UT Dallas

Software Quality and Software Testing

Part 1 – The Big Picture (How Quality Relates to Testing)
Part 2 – Fundamental Concepts of Measurement and Data Analysis
Part 3 – Defect Containment
Part 4 – Measuring Software Structure
Part 5 – Measuring Software Complexity

Dennis J. Frailey

Retired Principal Fellow - Raytheon Company

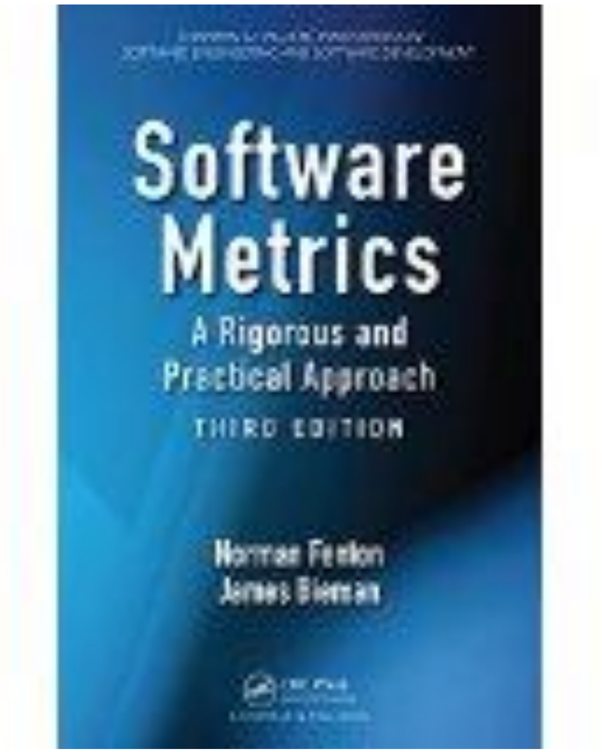
PhD Purdue, 1971, Computer Science
Assistant Professor, SMU, 1970-75
Associate Professor, SMU, 1975-77
(various titles), Texas Instruments, 1974-1997;
Raytheon Co. 1997-2010
Adjunct Associate Professor, UT Austin, 1981-86
Adjunct Professor, SMU, 1987-2019
Adjunct Professor, UT Arlington, 2014-present

Areas of specialty: software development process, software project management, software quality engineering, software metrics, compiler design, operating system design, real-time system design, computer architecture

A Recommended Book on Measurement

Some of this material is taken *from this book*.

Although not a book on testing, it is a very good book on measurement and addresses several aspects of testing.



By Norman Fenton and James Bieman

More Recommended References

***SWX – The Software Extension to the Project Management Body of Knowledge*, available from PMI (www.pmi.com) and the IEEE Computer Society (www.computer.org).**

– This is a general reference that may be important if you want to apply some of today's techniques in project management.

***SWEBOK – The Guide to the Software Engineering Body of Knowledge*, available from the IEEE Computer Society and also at www.swebok.org**

– This is another general reference that gives an overall picture of software engineering knowledge and summarizes topics that any software engineer should know about.

Part 1

The Big Picture – How Quality Relates to Testing and Other Aspects of Software Engineering

Test and Evaluation

***Evaluation*: Appraising a product through one of the following:**

- Examination, analysis, demonstration
- Testing
- or other means

***Testing*: Exercising a system to improve confidence that it satisfies requirements or to identify variations between desired and actual behavior.**

"Evaluation" is the broader term.

But What Are We Appraising?
What is "Desired Behavior"?

- Satisfies requirements
- Works correctly
- Does what I want it to do
- Does no harm
- Reliable – I can depend on it
- Easy to use
- Portable
- Easy to update and maintain
- Easy to test
- Runs efficiently / fast
- Consistent
- …

Can we test for these characteristics?

Can we measure them?

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But What Are We Appraising? What is "Desired Behavior"?

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- **Does no harm**
- **Reliable – I can depend on it**
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- **Portable**
- **Easy to update and maintain**
- **Easy to test**
- **Runs efficiently / fast**
- **Consistent**
- **…**

These are all characteristics of Software Quality

I.e., testing is one way to assess software quality.

SWEBOK® V3.0

Guide to the Software Engineering Body of Knowledge

Editors

Pierre Bourque
Richard E. (Dick) Fairley

IEEE
IEEE computer society

Downloadable at:
www.swebok.org

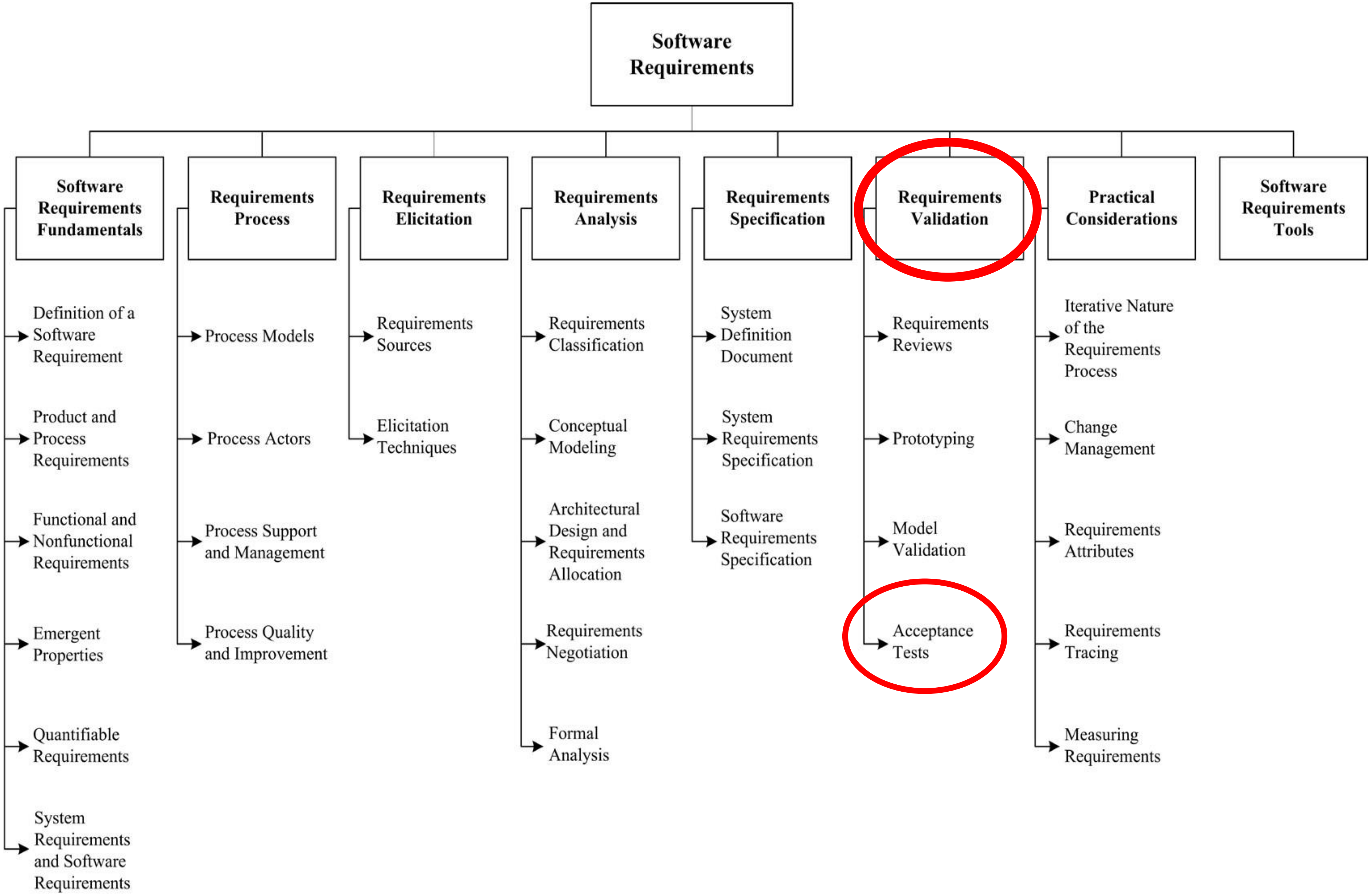
SWEBOK Facts

- **3 Editions have been produced since 1998**
- **2 Editors: Pierre Bourque and Richard Fairley**
- **8 Contributing and Co-Editors**
- **15 Knowledge Areas, each with its own Editors**
  - Each aligned with related ISO and IEEE standards
- **9-person Change Control Board**
- **Over 300 reviewers (chosen due to their expertise in various aspects of software engineering)**
  - Over 1500 comments received and adjudicated on various drafts (3rd edition)
- **36 Items in Consolidated Reference List**

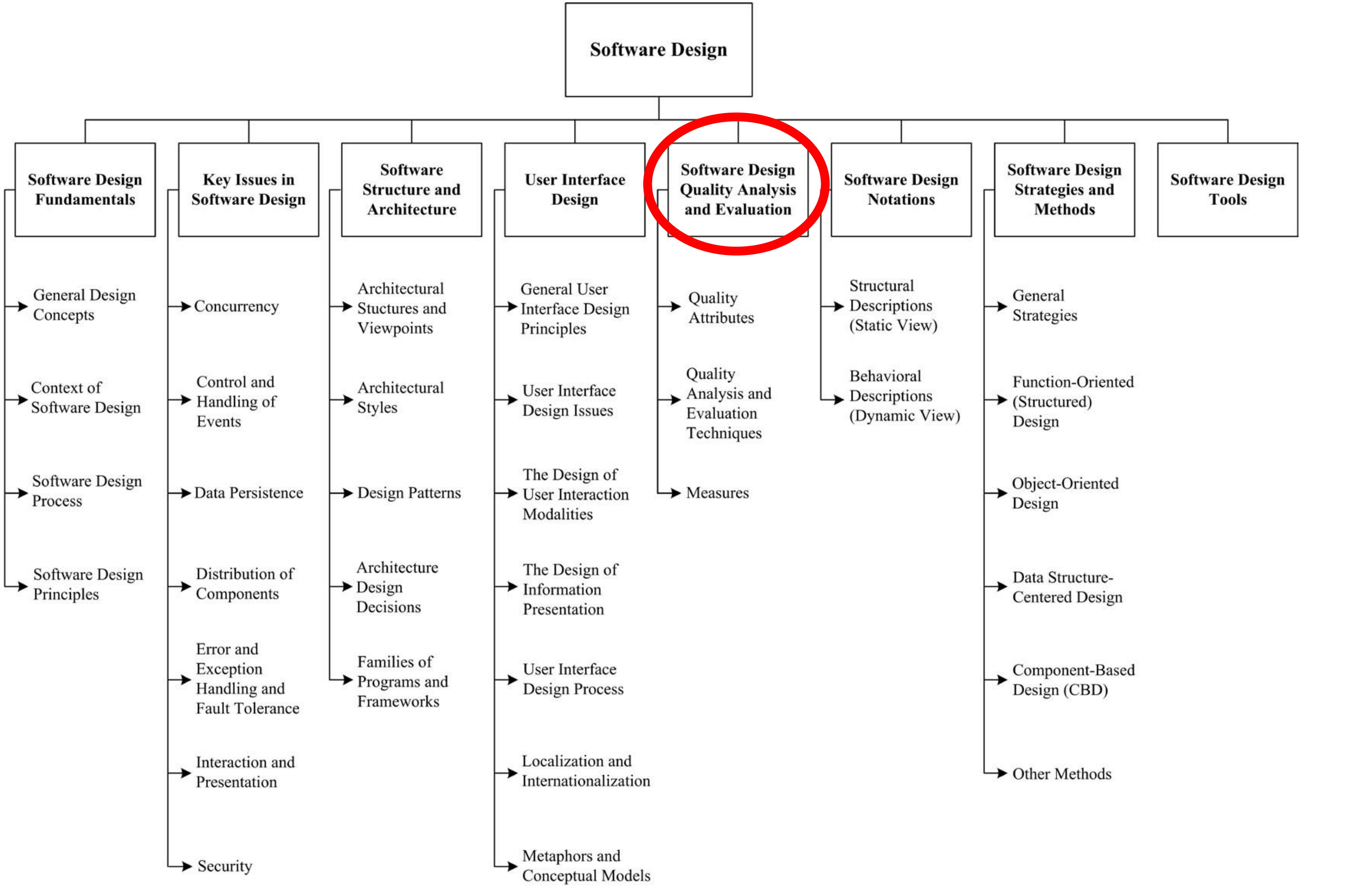
The 15 SWEBOK Knowledge Areas

- Software Requirements
- Software Design
- Software Construction
- Software Testing
- Software Maintenance
- Software Configuration Management
- Software Engineering Management
- Software Engineering Process
- Software Engineering Models and Methods
- Software Quality
- Software Engineering Professional Practice
- Software Engineering Economics
- Computing Foundations
- Mathematical Foundations
- Engineering Foundations

Software Requirements

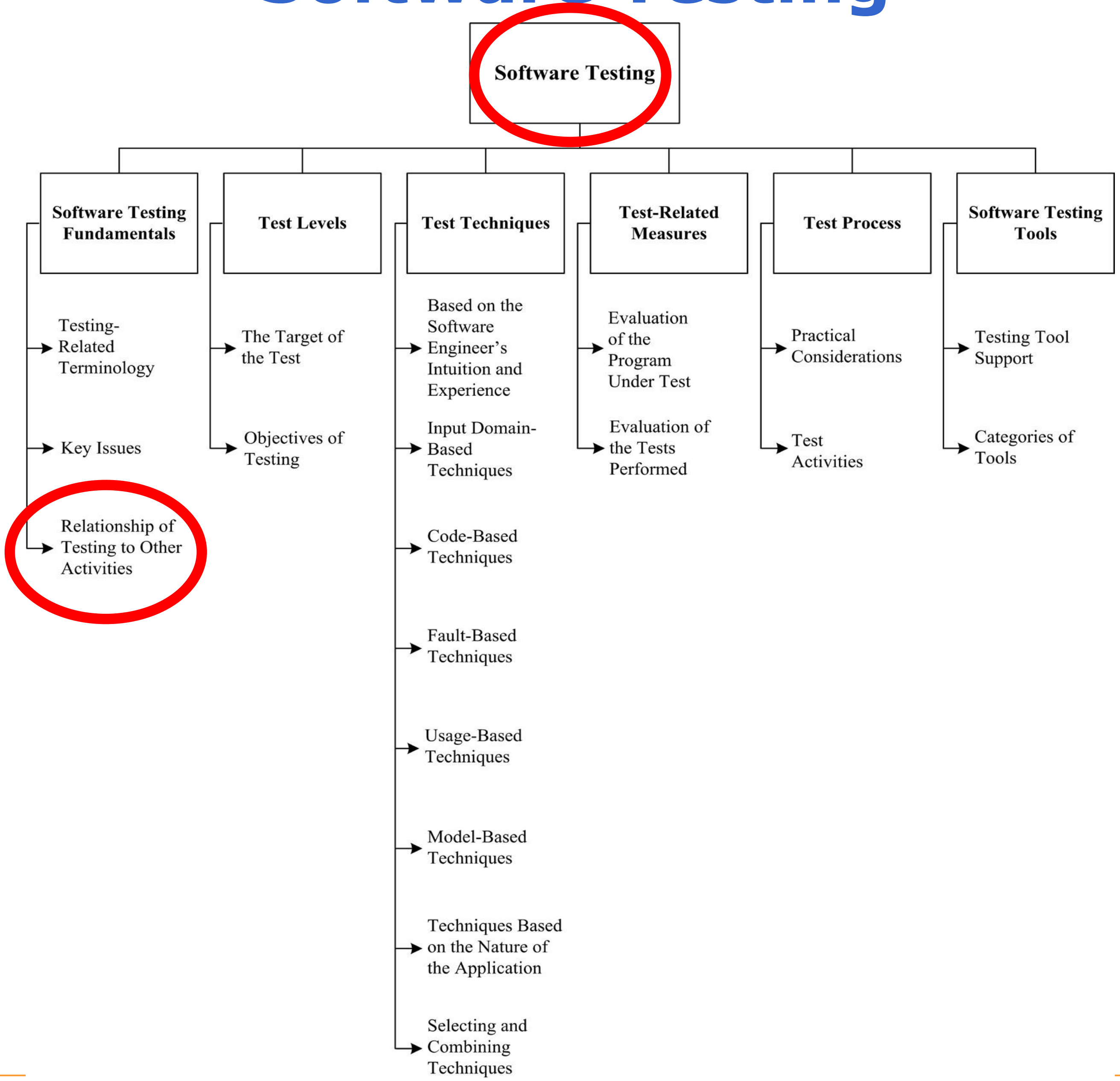


Software Design

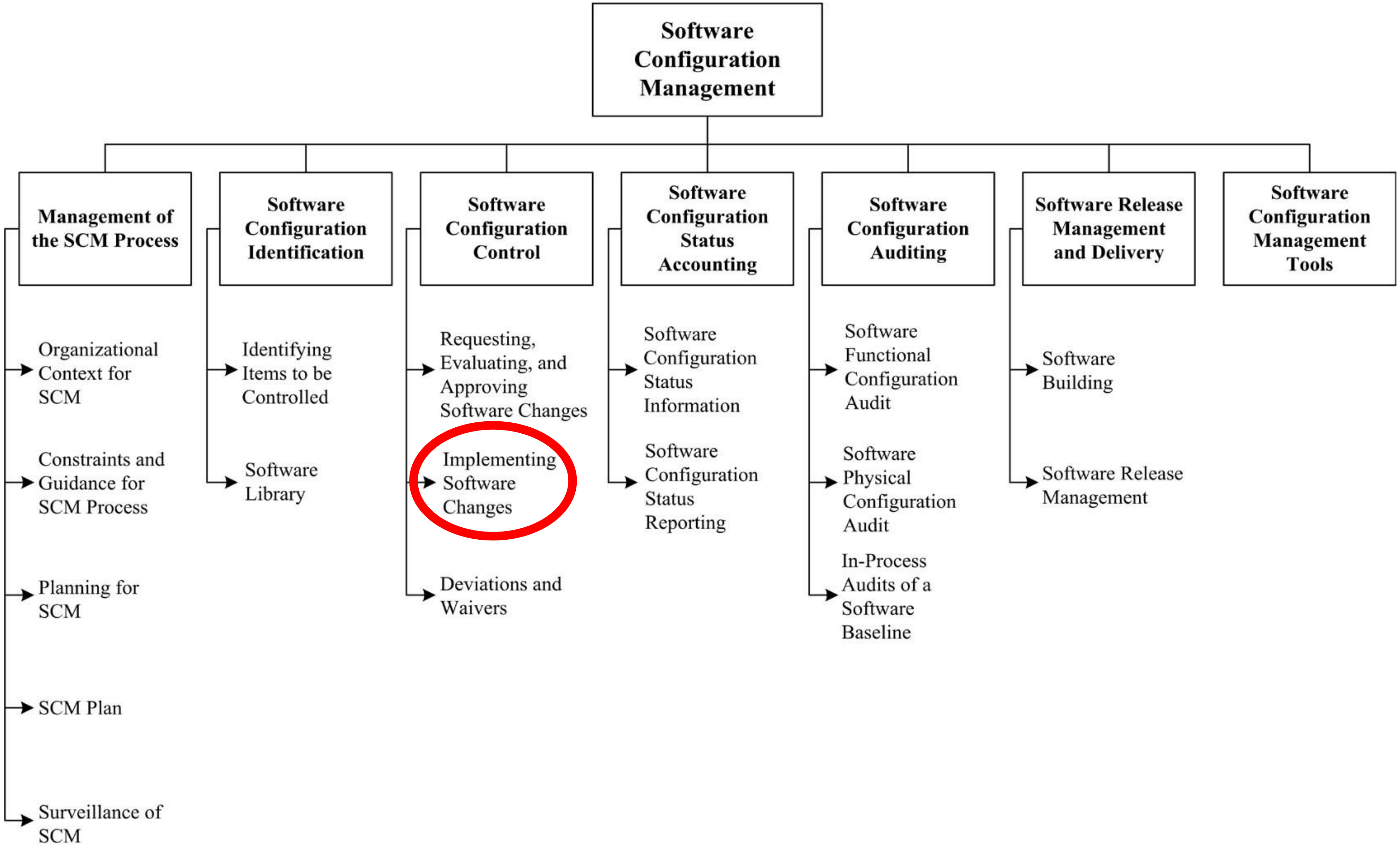


Software Construction

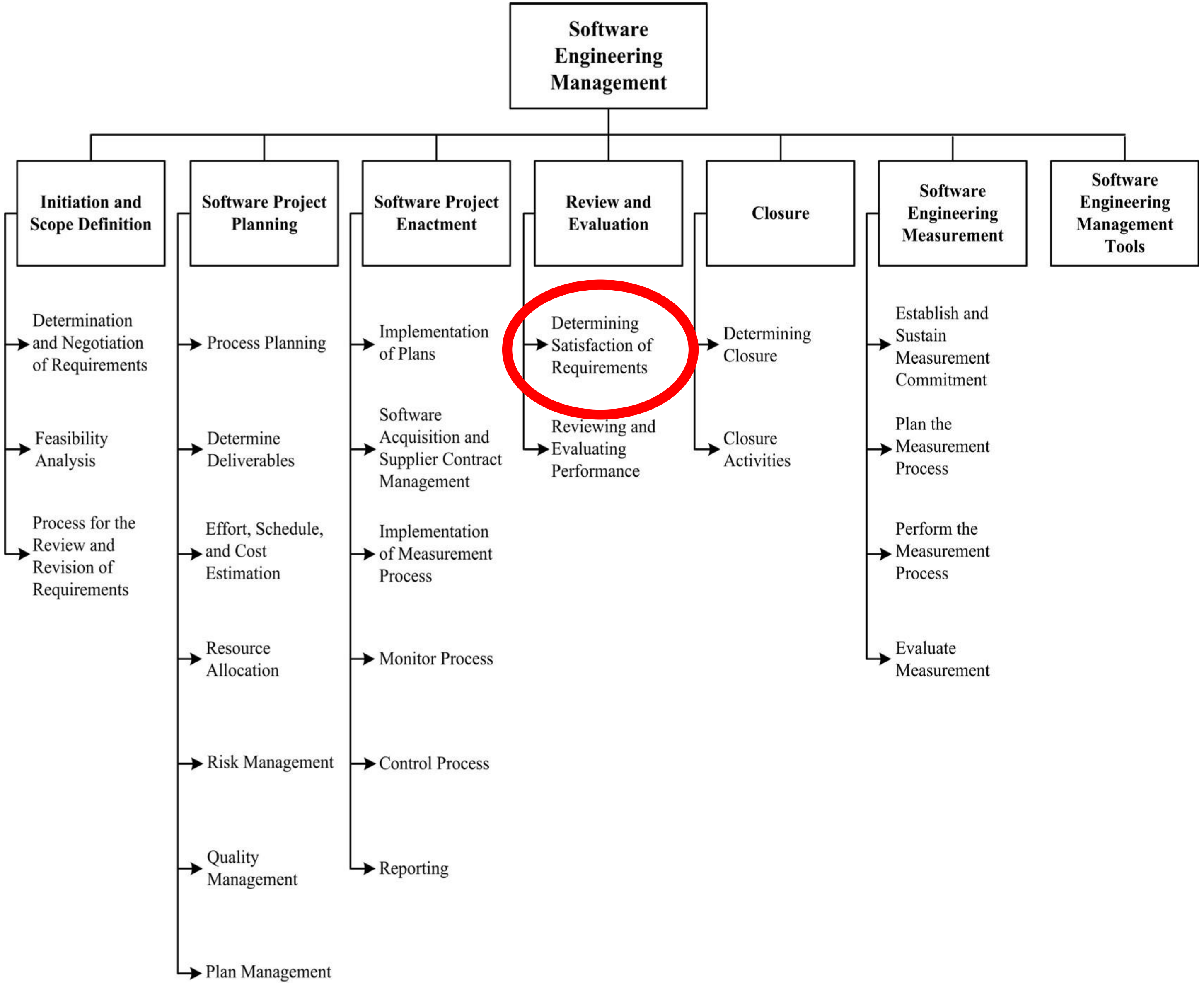
Software Testing



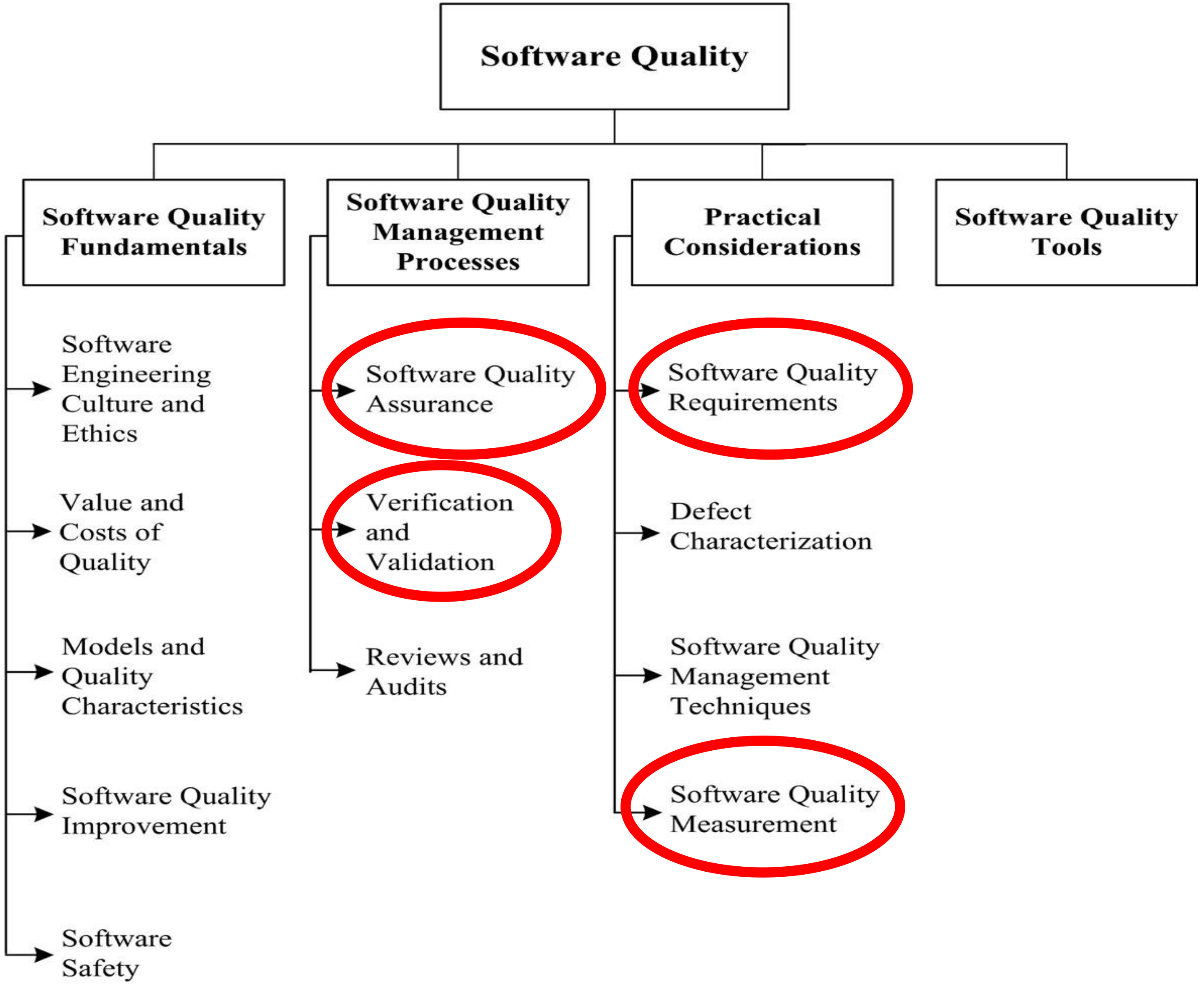
Software Configuration Management



Software Engineering Management



Software Quality



What Do We Mean by Quality?

Concepts of Quality for Products

"Quality is *conformance to requirements*"
Crosby

"Quality is *fitness for intended use*"
Juran

"Quality is *value to someone*"
Weinberg

"Quality is Conformance to Requirements"

- **If *testable requirements* can be established, then it is possible to decide *whether the product satisfies the requirements* – by testing it.**
- **If *measurable quality characteristics* can be established, then it is possible to decide on *the extent to which the product satisfies the requirements* – by measuring it.**
- **Thus you can avoid disputes and have workable contractual relationships**

HOWEVER …

Issues with "Conformance to Requirements" (1 of 4)

Who establishes the requirements?

