Feedback Control With Posicast

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***Abstract*—An alternative way to use Posicast to damp oscillations in lightly damped control systems is proposed in this paper. Specifically, it is suggested that the Posicast technique be used *within* a feedback system, instead of the classical feedforward configuration. There are many types of Posicast, but discussion and analysis are focused here on the classical "half-cycle" Posicast form. Theoretical analysis and a design example are used to compare classical Posicast and the proposed feedback control method. An experimental result from a power converter application is also presented.**

***Index Terms*—Lightly damped system, Posicast.**

I. Introduction

POSICAST was originally proposed by O. J. M. Smith to cancel the oscillatory behavior of lightly damped systems [1]. One of the earliest textbook descriptions of Posicast is found in [2]. Smith showed how accurate knowledge of the system damping and damped natural frequency could be used to design a *feedforward* dynamic compensator that cancels overshoot in the system step response. Posicast reshapes the step input command into two parts. The first part is a scaled step that causes the first peak of the oscillatory response to precisely meet the desired final value. The second part of the reshaped input is scaled and time-delayed to precisely cancel the remaining oscillatory response, thus causing the system output to stay at the desired value.

Posicast was further examined in the early 1960s by researchers trying to minimize vibration in various types of lightly damped systems. Cook described methods to extend the method to higher order systems [3], including flexible structures [4]. More recently, Singer and Seering revisited Posicast in a paper on vibration control by reshaping the reference input [5]; their contribution was further illuminated by Cook [6]. Application of Posicast to nonlinear dynamic is also effective when the nonlinear is slowly time varying. Singer and Seering's simulation study involved dynamics of a large flexible-link robotic arm [5]. When the dominant lightly damped poles of the system are well known, classical Posicast yields excellent results. Researchers have also shown that Posicast is very sensitive to inaccurate knowledge of the damped resonant frequency; such performance sensitivity is common to many feedforward control methods that rely on dynamic cancellation or model inversion. Posicast compensation can be more useful if the sensitivity to parameters can be reduced.

In this paper, the author shows that the sensitivity problem can be reduced if Posicast compensation is applied *within a feedback system* rather than in the classical feedforward configuration. The presentation is divided into several parts. Classical Posicast is described in Section II. Then, the proposed method for using Posicast in feedback control is introduced (Section III). Analytical support based on sensitivity function analysis follows in Section IV. The theoretical analysis shows that the sensitivity of Posicast can be reduced by a properly designed feedback control. Next, a design example and simulation results are presented (Section V). The proposed control method has also been experimentally verified in a power converter control application, so a brief description of the experimental system and a sample response are given in Section VI. Other applications for the proposed control method and directions for additional research are suggested in the concluding remarks (Section VII).

Manuscript received February 21, 2001; revised May 9, 2002. Abstract published on the Internet September 13, 2002.

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Digital Object Identifier 10.1109/TIE.2002.804979

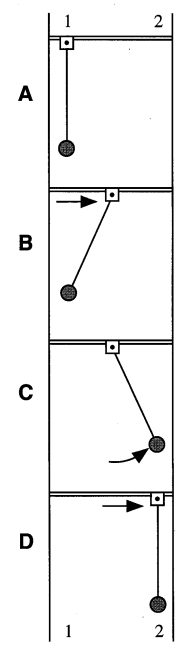

Fig. 1. Application of half-cycle Posicast.

II. Review of Classical Posicast

Half-cycle Posicast is described here using the example originally presented by Smith [2] and Cook [3]. Consider the problem of moving a load suspended by a cable attached to a gantry. The sequence of movements is illustrated in Fig. 1. In the uppermost frame A, the gantry and the load are both at position 1. The motion starts in the second frame B, with the gantry moving to midway between positions 1 and 2, causing the load to swing toward position 2. In the third frame C, the load has swung past the gantry to position 2, and is about to swing back. The gantry immediately moves to position

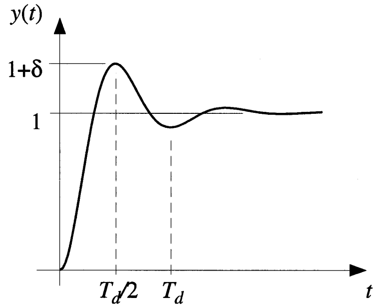

Fig. 2. Step response of lightly damped system.

