$$\mathrm{Im}[\chi^{\mathrm{abc}}_{i,\omega}] = \frac{\pi|e|^3}{2\hbar^2}\int\frac{d^3k}{8\pi^3}\sum_{cv}\frac{1}{(\omega^{\Sigma}_{cv})^2}\left[\mathrm{Re}\left[\left\{r^{\mathrm{b}}_{cv}\left(\mathcal{V}^{\Sigma,\mathrm{a}}_{vc}\right)_{;k^{\mathrm{c}}}\right\}\right] + \frac{\mathrm{Re}\left[\mathcal{V}^{\Sigma,\mathrm{a}}_{vc}\left\{r^{\mathrm{b}}_{cv}\Delta^{\mathrm{c}}_{cv}\right\}\right]}{\omega^{\Sigma}_{cv}}\right]\delta(\omega^{\Sigma}_{cv}-\omega)$$

$$\mathrm{Im}[\chi^{\mathrm{abc}}_{i,2\omega}] = \frac{\pi|e|^3}{2\hbar^2}\int\frac{d^3k}{8\pi^3}\sum_{vc}\frac{4}{(\omega^{\Sigma}_{cv})^2}\left[\mathrm{Re}\left[\mathcal{V}^{\Sigma,\mathrm{a}}_{vc}\left\{\left(r^{\mathrm{b}}_{cv}\right)_{;k^{\mathrm{c}}}\right\}\right] - \frac{2\mathrm{Re}\left[\mathcal{V}^{\Sigma,\mathrm{a}}_{vc}\left\{r^{\mathrm{b}}_{cv}\Delta^{\mathrm{c}}_{cv}\right\}\right]}{\omega^{\Sigma}_{cv}}\right]\delta(\omega^{\Sigma}_{cv}-2\omega)$$