International Conference on Control, Automation and Systems 2007
Oct. 17-20, 2007 in COEX, Seoul, Korea

Adaptive Fuzzy Nonlinear Anti-Sway Trajectory Tracking Control of Uncertain Overhead Cranes with High-Speed Load Hoisting Motion

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Abstract: In the present paper, an adaptive fuzzy nonlinear control (AFNC) law is proposed for the anti-sway trajectory tracking of uncertain overhead cranes with high-speed hoisting motion. In the propsoed AFNC, fuzzy systems are utilized in two different ways. One is for fuzzy uncertainty observer (FUO) and the other is for fuzzy nonlinear control (FNC). Two FUOs are employed to cope with system uncertainties in an adaptive manner, while an FNC is developed to reduce the high-frequency chattering effect of variable structure control. Through the stability analysis and numerical simulations, it is shown that the proposed AFNC guarantees robust asymptotic stability of trajectory tracking error dynamics in trolley and hoisting motion as well as in sway dynamics, against not only the coupling effect of hoisting velocity and hoisting rope length but also system uncertainties such as system parameter variations, external wind disturbance, and unknown actuator nonlinaerities.

Keywords: Adaptive fuzzy nonlinear control, overhead crane, anti-sway trajectory tracking, fuzzy uncertainty observer.

1. INTRODUCTION

As one of the most popular underactuated mechanical systems, crane systems have attracted a lot of interests of control engineers since the deficiency in actuator of sway dynamics makes the control of crane systems be much more challenging than fully actuated mechanical systems. For two-dimensional (2D) overhead cranes, the unactuated sway dynamics is heavily coupled with two actuated dynamics through trolley acceleration, hoisting velocity and hoisting rope length [1]. This property has been one of the major obstacles in anti-sway trajectory tracking control of 2D overhead cranes, of which main purpose is to make the load to be transferred track given trajectories in vertical and horizontal directions with as small sway as possible. In addition, 2D overhead cranes have weakly non-minimum phase character in that they have marginally stable zero-dynamics, which preclude the direct application of feedback linearization methods [2-3].

Over the past decades, extensive research has been performed toward the anti-sway control of overhead cranes. In recent years, there have been reported some reputable studies which are dedicated to improve the sway suppression performance. A feedback linearization and the Lyapunov stability theorem based nonlinear control law of a 2D pilot overhead crane was proposed in [4]. In the practical point of view, however, this scheme addressing *regulation problem* is not directly applicable to real cranes since they usually transfer loads over a long distance. On the other hand, several methods based on the Lyapunov stability theorem [5-6] and sliding mode control [7] were proposed, which have been paid *tracking problem* attention. The control laws in [5-6] were also extended to adaptive schemes to cope with system uncertainties.

Adaptive control has been one of the most attractive control schemes for the control of uncertain systems like overhead cranes, which are easily exposed to external wind disturbance, parameter variations, and unknown actuator nonlinearity. Recently, there have been some results on fuzzy logic based adaptive control to treat system uncertainties and external disturbances in robot manipulators [8-9]. Considering fully actuated manipulators, they have utilized fuzzy logic systems to monitor the lumped uncertainties in robot manipulators. In this case, the size of fuzzy systems becomes proportional to the number of degrees of freedom of manipulators. On the other hand, some of actuator nonlinearities, such as dead-zone characteristics, have been treated by several authors. While Gang *et al.* [10] have proposed adaptive control scheme based on the adaptive dead-zone inverse system, Jang [11] presented a fuzzy logic based dead-zone compensation scheme which requires an additional nonlinear function estimator. Although there exist some of the previous research treating system uncertainties, external disturbance, and actuator nonlinearities of fully actuated systems, few studies dealing with actuator nonlinearities in overhead cranes have been performed, to the best of our knowledge.

In this paper, we propose an adaptive fuzzy nonlinear control (AFNC) law for anti-sway trajectory tracking of 2D uncertain overhead cranes with high-speed hoisting motion. Uncertainties considered in the present paper include system parameter variations, wind disturbance, and actuator nonlinearity. In addition to tracking control law based on partial feedback linearization, a fuzzy nonlinear control (FNC) law is designed to guarantee the uniform boundedness and asymptotic stability of sway dynamics under high-speed hoisting motion. Then, two fuzzy uncertainty observers (FUOs) are employed to cope with uncertainties by utilizing the nonlinear function approximation property of Takagi-Sugeno (TS) fuzzy systems.

The remainder of this paper is organized as follows. In section 1, a brief review of fuzzy systems is given and 2D overhead crane dynamics with uncertainties and actuator nonlinearities is introduced in section 2. Then, an FNC is designed in section 4, followed by the design of AFNC in section 5. Finally, numerical simulation results for the evaluation of the proposed control law are given in section 6 and the conclusion of this study is drawn in section 7.

