

## *Test Generation based on Finite State Models*

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## *Speaker Biographical Sketch*

- Professor & Director of International Outreach  
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Computer Security Division  
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- Vice President, IEEE Reliability Society
- Secretary, ACM SIGAPP (Special Interest Group on Applied Computing)
- Principal Investigator, NSF TUES (Transforming Undergraduate Education in Science, Technology, Engineering and Mathematics) Project
  - *Incorporating Software Testing into Multiple Computer Science and Software Engineering Undergraduate Courses*
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# Learning Objectives

- What are Finite State Machines (FSM)?
- The W method for test generation
- The Wp method for test generation
- Automata theoretic versus control-flow based test generation
- What are Extended Finite State Machines (EFSM)?
- What are Communicating Extended State Machines (CEFSM)?
  - Architectural design in SDL
- EFSM-based test generation
- CEFSM-based test generation

# Where Are These Methods Used?

- Conformance testing of communications protocols
  - Finite state machines are widely used in modeling of different kinds of systems.
  - Testing of any system/subsystem modeled as a finite state machine, e.g. elevator designs, automobile components (locks, transmission, stepper motors, etc.), steam boiler control, etc.
  - Generation of tests from FSM assists in testing the  
conformance of implementations to the corresponding FSM model
- White box-based coverage testing for SDL design specifications, EFSMs, CFEMSs, reachability graphs, etc.

# What is an Finite State Machine?

- A finite state machine, abbreviated as FSM, is an **abstract representation** of behavior exhibited by some systems.
- An FSM is derived from application requirements. For example, a network protocol could be modeled using an FSM.
- **Not all aspects** of an application's requirements are specified by an FSM. For example, **real time requirements** and **performance requirements** cannot be specified by an FSM.

# Requirements Specification or Design Specification?

- An FSM could serve as a *specification* of the required behavior or as a *design* artifact according to which an application is to be implemented.
  - The role assigned to an FSM depends on whether it is a part of the *requirements specification* or of the *design specification*.
  - FSMs are part of UML 2.0 design notation.

# Finite State Machines with Output

- Mealy Machine (due to G. H. Mealy -1955 publication)
  - Outputs corresponds transitions between states.
- Moore Machine (due to E. F. Moore -1956 publication)
  - Outputs are determined only by the states

# Mealy Machine (1)

- A Mealy machine  $M = \{S, I, O, f, g, s_o\}$ 
  - $S$ : a finite set of states
  - $s_o$ : the start state ( a.k.a. initial state) contained in  $S$
  - $I$ : a finite input alphabet
  - $O$ : a finite output alphabet
  - $f$ : **a transition function** that maps a state/input pair to the next state  $(S \times I \rightarrow S)$
  - $g$ : **an output function** that maps a state/input pair to an output  $(S \times I \rightarrow O)$



# State Diagram Representation of a Mealy FSM

- A state diagram is a *directed graph* where each node is a state and each edge is a transition between states
- Each transition from a state can be triggered by an input and produce an output

– Input  $x$  Current State  $\rightarrow$  Output  $y$  Next State

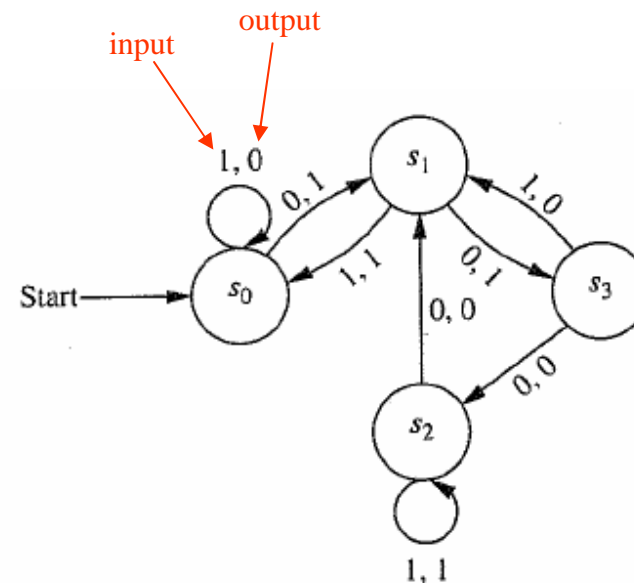
# Tabular Representation of a Mealy FSM

- A table is often used as an alternative to the state diagram
- The table consists of two sub-tables that consist of one or more columns each.
  - The left sub-table is the *input* sub-table.
  - The rows are labeled by the states of the FSM.
  - The right sub-table is the *output* sub-table.

## Mealy Machine (2)

- An example of a Mealy machine

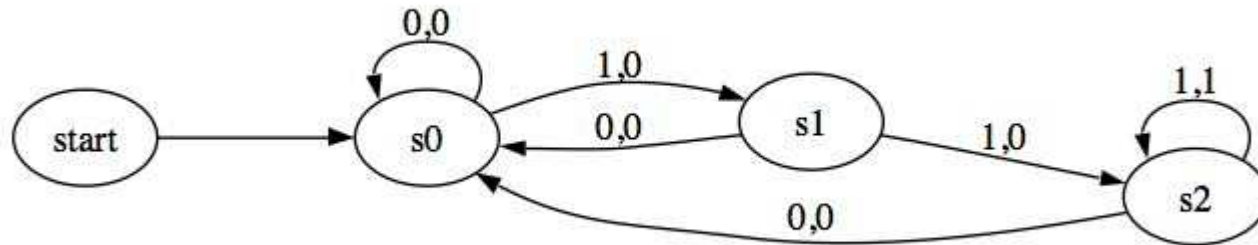
TABLE				
State	<i>f</i>		<i>g</i>	
	<i>Input</i>		<i>Input</i>	
	0	1	0	1
<i>s</i> <sub>0</sub>	<i>s</i> <sub>1</sub>	<i>s</i> <sub>0</sub>	1	0
<i>s</i> <sub>1</sub>	<i>s</i> <sub>3</sub>	<i>s</i> <sub>0</sub>	1	1
<i>s</i> <sub>2</sub>	<i>s</i> <sub>1</sub>	<i>s</i> <sub>2</sub>	0	1
<i>s</i> <sub>3</sub>	<i>s</i> <sub>2</sub>	<i>s</i> <sub>1</sub>	0	0



The state diagram for the FSM shown in the table

## Mealy Machine (3)

- A Mealy machine that outputs 1 if and only if the input string read so far ends with 111. This machine is a *language recognizer*.



