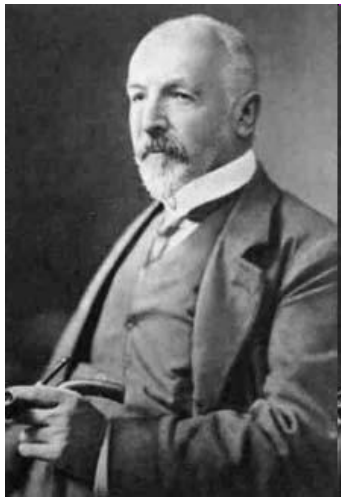
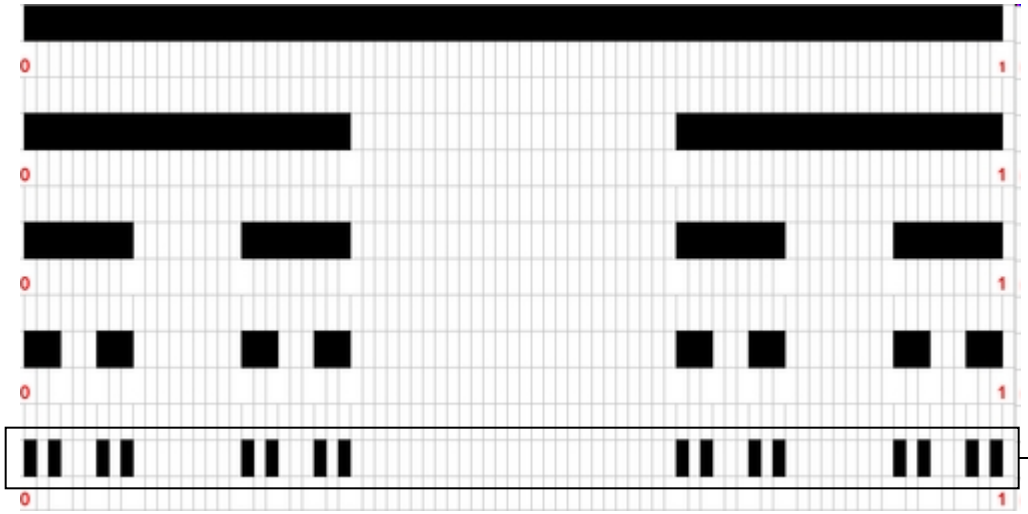
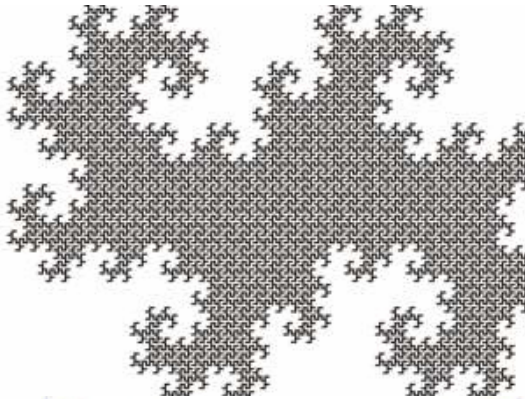
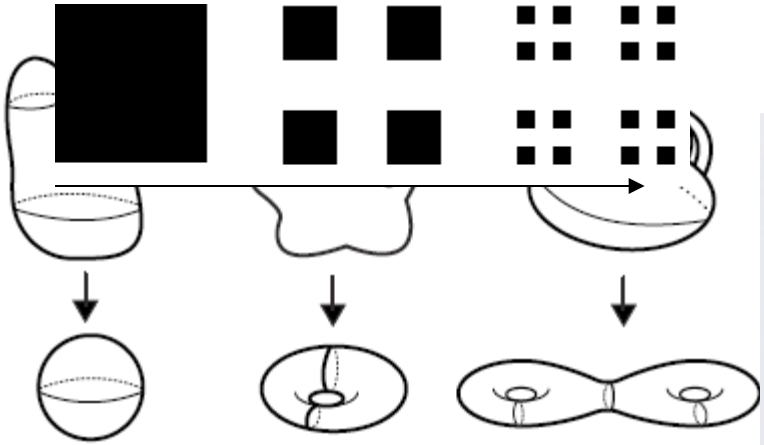
Fractais: Dimensões Generalizadas, Espectros e Expoentes de Escala