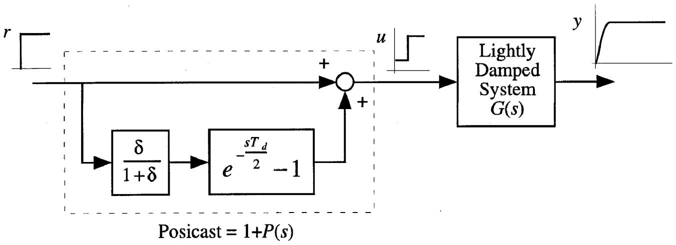

Fig. 3. Classical application of half-cycle Posicast.

2, so that the load stays at position 2 without overshoot or oscillations (fourth frame D).

The analytical structure of Posicast can be explained using the step response diagram of Fig. 2 and the block diagram shown in Fig. 3. Posicast is the portion encircled by the dashed curve, and is given by the the function $1 + P(s)$ where $P(s)$ is given by

$$P(s) = \frac{\delta}{1+\delta}\left(e^{-s(T_d/2)} - 1\right). \tag{1}$$

Parameters of the function $P(s)$ are the step response overshoot δ and the damped response period T_d. Posicast $1 + P(s)$ essentially reshapes the step reference signal into two parts. Initially, the controller subtracts a scaled amount from the reference signal, so that the peak of the lightly damped response coincides with the desired final value of the system response. The time to the peak of the step response is one-half the natural damped period. After this delay period, the full value of the step reference is applied to the system $G(s)$ so that the output remains at the desired final value. Another interpretation is that the reference amount originally subtracted from the input now cancels undesired overshoot because it is delayed by precisely one-half the damped natural period.

Half-cycle Posicast is equivalent to an all-zero filter, with an infinite set of zeros spaced at odd multiples of the damped natural frequency [2], [3]. Solving for the roots of $1 + P(s) = 0$ with $s = \sigma + j\omega$ yields the following relations:

$$\sigma = \frac{2}{T_d}\ln\delta \tag{2}$$

$$\omega = \frac{2\pi}{T_d}(2n+1), \qquad n = 0, 1, \ldots. \tag{3}$$

The frequency response of Posicast $1 + P(s)$ with $\delta = 0.8$ and $T_d = 1$ is shown in Fig. 4. The first pair of zeros cancels the dominant pair of poles in the lightly damped system

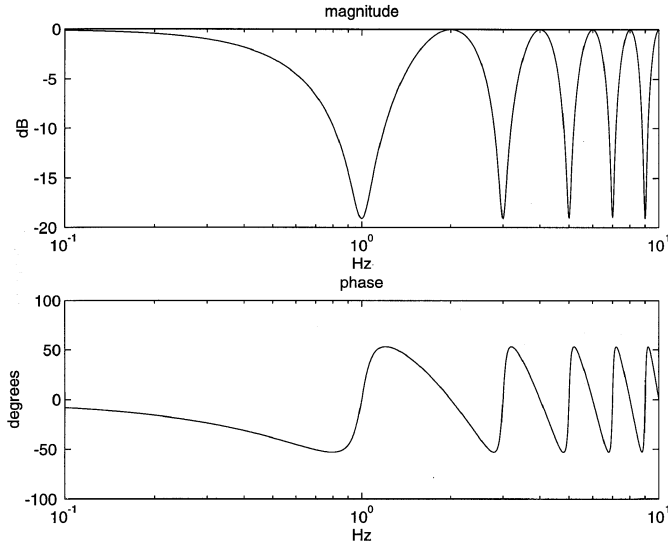

Fig. 4. Frequency response $1 + P(s)$ for $\delta = 0.8, T_d = 1$.

