International Conference on Control, Automation and Systems 2007
Oct. 17-20, 2007 in COEX, Seoul, Korea

Anti-sway Control for Overhead Crane

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Abstract: The models of overhead crane system have been studied in many regulation methods; hence bring about the variation parameter. However, there are have the similar conjecture of angular velocity ($\dot{\theta}$) in the mathematics models of pendulum system. The angular velocity can be deduced to be zero to perform the simply system as the linear equation form. In this paper, the different direction of mathematical analysis of an overhead crane model was study what the angular velocity ($\dot{\theta}$) was set as in the same value of natural angular velocity (ω_n) instead of neglected to return this values. These equations are still having the linear equation properties for the state space equation. The purpose of this study is to reduce the error of controller design to give the equation to be close to the real system. A design recommendation for optimal asymptotic linear quadratic (LQ) controllers with fixed gain and when follow to changed mass load are present to improve the stabilities. The results show the over shoot value was diminished. And, the extended gain values have less than the previous values of calculations. Whereas, the period of time of the system descended to the steady state have the same value. Which give the advantage to obviate the problem of over feed extended gain to the system.

Keywords: Linear Quadratic (LQ) Controller, Algebraic Riccati equation, Lagrangian Dynamics, Pendulum

1. INTRODUCTION

The pendulum dynamic system has been studied over the years and widely applied to the industry. In general, the important variables in the analysis of the crane system are angle, mass of cart, mass of object hang with cart, mass of rope and length of rope. Length of rope is set as constant to make the calculation simply [1,2]. In addition, some variables in the analysis of the crane system are omitted due to such variables are difficult to calculate and their effect to the system is insignificant. As know, pendulum dynamic system has been studied for a long time. However, the equations that can explain the system exactly still required. Therefore, further studies of pendulum dynamic system has been performed continuously. The analysis of the resultant force was proposed in several methods. The frequently used method is the Lagrangian Dynamic, which exploits the differentiation. This method provides the similar solutions, compared to the Newton's method that is more sophisticate. There are many methods to calculate the valued of control system. Among those, Linear Quadratic Regulator (LQR) method is widely used that is designed to generate the least capacity index function. Weighing factors using Riccati equation has to be selected by the designer. The purpose of this study is to provided the equation with pendulum angle ($\dot{\theta}$) adjusted by using Lagrange method [3-5]. Fixed gain linear quadratic controller by Routh Hurwitz stability is also used to perform in this work.

2. Mathematical Model of an Overhead Crane

2.1 Mathematical Model

Mathematical model of an overhead crane by Lagrangian Dynamic model is illustrated in Fig. 1. The equations are as to analyze as follow

$$ml^2\ddot{\theta} - ml\ddot{r}\cos(\theta) + mgl\sin(\theta) = 0 \qquad (1)$$

$$(M+m)\ddot{r} - ml\ddot{\theta} + ml\dot{\theta}^2\theta = 0 \qquad (2)$$

When M is mass of cart., m is mass of load., l is the length of rope between cart and mass of load., r is scattered distance that cart can move pass at the zero time., g is the earth gravity .

Since the angular angle is of a small value, so we can assume, $\cos(\theta) \approx 1$ and $\sin(\theta \approx \theta)$. To analyze equation 2 by using the same expectation method that $\dot{\theta}^2 \cdot \theta$ very least can be deduced to be zero. However, in general of pendulum analysis theorem, angular velocity is ω which is equal to $\omega = \sqrt{g/l}$. If deduce $\dot{\theta}^2 \approx \omega^2$, the new equation can be written as :

