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EXTREME EVENTS IN NATURE AND SOCIETY

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5 Endogenous versus Exogenous Origins of Crises

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Summary. Are large biological extinctions such as the Cretaceous/Tertiary KT boundary due to a meteorite, extreme volcanic activity or self-organized critical extinction cascades? Are commercial successes due to a progressive reputation cascade or the result of a well orchestrated advertisement? Determining the chain of causality for Xevents in complex systems requires disentangling interwoven exogenous and endogenous contributions with either no clear signature or too many signatures. Here, I review several efforts carried out with collaborators which suggest a general strategy for understanding the organizations of several complex systems under the dual effect of endogenous and exogenous fluctuations. The studied examples are: internet download shocks, book sale shocks, social shocks, financial volatility shocks, and financial crashes. Simple models are offered to quantitatively relate the endogenous organization to the exogenous response of the system. Suggestions for applications of these ideas to many other systems are offered.

5.1 Introduction

Xevents are pervasive in all natural and social systems: earthquakes, volcanic eruptions, hurricanes and tornadoes, landslides and avalanches, lightning strikes, magnetic storms, catastrophic events of environmental degradation, failure of engineering structures, crashes in the financial stock markets, social unrests leading to large-scale strikes and upheaval and perhaps to revolutions, economic drawdowns on national and global scales, regional and national power blackouts, traffic gridlocks, diseases and epidemics, and so on.

Can we forecast them, manage, mitigate or prevent them? The answer to these questions requires us to investigate their origin(s).

Self-organized criticality, and more generally, complex system theory contends that out-of-equilibrium slowly driven systems with threshold dynamics relax through a hierarchy of avalanches of all sizes. Accordingly, Xevents are seen to be endogenous [1, 2], in contrast with previous prevailing views. In addition, the preparation processes before large avalanches are almost undistinguishable from those before small avalanches, making the prediction of the former seemingly impossible (see [3] for a discussion). But how can one assert with 100% confidence that a given Xevent is really due to an endogenous self-organization of the system, rather than to the response to an external shock? Most natural and social systems are indeed continuously subjected to external stimulations, noises, shocks, solicitations, forcing, all of which can widely vary in amplitude. It is thus not clear a priori whether a given large event is due to a strong exogenous shock, to the internal dynamics of the system organizing in response to the continuous flow of small solicitations, or maybe to a combination of both. Addressing this question is fundamental to gaining an understanding of the relative importance of self-organization versus external forcing in complex systems and for the understanding and prediction of crises.

This leads to two questions:

1. Are there distinguishing properties that characterize endogenous versus exogenous shocks?
2. What are the relationships between endogenous and exogenous shocks?

Actually, the second question has a long tradition in physics. It is at the basis of the interrogations that scientists perform on the enormously varied systems they study. The idea is simple: subject the system to a perturbation, a "kick" of some sort, and measure its response as a function of time, of the nature of the solicitations and of the various environmental factors that can be controlled. In physical systems at thermodynamic equilibrium, the answer is known as the theorem of fluctuation-dissipation, sometimes also referred to as the theorem of fluctuation-susceptibility [4]. In a nutshell, this theorem relates quantitatively, in a very precise way, the response of the system to an instantaneous kick (exogeneous) to the correlation function of its spontaneous fluctuations (endogenous). An early example of this relationship is found in Einstein's relation between the diffusion coefficient D of a particle in a fluid subjected to the chaotic collisions of the fluid molecules and the coefficient η of viscosity of the fluid [5,6]. The coefficient η controls the drag; the response of the particle velocity when subjected to an exogenous force impulse. The coefficient D can be shown to be a direct measure of the (integral of the) correlation function of the spontaneous (endogenous) fluctuations of the particle velocity.

In out-of-equilibrium systems, the existence of a relationship between the response function to external kicks and spontaneous internal fluctuations has not been settled [7]. In many complex systems, this question amounts to distinguishing between endogeneity and exogeneity and is important for understanding the relative effects of self-organization versus external impacts. This is difficult in most physical systems because externally imposed perturbations may lie outside the complex attractor, which itself may exhibit bifurcations. Therefore, observable perturbations are often misclassified.

It is thus interesting to study other systems in which the dividing line between endogenous and exogenous shocks may be clearer in the hope that it will lead to insights into complex physical systems. The investigations of the two questions above may also bring a new understanding of these systems.

The systems to which the endogenous-exogenous question (which we will refer to as "endo-exo" for short) is relevant include the following:

– Biological extinctions, such as the Cretaceous/Tertiary KT boundary (meteorite versus extreme volcanic activity (Deccan traps) versus self-organized critical extinction cascades)
– Immune system deficiencies (external viral/bacterial infections versus internal cascades of regulatory breakdowns)
– Cognition and brain learning processes (role of external inputs versus internal self-organization and reinforcements)
– Discoveries (serendipity versus the outcome of slow endogenous maturation processes)
– Commercial successes (progressive reputation cascade versus the result of a well orchestrated advertisement)
– Financial crashes (external shocks versus self-organized instability)
– Intermittent bursts of financial volatility (external shocks versus cumulative effects of news in a long-memory system)
– The aviation industry recession (9/11/2001 terrorist attack versus structural endogenous problems)
– Social unrests (triggering factor or decay of social fabric)
– Recovery after wars (internally generated (civil wars) versus imported from the outside) and so on

It is interesting to mention that the question of exogenous versus endogenous forcing has been hotly debated in economics for decades. A prominent example is the theory of Schumpeter on the importance of technological discontinuities in economic history. Schumpeter argued that "evolution is lopsided, discontinuous, disharmonious by nature ... studded with violent outbursts and catastrophes ... more like a series of explosions than a gentle, though incessant, transformation" [8]. Endogeneity versus exogeneity is also paramount in economic growth theory [9]. Our analyses, reviewed below, suggest a subtle interplay between exogenous and endogenous shocks, which may cast a new light on this debate.

In the following, we review the works of the author with his collaborators, in which the endo-exo question is investigated in a variety of systems.

5.2 Exogenous and Endogenous Shocks in Social Networks

One defining characteristics of humans is their organization in social networks. It is probable that our large brains have been shaped by social interactions, and may have co-evolved with the size and complexity of social groups [10, 11]. A single individual may belong to several intertwined social networks, associated with different activities (work colleagues, college alumni societies, friends, family members, and so on). The formation and the evolution of social networks and their mutual entanglements control the hierarchy of interactions between humans, from the individual level to society and to culture. In this section, we review a few original probes of several social networks which unearth a remarkable universality: the distribution of human decision times in social networks seem to be described by a power law $1/t^{1+\theta}$ with $\theta = 0.3 \pm 0.1$. This constitutes an essential ingredient in models describing how the cascade of agent decisions leads to the bottom-up organization of the response of social systems. We first present such a model in terms of a simple epidemic process of word-of-mouth effects [12–14] and then discuss the different data sets.

5.2.1 A Simple Epidemic Cascade Model of Social Interactions

Let us consider an observable characterizing the activity of humans within a given social network of interactions. This activity can be the rate of visits or downloads on an internet website, the sales of a book or the number of newpaper articles on a given subject.