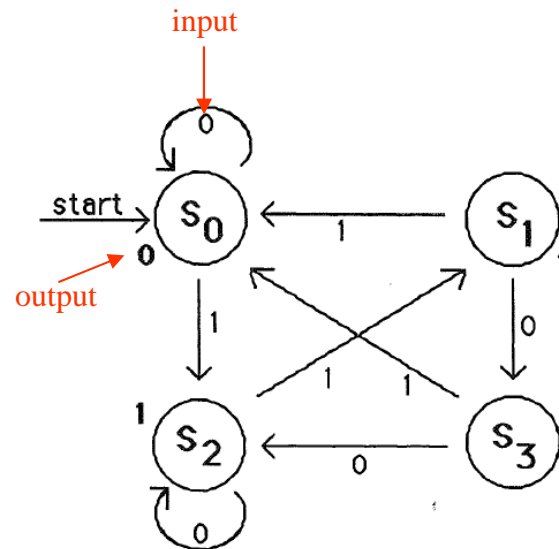
# Moore Machine (1)

- A Moore machine  $M = \{S, I, O, f, g, s_o\}$ 
  - $S$ : a finite set of states
  - $S_o$ : the start state ( a.k.a. initial state) contained in  $S$
  - $I$ : a finite input alphabet
  - $O$ : a finite output alphabet
  - $f$ : a transition function that maps a (current) state/input pair to the next state  
 $(S \times I \rightarrow S)$
  - $g$ : an output function that maps a state to an output  $(S \rightarrow O)$

## Moore Machine (2)

- In a Moore machine, the output at each transition is only dependent on the *state*, rather than a state/input pair
- In other words, *the output after transition is determined solely by the final state of the transition.*

State	$f$		$g$
	Input		
	0	1	
$s_0$	$s_0$	$s_2$	0
$s_1$	$s_3$	$s_0$	1
$s_2$	$s_2$	$s_1$	1
$s_3$	$s_2$	$s_0$	1



# Deterministic versus Nondeterministic

- A **deterministic** finite automaton: For each pair of state and input value, there is **a unique next state** given by the transition function.
- A **nondeterministic** finite automaton: There may be *several possible* next states for each pair of state and input value.
  - If the language  $L$  is recognized by a nondeterministic finite state automaton  $\mathcal{M}_1$ , then  $L$  is also recognized by a deterministic finite state automaton  $\mathcal{M}_2$ .

# Properties of FSM

- **Completely specified:** An FSM  $\mathcal{M}$  is said to be completely specified if from each state in  $\mathcal{M}$  there exists a transition for each input symbol.
- **Strongly connected:** An FSM  $\mathcal{M}$  is considered strongly connected if for each pair of states  $(s_i, s_j)$  there exists an input sequence that takes  $\mathcal{M}$  from state  $s_i$  to  $s_j$ .



## V-Equivalence of Two States

- **V-equivalence**: Let  $\mathcal{M}_1=(I, O, S_1, s_1^0, f_1, g_1)$  and  $\mathcal{M}_2=(I, O, S_2, s_2^0, f_2, g_2)$  be two FSMs (Mealy Machines). Let V denote a set of non-empty strings over the input alphabet I, that is,  $V \subseteq I^+$ .
- Let  $s_i$  and  $s_j$ ,  $i \neq j$ , be two states of machines  $\mathcal{M}_1$  and  $\mathcal{M}_2$ , respectively.  $s_i$  and  $s_j$  are considered **V-equivalent** if  $g_1(s_i, v)=g_2(s_j, v)$  for all  $v$  in V.
- Stated differently, states  $s_i$  and  $s_j$  are considered **V-equivalent** if  $\mathcal{M}_1$  and  $\mathcal{M}_2$ , when excited in states  $s_i$  and  $s_j$ , respectively, yield **identical output sequences**.

# Equivalence of Two States

- States  $s_i$  and  $s_j$  are said to be *equivalent* if  $g_1(s_i, v) = g_2(s_j, v)$  for **any set  $V$** 
  - If  $s_i$  and  $s_j$  are not equivalent then they are said to be *distinguishable*
- We write  $s_i = s_j$  if states  $s_i$  and  $s_j$  are equivalent, and  $s_i \neq s_j$  when they are distinguishable

# Equivalence of Two FSMs

- Machines  $\mathcal{M}_1$  and  $\mathcal{M}_2$  are said to be *equivalent* if
  - For each state  $\alpha$  in  $\mathcal{M}_1$  there exists a state  $\alpha'$  in  $\mathcal{M}_2$  such that  $\alpha$  and  $\alpha'$  are equivalent
  - For each state  $\beta$  in  $\mathcal{M}_2$  there exists a state  $\beta'$  in  $\mathcal{M}_1$  such that  $\beta$  and  $\beta'$  are equivalent
- Machines that are not equivalent are considered *distinguishable*.
- If  $\mathcal{M}_1$  and  $\mathcal{M}_2$  are strongly connected, then they are equivalent if their respective initial states,  $s^1_0$  and  $s^2_0$ , are equivalent
- We write  $\mathcal{M}_1 = \mathcal{M}_2$  if machines  $\mathcal{M}_1$  and  $\mathcal{M}_2$  are equivalent, and  $\mathcal{M}_1 \neq \mathcal{M}_2$  when they are distinguishable

# K-Equivalence

- States  $s_i \in S_1$  and  $s_j \in S_2$  are considered *k-equivalent* if when excited by any input of length *k*, yield identical output sequences
  - States that are not k-equivalent are considered *k-distinguishable*
- It is also easy to see that if two states are *k-distinguishable* for any  $k > 0$  then they are also distinguishable for any  $n \geq k$

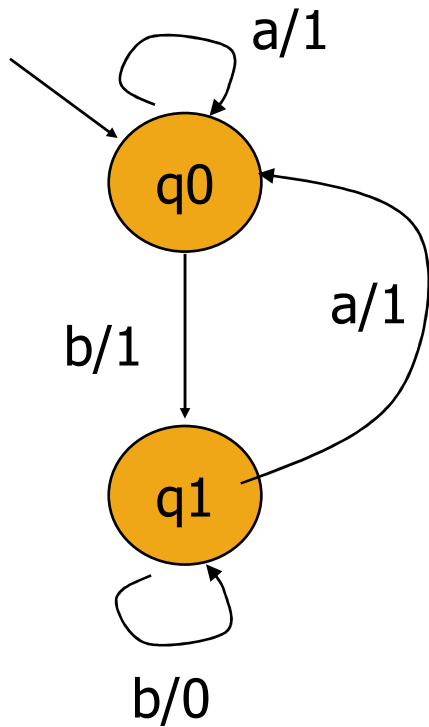
# Minimal Machine

- An FSM  $\mathcal{M}$  is considered *minimal* if the number of states in  $\mathcal{M}$  is less than or equal to any other FSM equivalent to  $\mathcal{M}$ .

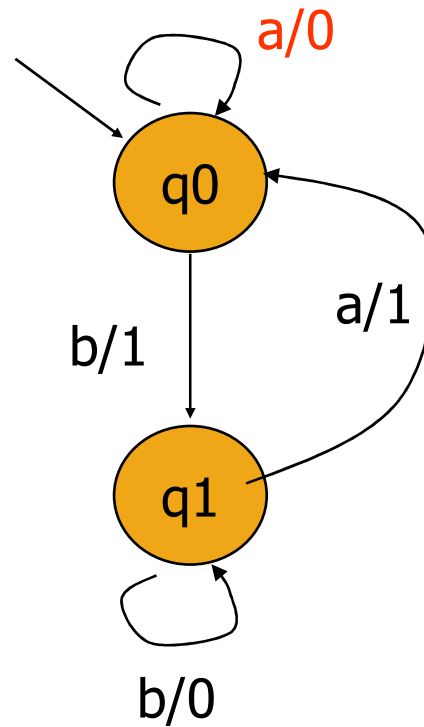
# Faults in Implementation

- An FSM serves to specify the correct *requirement* or *design* of an application. Hence tests generated from an FSM target faults related to the FSM itself.
- *What faults are targeted by the tests generated using an FSM?*