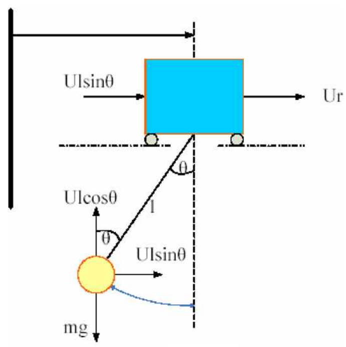

Figure1. Overhead Crane Model

$$\ddot{\theta} = \frac{-(M+2m)g\theta}{Ml} + \frac{Ur}{Ml} \qquad (3)$$

$$\ddot{r} = \frac{-2mg\theta}{M} + \frac{Ur}{M} \qquad (4)$$

2.2 State Space Equation

Simple for the to simplify for arrange equations in state space form

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{r} \\ \ddot{r} \end{bmatrix} \begin{matrix} \text{Pendulum angular velocity} \\ \text{Pendulum angular acceleration} \\ \text{Cart velocity} \\ \text{Cart acceleration} \end{matrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(M+2m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-2mg}{M} & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \frac{1}{Ml} & 0 & \frac{1}{M} \end{bmatrix}^T \qquad (5)$$

3. Design of a LQ controller

3.1 Minimize the performance index

Due to an overhead system crane is operated by Single Input Two Output (SITO) system. Thus, the control law type of Multi Input and Multi Output (MIMO) system was applied to use for this study.

$$U = -Kx \qquad (6)$$

Where K is gain value, U is signal input to system by choosing gain value by Performance Index Criteria type Integral Time Multiplied Absolute Error (ITAE)

$$J = \int_0^\infty (x^T Q x + u^T R u)dt \qquad (7)$$

Where Q and R are weighting factors; Q is only a positive definite or positive semi-definite and is a real symmetric matrix; R is a positive definite symmetric matrix. In this application with a single input, R ; Thus R was set by scalar (r) that can be solved by the algebraic Riccati equation.

3.2 Algebraic Riccati Equation

Algebraic Riccati Equation is given as

$$A^TP+PA-PBr^{-1}B^TP+Q = 0 \qquad (8)$$

Where P is a positive symmetric matrix and K is gain matrix.

$$K = r^{-1}B^TP \quad ; K = [\ k1, k2, k3, k4\ ] \qquad (9)$$

4. Fixed Gain LQ Controller By Routh Hurwitz Stability Theorem

4.1 Fixed gain LQ controller

Pole Placement Theorem applied with Performance index criteria type ITAE of the closed-loop system function was used to design controller system. Find the coefficient characteristic equation as below ;

det(sI-(A-BK))

$$s^4+(ck_2+dk_4)s^3+(ck_1+dk_3-a)s^2+(bck_4-adk_4)s+(bck_3-adk_3) = 0$$

The re- written form that the coefficient characteristic equation as follow ;

$$s^2 + \alpha s^3 + \beta s^2 + \gamma s + \delta = 0 \qquad (10)$$

4.2 Analyze Routh Hurwitz Stability

The method of fixed gain Controller By Routh Hurwitz Stability Theorem was applied to study stability analysis for find the limited of gain signal that have two criteria as show below ;

$$\text{i} = \frac{\alpha\beta - \gamma}{\alpha} > 0$$

$$\text{ii} = \frac{\frac{(\alpha\beta-\gamma)\gamma}{\alpha} - \frac{\alpha^2\delta}{\alpha}}{\frac{\alpha\beta-\gamma}{\alpha}} > 0 \qquad (11)$$

5. Mathematical Simulation

The Riccati equation was complied by mathematical program Maple 9.5. 8^{th} Riccati algebraic equation can spread the variable generate metric as Table 1.

By comparing equation 10 and equation 12, the value Q and R can be solved from equation 7 in which variables are given as follows : M = 2 kg., m = 1 kg., l = 1 m., g = 9.81 m/s^2.

Table 1 Expaned Riccati Equation Result

$$\begin{bmatrix} 2*a*p2+2*b*p4-R*k1\hat{}2+mu & p1+a*p5+b*p7-R*k1*k2 & a*p6+b*p9-R*k1*k3 & p3+a*p7+b*p10-R*k1*k4 \\ p1+a*p5+b*p7-R*k1*k2 & 2*p2-R*k2\hat{}2 & p3-R*k2*k3 & p6+p4-R*k2*k4 \\ a*p6+b*p9-R*k1*k3 & p3-R*k2*k3 & -R*k3\hat{}2+1 & p8-R*k3*k4 \\ p3+a*p7+b*p10-R*k1*k4 & p6+p4-R*k2*k4 & p8-R*k3*k4 & 2*p9-R*k4\hat{}2 \end{bmatrix} = 0$$