We envision that the instantaneous activity results from a combination of external forces such as news and advertisement, and from social influences in which each past active individual may prompt other individuals in her network of acquaintances to act. This impact of an active individual on other humans is not instantaneous, as people react on a variety of timescales. The time delays capture the time interval between social encounters, the maturation of the decision process, which can be influenced by mood, sentiments, and many other factors and the availability and capacity to implement the decision. We postulate that this latency can be described by a memory kernel $\phi(t - t_i)$, giving the probability that an action at time t_i leads to another action at a later time t by another person in direct contact with the first active individual. We consider the memory function $\phi(t - t_i)$ as a fundamental macroscopic description of how long it takes for a human to be triggered into action, following the interaction with an already active human.

Then, starting from an initial active individual (the "mother") who first acts (either from exogenous news or by chance), she may trigger actions by first-generation "daughters," which themselves prompt the actions of their own friends, who become second-generation active individuals, and so on. This cascade of generations can be shown to renormalize the memory kernel $\phi(t - t_i)$ into a dressed or renormalized memory kernel $K(t - t_i)$ [12, 13, 15], giving the probability that an action at time t_i leads to another action by another person at a later time t through any possible generation lineage. In physical terminology, the renormalized memory kernel $K(t)$ is nothing but the response function of the system to an impulse. This is captured by the following equations:

$$A(t) = s(t) + \int_{-\infty}^{t} d\tau\ A(\tau)\ \phi(t - \tau) = \int_{-\infty}^{t} d\tau\ s(\tau)\ K(t - \tau)\ . \tag{5.1}$$

The meaning of these two equivalent formulations is as follows. The $s(t)$'s are the spontaneous exogenous activations. The integral $\int_{-\infty}^{t} d\tau\ A(\tau)\ \phi(t-\tau)$ gives the additional contribution due to past activities $A(\tau)$, whose influences on the present are mediated by the direct influence kernel ϕ of the first generation. The last integral $\int_{-\infty}^{t} d\tau\ s(\tau)\ K(t-\tau)$ expresses the fact that the present activity $A(t)$ can also be seen as resulting from all past exogenous sources $s(\tau)$ mediated to the present by the renormalized kernel K, which takes into account all of the generations of cascades of influences.

The following functional dependence is found to provide an accurate description, as we shall discuss below:

$$K(t) \sim 1/(t-t_c)^p\ , \quad \text{with}\ \ p = 1-\theta\ . \tag{5.2}$$

The dependence (5.2) implies that ([12, 13, 15]):

$$\phi(t) \sim 1/(t-t_c)^{1+\theta}\ . \tag{5.3}$$

We should stress that the renormalization from the usually (but not always) unobservable "bare" response function $\phi(t)$ with exponent $1+\theta$ in (5.3) to the observable "renormalized" response function $K(t)$ in (5.2) with exponent $1-\theta$ is obtained if the network is close to critical; in other words if the average branching ratio n is close to 1 (n is defined as the average number of daughters of the first generation per mother). In other words, there is on average approximately one triggered daughter per active mother. This condition of criticality ensures, in the language of branching processes, that avalanches of active people triggered by a given mother are self-similar (power law distributed). In contrast, for $n < 1$, the cascade of triggered actions is "sub-critical" and avalanches die off more rapidly. It can be shown [12, 13, 15] that in this case there is a characteristic timescale

$$t^* \sim \frac{1}{(1-n)^{1/\theta}} \tag{5.4}$$

acting like a correlation time, which separates two regimes:

- for $t < t^*$, the renormalized response function $K(t)$ is indeed of the form (5.2);
- for $t > t^*$, the renormalized response function $K(t)$ crosses over to an asymptotic decay with exponent $1+\theta$, of the form of $\phi(t)$ in (5.3).

For $n > 1$, the epidemic process is supercritical and has a finite probability of growing exponentially. We will not be concerned with this last regime, which does not seem relevant in the data discussed below.

In the absence of strong external influences, a peak in social activity can occur spontaneously due to the interplay between a continuous stochastic flow of small external news and the amplifying impact of the epidemic cascade of social influences. It can then be shown that, for n close to 1 or equivalently for $|t - t_c| < t^*$, the average growth of the social activity before such an "endogenous" peak and the relaxation after the peak are proportional to [13,16]

$$\int_0^{+\infty} K(t - t_c + u) K(u) du \sim 1/|t - t_c|^{1-2\theta} , \tag{5.5}$$

where the right-hand-side of the expression holds for $K(t)$ of the form (5.2). The prediction that the relaxation following an exogeneous shock should happen faster (larger exponent $1 - \theta$) than for an endogeneous shock (with exponent $1 - 2\theta$) agrees with the intuition that an endogeneous shock should have inpregnated the network much more and should thus have a longer lived influence. In a nutshell, the mechanism producing the endogenous response function (5.5) is the constructive interference of accumulated small news cascading through the social influence network. In other words, the presence of a hierarchy of nested relaxations $K(t)$ given by (5.2), each one associated with each small news, creates the effective endogenous response (5.5).

Dodds and Watts have recently introduced a general contagion model which, by explicitly incorporating memories of past exposures to, for example, an infectious agent, a rumour, or a new product, includes the main features of existing contagion models and interpolates between them [17].

5.2.2 Internet Download Shocks

In [18], Johansen and Sornette report the following experiment. The authors were interviewed by a journalist from the leading Danish newspaper Jyllands Posten on a subject of rather broad interest, namely stock market crashes. The interview was published on April 14, 1999 in both the paper version of the newspaper as well as in the electronic version (with access restricted to subscribers) and included the URLs where the authors' research papers on the subject could be retrieved. It was hence possible to monitor the number of downloads of papers as a function of time since the publication date of the interview. The rate of downloads of the authors' papers as a function of time was found to obey a $1/t^p$ power law, with exponent $b = 0.58 \pm 0.03$, as shown in Fig. 5.1.

Within the model of epidemic word-of-mouth effect summarized in Sect. 5.2.1, the relaxation of the rate of downloads after the publication of the interview characterizes the response function $K(t)$ given by (5.2) with respect to an exogenous peak: prior to the publication of the interview, the rate of downloads was slightly less than one per day; it suddenly jumped to several tens of downloads per day in the first few days after the publication and then relaxed slowly according to (5.2). The reported power law with exponent $p \simeq 0.6$ is compatible with the form of (5.2) with $\theta = 0.4$, which is within the range of other values: $\theta = 0.3 \pm 0.1$.