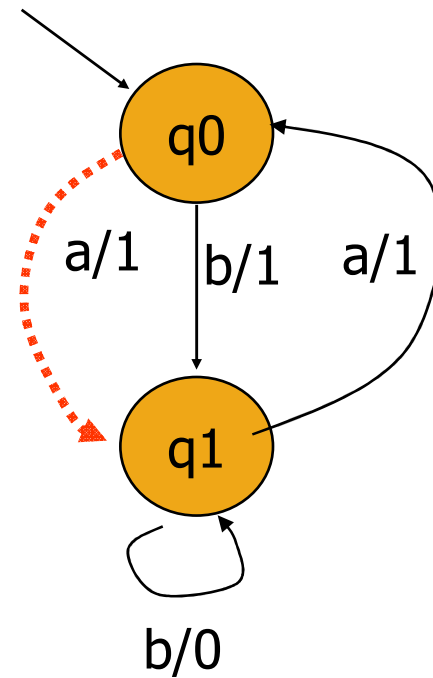
# Fault Model (1)



Correct design

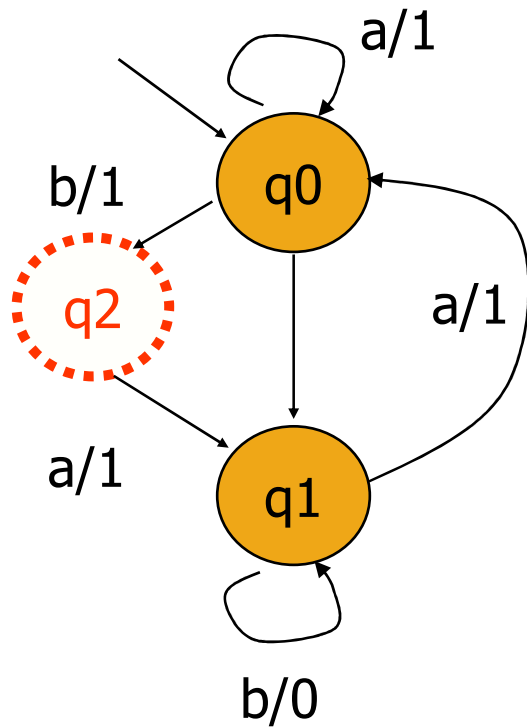


Operation error

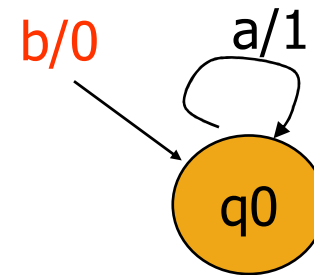


Transfer error

## Fault Model (2)



Extra state error



Missing state error





## Test Generation using the W Method

# Assumption

- $\mathcal{M}$  is *completely* specified, *minimal*, *connected*, and *deterministic*
- $\mathcal{M}$  starts in *a fixed initial state*
- $\mathcal{M}$  can be *reset accurately to the initial state*. A *null* output is generated during the reset operation
- $\mathcal{M}$  and the implementation under test have the *same input alphabet*

# Steps of the W Method

- Step 1: Estimate the maximum number of states ( $m$ ) in the correct implementation of the given FSM  $\mathcal{M}$
- Step 2: Construct the characterization set  $W$  for  $\mathcal{M}$
- Step 3: (1) Construct the *testing tree* for  $\mathcal{M}$  and  
(2) Generate the transition cover set  $P$  from the testing tree
- Step 4: Construct set  $Z$  from  $W$  and  $m$
- Step 5: Test generation and execution



Step 1: Estimation of  $m$

## Estimation of $m$

- This is based on a knowledge of the implementation. In the absence of any such knowledge, let  $m = |S|$  (number of states in the given FSM)



## Step 2: Construction of $W$

Step 2.1: Construction of the  $K$ -Equivalence Partitions

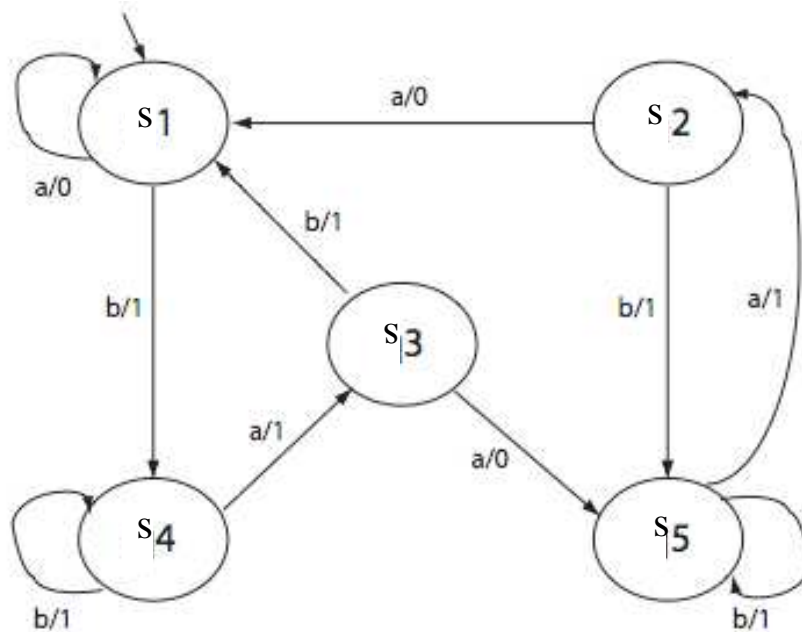
Step 2.2: Derivation of the Characterization Set

# What is a Characterization Set ?

- Let  $\mathcal{M}=(I, O, S, s_0, f, g)$  be a *minimal* and *complete* FSM
- $W$  is a finite set of input sequences that distinguish the behavior of any pair of states in  $\mathcal{M}$ .
  - Each input sequence in  $W$  is of finite length
- Given states  $s_i$  and  $s_j \in S$ ,  
 $W$  contains a string  $\alpha$  such that  $g(s_i, \alpha) \neq g(s_j, \alpha)$

## An Example of $W$

- For the following  $\mathcal{M}$ , we have  $W=\{baaa,aa,aaa\}$
- For example,  $baaa$  distinguishes state  $s_1$  from  $s_2$  as  $g(s_1, baaa) \neq g(s_2, baaa)$   
More precisely,  $g(s_1, baaa) = 1101$  and  $g(s_2, baaa) = 1100$







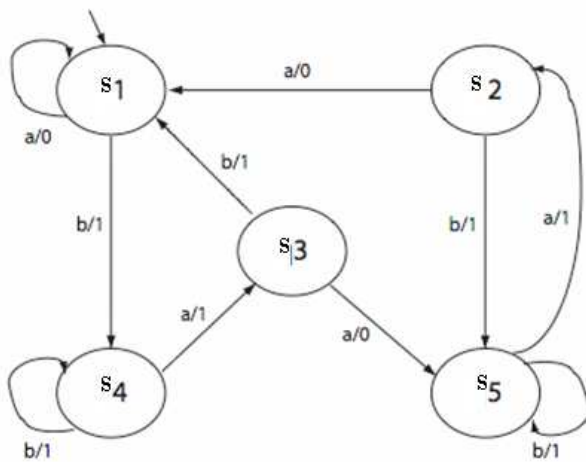
## Step 2.1: Construction of the K-Equivalence Partitions