- ***Sponsor*** - The one who pays for the product
- ***End User*** - The one who will use the product
- ***Sales or Marketing*** - The one who will sell the product
- ***Engineering*** - The ones who will design and build it

What the end user wants

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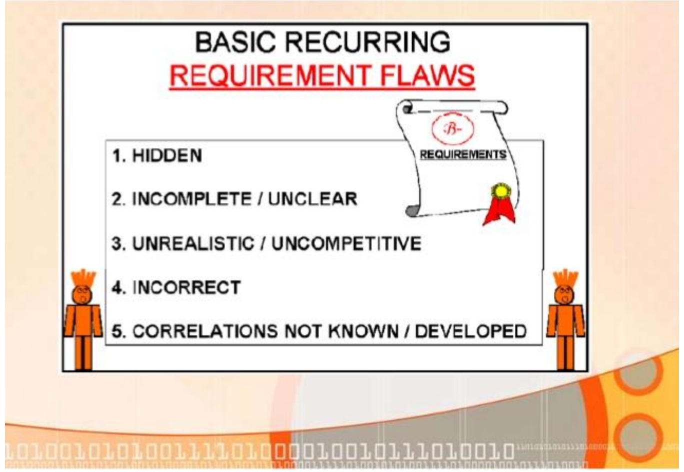
What the engineer builds

Issues with "Conformance to Requirements" (2 of 4)

Are the requirements right?

- consistent
- complete
- visible
- correct

Flaws



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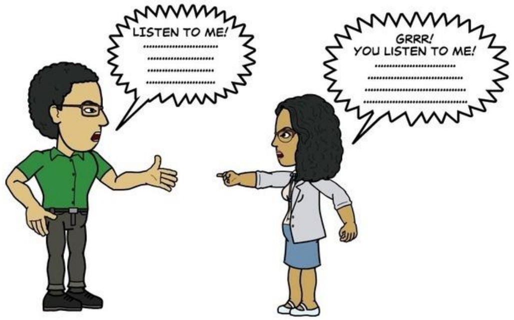
- **Who determines whether the requirements are right?**



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- **What if you discover a problem later on?**

Issues with "Conformance to Requirements" (3 of 4)

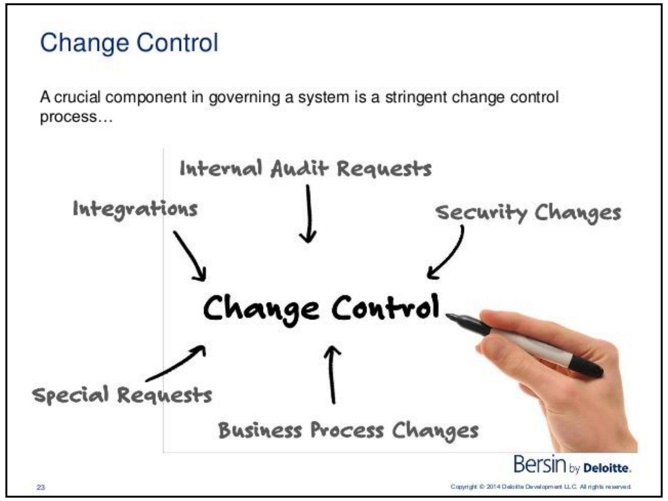
What about implicit vs. explicit requirements?

- ***Explicit requirement***: pizza should be hot and flavorful
- ***Implicit requirement***: **not harmful**

Issues with "Conformance to Requirements" (4 of 4)

What about when requirements change during the development process?

- ***Who makes the changes***?
- ***Who controls and authorizes the changes***?
- ***Who pays*** for the ***consequences*** of changes?



"Quality is Fitness for Intended Use"

- **This definition is based on *a fundamental concept of law* - that *a product should be suitable* for the use that it is intended for.**

- **This definition accommodates the fact that *we may not be able to fully define the requirements*.**

HOWEVER …

Issues with "Fitness for Intended Use" (1 of 3)

Who defines *fitness*?

- Consider a TV set
  - which fitness characteristics are not understood by
    - Typical User
    - Engineer
    - Sales Personnel

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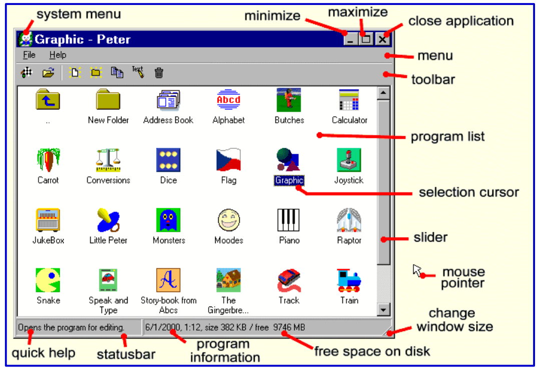
- Consider a software program
  - which fitness characteristics are not understood by the typical software developer?



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Issues with “Fitness for Intended Use” (2 of 3)

Different users have different definitions of fitness

- Ease of use for novices
- Control of fine details for experts
- Ease of maintenance for support staff
- Able to survive power failures
- Compatibility with previous system

Theodysseyonline.com

- **Uses change as users grow in experience**
  - Too many “ease of use” and “automatic” features may frustrate an expert

Issues with “Fitness for Intended Use” (3 of 3)

The “*pleasant surprise*” concept

User gets more than he or she expected

They really knew what they were doing when they designed this software

There is often tension between the engineer knowing better than the customer and the customer knowing better than the engineer

"Quality is Value to Someone"

- **This definition incorporates the idea that *quality is relative***
- **And it places increased emphasis on understanding *what quality means to the intended user* of the software**

HOWEVER …

Issues with "Value to Someone" (1 of 4)

Whose opinion counts?

What features do you want?

Can it survive spilled drinks?

How is the financial software?

I want hot games

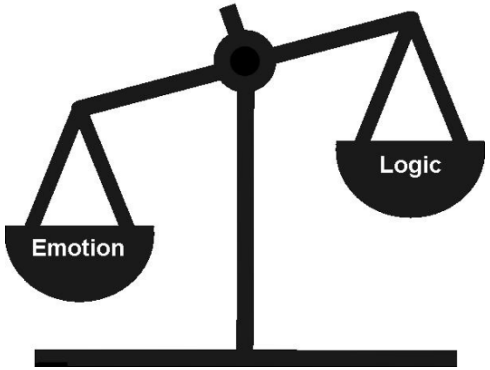
Does it have Facebook and Twitter?

- You may need to weigh different opinions

Issues with "Value to Someone" (2 of 4)

Logic vs **Emotion**

– "Glitz" v. "Substance"

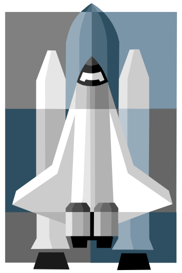


Which Car is Best for Our Family?

Issues with "Value to Someone" (3 of 4)

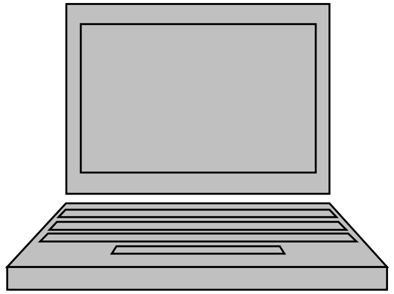
Value depends on What Features are Most Important

- Space Shuttle
  - 0 defects
  - Reliability



- Video Game
  - Good user interface
  - High performance

- School Laptop
  - Rugged
  - Fast
  - Good Battery Life
  - Good Software

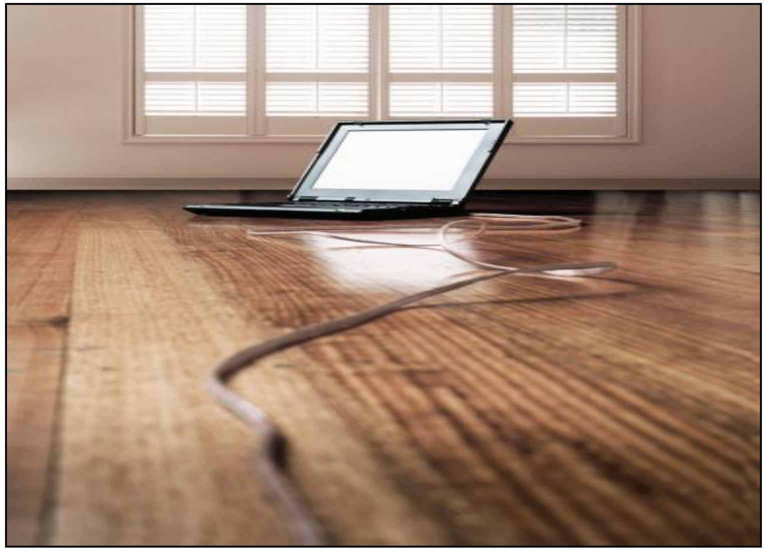


Issues with "Value to Someone" (4 of 4)

Some Needs are Implicit (unstated)

Explicit

- I need an office
- It must have a computer
- And lots of space



Implicit

- I need a desk
- And a chair
- And convenient electrical outlets

Definitions of Software Quality

IEEE: **The degree to which the software *possesses a desired combination* of *attributes***

Crosby: **The degree to which a *customer perceives* that software *meets* composite *expectations***

Note that both definitions imply multiple expectations

Summary of Quality Definition Issues

- **You Must *Define Quality***
  - Before you can **engineer it** into your product
  - … and before you can **measure it**
  - … or **test** whether the product has the desired quality attributes
- **Quality has *Multiple Elements***
  - It reflects a multitude of expectations
- **Quality is *Relative***
  - Quality is in the eye of the customer
- **Quality encompasses *fitness, value, and other attributes***

Quality Attributes are Seldom Directly Measurable

- **Fitness for intended use**
- **Value to someone**
- **Satisfaction of requirements**
  - Including implicit, unstated requirements
- **Maintainability**
- **Reliability**
- **Supportability**
- **Testability**
- **…**

How can these be measured?

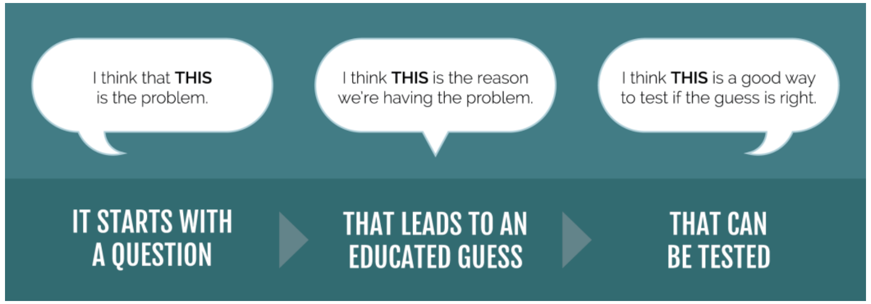
We need to find suitable *ways to measure* these attributes.

Some Attributes Are Measurable

Examples

- Water boils at 100º Centigrade
- My new application will complete at least 10 searches per minute
- Code written in C takes less memory space than code written in Python

The above statements may or may not be true, but they can all be tested because they are all measurable.



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Some Attributes are Not Measurable

Examples:

- Joe’s code is better than Jan’s code
- Lisp is a superior programming language to C#
- Object oriented design produces code that is more maintainable

The above cannot be measured unless we define what we mean by:

- Better than
- Superior
- Maintainable

In a measurable way!

Surrogates

In order to measure an un-measurable attribute

- such as “quality” or “maintainability”

We may need to measure indirectly

- we measure something else that is associated with that attribute
- such as “defects” or “repair cost”

This alternative, measurable attribute is called a *surrogate*.

Surrogates Are Not the Real Thing

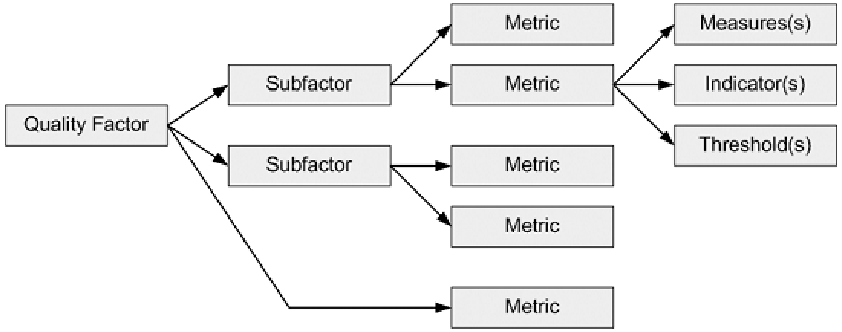
A surrogate may or may not accurately reflect the desired attribute

Examples:

- **Defects** are a common surrogate for quality
- But defects may or may not reflect **quality**.
  - Defects may reflect failure to do effective testing
  - Or failure of the customer to use the product
- **Repair cost** may or may not reflect **maintainability** of the software
  - Perhaps "repair" included many changes to the software to add new features
  - Or perhaps the maintenance staff are not competent

There Are Systematic Ways to Identify Surrogates

- **Decomposition Approaches**
  - Fixed models
  - Individualized models



- **Standardized Approaches**
  - These enable comparisons of software from different organizations
  - But may not fit the desired quality characteristics of some software

Quality Characteristic	Borhm	McCall	FURPS	ISO 9126	Dromey
Testability	X	X		X	
Correctness		X			
Efficiency	X	X	X	X	X
Understandability	X		X	X	X
Reliability	X	X	X	X	X
Flexibility		X	X		
Functionality			X	X	X
Human Engineering	X				
Integrity		X		X	
Interoperability		X		X	
Process Maturity					X
Maintainability	X	X	X	X	X
Changeability	X				
Portability	X	X		X	X
Reusability		X			X

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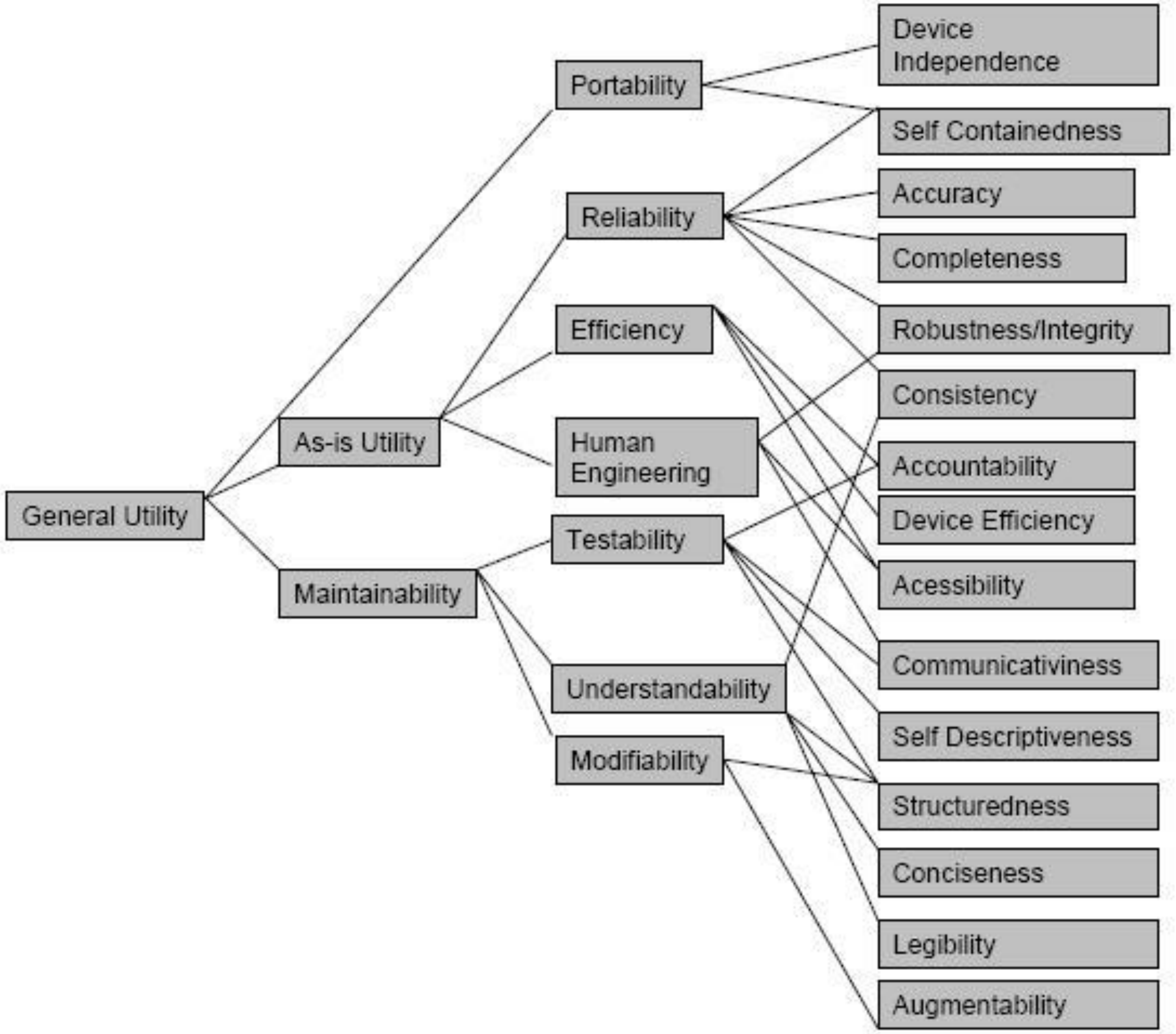
There is little consensus on how to measure quality attributes, so most organizations define them in ways that fit their specific customer needs.

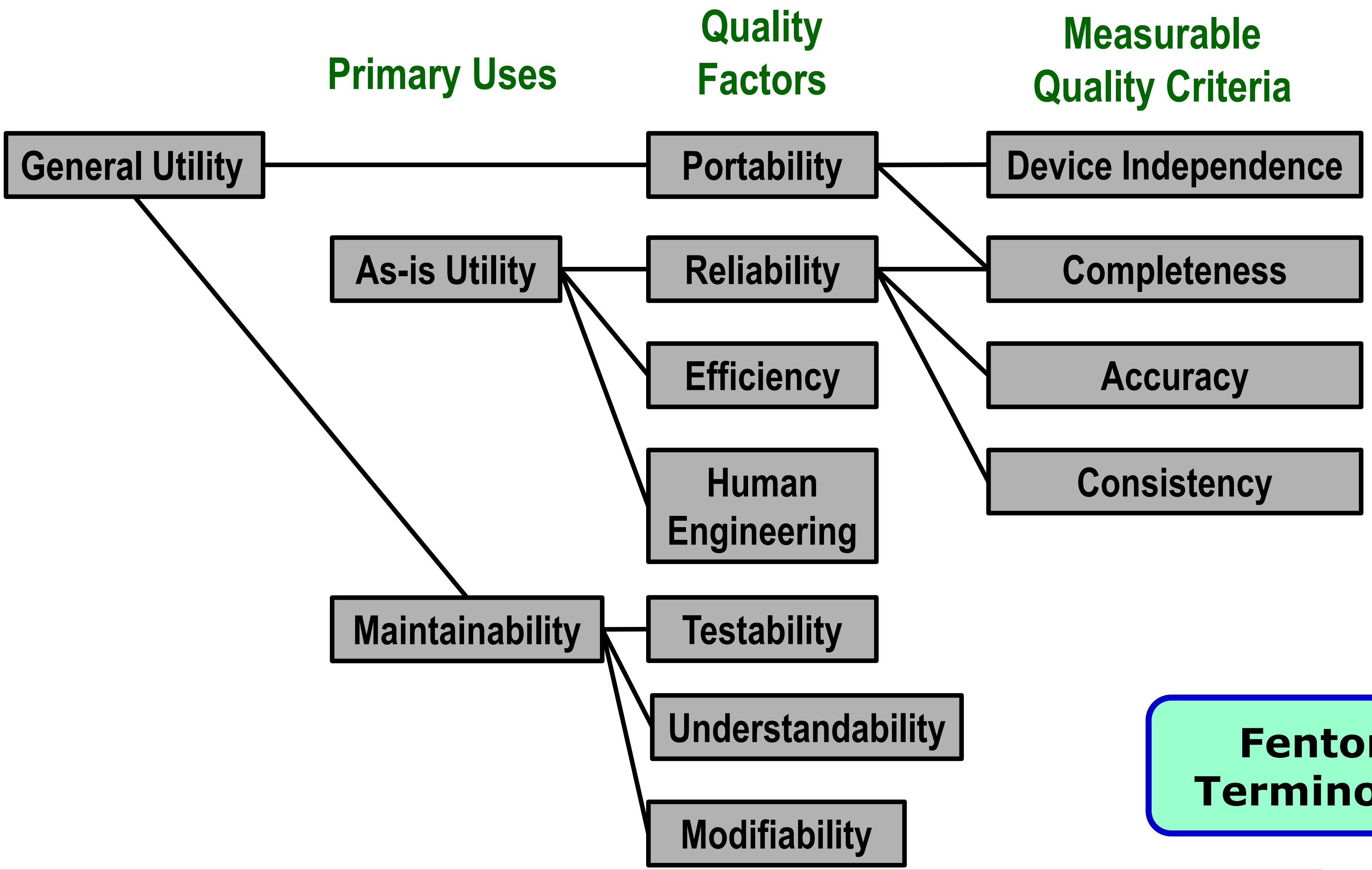
Decomposition Approaches

Boehm Software Quality Model



The concept here is to *decompose quality attributes or factors into subfactors* until you find factors that are measurable.

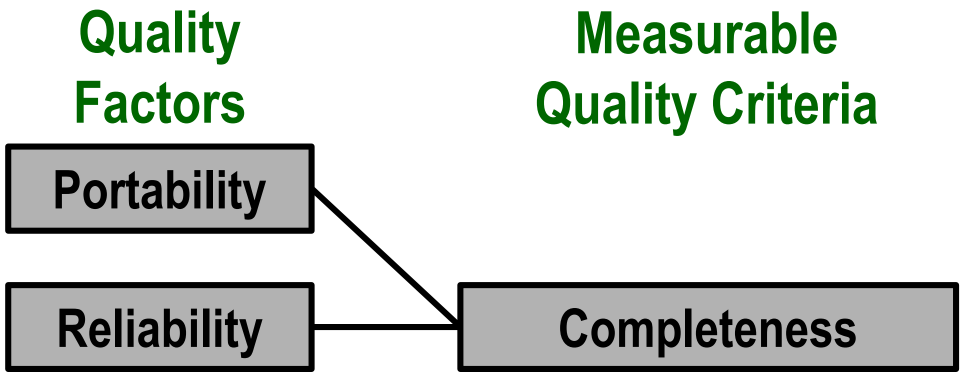
A Closer Look at the Boehm Model



Fenton's Terminology

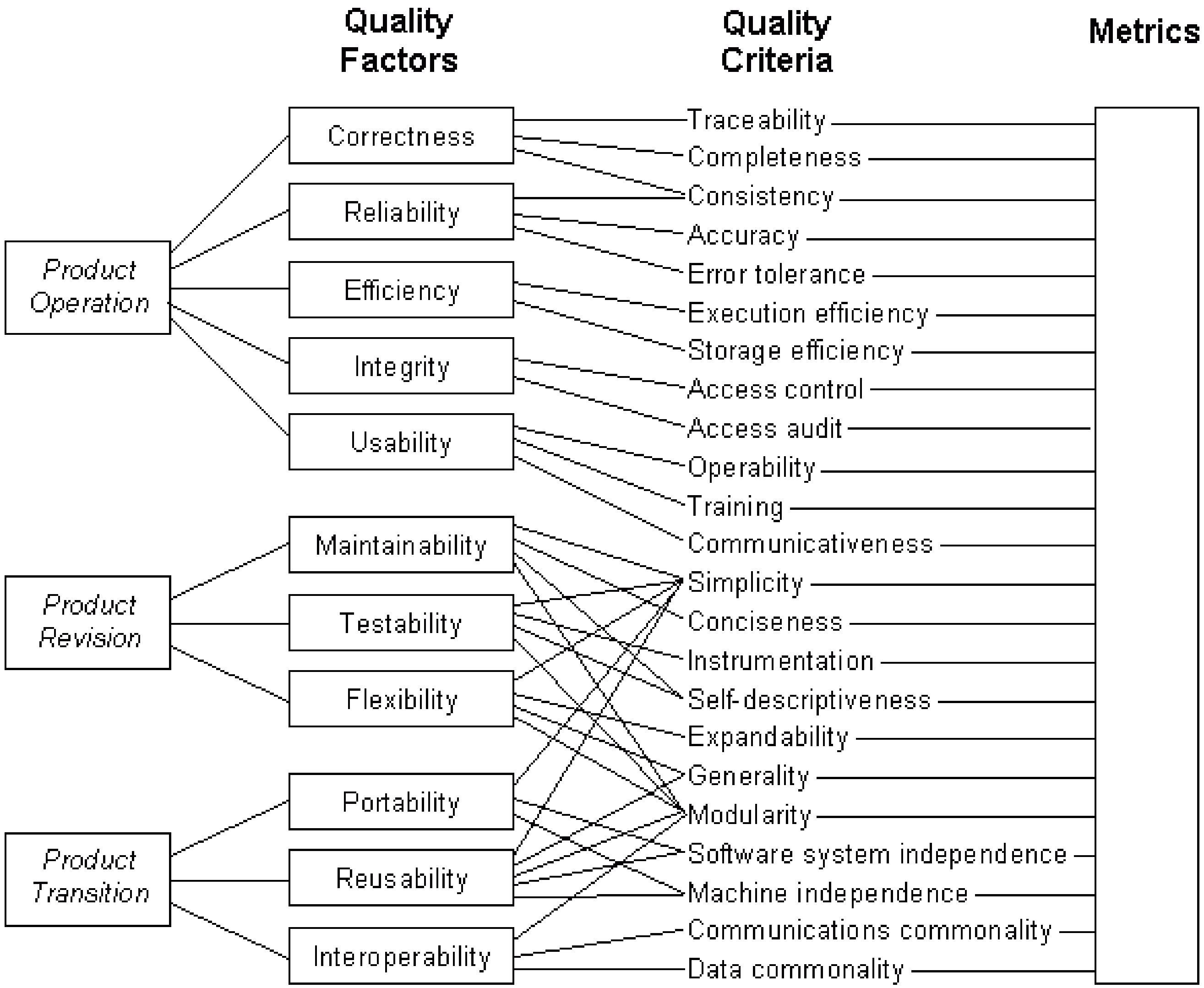
Comments on Boehm's Model

- **This is a way to *decompose what we mean by "quality"* until we have measurable attributes (quality criteria)**
- **These quality criteria are *surrogates* for quality**
  - There are many of them
  - Some of them relate to multiple quality factors



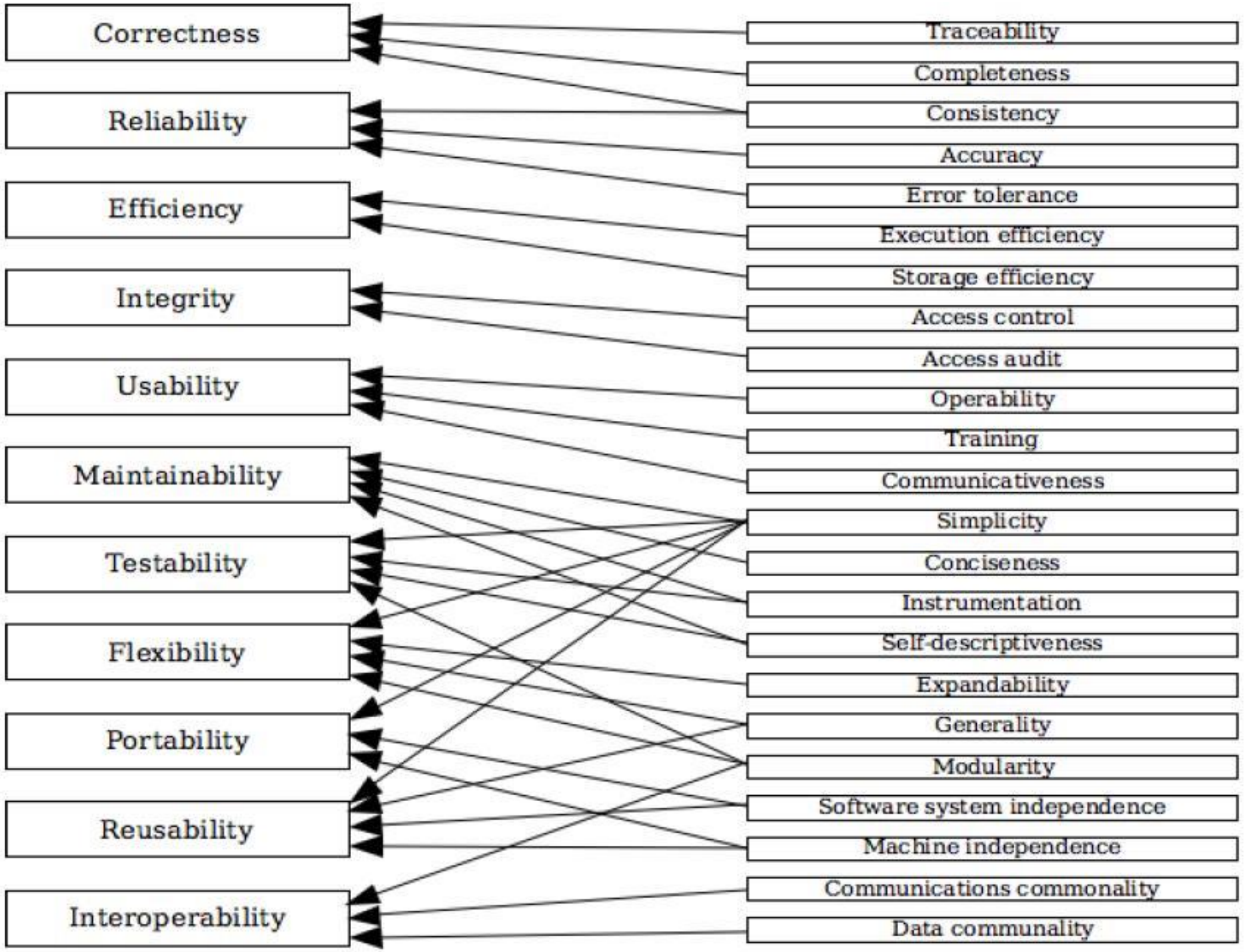
Decomposition Approaches

McCall Software Quality Model



As you can see, it's possible to establish a lot of criteria related to quality

McCall Model – Quality Factors and (Measurable) Criteria



As with the Boehm model, some criteria relate to multiple quality factors

Do I Really Need to Measure So Many Attributes?

