

$$\text{Im}[\chi_{i,\omega}^{\text{abc}}] = \frac{\pi|e|^3}{2\hbar^2} \int \frac{d^3k}{8\pi^3} \sum_{cv} \frac{1}{(\omega_{cv}^\Sigma)^2} \left[ \text{Re} \left[ \left\{ r_{cv}^b \left( \mathcal{V}_{vc}^{\Sigma,a} \right)_{;k^c} \right\} \right] + \frac{\text{Re} \left[ \mathcal{V}_{vc}^{\Sigma,a} \left\{ r_{cv}^b \Delta_{cv}^c \right\} \right]}{\omega_{cv}^\Sigma} \right] \delta(\omega_{cv}^\Sigma - \omega)$$

$$\text{Im}[\chi_{i,2\omega}^{\text{abc}}] = \frac{\pi|e|^3}{2\hbar^2} \int \frac{d^3k}{8\pi^3} \sum_{vc} \frac{4}{(\omega_{cv}^\Sigma)^2} \left[ \text{Re} \left[ \mathcal{V}_{vc}^{\Sigma,a} \left\{ \left( r_{cv}^b \right)_{;k^c} \right\} \right] - \frac{2\text{Re} \left[ \mathcal{V}_{vc}^{\Sigma,a} \left\{ r_{cv}^b \Delta_{cv}^c \right\} \right]}{\omega_{cv}^\Sigma} \right] \delta(\omega_{cv}^\Sigma - 2\omega)$$