Johansen [19] has reported another similar observation following another web interview on stock market crashes, which contained the URL of his articles on the subject. He again found a power law dependence (5.2), but with

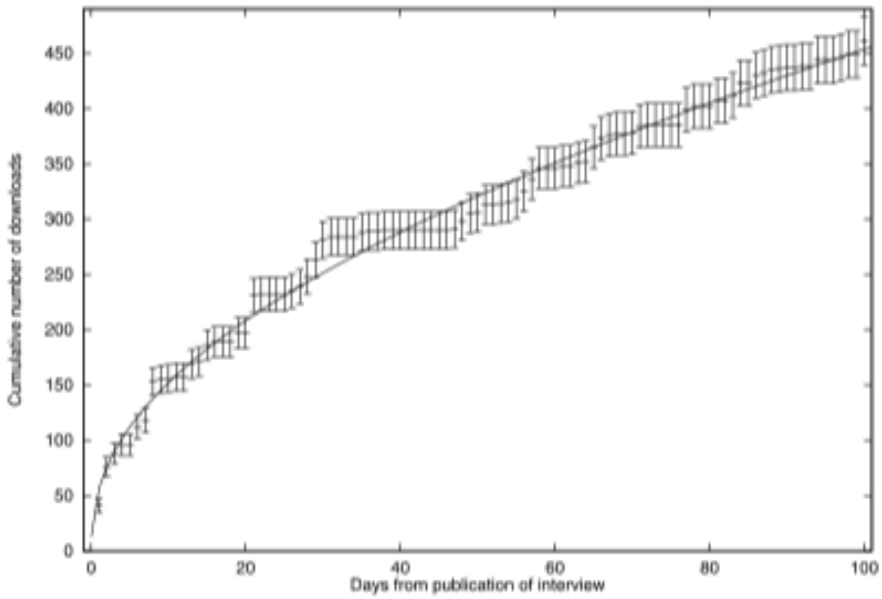

Fig. 5.1. Cumulative number of downloads N as a function of time t from the appearance of the interview on Wednesday 14th April 1999. The *fit* is $N(t) = \frac{a}{1-p}t^{1-p} + ct$ with $b \approx 0.58 \pm 0.03$. Reproduced from [18]

an exponent p close to 1, leading in the terminology of the model of epidemic word-of-mouth effect to $\theta \simeq 0$. Two interpretations are possible: (i) the exponent θ is non-universal; (ii) the social network is not always close to criticality ($n \simeq 1$) and the observable response function $K(t)$ is then expected to cross over smoothly from a power law with exponent $1 - \theta$ to another asymptotic power law with exponent $1 + \theta$. According to this second hypothesis, the exponent p of the relaxation kernel $K(t)$ may be found in the range $1 - \theta$ to $1 + \theta$, depending upon the range of investigated timescales and the proximity of $1 - n$ to criticality. We find hypothesis (ii) more attractive as it places the blame on the non-universal parameter n, which embodies the connectivity structure, static and dynamic, of social interactions at a given moment. It does not seem unrealistic to think that n may not always be at its critical value 1, due to many other possible social influences. In constrast, one could postulate that the power law (5.3) for the direct influence function $\phi(t)$ between two directly linked humans may reflect a more universal character. But, of course, only more empirical investigations will allow us to shed more light on this issue.

Eckmann, Moses and Sergi [20] also report on an original investigation probing the temporal dynamics of social networks using email networks in their universities. They find a distribution of response times for answering a message that seems to be a power law with an exponent of less than 1 for rapid response times (one hour) to another power law with an exponent larger than 1 at slower response times (days), which could be a direct evidence of the direct response function $\phi(t)$ defined in (5.3). The relationship between their investigation and the previous works using web downloads [18, 19] has been noted by Johansen [21].

5.2.3 Book Sale Shocks

Sornette, Deschatres, Gilbert and Ageon have used a database of sales from Amazon.com as a proxy for commercial growth and successes [14]. Figure 5.2 shows about 1.5 years of data for two books, Book A ("Strong Women Stay Young" by Dr. M. Nelson) and Book B ("Heaven and Earth (Three Sisters Island Trilogy)" by N. Roberts), which are illustrative of the two classes found in this study. On 5th June 2002, Book A jumped from a sales rank of over 2,000 to a rank of six in less than 12 hours. On 4th June 2002, the New York Times published an article crediting the "groundbreaking research done by Dr. Miriam Nelson" and advising the female reader, interested in having a youthful postmenopausal body, to buy the book and consult it directly [22]. This case is the archetype of an "exogenous" shock. In contrast, the sales rank of Book B peaked at the end of June 2002 after slow and continuous growth, with no such newspaper article, followed by a similar almost symmetrical decay, the entire process taking about four months. We will show below that the peak for Book B belongs to the class of endogenous shocks. This endogeneous growth is well explained qualitatively in [23] by taking the example of the book "Divine Secrets of the Ya-Ya Sisterhood" by R. Wells, which became a bestseller two years after publication, with no major advertising campaign. After reading this (originally) small budget book, "Women began forming *Ya-Ya* Sisterhood groups of their own [...]. The word about *Ya-Ya* was spreading [...] from reading group to reading group, from *Ya-Ya* group to *Ya-Ya* group" [23]. Generally, the popularity of a book is based on whether the information associated with that book will be able to propagate far enough into the network of potential buyers.

Another dramatic example of exogenous shocks is shown in Fig. 5.3. Here, the personal trainer of Oprah Winfrey had his book presented seven or eight times during the Oprah Winfrey Show, leading to dramatic overnight jumps in sales.

The declines in the sales of about 140 books that reached the top 50 in the Amazon.com ranking system have been analysed and shown to fall into two categories: relaxations described by a power law with an exponent close to $0.7 = 1 - \theta$, and relaxations described by a power law with an exponent close to $0.4 = 1 - 2\theta$, for $\theta \simeq 0.3$. Examples of these fits for the two books shown in Fig. 5.2 are presented in Fig. 5.4. In addition, Sornette et al. [14] checked that an overwhelming majority of those sale peaks classified as exogenous from the value of their exponent $\simeq 0.7 = 1 - \theta$ were preceded by an abrupt jump, in agreement with the epidemic cascade model of social interactions described in Sect. 5.2.1. In contrast, those sale peaks that fell into the endogenous class

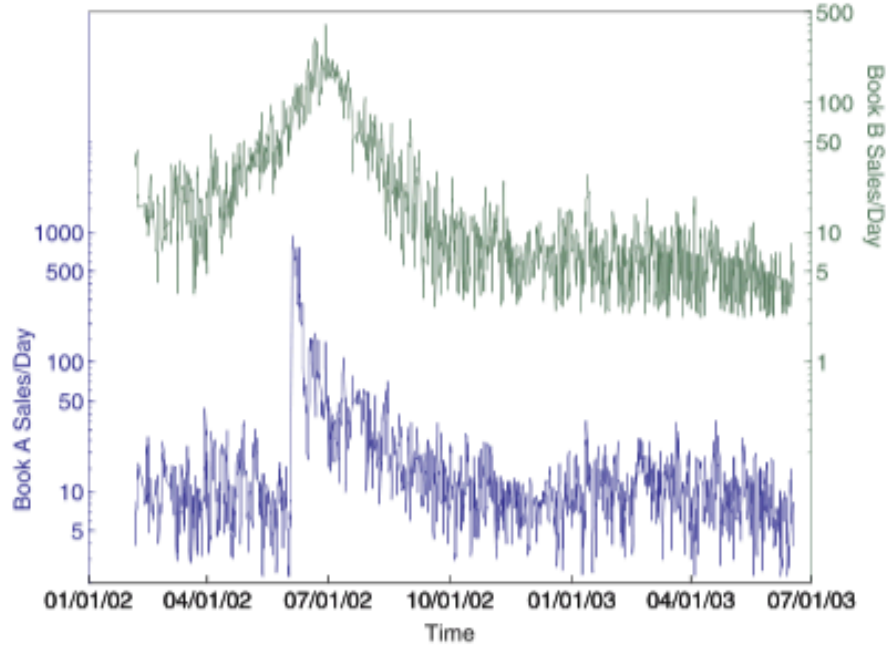

Fig. 5.2. Time evolution over a year and a half of the sales per day of two books: Book A (*bottom, blue, left scale*) is "Strong Women Stay Young" by Dr. M. Nelson and Book B (*top, green, right scale*) is "Heaven and Earth (Three Sisters Island Trilogy)" by N. Roberts. The difference in the patterns is striking, Book A undergoing an exogenous peak on 5th June 2002, and Book B endogenously reaching a maximum on 29th June 2002. Reproduced from [14]

according to the exponent $\simeq 0.4 = 1 - 2\theta$ of their relaxation after the peak were found to be preceded by approximately symmetric growth described by a power law with the same exponent, as predicted by (5.5). An example is shown also for Book B in Fig. 5.4.

