

## ANÁLISE ESTATÍSTICA E ESPECTRAL DE PROCESSOS ESTOCÁSTICOS

Uma Abordagem Computacional  
[reinaldo.rosa@inpe.br](mailto:reinaldo.rosa@inpe.br)

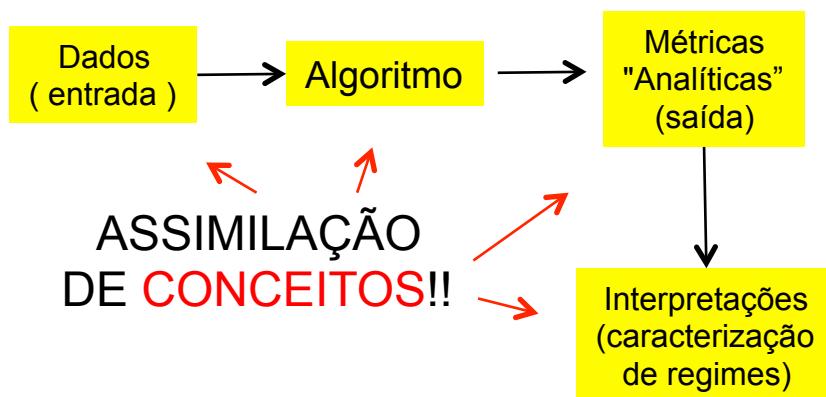
2020

### MÓDULO B1

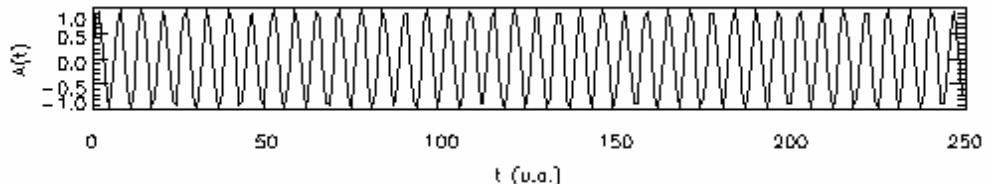
1. Eventos, Variáveis Aleatórias & PE
2. Espaço de Probabilidades
3. Fórmula de Bayes
4. Histogramas & PDF's
5. Momentos & Estacionaridade
6. Teoremas dos Limites Amostrais
7. Espaço de Cullen-Frey
8. Processos Estocásticos – p model  
(eventos extremos)

### MÓDULO B2:

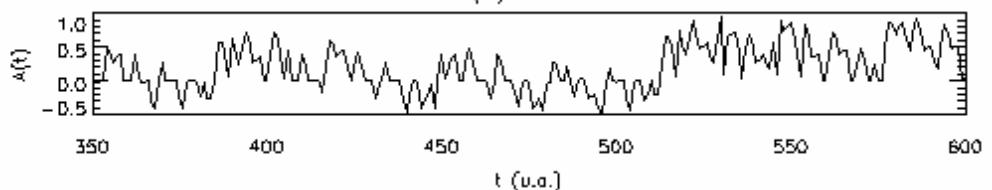
1. Função de Autocorrelação
2. Transformações para o Domínio Espectral
3. Transformada de Fourier
4. Transformada Rápida de Fourier
5. Densidade Espectral de Potências
6. Teoremas WK&P e Exponente de Holder
7. Espectro DFA
8. Espectro de Singularidades (MFDFA)
9. SOC
10. Global Wavelet Spectra



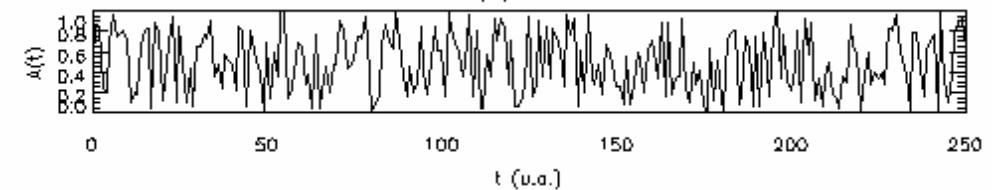
(a) Periódico



(b) MultiP

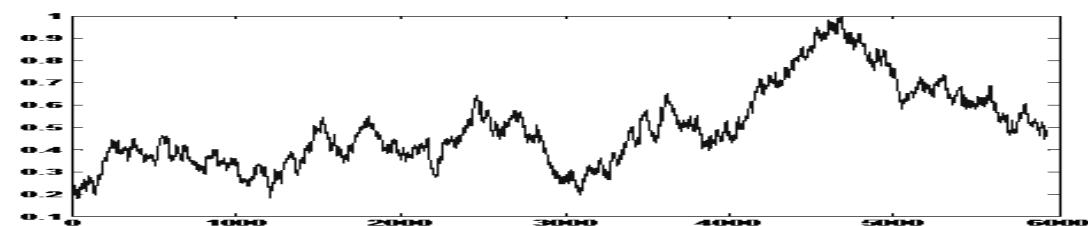
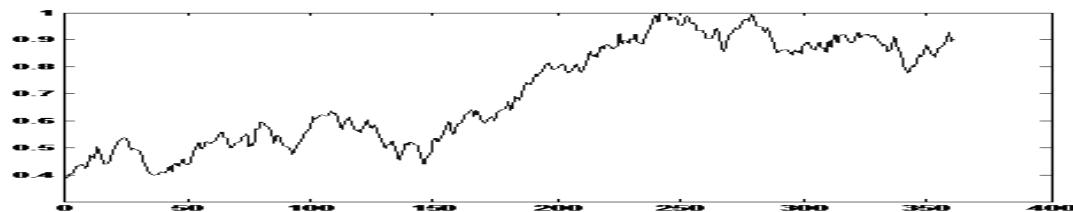


(c) Rand

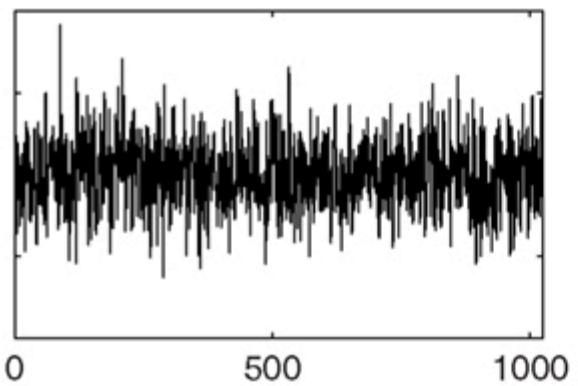


Ex. Caóticos

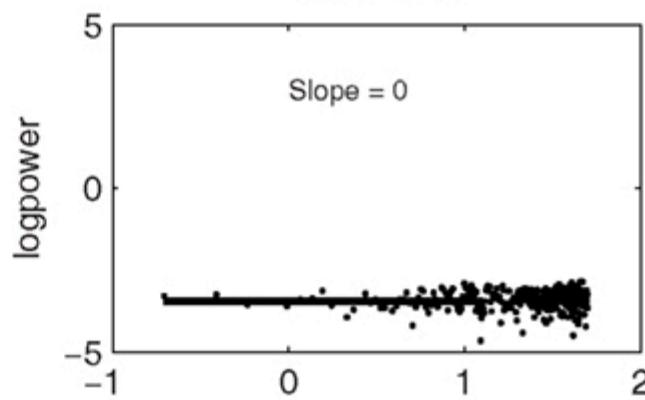
Ex. Estocásticos



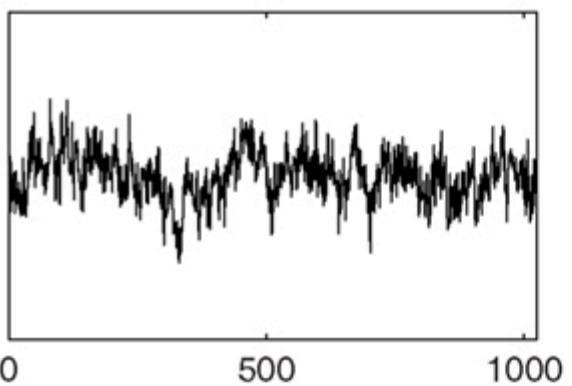
White Noise



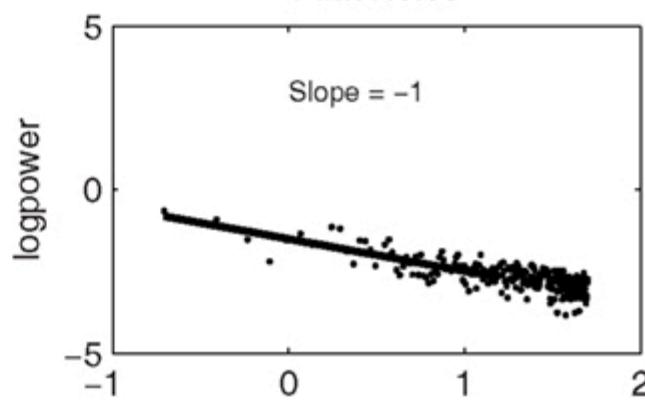
White Noise



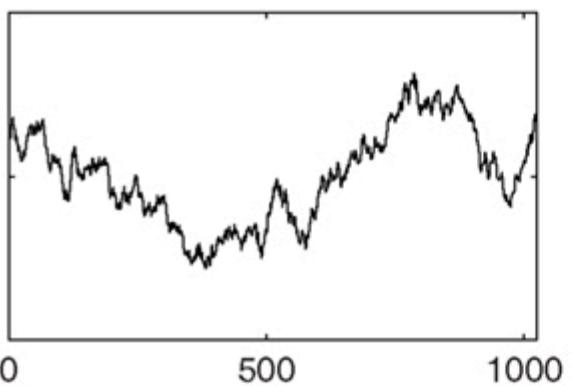
Pink Noise



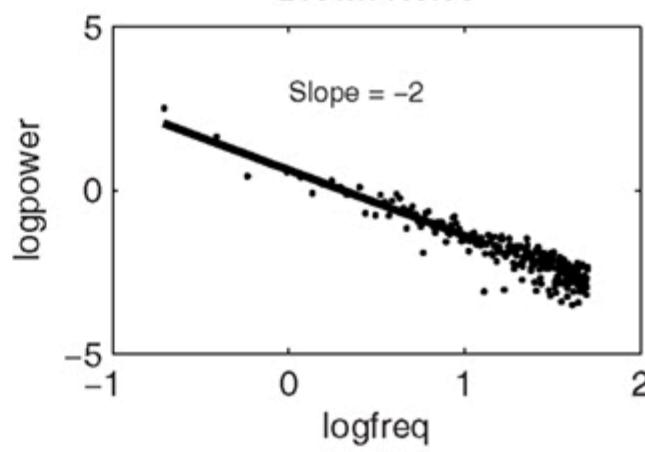
Pink Noise



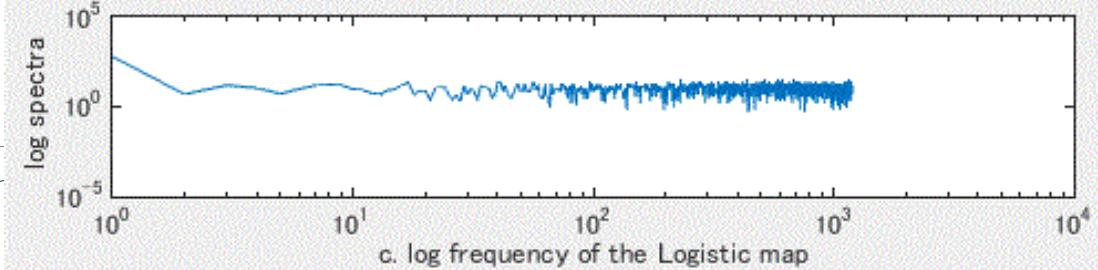
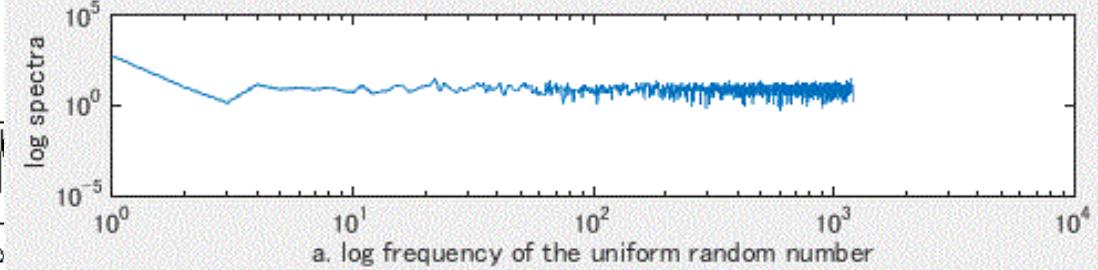
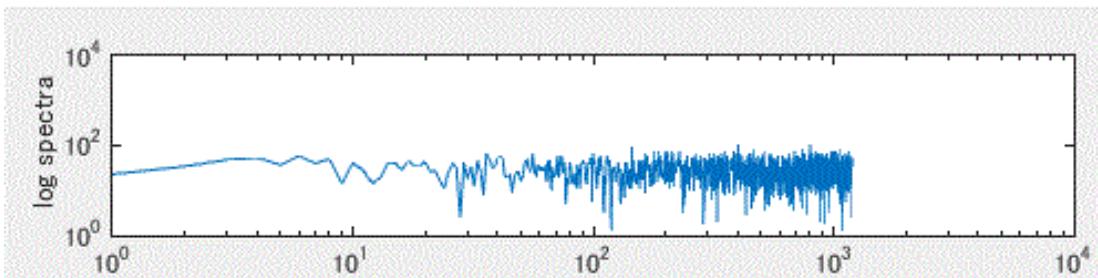
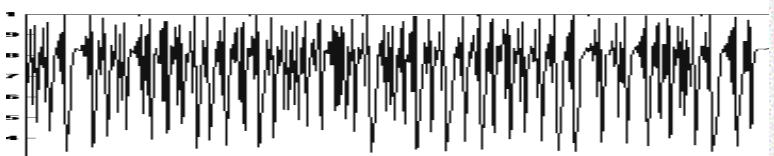
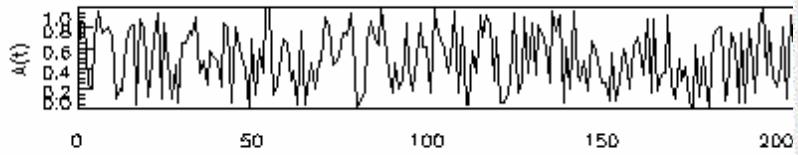
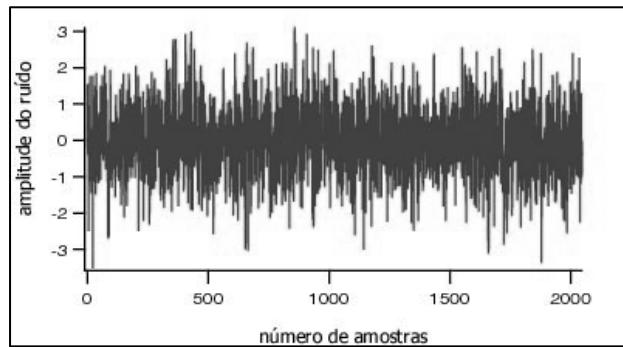
Brown Noise



