# Razones

Del artículo [1] tnemos que $\mu^{\rm abcd} = (\mu^{\rm abdc})^*$ y que además $\mu$ es real. De las notas tenemos que

$$\begin{aligned}
K^{\rm ab}_{nm}(-\mathbf{k}) &= \sum_l v^{\rm a}_{nl}(-\mathbf{k})S^{\rm b}_{lm}(-\mathbf{k}) = \sum_l (-v^{\rm a}_{ln}(\mathbf{k}))(-S^{\rm b}_{ml}(\mathbf{k})) \\
&= \sum_l v^{\rm a*}_{nl}(\mathbf{k})S^{\rm b*}_{lm}(\mathbf{k}) = \sum_l (v^{\rm a}_{nl}(\mathbf{k})S^{\rm b}_{lm}(\mathbf{k}))^* = K^{\rm ab*}_{nm}(\mathbf{k}). \qquad (1)
\end{aligned}$$

$$\begin{aligned}
\dot{K}^{\rm ab} &= \frac{e^2}{i\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'} K^{\rm ab}_{c'c} r^{\rm c}_{cv} r^{\rm d}_{vc'}\left(\frac{1}{\omega-\omega_{c'v}-i\epsilon} - \frac{1}{\omega-\omega_{cv}+i\epsilon}\right)E^{\rm c}(\omega)E^{\rm d\star}(\omega) \\
&= \frac{e^2}{i\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'}\Big((K^{\rm ab}_{c'c} r^{\rm c}_{cv} r^{\rm d}_{vc'})\big|_{\mathbf{k}>0} + (K^{\rm ab}_{c'c} r^{\rm c}_{cv} r^{\rm d}_{vc'})\big|_{\mathbf{k}<0}\Big) \\
&\dots \\
&= \frac{\pi e^2}{\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'} \mathrm{Re}\Big[K^{\rm ab}_{c'c} r^{\rm c}_{cv} r^{\rm d}_{vc'} + K^{\rm ab}_{cc'} r^{\rm c}_{c'v} r^{\rm d}_{vc}\Big]\delta(\omega-\omega_{cv})E^{\rm c}(\omega)E^{\rm d\star}(\omega), \qquad \dots \\
&= \frac{\pi e^2}{\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'} \mathrm{Re}\Big[K^{\rm ab}_{c'c} r^{\rm c}_{cv} r^{\rm d}_{vc'} + K^{\rm ab}_{cc'} r^{\rm c}_{vc'} r^{\rm d}_{cv}\Big]\delta(\omega-\omega_{cv})E^{\rm c}(\omega)E^{\rm d\star}(\omega) \\
&= \frac{\pi e^2}{\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'} \mathrm{Re}\Big[K^{\rm ab}_{c'c}\Big(r^{\rm c}_{cv} r^{\rm d}_{vc'} + r^{\rm d}_{cv} r^{\rm c}_{vc'}\Big)\Big]\delta(\omega-\omega_{cv})E^{\rm c}(\omega)E^{\rm d\star}(\omega) \\
&= \frac{\pi e^2}{\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'} \mathrm{Re}\Big[K^{\rm ab}_{c'c}\Big(r^{\rm c}_{cv} r^{\rm d}_{vc'} + ({\rm c}\leftrightarrow{\rm d})\Big)\Big]\delta(\omega-\omega_{cv})E^{\rm c}(\omega)E^{\rm d\star}(\omega) \\
&= \frac{\pi e^2}{\hbar^2}\int \frac{d^3k}{8\pi^3}\sum_{vcc'} \mathrm{Re}\Big[K^{\rm ab}_{c'c}\Big(r^{\rm c}_{vc'} r^{\rm d}_{cv} + ({\rm c}\leftrightarrow{\rm d})\Big)\Big]\delta(\omega-\omega_{cv})E^{\rm c}(\omega)E^{\rm d\star}(\omega). \qquad (2)
\end{aligned}$$

$$\dot{K}^{\rm ab} = \mu^{\rm abcd}E^{\rm c}(\omega)E^{\rm d\star}(\omega), \qquad (3)$$

$$\mu^{\rm abcd} = \frac{\pi e^2}{\hbar^2}\int \frac{d^3k}{8\pi^3}{\sum_{vcc'}}' \mathrm{Re}\Big[K^{\rm ab}_{c'c}\Big(r^{\rm c}_{vc'} r^{\rm d}_{cv} + ({\rm c}\leftrightarrow{\rm d})\Big)\Big]\delta(\omega-\omega_{cv}), \qquad (4)$$

donde se puede ver que $\mu^{\rm abcd}$ es real.

Entonces,

$$\begin{aligned}
\dot{K}^{\rm ab} &= \mu^{\rm abcd}E^c(-\omega)E^d(\omega) \\
&= \mu^{\rm abcd}E^{c*}E^d + \mu^{\rm abdc}E^{d*}E^c \\
&= \mu^{\rm abcd}E^{c*}E^d + (\mu^{\rm abcd})^*E^{d*}E^c \qquad (5)
\end{aligned}$$

$$
\begin{aligned}
&= \mu^{\mathrm{abcd}} E^{c*} E^{d} + \mu^{\mathrm{abcd}} E^{d*} E^{c} \\
&= \mu^{\mathrm{abcd}} E^{c*} E^{d} + \mu^{\mathrm{abcd}} (E^{d} E^{c*})^{*} \\
&= \mu^{\mathrm{abcd}} [E^{c*} E^{d} + (E^{d} E^{c*})^{*}] \\
&= 2\mu^{\mathrm{abcd}} \mathrm{Re}[E^{c*} E^{d}]
\end{aligned}
\tag{6}
$$

$$
E^{a} = E_0 e^{i\varphi_a} \Rightarrow \quad
\begin{aligned}
E^{c*} &= E_0 e^{-i\varphi_c} \\
E^{d} &= E_0 e^{i\varphi_d}
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
E^{c*} E^{d} &= E_0^2 e^{i(\varphi_d - \varphi_c)} \\
E^{c*} E^{d} &= E_0^2 [\cos(\varphi_d - \varphi_c) + i \sin(\varphi_d - \varphi_c)] \\
\mathrm{Re}[E^{c*} E^{d}] &= E_0^2 \cos(\varphi_d - \varphi_c)
\end{aligned}
\tag{8}
$$

Entonces tenemos que

$$
\dot{K}^{\mathrm{ab}} = 2\mu^{\mathrm{abcd}} E_0^2 \cos(\varphi_d - \varphi_c) \tag{9}
$$

que tiene valor máximo cuando $\varphi_d = \varphi_c$

# References


[1] R. D. R Bhat, F. Nastos, Ali Najmaie, and J. E. Sipe. Pure spin current from one-photon absorption of linearly polarized light in noncentrosymmetric semiconductors. *Phys. Rev. Lett.*, 94:096603, Mar 2005.