# What is a $k$ -equivalence partition of $S$ ?

- Given an FSM  $\mathcal{M}=(I, O, S, s_0, f, g)$   $k$ -equivalence partition of  $S$ , denoted as  $P_k$ , is a collection of  $n$  finite sets  $\Sigma_{k1}, \Sigma_{k2} \dots \Sigma_{kn}$  such that
  - $\cup_{i=1}^n \Sigma_{ki} = S$
  - states in  $\Sigma_{ki}$  for  $1 \leq i \leq n$  are  $k$ -equivalent
  - If state  $u$  is in  $\Sigma_{ki}$  and  $v$  in  $\Sigma_{kj}$  for  $i \neq j$ , then  $u$  and  $v$  are  $k$ -distinguishable

# Construction of One-Equivalence Partition (1)

- Computing the one-equivalence partition,  $P_1$ , for the following FSM
- Start with a tabular representation of  $\mathcal{M}$



Current state	Output		Next state	
	a	b	a	b
s1	0	1	s1	s4
s2	0	1	s1	s5
s3	0	1	s5	s1
s4	1	1	s3	s4
s5	1	1	s2	s5

## Construction of One-Equivalence Partition (2)

- Group states identical in their *output* entries. This gives us 1-partition  $P_1$  consisting of  $\Sigma_1 = \{s_1, s_2, s_3\}$  and  $\Sigma_2 = \{s_4, s_5\}$

$\Sigma$	Current state	Output		Next state	
		a	b	a	b
1	s1	0	1	s1	s4
	s2	0	1	s1	s5
	s3	0	1	s5	s1
2	s4	1	1	s3	s4
	s5	1	1	s2	s5

$P_1$  Table

- We have the one-equivalence partition as follows
  - $P_1 = \{1, 2\}$
  - Group 1 =  $\Sigma_{11} = \{s_1, s_2, s_3\}$
  - Group 2 =  $\Sigma_{12} = \{s_4, s_5\}$

# Construction of Two-Equivalence Partition (1)

- **Rewrite  $P_1$  Table.** Remove the output columns. Replace a state entry  $s_i$  by  $s_{ij}$  where  $j$  is the group number in which lies state  $s_i$

$\Sigma$	Current state	Next state	
		a	b
1	s1	s11	s42
	s2	s11	s52
	s3	s52	s11
2	s4	s31	s42
	s5	s21	s52

group number

state  $s_5$  is in group 2

state  $s_1$  is in group 1

# Construction of Two-Equivalence Partition (2)

- **Construct  $P_2$  Table.** Group all entries with *identical second subscripts* under the *next state column*. This gives us the  $P_2$  table.
  - Note the change in second subscripts
  - We have three groups in the  $P_2$  table

$\Sigma$	Current state	Next state	
		a	b
1	s1	s11	s43
	s2	s11	s53
2	s3	s53	s11
3	s4	s32	s43
	s5	s21	s53

$P_2$  Table

state  $s_5$  is in group 3

# Construction of Three-Equivalence Partition

- **Construct  $P_3$  Table.** Group all entries with *identical second subscripts* under the next state column. This gives us the  $P_3$  table.
  - Note the change in second subscripts
  - We have four groups in the  $P_3$  table

$\Sigma$	Current state	Next state	
		a	b
1	s1	s11	s43
	s2	s11	s54
2	s3	s54	s11
3	s4	s32	s43
4	s5	s21	s54

$P_3$  Table

state  $s_5$  is in group 4

# Construction of Four-Equivalence Partition

- **Construct  $P_4$  Table.** Continuing with regrouping and re-labeling

- Note the change in second subscripts
- We have five groups in the  $P_4$  table

$\Sigma$	Current state	Next state	
		a	b
1	s1	s11	s44
2	s2	s11	s55
3	s3	s55	s11
4	s4	s33	s44
5	s5	s22	s55

$P_4$  Table

state  $s_5$  is in group 5



## k-equivalence partition: Convergence

- The process is guaranteed to converge
- When the process converges, and the machine is minimal, each state will be in a separate group
- The next step is to obtain the *distinguishing strings for each state*



## Step 2.2: Using the W-Procedure to derive the Characterization Set $W$

A procedure to derive  $W$  from a set of partition tables  
constructed at Step 2.1

# The W-Procedure (1)

- Let  $\mathcal{M} = \{S, I, O, f, g, s_0\}$  be the FSM for which  $P = \{P_1, P_2, \dots, P_n\}$  is the *set of  $k$ -equivalence partition tables* for  $k = 1, 2, \dots, n$   
Initialize  $W = \emptyset$
- Traverse the  $k$ -equivalence partitions in reverse order to obtain distinguishing sequence for each pair of states

# Finding the distinguishing sequences (1)

- Let us find a distinguishing sequence for states  $s_1$  and  $s_2$
- Find Tables  $P_i$  and  $P_{i+1}$  such that  $(s_1, s_2)$  are in the same group in  $P_i$  and different groups in  $P_{i+1}$ . We get  $P_3$  and  $P_4$
- Initialize  $z = \epsilon$ . Find the input symbol that distinguishes  $s_1$  and  $s_2$  in  $P_3$ . This symbol is  $b$ . We update  $z$  to  $z.b$ . Hence  $z$  now becomes  $b$ .

## Finding the distinguishing sequences (2)

- The next states for  $s_1$  and  $s_2$  on  $b$  are, respectively,  $s_4$  and  $s_5$ .
- We find that  $s_4$  and  $s_5$  are in the same group in  $P_2$  and different groups in  $P_3$ .
- We move to the  $P_2$  table and find the input symbol that distinguishes  $s_4$  and  $s_5$ . Let us select  $a$  as the distinguishing symbol. Update  $z$  which now becomes  $ba$ .
- Refer to Table  $P_2$ , the next states for states  $s_4$  and  $s_5$  on symbol  $a$  are, respectively,  $s_3$  and  $s_2$ . These two states are distinguished in  $P_1$  by  $a$  and  $b$  (i.e.,  $s_2$  and  $s_3$  are in the same group in  $P_1$  and different groups in  $P_2$ ). Let us select  $a$ . We update  $z$  to  $baa$ .

## Finding the distinguishing sequences (3)

- Refer to Table  $P_1$ . The next states for  $s_2$  and  $s_3$  on **a** are, respectively,  $s_1$  and  $s_5$ .
- We find that  $s_1$  and  $s_5$  are in the same group in the original table and different groups in  $P_1$ .
- Moving to the original state transition table we obtain **a** as the distinguishing symbol for  $s_1$  and  $s_5$
- We update  $z$  to **baaa**. This is the *farthest we can go backwards* through the various tables. **baaa** is the desired distinguishing sequence for states  $s_1$  and  $s_2$ . Check that  $g(s_1, \text{baaa}) = 1101$  and  $g(s_2, \text{baaa}) = 1100$ . We have  $g(s_1, \text{baaa}) \neq g(s_2, \text{baaa})$

## Finding the distinguishing sequences (4)

- Using the procedure analogous to the one used for  $s_1$  and  $s_2$ , we can find the distinguishing sequence for each pair of states. This leads us to the following characterization set for our FSM

- We have the distinguishing sequences as follows

- |                               |                            |
|-------------------------------|----------------------------|
| – $s_1, s_2 \rightarrow$ baaa | $s_2, s_4 \rightarrow$ a   |
| – $s_1, s_3 \rightarrow$ aa   | $s_2, s_5 \rightarrow$ a   |
| – $s_1, s_4 \rightarrow$ a    | $s_3, s_4 \rightarrow$ a   |
| – $s_1, s_5 \rightarrow$ a    | $s_3, s_5 \rightarrow$ a   |
| – $s_2, s_3 \rightarrow$ aa   | $s_4, s_5 \rightarrow$ aaa |