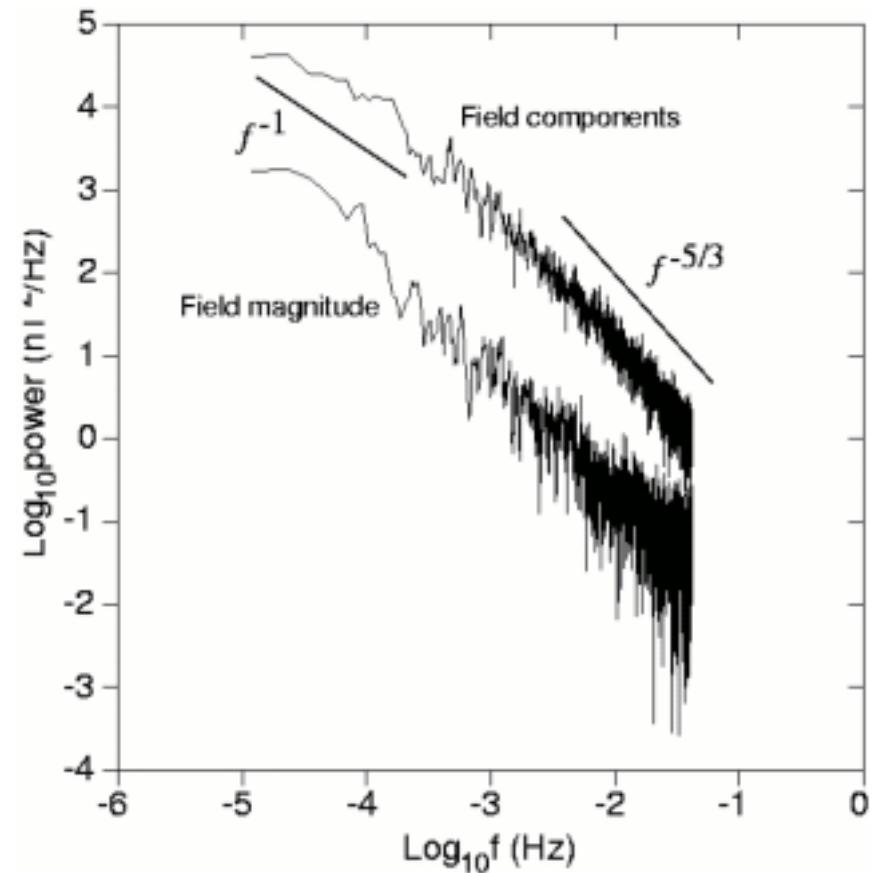
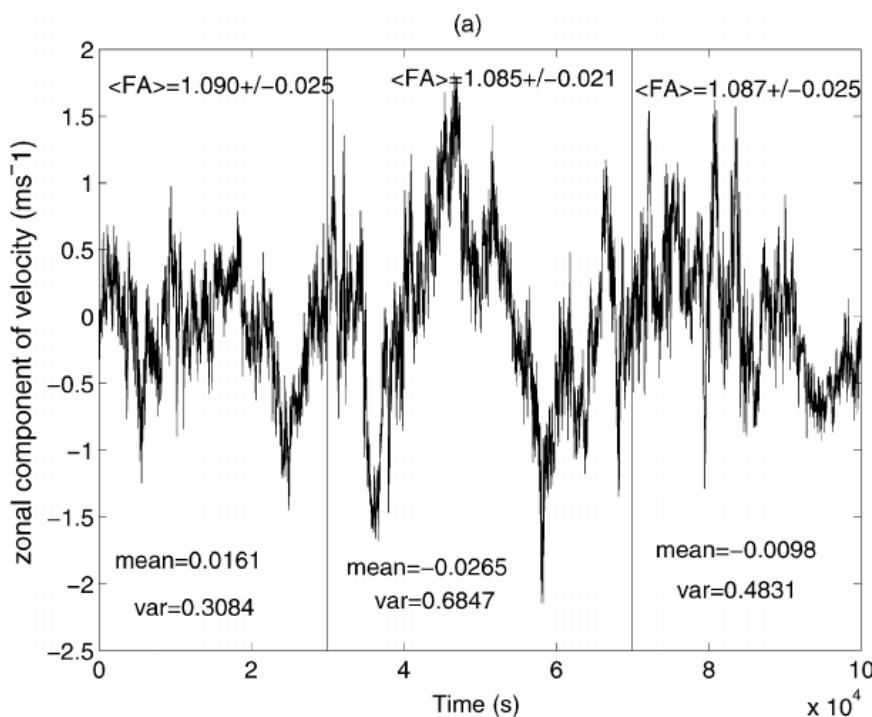
Brown Noise



# Espectros de Regimes Antipersistentes

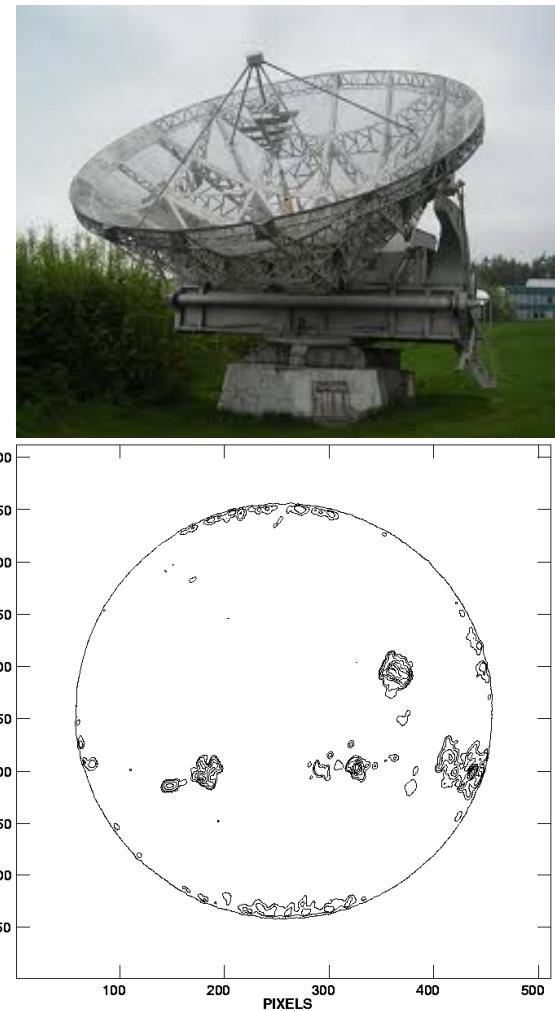
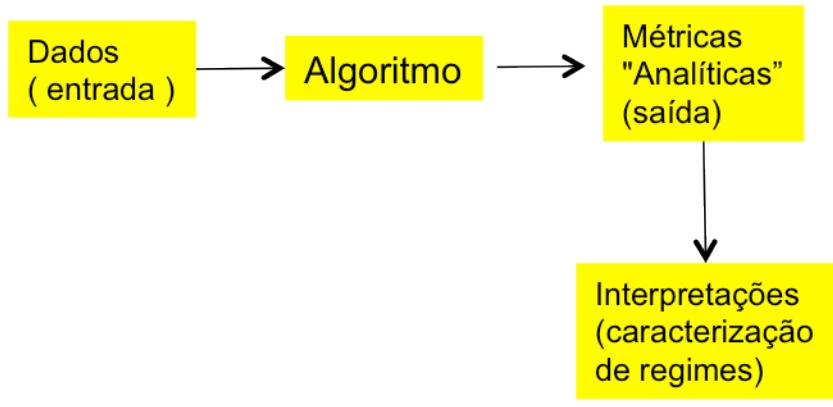
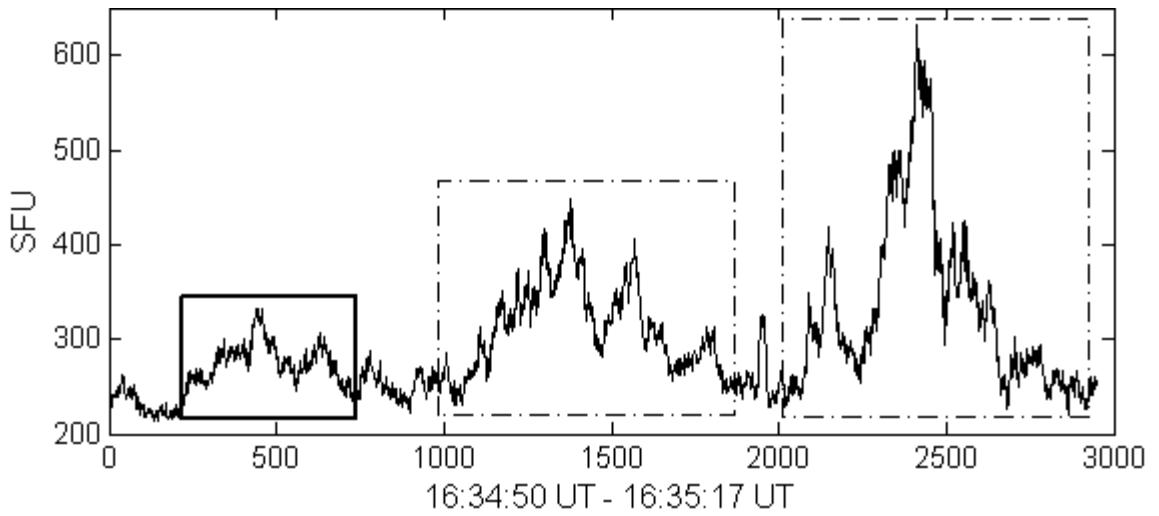
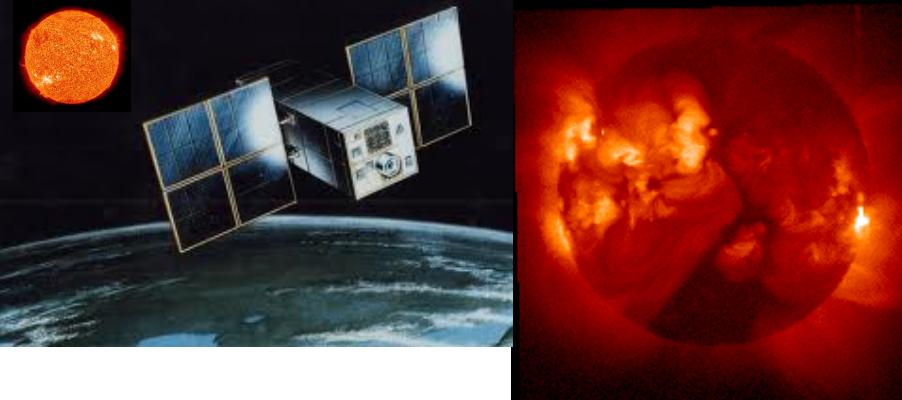


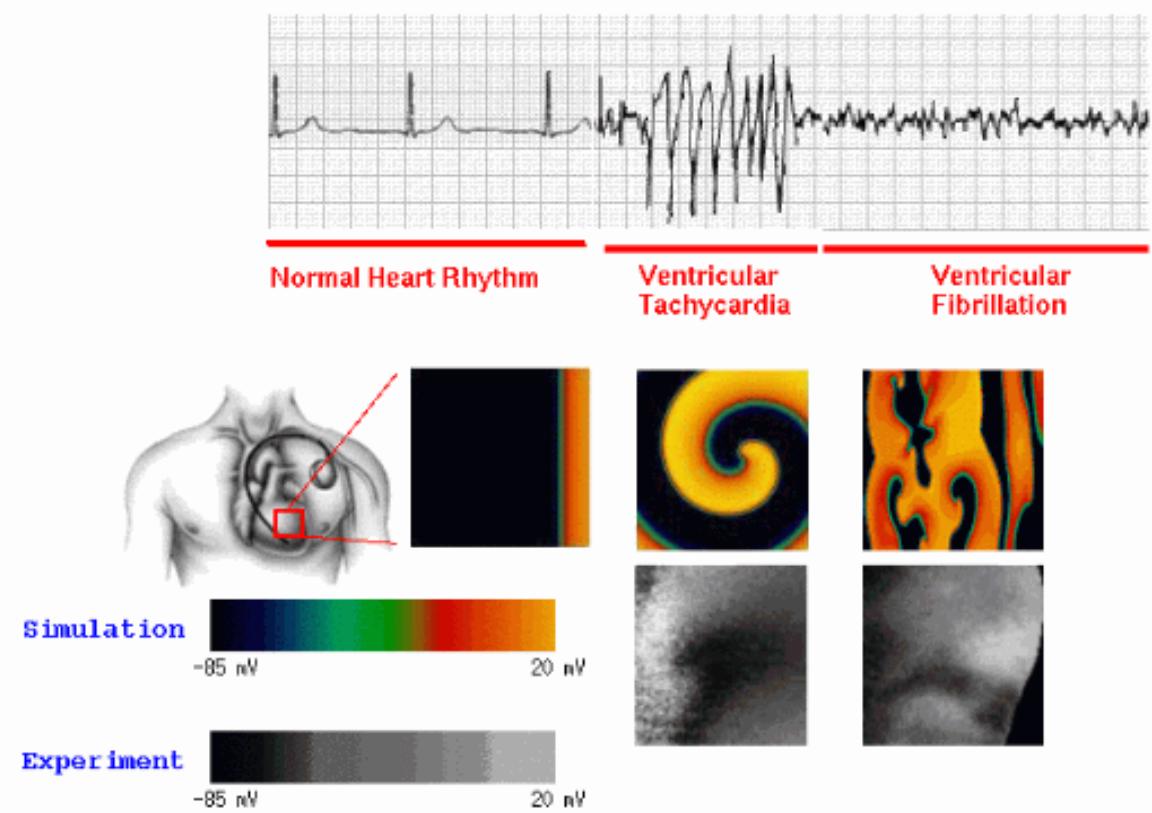
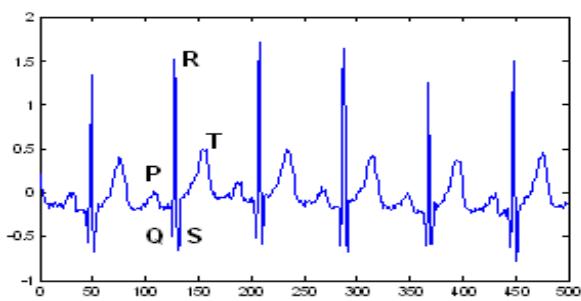
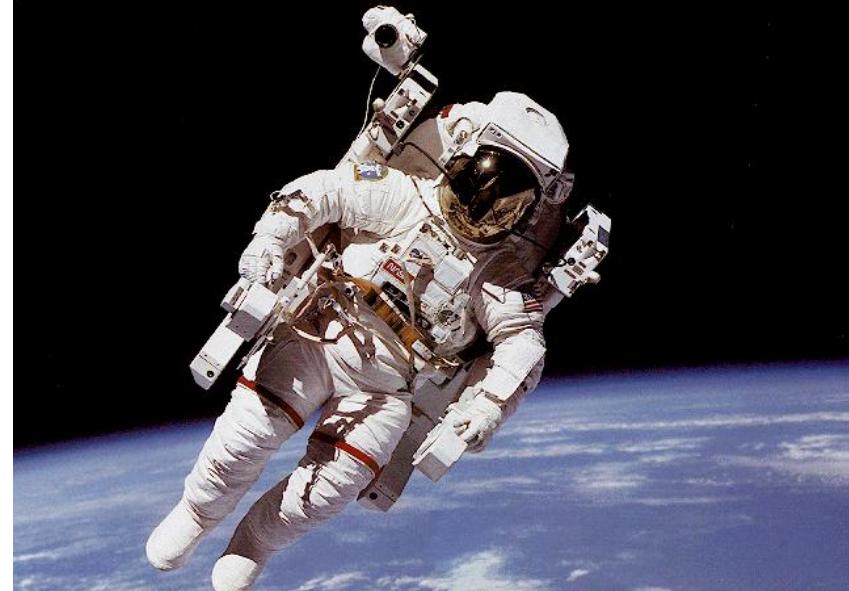
# Espectro de um Regime Turbulento K44 (Estocástico, Não-estacionário e Antipersistente)



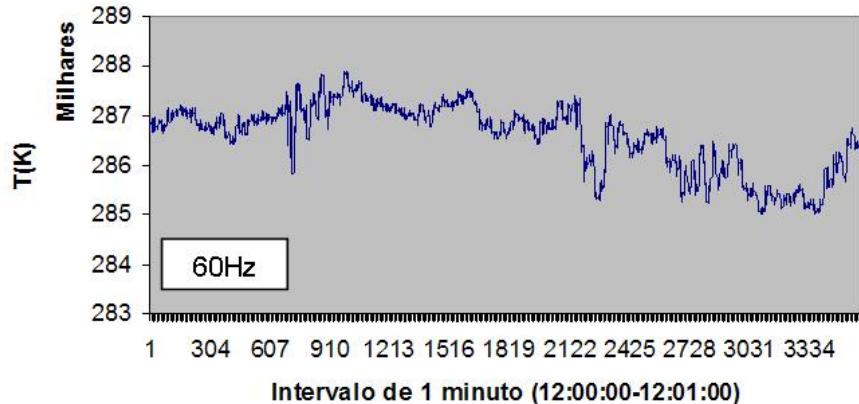
Assireu, Rosa, Ramos, Physica D, 2002  
DOI: 10.1016/S0167-2789(02)00527-4

# ST no Mundo Real

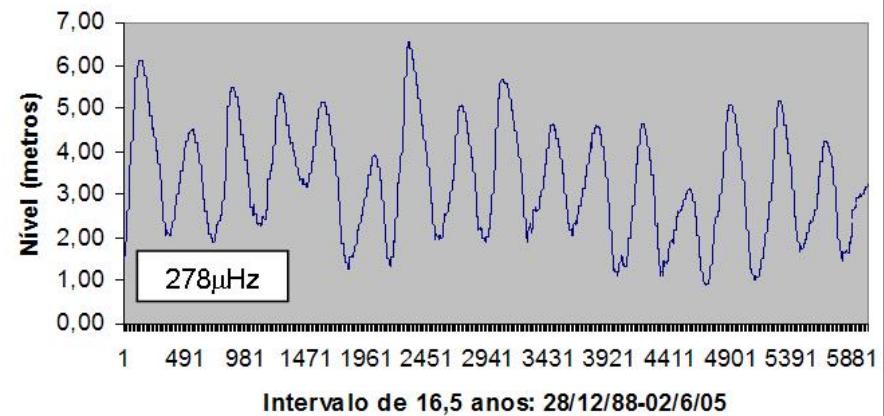




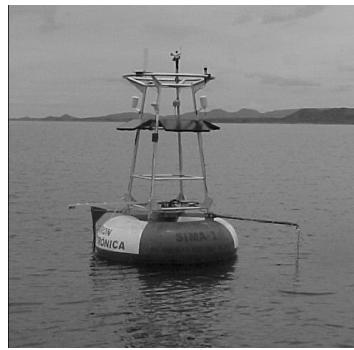
### Floresta Amazônica-Copa



### Cotas do Rio Paraguay/Pantanal



### ST no Mundo Real



# Transformações de Domínios

Visão Genérica sobre *Transformações* entre domínios para Caracterização de Componentes/Escalas que geram uma função em seu domínio fundamental.