From equation 9, K can be calculated as :

$$K = [\ 7.848 \quad -3.7585 \quad 19.62 \quad 16.9133\ ] \tag{12}$$

In matrix Q, u is a variable which is in the position of the variable of interest ; θ

$$Q = \begin{bmatrix} \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By the substitution of equation 12 to equation Table 1. mu and r can be calculated as show below.

$$Q = \begin{bmatrix} 1.685 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$r = 0.0025978 \tag{13}$$

The variables derived from the previous section are replaced in MATLAB.

6. Results and Discussions

We made comparison of the Response between the old overhead crane system and the new overhead crane system one after we feed the step input. Gain can be illustrated in 2 ways.

6.1 Adjust gain follow parameter that modified

In case that the mass of load is almost equal to mass of cart, the value position of cart and angle pendulum between old overhead crane are quite similar as show in Fig. 2. If the mass of load was increased repeatedly and gain was observed at every changed of mass of load, the values of pendulum of load would contract, the angle would oscillate more frequently. There were have the same value positions of cart. The values of position of the cart of new overhead crane system will have a long response period, compared to old overhead crane system as shown in Fig. 3-6.

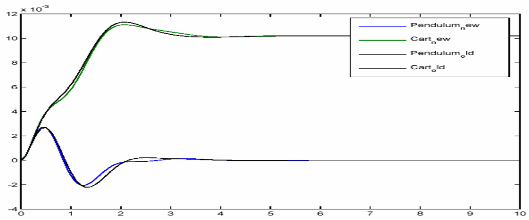

Fig. 2 Response of position cart and angle pendulum when m = 1 kg., Knew = [11.0199 -3.1082 19.6200 14.1788], Kold =[17.0617 -1.5239 19.6200 13.6370]

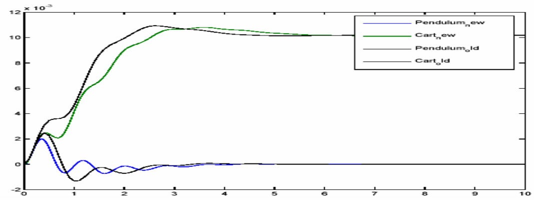

Fig. 3 Response of position cart and angle pendulum when m = 5 kg., Knew = [-6.4964 -13.8472 19.6200 21.0925], Kold =[0.7775 -7.6343 19.6200 16.6670]

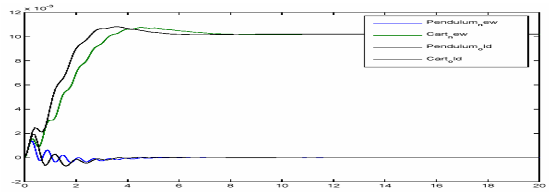

Fig. 4 Response of position cart and angle pendulum when m = 10 kg., Knew = [-11.9603 -23.0207 19.6200 28.5559], Kold =[-6.4964 -13.8472 19.6200 21.0925]

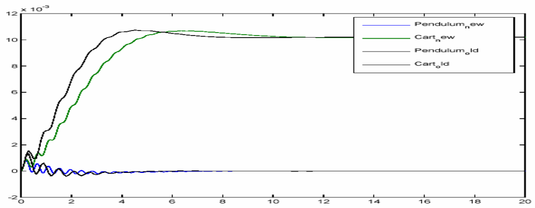

Fig. 5 Response of position cart and angle pendulum when m = 20 kg., Knew = [-15.4467 -35.7425 19.6200 39.8283], Kold =[-11.9603 -23.0207 19.6200 28.55590925]

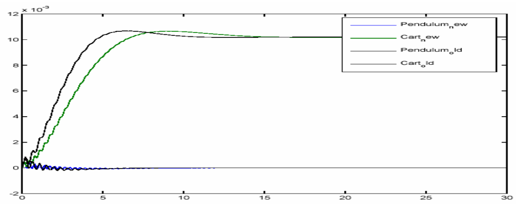

Fig. 6 Response of position cart and angle pendulum when m = 40 kg., Knew = [-17.4373 -53.1517 19.6200 56.1064], Kold =[-15.4467 -35.7425 19.6200 39.8283]