2. FUZZY SYSTEMS

A basic fuzzy system consists of fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier. The fuzzy inference engine performs a mapping from input vector $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbf{R}^n$ to an output variable $y \in \mathbf{R}$ for single output cases, which is based on the fuzzy IF-THEN rules in the rule base and compositional rule of inference. The ith IF-THEN rule of k fuzzy rules defining fuzzy implications can be written as

$$Rule^i: \text{If } x_1 \text{ is } A_1^i \text{ and} \ldots \text{and } x_n \text{ is } A_n^i, \quad \text{then } y \text{ is } y^i. \tag{1}$$

where A_j^i and y^i denote a fuzzy variable for premise part and a Singleton for consequent part, respectively. Then, the fuzzy system with center-average defuzzifier, product inference engine, and singleton fuzzifier can be expressed as

$$y(\mathbf{x}) = \frac{\Sigma_{i=1}^k y^i (\Pi_{j=1}^n \mu_{A_j^i}(x_j))}{\Sigma_{i=1}^k (\Pi_{j=1}^n \mu_{A_j^i}(x_j))} = \hat{\phi}^T \boldsymbol{\xi}(\mathbf{x}) \tag{2}$$

with $\boldsymbol{\xi}^T = (\xi^1, \xi^2, \ldots, \xi^k)$, where $\hat{\phi}^T = (y^1, y^2, \ldots, y^k)$ is an adjustable parameter vector; $\mu_{A_j^i}(\cdot)$ is the membership function of fuzzy variable A_j^i; and ξ^i s are fuzzy basis functions (FBFs) given as

$$\xi^i = (\Pi_{j=1}^n \mu_{A_j^i}(x_j)) / \Sigma_{i=1}^k (\Pi_{j=1}^n \mu_{A_j^i}(x_j)),$$

In this paper, we consider two important functions of the fuzzy system (2) among its diverse potential. The first one is universal approximation capability which is used as fuzzy uncertainty observers. This is the most powerful function of (2) and has been utilized in various applications [12]. The other one is the weighted averaging capability of its output Singletons with input-dependent weights (*i.e.*, membership functions of input fuzzy variables) due to the employment of the center-average defuzzifier. The latter, moreover, can also be interpreted as a convex combination of the output Singletons, provided that input fuzzy variable is a scalar (*i.e.*, $n = 1$) and the sum of its all membership functions equals uniformly to one as follows:

$$\Sigma_{i=1}^k \mu_{A^i}(x) = 1, \quad \forall x \in \mathbf{R} \tag{3}$$

This property will be utilized for the smooth transition in variable structure control by reducing high-frequency chattering phenomena.

2. OVERHEAD CRANE DYNAMICS

Figure 1 shows the schematic of a 2D overhead crane traveling on a rail with its load at end of rope. In this figure, $x(t)$, $l(t)$, and $\theta(t)$ are trolley position, hoisting rope length, and load sway angle, respectively; $u_x(t)$ and $u_l(t)$ are trolley driving and load hoisting forces, respectively.

For the overhead crane in figure 1, the equations of motion are given as follows [1]:

$$\begin{aligned} \gamma\ddot{x} + \beta\cos(\theta)\ddot{\theta} + m\sin(\theta)\ddot{l} + 2m\cos(\theta)\dot{l}\dot{\theta} - \beta\sin(\theta)\dot{\theta}^2 &= u \\ \kappa\ddot{l} + m\sin(\theta)\ddot{x} - \beta\dot{\theta}^2 + (\eta / l)\cos(\theta) &= u_l \\ \alpha\ddot{\theta} + \beta\cos(\theta)\ddot{x} + 2\beta\dot{l}\dot{\theta} - \eta\sin(\theta) &= d_\theta \end{aligned} \tag{4}$$

where $\alpha = ml^2$, $\beta = ml$, $\gamma = M + m$, $\eta = -mgl < 0$, and $\kappa = (m_l + m)$; M, m, m_l, and g stand for the total mass of trolley, the mass of load, the mass of hoisting equipment, and gravitational acceleration, respectively; and d_θ is external disturbance due to wind gust.