Considere

- Função Genérica  $f(\mu_1)$

- Função Especial  $s(\mu_1, \mu_2)$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \dots$$

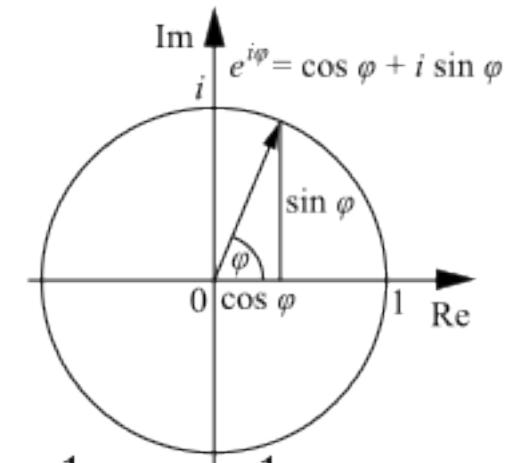
→ transformação genérica:

$$T [ f(\mu_1) ] = \int f(\mu_1) s(\mu_1, \mu_2) d \mu_1$$

$$s(\mu_1, \mu_2) = e^{(-2\pi i \mu_1 \mu_2)} = 1 / (\cos 2\pi \mu_1 \mu_2)$$

$$\mu_1 = t$$

$$\mu_2 = L \text{ ou } 1/L = k \text{ ou } f$$



$$A(f) = \int_{-\infty}^{+\infty} e^{-i\omega t} A(t) dt$$





# Análise Espectral

- Joseph Fourier (1807)
- Alfred Harr (1910)
- Alberto Calderón e Antoni Zygmund (1952)
- George Zweig (1975)
- Jean Morlet e Alexander Grossmann (1980) (formalismo)
- J.S.Liénard e X.Rodet (1980)
- Morlet, Arens, Fourseau, Giard (1982)



Geophysics 47, 203-206

- Y.Meyer
- I. Daubechies
- S.Mallat
- R. Fargé
- W. Gao e B.L. Li

> 1985



# Transformações de Domínios

Visão Genérica sobre *Transformações* entre domínios para  
Caracterização de Componentes/Escalas  
que geram uma função em seu domínio fundamental.

Considere

- Função Genérica  $f(\mu_1)$
- Função Especial  $s(\mu_1, \mu_2) \rightarrow$  transformação genérica:

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$$s(\mu_1, \mu_2) = e^{(-2\pi i \mu_1 \mu_2)} = 1 / (\cos 2\pi \mu_1 \mu_2)$$

$$\mu_1 = t$$

$$\mu_2 = \omega$$

Exercício



A transformada wavelet opera em uma  $f(\mu_1)$  utilizando uma família de  $s(\mu_1, \mu_2)$  que mantém o domínio fundamental na transformação através de suas propriedades de dilatação (coeficiente a) e translação (coeficiente b).

$$S_{a,b}(\mu_1, \mu_2) = \frac{1}{a} \Psi\left(\frac{t - b}{a}\right)$$

O termo ondeleta refere-se a um conjunto de funções com forma de pequenas ondas geradas por dilatações e translações de uma função geradora simples, a **ondeleta-mãe**, através da transformação:

$$\Psi_{a,b}(t) = \frac{1}{2} \Psi\left(\frac{t - b}{a}\right)$$

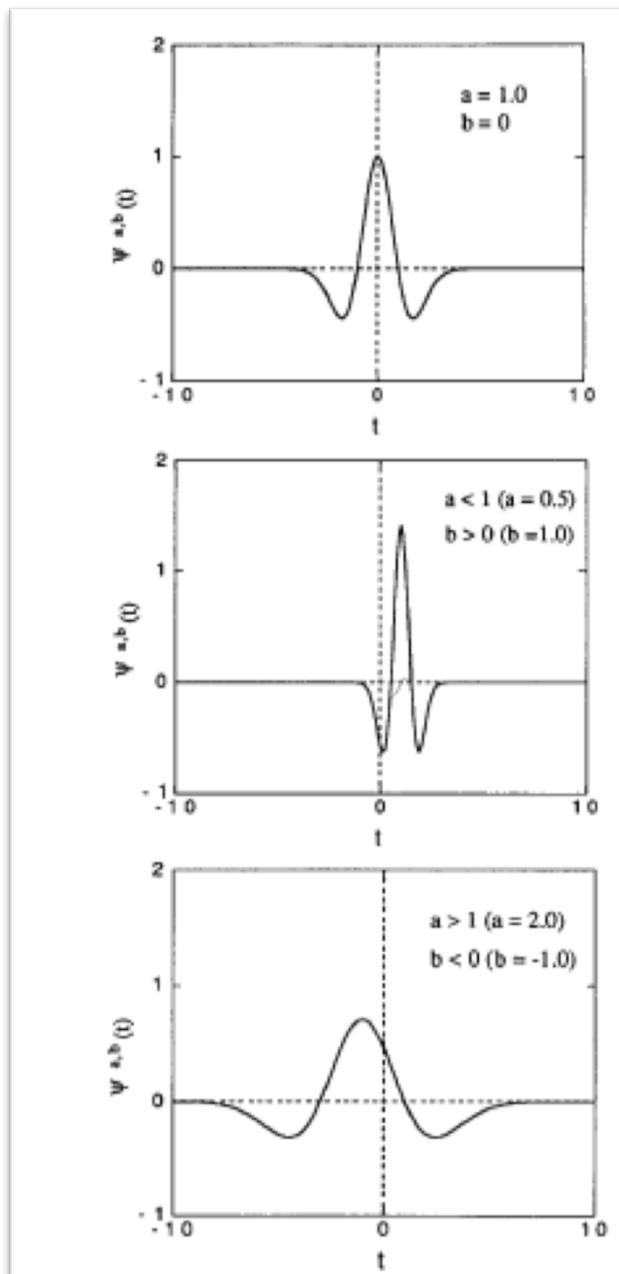


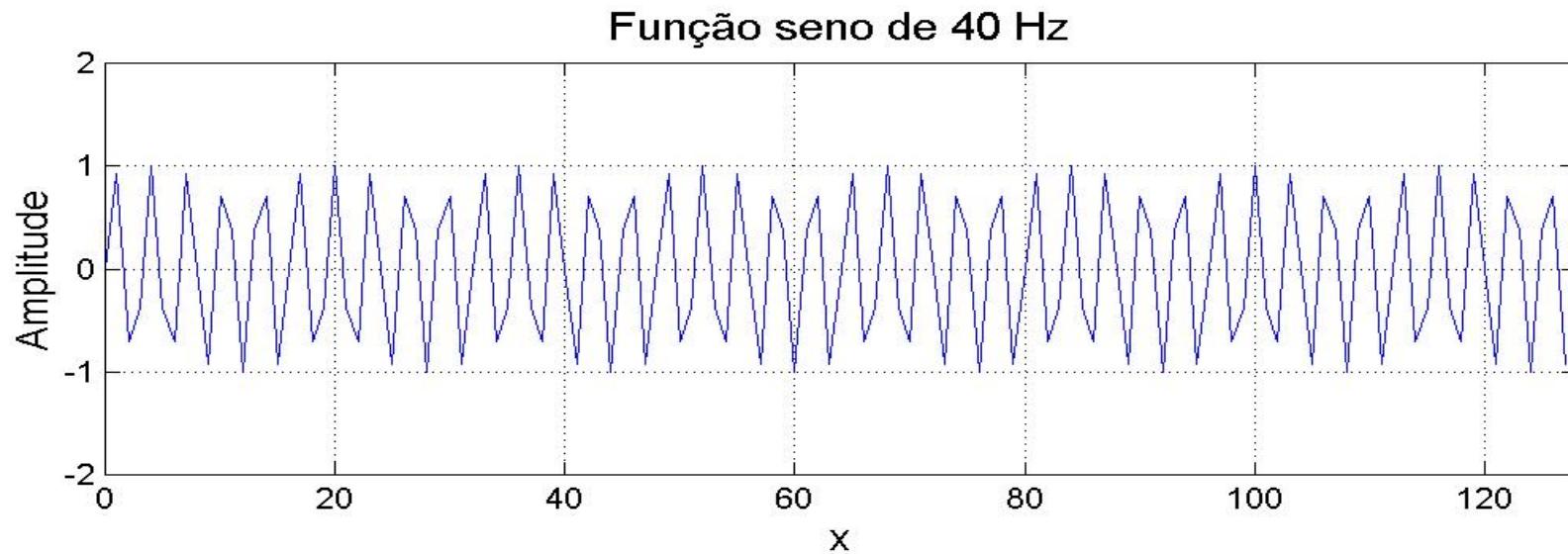
FIG. 1. Changes in the shape of the wavelet function  $\psi^{a,b}(t)$  with selected values of  $a$  and  $b$  for the "mother wavelet" given by the "Mexican hat" function  $\psi(t') = (1 - t'^2) \exp(-t'^2/2)$ .

# Transformada em Ondeleta

- Portanto, pode-se definir a Transformada em Ondeletas (TO) de uma função temporal  $f(t)$ , representada por  $W(a,b)$ , usando a equação anterior como função moduladora, de modo análogo ao que foi feito para a TF, através de:

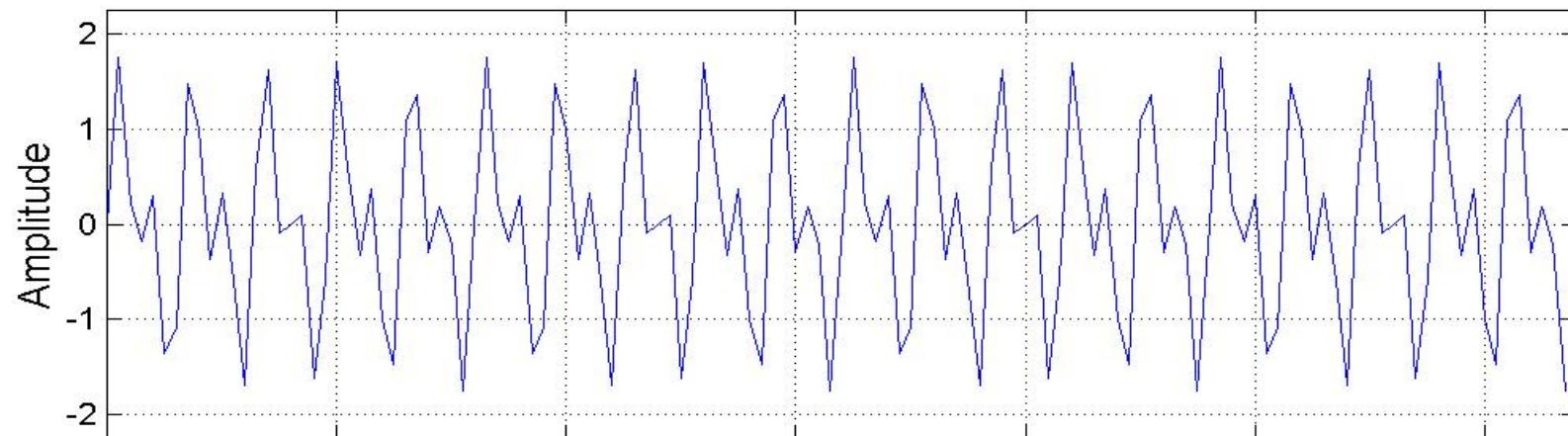
$$W(a,b) = \langle f, \Psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t) \Psi_{a,b}(t) dt$$

# VANTAGEM 1

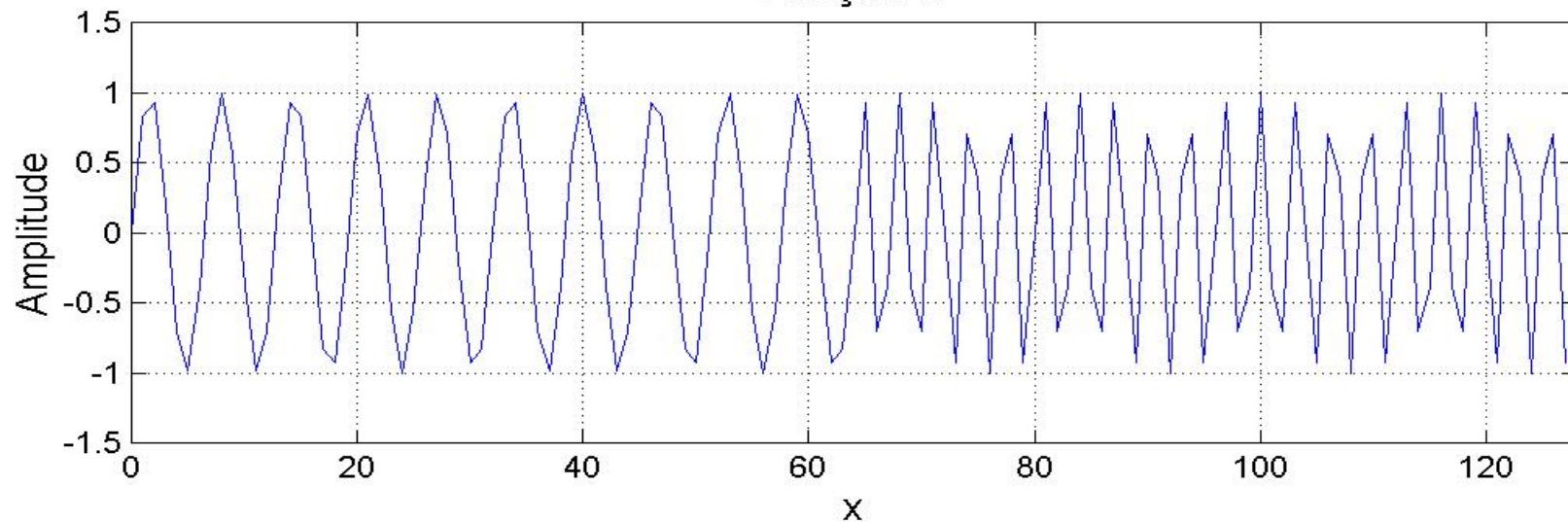


# COMPONDO SINAIS

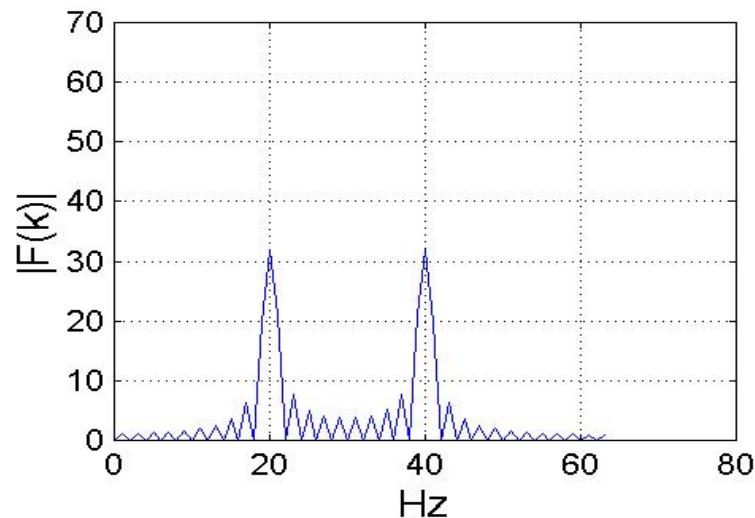
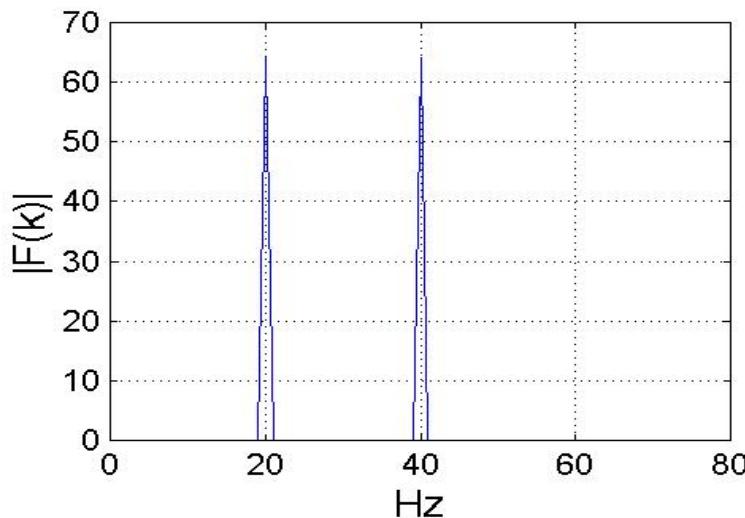
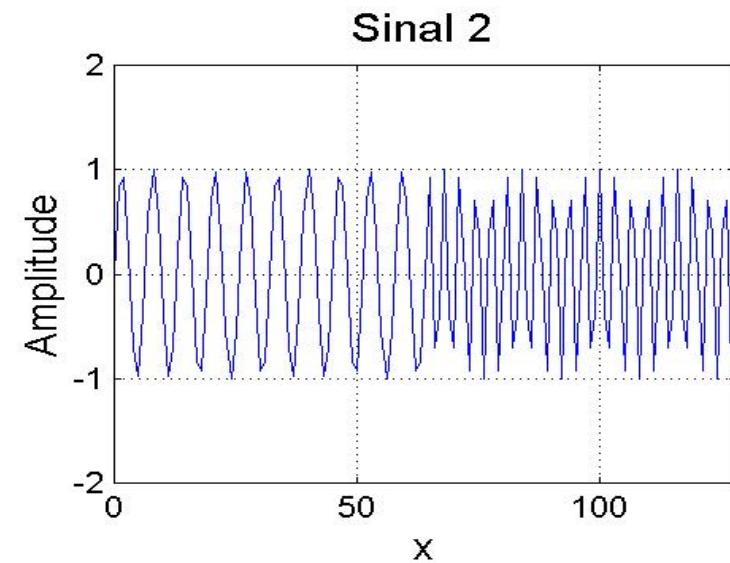
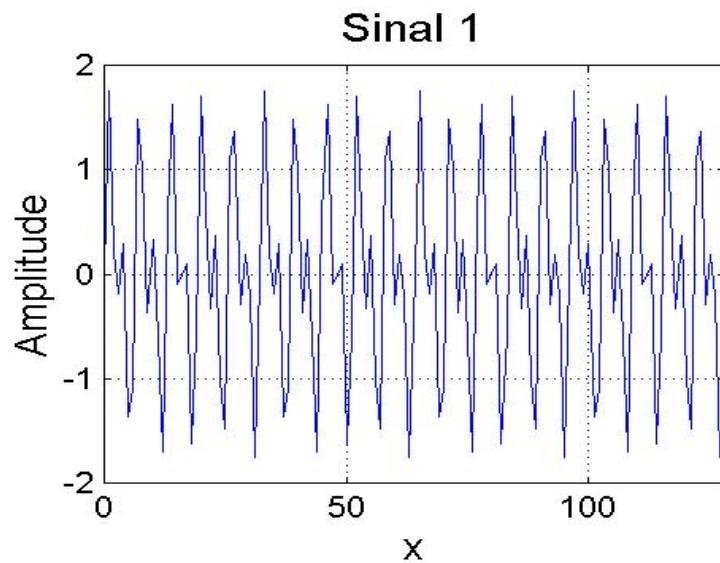
Função 1