$G(s)$. Posicast is not the same, however, as canceling poles by model inversion. Model inversion exhibits noise sensitivity because the typical inverse model has increasing gain at high frequency. In contrast, Posicast has limited high-frequency gain. However, effective Posicast still relies on cancellation of poles and zeros, so inaccurate knowledge of the plant dominant poles will result in residual oscillation. For this reason, Posicast has not been widely used in practice. Singer and Seering propose a method to reduce sensitivity [5]; their method has been interpreted as an extended or higher order Posicast [4], having more than one step. Their technique encompasses and extends Smith's quarter-cycle Posicast [2].

The underlying philosophy of classical Posicast is to eliminate oscillations in a control system that has already been otherwise designed to give the best possible performance. Therefore, classical Posicast is designed subsequent to a feedback controller design that achieves all other performance requirements (except for the lightly damped response). Since Posicast is a feedforward control, load disturbances are not compensated directly. If the system $G(s)$ is a feedback system, then it is possible to incorporate Posicast to counteract disturbance affects. A second Posicast is designed and placed between the disturbance input and $G(s)$. Since the disturbance input is not measurable, the second Posicast must be moved to a physically realizable place in the model. Smith employs block diagram algebra to move the second Posicast into the feedback loop that represent the lightly damped system $G(s)$ [2]. The control effect is still based on feedforward compensation, however, so parametric sensitivity remains a problem. In the next section, the author introduces a method to reduce the sensitivity effects.

III. An Alternative Method of Posicast Compensation

To the author's knowledge, Posicast and its variations have been used exclusively as feedforward compensation for lightly damped systems. It is proposed in this paper that Posicast be used in conjunction with other classical feedback control schemes. A hybrid control scheme is proposed to take

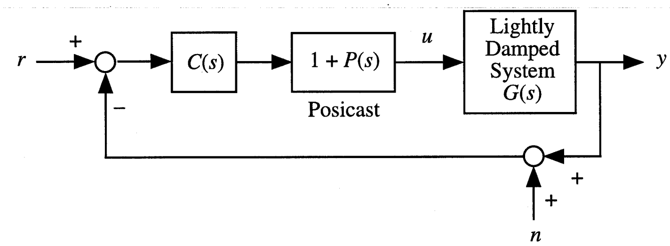

Fig. 5. Proposed hybrid feedback control using Posicast.

advantage of the superior damping qualities of Posicast, while reducing parametric sensitivity and load sensitivity through feedback. A block diagram explaining the control method is shown in Fig. 5. Whereas previous applications placed Posicast *before* the lightly damped system, it is now suggested that Posicast be used *within* a feedback system. The proposed control method is a significant departure from classical Posicast. Note that the overall system characteristic polynomial using classical half-cycle Posicast is found by simply removing the dominant lightly damped poles of $G(s)$. In the proposed method, the closed-loop characteristic polynomial is given by $1 + C(s)[1 + P(s)]G(s)$. The primary purpose of the Posicast function is to cancel undesirable plant poles, thus minimizing the effect of lightly damped poles in the closed-loop response. Poles of the closed-loop system would be determined by the remaining open-loop poles and zeros. A properly designed compensator $C(s)$ reduces the effect of imperfect Posicast compensation.

The design method for the hybrid system has two steps. First, the function $P(s)$ is designed for the lightly damped system $G(s)$. For half-cycle Posicast, two open-loop step response parameters are required: the overshoot δ and the time to first peak T_d. Next, the feedback controller $C(s)$ is designed based the combined model $[1 + P(s)]G(s)$. Classical frequency domain techniques can be used.

Analysis of the proposed method's benefit is discussed in the next section.

IV. Sensitivity Analysis

The effect of using feedback with Posicast compensation can be examined using sensitvity analysis. The sensitivity function of a system using classical feedforward Posicast is developed first, then compared to the case where feedback is used.