This gives  $W = \{a, aa, aaa, baaa\}$

## Where Are We?

- Step 1: Estimate the maximum number of states ( $m$ ) in the correct implementation of the given FSM  $\mathcal{M}$
- Step 2: Construct the characterization set  $W$  for  $\mathcal{M}$
- Step 3: (a) Construct the *testing tree* for  $\mathcal{M}$  and (b) generate the transition cover set  $P$  from the testing tree
- Step 4: Construct set  $Z$  from  $W$  and  $m$
- Step 5: Desired test set is  $P.Z$



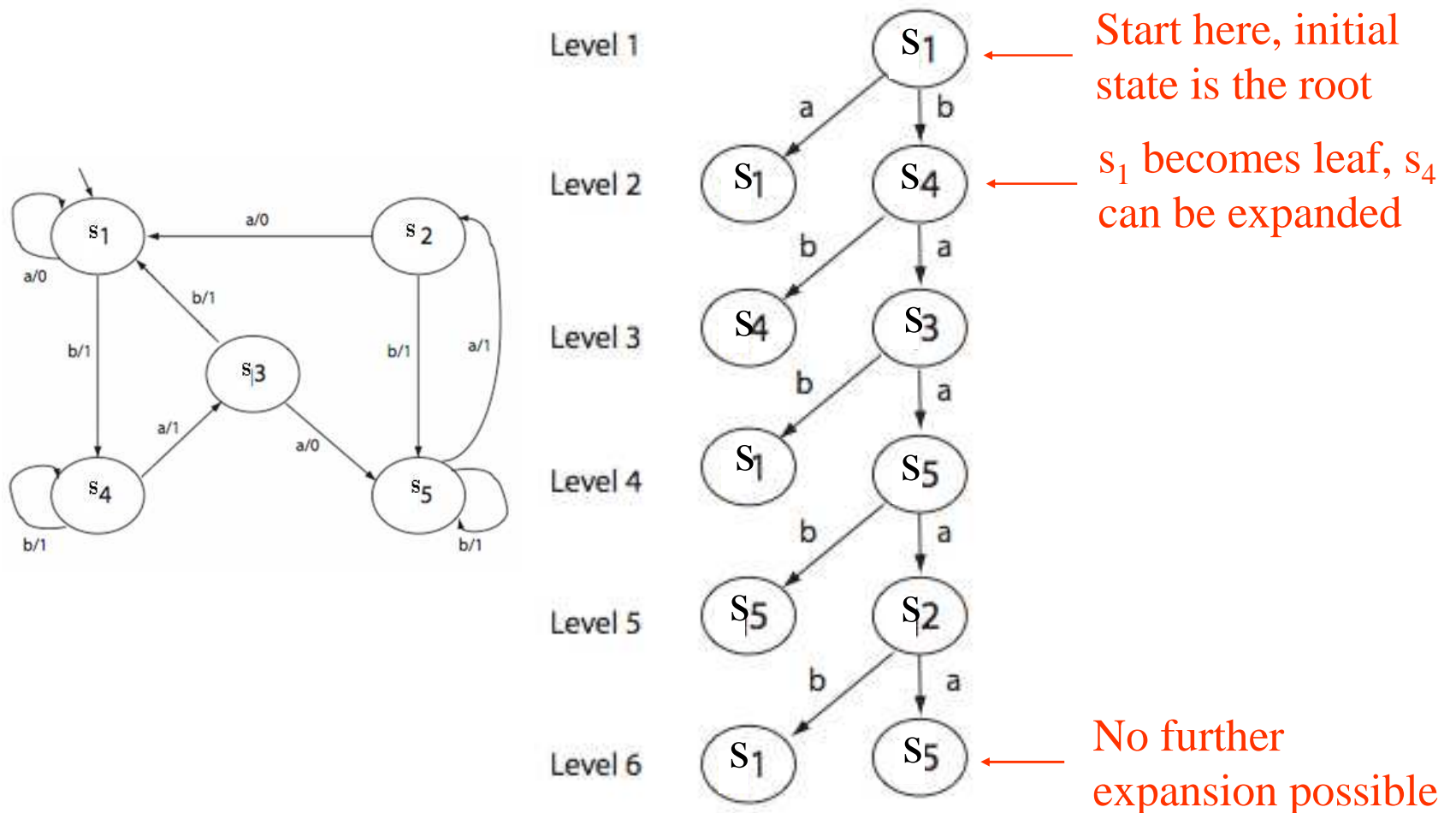


### Step 3.1: Construction of the Testing Tree for $\mathcal{M}$

# Tree Construction (1)

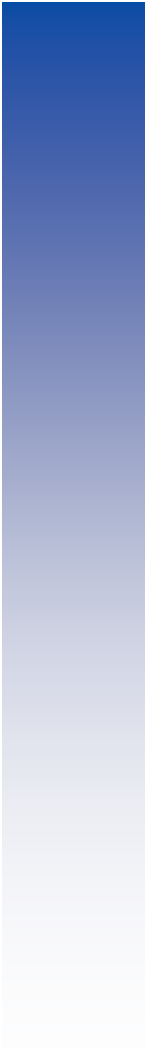
- A **testing tree** of an FSM,  $\mathcal{M}=\{S, I, O, f, g, s_0\}$ , is a tree *rooted at the initial state*. It contains *at least one path from the initial state to the remaining states* in  $\mathcal{M}$ .
- Construction procedure
  - State  $s_0$ , the initial state, is the **root** of the testing tree.  
Suppose that the testing tree has been constructed until **level  $k$**   
The  $(k+1)^{\text{th}}$  level is built as follows
  - Select a node  $n$  at level  $k$ .  
If  $n$  appears at any level from 1 through  $k$ , then  $n$  is a leaf node and is not expanded any further.  
If  $n$  is not a leaf node then we expand it by adding a branch **from node  $n$  to a new node  $m$  if  $f(n, \alpha)=m$  for  $\alpha \in I$** .  
This branch is labeled as  $\alpha$ .  
This step is repeated for all nodes at level  $k$ .

## Tree Construction (2)



## Tree Construction (3)

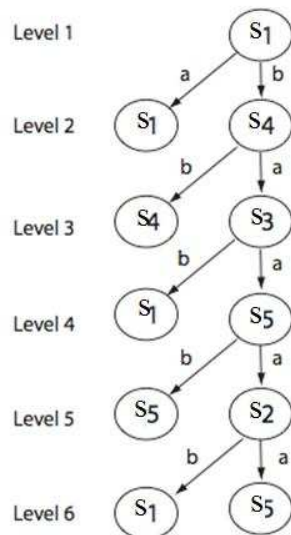
- The testing tree is initialized with the initial state  $s_1$  as the root node.
- This is level 1 of the tree.
- We note that  $g(s_1, a) = s_1$  and  $g(s_1, b) = s_4$ . Hence, we create two nodes at the next level and label them as  $s_1$  and  $s_4$ .
- The branches from  $s_1$  to  $s_1$  and  $s_4$  are labeled, respectively, a and b.
- As  $s_1$  is the only node at level 1, we now proceed to expand the tree to form level 2.
- At level 2, we first consider the node labeled  $s_1$ . However, another node labeled  $s_1$  already appears at level 1; hence, this node becomes a leaf node and is not expanded any further.
- Next, we examine the node labeled  $s_4$ . We note that  $g(s_4, a) = s_3$  and  $g(s_4, b) = s_4$ . We, therefore, create two new nodes at level 3 and label these as  $s_3$  and  $s_4$  and label the corresponding branches as a and b, respectively.



