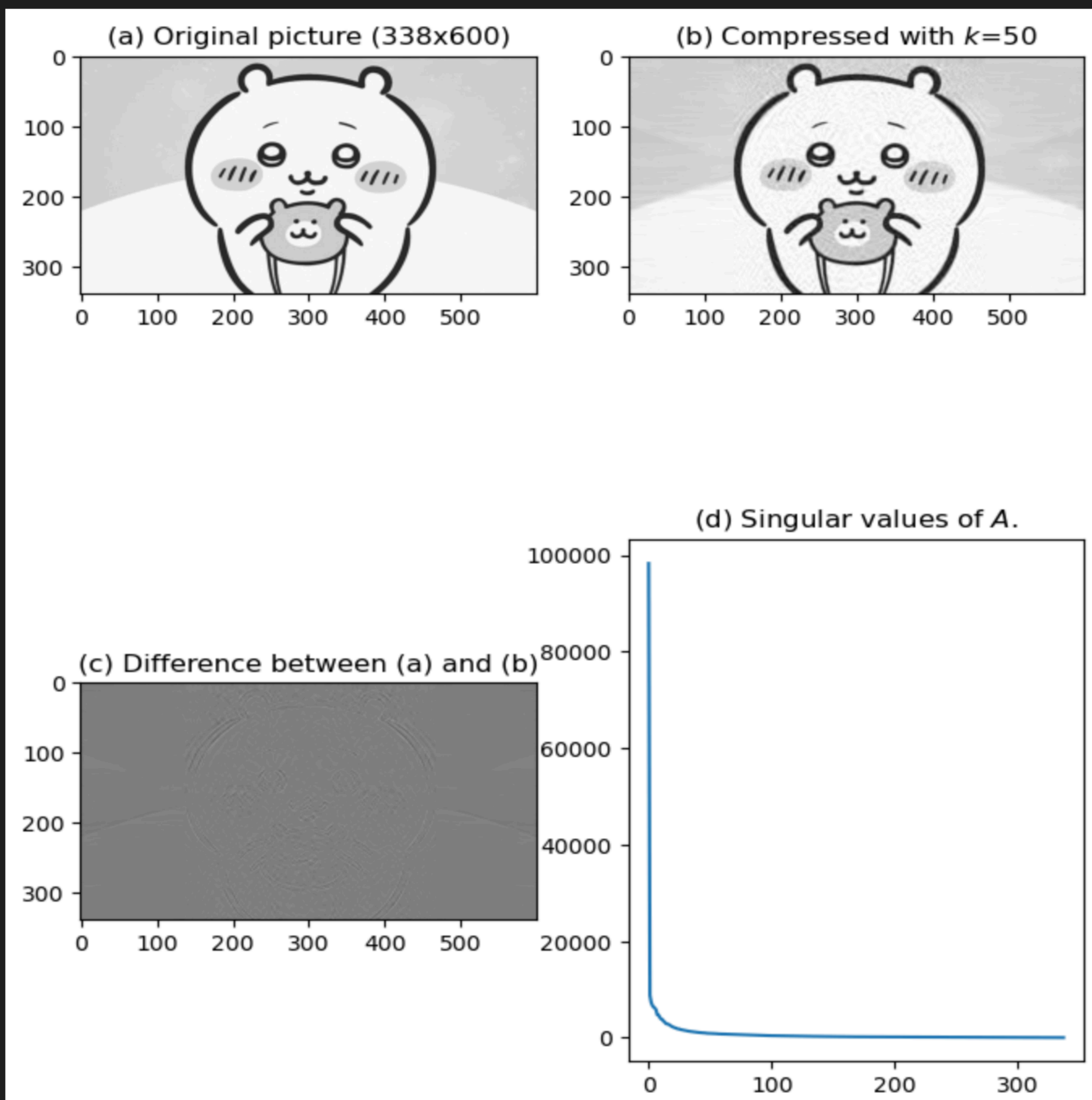