The transfer function for a system compensated using feedforward Posicast can be rewritten as

$$T_1(s) = [1 + P(s)]\, G(s) = \frac{\hat{D}_u(s)}{D_u(s)} G'(s) \tag{4}$$

where

- $D_u(s)$ undesirable lightly damped poles in $G(s)$ that should be cancelled by Posicast;
- $\hat{D}_u(s)$ the zeros of Posicast that should cancel $D_u(s)$;
- $G'(s)$ remaining zeros and poles of Posicast and the plant.

Under ideal conditions, the functions $D_u(s)$ and $\hat{D}_u(s)$ are identical, such that $D_u(s)/\hat{D}_u(s) = 1$. In practice, the functions are not coprime, so the ratio can be written as

$$\frac{\hat{D}_u(s)}{D_u(s)} = 1 + \delta(s) \tag{5}$$

where $\delta(s)$ represents the mismatch between the functions $D_u(s)$ and $\hat{D}_u(s)$.

The sensitivity of the transfer function $T_1(s)$ with respect to the mismatch $\delta(s)$ is given by

$$S_\delta^{T_1} = \frac{\partial T_1}{\partial \delta}\frac{\delta}{T_1} = \frac{\delta}{1+\delta}. \tag{6}$$

Therefore, the sensitivity of classical Posicast approaches unity for large mismatches, i.e., if the Posicast parameters and plant parameters are not well matched.

Next, observe that the transfer function of the proposed system shown in Fig. 5 is given by

$$\begin{aligned} T_2(s) &= \frac{C(s)[1+P(s)]G(s)}{1+C(s)[1+P(s)]G(s)} \\ &= \frac{C(s)[1+\delta(s)]G'(s)}{1+C(s)[1+\delta(s)]G'(s)} \end{aligned} \tag{7}$$

from which the sensitivity of the transfer function $T_2(s)$ with respect to the mismatch $\delta(s)$ can be shown to be

$$\begin{aligned} S_\delta^{T_2} &= \frac{\partial T_2}{\partial \delta}\frac{\delta}{T_2} \\ &= \frac{\delta}{1+\delta}\frac{1}{1+C(1+\delta)G'} \\ &= \frac{S_\delta^{T_1}}{1+C(1+\delta)G'}. \end{aligned} \tag{8}$$

Comparing the sensitivity functions (6) and (8), it is clear that feedback control with Posicast can yield a system whose sensitivity with respect to Posicast parameters is reduced by the system loop gain. The sensitivity function (8) shows that sensitivity of system characteristics is reduced for frequencies within the system bandwidth. Hence, the design goal is to increase the loop gain without destabilizing the system. A design example is presented in the next section, followed by a sample of experimental verification.

V. Design Example

Two lightly damped systems are considered

$$G_0(s) = \frac{26}{s^2 + 2s + 26} \tag{9}$$

$$G_1(s) = \frac{2626}{(s^2 + 2s + 26)(s^2 + 2s + 101)}. \tag{10}$$

The proposed controller is designed based on dynamic system (9) and examined for sensitivity using system (10). System $G_1(s)$ is similar to $G_0(s)$, but has an additional pair of lightly damped poles. The dominant lightly damped poles of $G_0(s)$ are located at $s = -1 \pm j5$, from which the step response overshoot and time to first peak are calculated