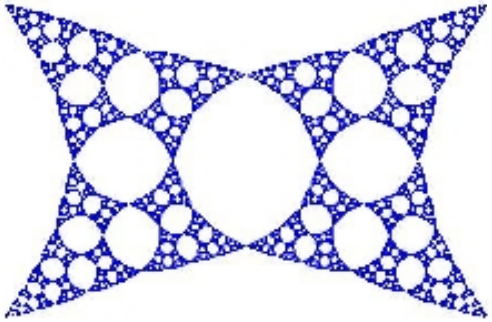
G. Cantor, 1883



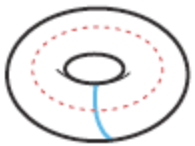


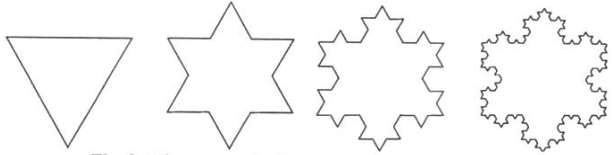


Co-dimension: **d^*=D-d**

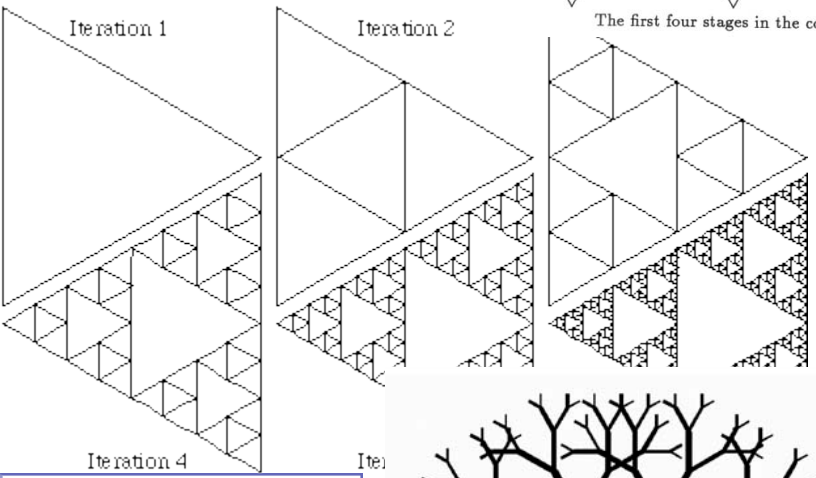


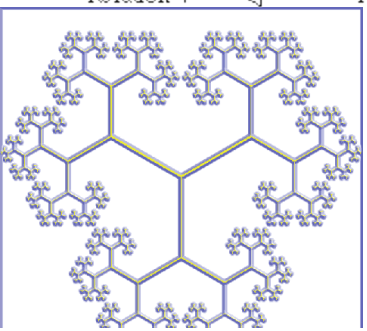
Topologias Euclidianas
(Geometrias com D∈ℵ)

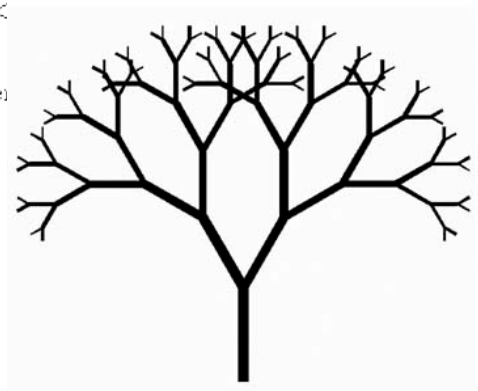


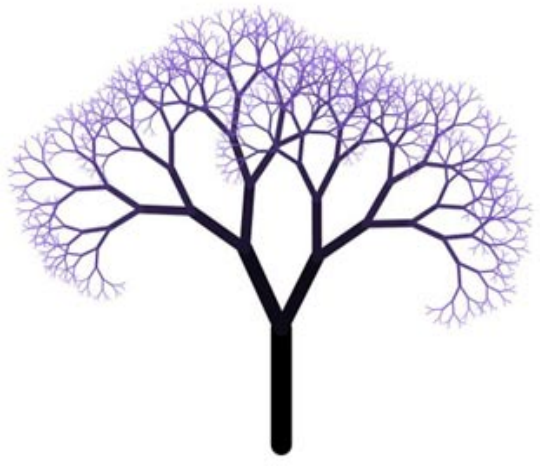


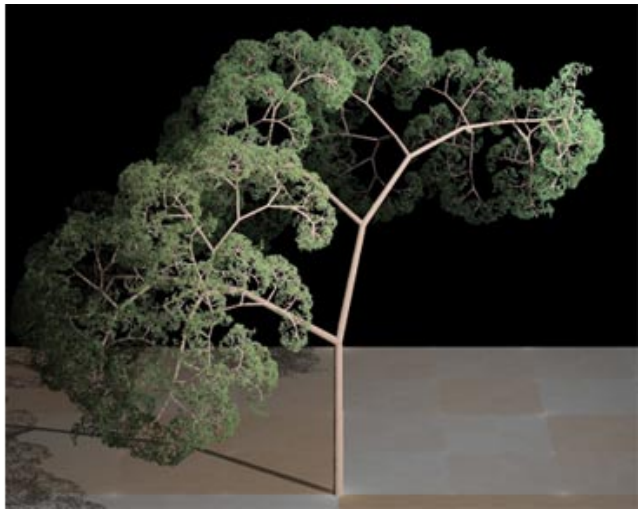
The first four stages in the construction of the Koch snowflake