The small values of the exponents (close to $1 - \theta$ and $1 - 2\theta$) for both exogenous and endogenous relaxations imply that the sales dynamics are dominated by cascades involving higher-order generations rather than by interactions that stop after first-generation buy triggering. Indeed, if buys were initiated mostly due to news or advertisements, and not much by triggering cascades in the acquaintance network, the cascade model predicts that we should then measure an exponent $1 + \theta$ given by the "bare" memory kernel $\phi(t)$, as already said. This implies that the average number n (the average branching ratio in the language of branching models) of prompted buyers per initial buyer in the social epidemic model is on average very close to the critical value 1, because the renormalization from $\phi(t)$ to $K(t)$ given by (5.2) only operates close to criticality, as characterized by the occurrence of large cascades of buys. Reciprocally, a value of the exponent p that is larger than 1 suggests that the associated social network is far from critical. Such instances can actually be observed. Examples of crossovers from the renormalized re-

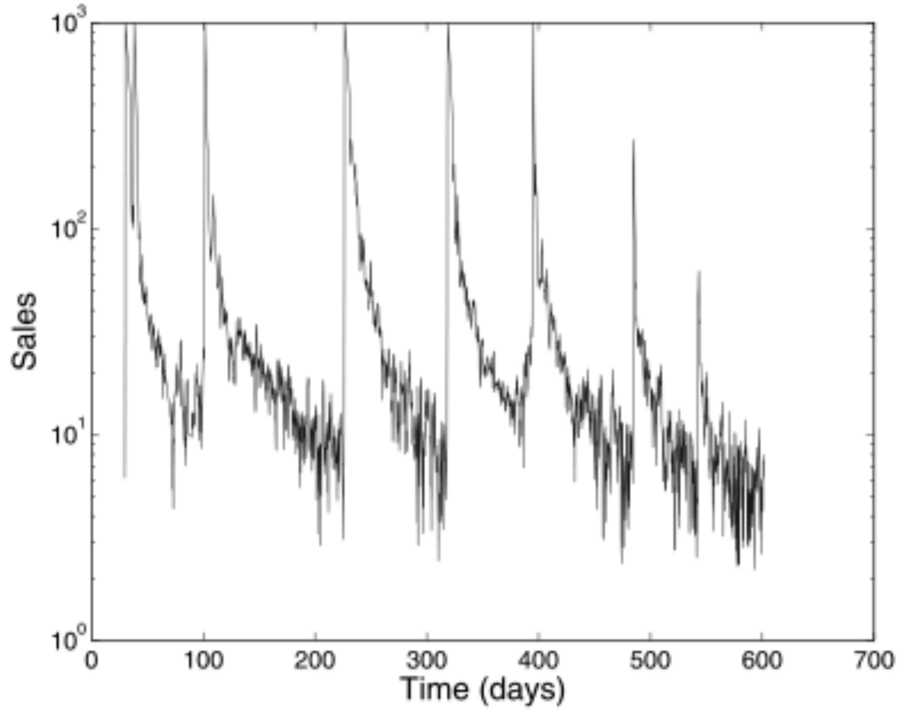

Fig. 5.3. Time evolution of the book entitled "Get with the Program." Each time the book appeared on the Oprah Winfrey Show (B. Greene was Oprah Winfrey's personal trainer), the sales jumped overnight

sponse function $K(t)$ (5.2) to $\phi(t)$ in (5.3) with an asymptotic decay with exponent $1+\theta$ have been documented ([14], Deschatres, F. and D. Sornette, in preparation). Note that it is possible to give an analytical description of this crossover exhibited by $K(t)$ as a function of n [12], thus allowing us, in principle, to invert for n for a given data set. This opens up the tantalizing possibility of measuring the dynamical connectivity of the social network, and possibly of monitoring it as a function of time.

These findings open up other interesting avenues of research. While this first investigation has emphasised the distinction between exogenous and endogenous peaks, setting the fundamentals for a general study, repeating peaks as well as peaks that may not be pure members of a single class are also frequent. In a sense, there are no real "endogenous" peaks, one could argue, because there is always a source or a string of news impacting upon the network of buyers. What Sornette et al. [14] have done is to distinguish between two extremes, the very large news impact and the structureless flow of small news amplified by the cascade effect within the network. One can imagine and actually observe a continuum between these two extremes, with feedbacks between the development of endogeneous peaks and the increased interest of the media as a consequence, feeding back and providing a kind of exogenous boost, and so on. In those and in more complicated cases, the

