

$$\text{Im}[\chi_{e,\omega}^{\text{abc}}] = \frac{\pi|e|^3}{2\hbar^2} \int \frac{d^3k}{8\pi^3} \sum_{vc} \sum_{\mathbf{q} \neq (v,c)} \frac{1}{\omega_{cv}^\Sigma} \left[\frac{\text{Im}[\mathcal{V}_{\mathbf{q}c}^{\Sigma,a} \{r_{cv}^b r_{v\mathbf{q}}^c\}]}{(2\omega_{cv}^\Sigma - \omega_{c\mathbf{q}}^\Sigma)} - \frac{\text{Im}[\mathcal{V}_{v\mathbf{q}}^{\Sigma,a} \{r_{\mathbf{q}c}^c r_{cv}^b\}]}{(2\omega_{cv}^\Sigma - \omega_{\mathbf{q}v}^\Sigma)} \right] \delta(\omega_{cv}^\Sigma - \omega)$$

$$\text{Im}[\chi_{e,2\omega}^{\text{abc}}] = -\frac{\pi|e|^3}{2\hbar^2} \int \frac{d^3k}{8\pi^3} \sum_{vc} \frac{4}{\omega_{cv}^\Sigma} \left[\sum_{\mathbf{v}' \neq v} \frac{\text{Im}[\mathcal{V}_{vc}^{\Sigma,a} \{r_{c\mathbf{v}'}^b r_{\mathbf{v}'v}^c\}]}{2\omega_{c\mathbf{v}'}^\Sigma - \omega_{cv}^\Sigma} - \sum_{\mathbf{c}' \neq c} \frac{\text{Im}[\mathcal{V}_{vc}^{\Sigma,a} \{r_{c\mathbf{c}'}^c r_{\mathbf{c}'v}^b\}]}{2\omega_{\mathbf{c}'v}^\Sigma - \omega_{cv}^\Sigma} \right] \delta(\omega_{cv}^\Sigma - 2\omega)$$