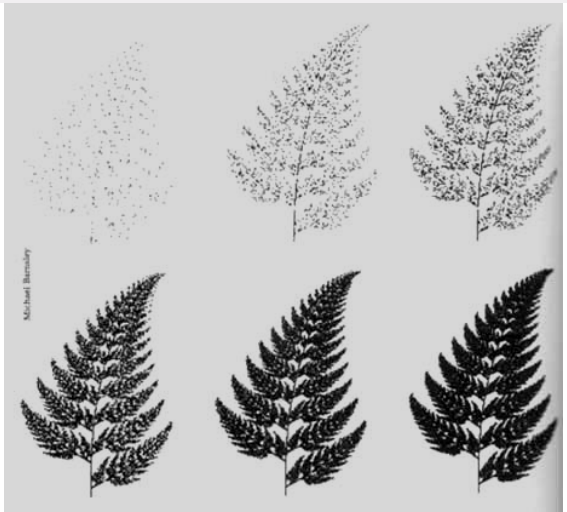


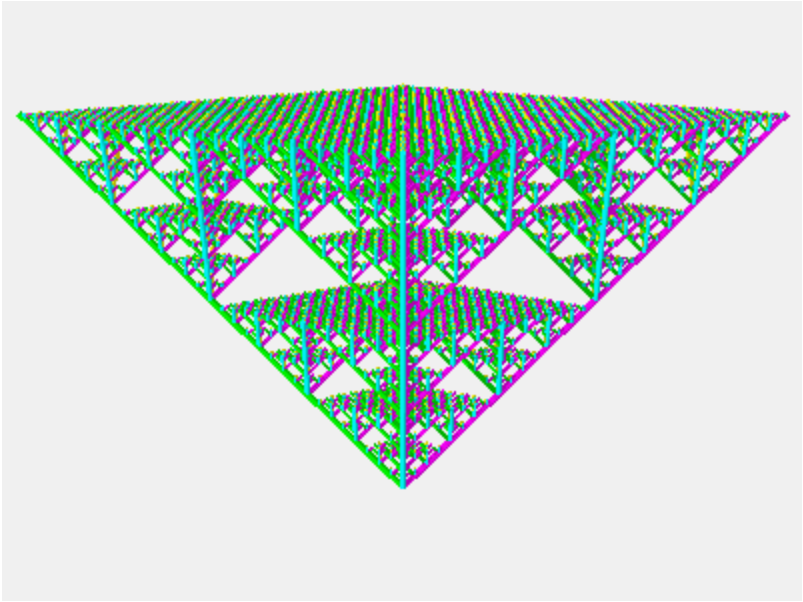










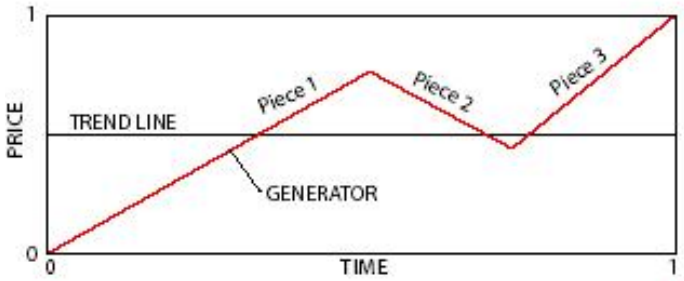


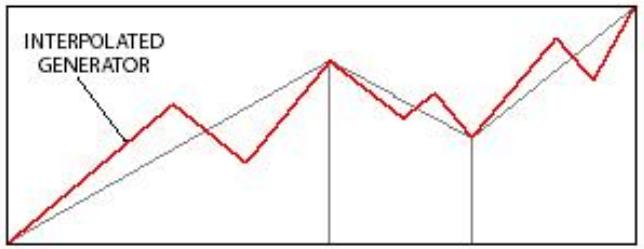
Felix Hausdorff, 1918

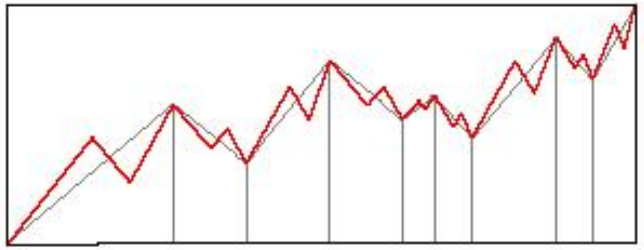
Hausdorff–Besicovitch dimension

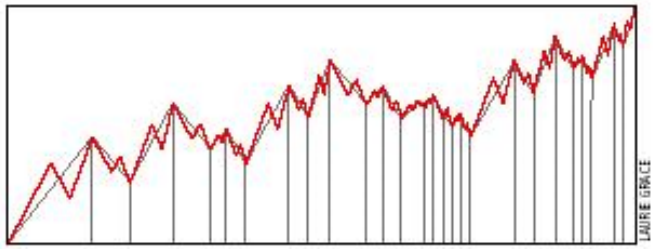
Topologias de geometrias não-euclidianas

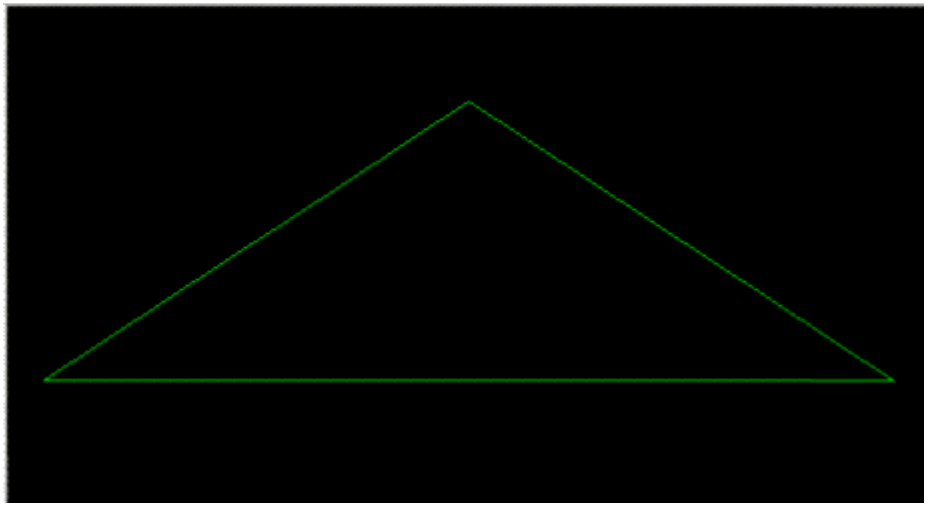
A. S. Besicovitch, *On Linear Sets of Points of Fractional Dimensions*, Mathematische Annalen 101 (1929).