- **The various models tend to be comprehensive**
  - But you may need to ***use only a portion*** of a model for your specific situation
  - Ultimately you need to ***measure only what will actually be used*** and be ***useful***

Measures of Software Quality

Based on

Defects or Faults or Failures

Quality = Lack of Defects (or Lack of Faults or Lack of Failures)[1]

The advantage of this approach is that it is often *easier to test for* defects or failures and *easier to measure* them than many other measures of quality

- However this approach ***may not capture what quality means to the end user***
  - Ease of use
  - Speed
  - …
- And it ***may not reflect all that the developer considers important***
  - Maintainability
  - Supportability
  - …

[1] Defects and faults usually mean the same thing – causes of failures.

Defect Density[1]

$$\textit{Defect Density} = \frac{\textit{Number of Defects}}{\textit{Size of Software Product}}$$

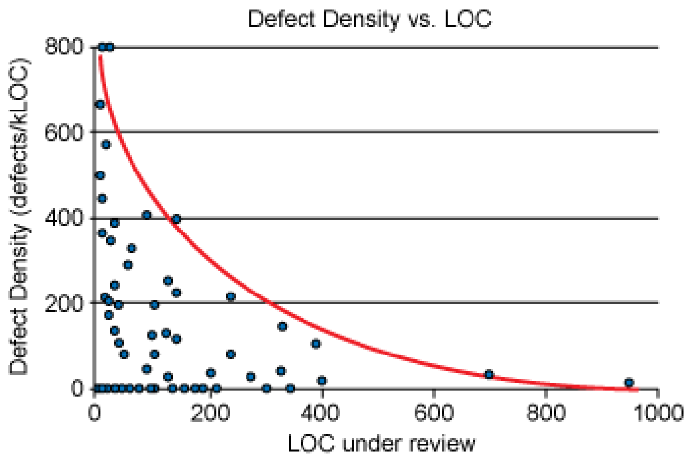
Variations:

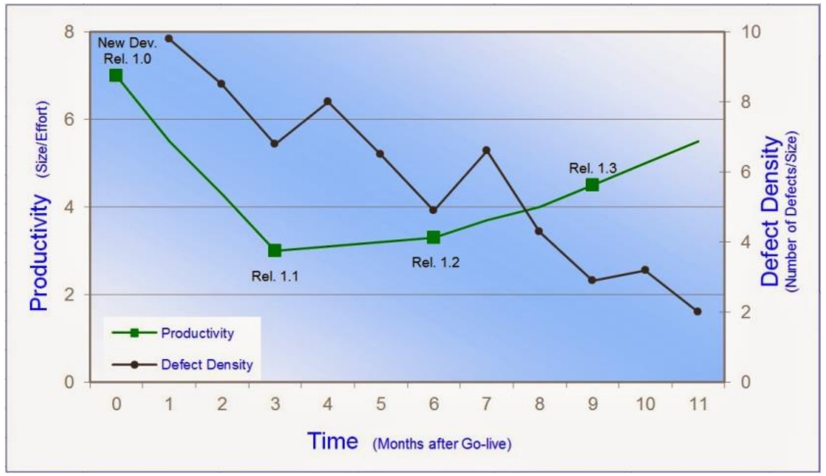
- *Failure Density (instead of defects)*
- *Number of Defects (this can be defined in different ways)*
  - *Known Defects*
  - *Total Defects (Known Defects + Latent Defects[2])*
- *Size of Software Product (can be defined in different ways)*
  - *It depends on the definition of size*

[1] Sometimes called "defect rate", although this is inaccurate
[2] Latent defects are defects we have not yet discovered

Defect Density Advantages

- **Easily measured, compared with other options**
- **Gives a good, *general idea of the overall quality* of the software**
- **This measure has been used for over 50 years to measure software, and overall the defect density has correlated well with perceived quality of products**





Defect Density Drawbacks

- **People can't always agree on *what constitutes a defect***
  - Failure in operation vs mistake in the code
  - Post-release defects vs defects found during development
  - Discovered vs latent defects
- ***Severity of problems* caused by defects may be *hard to assess***
  - Some software defects have no significant impact on customer's perception of quality
  - Different customers use the software in different ways

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Example from IBM[1]

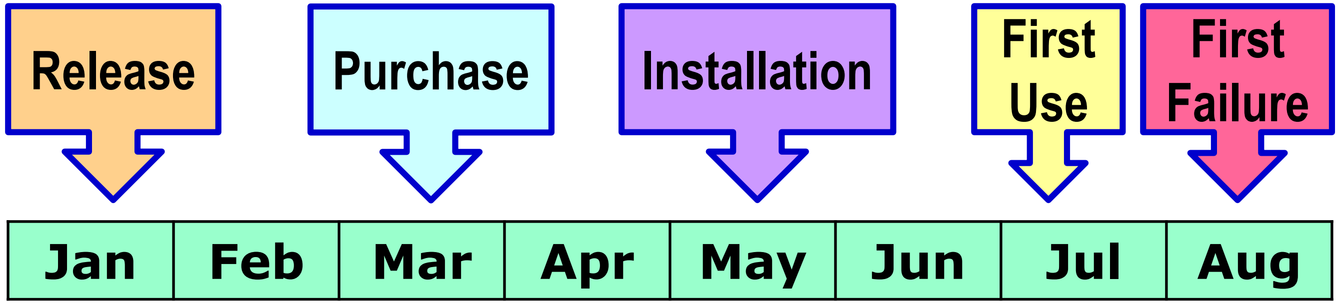
- **Approximately *one out of three defects* will only cause a user failure *once in 500 years*.**
- **A *very small portion* of defects (<2%) *cause the most important user failures***

Number of defects <u>may</u> <u>not</u> be strongly correlated to the frequency or severity of end user failures.

[1] See Adams in reference list.

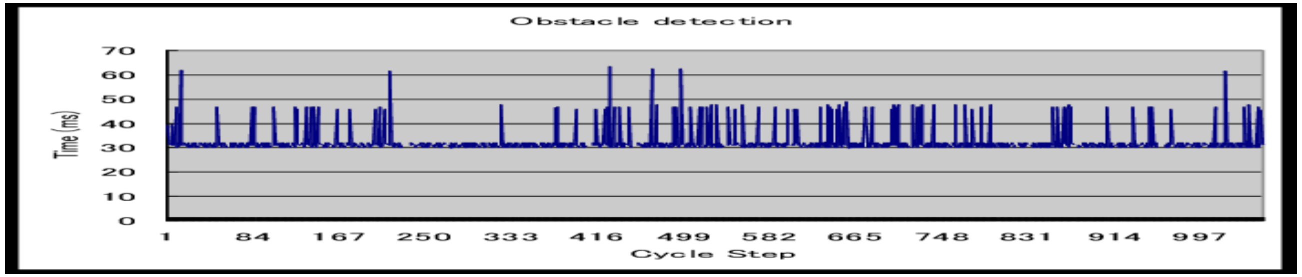
Defect Density Drawbacks
(2 of 3)

- ***Different measures of the time scale***
  - Amount of ***time since release of product***



  - Amount of ***time the product is actually used***

  - ***Processing time*** actually used by the product



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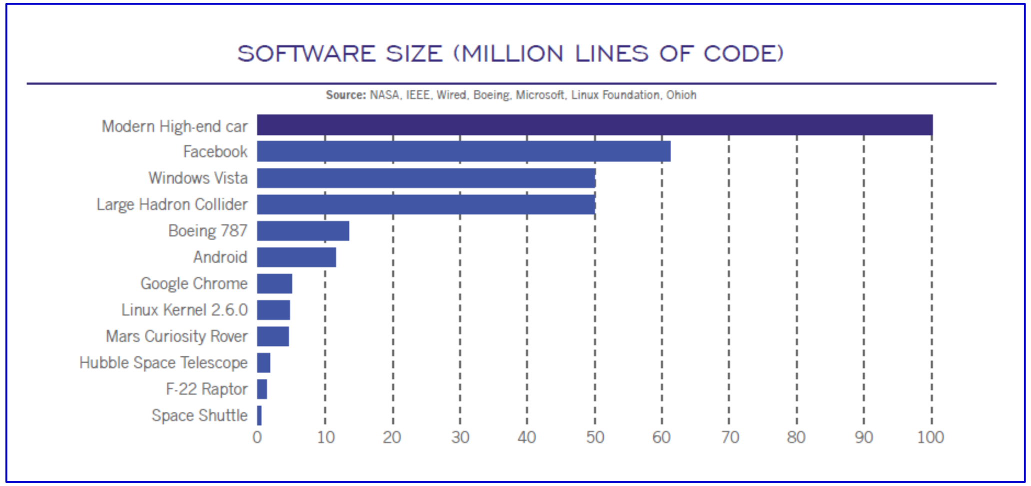
Defect Density Drawbacks
(3 of 3)

- ***Different measures of size***
  – This can make it hard to compare different projects or processes or development methods or organizations



- ***What is defect density telling us*?**
  – The quality of our product?

    or

  – The effectiveness of our defect detection and correction process?

Despite These Drawbacks, Defect Density is Very Widely Used

Some metrics that incorporate defect density

- Cumulative defect density
  - During development or after delivery
- Total serious defects found
- Mean time to fix serious defects
- Defects found during design reviews per KLOC
- Code inspection or peer review defects found per KLOC
- System test errors found per KLOC
- Customer-discovered problems per KLOC or per product

Usability
Hard to Test For & Hard to Measure

Formal Definition:

Usability is the degree to which a system can be used by specified users to achieve specified goals with effectiveness, efficiency and satisfaction in a specified context of use.

ISO/IEC 25010 (2011)

Commonly used concepts of usability:

- User Friendliness
- Ease of use

This is a very complex concept that is hard to measure, but important to most end users

Three Categories of Usability[1]

- **Effectiveness**
  - Can users complete the tasks correctly?
  - Example: $\textbf{Effectiveness} = \dfrac{\boldsymbol{Quantity * Quality}}{\mathbf{100}}$
- **Efficiency**
  - Time required to complete the tasks
  - Example: $\textbf{Efficiency} = \dfrac{\boldsymbol{Effectiveness}}{\boldsymbol{Task\ Time}}$
- **Satisfaction**
  - Degree to which the end user likes the software
    - ***This is a very subjective measure***

[1] See Fenton, section 10.3 for further details

Internal Attributes Generally Viewed as Related to Usability

These are more readily measured and can be measured before the software is released

- Good use of menus
- Good use of graphics
- Good help functions
- Consistent interfaces
- Well-organized reference manuals

Researchers have been unsuccessful in relating these to effectiveness, efficiency or customer satisfaction.

Use of these to predict usability is not recommended.

Testability

A product is testable if:

- It can be tested in a reasonable way (readily testable)
- The tests are well defined, comprehensive, and not overly redundant
- Each test can be directly traced to and from:
  - product requirements,
  - derived requirements resulting from design decisions, or
  - design or coding elements calling for specific testing
- Each test failure can be directly traced to:
  - a requirement that is not being met, or
  - A design element that was not properly implemented, or
  - A portion of the code that has a programming error

Good testing starts with testable requirements and designs.

Testing is unsuitable when …

- It would destroy the product
- It is too dangerous
- It is too costly
- It cannot reasonably be expected to provide confidence that requirements are satisfied
- It cannot be done

Evaluation Techniques
(other than testing)

- ***Examination***
  - For example, reading designs or code or other documents to check for errors
- ***Demonstration***
  - e.g. flying an airplane to show that it can fly
  - e.g. running a program to show that it works
- **Other techniques (examples)**
  - providing a ***formal proof*** that a program is correct
  - ***measuring*** something
  - showing through ***statistical analysis*** that the probability of a defect is below a threshold

Reasons why Requirements/Designs May be Hard to Test

- **Requirements may not be well understood**
- **Requirements may not be well documented**
- **What seems obvious to the customer or the system designer may not seem clear or obvious to the software developer or tester**
  - Different kinds of knowledge
  - Unstated assumptions
- **The customer and the software developer may not agree on what constitutes an acceptable test**
- **Changes made during software development may not be communicated to the software team**

Suggestions (slide 1 of 3)

- **A requirement or design feature is not complete until you have reached agreement on how it is to be tested**
  - For each requirement, reach agreement between the software team and the customer or system engineer on how the requirement is to be tested
  - For each design feature, reach agreement between the software designer and the software test team on how the design feature is to be tested

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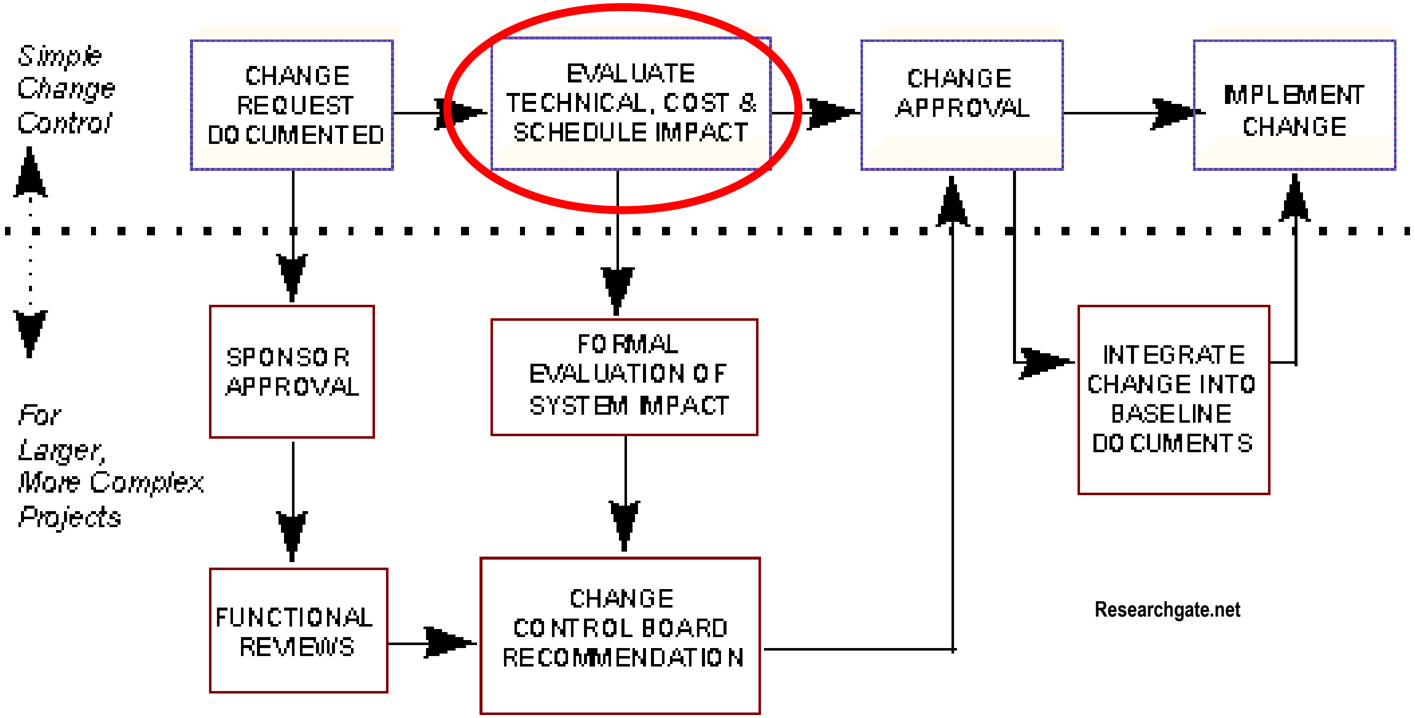
Suggestions (slide 2 of 3)

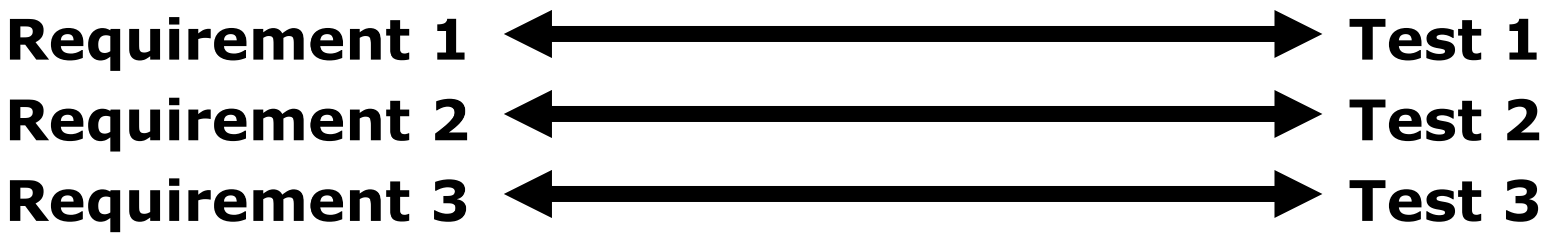
- **Control changes to requirements and design**
  - Don't allow a requirements or design change without a clear understanding of the effect of the change on the software cost, schedule and technical development
  - For each change to requirements or design, indicate how the corresponding tests must be changed.



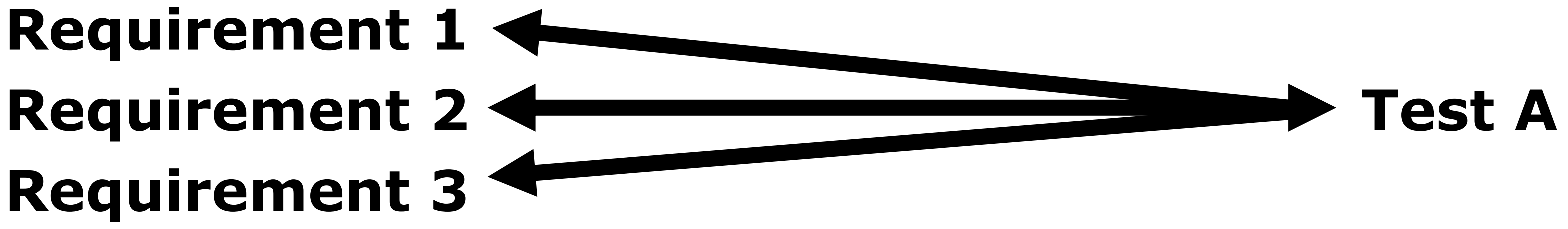
Suggestions (slide 3 of 3)

- **Keep track of which tests correspond to which requirements or design elements *(traceability)***

Ideal

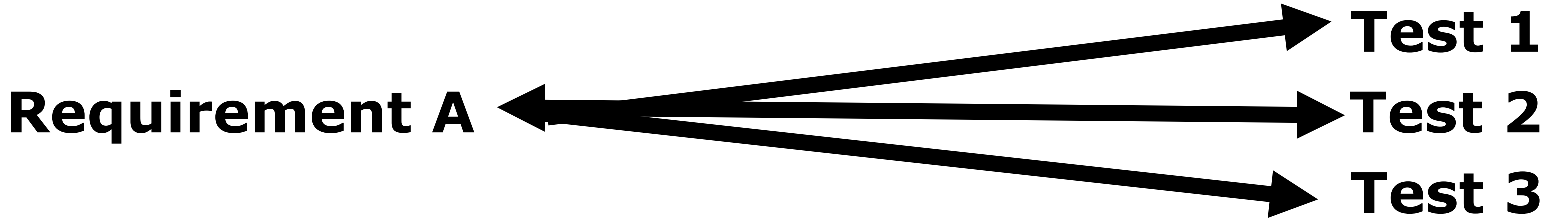


Acceptable

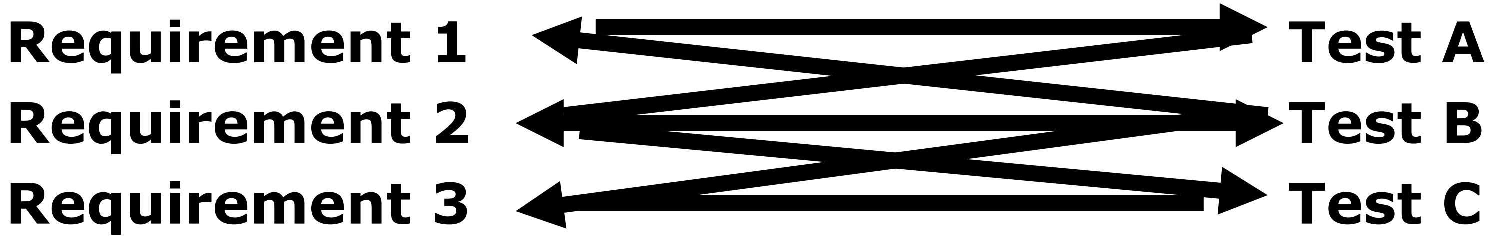


Other Traceability Options

Acceptable



Undesirable



Reasons Why Code May Be Difficult to Test

- **Code is not well structured**
  - Needlessly complex
  - Poorly organized

We will address this in parts 4 and 5

- **Code elements do not trace directly to requirements or design elements**
  - So when the code causes a failure, it is hard to determine whether the problem is with the code or the design or the requirement

- **Code is not well documented or does not follow coding conventions**
  - Hard to understand
  - Error prone

Seeding and Tagging
A way to assess Testing Progress

Seeding and Tagging

Purpose: **To help you estimate how many undetected errors (defects) are in your code**

When to do this: **During test planning and during the testing process**

Suppose**: You have been testing your code and have discovered D_1 errors (defects).**

Question**: How many errors are left?**

Technique**: Seeding and Tagging**

Concept**: Introduce extra errors and see how many of them your test process has found.**

Seeding and Tagging Details

- **Introduce a given number of extra errors into the software -- say *E* of them**
- **Run standard tests, detecting *D_2* of them**
- **Compute *D_2/E* = % of errors detected**
- **Suppose *D_1* = number of genuine errors already detected**
- **Then you assume the total number of errors in the software is**

$$D_1 * E / D_2$$

Example of Seeding and Tagging

- 200 defects found so far
- You have injected 20 extra defects
- You have found 12 of these extra defects
- Therefore, assume total defects =

  200 * 20 / 12 = 4000 / 12 = 333 total defects

  => 333 - 200 = 133 defects remaining

By performing this analysis from time to time, you can estimate your defect density and your testing progress over time.

Part 2

Some Basic Principles of Measurement and Data Analysis

Contents

- **Definitions**
- **Scales**
- **Basic Analysis Approaches**
- **Statistical Distributions**
- **Other Statistical Concepts**

Definition of Measurement

Measurement is ...

... the process by which numbers or symbols are assigned to attributes of entities in the real world in such a way so as to describe them according to clearly defined rules.

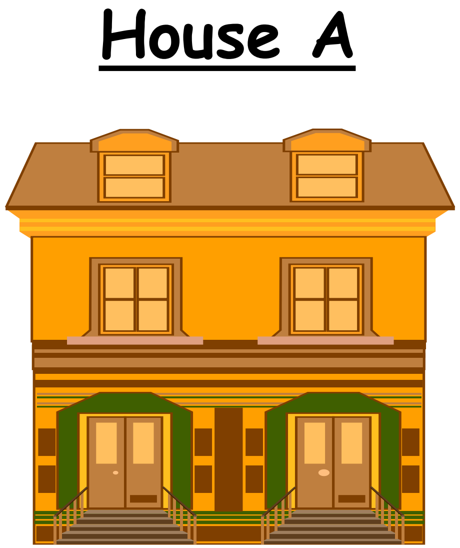
- **The assignment of numbers must preserve intuitive and empirical observations about the attributes and entities**

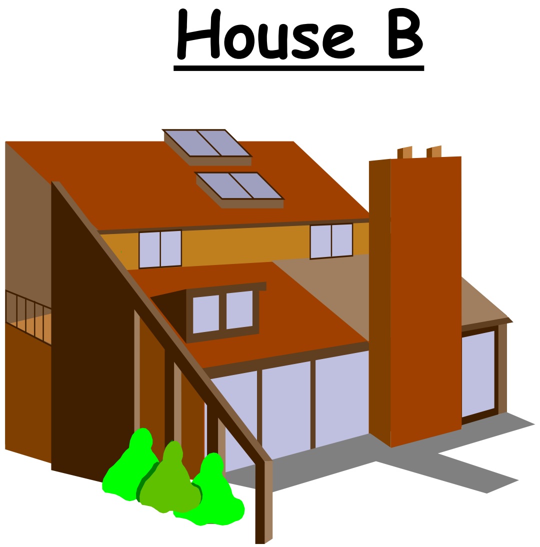
Source: Fenton, page 5

Preservation of Attributes Example

"***House A is bigger than House B***"

This is a meaningful statement only if the number we assign to "size" preserves our intuitive notion of houses and their sizes.





But Intuitions Vary

What do we really mean by "size"?

Before we can measure size, we must define a *model* that reflects a specific *viewpoint*

– The model must specify an **entity** to be measured

and

– an **attribute** of that entity.