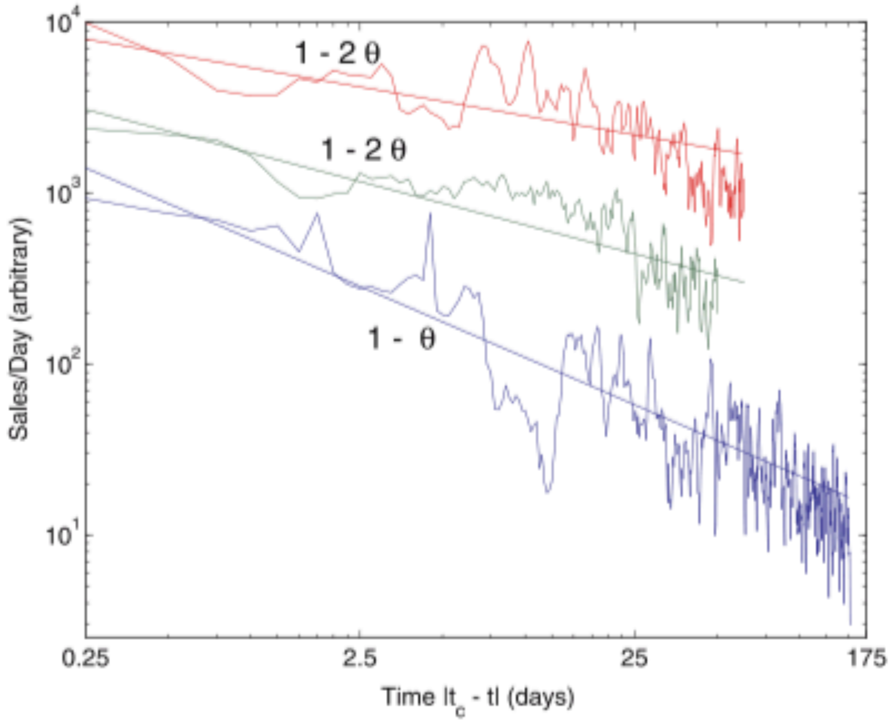

Fig. 5.4. The *bottom curve* (*blue*) shows the relaxation in the sales of Book A after the sales peak at t_c = 5th June 2002 as a function of the time $t - t_c$ from the time of the peak. A least squares best fit with a power law gives a slope of ≈ -0.7. Since this peak is identified as exogenous with a theoretical slope of $1 - \theta$, we obtain the estimate $\theta = 0.3 \pm 0.1$. *The curve in the middle* (*green*, shifted up by a factor 6 compared to the bottom curve) shows the relaxation in sales of Book B after the peak at t_c = 29th June 2002 as a function of the time $t - t_c$ from the time of the peak. The least squares fit gives a slope of ≈ -0.4, which provides the independent estimate $\theta = 0.3 \pm 0.1$ from the theoretical endogenous exponent $1 - 2\theta$. The *top curve* (*red*, shifted up by a factor 25 with respect to the bottom curve) shows the acceleration in sales of Book B leading to the same peak at t_c = 29th June 2002 as a function of the time $t_c - t$ to the time of the peak. The time on the x-axis has been reversed to compare the precursory acceleration with the aftershock relaxation. The least squares slope is ≈ -0.3, not far from the predicted $1 - 2\theta$ of the cascade model, with $\theta = 0.3 \pm 0.1$

epidemic model of word-of-mouth effects should provide a starting platform for predicting the sales dynamics as a function of an arbitrary set of external sources. By dynamically tracking the connectivity $n(t)$ of each social network relevant to a given product, it should also be possible to target the most favourable times, corresponding to the largest $n(t)$, for promoting or sustaining the sales of a given product, with obvious consequences for marketing and advertisement strategies. An additional extension includes the possible feedback of the marketing strategy into the control parameter $n(t)$, which could be manipulated so as to keep the system critical, an ideal situation from the point of view of marketers and firms. Quantifying this effect requires us to extend the simple epidemic model in the spirit of mechanisms leading to self-organized criticality by positive feedbacks of the order parameter onto the control parameter [24, 25]. The results of Sornette et al suggest that social networks have evolved to converge very close to criticality. As Andreas S. Weigend, chief scientist of Amazon.com (2002–2004) wrote on his webpage: "Amazon.com might be the world's largest laboratory to study human behaviour and decision making." I share this viewpoint.

Actually, I envision that an extension of the study of Sornette et al to a broad database of sales from all products sold by e-retailers like Amazon.com could give access to the equivalent of the "social climate" of a country like the USA and its evolution as a function of time under the various exogenous and endogenous factors at work. Indeed, Amazon.com categorizes its products into different (tradable) compartments of possible interest, such as

- Books, Music, DVD,
- Electronics (audio and video, cameras and photography, software, computers and video games, cell phones...)
- Office
- Children and Babies
- Home and Garden (which includes pets)
- Gifts, Registries, Jewellery and Watches
- Apparel and Accessories
- Food
- Health, Personal Care, Beauty
- Sports and Outdoors
- Services (movies, restaurants, travel, cars, ...)
- Arts and Hobbies
- Friends and Favourites

with many subcategories. Monitoring and analysing the sales as a function of time in these different categories is like getting the temperature, wind velocity, humidity in meteorology in many different locations. The flow of interest of society at large and of subgroups could in principle tell us how society is responding in its spending habits to large scale influences. As an illustrative example, it has been shown that, during bullish periods characterized by strong stock market gains (bubble regimes), the number of books written and sold related to financial investments soar [26, 27].

Another potentially fruitful application is the music industry and the impact upon sales of internet piracy, the quality of performers (endogenous effect on the network of potential buyers who can promote a CD by word-of-mouth in the network of potential buyers), as well as the promotion campaigns of short-lived performers and their one-hit wonders [28]. Indeed, according to an internal study performed by one of the big companies that dominate the production and distribution of music, the drop in sales in America may have less to do with internet piracy than with other factors, among them the decreasing quality of music itself. The days of watching a band develop slowly over time with live performances are over, according to some professionals. Even Wall Street analysts are questioning quality. If CD sales have shrunk, one reason could be that people are less excited by the industry's product. A poll by Rolling Stone magazine found that fans believe that relatively few "great" albums have been produced in recent years [28]. This is clearly an endo-exo question that can be analysed with databases available on the Internet.

5.2.4 Social Shocks

Roehner, Sornette and Andersen [29] have used the concept of exogeneous shocks to propose a general method for quantifying the response function in order to advance the social sciences. By using a database of newspaper articles called Lexis-Nexis, which is available in many departments of political science or sociology, they have quantified the response to shocks, such as the following:

- On 31st October 1984, the Prime Minister of India, Indira Gandhi, was assassinated by two of her Sikh bodyguards. This event triggered a wave of retaliations against Sikh people and Sikh property, not only in India (particularly in New Delhi), but in many other countries as well.
- In the early hours of 6th December 1992, thousands of Hindus converged on the holy city of Ayodhya in northern India and began to destroy the Babri mosque which was said to be built on the birthplace of Lord Rama. The old brick walls came down fairly easily and soon the three domes of the mosque crashed to the ground. This event triggered a wave of protestations and retaliations which swept the whole world from Bangladesh to Pakistan, to England and the Netherlands. In all of these countries, Hindu people were assaulted and Hindu temples were firebombed, damaged or destroyed.
- On 11th September 2001, two planes crashed into the twin towers of the World Trade Center in New York. This event triggered a wave of reactions against Islamic people and property, not just in the United States.

For these different events, Roehner et al. [29] show that different quantitative measures of social responses exhibit an approximately universal behaviour, again characterised by a power law, as shown in Fig. 5.5. This figure gives the time evolution after 11th September 2001 of newspaper articles, anti-Arab incidents and the Dow Jones Industrial Average, which are approximated by a power law $\sim 1/t^p$. Due to the coarseness of the measures, the exponent p is not well-constrained: $p = -1.8 \pm 0.7$ (newspaper articles), $p = -1.4 \pm 0.5$ (anti-Arab incidents) and $p = -2.2 \pm 1.6$ (DJI). Comparing the reaction to 11th September 2001 in different countries such as Canada, Great Britain and the Netherlands, Roehner et al. [29] have suggested that