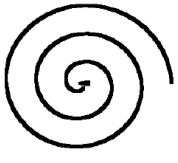


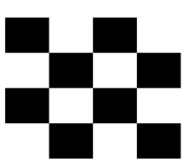




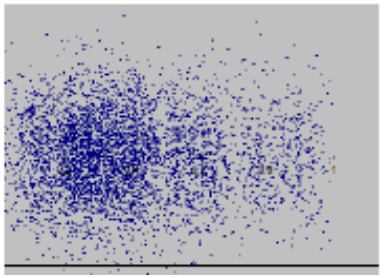




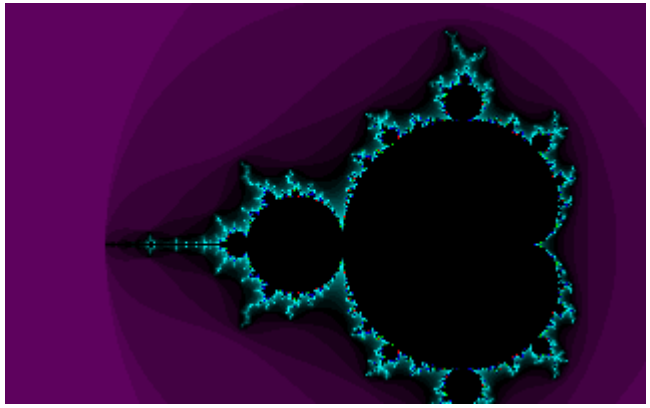




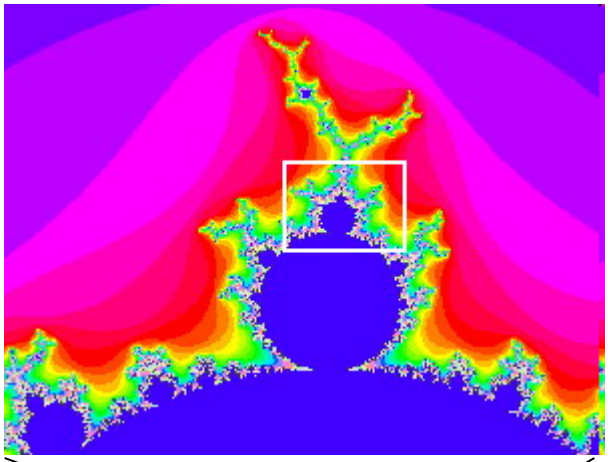
1 2 1.7

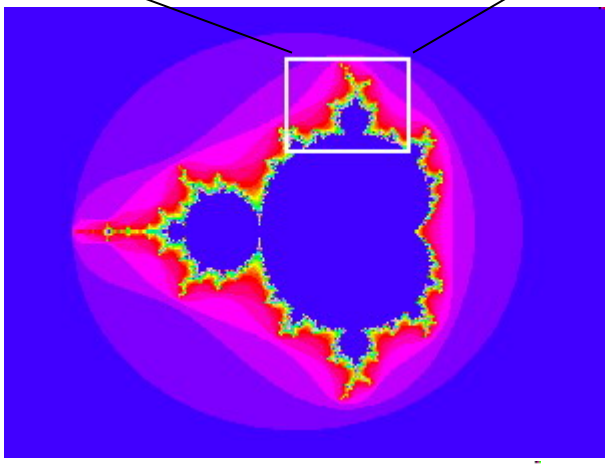


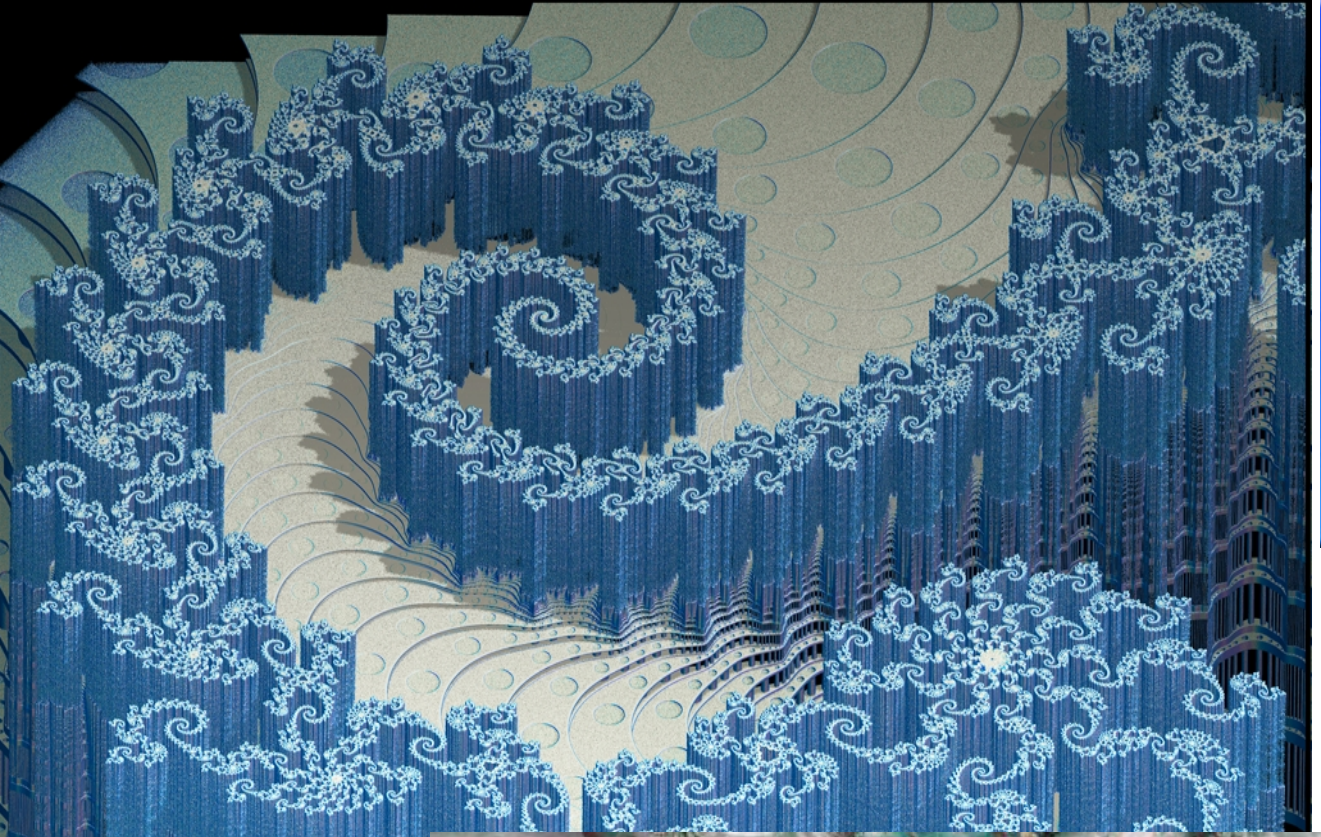
Felix Hausdorff, 1918

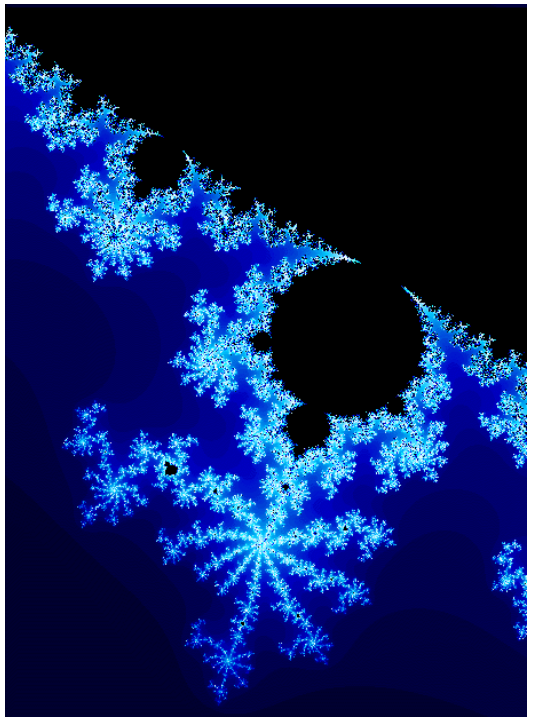