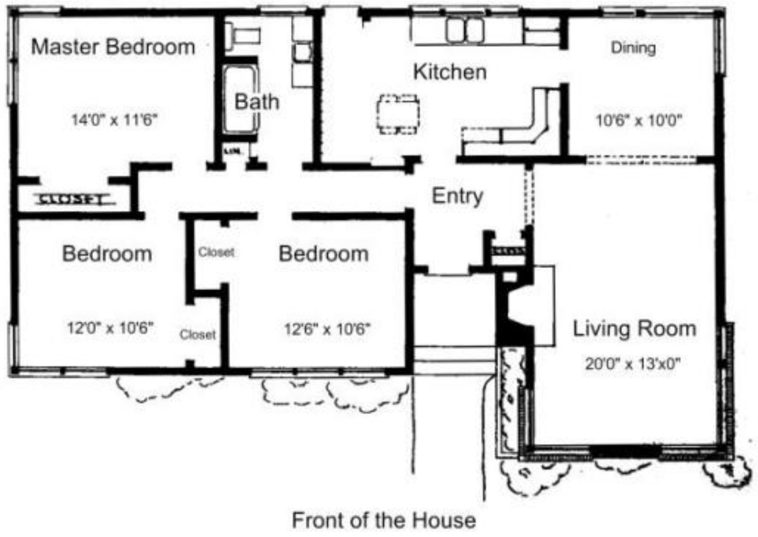
- I.e.,
  - **what do you want to measure?** and
  - **what do you want to know about it?**

– Examples (next two slides)

Functionality Model for Size of a House

In this model, the size of a house is based on *how many people can comfortably live there*

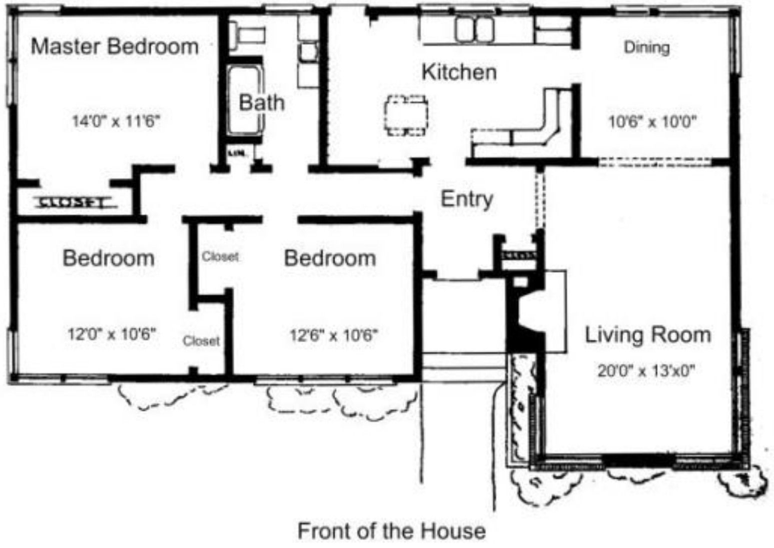
- The size of a house is measured by the ***number of bedrooms and the number of bathrooms***



Cost Model for Size of a House

In this model, the size of a house is based on *how much it will cost to construct it*

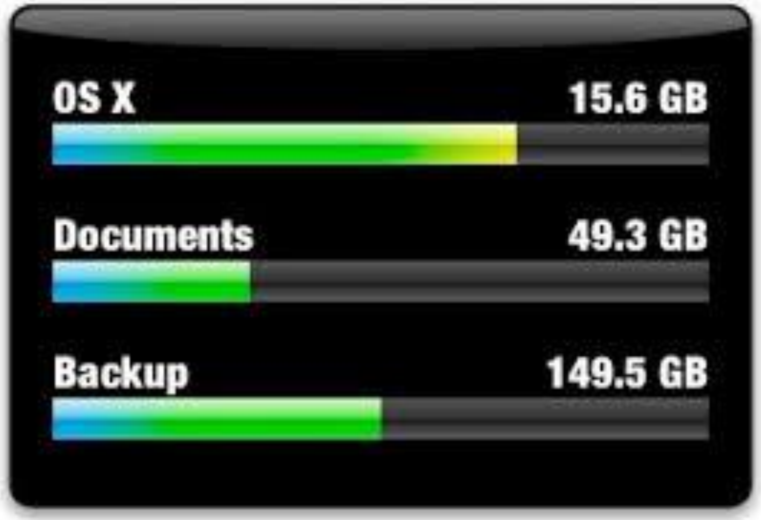
- The size of a house is measured by the ***square feet of living space***



Storage Space Model for Size of Software Used on a PC

In this model, the size of software is based on *how much disk space it occupies*

- The size of software is measured by the ***number of bytes of memory*** it requires when stored on a disk



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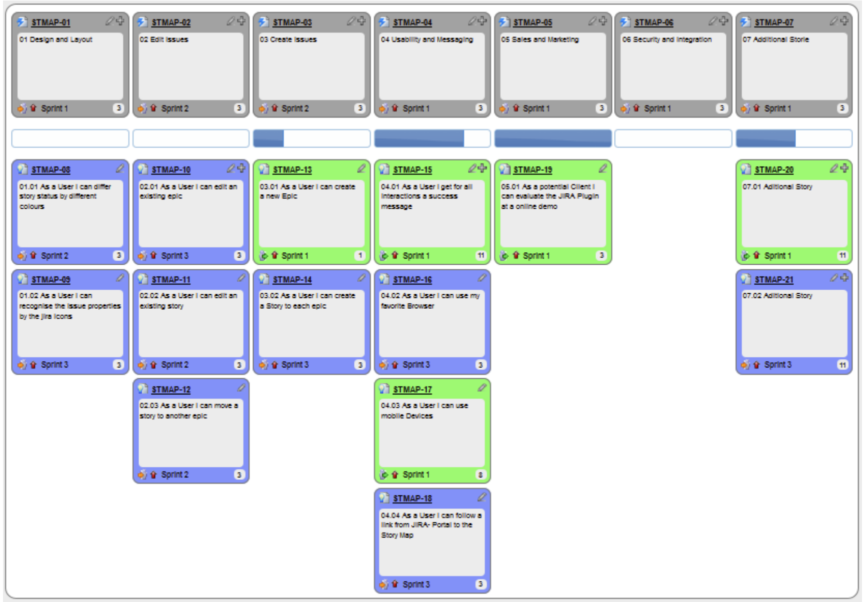
Cost Model for Size of Software Used on a PC

In this model, the size of software is based on *how much it will cost to construct it*

- The size of software is measured by the ***number of user stories*** used to establish the requirements.



https://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0ahUKEwikptj9kdfSAhWKz4MKHfSNAfEQjRwIBw&url=https%3A%2F%2Fen.wikipedia.org%2Fwiki%2FUser_story&bvm=bv.149397726,d.cGc&psig=AFQjCNFdWIGXMDrGnfsbmrgiNdbLFEGADg&ust=1489620239765305

Contents

- Definitions
- **Scales**
- Basic Analysis Approaches
- Statistical Distributions
- Other Statistical Concepts

Measures Allow Us to Categorize and Analyze the Attributes of Entities

But measures do not have to be numbers.

Example: Road Signs – shape and color are used to categorize road signs

Warning

Information

Definitions: Scales of Measure

A ***scale of measure***

or

A ***level of measurement***

or simply

A ***scale,*** is:

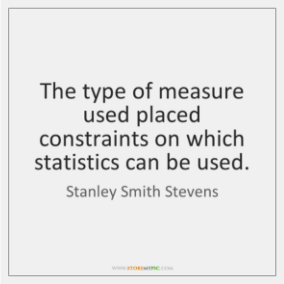
- A ***collection of symbols or numbers*** used to **classify** attributes of entities or variables
- A ***classification system*** for **describing** the nature of information

- **There are various scales in use and the properties of those scales are important**
  - The properties help determine which forms of analysis are appropriate

Classification of Scales

The classification system most widely adopted in data analysis and statistics was originated by Stanley Smith Stevens[1] for use in psychological research.

www.fixquote.com



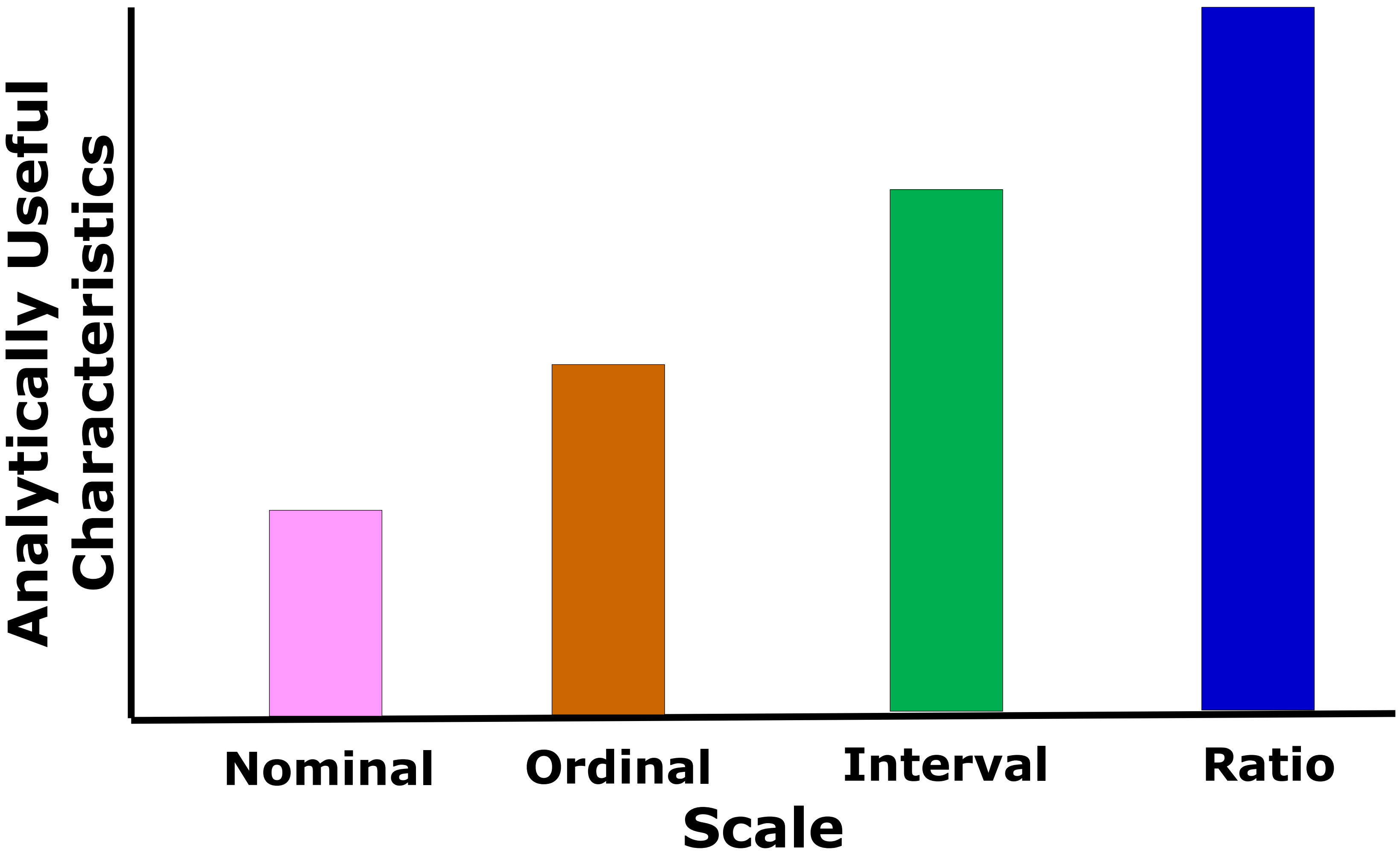
www.storemypic.com

- **This classification includes four levels of scales:**
  - Nominal
  - Ordinal
  - Interval
  - Ratio

Researchers have debated Stevens' classification and proposed other classification systems, but no other scale classification has achieved such widespread use.

[1]Stevens, S. S. (7 June 1946). "On the Theory of Scales of Measurement". Science. 103 (2684): 677–680.

Stevens' Types of Scales



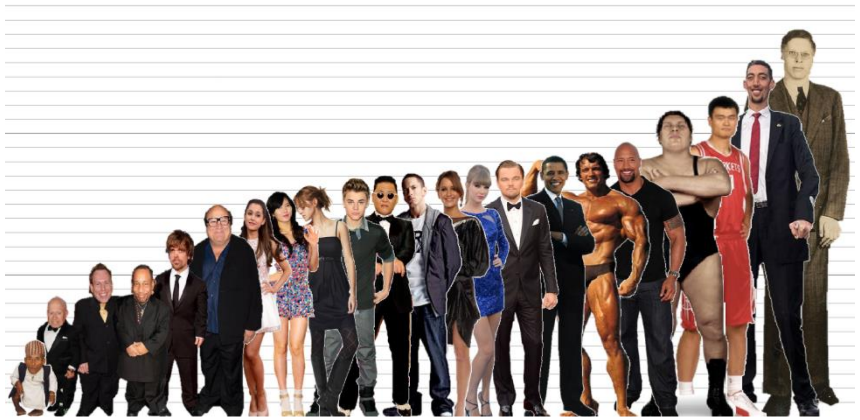
Scales Help Us Understand What Kinds of Analysis are Meaningful

Color

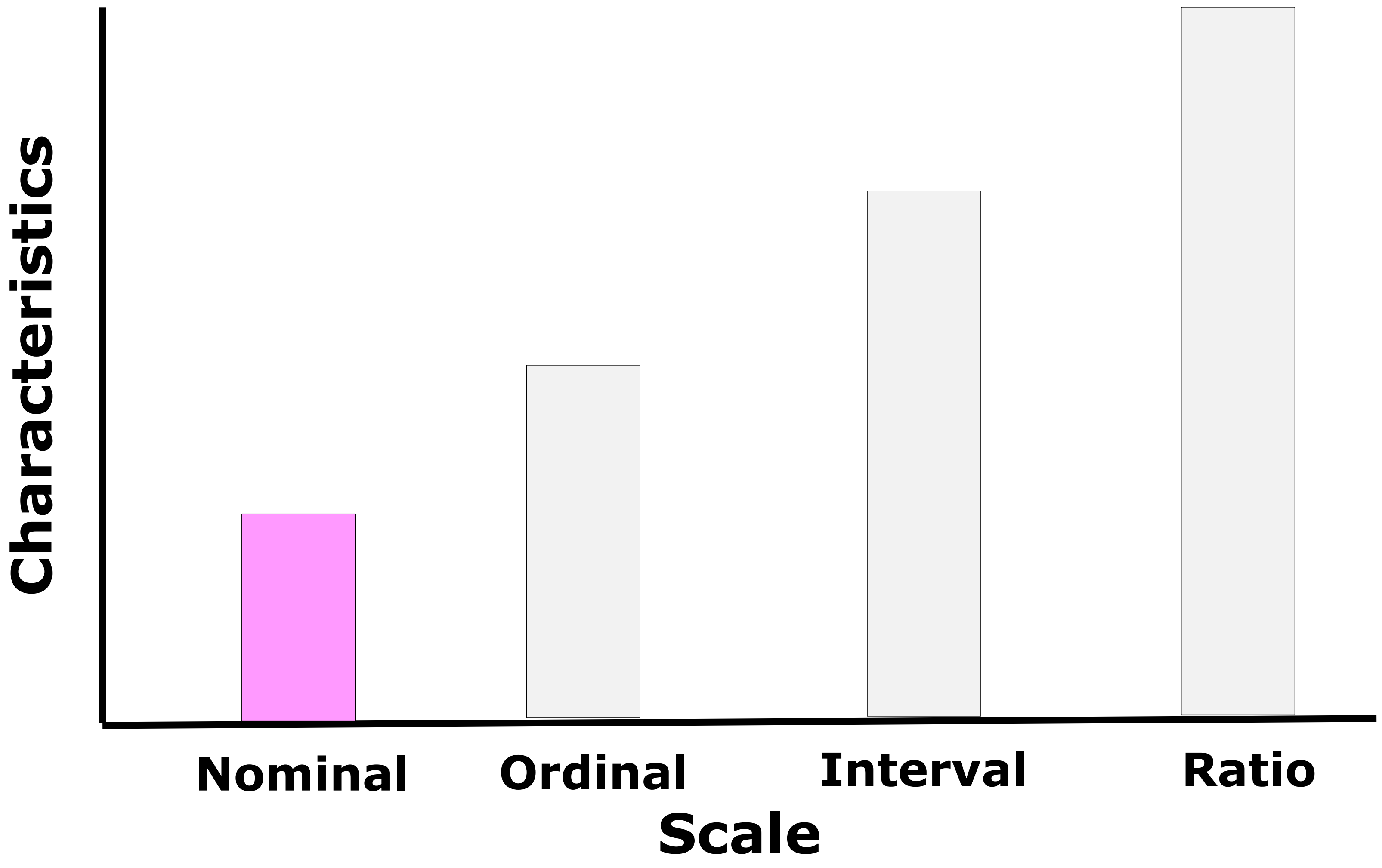
No Natural Order

Height

Natural Order



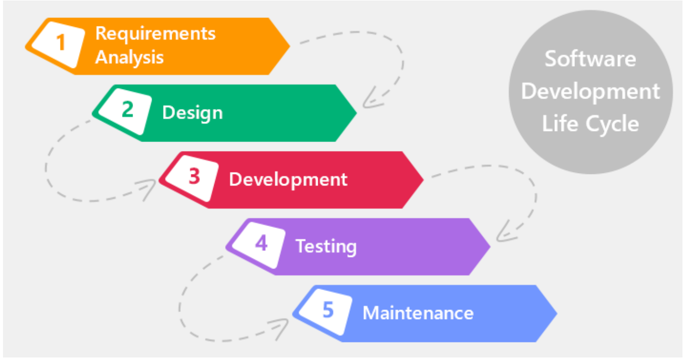
Nominal Scales



Nominal: a scale that places entities in ***categories*** – but without any ordering

Example: Color
– marbles are
- Blue, or
- Black, or
- Red, or
- Yellow

Software Example: Defect Origin
- defects discovered while testing are
- Requirements related, or
- Design related, or
- Development related, or
- Testing related.



www.xbsoftware.com

Nominal Scales - Characteristics

- ***We can categorize the entities***
  - *Often in terms of specific qualities such as color or shape*
- ***We can count the size or frequency of each category***
- ***We can determine the most frequent category (the mode)***
- ***But there is no natural order or ranking to the categories***
- ***And there's no such thing as the median or average***

Other Examples of Nominal Scales

- Gender
- Nationality
- Ethnicity
- Language
- Genre
- Style
- Biological Species
- Form or Shape
- Parts of Speech (in grammar)

We can use numbers to classify attributes or variables in a nominal scale

But they do not have any numerical values or relationships other than equality or inequality

Example: Suppose We Have Three Animal Species

1. Tigers

Mynicetime23.wordpress.com

Just because we have numbered them in a particular order does not mean anything.

2. Elephants

True-wildlife.blogspot.com

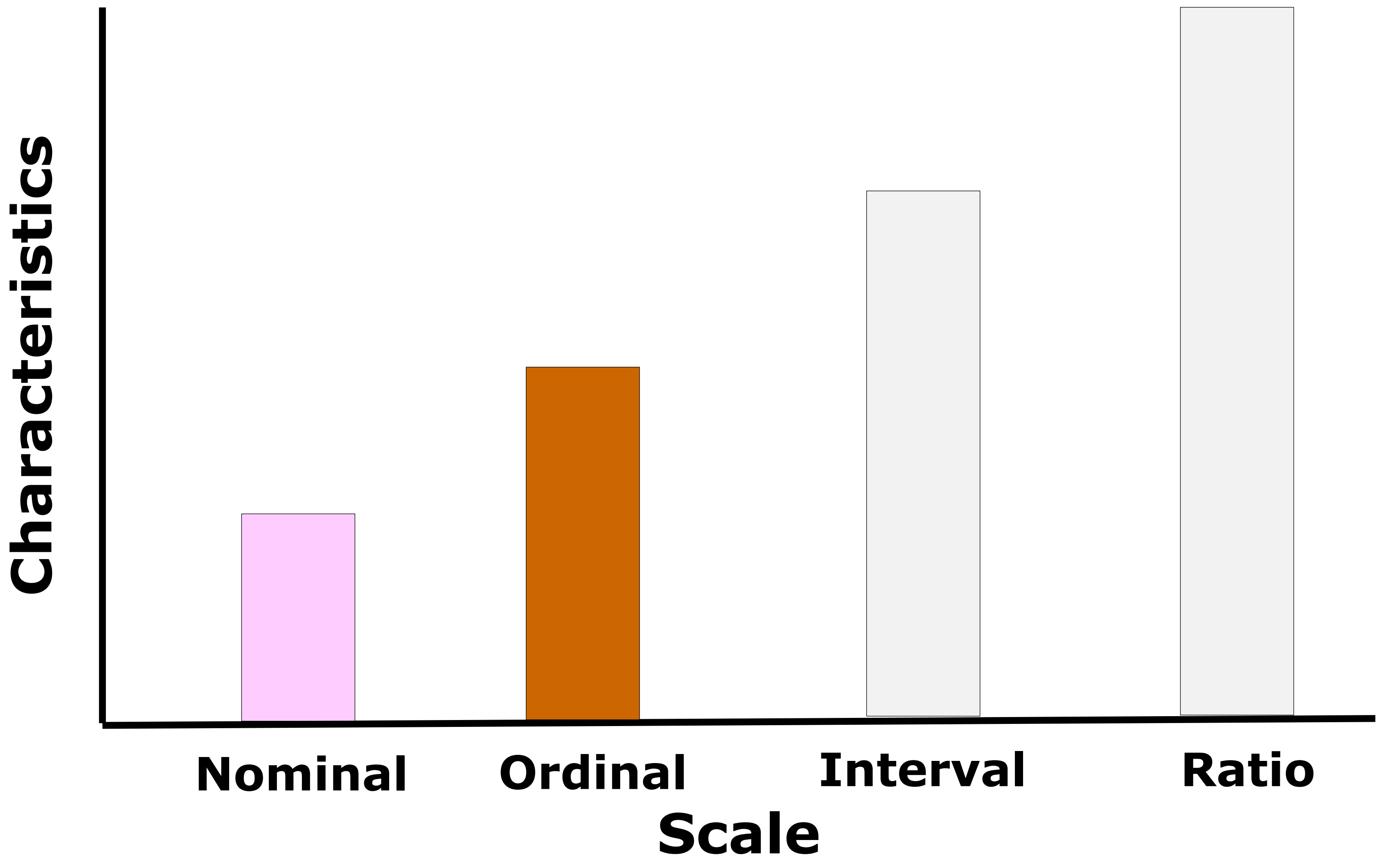
Tigers are not more important because they are listed first

3. Horses

kidssearch.com

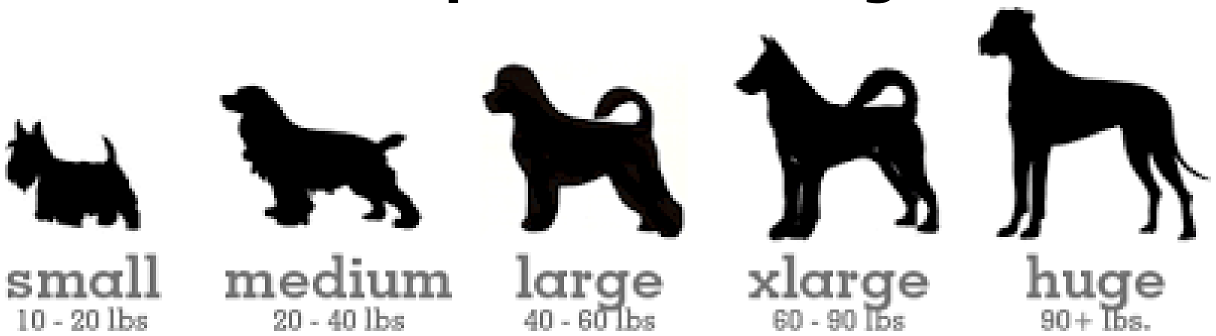
Horses are not 3 times as valuable because we gave them the number three

Ordinal Scales



Ordinal: there is a *ranking* or *ordering*

- **Example: military rank**
  - General
  - Colonel
  - Lieutenant
  - Sergeant
- **SW Example: defect severity**
  - minor
  - significant
  - major
- **Example: size of dogs**



https://cdn.shopify.com/s/files/1/0118/8082/files/dog_sizes_antler.png?373

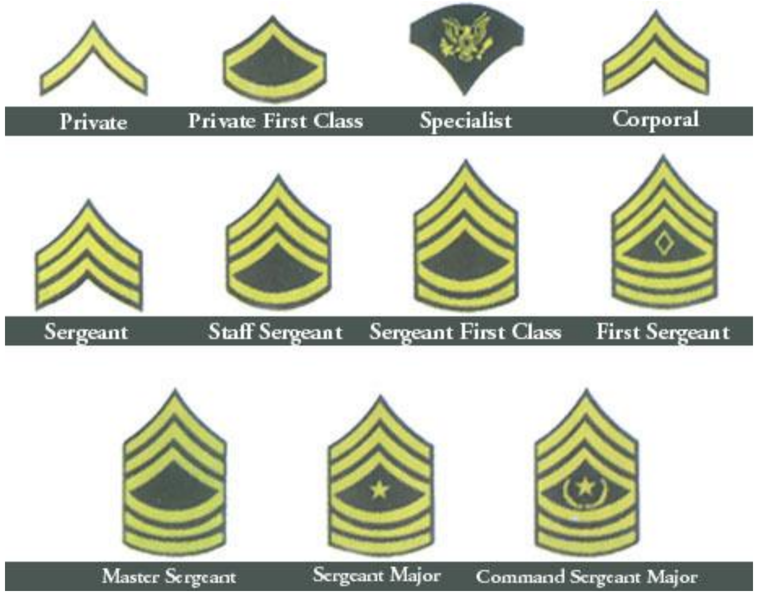
Ordinal Scales - Characteristics

- You can place them in ***order*** (sorting)
- You can determine the ***median*** (middle) item in a list

➢ But you **cannot compute an average**

➢ And there is **no mathematical relationship between categories**

➢ (sergeant is not “twice as” high as “private”)



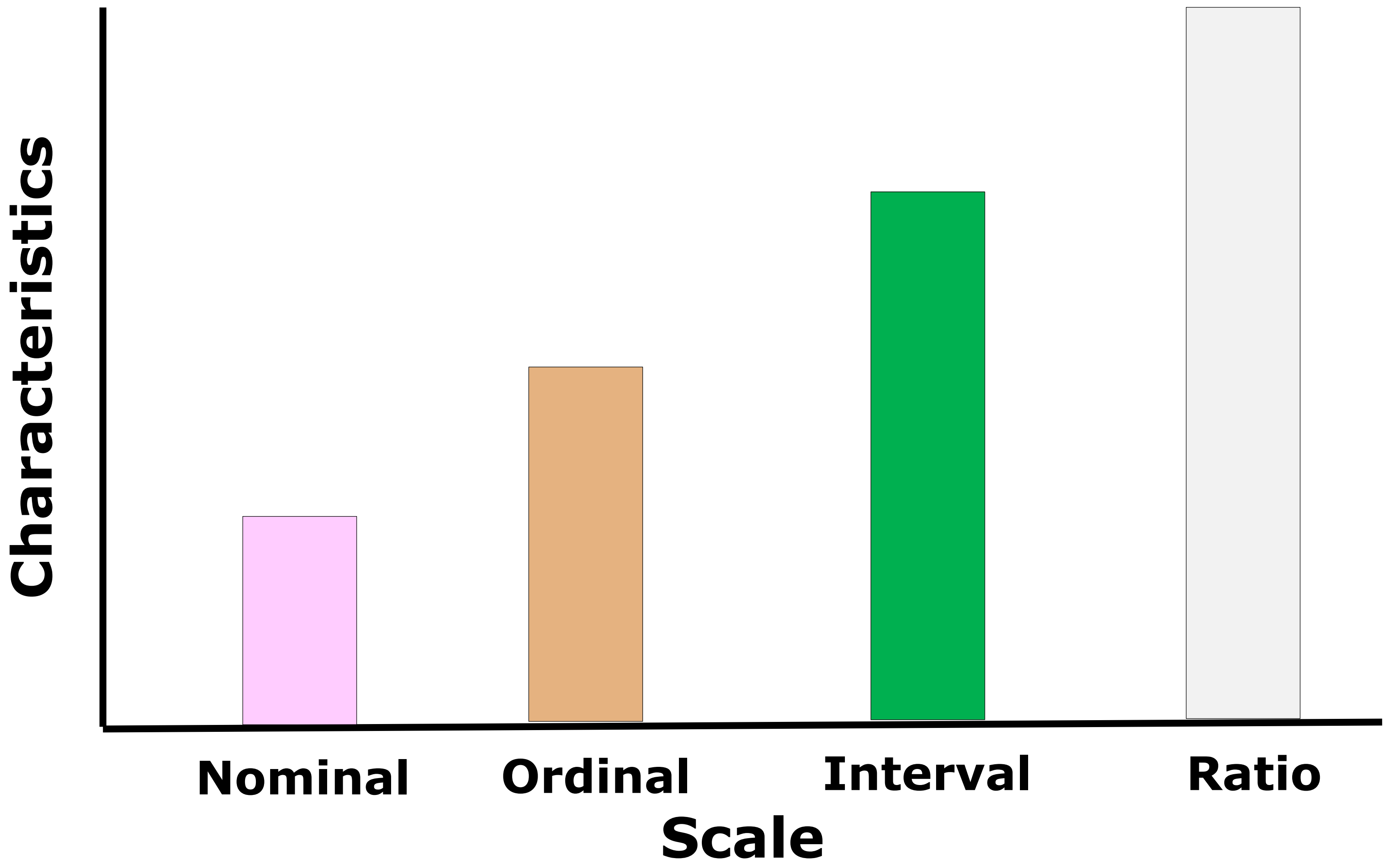
Ordinal Scales – Other Examples

- **Rank in a contest or race**
- **Degree of health**
  - Critical Serious Fair Good Excellent
- **Comparative restaurant ratings**
  - Excellent Very Good Good Poor Terrible

The degree of distance between successive categories is not defined

You can determine the middle item (median) but not the "average" (mean)

Interval Scales



Interval: there is a *fixed distance* between consecutive members of a sequence

- **Example: Dates**
  - 1/1/2012, 1/2/2012, …

Given any two dates, we can count the distance between them in days

- **Example: Temperature in Farenheit**
  - 10 degrees, 30 degrees, etc.