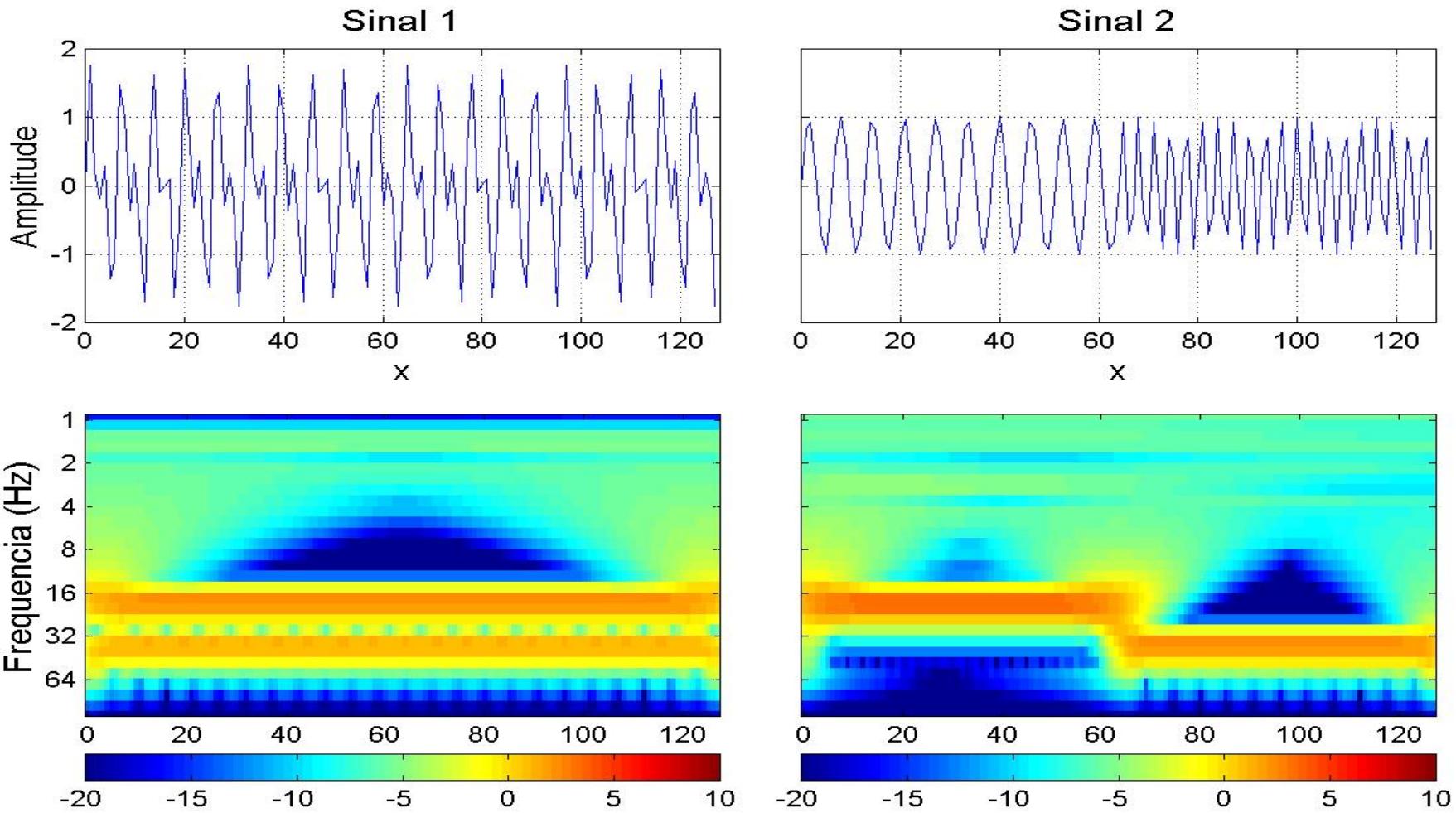
Função 2



# ESPECTRO DE ENERGIA



# PERIODOGRAMA DE ONDELETA



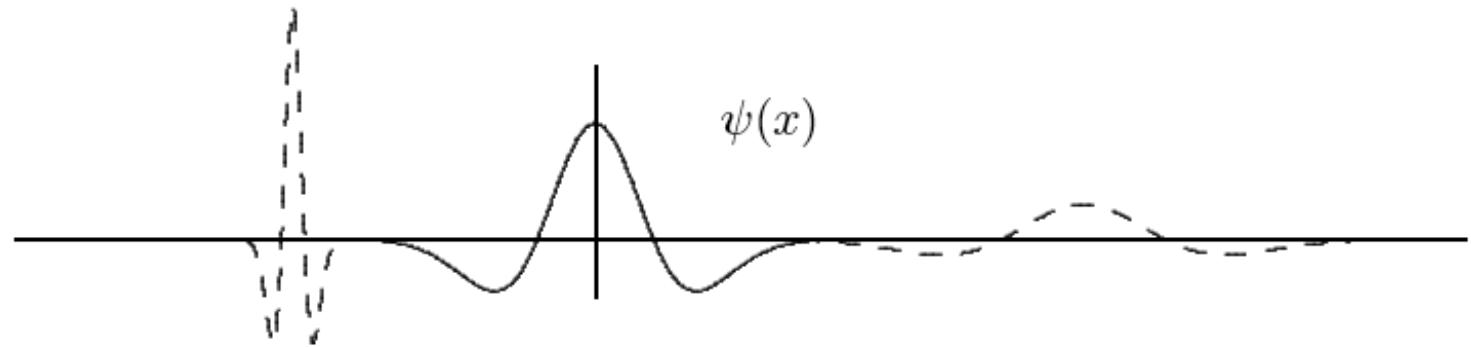
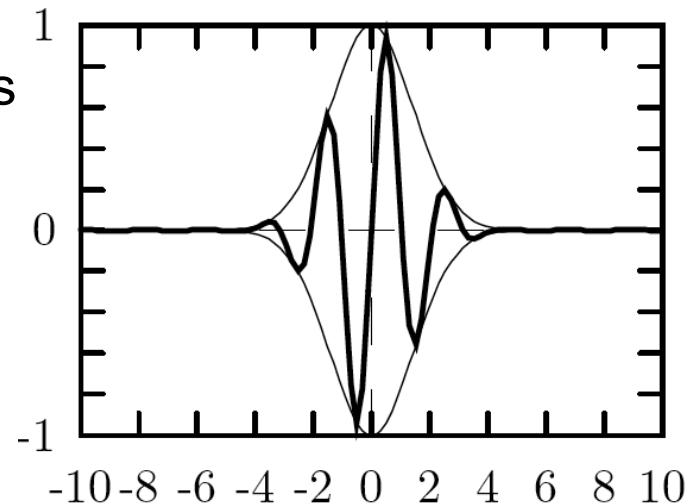
# Origem: Transformações de Gabor

- Windowed fourier transform or Gaborlets

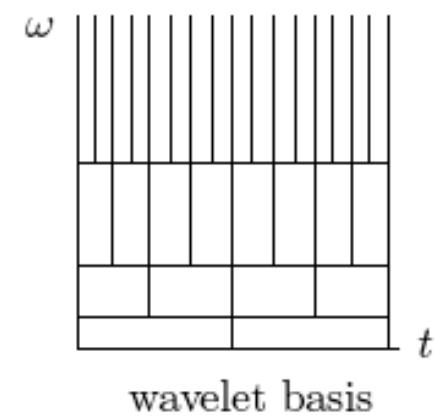
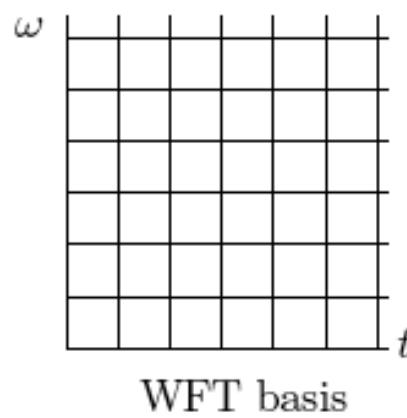
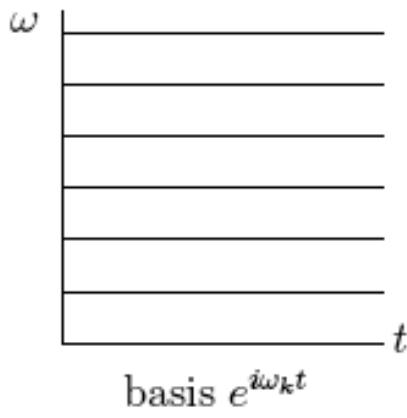
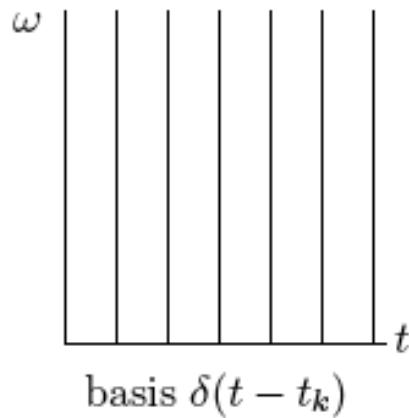
$$\psi_{\omega,b}(t) = g(t-b)e^{i\omega t}$$

- Wavelets :

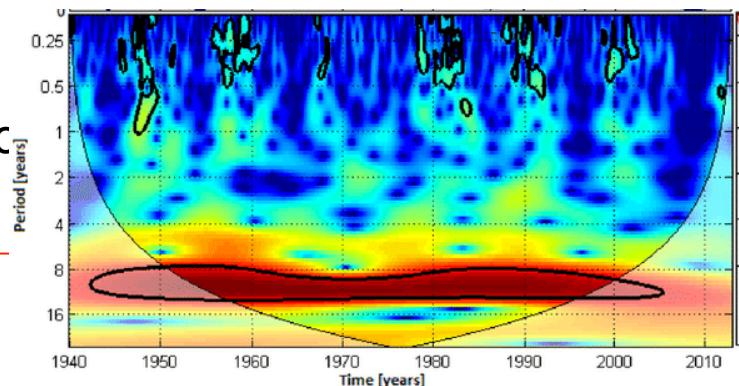
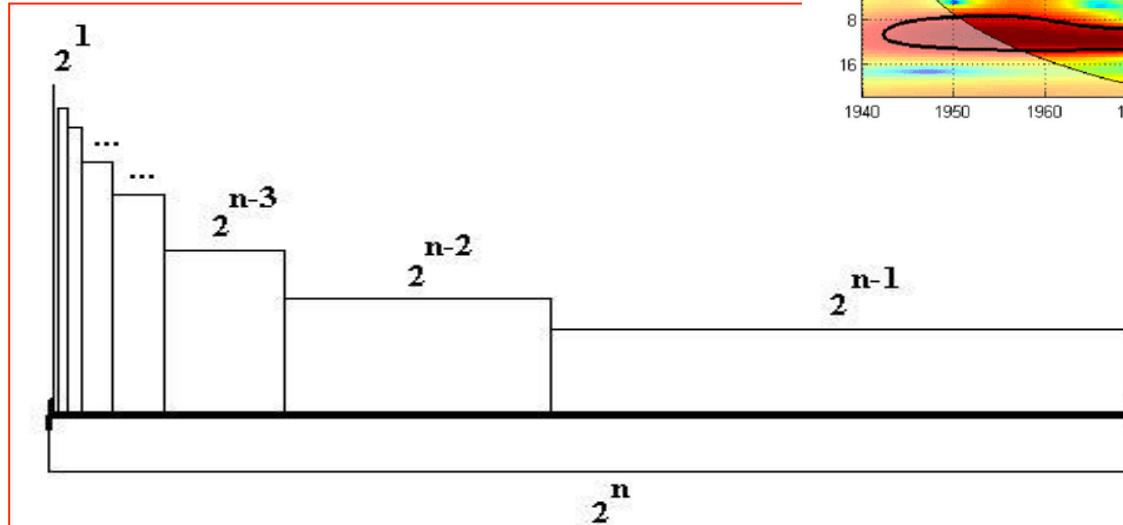
$$\psi_{a,b}(t) = \frac{1}{\sqrt{b}}\psi\left(\frac{t-a}{b}\right)$$



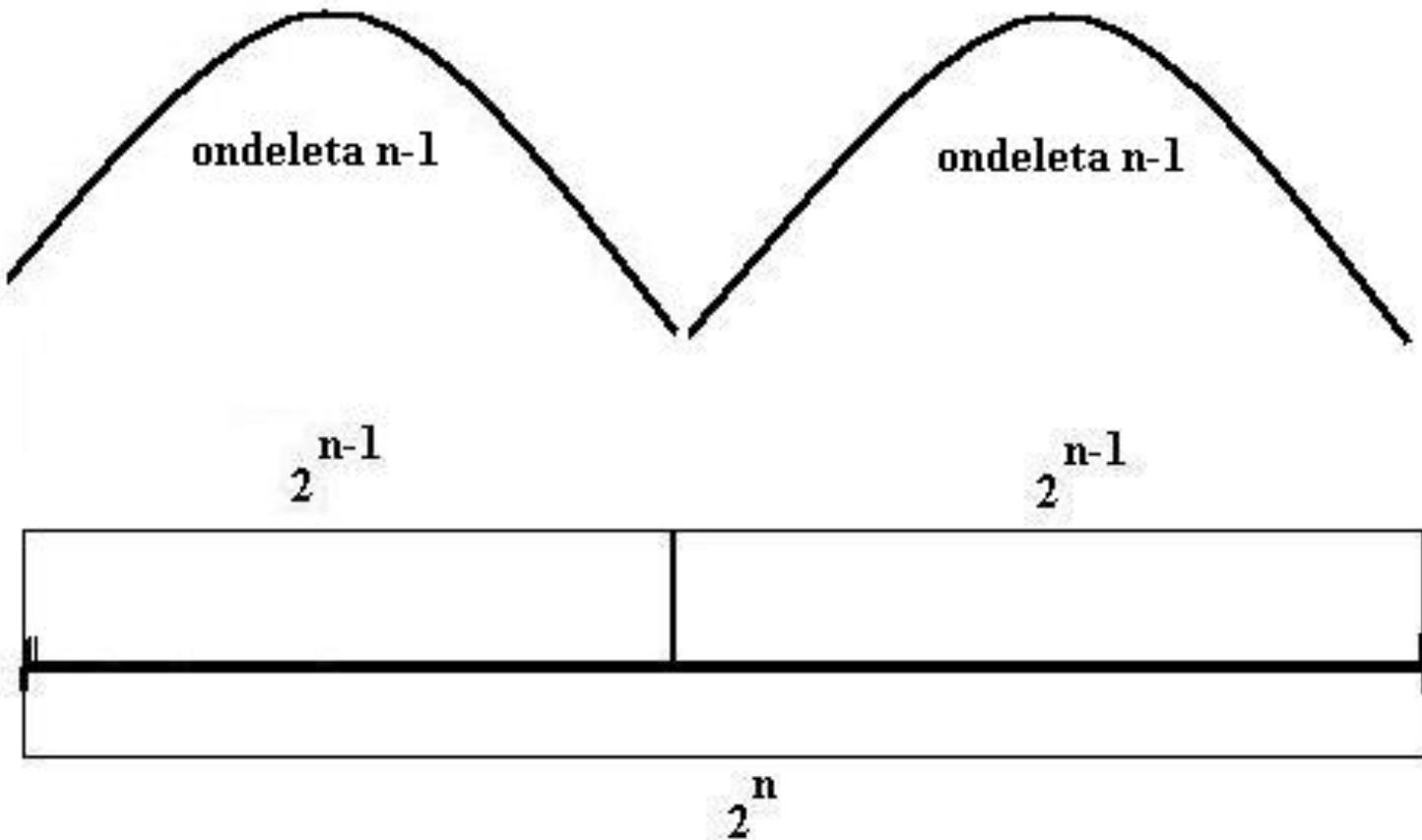
# Bases de Núcleos para Transformações de Domínios