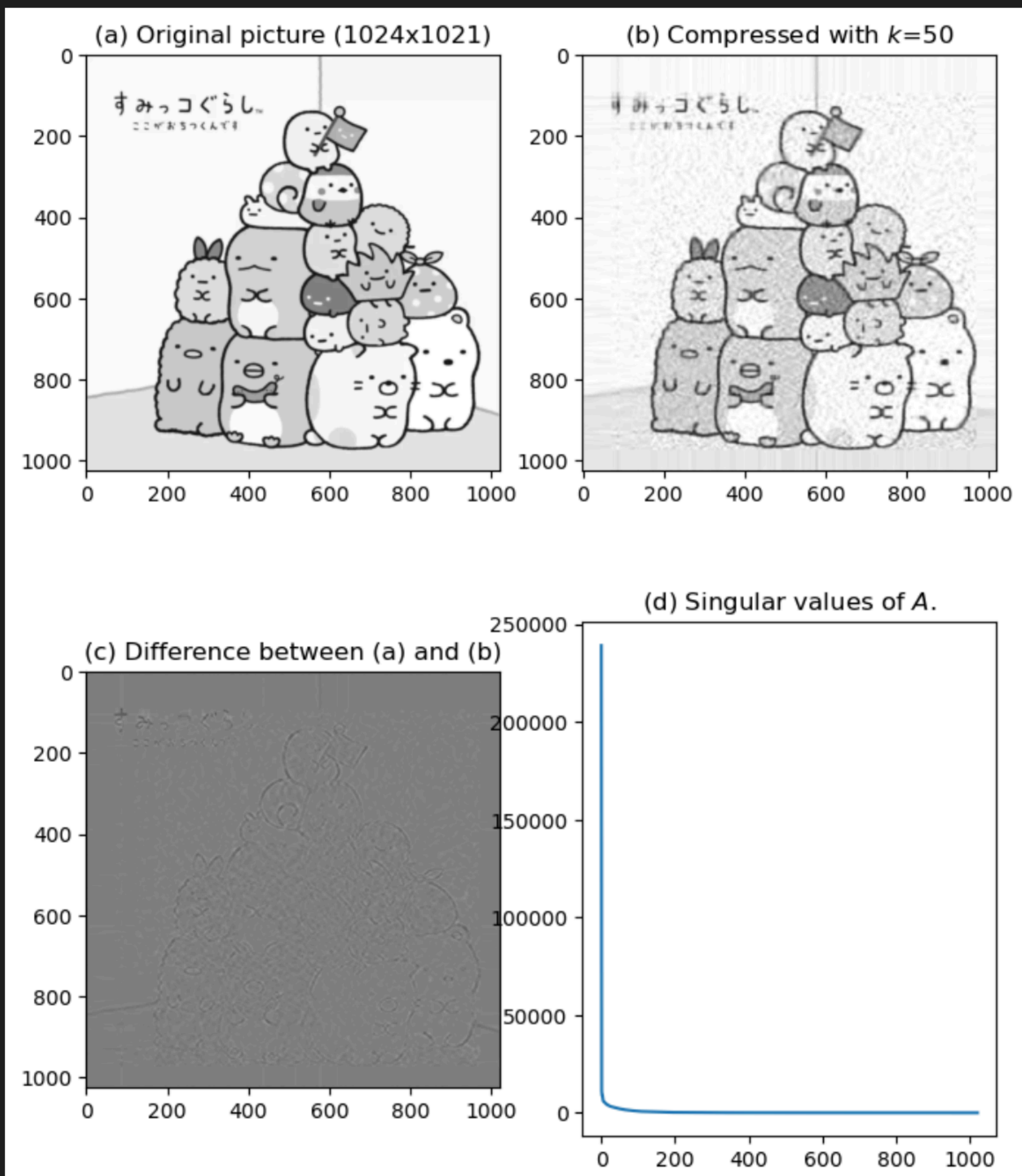
HW4_110062272