Given any two temperatures, we can count the distance between them in degrees

Interval Scales - Characteristics

- **There is an *ordering***
- **You can quantify the *distance* (degree of difference) between any two values**
- **You can *add or subtract* values**

You can tell if one is larger than another

Given any two dates, we can count the distance between them in days

30º plus 10º = 40º

3/5/2019 – 3/1/2019 = 4 days

➢ **But you cannot multiply or compute a ratio**
   – See next slide

With Interval Scales We Cannot Multiply or Divide

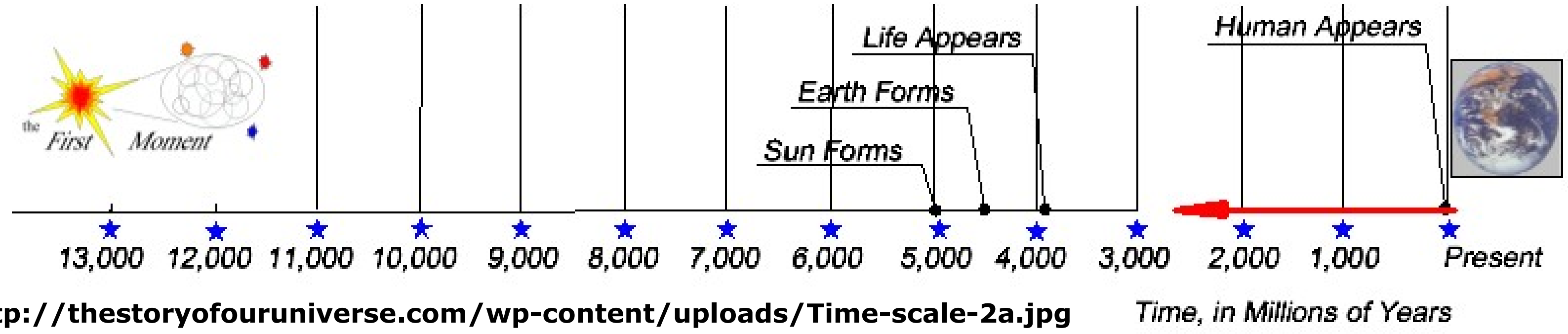
– Example: Dates
  - 1/1/2012, 1/5/2014, 1/25/2019, …

It does not make sense to say one date is "five times" another date

Mathematically, this means there is no "true 0" value -- we can place the "0" value anywhere in the sequence

Interval Scale Example - Time

- **We lack a precise measure of when time began**
- **So we measure time using an arbitrary "0" point**
  - Years are measured since an arbitrary date
    - Microsoft Excel dates are measured in days since 12/31/1899
  - Scientists often measure time in years backwards from the present



http://thestoryofouruniverse.com/wp-content/uploads/Time-scale-2a.jpg

Other Examples of Interval Scales

- **Temperature (on Centigrade or Fahrenheit scale)**
- **Map coordinates (longitude or latitude)**
- **Map direction (degrees from North)**

Note: a common error is to assume you can compute ratios for items in an interval scale.

See next slide for more information on this.

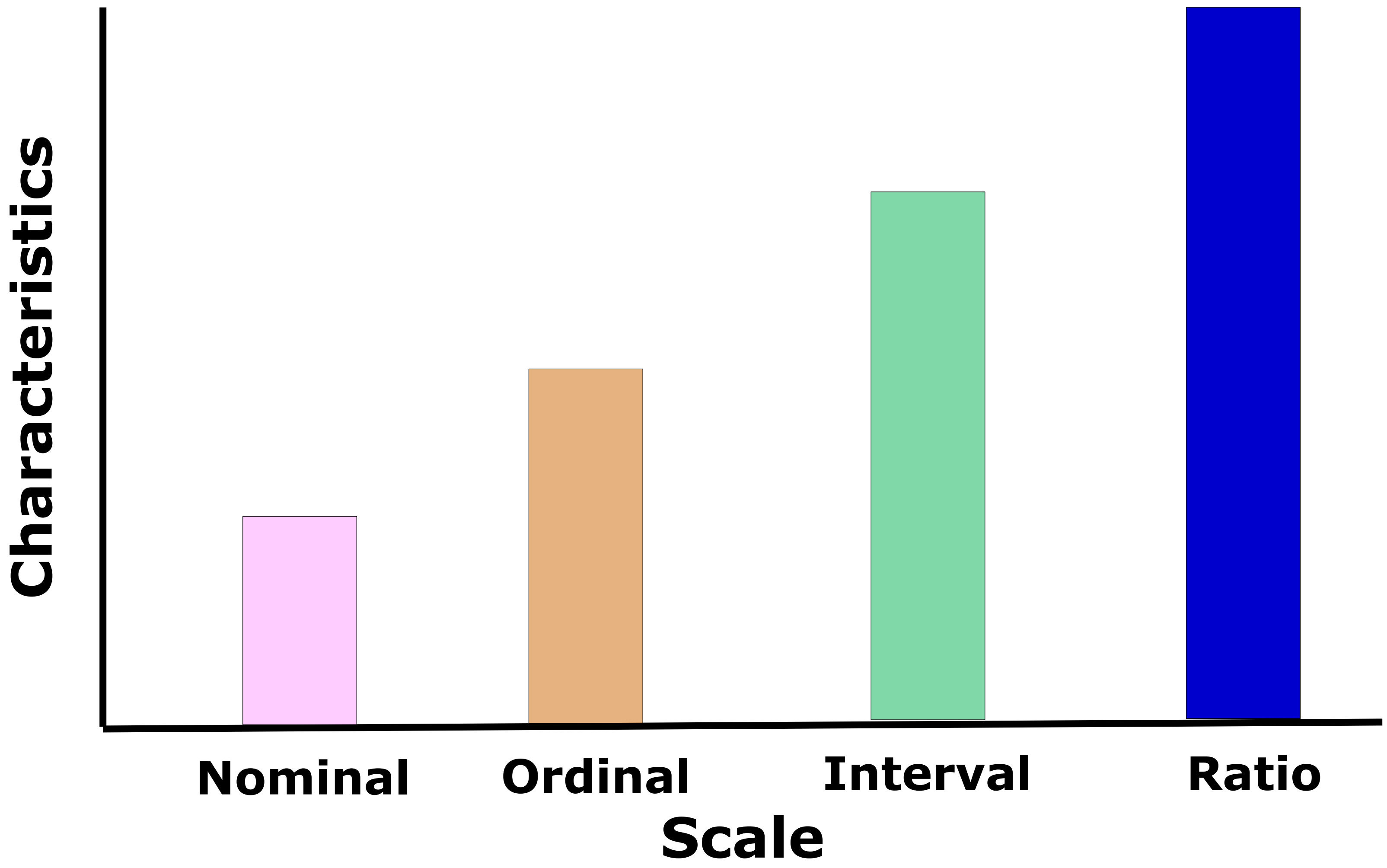
Computing Ratios for Interval Scales

- **The ratio between two items on an interval scale cannot be determined**
- **But you can determine the ratios between <u>differences</u>**

Example: the parent's age could be twice that of the child's:

Each age is the difference between the current date and the date of birth

Ratio Scales



Ratio: There is a fixed distance between consecutive sequence members, AND *multiplication is meaningful*

This means that ratios are meaningful

- **Examples:**
  - Size
  - Weight
  - Length
  - Duration
  - Electric Charge

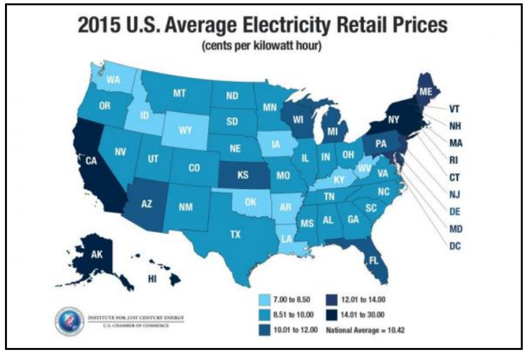
http://www.sixsigmatrainingconsulting.com/wp-content/uploads/2010/10/ratio-scale.bmp

60 mph is twice as fast as 30 mph

Note that there is a "true 0" value

With a Ratio Scale We Can Compute an Average or Mean

- Because we can multiply and divide



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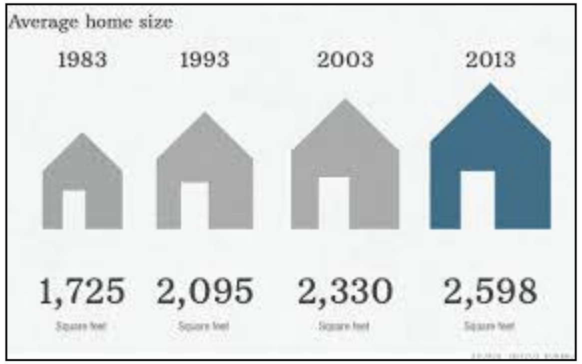

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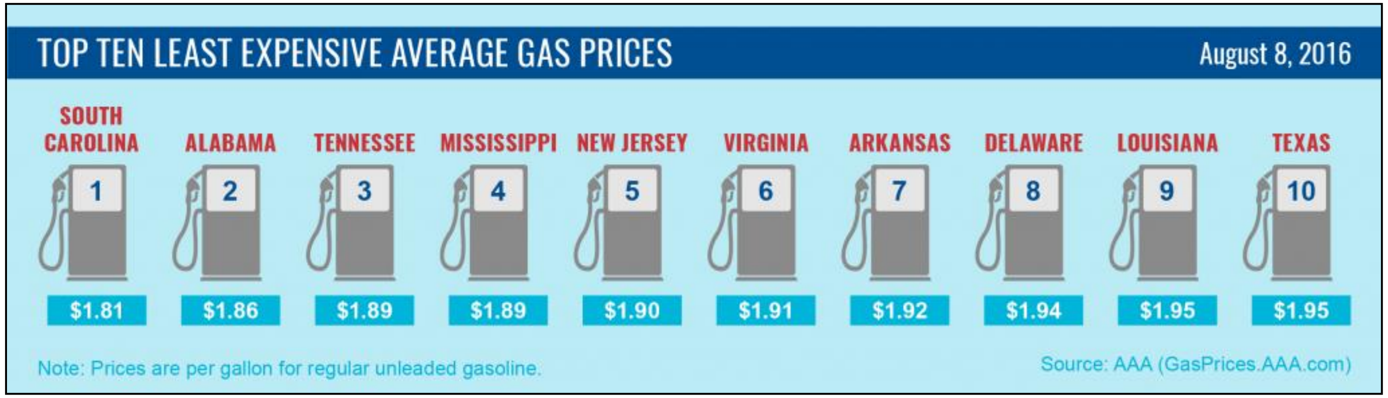

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One Other Scale is Often Used in Data Analysis

Absolute Scales

- ***Absolute*: all mathematical operations are meaningful**
  - Square root
  - Exponentiation
  - Etc.
- **In some definitions, only positive values are permitted**
  - (i.e., there is an absolute 0, marking the starting point of the scale)

Test Yourself – What Scale is Clothing Size?

https://rules.ssw.com.au/PublishingImages/size-stories-bad-example.jpg

Scales - Summary

Characteristics	Nominal	Ordinal	Interval	Ratio
Categorization, Classification	☑	☑	☑	☑
Mode (most frequent)	☑	☑	☑	☑
Order, Comparison, Sorting		☑	☑	☑
Median (middle)		☑	☑	☑
Fixed Distance, Add, Subtract			☑	☑
Ratio of differences			☑	☑
Multiply and Divide, Ratio				☑
True 0				☑
Mean				☑

Misuse of Scales

Example - Assigning a Scale to Test Failures

{**Blue**, **Green**, **Yellow**, **Red**}

Fewest ⟷ **Most**

This is an ordinal scale

- It provides a ranking but not ratios
- There is not a fixed difference between values
  - The difference between "red" and "yellow" is not comparable to the difference between "yellow" and "green"
- It makes no sense to add, subtract, multiply or divide the values.

But Suppose we Replace with a Numeric Scale

4 = Blue **3 = Green**

2 = Yellow **1 = Red**

We are tempted to make meaningless or misleading statements like these:

"The average test error improved from 2.2 to 3.1"

"The average test error improved by 47%"

Another Example – Customer Survey

The average response from our customers is "good" [on a scale of very poor, poor, good, very good]

- But the scale is not a ratio scale (or even an interval scale), so what does "average" mean?
- Does "half very good and half poor" mean "good"?

http://www.brecoflex.com/wp-content/uploads/2016/10/survey.jpg

UTD

Assigning Numbers Can be Misleading

The properties of the number system may not necessarily apply to the attribute being measured

Consider the attribute "temperature":

Scale	Yesterday	Today
Centigrade	0	18
Fahrenheit	32	64

Is it twice as hot today as it was yesterday?

Twice as …

"Twice as" is a meaningful concept for numbers

It is not necessarily a meaningful concept for temperature

- Because centigrade and Fahrenheit are interval scales, not ratio scales

The error we make is assuming that properties of the number system apply to the attribute being measured

Footnote: "twice as" is a meaningful concept for temperature measured on the Kelvin scale because the volume of a gas is proportional to its temperature on that scale.

A Very Common Example: Student Grade Point Average

Student 1

- B – 3 points
- B – 3 points
- B – 3 points
- B – 3 points
- B – 3 points

- Average: 15/5 = 3.0

Student 2

- A – 4 Points
- A – 4 Points
- B – 3 Points
- B – 3 Points
- D – 1 Point

- Average: 15/5 = 3.0

Grade Point Average is an example of a *Descriptive Statistic*.

Not technically accurate, but does give a useful indication.

Are These Students Really Equal?

Contents

What Do We Have, After Data Collection?

We have a collection of data values, known as a *dataset* or a *batch*.

These values are *measurements*

of

attributes

of

entities

Play golf dataset

Independent variables				Dep. var
OUTLOOK	TEMPERATURE	HUMIDITY	WINDY	PLAY
sunny	85	85	FALSE	Don't Play
sunny	80	90	TRUE	Don't Play
overcast	83	78	FALSE	Play
rain	70	96	FALSE	Play
rain	68	80	FALSE	Play
rain	65	70	TRUE	Don't Play
overcast	64	65	TRUE	Play
sunny	72	95	FALSE	Don't Play
sunny	69	70	FALSE	Play
rain	75	80	FALSE	Play
sunny	75	70	TRUE	Play
overcast	72	90	TRUE	Play
overcast	81	75	FALSE	Play
rain	71	80	TRUE	Don't Play

What Do We Want to Know?

Characteristics **of attributes / entities of the same type**

Relationships **between attributes / entities of the same or different types**

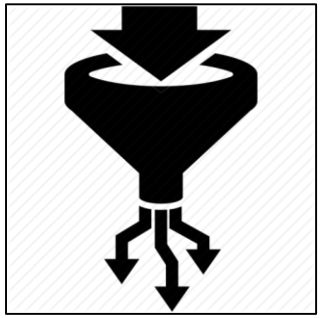
Examples:

– Which method is faster?
– How many defects?
– Do effective peer reviews reduce customer complaints?

The First Step is to Organize the Data for Analysis

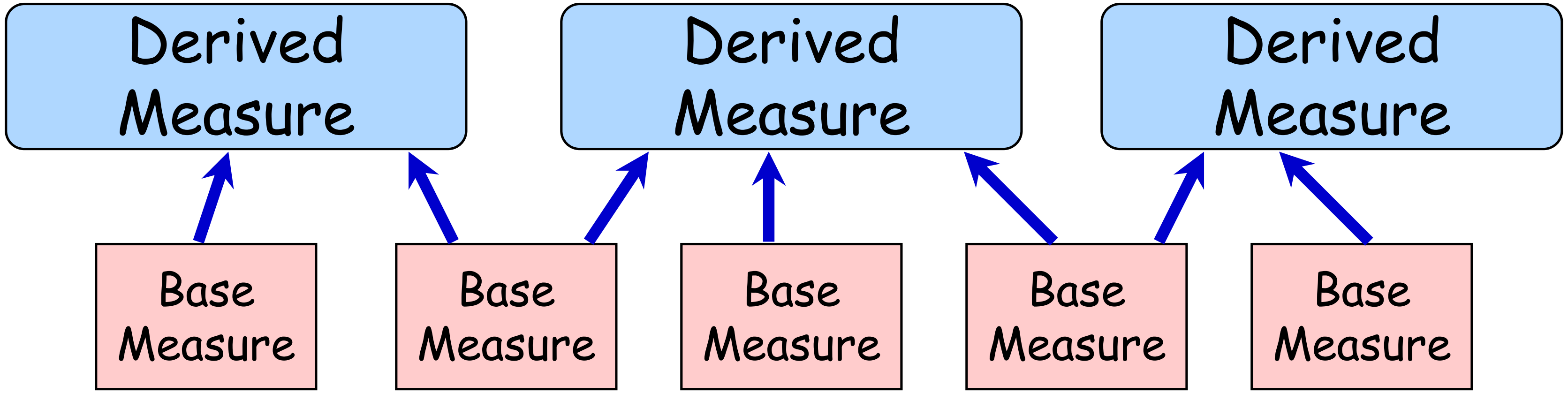
First, we <u>refine</u> & <u>compress</u> the data

- Eliminate duplicates, errors, etc.
- Compute totals, sort data, etc.
- Perform appropriate data compression

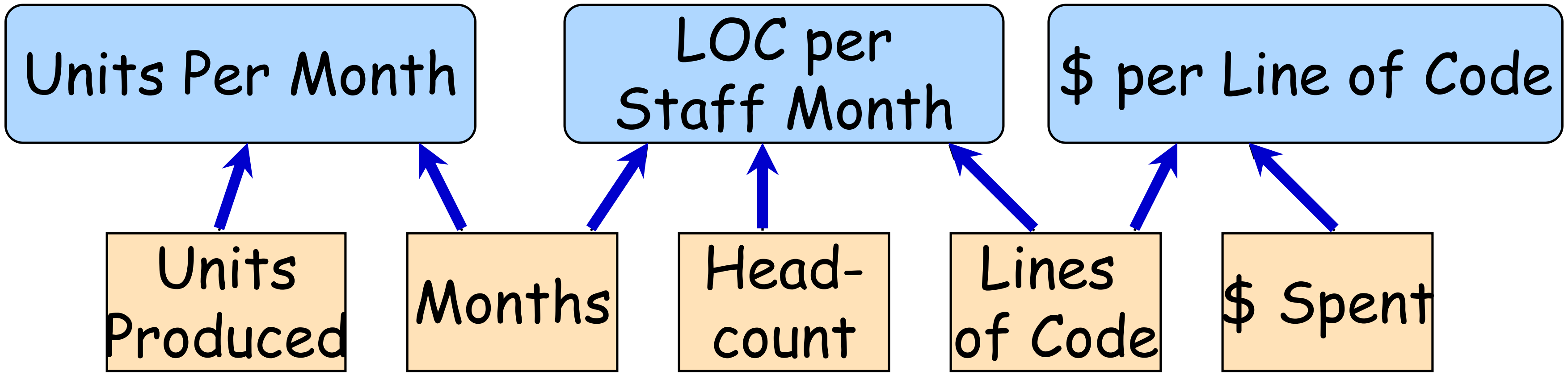


Thenounproject.com

Then, we compute various <u>derived measures</u> (computations)



Examples of Derived Measures



Exactly what derived measures we want is determined by our metric selection process (based on our information needs).

The Next Step is Analysis

One option is to simply look at the data:

We can look at the data as we have it

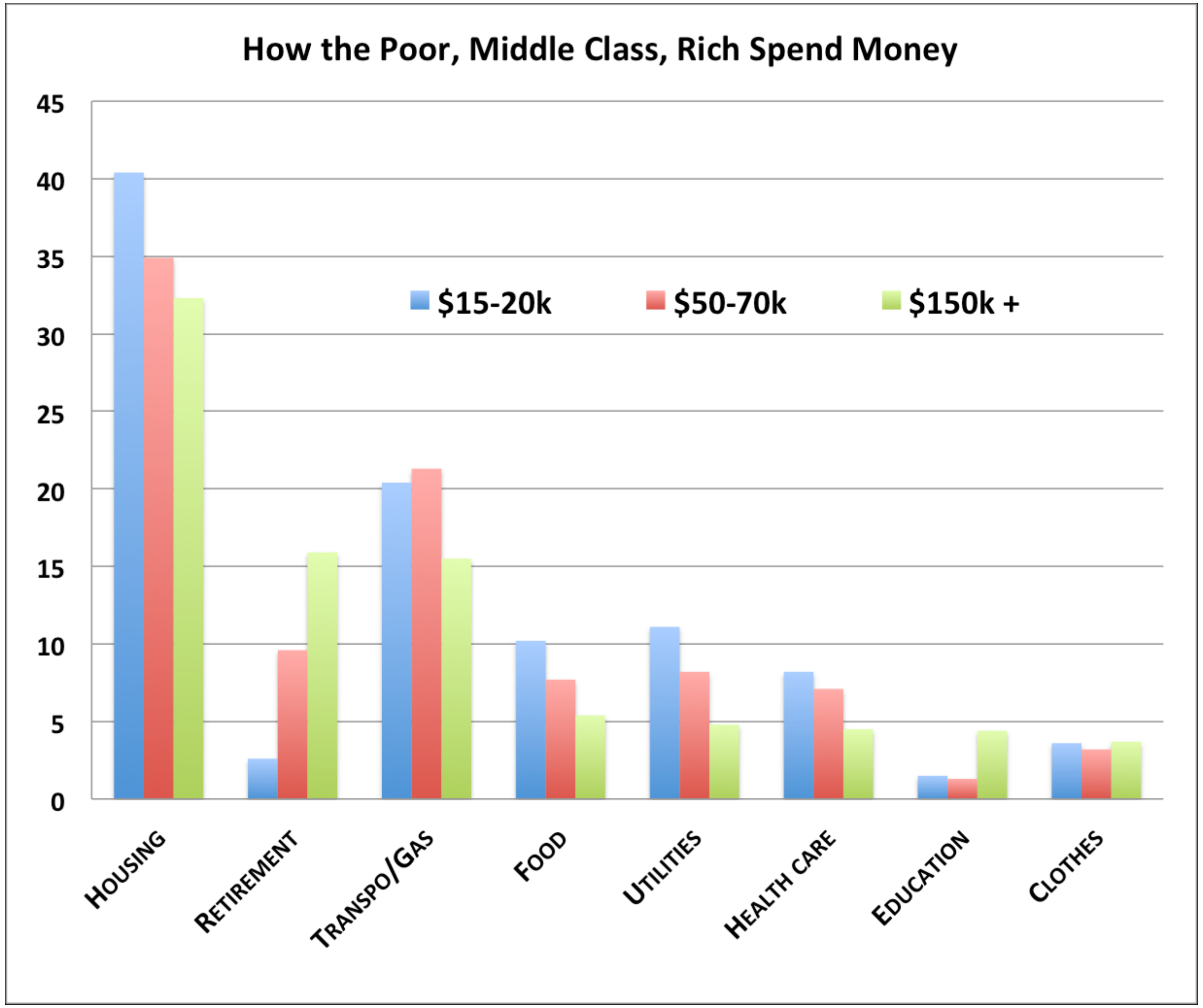
Microsoft Excel - sorttest.xls

	A	B	C
1	Division	Employees	Sales Total
2	Africa	712	33,000,000
3	Asia	381	48,000,000
4	Europe	453	51,000,000
5	North America	698	44,000,000
6	South America	833	72,000,000
7			

We may want to sort the data or do other simple processing to make sense of it

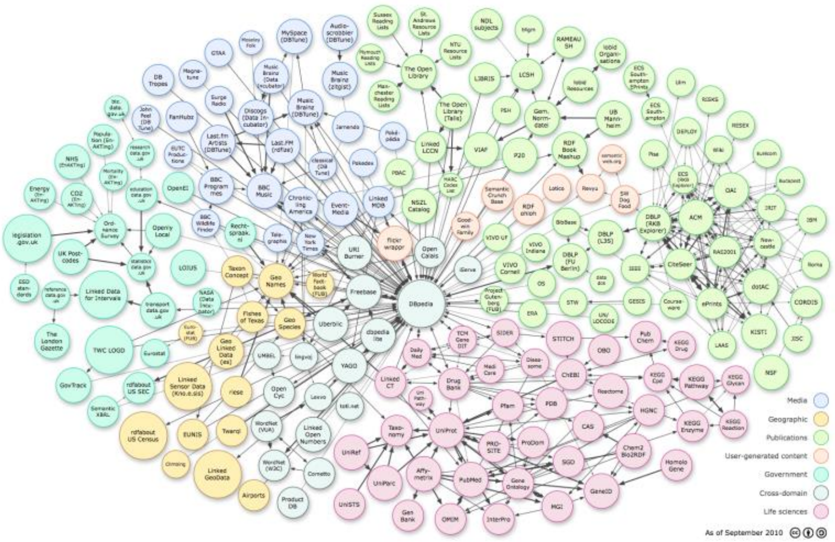
3	BBC 1	
4		Millions
5	Casualty	8.82
6	Antiques Roadshow	7.68
7	Changing Rooms	7.38
8	DIY SOS	7.20
9	Lenny Henry	7.03
10	Holby City	6.99
11	Ground Force	6.71
12	Vicar of Dibley	6.34
13	Ready Steady Cook	6.33
14	National Lottery	6.27
15		

Another Option is to Graph the Data



Often, a graph or chart makes it very easy to see what the data are telling us.