6.2 By ignoring the system parameters which have been changed variable parameter by choosing m = 1kg.

Knew = [11.0199 -3.1082 19.6200 14.1788], Kold =[17.0617 -1.5239 19.6200 13.6370]

The comparison of the old overhead crane system and the new overhead crane system gave the same results as section 6.1, when the equality of the mass of load and the mass of cart was given as shown in Fig. 2. If the mass of load was increased and gain was fixed at m = 1 kg. , the values of angle pendulum of load would contract. Then, the values of angle would less oscillate and give the same value when a comparison of old and new overhead crane system was considered.

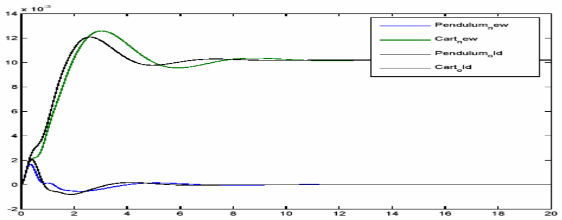

Fig. 7 Response of position cart and angle pendulum when m = 5 kg.

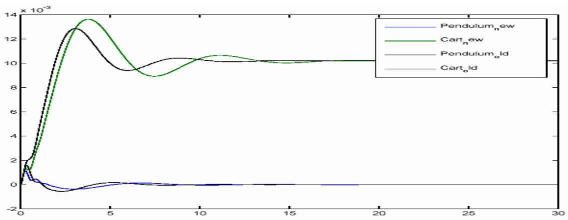

Fig. 8 Response of position cart and angle pendulum when m = 10 kg.

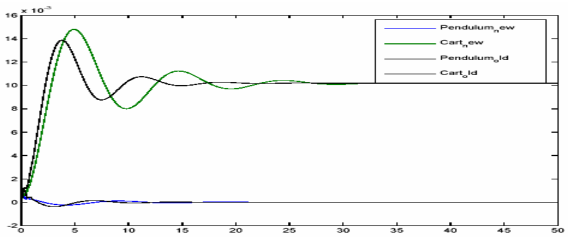

Fig. 9 Response of position cart and angle pendulum when m = 20 kg.

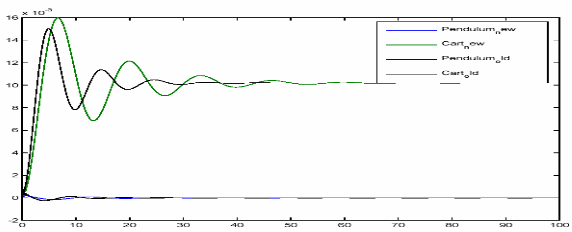

Fig. 10 Response of position cart and angle pendulum when m = 40 kg.

If the gain was chosen from the mass of load which have more than 3 times of real mass, the response would be unstable for both of system as shown in Fig. 11

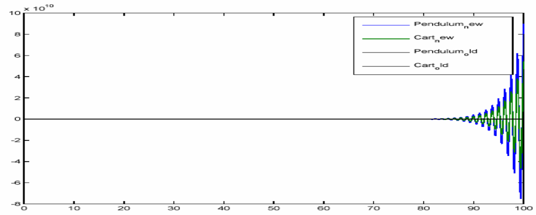

Fig. 11 Response of position cart and angle pendulum when use gain k at m = 5 kg., but mass load is m = 1 kg.

7. Conclusions and Recommendations

The results of the comparison between method 6.1 and 6.2 in terms of fix gain by ignoring the change of the mass of load will create more over shooting to cart which is not useful to the new and old overhead crane system, the consequence of this method has shown that the new overhead crane system has lower response than the old one because of the equation of mass of load of new method has double increased of the mass of load in the old method. As to the result, using the former system to analysis of gain will affect less stability due to over gaining of mass of load as shown in Fig. 11.

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