1. Find five different styled pictures whose sizes are between 256x256 to 1024x1024, and perform SVD compression on them.

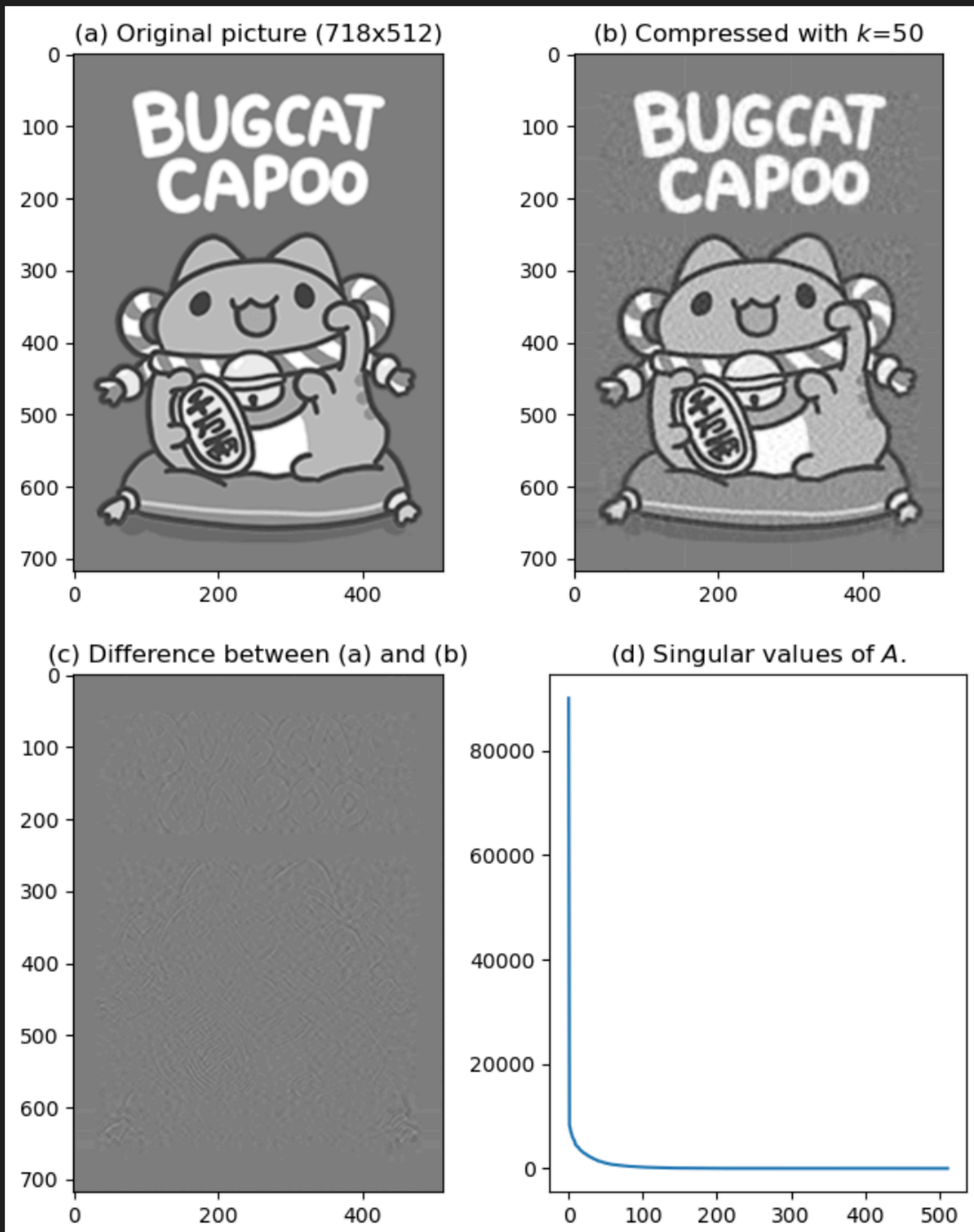
```
The size of the image is: 338 x 600
Compression ratio is: 0.231509
Average relative difference is: 0.075928
```