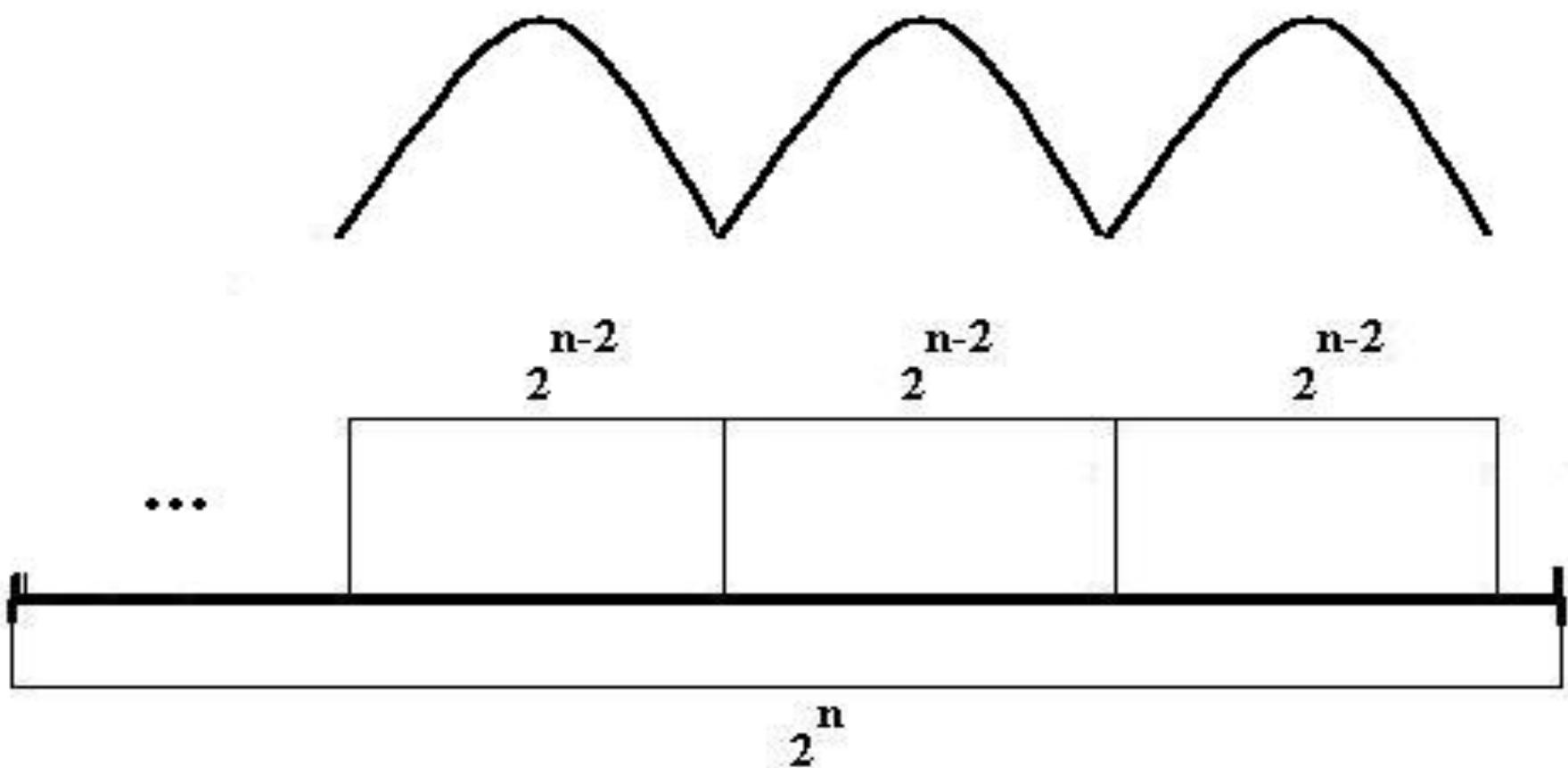
Caso Discreto: cascata **diádica** = número de pc  
então será o número de freqüências possíveis



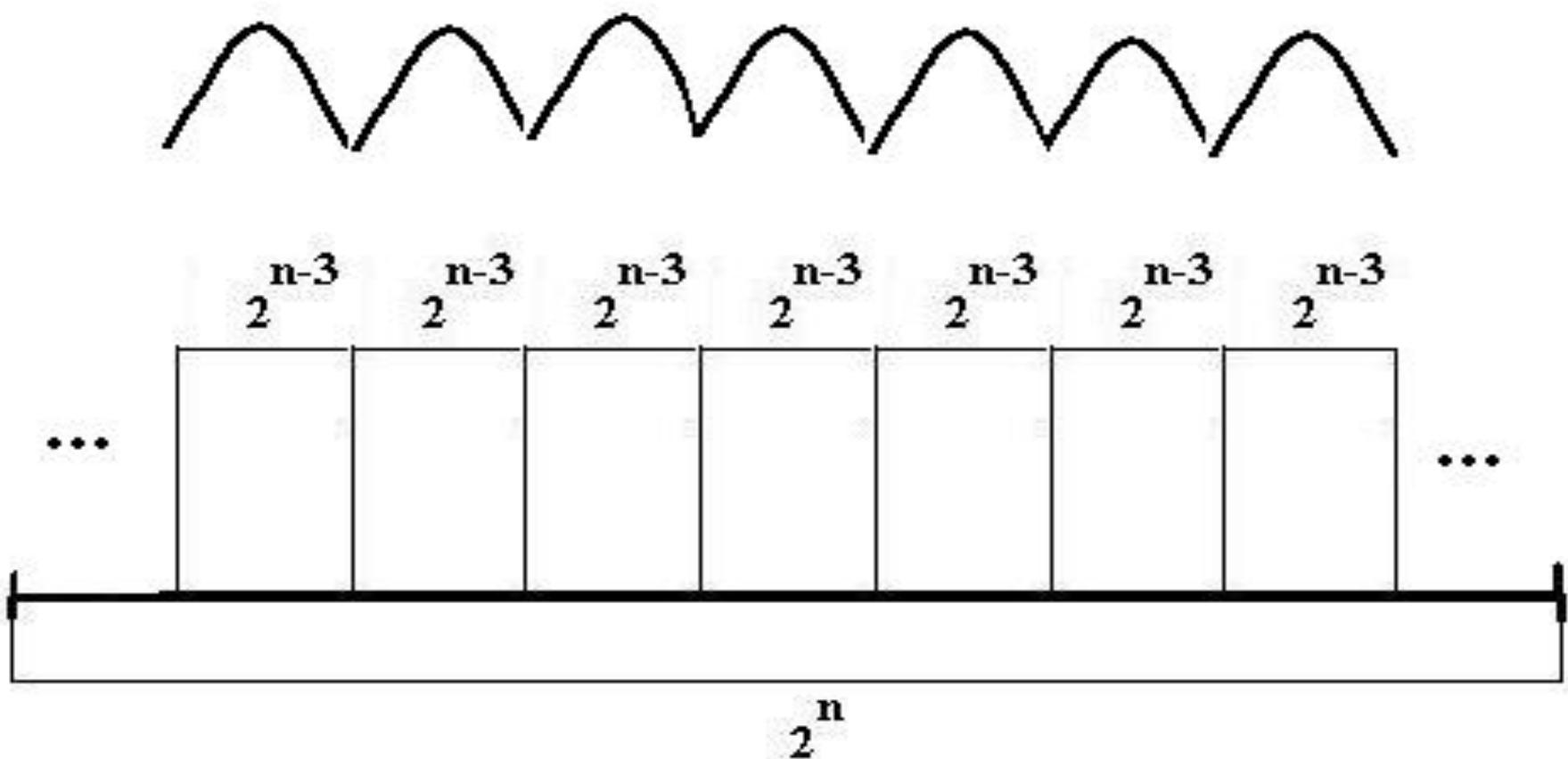
# Translação e Dilatação da Ondeleta Escala $n-1$



# Translação e Dilatação da Ondeleta Escala $n-2$

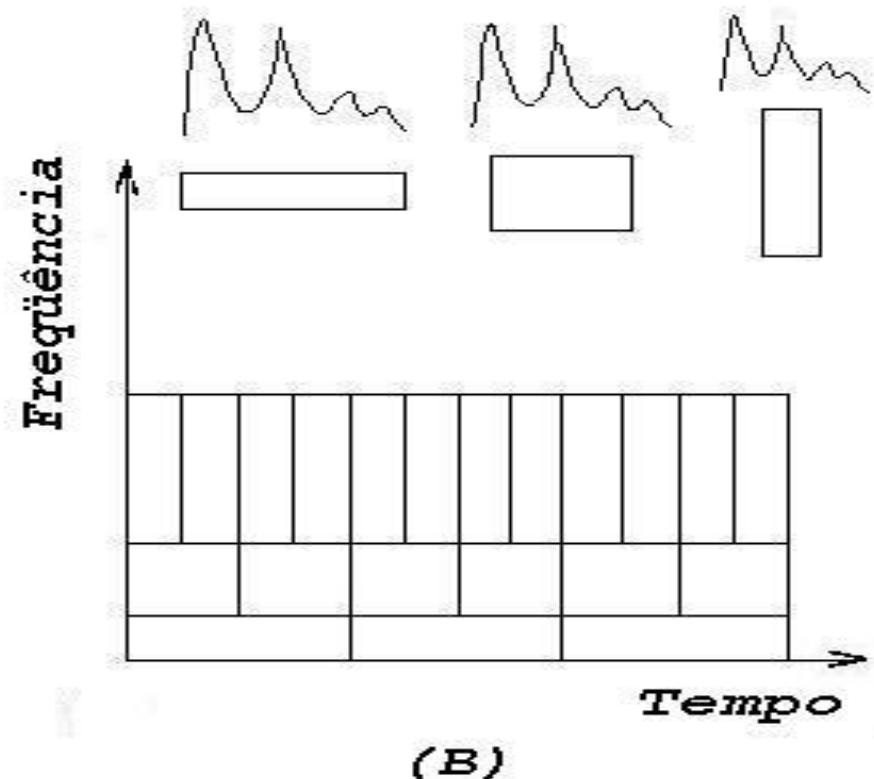
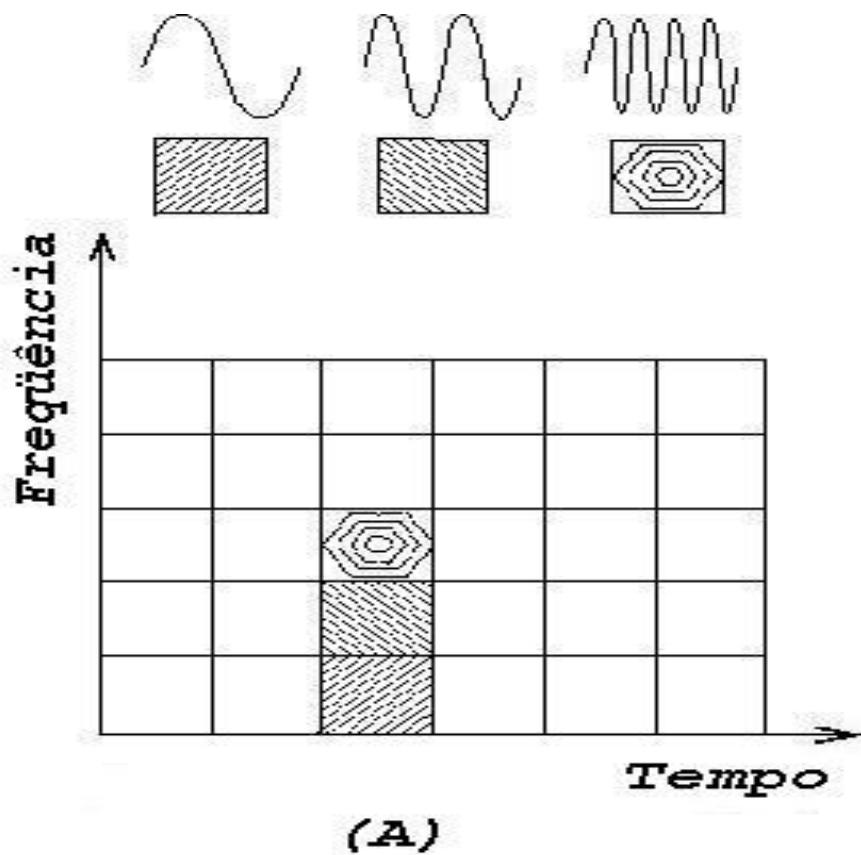


# Translação e Dilatação da Ondeleta Escala $n-3$



# Fourier e Ondeleta

- Análise em Tempo e Frequencia



# TIPOS DE FUNÇÕES ONDELETAS

Discretas: Biortogonal, Haar e etc

1. Utilização para filtragem e decomposição de sinais

Contínuas: Chapéu Mexicano, Morlet e etc

1. Construção de periodogramas para análise em tempo e escala.

## 2.3. Ergodicidade (Teorema de Birkhoof)

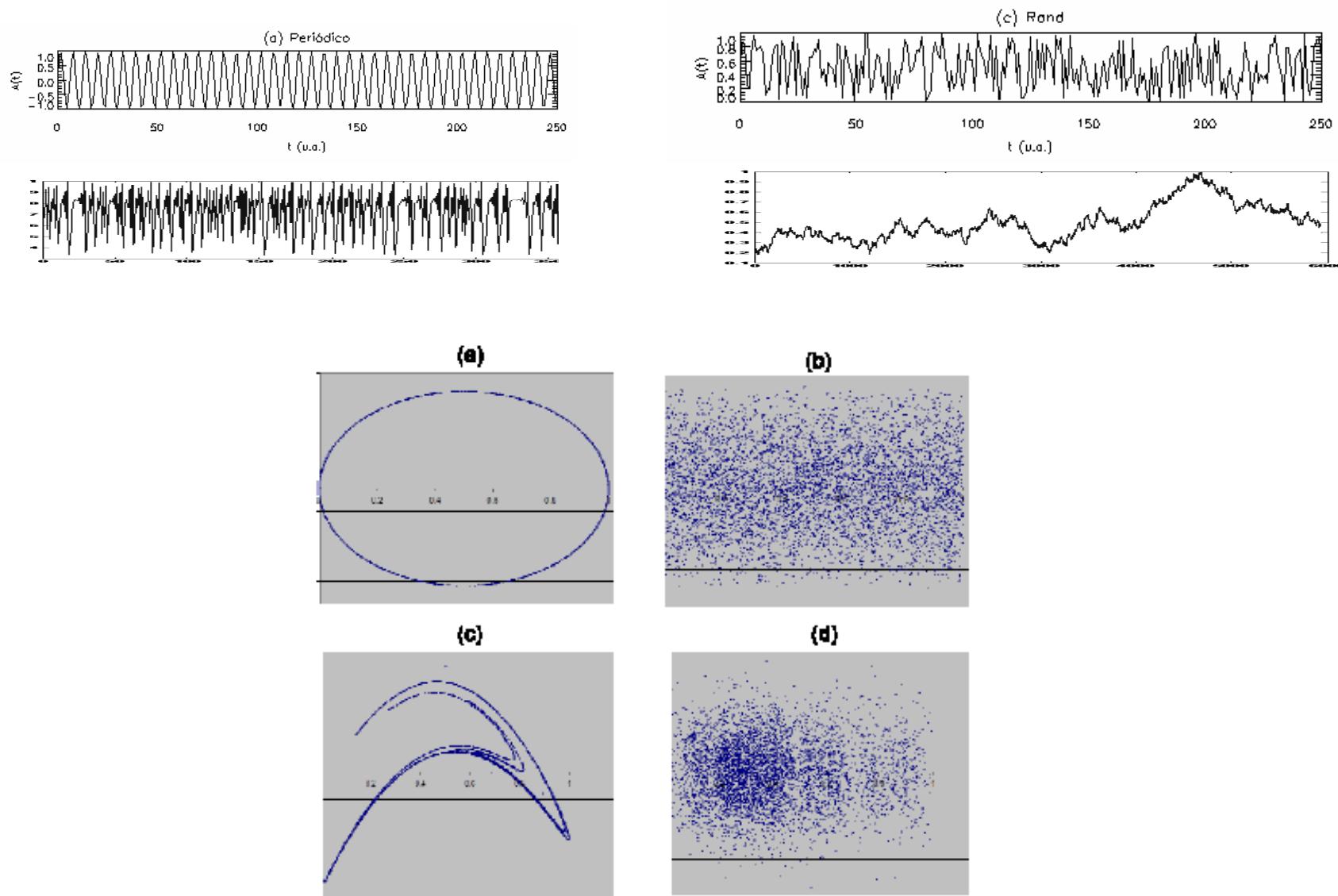
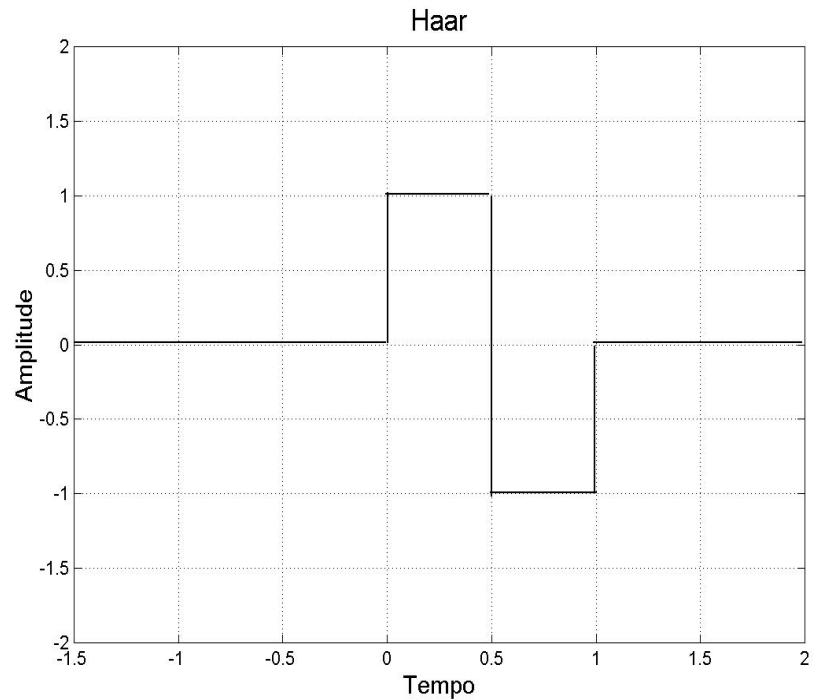


Figura 8.12 - Resultados da reconstrução de 1<sup>a</sup> ordem (*Portraits*) para os processos: (a) oscilatório periódico; (b) pseudo-aleatório; (c) Hénon caótico ( $k_1=1.4$  e  $k_2=0.3$ ); e (d) multifractal Br  $1/\omega^2$ .