But Often There is Too Much Data or the Relationships Are Not So Easy to See



Statistical methods are often helpful in situations like these

Contents

- Definitions
- Scales
- Basic Analysis Approaches
- Statistical Distributions
- Other Statistical Concepts

Statistical Techniques Help Us Deal with Many Situations

Statistical techniques **can often be used to describe the attributes and relationships**

Examples:

- High and Low values (and how often each occurs)
- Average (Mean), Median and Mode
- Mathematical relationships between values
  - For example, if you double the number of programmers, how much do you reduce the schedule?
- Patterns of values
  - For example, do most defects result from errors in requirements, errors in design, or errors in coding?
- Overall distribution of values

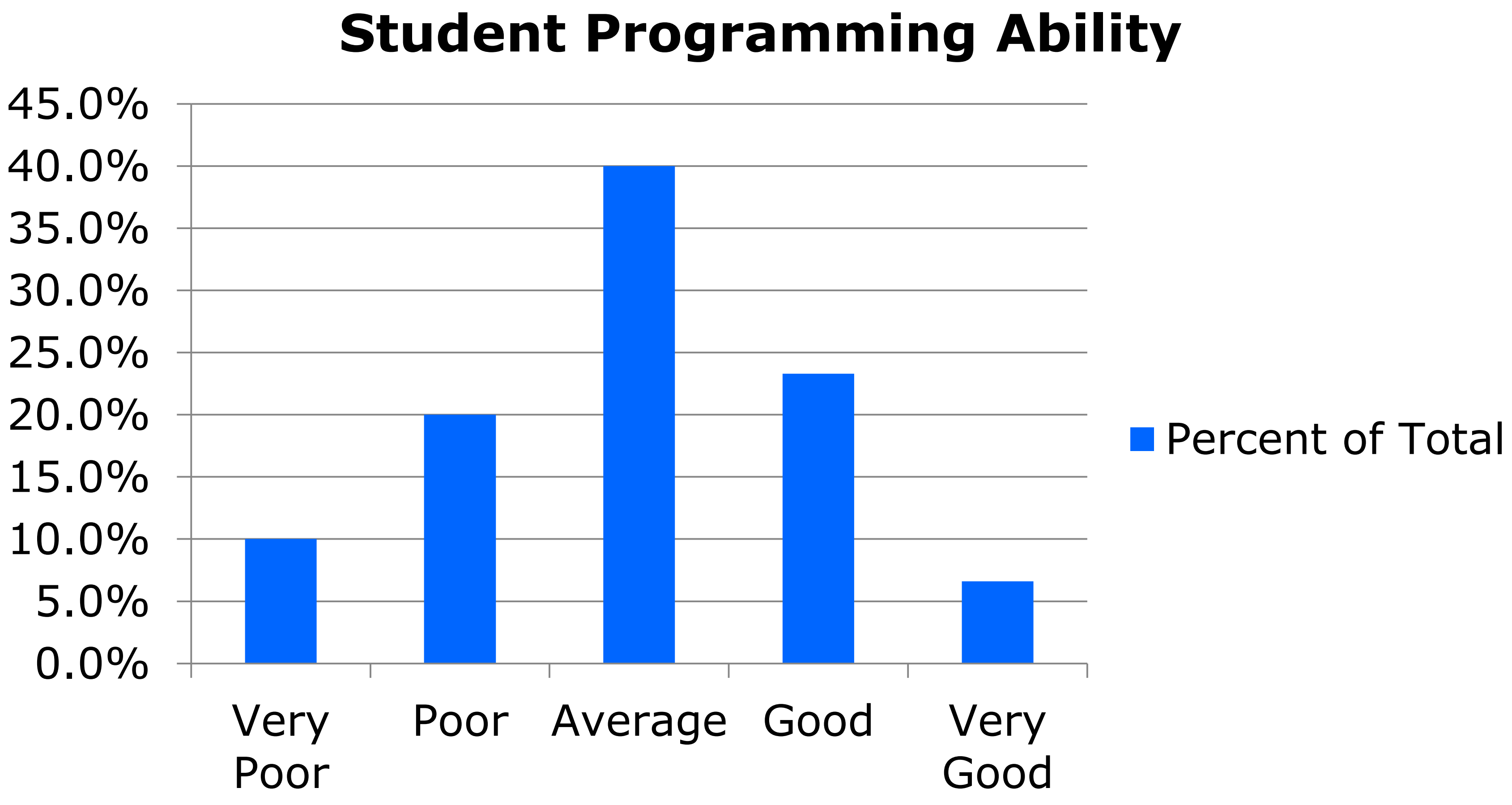
Distributions

Suppose you evaluate 30 students on their programming ability and come up with the following values:

Overall Ability	Total	Percent
Very Poor	3	10.0%
Poor	6	20.0%
Average	12	40.0%
Good	7	23.3%
Very Good	2	6.6%
Total	30	100%

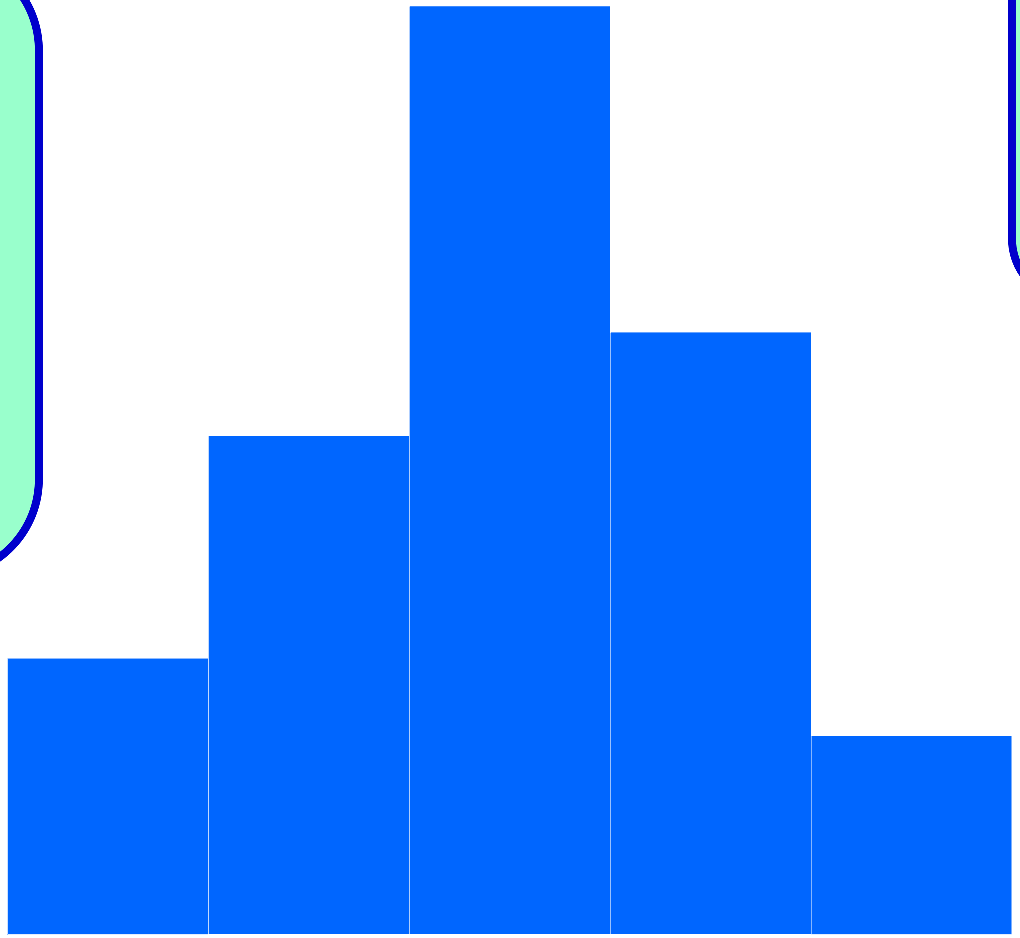
Since all students are accounted for, this is called a *distribution* of student ability values.

The Distribution as a Bar Graph



The Distribution as a Histogram

A histogram looks like a bar chart with no spaces between the bars.



Histograms are often used to show distributions.

Probability Distributions

Suppose one student is selected at random from the group of 30.

What is the probability that the student will have any given ability level?

Overall Ability	Total	Probability
Very Poor	3	.1
Poor	6	.2
Average	12	.4
Good	7	.233
Very Good	2	.066
Total	30	1.0

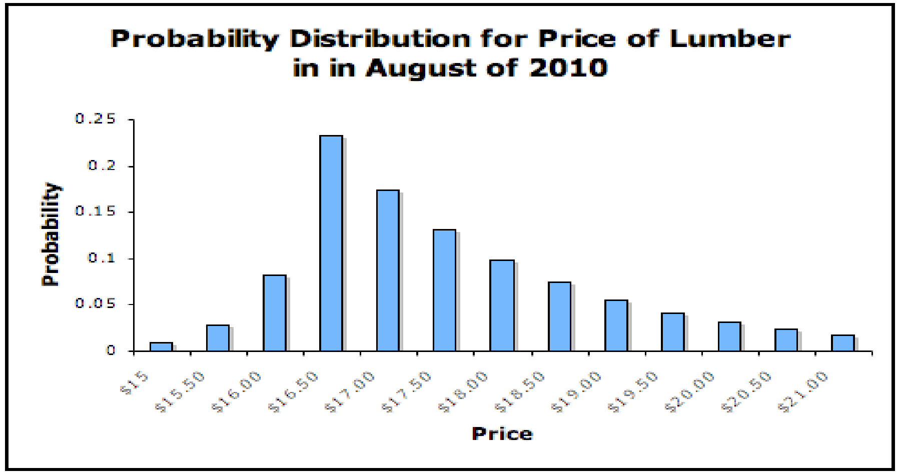
If you represent the percentage as a probability, this is called a *probability distribution* of student ability values.

Requirements for a Valid Probability Distribution

A probability distribution is the *assignment of probability values* to *each of the possible outcomes*

- **The probabilities must be numbers between 0 and 1**
- **The probabilities must add up to 1**

The probability distribution represents the *likelihood that a randomly chosen value will have a given outcome*.



Calculating Probability of Other Situations

Suppose you want to know if a randomly chosen student will be *at least average* in ability.

Overall Ability	Total	Probability
Very Poor	3	.10
Poor	6	.20
Average	12	.40
Good	7	.233
Very Good	2	.066
Total	30	1.0

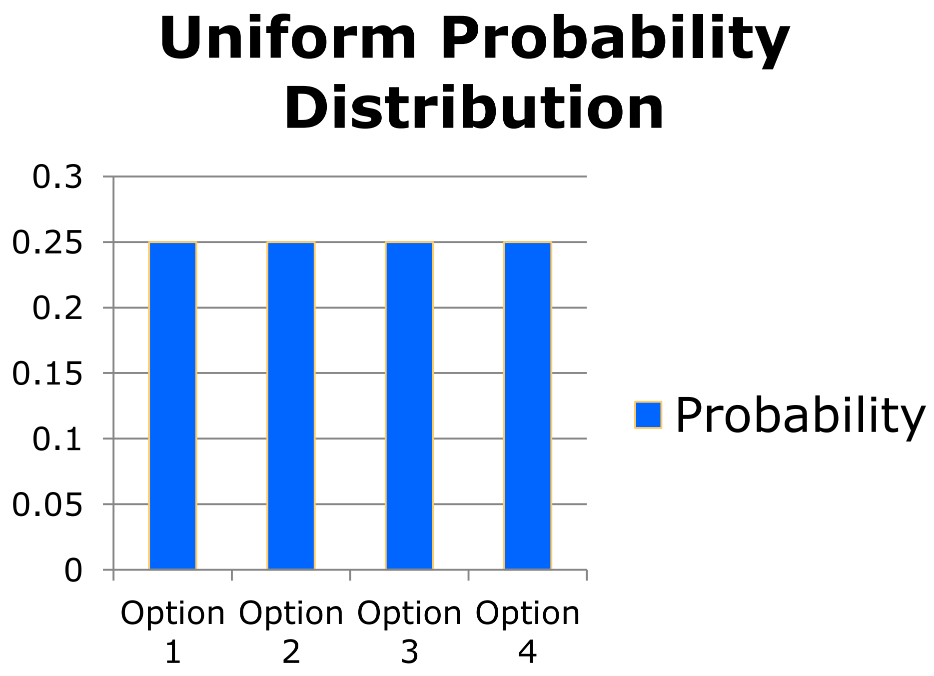
These are average or higher

Add up the probabilities:
.40 + .233 + .066 = .70

The probability of a student being at least average is .70

Uniform Distribution

- **If all events are equally likely, this is called a *uniform distribution*.**
- **If there are N options, the probability of any of them is 1/N**



Suppose There are Many Possible Values

Example: The ages of the software development staff

- Total staff size is 200 people
- Range of ages is from 18 to 72

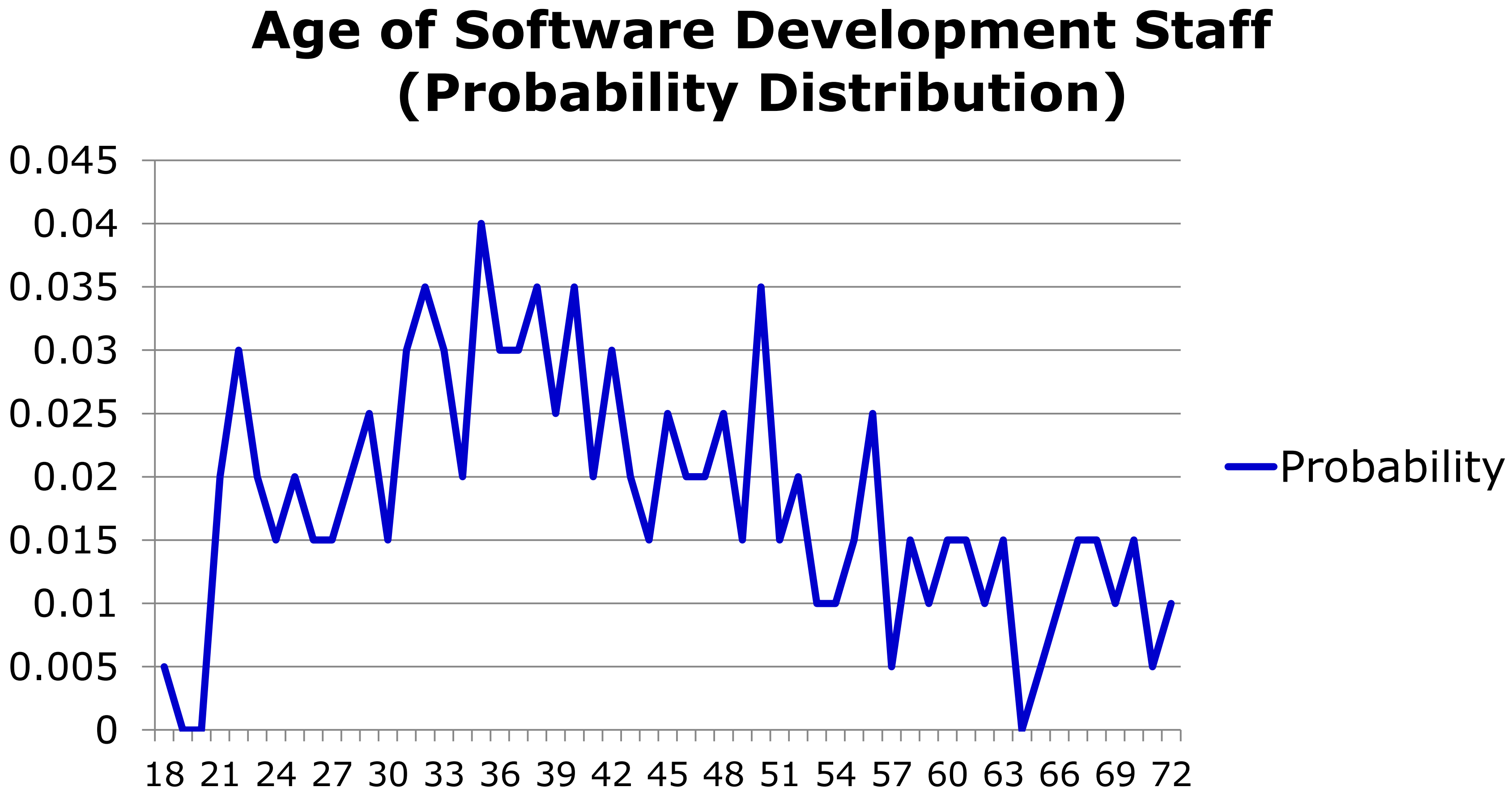
There are 55 possible values

Age	Total	Probability
18	1	.005
19	0	.000
20	2	.010
21	0	.000
22	1	.005
…	…	…
Total	200	1.0

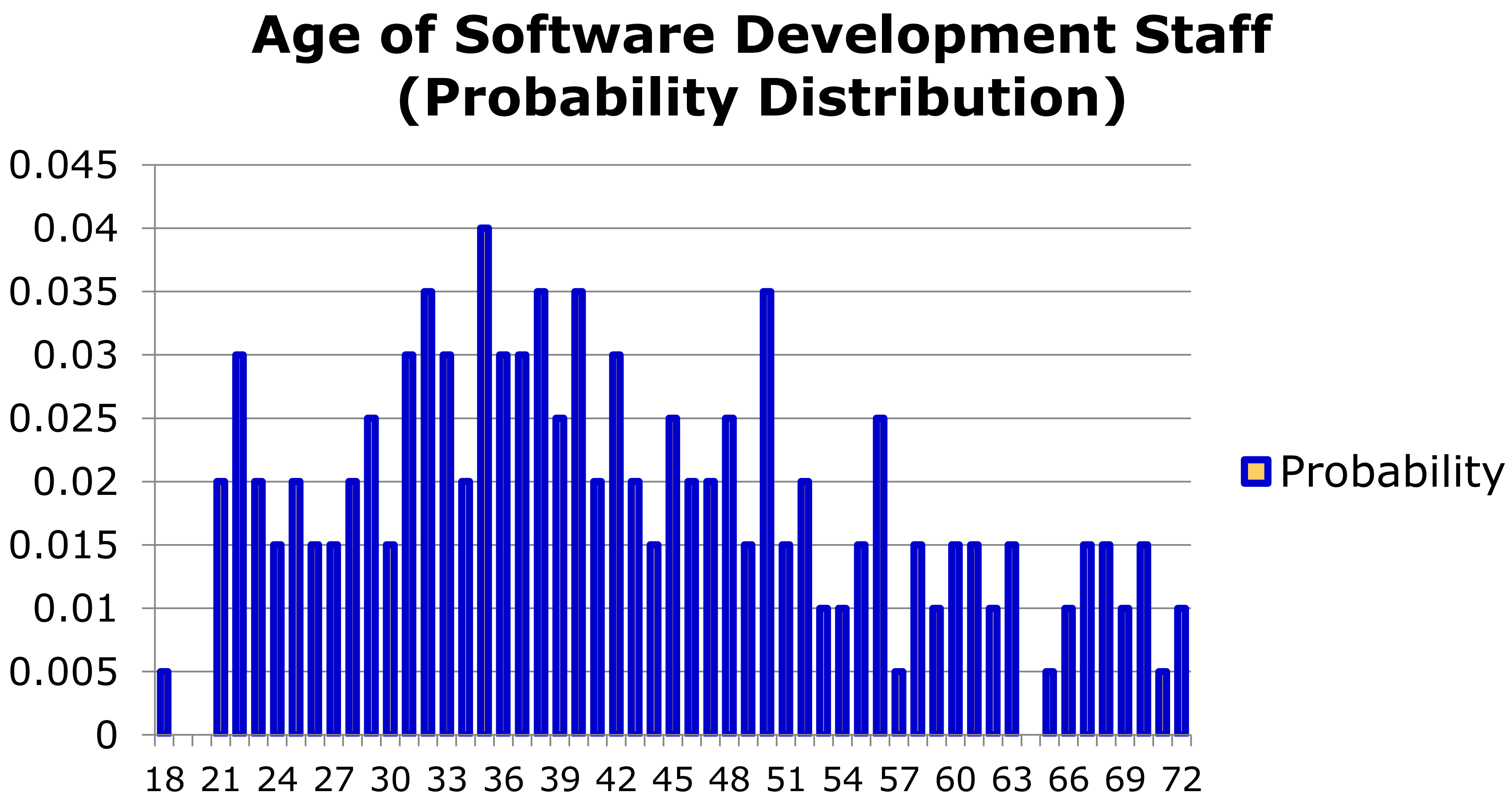
There would be a *very long list* and *very small probabilities* for most cases.

This might be very cumbersome to analyze.

Graph of Probability Distribution – <u>Line Chart</u>



Graph of Probability Distribution – <u>Bar Chart</u>



What If There Are an Infinite Number (or a very large number) of Possible Values

Examples:

- Number of lines of code in each software product.
  - Range: 11 to 128,045

In this case, it could be *meaningless to have a separate probability* for each possible value.

Solution 1: *Discretization*

- Divide the possibilities into discrete ranges that make sense for your purposes

Solution 2: *Continuous Function*

- Represent the values with a continuous function

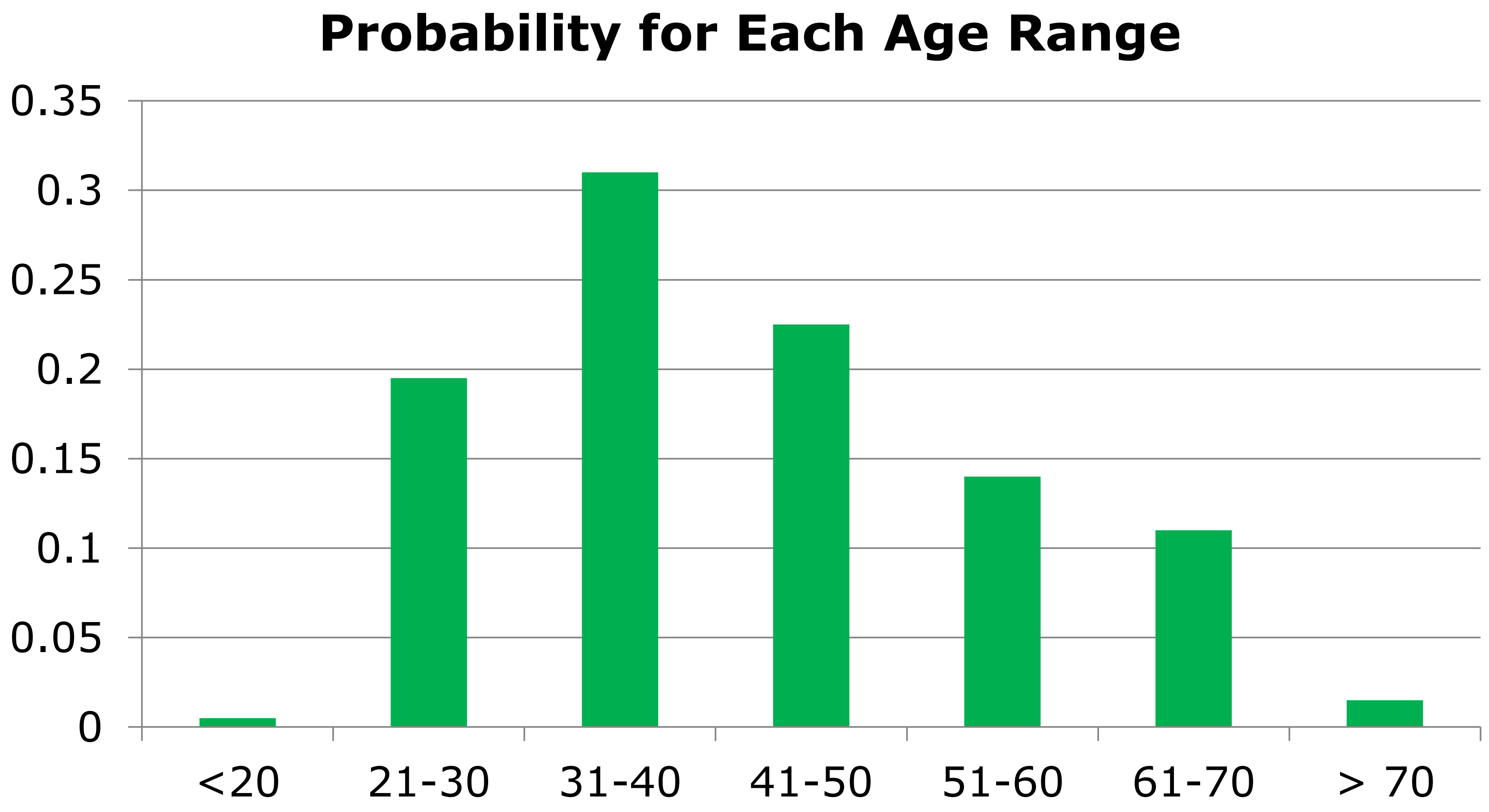
Discretization

Divide the list into discrete intervals

Age Range	Total	Probability
20 or less	1	.005
21-30	39	.195
31-40	62	.310
41-50	45	.225
51-60	28	.140
61-70	22	.110
Over 70	3	.015
Total	200	1.0

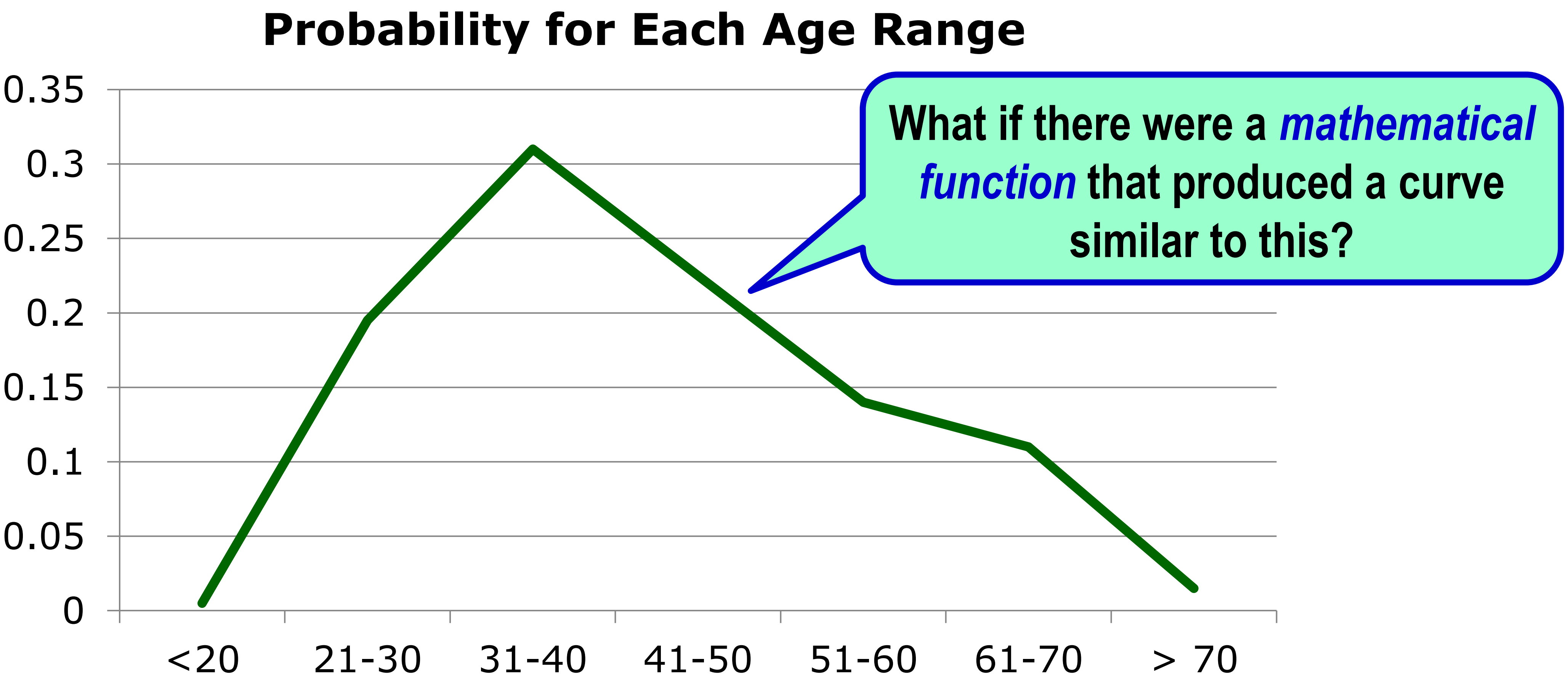
For many purposes, these categories would be useful and having only 7 categories makes analysis a lot easier than having 55 of them.

Bar Graph of Discretized Distribution Data for Programmer Age



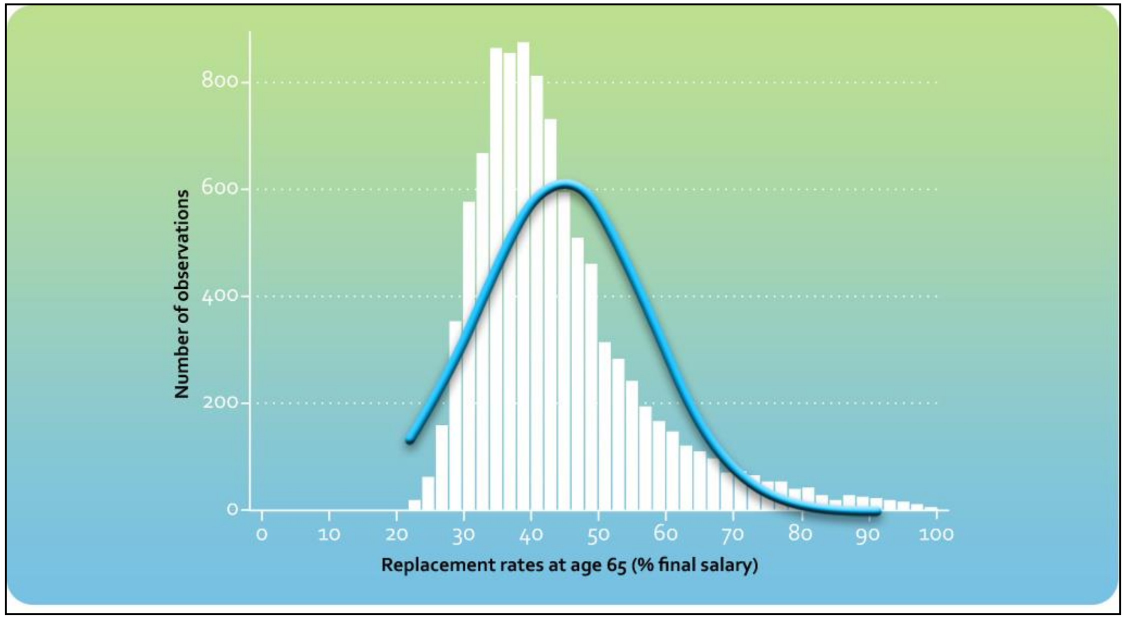
Note that this discretized graph tells us things that were harder to see before.

Same Graph Using a Line Chart



Use of a Continuous Distribution Function

For very large numbers (or infinite possibilities), it is sometimes convenient to use a *continuous function* whose shape approximates the distribution of the data.



The area under the curve is equal to 1.0 – the sum of all probabilities.

