

Spherical Coordinates

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Spherical coordinates

The use of symbols and the order of the coordinates differs between sources. In one system frequently encountered in physics (r, θ, φ) gives the radial distance, polar angle, and azimuthal angle, whereas in another system used in many mathematics books (r, θ, φ) gives the radial distance, azimuthal angle, and polar angle.

The coordinates are defined as:

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} & r &\geq 0 \\ \theta &= \arccos\left(\frac{z}{r}\right) = 2 \arctan\left(\frac{\sqrt{1 - (z/r)^2}}{1 + z/r}\right) & 0 &\leq \theta \leq \pi \\ \varphi &= \arctan\left(\frac{y}{x}\right) & 0 &\leq \varphi \leq 2\pi \\ x &= r \sin \theta \cos \varphi \\ y &= r \sin \theta \sin \varphi \\ z &= r \cos \theta \end{aligned}$$

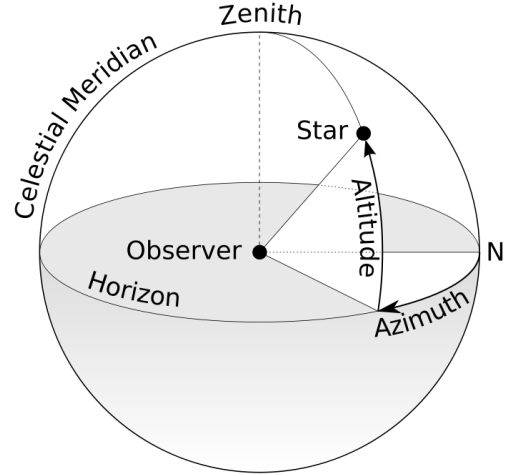
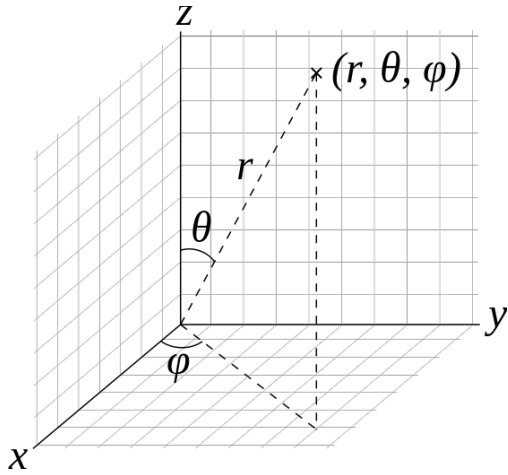


Figure 1: (a) Spherical coordinates (r, θ, φ) as commonly used in physics. (b) The azimuth.