Step 3.2: Generation of the transition cover set  $P$   
from the testing tree

# Find The Transition Cover Set from a Testing Tree

- A **transition cover set**  $P$  is a set of all strings *representing sub-paths*, starting at the root, *in the testing tree*.
  - A sub-path is a path starting from the root of the testing tree and terminating in any node of the tree.
  - Concatenation of the labels along the edges of a sub-path is a string that belongs to  $P$ .
  - The empty string ( $\epsilon$ ) also belongs to  $P$ .



$P = \{\epsilon, a, b, bb, ba, bab, baa, baab, baaa, baaab, baaaa\}$

## Why is it called a Transition Cover Set?

- Exciting an FSM in  $s_0$ , the initial set, with an element of P, forces the FSM into some state.
- After the FSM has been excited with all elements of P, each time starting in the initial state, the FSM has reached every state.
- Thus, exciting an FSM with elements of P ensures that *all states are reached* and *all transitions have been traversed at least once*.
  - The empty input sequence does not traverse any branch but is useful in constructing some desired test sequence.



Step 4: Construction of  $Z$  from  $W$  and  $m$



## Construction of Z from W and $m$

- Given that  $I$  is the input alphabet and  $W$  the characterization set. Suppose that the number of states estimated to be in the implementation under test is  $m$ , and the number of states in the design specification is  $n$ ,  $m > n$ .
- We compute  $Z$  as
$$Z = (I^0.W) \cup (I^1.W) \cup \dots \cup (I^{m-1-n}.W) \cup (I^{m-n}.W)$$
  - Recall that  $I^0 = \{\epsilon\}$ ,  $I^1 = I$ ,  $I^2 = I . I$ , and so on, where  $(.)$  denotes string concatenation.
- For  $m = n$ , we get  $Z = I^0.W = W$
- For  $m < n$ , we use  $Z = IW$



## Step 5.1: Test Generation

# Generation of a Test Set from P and Z

- The test inputs based on the given FSM  $\mathcal{M}$  can now be derived as  $T=P.Z$
- Let's use the same example. Suppose  $m = n = 5$ . We also have  $I=\{a, b\}$  and  $W$  (the characterization set)  $=\{a, aa, aaa, baaa\}$  (see slide 46)
  - Concatenating P with Z, we obtain the desired test set as follows
    - $Z = I^0.W = \{a, aa, aaa, baaa\}$  (Note:  $I^0 = \{\epsilon\}$  see slide 56)
    - $T = P . Z = \{\epsilon, a, b, bb, ba, bab, baa, baab, baaa, baaab, baaaa\} . \{a, aa, aaa, baaa\}$
- If we assume that the implementation has one extra state, that is,  $m = 6$ , then we have
  - $Z = I^0.W \cup (I^1.W) = \{\underline{a, aa, aaa, baaa}, \underline{\cancel{aa, aaa, aaaa, abaaa}}, \underline{ba, baa, baaa, bbaaa}\}$
  - $T = P . Z = \{\epsilon, a, b, bb, ba, bab, baa, baab, baaa, baaab, baaaa\} . \{a, aa, aaa, baaa, aa, aaa, aaaa, abaaa, ba, baa, baaa, bbaaa\}$

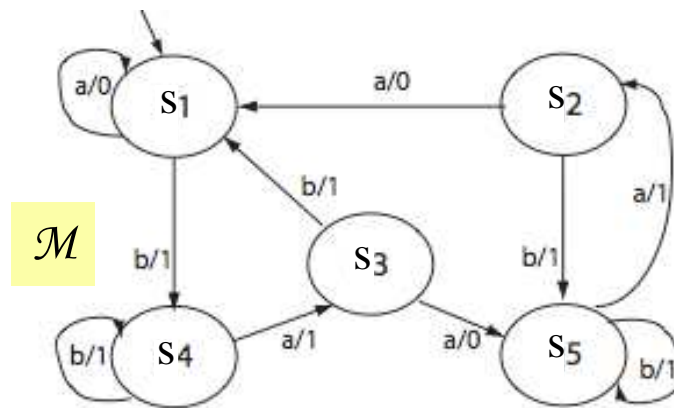


## Step 5.2: Test Execution

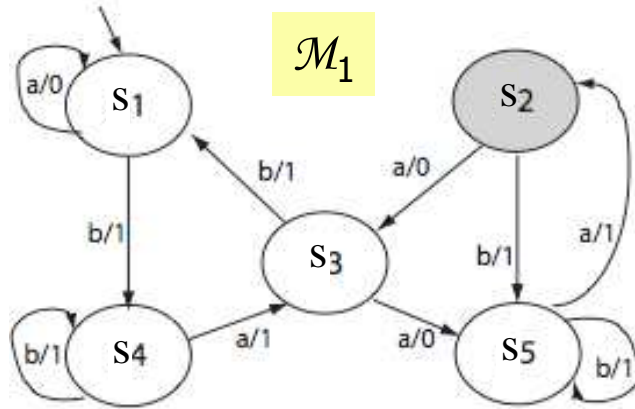
# Execution of the Generated Test Set

- To test the given implementation against its specification  $M$ , we do the following for each test input
  - Find the expected response to each element of  $T$
  - Generate test cases for the application. Note that even though the application is modeled by  $\mathcal{M}$ , there might be variables to be set before it can be exercised with elements of  $T$ .
  - Execute the application and check if the response matches.  
**Reset the application to the initial state after each test.**
  - **A mismatch between the expected and the actual response does not necessarily imply an error in the implementation.**
    - Is the specification error free?
    - Are the expected and actual responses determined without any error?
    - Is the comparison between them correct?If the answer is YES to all these questions, then a mismatch implies an error in the implementation.