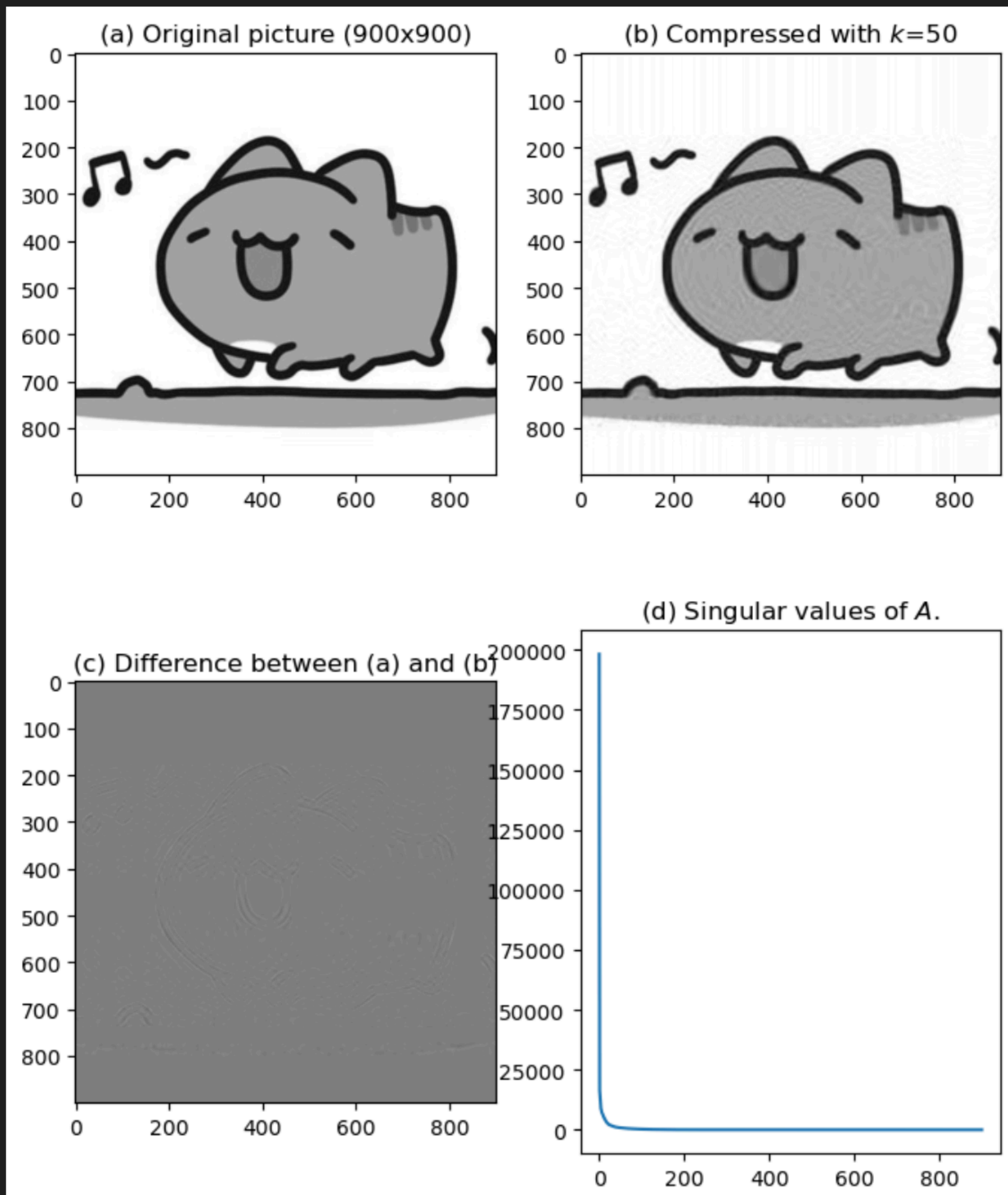
```
The size of the image is: 1024 x 1021
Compression ratio is: 0.097848
Average relative difference is: 0.032469
```