Benoit Mandelbrot, 1975



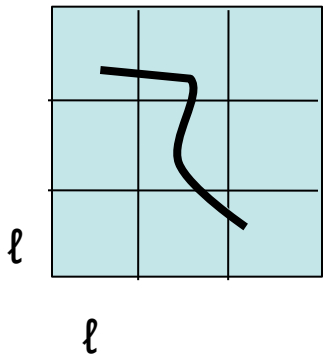






Dimensão de Hausdorff:

N *sub-sets* de escala ℓ para cobrir qualquer conjunto finito de pontos no $\Re_n$: $N(\ell) \sim K\,\ell^{-D}$



➔ $D = \log N(\ell) / \log \ell$

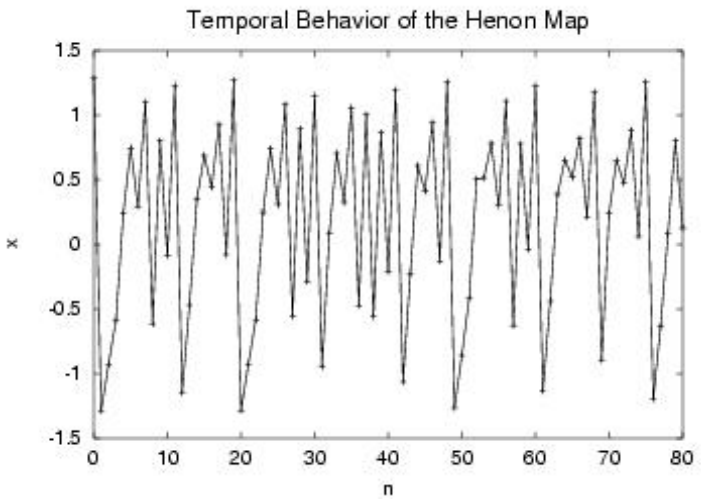
- Expoente de Lyapunov : $|\delta Z(t)| \approx \exp(\lambda t)\,|\delta Z_0|$