$$\delta = e^{-\pi/5} \tag{11}$$

$$T_d = \frac{2\pi}{5}. \tag{12}$$

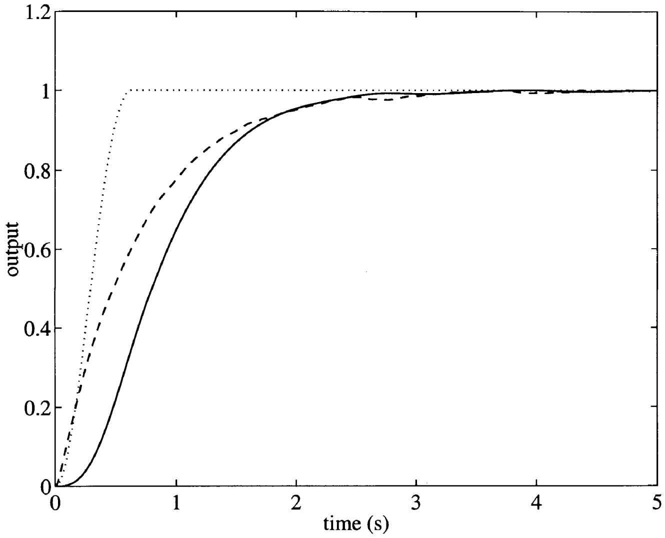

Fig. 6. Output responses of $G_0(s)$: proposed (solid), PID (dashed), and classical Posicast (dotted).

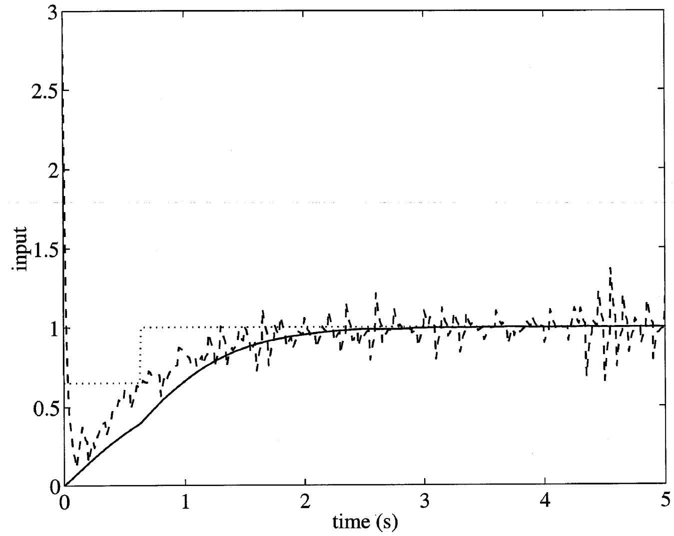

Fig. 7. Controller responses with band-limited white noise n: proposed (solid), PID (dashed), and classical Posicast (dotted).

Using (11) and (12), half-cycle Posicast is characterized by

$$\begin{aligned} 1+P(s) =& 1+\frac{\delta}{1+\delta}\left(e^{-sT_d/2}-1\right) \\ \approx& 1+0.348\left(e^{-0.628s}-1\right). \end{aligned} \tag{13}$$

A compensator is designed for the augmented system $[1+P(s)]G_0(s)$. To counteract steady-state disturbances, a pure integral control can be designed

$$C(s)=\frac{K}{s}. \tag{14}$$

The gain K is chosen as large as possible to minimize settling time while keeping overshoot to a minimum. The hybrid controller transfer function is given by $C(s)[1+P(s)]$.

A classical proportional–integral–derivative (PID)-type feedback controller is also designed for purposes of comparison. Zeros of the PID controller are designed to cancel the lightly damped dominant poles of $G_0(s)$, since half-cycle Posicast has the same effect. An instrumentation pole is added to the PID controller so that its transfer function would be physically realizable. The instrumentation pole is placed at $s=-50$, corresponding to a corner frequency that is one decade higher than the damped natural frequency of the PID zeros

$$C_{\text{PID}}(s)=K_c\frac{s^2+2s+26}{s(s+50)}. \tag{15}$$

Three simulated output responses are plotted in Fig. 6. The response due to the proposed controller $C(s)[1+P(s)]$ is plotted with a solid curve. The response due to classical feedforward half-cycle Posicast control is plotted with the dotted curve. The response due to a PID controller (15) is plotted with a dashed curve. Classical Posicast control yields the fastest rise time and absolutely no overshoot. The Posicast controlled response cannot be tuned, however, since the function $1+P(s)$ is uniquely determined by the system overshoot δ and damped natural period T_d. The proposed hybrid controller gain K and PID-based controller gain K_c are tuned to yield similar settling times. The PID controller yields slightly faster risetime than the proposed controller, and can be tuned to give even faster response. Both feedback systems exhibit some deviation from steady state due to measurement noise that has been included in the simulation.