Advantages and Drawbacks of Continuous Functions

Advantages

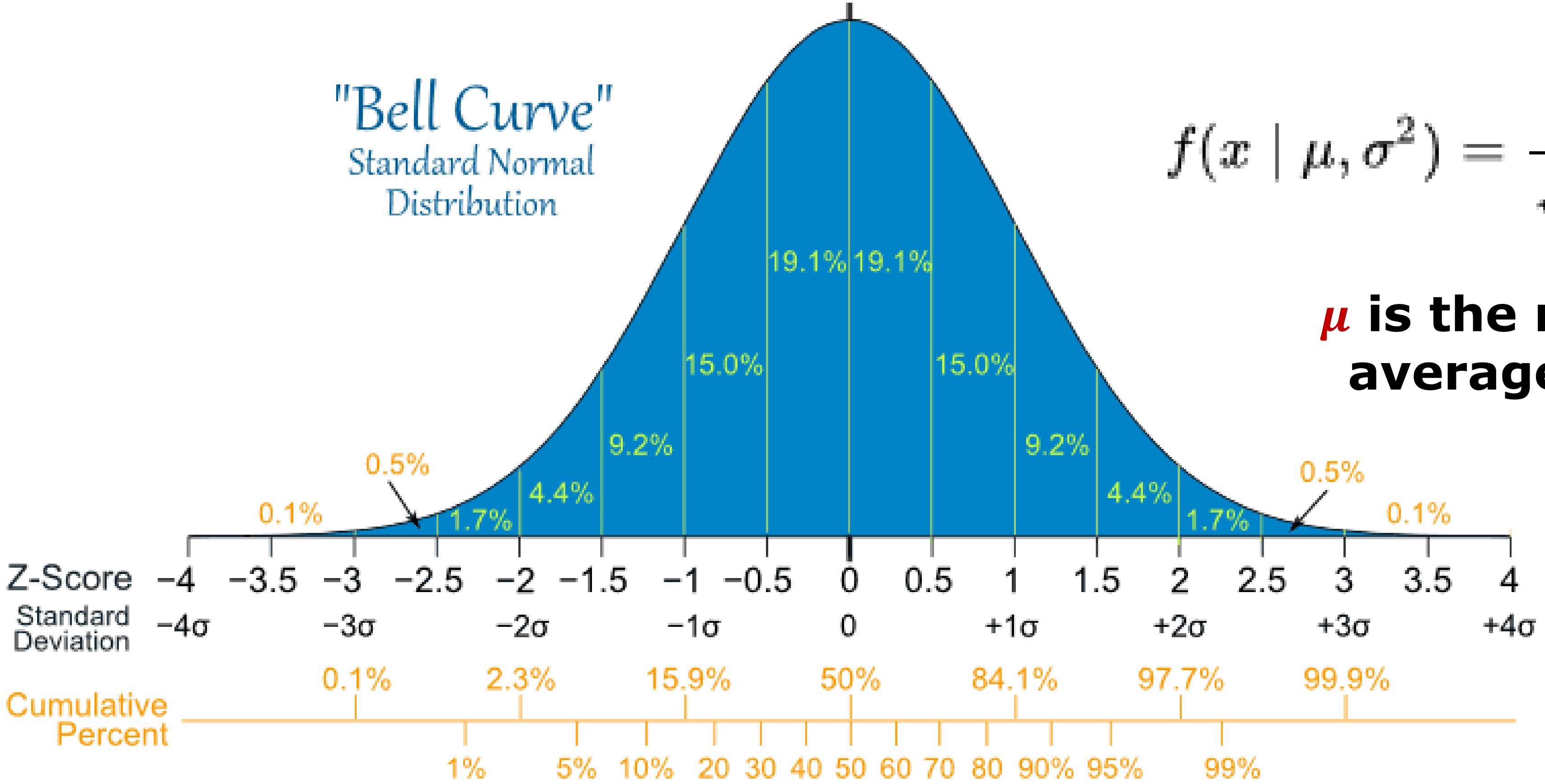
- ***Many mathematical and statistical analyses*** can be performed on the functions
  - You can often make good predictions
  - Or combine multiple independent variables and determine their relationships

Drawbacks

- You ***cannot draw conclusions*** about ***individual values***
  - For example: what is the probability that a programmer is exactly 34.25 years old?
- You must draw conclusions about ***ranges of values***
  - For example: what is the probability that a programmer is between 34 and 35 years old?

Normal Distribution (Bell Curve)

This is a widely used continuous distribution because many phenomena fit this shape.

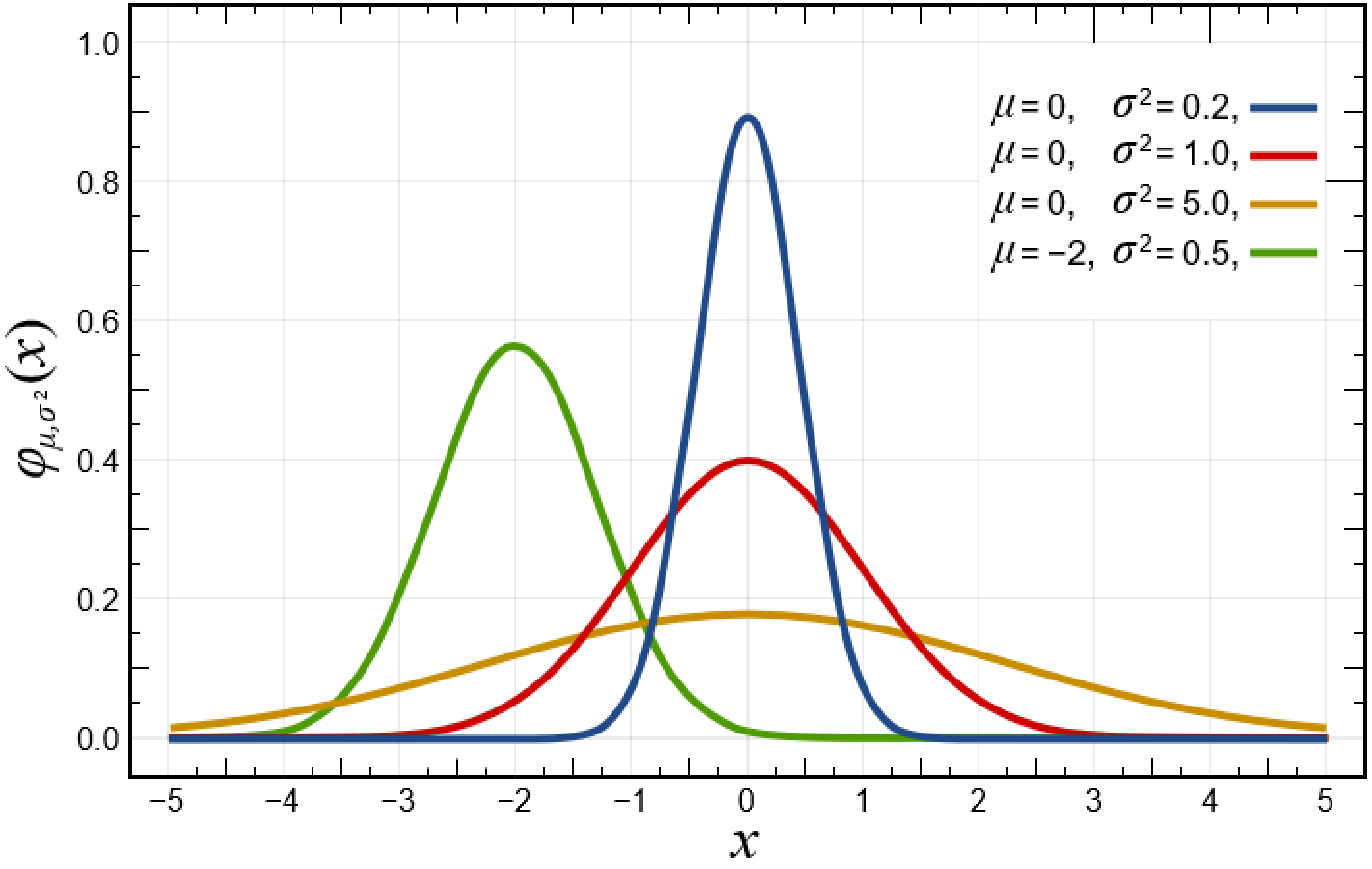


$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ is the mean or average value

Normal Distribution Variations

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



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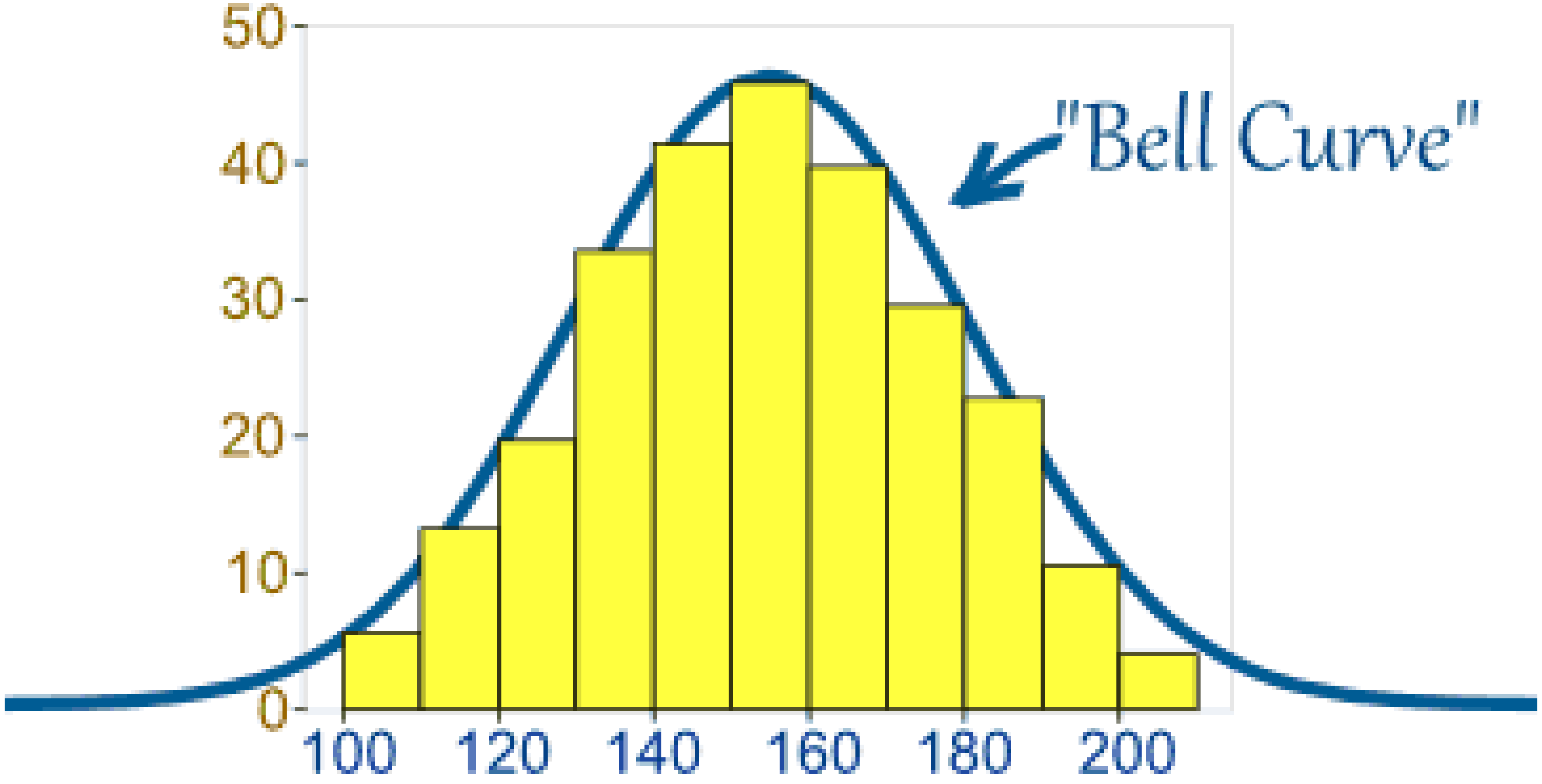
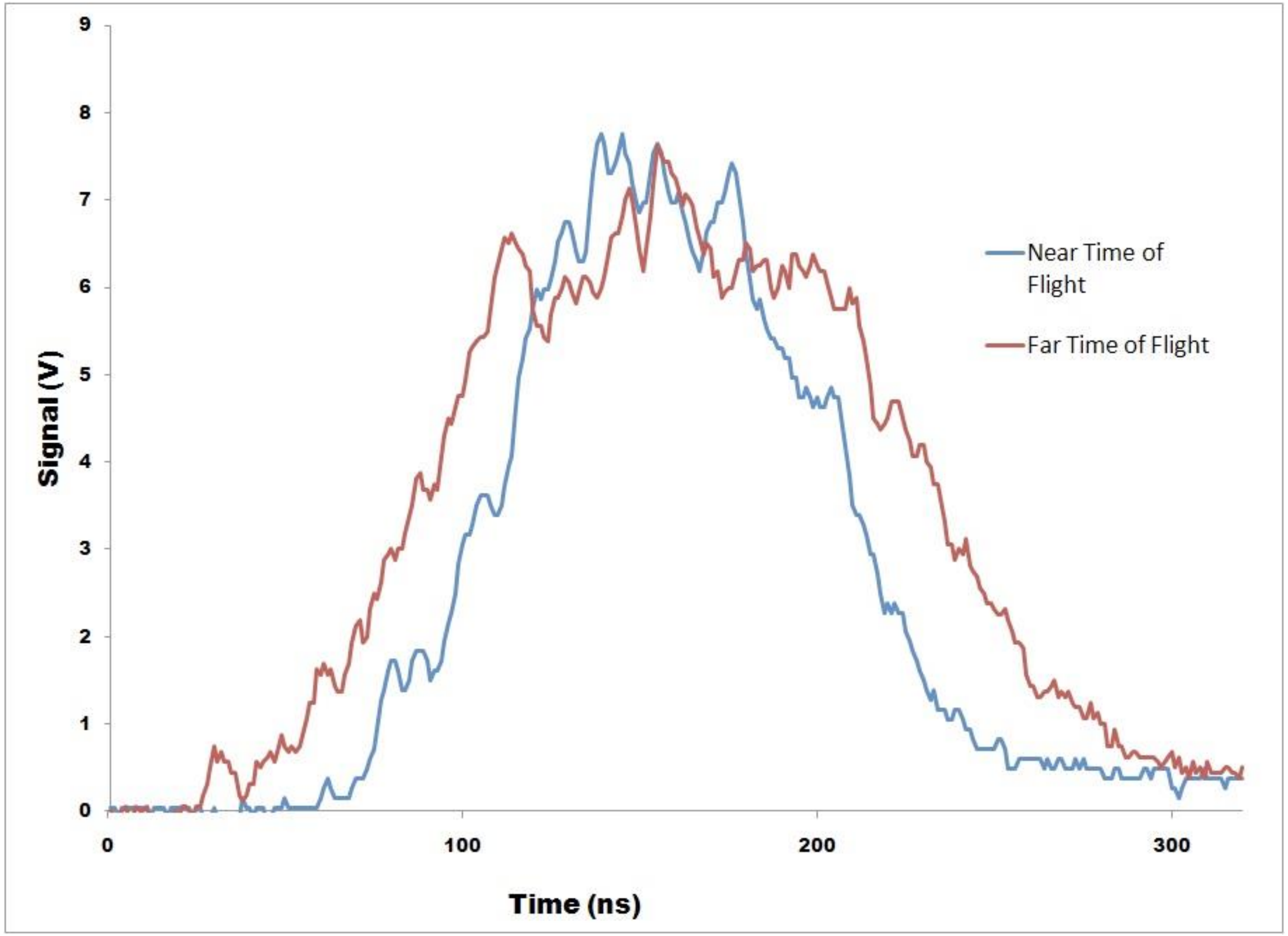
Example
Bell Curve Approximates Data



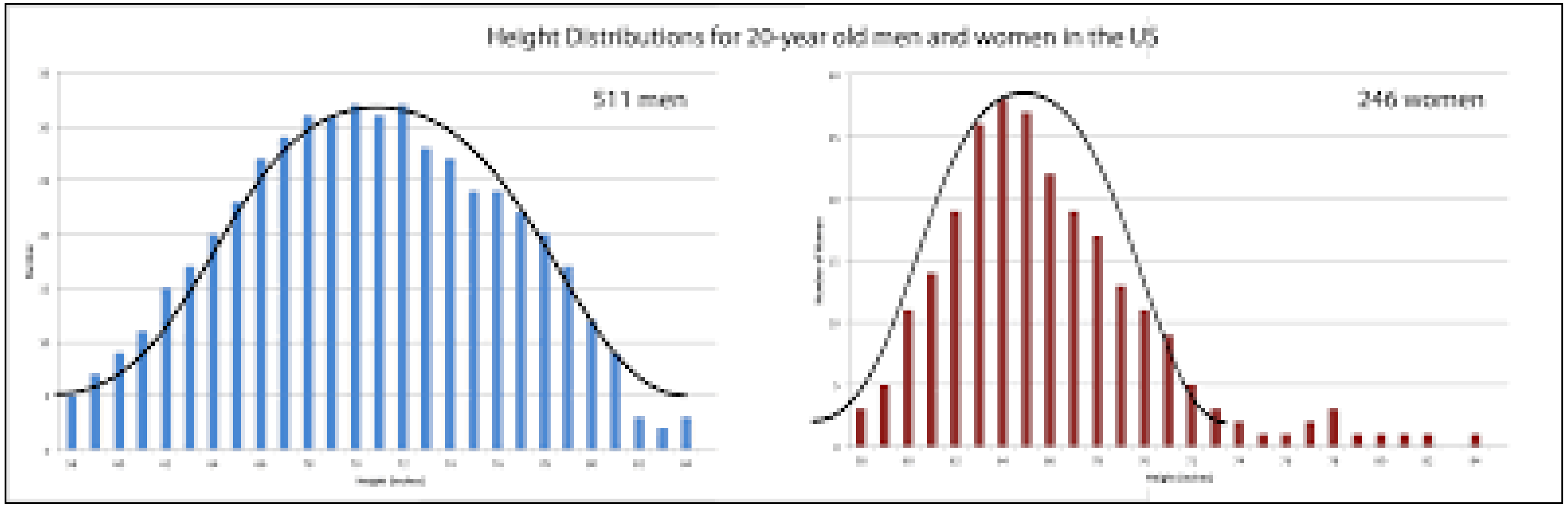
Real Data With Approximately Normal Distribution



Uses of Normal Distribution (1 of 2)

If the data are distributed in a pattern that is similar to normal distribution, *we can often reach conclusions about the data* from known facts about the normal distribution

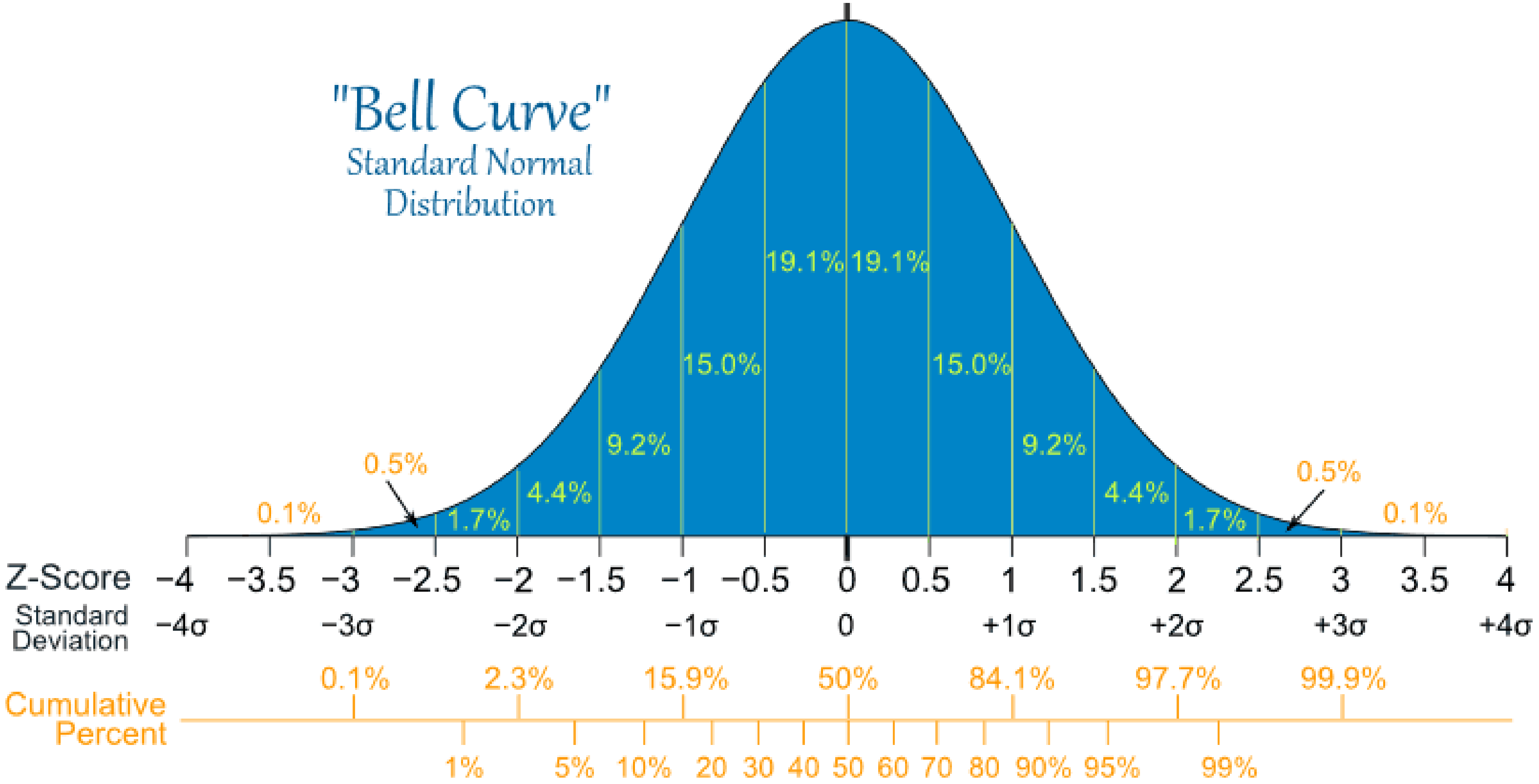
- This happens often with many natural phenomena
- For example: height



Uses of Normal Distribution (2 of 2)

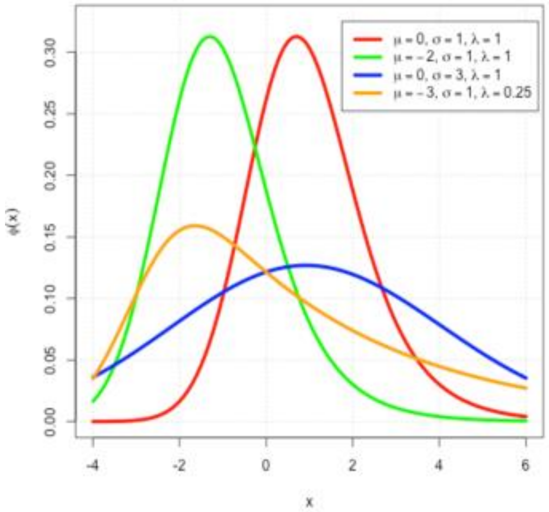
Many statistical concepts are easy to explain using the normal distribution

– Because it has many very convenient statistical and mathematical properties



Normal Distribution Variants

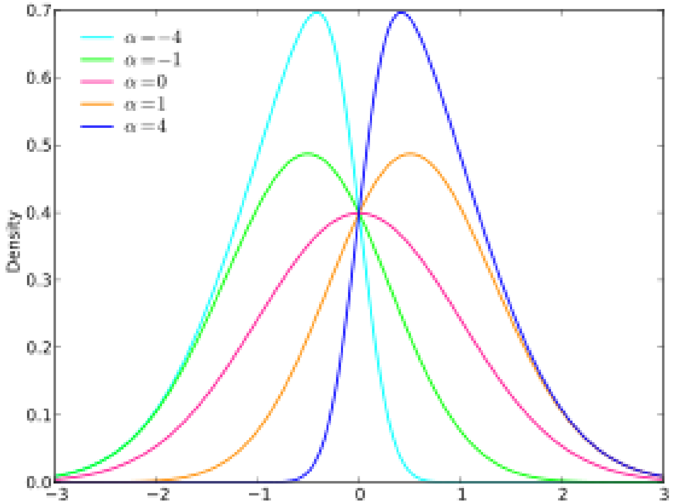
The Normal Distribution is a type of Gaussian Distribution



Various Gaussian Distributions

There are many ways to modify a Gaussian distribution to fit specific datasets

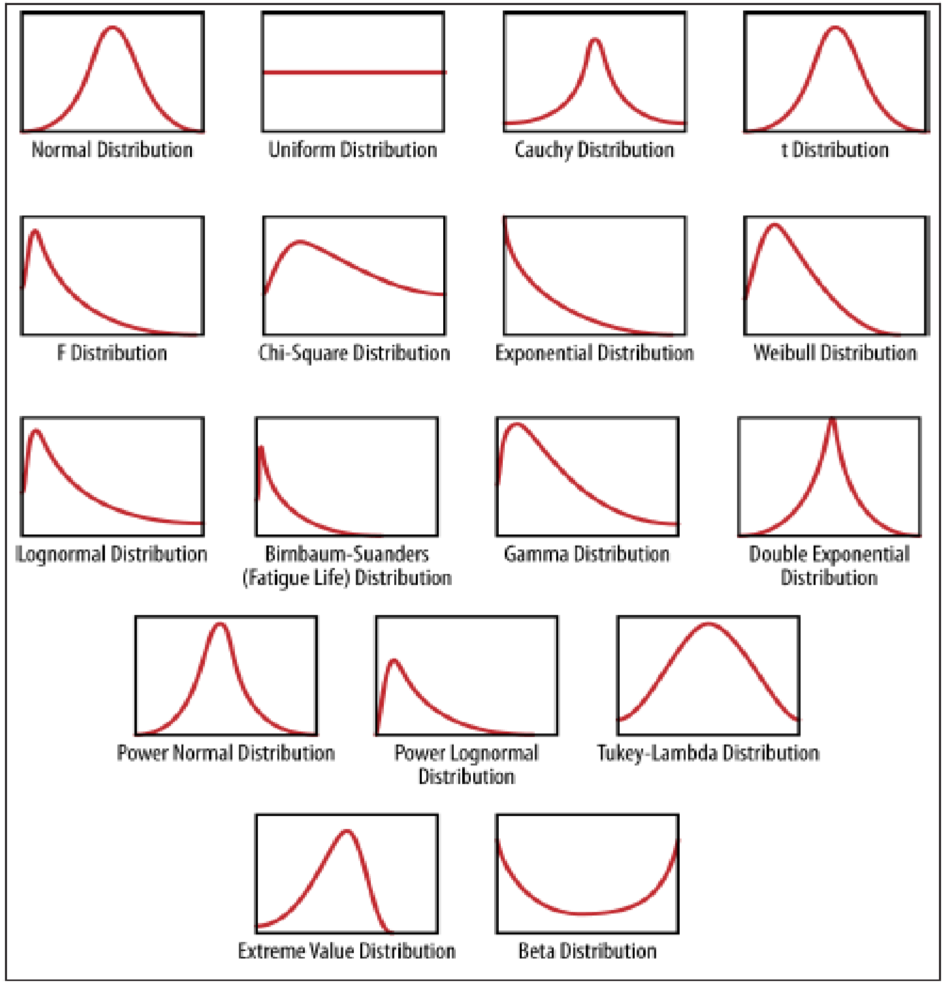
The Gaussian Distribution is a type of Elliptical Distribution



Various Elliptical Distributions

For some situations, an Elliptical distribution will fit the data

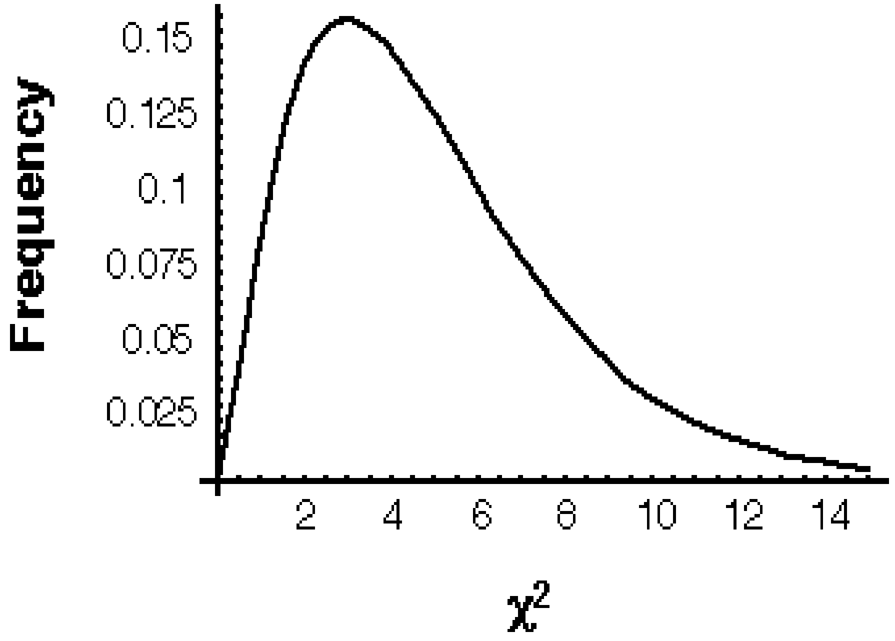
Other Examples of Continuous Distributions



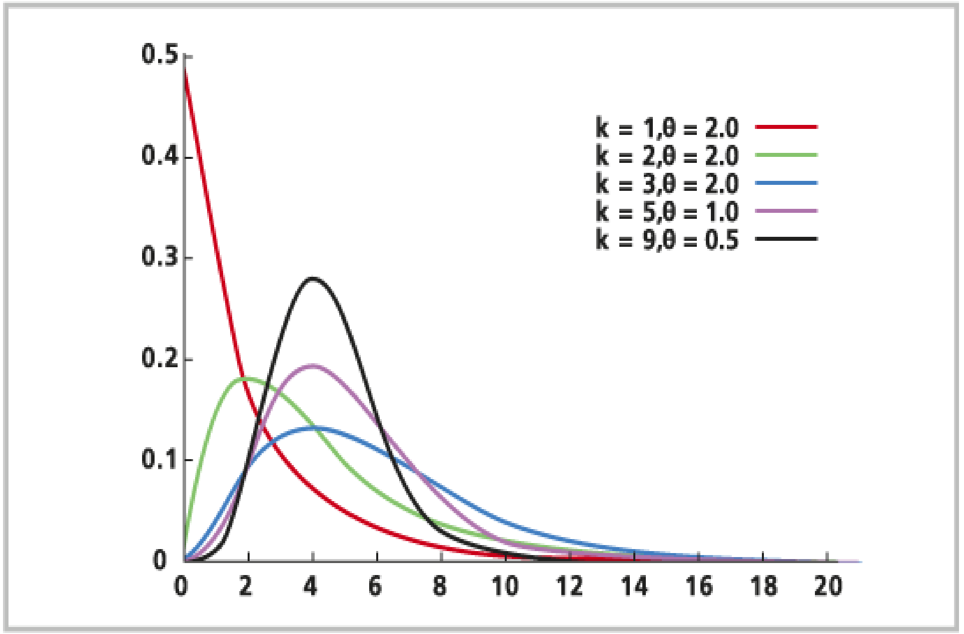
For many phenomena there are continuous distributions that approximate the actual distributions.