- $M-1 < D < M$
- $\exists$ M expoentes λ

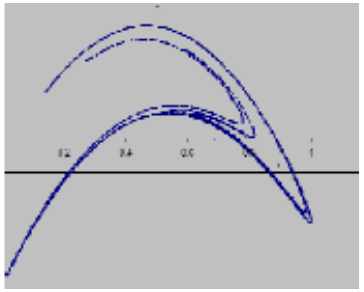
$x_{n+1} = 1 + y_n - a\,x_n^2$

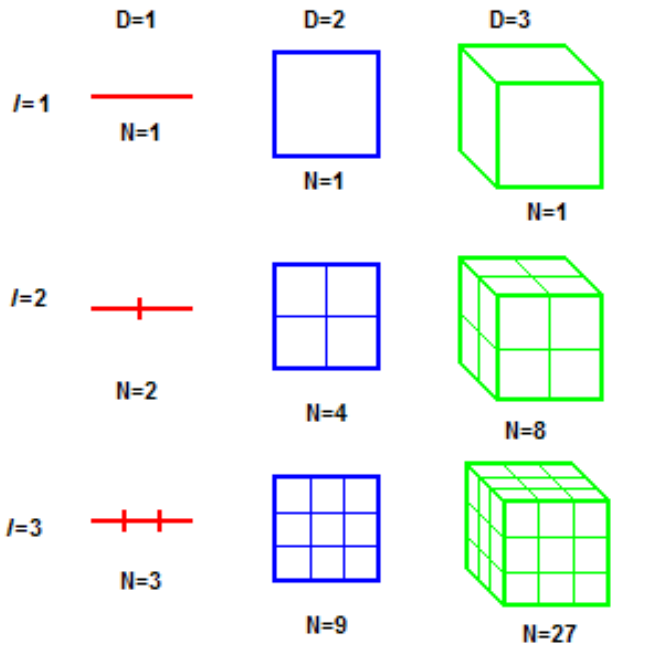
$y_{n+1} = b\,x_n$

a=1.4; b=0.3

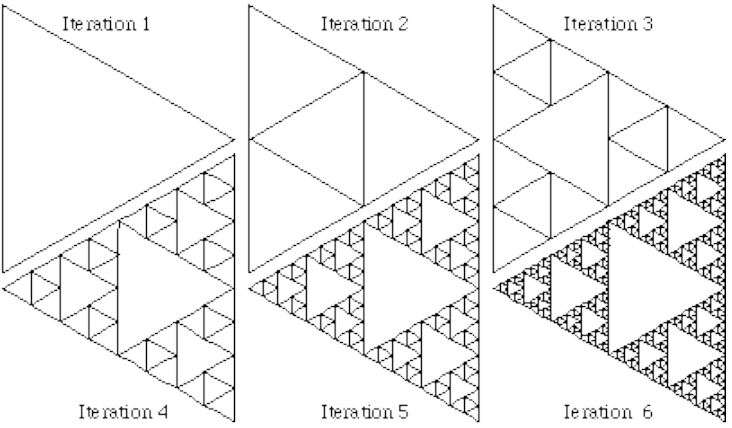


Ex. Mapa de Henon
M=2 e 1.25 < D < 1.45





Dimensão de Hausdorff: $D = \frac{\log N(l)}{\log l}$



Dimensão Generalizada de Rényi:

$$D_\alpha = \lim_{\epsilon \to 0} \frac{\frac{1}{1-\alpha} \log\left(\sum_i p_i^\alpha\right)}{\log \frac{1}{\epsilon}}$$

Dimensão de Similaridade (Fractal): $D_0 = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log \frac{1}{\epsilon}}$

Dimensão de Informação: $D_1 = \lim_{\epsilon \to 0} \frac{-\langle \log p_\epsilon \rangle}{\log \frac{1}{\epsilon}}$

Dimensão de Correlação: $D_2 = \lim_{\epsilon \to 0, M \to \infty} \frac{\log(g_\epsilon / M^2)}{\log \epsilon}$

Propriedades:

- $D_0 \geq D_1 \geq D_2 \geq \ldots \geq D_\infty$
- $D_0 > D_1 > D_2 > \ldots > D_\infty$ ➔ ***Multifractal***

Ex: Tapete de Sierpinski

$$D = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log\left(\frac{1}{\epsilon}\right)} = \lim_{k \to \infty} \frac{\log 3^k}{\log 2^k} = \frac{\log 3}{\log 2} \approx 1.585$$

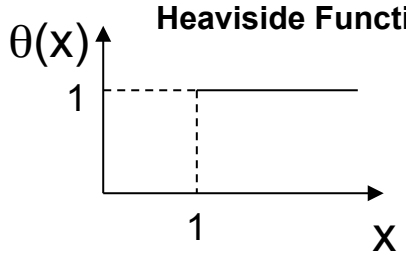
Cálculo Computacional da Dimensão de Correlação D_3=D

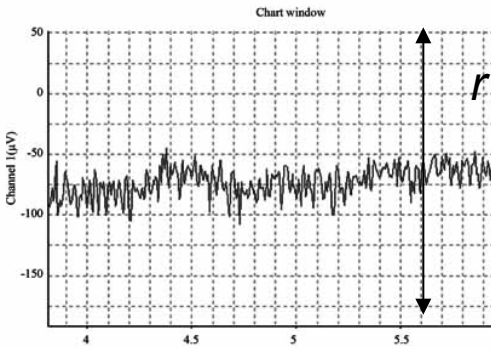
http://www.scholarpedia.org/article/Grassberger-Procaccia_algorithm

Peter Grassberger, 1980

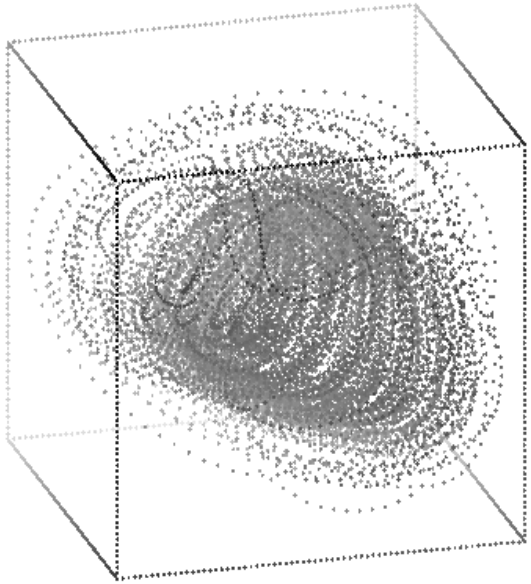
$\exists$ C (r) | C (r) ~ r ^{D} ➔ $D = \lim_{r \to 0} \frac{\log C(r)}{\log r}$

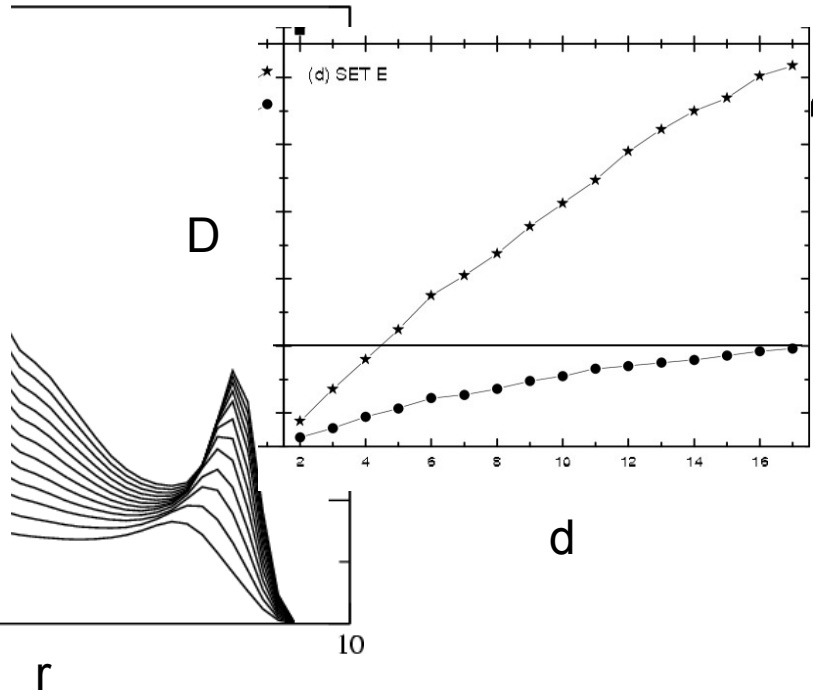
$$\hat{C}(r) = \frac{2}{N(N-1)} \sum_{i<j} \theta(r - |\mathbf{x}_i - \mathbf{x}_j|),$$