The respective inputs to $G_0(s)$ are plotted in Fig. 7. Observe how classical half-step Posicast results in a reshaped step input (dotted lines). The input resulting from PID control exhibits very high initial rate of change due to the derivative action. The proposed control is much less aggressive. The curves in Fig. 7 also show the effect of measurement noise, which is simulated by additive band-limited white noise n at the system output. Classical half-cycle Posicast is not affected by output noise, since it is a feedforward compensation. The input signal computed by PID control (15) is much noisier than the proposed controller (13) and (14) because the PID controller has greater high-frequency gain. The PID controller's noise sensitivity can be reduced, however, by adding more instrumentation poles to reduce the high-frequency gain.

Sensitivity to unmodeled dynamics is examined by simulating the controllers with the system $G_1(s)$ rather than $G_0(s)$. The second pair of lightly damped poles in $G_1(s)$ have a natural frequency that is double that of the dominant pair. These poles are not cancelled by any of the half-cycle Posicast zeros. Therefore, classical Posicast is very sensitive to these poles, as evidenced by the system reponses plotted in Fig. 8 (dotted curve). The PID-based feedback controller does not perform much better (dashed curve) than half-cycle Posicast. Furthermore, the performance of the PID controller cannot be improved by retuning the gain K_c. Both of these responses exhibit oscillations at the natural frequency of the second pair of "unmodeled" lightly damped poles. The proposed controller with Posicast inside the feedback loop performs much better (solid curve).

VI. A Sample Experimental Response

The type of controller discussed in the previous section has been studied in laboratory experiments with a buck-type dc–dc

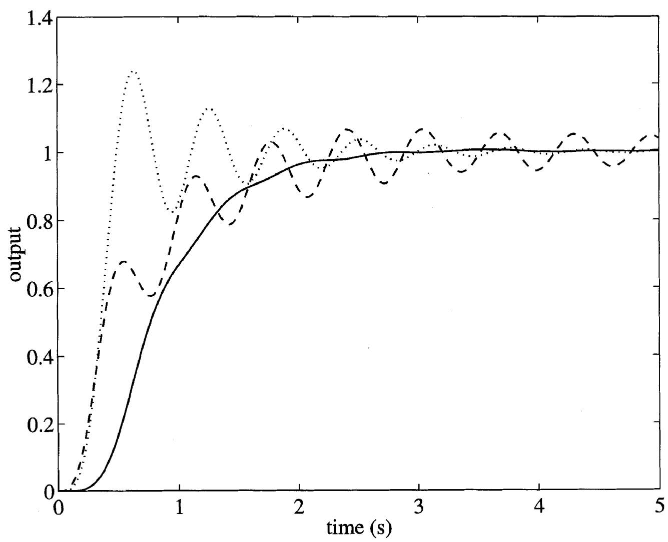

Fig. 8. Output responses of $G_1(s)$: proposed (solid), PID (dashed), and classical Posicast (dotted).