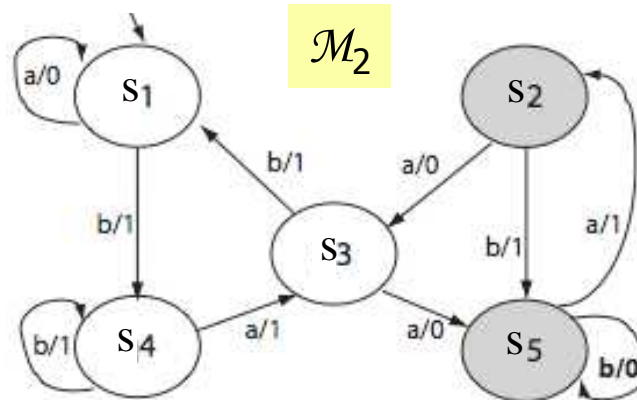
# Example One (1): $n = m = 5$



(a) Correct design



(b) Transfer error in state  $s_2$

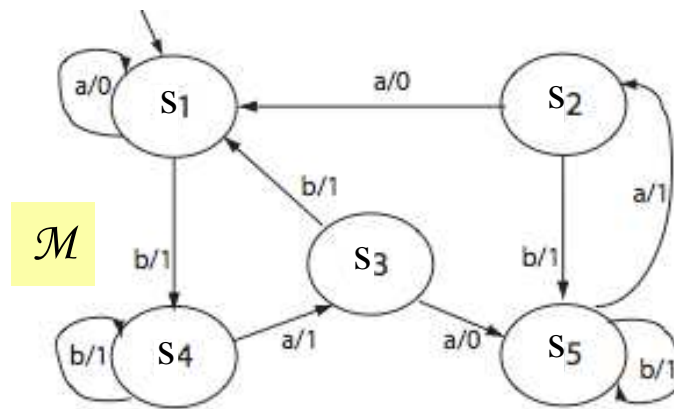


(b) Transfer error in state  $s_2$  and operation error in state  $s_5$

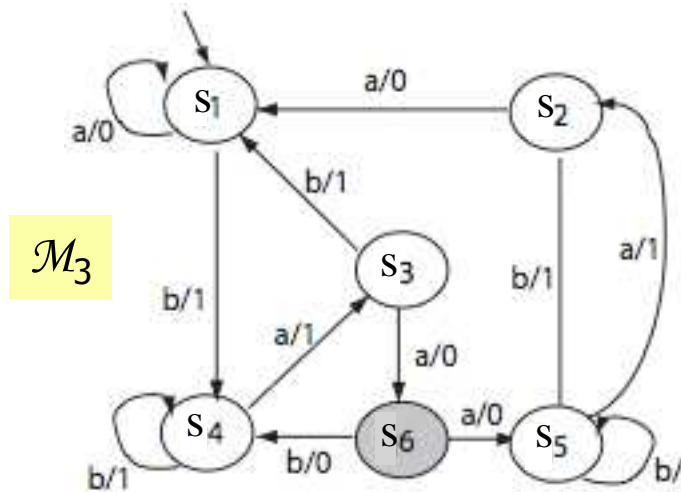
## Example One (2): $n = m = 5$

- To test  $\mathcal{M}_1$  against  $\mathcal{M}$ , we apply each test  $t$  from the set P.Z and compare  $\mathcal{M}(t)$  with  $\mathcal{M}_1(t)$ .
  - We find that when  $t = \text{baaaaaa}$ ,  $\mathcal{M}(t) = 1101000$  and  $\mathcal{M}_1(t) = 1101001$ .  
Hence, the input sequence baaaaaa reveals the transfer error in  $\mathcal{M}_1$ .
- Similarly, to test  $\mathcal{M}_2$  against  $\mathcal{M}$ , we apply each test  $t$  from the set P.Z and compare  $\mathcal{M}(t)$  with  $\mathcal{M}_2(t)$ .
  - We find that when  $t = \text{baaba}$ ,  $\mathcal{M}(t) = 11011$  and  $\mathcal{M}_2(t) = 11001$ .  
Hence, the input sequence baaba reveals the transfer error in  $\mathcal{M}_2$ .

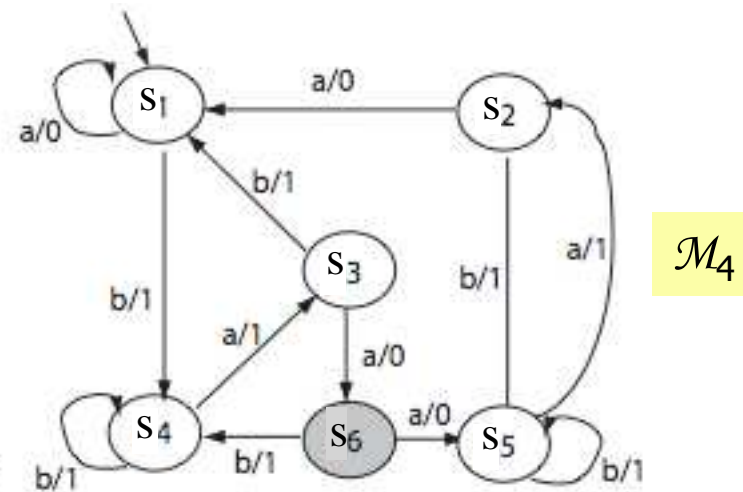
## Example Two (1): $n = 5$ & $m = 6$



(a) Correct design



(d) an extra state



(e) an extra state




## Example Two (2): $n = 5$ & $m = 6$

- To test  $\mathcal{M}_3$  against  $\mathcal{M}$ , we apply each test  $t$  from the set P.Z and compare  $\mathcal{M}(t)$  with  $\mathcal{M}_3(t)$ .
  - We find that when  $t = \text{baaba}$ ,  $\mathcal{M}(t) = 11011$  and  $\mathcal{M}_3(t) = 11001$ .  
Hence, the input sequence baaba reveals the transfer error in  $\mathcal{M}_3$ .
- Similarly, to test  $\mathcal{M}_4$  against  $\mathcal{M}$ , we apply each test  $t$  from the set P.Z and compare  $\mathcal{M}(t)$  with  $\mathcal{M}_4(t)$ .
  - We find that when  $t = \text{baaaa}$ ,  $\mathcal{M}(t) = 1101$  and  $\mathcal{M}_4(t) = 1100$ .  
Hence, the input sequence baaaa reveals the transfer error in  $\mathcal{M}_4$ .

# Test Sets Generated Using the W-Method

- The W-method is used for *constructing a test set* from a given FSM  $\mathcal{M}$ .
- The test set so constructed is *a finite set of sequences* that can be input to a *program* whose *control structure is modeled by  $\mathcal{M}$* .
- The tests can also be input to a *design* to test its correctness with respect to some specification.
- Most software systems *cannot be modeled 100% accurately* using an FSM. However, the *global control structure* of a software system can be modeled by an FSM.
- Tests generated using the W-method, or any other method based exclusively on a finite-state model of an implementation is likely to *reveal only certain types of faults*. (see slide 4)



# Automata Theoretical versus Control-Flow-Based Techniques

## Question

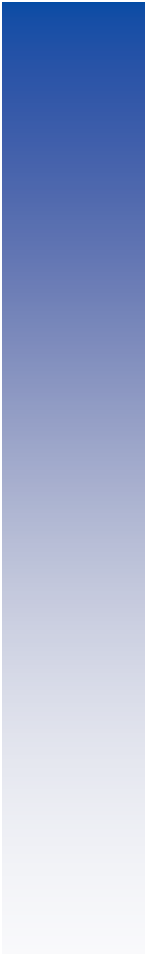


- The W method is an *automata theoretic* method for test generation.
- In contrast, many books on software testing mention *control flow-based techniques* for test generation.
- What is the difference between the two types of techniques and their fault detection abilities?

# Control-Flow-Based Techniques (1)

- **State cover**: A test set  $T$  is considered adequate with respect to the *state cover* criterion for an FSM  $\mathcal{M}$  if the execution of  $\mathcal{M}$  against each element of  $T$  causes *each state in  $\mathcal{M}$*  to be visited at least once
- **Transition cover**: A test set  $T$  is considered adequate with respect to the *branch/transition cover* criterion for an FSM  $\mathcal{M}$  if the execution of  $\mathcal{M}$  against each element of  $T$  causes *each transition in  $\mathcal{M}$*  to be taken at least once
- **Switch cover**: A test set  $T$  is considered adequate with respect to the *1-switch cover* criterion for an FSM  $\mathcal{M}$  if the execution of  $\mathcal{M}$  against each element of  $T$  causes *each pair of transitions  $(tr_1, tr_2)$  in  $\mathcal{M}$  to be taken at least once*, where for some input substring *ab*  $tr_1: f(s_j, a) = s_i$  and  $tr_2: f(s_i, b) = s_k$