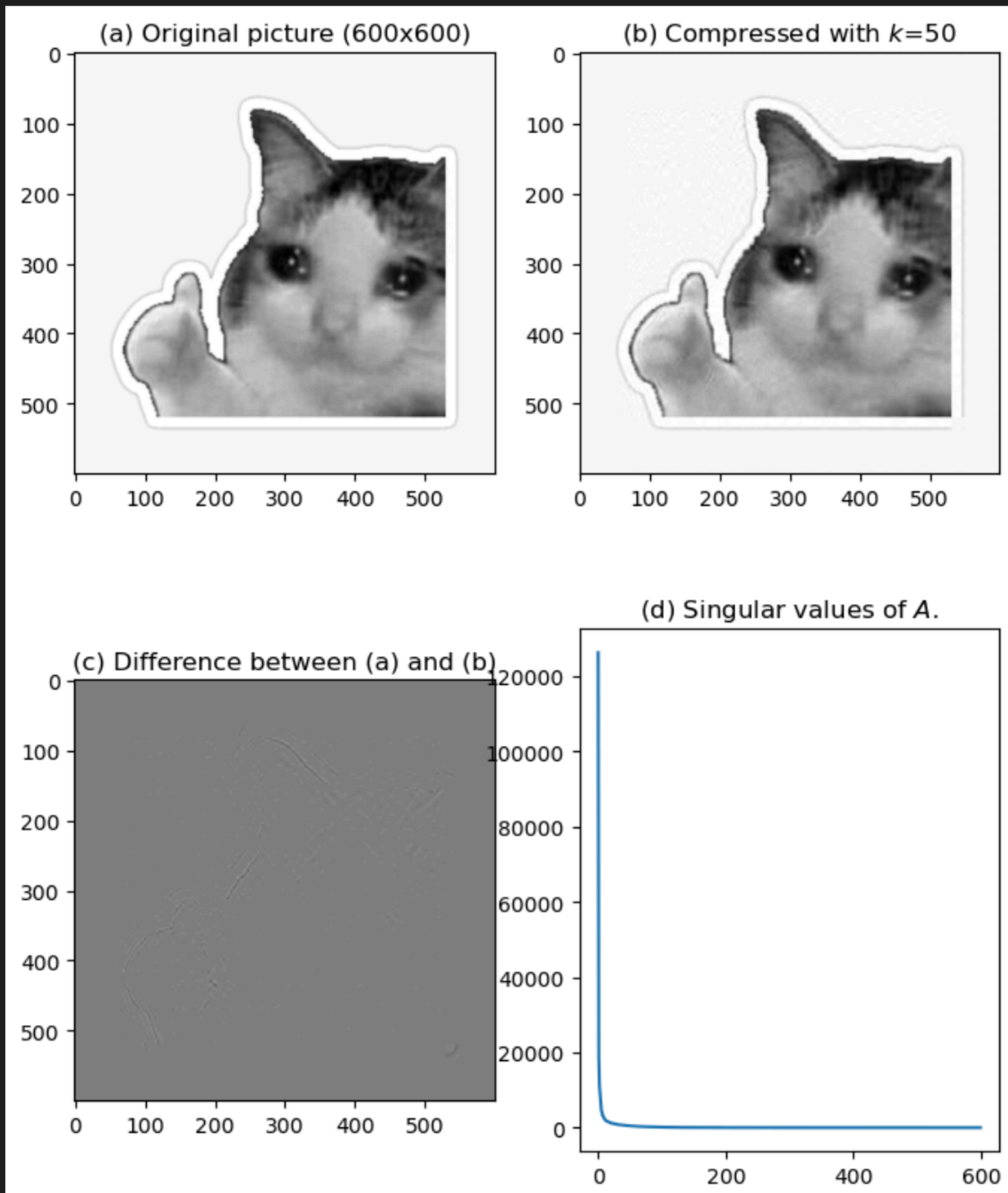
```
The size of the image is: 718 x 512
Compression ratio is: 0.167430
Average relative difference is: 0.036680
```



```
The size of the image is: 900 x 900
Compression ratio is: 0.111173
Average relative difference is: 0.014124
```



```
The size of the image is: 600 x 600
Compression ratio is: 0.166806
Average relative difference is: 0.014606
```



2. For each picture, discuss the relation of compression ratio ρ and the average relative difference δ. For example, what δ would be if we want $\rho < 1$ or $\rho < 0.5$, or what ρ is if we want to keep $\delta < 5\%$.