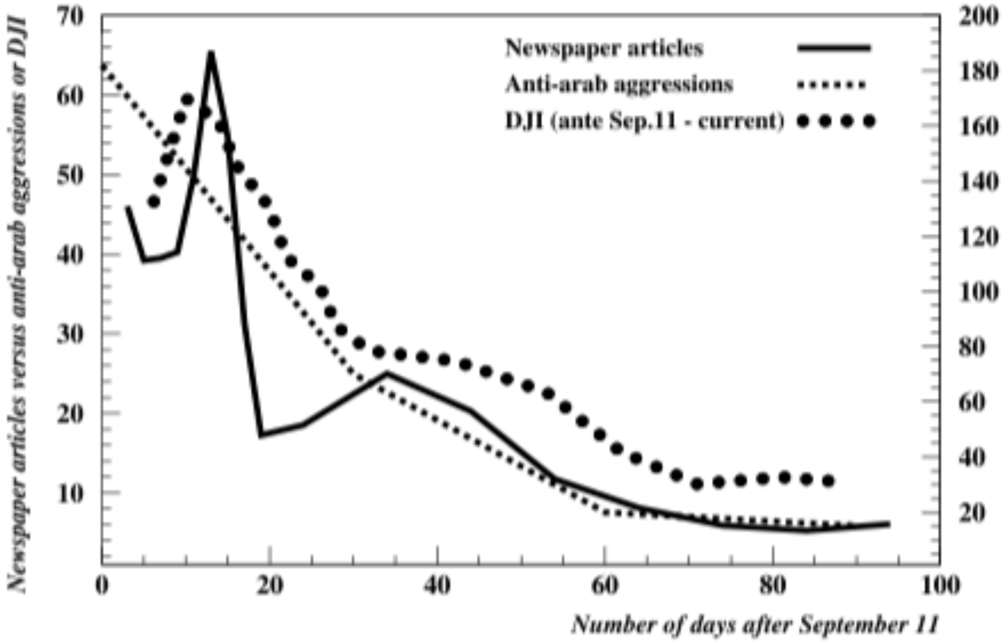

Fig. 5.5. Relaxation of three different social variables after the events of 11th September 2001. The *solid line curve* is the number of articles reporting on the destruction of mosques after the event; the *broken line* (scale on the right-hand side) shows the number of anti-Arab incidents in California in the three months after 11th September; the *dotted line* shows the changes in the level of the Dow Jones Index with respect to its pre-9/11 level, as given by the difference DJI(pre-9/11)−DJI(current). Source: California's Attorney General Office; published in the San Jose Mercury News, 11th March 2002. Reproduced from [29]

the response function actually expresses information on "cracks" that preexisted in the social networks of the corresponding countries. For instance, the number of attacks on Mosques was larger in the Netherlands, which is in line with other information on the concern expressed at high political levels (private communication to the authors) about the integrity of the social fabric of the Netherlands, a fact illustrated more recently on the political scene by the rapid rise and then assassination of the rightist politician Fortuyn in May 2002. This line of evidence can be quantified within the epidemic model of social influence by different values of the connectivity parameter n in different countries.

Burch, Emery and Fuerst [30] have used also the unique opportunity offered by the 9/11 terrorist attack to clearly confirm the hypothesis that closed-end mutual fund discounts from fund net asset values reflect small investor sentiment. Carter and Simkins [31] investigated the reaction of airline stock prices to the 9/11 terrorist attack and found that the market was concerned about the increased likelihood of bankruptcy in the wake of the attacks and distinguished between airlines based on their ability to cover short-term obligations (liquidity).

5.3 Exogenous and Endogenous Shocks in Financial Markets

5.3.1 Volatility Shocks

Standard economic theory maintains that the complex trajectory of stock market prices is the faithful reflection of the continuous flow of news that is interpreted and digested by an army of analysts and traders. Accordingly, large shocks should result from really bad surprises. It is a fact that exogenous shocks exist, as epitomized by the recent events of 11th September 2001, and there is no doubt about the existence of utterly exogenous bad news that moves stock market prices and creates strong bursts of volatility. One case that cannot be refuted is the market turmoil observed in Japan following the Kobe earthquake of 17th January 1995, the estimated cost of which was around $200 billion dollars. Indeed, so longinfancy, destructive earthquakes cannot be not endogenized in advance in stock market prices by rational agents ignorant of seismological processes. One may also argue that the invasion of Kuwait by Iraq on 2nd August 1990 and the coup against Gorbachev on 19th August 1991 were strong exogenous shocks. However, some could also argue that precursory fingerprints of these events were known to some insiders, suggesting the possibility that the action of these informed agents may have been reflected in part in stock markets prices. Even more difficult is the classification (endogenous versus exogenous) of the hierarchy of volatility bursts that continuously shake stock markets. While it is a common practice to associate the large market movements and strong bursts of volatility with external economic, political or natural events [32], there is no convincing evidence to support this.

Perhaps the most robust observation in financial stock markets is that volatility is serially correlated with long-term dependence (approximately power law-like). Volatility autocorrelation is typically modelled using autoregressive conditional heteroskedasticity (ARCH) [33], generalized ARCH [34], stochastic volatility [35], Markov switching [36, 37], nonparametric [38] and extensions of these models (see [39] for comparisons). Recent powerful extensions include the Multifractal Random Walk model (MRW) introduced by Muzy, Bacri and Delour [40, 41], which belongs to the class of stochastic volatility models. Using the MRW, Sornette, Malevergne and Muzy [42] have shown that it is possible to distinguish between an endogenous and an exogenous originated volatility shock. Tests on the October 1987 crash on a hierarchy of volatility shocks and on a few of the obvious exogenous shocks have validated the concept. This study shows that the relaxation with time of a burst of volatility is distinctly different after a strong exogenous shock compared with the relaxation of volatility after a peak with no identifiable exogenous sources. This study does not explain the origin of volatility correlation. But it identifies the "natural" response function of the system to an external shock, from which the stationary long-term dependence structure of the volatility and its intermittent bursts derive automatically. In other words, the study of Sornette et al. leads to the view that the properties of the volatility can be largely understood from a single characteristic, which is the response of the agents to a new piece of news. This response function must ultimately be derived from the behaviour of financial agents, for instance taking into account their sensitivity to changes in wealth, their loss aversion as well as their finite-time memory of past losses that may impact their future decisions [44].

The multifractal random walk is an autoregressive process with a long-range memory decaying as $t^{-1/2}$, which is defined using the logarithm of the volatility. Using the MRW model for the dependence structure of the volatility, Sornette et al. predict that exogenous volatility shocks will be followed by a universal relaxation

$$\simeq \lambda/t^{1/2} , \tag{5.6}$$

where λ is the multifractal parameter, while endogenous volatility shocks relax according to a power law

$$\simeq 1/t^{p(V_0)} , \qquad \text{with} \;\; p(V_0) \simeq \lambda^2 \;\ln(V_0) , \tag{5.7}$$

with an exponent $p(V_0)$ which is a linear function of the logarithm $\ln(V_0)$ of the shock of volatility V_0. The difference between these behaviours and those reported above modelled by the epidemic process with long-term memory stems from the fact that the stock market returns $r_{\Delta t}(t)$ at timescale Δt at a given time t can be accurately described by the following process [40,41]:

$$r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)} , \tag{5.8}$$

where $\epsilon(t)$ is a standardized Gaussian white noise independent of $\omega_{\Delta t}(t)$, and $\omega_{\Delta t}(t)$ is a near-Gaussian process with mean and covariance

$$\mu_{\Delta t} = \frac{1}{2} \ln(\sigma^2 \Delta t) - C_{\Delta} t(0) \tag{5.9}$$

$$C_{\Delta} t(\tau) = Cov[\omega_{\Delta t}(t), \omega_{\Delta t}(t+\tau)] = \lambda^2 \ln\left(\frac{T}{|\tau| + e^{-3/2}\Delta t}\right) . \tag{5.10}$$

where $\sigma^2 \Delta t$ is the return variance at scale Δt and T represents an "integral" (correlation) timescale. λ is called the multifractal parameter: when it vanishes, the MRW reduces to a standard Wiener process (standard continuous random walk). Such a logarithmic decay of the log-volatility covariance at different timescales has been shown empirically in [40,41]. Typical values for T and λ^2 are respectively one year and 0.04.