Δt





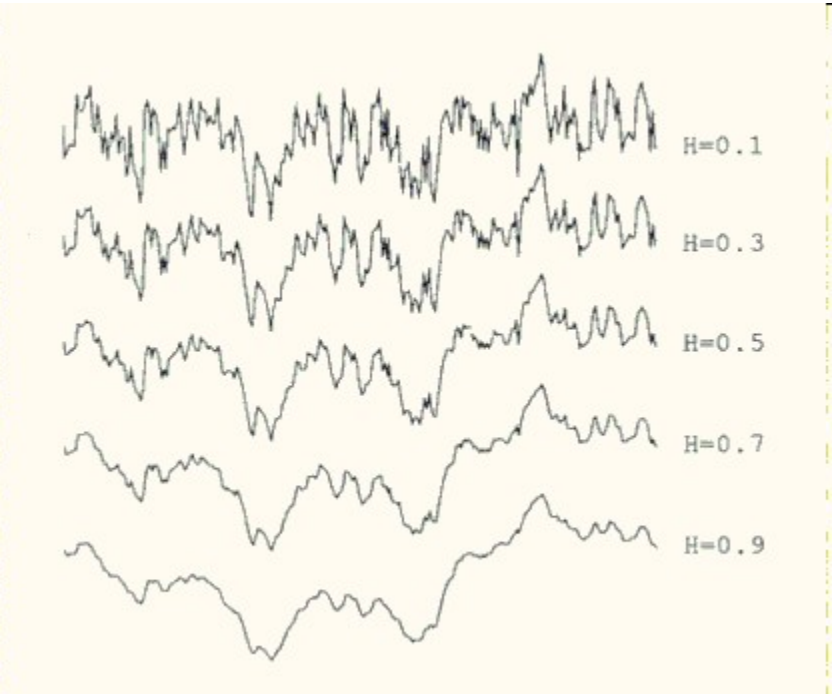
Ver Artigo: Campanharo et al.

$\exists$ C (r) | C (r) ~ r ^{D} ➔ $D = \lim_{r \to 0} \frac{\log C(r)}{\log r}$

D: para ST, difícil calcular!

Harold Edwin **Hurst** (1880-1978) and Ludwig Otto **Hölder** (1859-1937) (1965):

$S_q = \langle | A(t+\tau) - A(\tau) |^{q} \rangle \; T \sim \tau^{q\,H(q)}$; $T >> \tau$ ➔ **0<H<1 (domínio geral)**



D = 2 – H e α = 2 H + 1

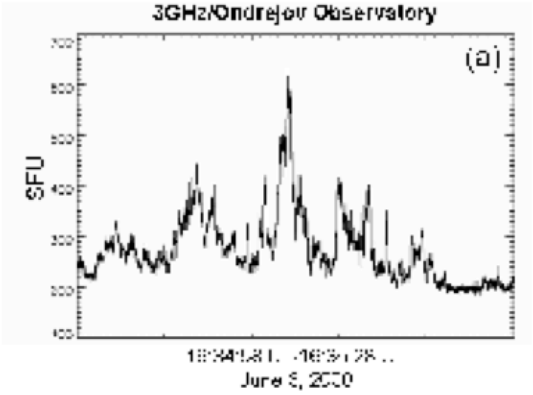
Na Prática:

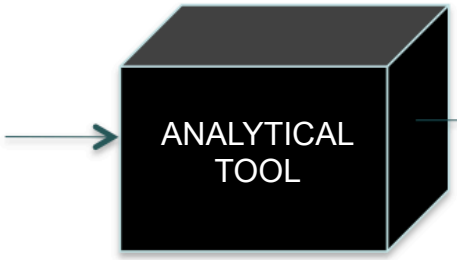
1. Obter β (powspecscale.m)
2. Obter α (dfa.m)
3. Conferir Relação entre β e α
4. Estimar H
5. Estimar D
6. Conferir Confiabilidade (*bias-N*)

Critério de Tsonis: *N* ~ 10 $^{(2 + 0.4D)}$

Caracterizações

V - OK? - F → Descartar ou aumentar N





STATISTICAL PARAMETERS (PDF MODELS)
and / or
SPECTRAL INDICES

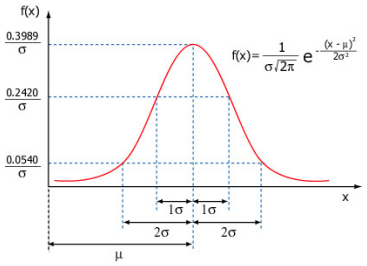
Parte 1:
Análise Estatística

Parte 2:
Análise Espectral

Fundamental

Teorema do Limite Central

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

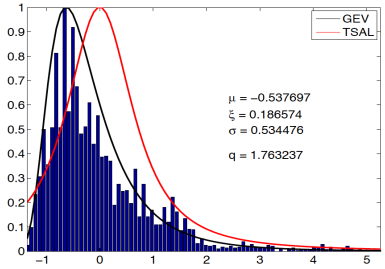


Avançada

Teorema do Valor Extremo

$$G_{(\xi,\sigma,\mu)}(A) = \begin{cases} 1-\left(1+\dfrac{\xi(A-\mu)}{\sigma}\right)^{-\frac{1}{\xi}} & p/\ \xi \neq 0 \\ 1-exp\left(-\dfrac{A-\mu}{\sigma}\right) & p/\ \xi = 0 \end{cases}$$