Looking at the formulas:

- Compression ratio:

$$\rho = \frac{m \times k + k + k \times n}{m \times n}$$

- Average relative difference:

$$\delta = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{|A_{i,j} - \tilde{A}_{i,j}|}{A_{i,j} + \epsilon}, \epsilon = 10^{-8}$$

  - k is singular value
  - m and n are image width and height respectively

For the images in our dataset:

1. Largest Image (1024x1021):
- $\rho = 0.097848$, $\delta = 0.032469$
- Already achieves $\rho < 0.5$ and $\delta < 5\%$
- Could potentially reduce k further while maintaining $\delta < 5\%$
2. Second Largest Image (900x900):
- $\rho = 0.111173$, $\delta = 0.014124$
- Shows that larger images can maintain low δ with low ρ

3. Medium Image (718x512):
- $\rho = 0.167430$, $\delta = 0.036680$
- Still maintains $\delta < 5\%$ despite higher ρ
- Shows good balance between compression and quality
4. Small Image (600x600):
- $\rho = 0.166806$, $\delta = 0.014606$
- Similar ρ to medium image but better quality preservation
- Demonstrates that image content affects δ independently of size
5. Smallest Image (338x600):
- $\rho = 0.231509$, $\delta = 0.075928$
- Only image with $\delta > 5\%$
- Highest compression ratio due to smaller dimensions

This pattern clearly shows that:

1. Each image easily achieves $\rho < 0.5$ with k=50
2. To maintain $\delta < 5\%$, larger images can use lower k values (resulting in lower ρ)
3. Smaller images require higher k values to maintain the same quality level

When k = 30:

1. Largest Image (1024x1021):
- ρ: 0.058709 (from 0.097848)
- δ: 0.047465 (from 0.032469)
2. Second Largest Image (900x900):
- ρ: 0.066704 (from 0.111173)
- δ: 0.022655 (from 0.014124)
3. Medium Image (718x512):
- ρ: 0.100458 (from 0.167430)

- δ: 0.072188 (from 0.036680)
- Quality degradation more noticeable

4. Small Image (600x600):

- ρ: 0.100083 (from 0.166806)
- δ: 0.023686 (from 0.014606)
- Maintains good quality despite size

5. Smallest Image (338x600):

- ρ: 0.138905 (from 0.231509)
- δ: 0.110166 (from 0.075928)
- Most significant quality loss

This additional data with k=30 confirms our observations:

1. Larger images generally maintain better quality with reduced k
2. Quality impact (increase in δ) varies significantly by image size and content
3. The largest image still maintains reasonable quality (δ < 5%) even with k=30

Conclusion:

Compression ratio ρ is proportional to k, meaning lower k values result in lower compression ratios but might increase the relative difference δ. The impact of reducing k is generally less severe for larger images, though image content also plays a significant role in quality preservation.

3.

(a) What kind of pictures can be compressed easily by SVD (smaller relative difference)? And why?

Images that can be easily compressed by SVD are those with large uniform areas and simple geometric patterns, like cartoon-style images. This is because these images have high spatial correlation and can be well-represented by fewer singular values. In contrast, images with complex textures and fine details require more singular values to maintain quality, making them harder to compress effectively.

(b) which means they have low ρ and δ? Try to find its relation with the distribution of singular values of the original pictures.

The distribution of singular values significantly affects compression quality. Images with better compression (low ρ and δ) show a steep initial drop in singular values, with most information concentrated in the first few values. In contrast, images with poorer compression have a more gradual decline in singular values, meaning important information is spread across more values, leading to higher loss when truncated.

(c) Which part of the pictures have the largest difference of the compressed images? And why?

The largest differences in compressed images typically appear in edge regions and areas with sharp transitions or high contrast. This occurs because SVD compression performs a low-rank approximation that smooths out abrupt changes, making it difficult to preserve crisp edges and fine details which require more singular values to represent accurately.