# Ondeletas Discretas

- Função de Haar:

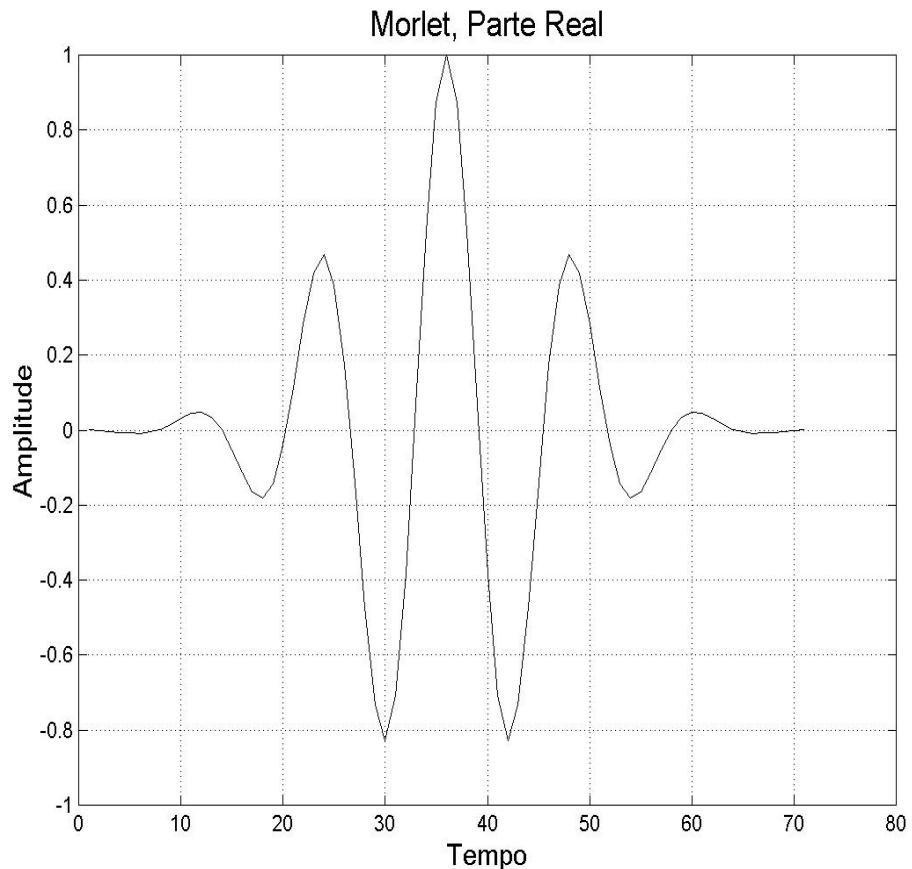
$$\Psi_{j,i}(t) = \begin{cases} 1, & 0 \leq t \leq 1/2 \\ -1, & 1/2 \leq t < 1 \\ 0, & \text{outros valores} \end{cases}$$



# Ondeletas Contínuas

- Função de Morlet:

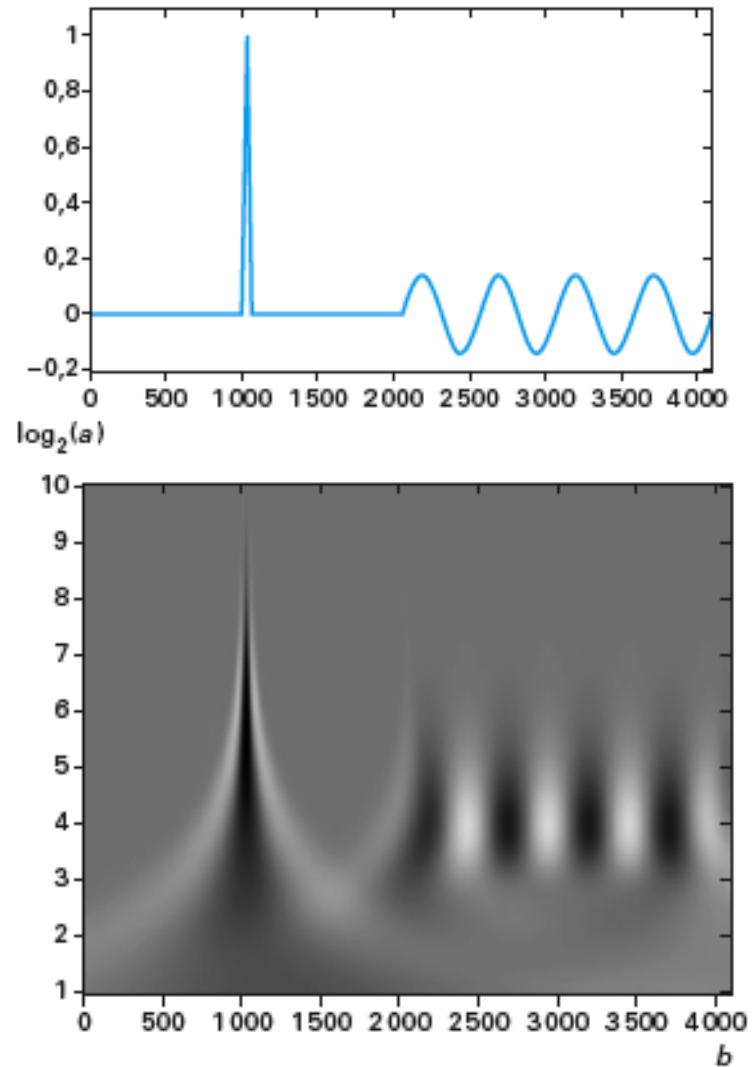
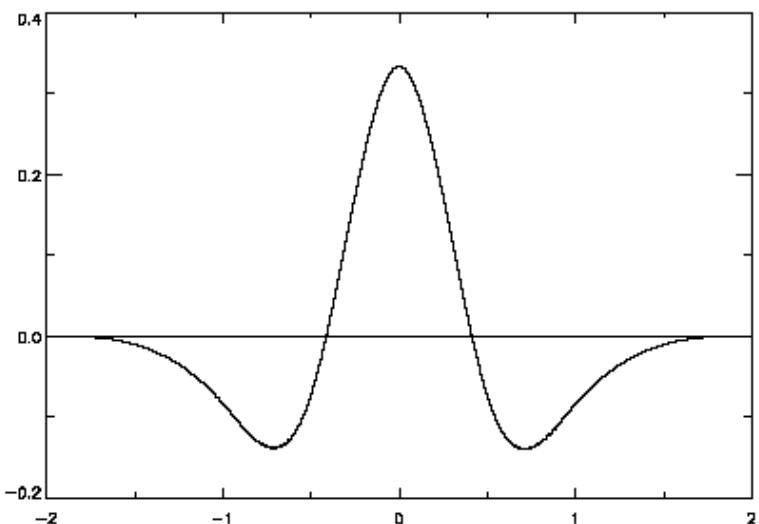
$$\Psi_{j,i}(t) = e^{iKt} e^{-t^2/2}$$



# Continuous Wavelet Transform

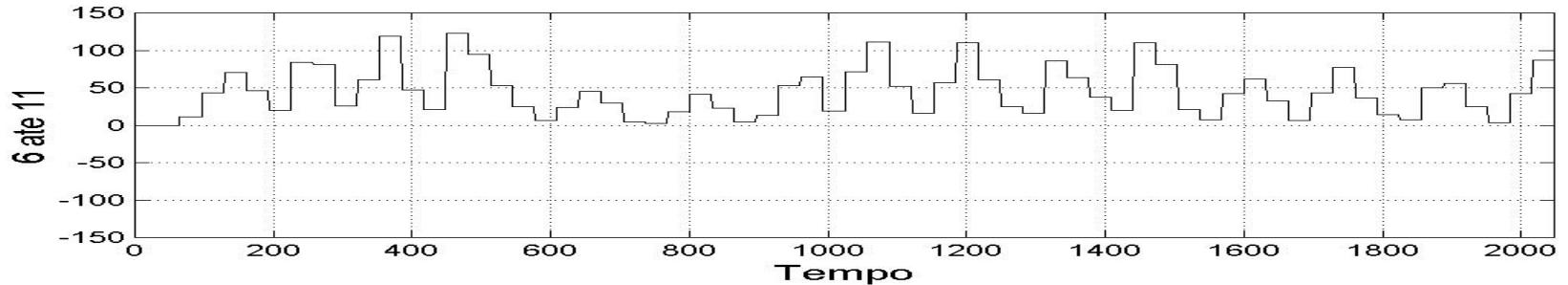
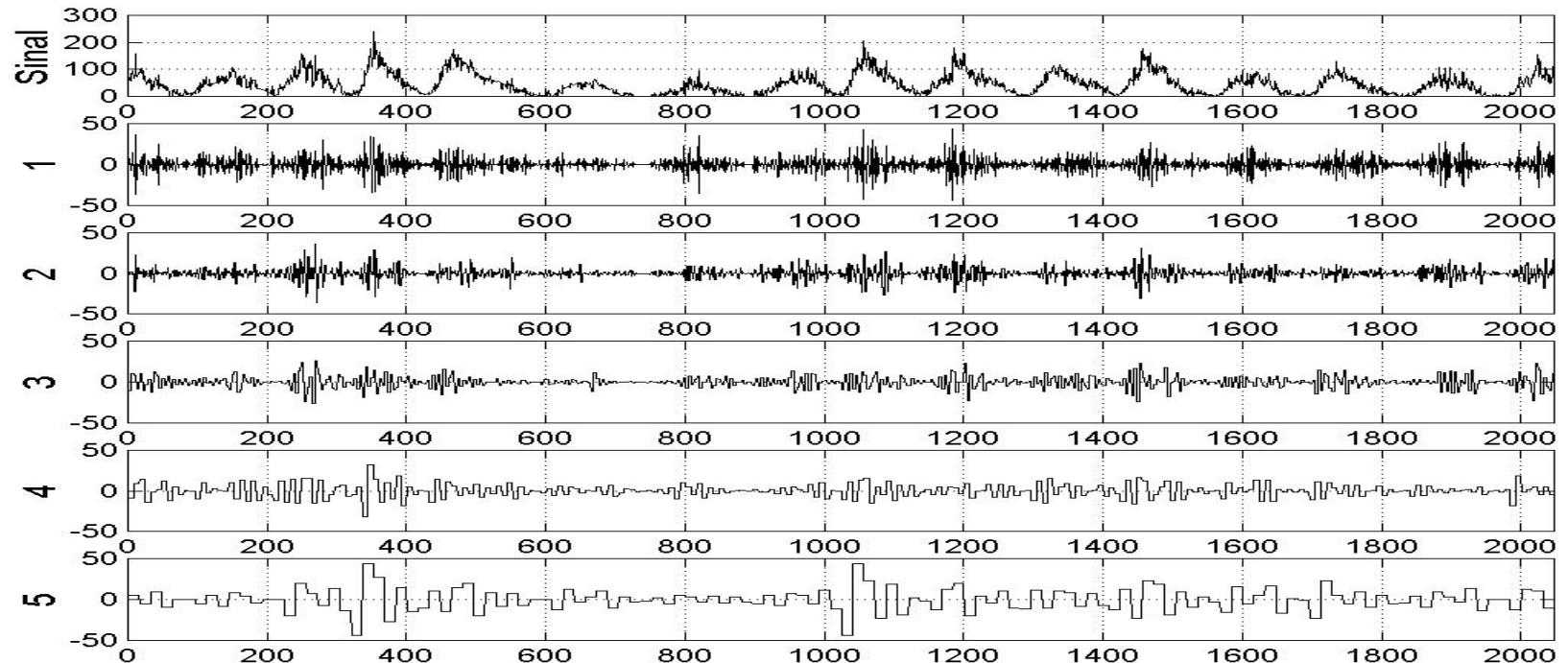
- Example : The mexican hat wavelet

$$g(x) = (1 - x^2)e^{-\frac{1}{2}x^2}$$

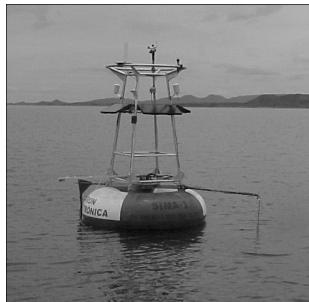


# Decomposição de Sinais

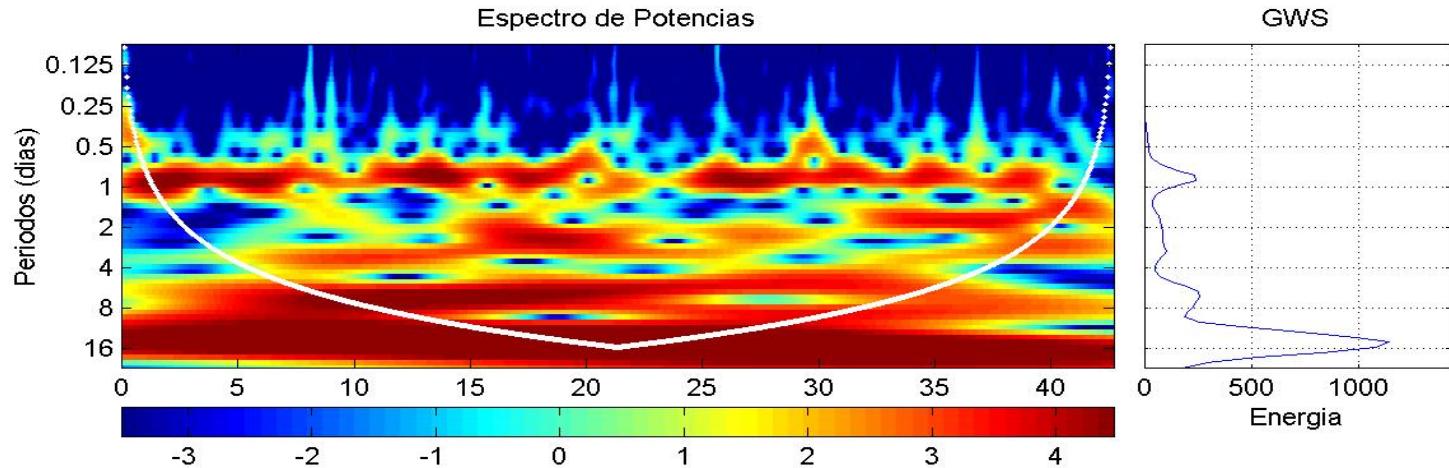
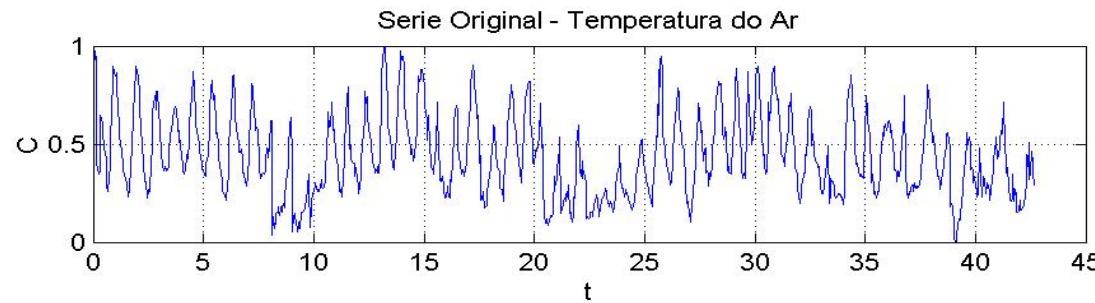
- Apenas as ondeletas discretas permitem esta