The MRW model can be expressed in a more familiar form, in which the log-volatility $\omega_{\Delta} t(t)$ obeys an auto-regressive equation whose solution reads

$$\omega_{\Delta} t(t) = \mu_{\Delta} t + \int_{-\infty}^{t} d\tau \; \eta(\tau) \; K_{\Delta} t(t-\tau) , \tag{5.11}$$

where $\eta(t)$ denotes a standardized Gaussian white noise and the memory kernel $K_{\Delta}t(\cdot)$ is a causal function, ensuring that the system is not anticipative. The process $\eta(t)$ can be seen as the information flow. Thus $\omega(t)$ represents the response of the market to incoming information up to the date t. At time t, the distribution of $\omega_{\Delta}t(t)$ is Gaussian with mean $\mu_{\Delta}t$ and variance $V_{\Delta}t = \int_0^{\infty} d\tau\ K_{\Delta}^2 t(\tau) = \lambda^2 \ln\left(\frac{Te^{3/2}}{\Delta t}\right)$. Its covariance, which entirely specifies the random process, is given by

$$C_{\Delta}t(\tau) = \int_0^{\infty} dt\ K_{\Delta}t(t) K_{\Delta}t(t + |\tau|)\ . \tag{5.12}$$

Performing a Fourier transform, we obtain

$$\hat{K}_{\Delta}t(f)^2 = \hat{C}_{\Delta}t(f) = 2\lambda^2\ f^{-1} \left[\int_0^{Tf} \frac{\sin(t)}{t} dt + O\left(f\Delta t \ln(f\Delta t)\right)\right]\ , \tag{5.13}$$

which shows, using (5.10), that for a small enough τ,

$$K_{\Delta}t(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for}\ \ \Delta t \ll \tau \ll T\ , \tag{5.14}$$

which is the previously stated exogenous response function (5.6). The slow power law decay (5.14) of the memory kernel in (5.11) ensures the long-range dependence and multifractality of the stochastic volatility process (5.8).

The main difference between the MRW model and the previous class of epidemic process is that the long-term memory appears in the logarithm of the variable in the former, as shown from (5.11). As a consequence, the MRW basically describes a variable which is the exponential of a long-memory process. It is the interplay between this strongly nonlinear exponentiation and the long-memory which gives multifractal properties to the MRW and, as a consequence, the shock amplitude dependence of the exponents $p(r)$ of the relaxation of the volatility following endogenous shocks. In contrast, the linear long-term memory structure (5.1) of the epidemic processes of Sect. 5.2.1 ensures universal exponents that are independent of the shock amplitudes (but not of the endo-exo nature). In the epidemic process (5.1), the relationship between exogenous and endogenous relaxations is expressed by the exponents of the power laws $\sim 1/t^{1-\theta}$ (exo) versus $\sim 1/t^{1-2\theta}$ (endo). In the MRW, notice that the relationship between exogenous (5.6) and endogenous relaxations (5.7) is through the multifractal parameter λ: the fact that an amplitude of the exogenous response function impacts the power law exponent of the endogenous relaxation is again a signature of the exponential structure of the multifractal model. The MRW extends the realm of possible relationships between endogenous and exogenous responses discussed until now.

5.3.2 Financial Crashes

The endo-exo question also appears to be crucial for understanding financial crashes. In contrast with the previous examples, the strongest distinction is not in the relaxation or recovery after the shock but rather in the precursory behaviour before the crash. An endogenous crash might be expected to end a period of strong price gains, due to speculative herding for instance. In contrast, an exogenous crash would be the response of the financial system to a very strong adverse piece of information.

Indeed, according to standard economic theory, the complex trajectory of stock market prices is the faithful reflection of the continuous flow of news that are interpreted and digested by an army of analysts and traders [45]. Accordingly, large market losses should result only from really bad surprises. It is indeed a fact that exogenous shocks exist, as epitomized by the recent events of 11th September 2001 and the coup in the Soviet Union on 19th August 1991, which move stock market prices and create strong bursts of volatility [42], as discussed above. However, is this always the case? A key question is whether large losses and gains are indeed slaved to exogenous shocks, or whether they may result from endogenous origins in the dynamics of that particular stock market. The former possibility requires the risk manager to closely monitor the world of economics, business, political, social, environmental news for possible instabilities. This approach is associated with standard "fundamental" analysis. The latter endogenous scenario requires an investigation of the signs of instabilities to be found in the market dynamics itself, and it could, in part, rationalize so-called "technical" analysis (see [43] and references therein).

Johansen and Sornette [47] have carried out a systematic investigation of crashes to clarify this question. They have proceeded in several steps:

1. They have developed a methodology to identify crashes as objectively and unambiguously as possible. Specifically, they have studied the distributions of drawdowns (runs of losses) in several markets: the two leading exchange markets (US dollar against the Deutsch and against the Yen), the major world stock markets, the U.S. and Japanese bond market and the gold market. By introducing and varying a certain degree of fuzziness in the definition of drawdowns, they have tested the robustness of the empirical distributions of drawdowns.
2. By carefully analysing these distributions, they have shown that the extreme tail belongs to a different population than the bulk (typically the top 1% (most extreme) drawdowns occur 10–100 times more often than would be predicted by an extrapolation of the distribution of the other 99% of the drawdowns).
3. The Xevents which seem to belong to a different population have been called "outliers" [46,48–50]. Others have referred to such events as "kings" or "black swans." Johansen and Sornette [47] have taken these kings to be