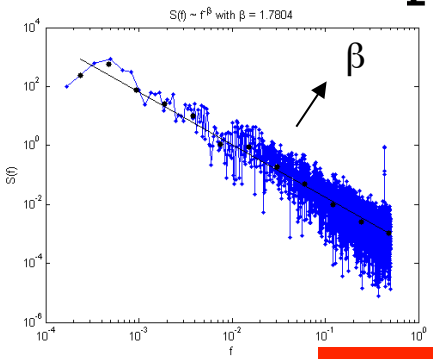
$\mu \in (-\infty, \infty)$ *localização*
$\sigma \in (0, \infty)$ *escala*
$\xi \in (-\infty, \infty)$ *forma*



Fundamental

Teorema de Wiener-Khinchin

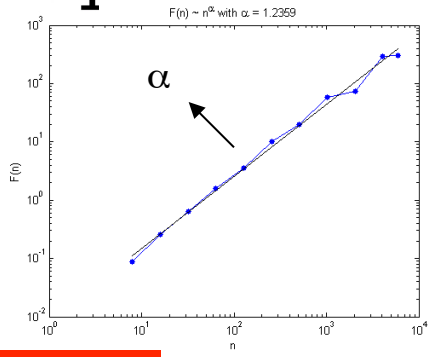
PSD = FFT [C(L)]



Avançada

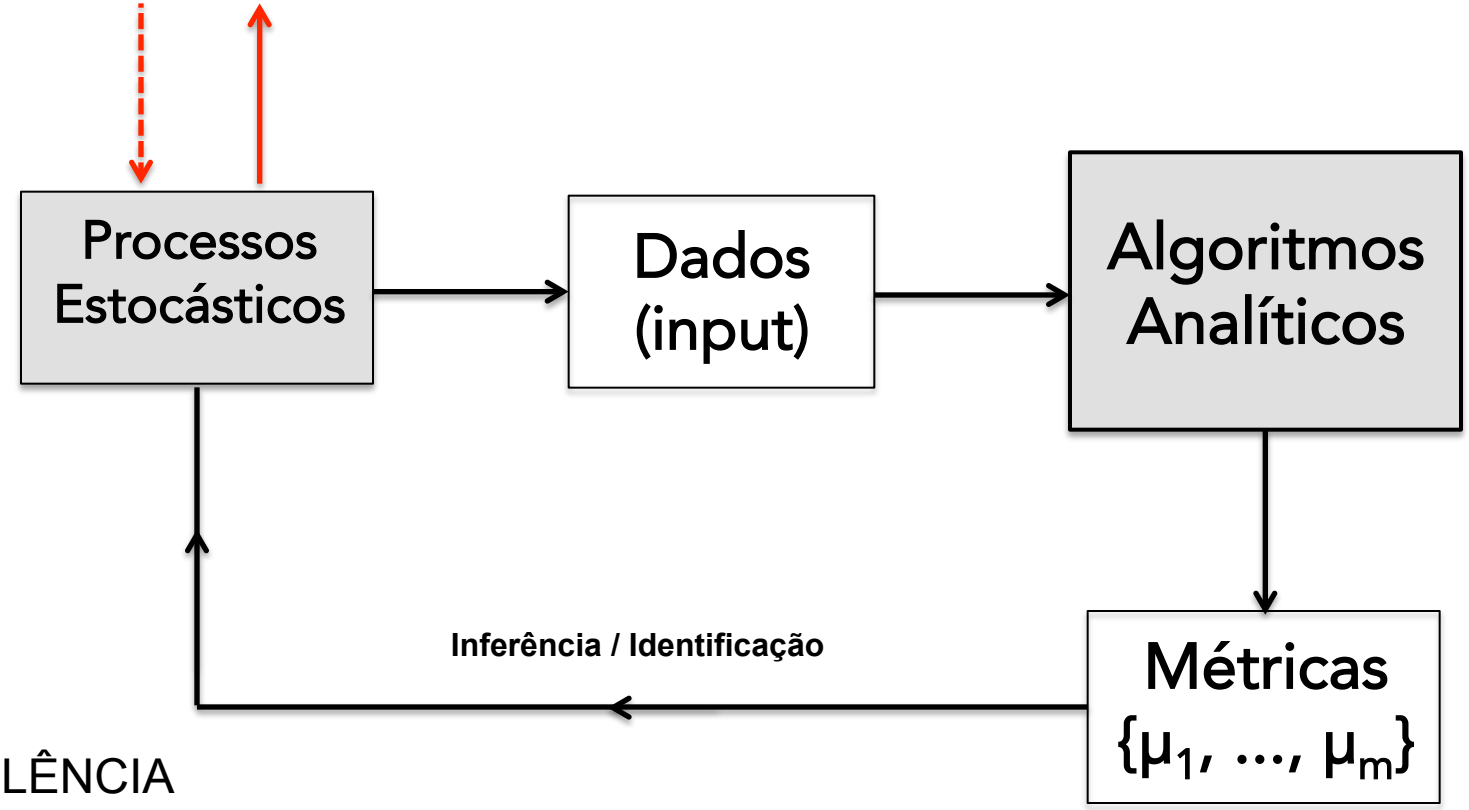
Teorema de Parceval

DFAS = FH [A(t)]



α= (β + 1) / 2

PROCESSOS DINÂMICOS
SUBJACENTE (TEORIAS/MODELOS)



EX: TURBULÊNCIA