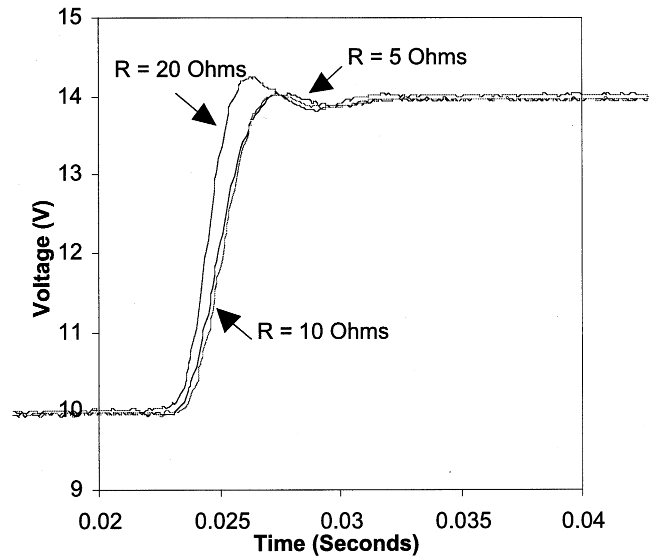

Fig. 9. Measured step responses for various load resistances R. See Table I for other controller parameters.

TABLE I
PARAMETERS OF THE EXPERIMENTAL SYSTEM

Symbol	Parameter	Value	Units
R	Load resistance	5 10 20	Ω
L	Series inductance	150	μH
R_L	Estimated Series Inductor Resistance	10	mΩ
C	Capacitance	1000	μF
R_C	Estimated Series Capacitor Resistance	30	mΩ
T_d	Damped natural period	2.44	ms
δ	Open loop overshoot (0-1)	0.8	
V_{in}	Input voltage	15-36	V
Fs	Converter switching frequency	20	KHz
T	Controller sampling period	50	μs
K	Gain in compensator $C(s)$	20-43	% duty cycle/V

voltage converter. From the ideal transfer function of the buck converter, the undamped natural frequency ω_n and damping factor ζ are computed. Then, the damped natural period T_d and step response overshoot δ are computed from the undamped natural period and damping factor; these are the Posicast parameters. The controller $C(s)[1+P(s)]$ is implemented on a Texas Instruments TMS320F240PQ digital signal processor (DSP). A first-in–first-out (FIFO) queue and the DSP internal timer are used to implement the time delay in the Posicast function $[1+P(s)]$, and the remaining parts of the controller are discretized using Euler's approximation. Parameters of the experimental buck converter and controller parameters are listed in Table I. A family of step responses for different load resistances is shown in Fig. 9. Additional implementation details, steady-state performance, and sensitivity of the transient response to controller parameters are detailed in the reference [7].

VII. CONCLUDING REMARKS

Classical Posicast control principles have been reviewed in this paper, and an alertnative approach to using Posicast within a feedback loop has been presented. The different variations of classical Posicast were originally used in a feedforward manner, by reshaping the system input to minimize oscillatory behavior of the lightly damped system that followed the Posicast function. Classical Posicast can be explained in the frequency domain as the cancellation of the dominant pair of lightly damped system poles. In the time domain, classical Posicast relies on accurate timing to be effective. For these reasons, Posicast has not found widespread application in its orginal form. The hybrid control approach proposed in this work combines the Posicast principle with classical feedback system design. A feedback compensator is designed to stabilize the combined Posicast and lightly damped system. Analysis shows that sensitivity to unmodeled dynamics is greatly reduced by the feedback mechanism for frequencies within the system bandwidth. The proposed controller can be much more effective than either classical Posicast or PID control in the presence of unmodeled dynamics. A simulation example and results of a laboratory experiment show good results.

Application of Posicast *within* a feedback loop can be applied to a wide range of industrial applications. The Posicast function $1+P(s)$ can be easily implemented in a digital controller using a FIFO queue in memory. Power converter control is one application that has been examined. For example, the typical buck dc–dc converter is characterized by lightly damped dynamics, yet the resonant frequency is fairly stable over a large load range. Another potential application is the control of compliant drive systems and flexible structures. The method described here is based on half-cycle Posicast; performance using other types of Posicast is still open to further study. Furthermore, other types of feedback control methods might be combined with Posicast. For example, it may be feasible to extend the application to lightly damped nonlinear systems, such as robot control.

ACKNOWLEDGMENT

The author thanks the anonymous reviewers for their insightful discussions and suggestions, which have improved the presentation of this work.

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