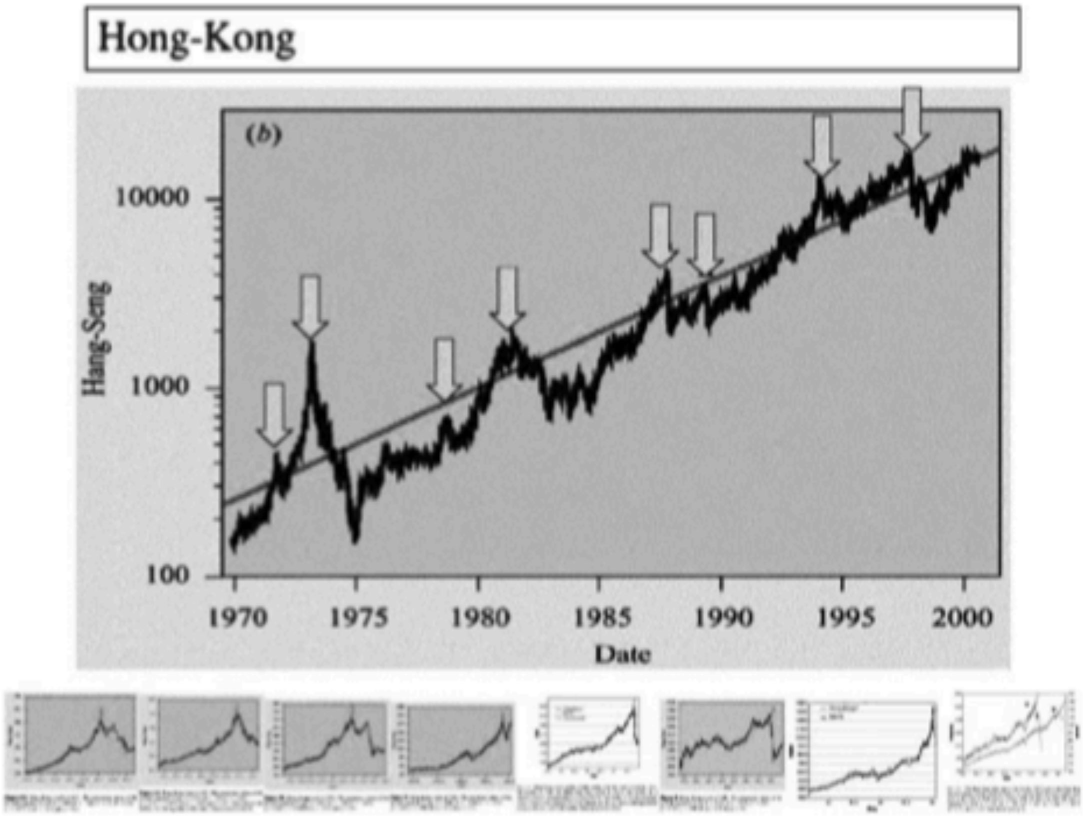

Fig. 5.6. The Hang-Seng composite index of the Hong Kong stock market from November 1969 to September 1999. Note the logarithmic scale on the *vertical axis*. The peaks of the bubbles followed by strong crashes are indicated by the *arrows* and correspond to the times Oct. 1971, Feb. 1973, Sept. 1978, Oct. 1980, Oct. 1987, April 1989, Jan. 1994 and Oct. 1997. This figure shows that the Hang-Sing index has grown exponentially on average at the rate of ≈ 13.6% per year, represented by the *straight line* corresponding to the best exponential fit to the data. Eight large bubbles (five of which are very large) can be observed as upward accelerating deviations from the average exponential growth, and are characterized by LPPL signatures ending in a crash, here defined as a drop of more than 15% in less than two weeks. The eight small panels at the *bottom* are given to show the LPPL price trajectory over a period of six months preceding each of these eight crashes. Constructed from [46] and other papers from the author

the crashes that need to be explained. Note that this procedure ensures that the definition of a crash is relative to the specific market rather than obeying such an arbitrary absolute rule.

4. Then, for each identified king, Johansen and Sornette [47] checked whether a specific market structure, called log-periodic power law (LPPL), is present in the price trajectory *preceding* the occurrence of the draw-down king. The rational for this approach was based on their previous works [46, 51–53], in which they documented the existence of such log-periodic power law signatures associated with speculative bubbles before

crashes. The work [47] is in this respect an out-of-sample test of the LPPL bubble-crash hypothesis applied to a population of financial time series selected according to a criterion (outlier test in the distribution of drawdowns) which is unrelated to the LPPL structure itself.

5. In this test, Johansen and Sornette [47] take the existence of a LPPL as the qualifying signature for an endogenous crash: a drawdown outlier is seen as the end of a speculative unsustainable accelerating bubble generated endogenously.
6. With these criteria fixed, Johansen and Sornette [47] identify two classes of crashes. Those that are not preceded by a LPPL price trajectory are classified as exogenous. For those, it was possible to identify what seems to have been the relevant historical event (a new piece of information of such magnitude and impact that it is reasonable to attribute the crash to it, following the standard view of the efficient market hypothesis). Such drawdown outliers are classified as having an exogenous origin.
7. The second class, characterized by LPPL price trajectories, is called endogenous. Figure 5.6 illustrates a series of endogenous crashes preceded by LPPL bubble trajectories on the Heng-Seng composite index of the Hong-Kong stock market, perhaps one of the most speculative markets in the world. All of the events shown belong to the endogenous class.
8. Globally over all of the markets analysed, Johansen and Sornette [47] identified 49 outliers, of which 25 were classified as endogenous, 22 as exogenous and two as associated with the Japanese "anti-bubble" that started in January 1990. Restricting to the world market indices, they found 31 outliers, of which 19 are endogenous, ten are exogenous and two are associated with the Japanese anti-bubble.

The combination of the two proposed detection techniques, one for drawdown outliers and the second for LPPL signatures, provides a novel and systematic taxonomy of crashes, further substantiating the importance of LPPL (see also [54–58] for reviews and extensions).

A more microscopic approach, formulated in terms of agent-based models has also allowed some mechanisms to be identified with the occurrence of Xevents, such as excess bias on nodes in the de Bruijn diagram of active agent strategies [59], or the decoupling of strategies which become transiently independent from the recent past [60].

5.4 Concluding Remarks

Let us end with a discussion of other domains of applications.

While the idea is not yet well developed, I think that beyond the products sold by e-retailers discussed above, which are proxies of reputation and commercial successes, the endo-exo question is relevant to understanding the characteristics of Initial Public Offerings (IPO) [62] and the movie industry [63]. In the latter, the mechanism of information cascade derives from the fact that agents can observe box office revenues and communicate via word-of-mouth about the quality of the movies they have seen.

Earthquakes are now thought to be caused by a mixture of spontaneous occurrences driven by plate tectonics and triggering by previous earthquakes. Within such a picture [12], which rationalizes much of the phenomenology of seismic catalogs, Helmstetter and Sornette have shown that there is a fundamental limit to earthquake predictability resulting from the "exogenous" class of earthquakes that are not triggered by other earthquakes [61]. Furthermore, the rate of foreshocks preceding mainshocks can be understood from the idea that mainshocks may result from endogenous triggering by previous events, as developed above in Sect. 5.2.1. The time dependence of the seismic rate of foreshocks is predicted and observed to follow (5.5). The memory kernels $\phi(t)$ given by (5.3), and $K(t)$ given by (5.2), correspond respectively in the present case to the bare and renormalized Omori law [64] for triggered aftershocks [12,15].

The weather and the climate also involve extremely complex processes, which are often too difficult to disentangle. This leads to major uncertainties about the important mechanisms that need to be taken into account, for instance, to forecast the future global warming of the earth due to anthropogenic activity coupled with natural variability. 9/11 has again offered a unique window. Travis and Carleton [65] noted the following: "Three days after suicide airplane hijackers toppled the World Trade Center in New York and slammed into the Pentagon in Washington, D.C., the station crew noted an obvious absence of airborne jetliners from their perch 240 miles (384 kilometers) above Earth. "I'll tell you one thing that's really strange: Normally when we go over the U.S., the sky is like a spider web of contrails", U.S. astronaut and outpost commander Frank Culbertson told flight controllers at NASA's Mission Control Center in Houston. "And now the sky is just about completely empty. There are no contrails in the sky," he added. "It's very, very weird." "I hadn't thought of that perspective," fellow astronaut Cady Coleman replied." Travis and Carleton [65] showed that there was a significant elevation of the average diurnal temperature of the US in the three days following 9/11, when most jetliners were grounded and no contrails were present. This is the archetype of an exogenous response. It remains to be seen if the endo-exo viewpoint will offer new fruitful perspectives that will allow us to make progress in understanding and in forecasting the weather and the climate.

Finally, from a theoretical viewpoint, another potentially interesting domain of research is to extend the concept of the response function to nonlinear systems [66,67] and to study its relationship with the internal fluctuations [7].

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