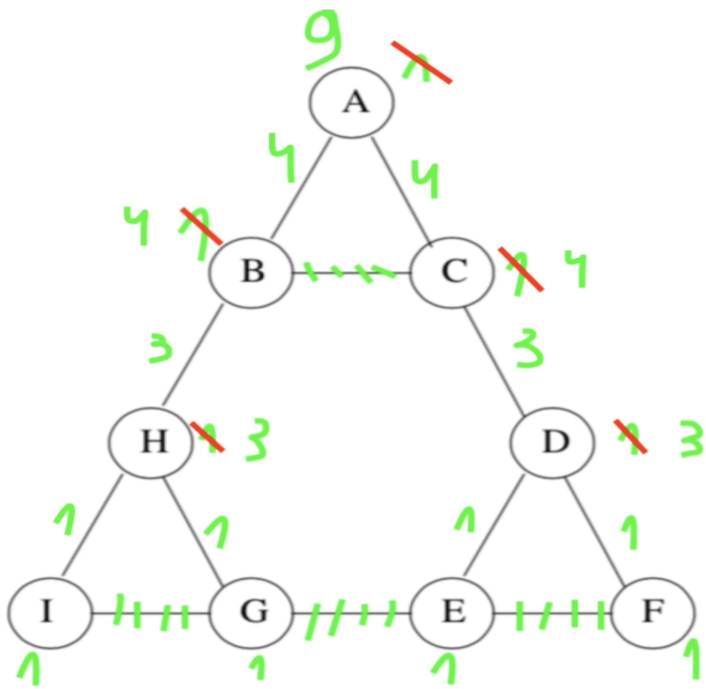
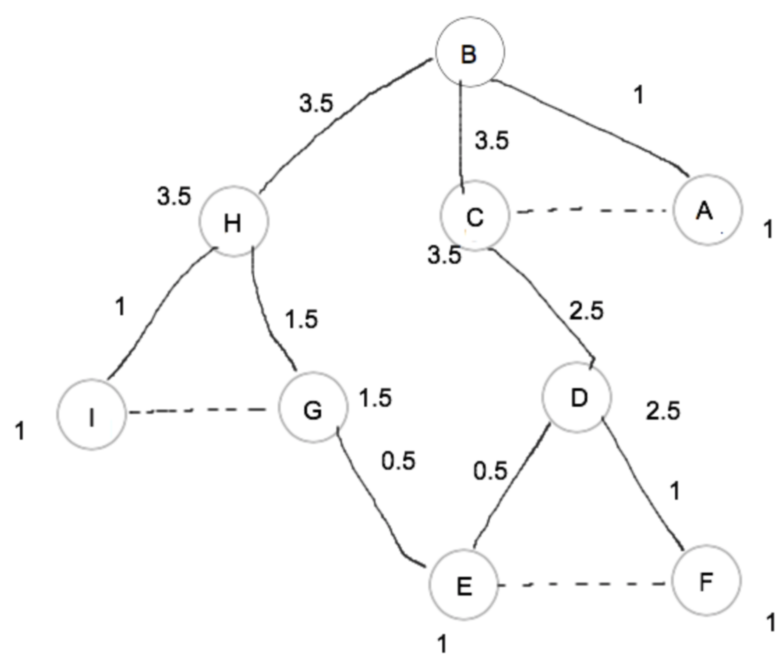
1. **Exercise 10.2.1**: Figure 10.9 is an example of a social-network graph. Use the Girvan-Newman approach to find the number of shortest paths from each of the following nodes that pass through each of the edges. (a) A (b) B.

   a)



   b)



2. **Exercise 10.2.2**: Using symmetry, the calculations of Exercise 10.2.1 are all you need to compute the betweenness of each edge. Do the calculation.

   Voor iedere node als root gaan we 1 van de 2 bovenste trees gebruiken. Dus neem een node als root, probeer deze te mappen op 1 van de 2 trees uit vorige oefening.
   (A,B)

   (A): 4
   (B): 1
   (C): /
   (D): /
   (E): /
   (F): /
   (G): 1
   (H): 1
   (I): 1
   (4 + 4 * 1) / 2 = 4

   we hebben nu 1 boog maar moeten alle bogen bekijken. Door de symetrie weten we dat (A,B) = (A,C) = (I,H) = (I,G) = (F,E) = (F,D)
   (G,E) = (B,H) = (C,D) = 9,5 (zelf nog berekenen, tijdens responsie werd enkel deze uitkomst gegeven)
   (B,C) = (H,G) = (D,E) = 6,5

3. **Exercise 10.2.3**: Using the betweenness values from Exercise 10.2.2, deter- mine reasonable candidates for the communities in Fig. 10.9 by removing all edges with a betweenness above some threshold.

   We hebben een treshhold groter als 6,5 en kleiner dan 9,5 nodig om de 3 hoeken samen te krijgen.

4. **Exercise 10.4.1** : For the graph of Fig. 10.9, construct:
   (a) The adjacency matrix.
   (b) The degree matrix.
   (c) The Laplacian matrix.

$$a = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$c = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 3 & -1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

5. **Exercise 10.5.1**: Suppose graphs are generated by picking a probability p and choosing each edge independently with probability p, as in Example 10.21. For the graph of Fig. 10.20, what value of p gives the maximum likelihood of seeing that graph? What is the probability this graph is generated?

   $\binom{4}{2} = 6$ mogelijke bogen (combinatie)
   we willen 4 van de 6 bogen, dus hebben we 4/6 nodig → p = 2/3.
   $P(G) = p^4 * (1-p)^2 = 16/729$

6. **Exercise 10.5.3**: Suppose we have a coin, which may not be a fair coin, and we flip it some number of times, seeing h heads and t tails.
   (a)If the probability p of getting a head on any flip is p, what is the MLE for p, in terms of h and t?

   $p^k * (1-p)^t$ voor het maximum te weten moeten we afleiden.
   $h * p^{h-1} * (1-p)^t + p^h * (-t) * (1-p)^{t-1} = 0$
   $p = h/(h+t)$