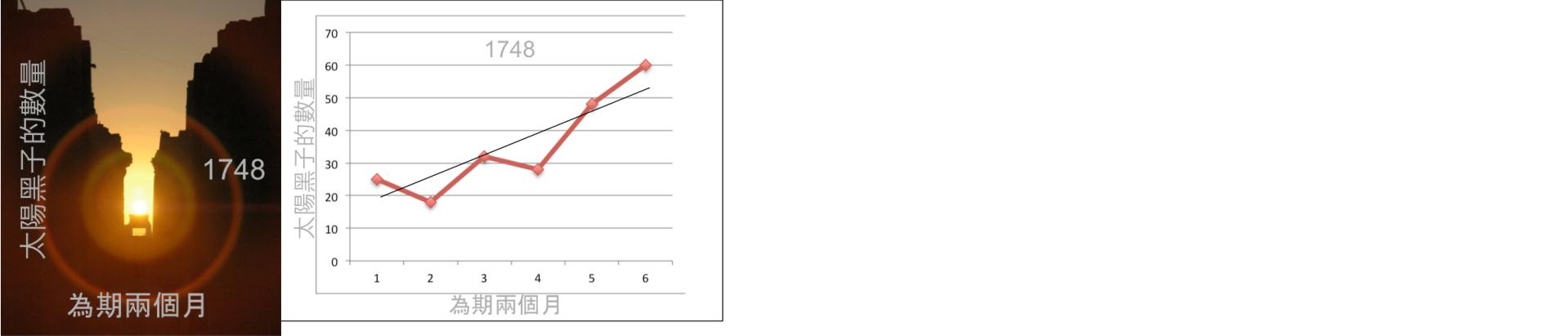


# SIMA

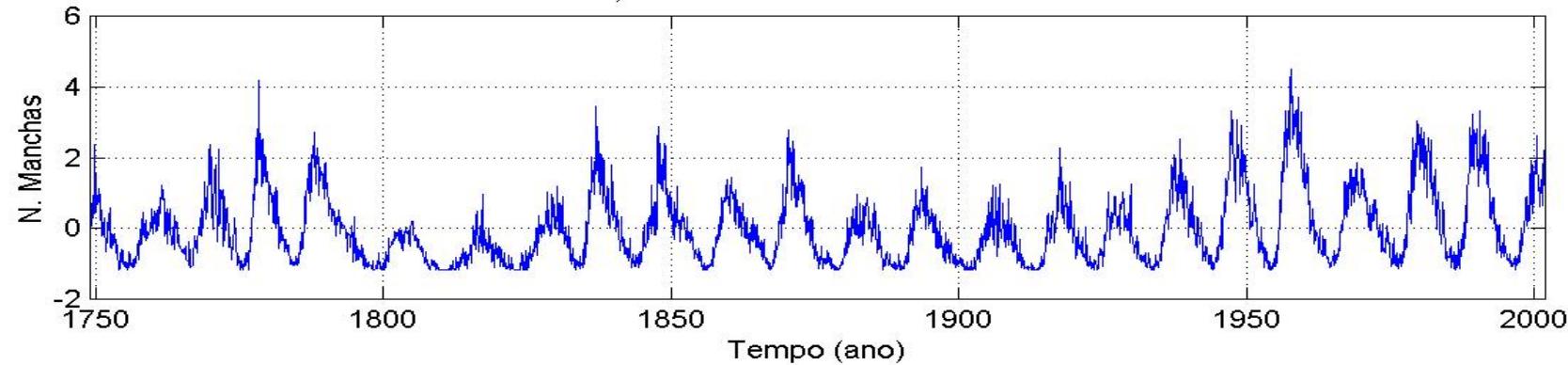


## Global wavelet spectrum

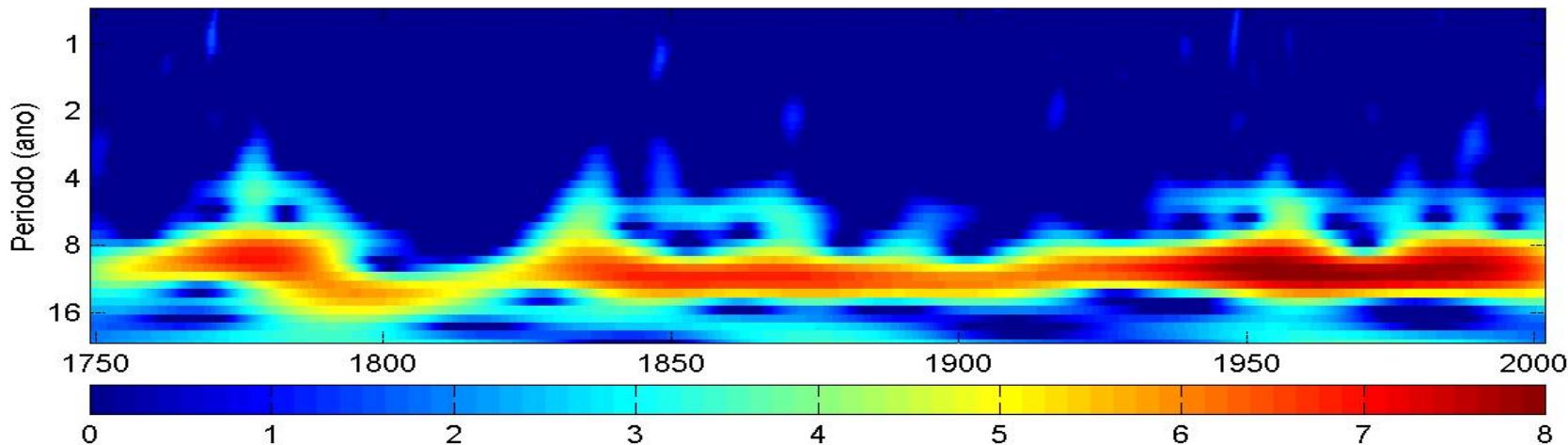


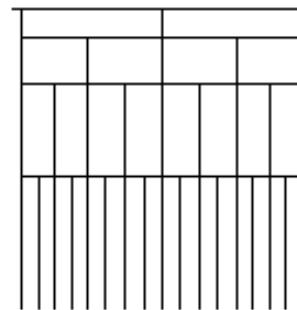
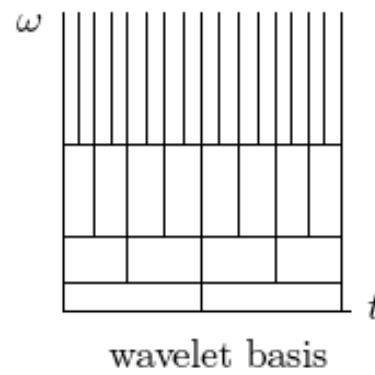
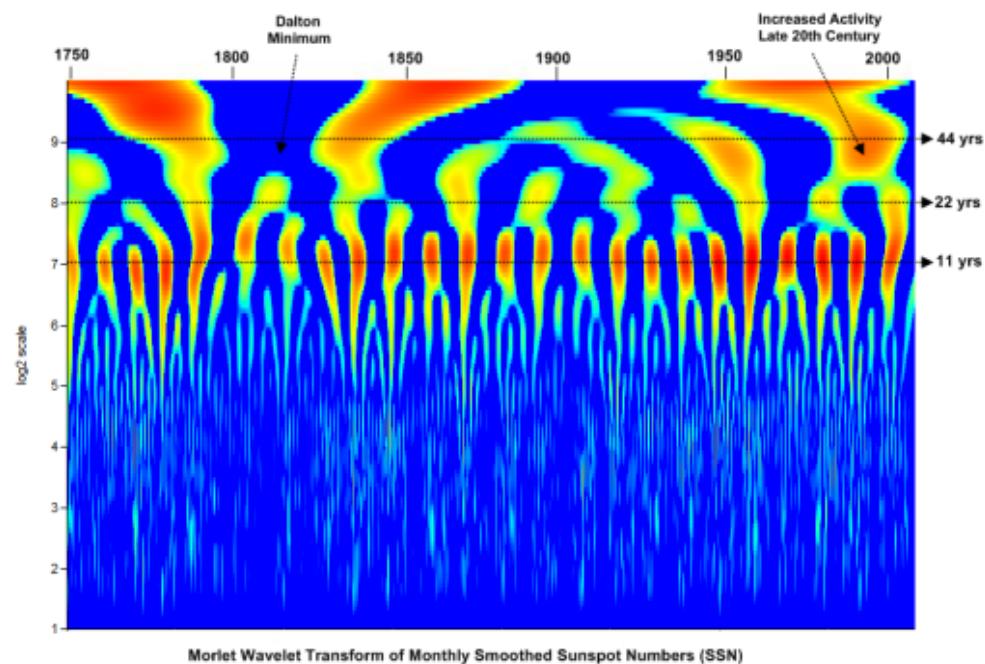
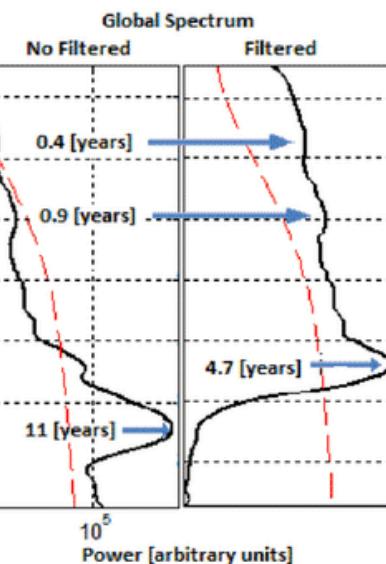
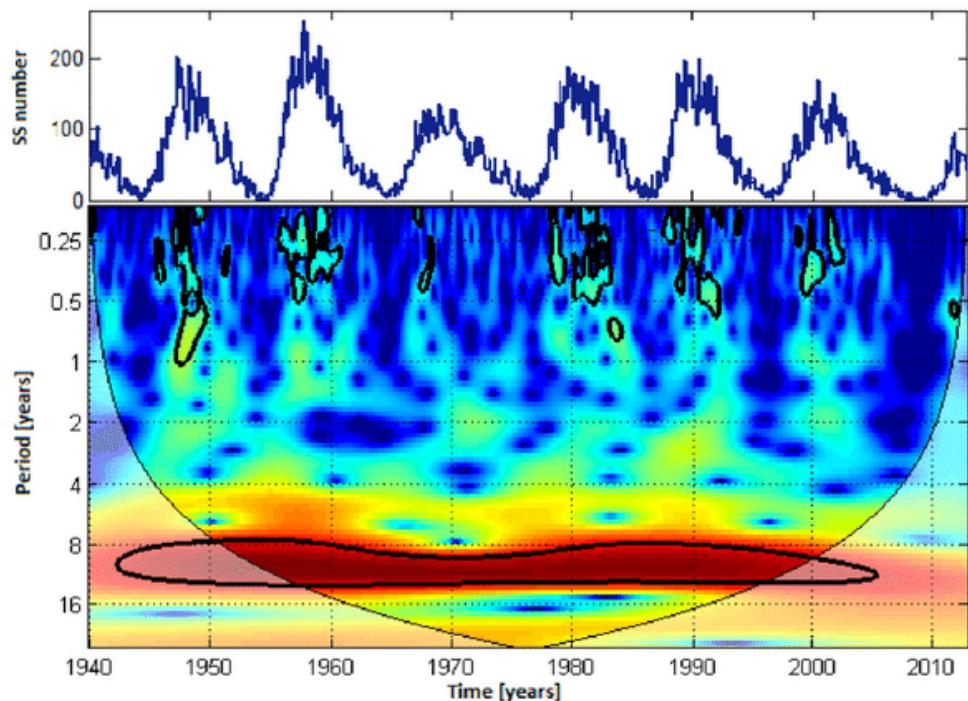


a) Numero de Manchas Solares

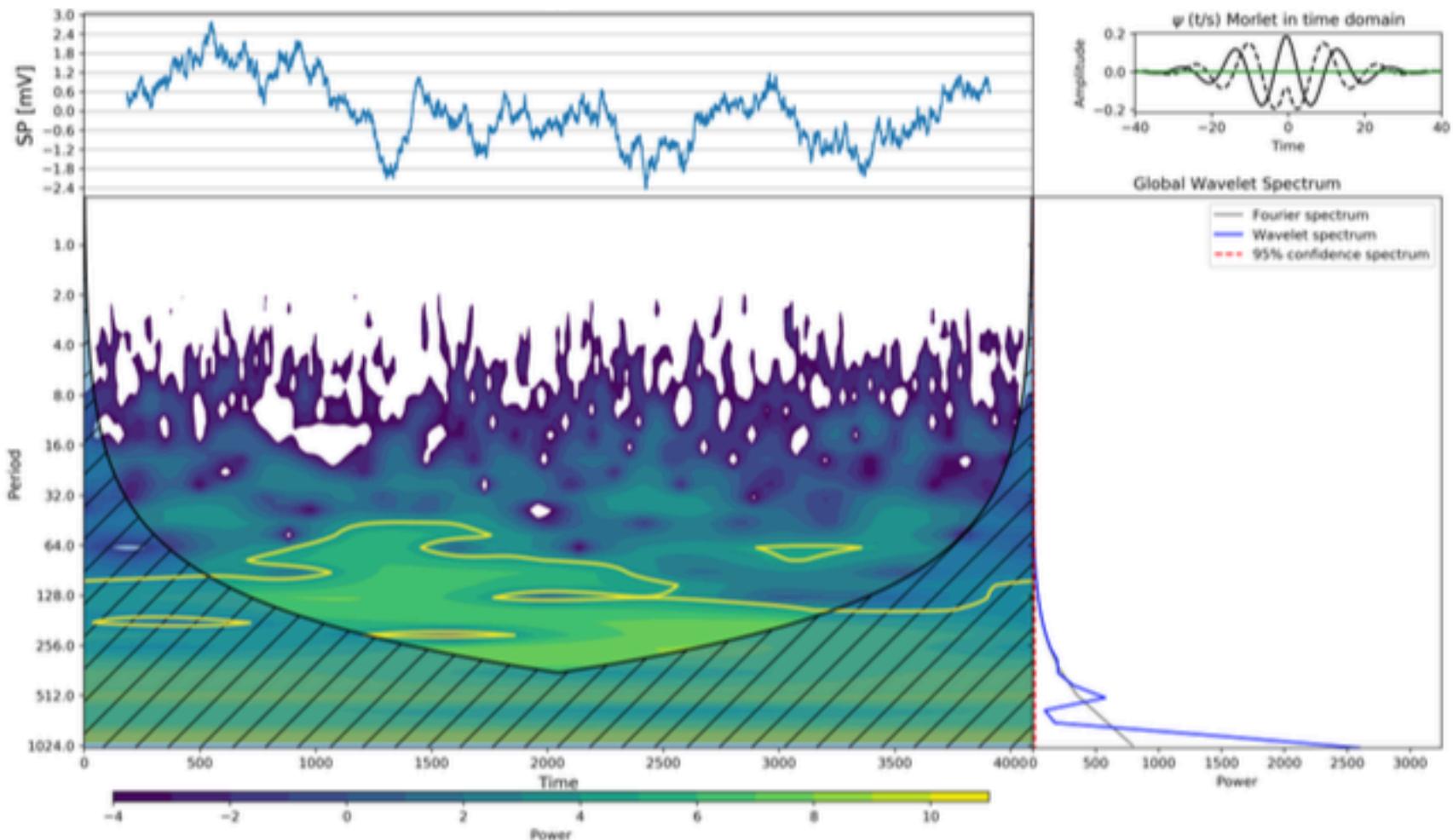


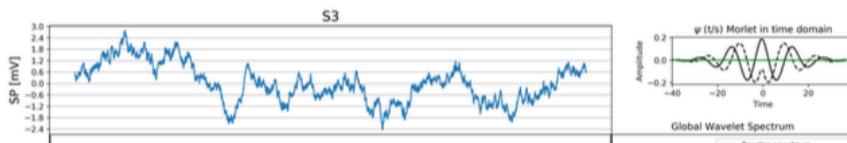
b) Periodograma de Wavelet p/ Manchas Solares





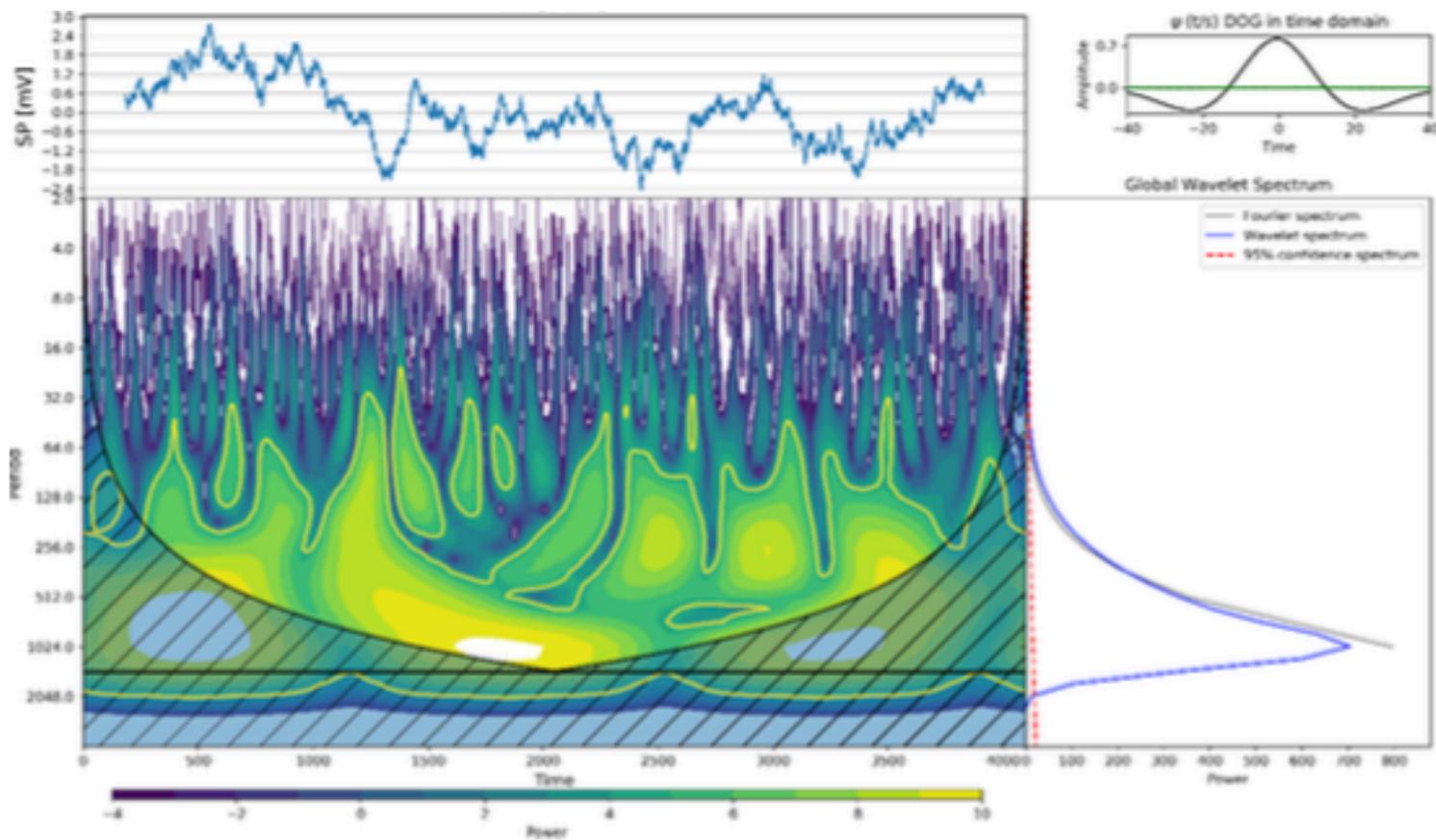
## Algoritmo Morlet\_Waipy.py (Python):