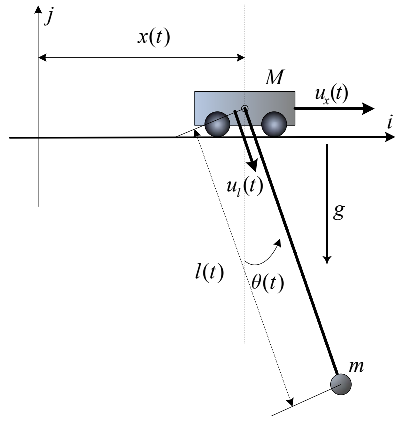

Figure 1: A 2D overhead crane system

For the equations of motion of the overhead crane (4), it is remarkable that: 1) the third equation reveals the coupled nature of sway dynamics, 2) that is, the sway dynamics is disturbed by hoisting velocity ($\dot{l}$) as well as trolley acceleration ($\ddot{x}$), and 3) the sway dynamics results in marginally stable zero dynamics when outputs are all zeros (*i.e.*, weakly minimum phase). In addition, we consider the following assumptions:

Assumption 1. Sway angle satisfies $|\theta(t)| \le \pi / 2$ for all $t \ge 0$, such that rope length is positive throughout the entire control.

Assumption 2. The trolley driving and load hoisting forces, generated by actuators built in trolley and hoisting equipment, are described as [10]

$$u_i = DZ(\upsilon_i) = \upsilon_i + \Delta\upsilon_i,$$

for $i = x, l$, where υ_i is ideal force generated by actuator; $\Delta\upsilon_i$ is the uncertainty of υ_i due to unknown nonlinearity of actuator; and $DZ(\cdot)$ stands for dead-zone characteristics.

Then, by defining a new variable $\mathbf{p} = (x, l)^T$ and considering system uncertainties, (4) can be rewritten as

$$\begin{aligned} \ddot{\mathbf{p}} &= \mathbf{F_p}(\mathbf{x}) + \mathbf{G_p}(\mathbf{x})\Upsilon + \mathbf{\Omega_p}(\mathbf{x}, \Upsilon) \\ \ddot{\theta} &= F_\theta(\mathbf{x}) + \mathbf{G}_\theta(\mathbf{x})\Upsilon + \Omega_\theta(\mathbf{x}, \Upsilon) \end{aligned} \tag{5}$$

where $\mathbf{x} = (x, \dot{x}, l, \dot{l}, \theta, \dot{\theta})$; all matrix functions are given in Appendix A; Υ is control vector; $\mathbf{\Omega_p}$ and Ω_θ are lumped uncertainties for actuated and unactuated subsystems, respectively.

As aforementioned, (4) reveals that three subsystems are coupled with each other and the sway dynamics is disturbed by trolley acceleration and hoisting velocity. That is, persistent trolley acceleration and high-speed hoisting velocity can give rise to unexpected oscillation of the sway dynamics. Moreover, two subsystems in (5) possess lumped uncertainties $\mathbf{\Omega_p}$ and Ω_θ, respectively. This motivates the objective of this study to design an adaptive anti-sway trajectory tracking control law such that the sway angle remains as small as possible while trolley and hoisting rope track the specified reference trajectories in the presence of the lumped uncertainties.

3. Trajectory Tracking and Anti-Sway Control

3.1 Trajectory Tracking

In this subsection, we design a trajectory tracking law based on feedback linearization [4].

First, the trajectory tracking error is defined as $\mathbf{e} \triangleq \mathbf{p} - \mathbf{p}_d$ where $\mathbf{p}_d$ denotes the reference trajectory vector satisfying the following assumption.

Assumption 3. The reference trajectory $\mathbf{p}_d \in C^2$ and its first and second time derivatives are uniformly bounded.

Then, one can design trajectory tracking control law as

$$\Upsilon = [\mathbf{G_p}]^{-1}(-\boldsymbol{\chi}_\mathbf{p} - \mathbf{F_p} + \Upsilon^{AFNC}) \tag{6}$$

with $\boldsymbol{\chi}_\mathbf{p} = (\boldsymbol{\chi}_{\mathbf{p}1}, \boldsymbol{\chi}_{\mathbf{p}2})^T = -\ddot{\mathbf{p}}_d + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e}$, where $\mathbf{K}_d = 2\mathbf{K_e}$, $\mathbf{K}_p = \mathbf{K}_\mathbf{e}^2$, $\mathbf{K_e} = \text{diag}(k_x, k_l)$; $k_x, k_l > 0$ are design parameters; and Υ^{AFNC} denotes adaptive anti-sway control law to be designed in the subsequent section. With the control law (6), (5) becomes as follows:

$$\begin{aligned} &\ddot{\mathbf{e}} + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} = \Upsilon^{AFNC} + \mathbf{\Omega_p}(\mathbf{x}, \Upsilon) \\ &\ddot{\theta} = \overline{\eta}\sin(\theta) - \mathbf{D}(-\boldsymbol{\chi}_\mathbf{p} + \chi_l + \Upsilon^{AFNC}) + \Omega_\theta(\mathbf{x}, \Upsilon) \end{aligned} \tag{7}$$

where $\overline{\eta} = \eta / \alpha < 0$; $\mathbf{D} = (D_1, D_2) = \overline{\mathbf{B}} / \alpha \geq 0$, and $\chi_l = (\chi_{l1}, \chi_{l2})^T = (2\beta\dot{l}\dot{\theta} / \alpha, 0)^T$.