You find out using curve fitting techniques

Many Natural Phenomena Fit the Chi Square Distribution (C2 or χ^2)



C2 Distribution



Variations on C2

Unlike the normal distribution, C2 is constrained to have no negative values.

Contents

- Definitions
- Scales
- Basic Analysis Approaches
- Statistical Distributions
- **Other Statistical Concepts**

Central Tendency Measures

A ***Measure of Central Tendency***

– is a

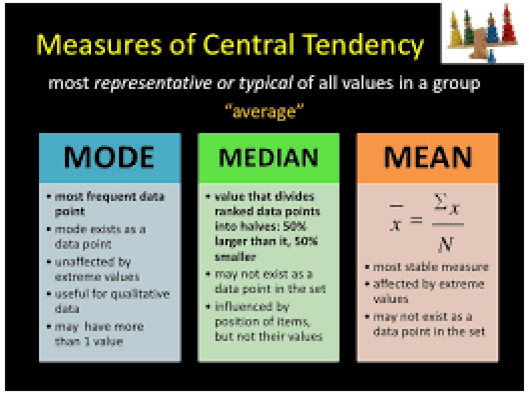
single value

– that

attempts to ***describe*** a set of data

– by

identifying the ***central position*** within that set of data.



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These are also sometimes called
summary statistics
or
measures of central location

Mode, Median and Mean

(Central Tendency Measures)

- ***Mode***: **The value that appears <u>most often</u>**
- ***Median***: **The value of the <u>middle</u> item**
- ***Mean (or   )***: **The <u>average</u> of all values**

Given these values: 1,2,2,3,4,7,9			
Term	**Description**	**Example**	**Result**
Mean	Sum / Total Number of Values	(1+2+2+3+4+7+9)/7	**4**
Median	Middle value	1,2,2,**3**,4,7,9	**3**
Mode	Most Frequent Value	1,**2**,**2**,3,4,7,9	**2**

Source: Wikipedia

Mode, Median and Mean for Various Distributions

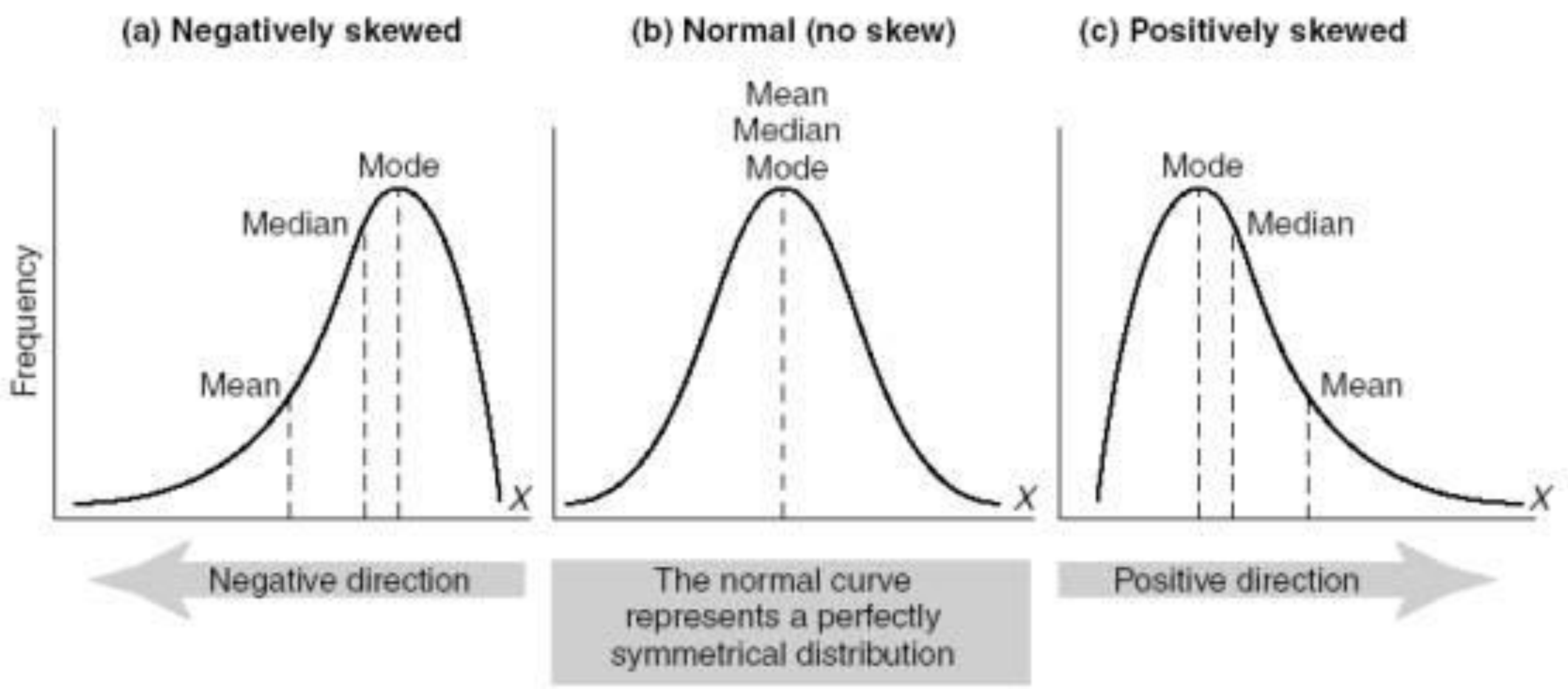
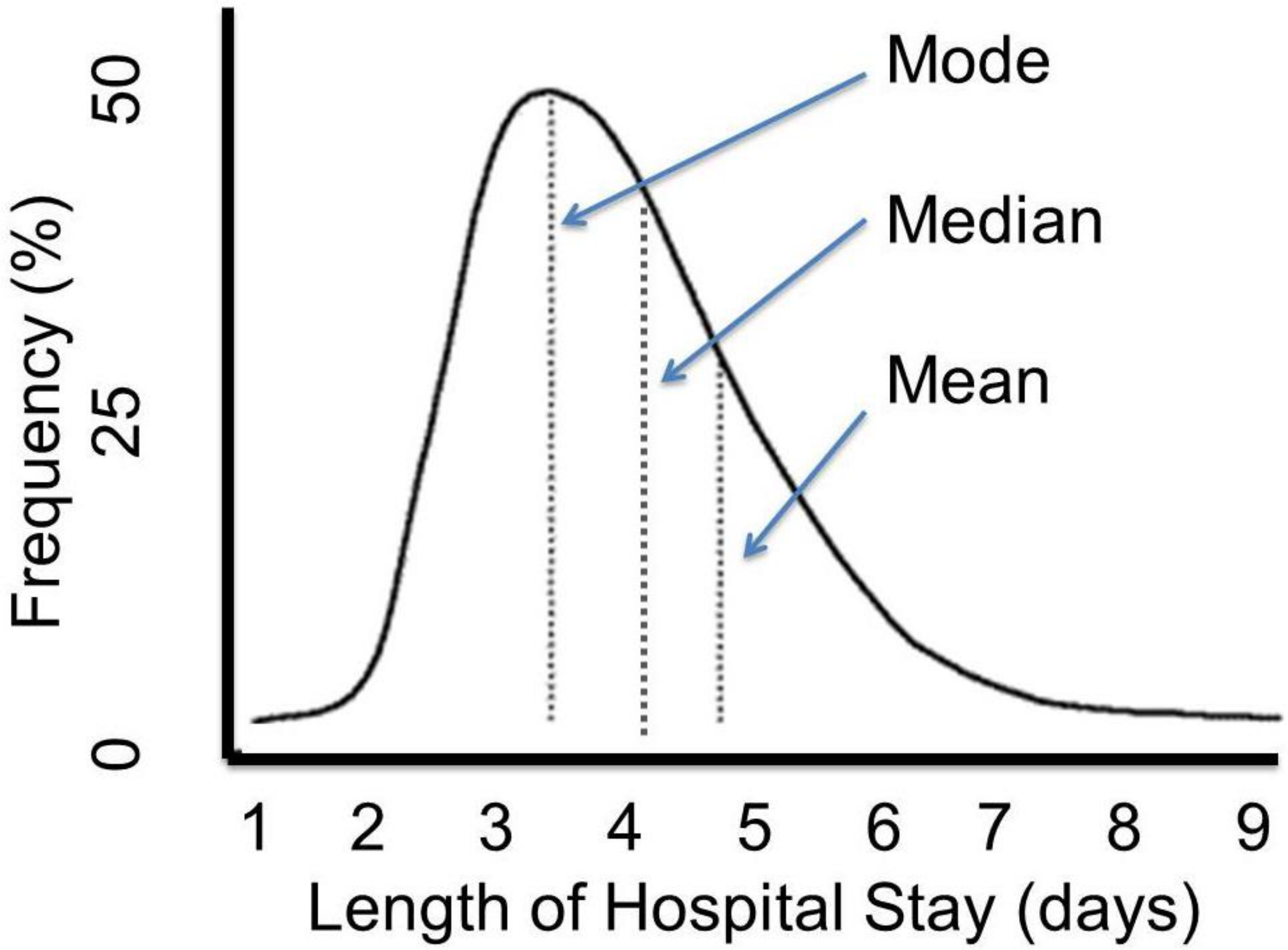


FIGURE 15.6 Examples of normal and skewed distributions

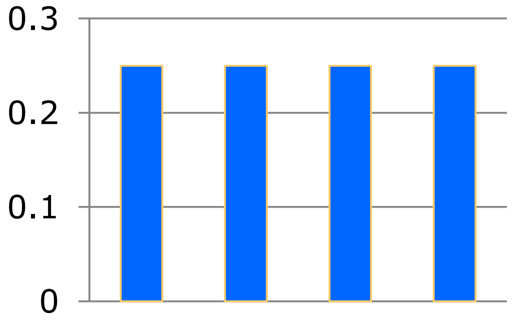
Example of Skewed Distribution

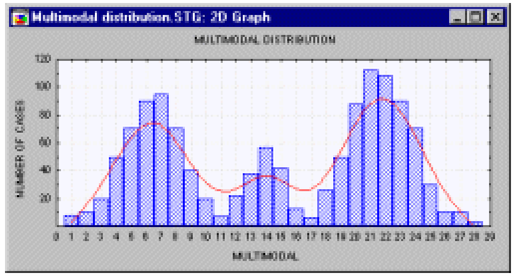


Notes about Mode (1 of 2)

1. **There can be no mode or more than one mode**

– no mode - Bimodal - Multimodal



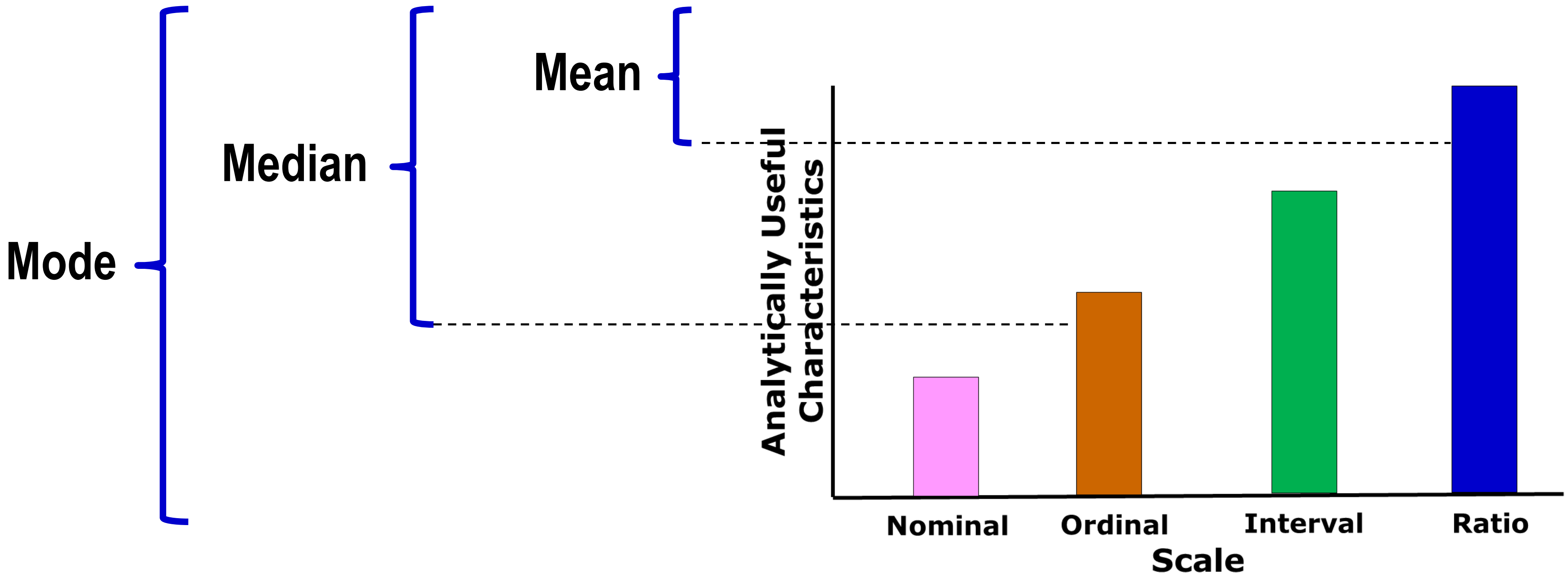


- **When there is a lot of data and there are several local maximum points, each is considered a mode**
  - i.e., *any relative high point* or peak in the data may be considered a mode.

Notes about Mode (2 of 2)

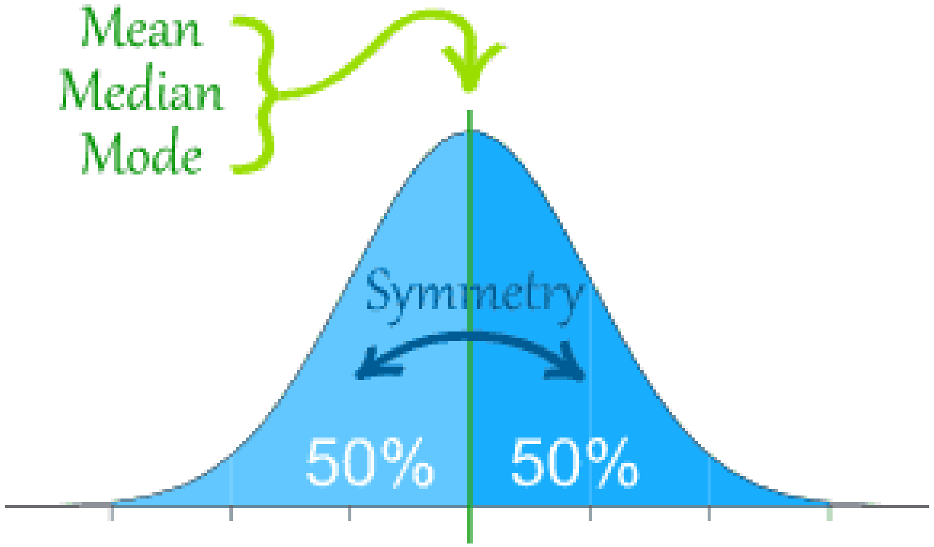
2. **The mode can be determined for data using nominal, ordinal, interval, ratio and absolute scales**

   - ***Mean & median require higher scales***



Notes about Mode, Median and Mean

For normally distributed data, these three are equal

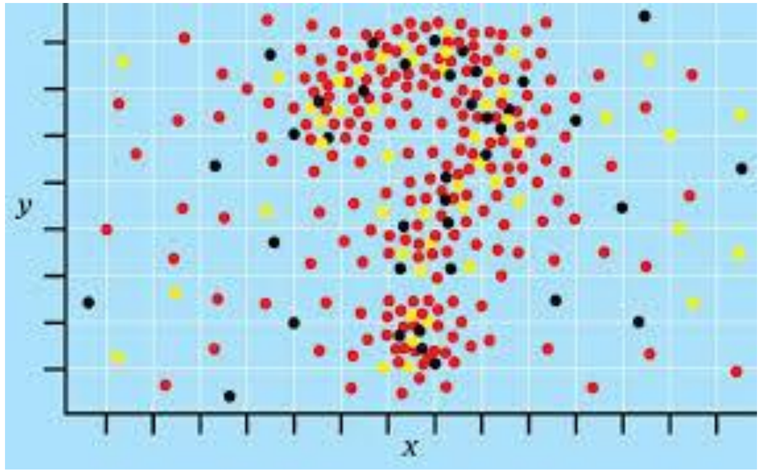


For other distributions they may not be.

- So one way to tell if the normal distribution is a possible fit to your data is to compute the mode, median and mean and see if they are the same.

Measures of Dispersion

- A ***Measure of Dispersion***
  - is a
- ***single value***
  - that
- attempts to ***describe*** a set of data
  - by
- identifying the ***spread of values*** within that set of data.



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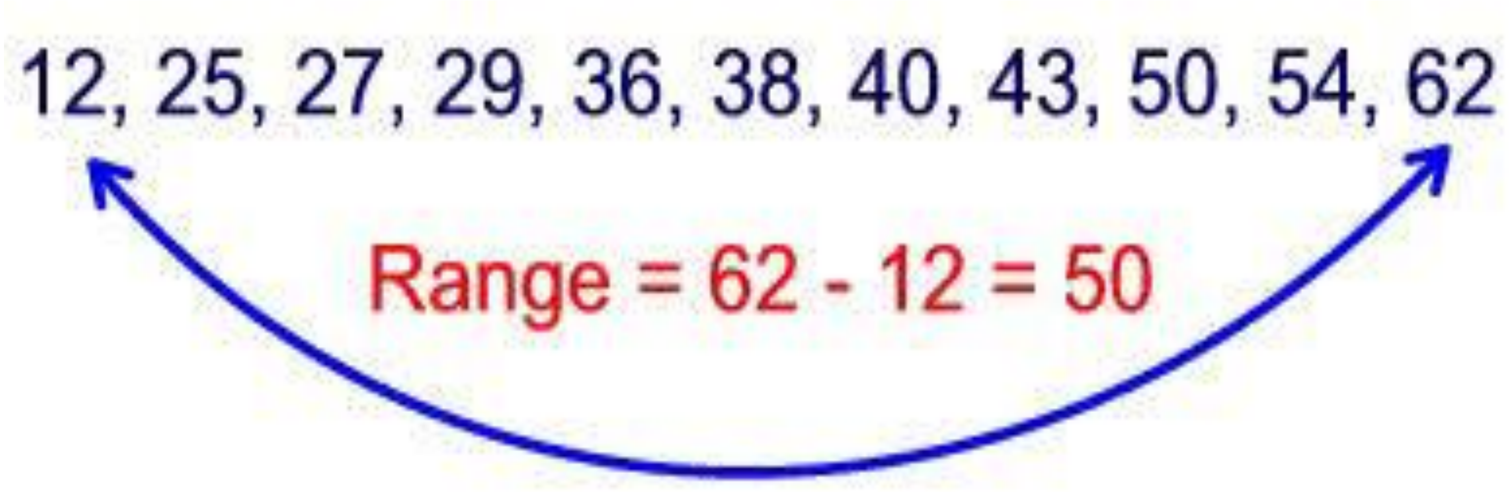
The most common measures of dispersion are:
Range
Variance, and
Standard deviation

The Range

- The ***Range*** is the ***distance*** between the ***largest*** and the ***smallest*** values in a set of data
  - (Requires an interval scale)



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The Variance
(requires a ratio scale)

In statistics, particularly when looking at probability distributions,

the *Variance* is a measure of *how far a set of numbers is spread out*.

- This is also known as the ***dispersion*** or ***variation***.

Variance = 0

means all the numbers are the same

Variance = a small number

means all the numbers are close to each other

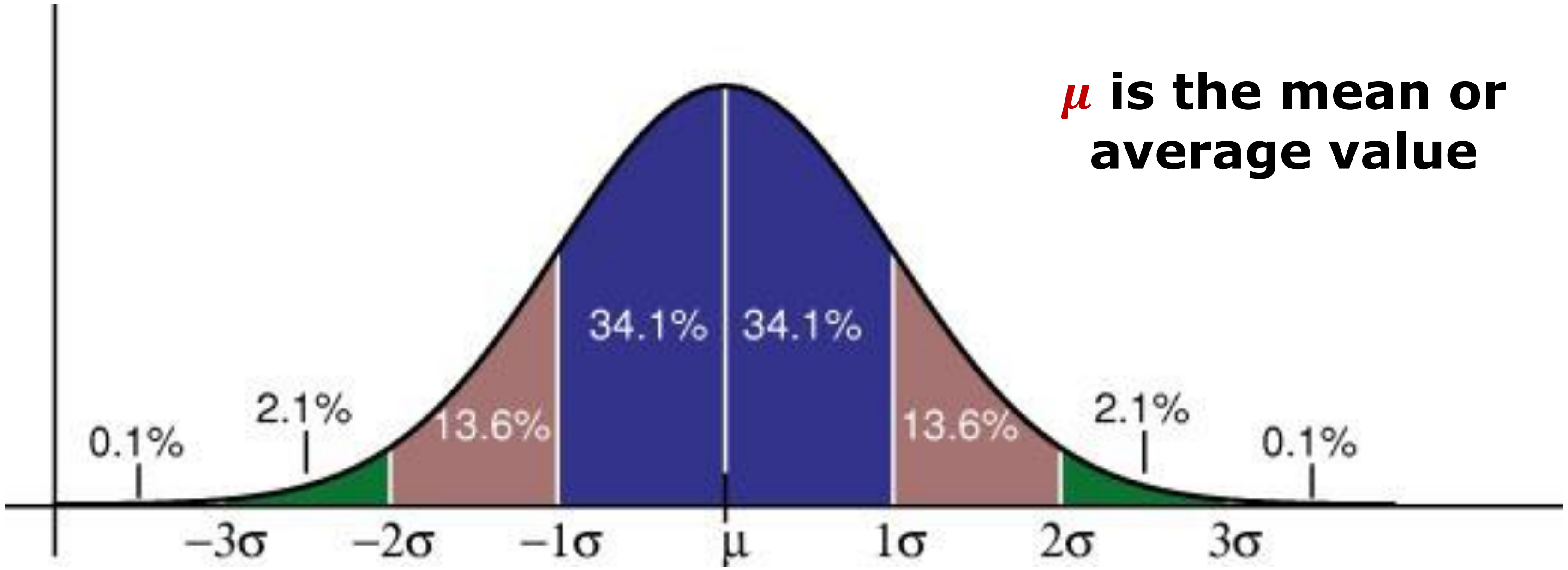
Variance = a large number

Means the numbers are widely dispersed

The Standard Deviation ( )

(square root of the variance)

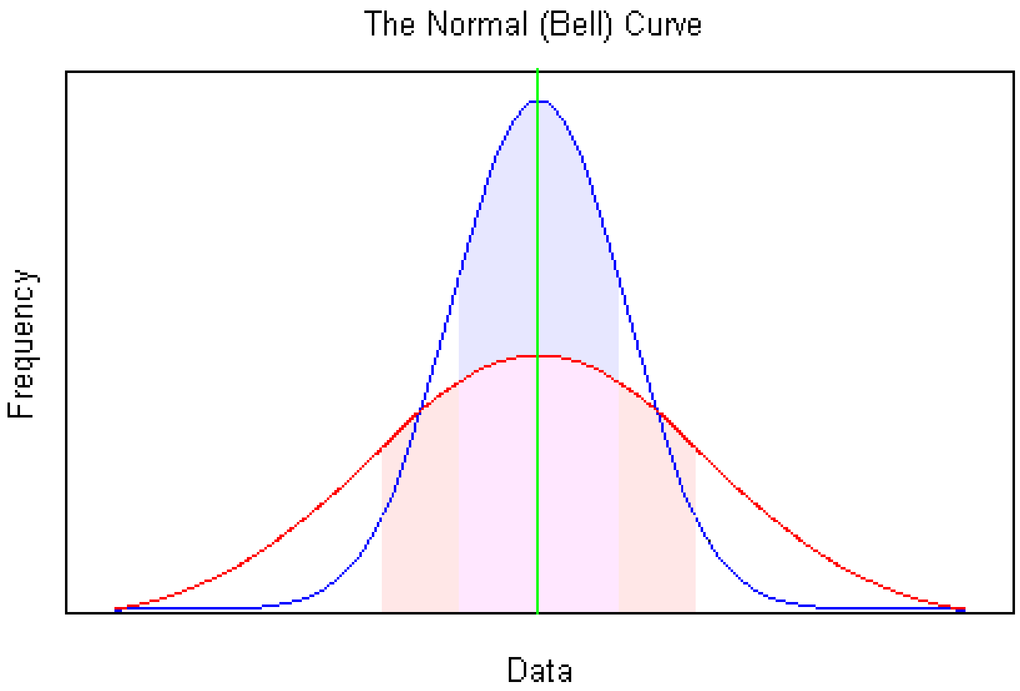
The Standard Deviation (denoted by the symbol σ or "sigma") is another measure used to represent the *amount of variation* or *dispersion* of a set of data values



μ is the mean or average value

Standard Deviation & Variance

- **If the standard deviation or variance is small, it means the data values are very close together (*blue curve*)**
- **If the standard deviation or variance is large, it means the data values are dispersed (*red curve*)**



Calculations for a Discrete, Random Variable x

Suppose you have an <u>ordered</u> collection of numbers, X, on a ratio scale

And suppose x is a random member of the collection

μ = **mean or average value**

x_i = **ith possible value**

p_i = **probability that x has the value x_i**

$Variance = \sum_{i=1}^{n} p_i * (x_i - \mu)^2$

$\sigma = \sqrt{Variance}$

There are many uses for these calculations when doing statistical analysis of measured data

Alternative Calculations for Variance

Suppose you have an ordered collection of numbers, X, on a ratio scale

And suppose N is the number of members (the size) of the collection

μ = **mean or average value**

x_i = **i^th value**

$$Variance = \frac{\sum (x_i - \mu)^2}{N}$$

$$\sigma = \sqrt{Variance}$$

For some data sets, this formula for variance may experience overflow, underflow or other calculation instabilities due to the sums of squares required. See "Algorithms for calculating variance" on Wikipedia.

Any Questions?

End of Lecture

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