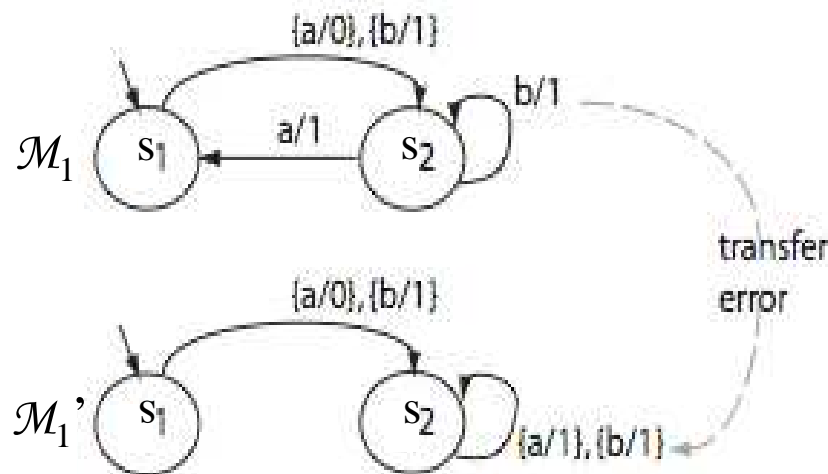
## Control-Flow-Based Techniques (2)

- **Boundary interior cover**: A test set  $T$  is considered adequate with respect to the *boundary-interior cover* criterion for an FSM  $\mathcal{M}$  if the execution of  $\mathcal{M}$  against each element of  $T$  causes *each loop (a self-transition) across states* to be traversed *zero times and at least once*.
  - Exiting the loop upon arrival covers the “*boundary*” condition
  - Entering it and traversing the loop at least once covers the “*interior*” condition

- 
- 
- 
- The following examples illustrate weaknesses of the state cover, branch/transition cover, switch cover and the boundary-interior cover test-adequacy criteria.

## Control-Flow-Based Techniques (3)

- Consider the following machines, a correct one ( $\mathcal{M}_1$ ) and one with a transfer error ( $\mathcal{M}_1'$ )
- $t=abba$  *covers all states* but does not reveal the error. Both machines generate the same output which is 0111.
- Will the tests generated by the W method reveal this error?
  - Check it out!



Refer to Example 3.38, Page 263



## Control-Flow-Based Techniques (4)

- Consider the following machines, a correct one ( $\mathcal{M}_2$ ) and one with a transfer error ( $\mathcal{M}_2'$ )
- There are 12 branch pairs, such as  $(tr_1, tr_2)$ ,  $(tr_1, tr_3)$ ,  $tr_6, tr_5$
- Consider the test set:  $\{bb, baab, aabb, aaba, abbaab\}$

- Does it cover all branches?

Yes

T also satisfies the 1-switch cover criterion

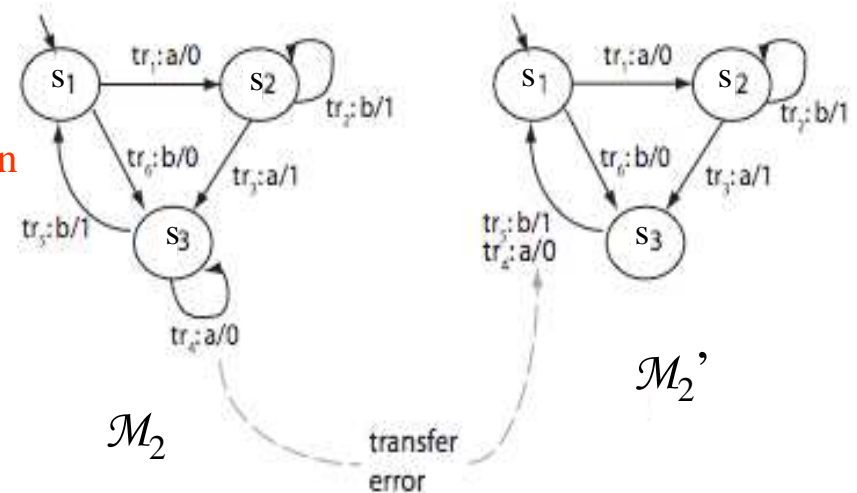
- Does it reveal the error?

No

- Are the states in  $\mathcal{M}_2$  1-distinguishable?

YES

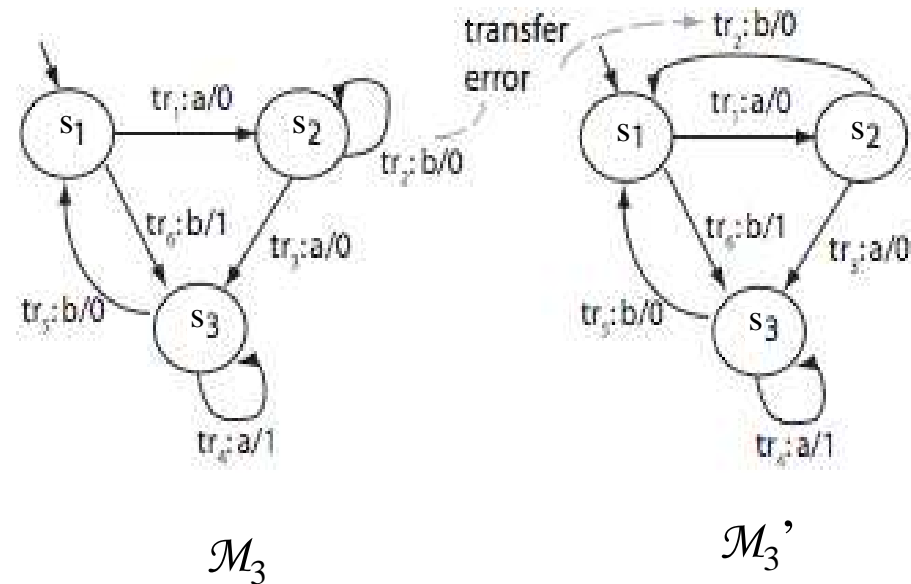
For each pair of states  $(s_i, s_j)$ ,  $i \neq j$ , there exists a string of length 1 that distinguishes  $s_i$  from  $s_j$



Refer to Example 3.38

# Control-Flow-Based Techniques (5)

- Consider the following machines, a correct one ( $\mathcal{M}_3$ ) and one with a transfer error ( $\mathcal{M}_3'$ )
  - There are two loops in  $\mathcal{M}_3$  one in state  $s_2$  and the other in  $s_3$
- Consider  $T = \{t_1: aab, t_2: abaab\}$ 
  - $T$  is adequate with respect to the **boundary-interior cover criterion**
  - $t_1$  causes both loops to exit without looping in either  $s_2$  or  $s_3$
  - $t_2$  causes each loop to be traversed once
- Is the error revealed by  $T$ ?
  - $g_{\mathcal{M}_3}(s_1, t_1) = g_{\mathcal{M}_3'}(s_1, t_1) = 000$
  - $g_{\mathcal{M}_3}(s_1, t_2) = g_{\mathcal{M}_3'}(s_1, t_2) = 00010$
  - $T$  cannot distinguish  $\mathcal{M}_3$  from  $\mathcal{M}_3'$  and hence does not reveal the error



# Summary

- Behavior of a large variety of applications can be modeled using finite state machines (FSM)
- The W method is an automata theoretic method to generate tests from a given FSM model
- Tests generated by using the W method are guaranteed to detect all operation errors, transfer errors, and missing/extra state errors in the implementation given that the FSM representing the implementation is complete, connected, and minimal.
  - What happens if it is not?
- Automata theoretic techniques generate tests superior in their fault detection ability than their control-theoretic counterparts