## 8- Global Wavelet Spectrum

- 8.1. Utilize o [Waipy](#) para obter o GWS (Morlet) de ~~todos~~ as ST do exercício 6.2.
- 8.2. Repita 9.1. utilizando uma Db8. **DoG**





## Fluctuation analysis of solar radio bursts associated with geoeffective X-class flares<sup>☆</sup>

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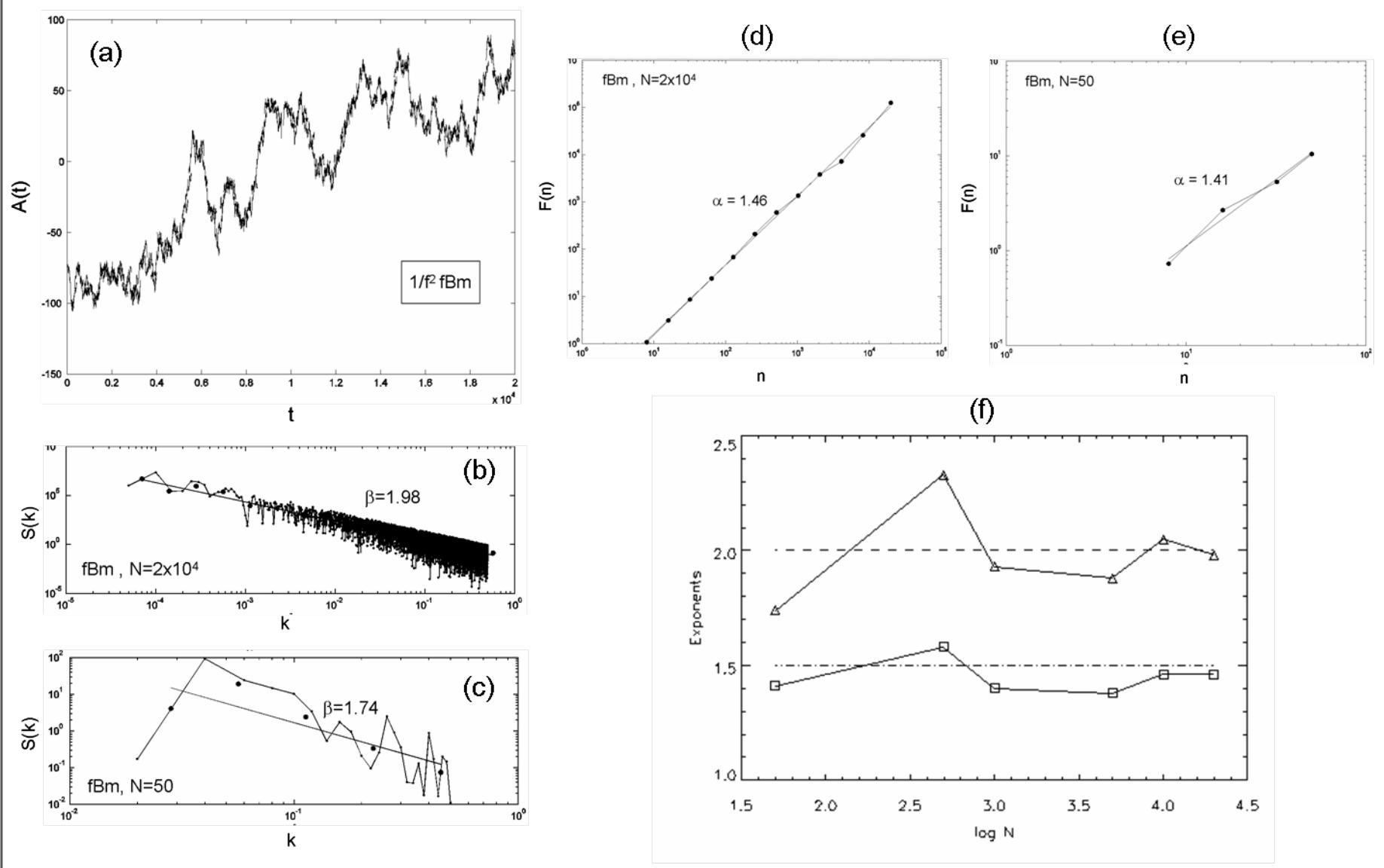
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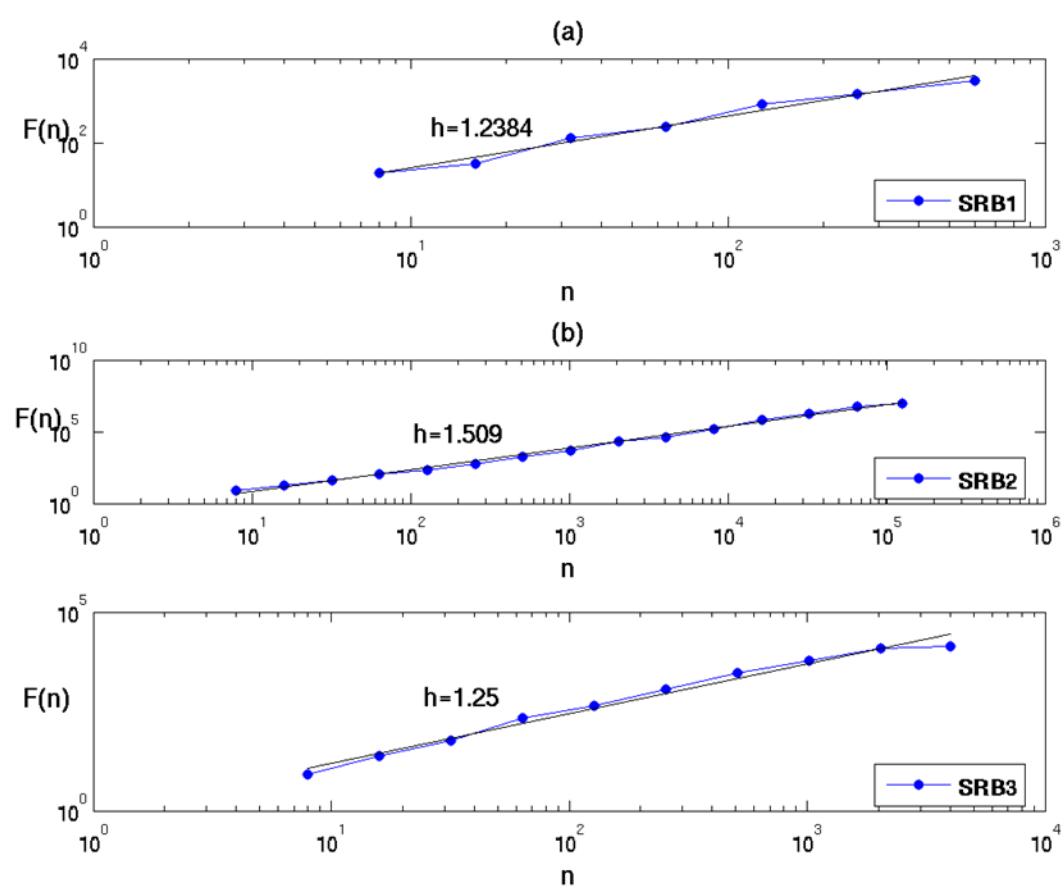
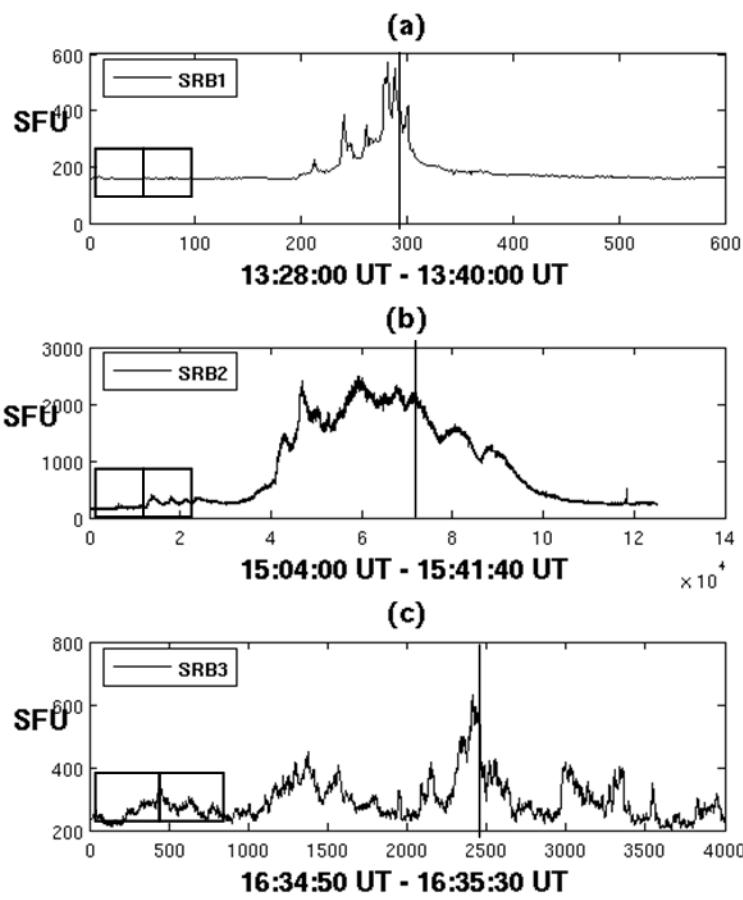
### ABSTRACT

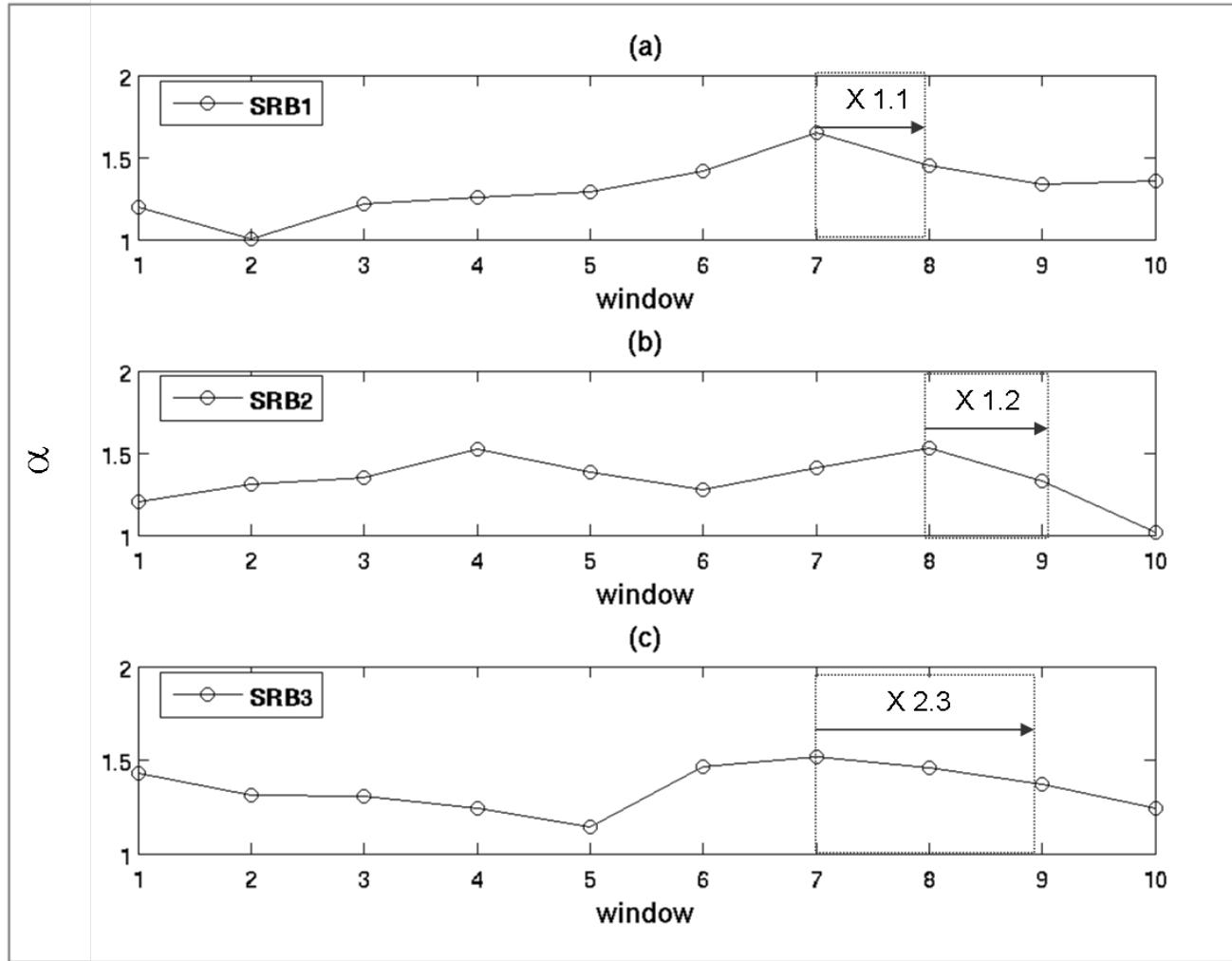
High temporal resolution solar observations in the decimetric range (1–3 GHz) can provide additional information on solar active regions dynamics and thus contribute to better understanding of solar geoeffective events as flares and coronal mass ejections. The June 6, 2000 flares are a set of remarkable geoeffective eruptive phenomena observed as solar radio bursts (SRB) by means of the 3 GHz Ondrejov Observatory radiometer. We have selected and analyzed, applying detrended fluctuation analysis (DFA), three decimetric bursts associated to X1.1, X1.2 and X2.3 flare-classes, respectively. The association with geomagnetic activity is also reported. DFA method is performed in the framework of a radio burst automatic monitoring system. Our results may characterize the SRB evolution, computing the DFA scaling exponent, scanning the SRB time series by a short windowing before the extreme event. For the first time, the importance of DFA in the context of SRB monitoring analysis is presented.

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# Auto-correlation $C(L)$ : Power Spectrum x Detrended Fluctuation Analysis







**Table 1**  
The basic characteristics of the June 6, 2000 SRBs observed at 3 GHz.

Event	$\tau$ (s)	$t_0$ (UT)	$\Delta t_d(N)$	$\Delta t_w(N_w)$	Class flare (UT)	$\Delta t_{sf}$	$\Delta t_{scme}$
SRB1	1.2	13:28:00	12 min (600)	1.2 min (60)	X1.1 (13:36:00)	8 min	6 h 14 min
SRB2	0.02	15:04:00	$\approx 42$ min ( $125 \times 10^3$ )	4.2 min (12,500)	X1.2 (15:36:00)	32 min	4 h 38 min
SRB3	0.01	16:34:50	40 s (4000)	4 s (400)	X2.3 (16:35:22)	32 s	3 h 08 min

## **Reformulação da Nossa Agenda (CAP239)**

**25/5: 10h30-12h30**

Aula: Programação da Ultima Aula de Conteúdo

1- Global Wavelet Spectra

2- Overview da Parte B da Disciplina

3- Esclarecimentos sobre a Lista de Exercícios

**27/5: 11h00 as 12h30**

1-Envio das Provas e Esclarecimentos sobre a logística

2-Esclarecimentos sobre o Trabalho

**01/6: 10h-12h30**

Live: Entrega das Provas (de forma sequencial e programada)

**03/6: 11h-12h30**

Live para Consulta ao Professor

**04/6: Prazo Final para Entrega da Lista de Exercícios**

**07/6: Prazo para Envio dos Trabalhos (até as 17h00)**

**08/6: 9h30 as 12h30**

Apresentação dos Trabalhos

**09/6: Emissão dos Conceitos Finais**

**10 a 20/6: fechamento do artigo (nesse período iremos trabalhar no fechamento do artigo para o Frontiers, todos estão convidados a colaborar pq serão co-autores).**