3.2 FNC for Anti-Sway Control

Considering the closed-loop sway dynamics given as second equation of (7), a discontinuous anti-sway control law is proposed as follows:

$$\Upsilon^C = \Upsilon^D + \overline{\Upsilon}^C \tag{8}$$

with

$$\Upsilon^D = -\chi_l + \frac{1}{D_1}\begin{bmatrix} k_\theta\dot{\theta} + (k_g l - 1)\overline{\eta}\sin\theta \\ 0 \end{bmatrix}, \ \overline{\Upsilon}^C = \begin{bmatrix} |\boldsymbol{\chi}_{\mathbf{p}1}|\text{sgn}(\chi_\theta) \\ 0 \end{bmatrix},$$

where $\chi_\theta = k_\theta\theta + 2\dot{\theta}$, k_θ and k_g are positive design parameters, $\text{sgn}(\cdot)$ denotes sign function. The control law (8) consists of a damping control term Υ^D dedicated to sway suppression and a switching compensator $\overline{\Upsilon}^C$ to cope with the effect of the trajectory tracking control, namely, a perturbation term $\boldsymbol{\chi}_{\mathbf{p}1}$. Unfortunately, this control law is not practical since it leads to high-frequency undesirable action in the closed-loop response, *i.e.*, chattering. In general, the sign function in a switching compensator can be replaced with a saturation function to reduce chattering effect. However, the saturation function introduces a boundary layer in the neighborhood of the switching surface at the expense of asymptotic stability. To this end, we propose an FNC with vanishing boundary layer for anti-sway control, which not only guarantees asymptotic stability of the sway angle but also renders the transition of control laws smoother than that of the discontinuous control law (8).

First, consider the following two control laws

$$\Upsilon^{IB} = \Upsilon^D, \qquad \Upsilon^{OB} = \Upsilon^D + \overline{\Upsilon}^C \tag{9}$$

where Υ^{IB} and Υ^{OB} are control laws corresponding to the cases when $\chi_\theta = 0$ and $\chi_\theta \neq 0$, respectively. In addition, two membership functions $\mu_{X_\theta^{IB}}(\chi_\theta)$ and $\mu_{X_\theta^{OB}}(\chi_\theta)$ are defined using the property (3) as

$$\mu_{X_\theta^{IB}}(\chi_\theta) \triangleq e^{-(\chi_\theta/\phi^2(t))}, \ \mu_{X_\theta^{OB}}(\chi_\theta) \triangleq 1 - \mu_{X_\theta^{IB}}(\chi_\theta) \tag{10}$$

where $\phi(t)$ is a boundary layer defined later, and X_θ^{IB} and X_θ^{OB} are fuzzy sets for χ_θ, meaning *in boundary layer* and *out of boundary layer*, respectively. Then, by using the two fuzzy rules given by

$$\begin{aligned} &Rule^1: \text{If } \chi_\theta \text{ is } X_\theta^{IB}, \text{then } \Upsilon^C \text{ is } \Upsilon^{IB}. \\ &Rule^2: \text{If } \chi_\theta \text{ is } X_\theta^{OB}, \text{then } \Upsilon^C \text{ is } \Upsilon^{OB}. \end{aligned} \tag{11}$$

an FNC is given as

$$\Upsilon^C = \frac{[\mu_{X_\theta^{OB}}(\chi_\theta) \ \ 0] \cdot \Upsilon^{OB} + [\mu_{X_\theta^{IB}}(\chi_\theta) \ \ 0] \cdot \Upsilon^{IB}}{\mu_{X_\theta^{OB}}(\chi_\theta) + \mu_{X_\theta^{IB}}(\chi_\theta)} \tag{12}$$

and rewritten as

$$\Upsilon^C = \Upsilon^D + \Upsilon^{FNC} = \Upsilon^D + \overline{\Upsilon}^C \cdot \mu_{X_\theta^{OB}}(\chi_\theta) \tag{13}$$

In (13), it can be shown that Υ^C consists of the same damping control as in (8) but $\overline{\Upsilon}^C$ is replaced with a fuzzy switching compensator Υ^{FNC} contributing a smooth transition of control laws. In addition, the vanishing boundary layer for the proposed Υ^{FNC} is defined as

$$\phi(t) \triangleq \begin{cases} 1/(a+b), & 0 \leq t \leq T \\ 1/(a + be^{c(t-T)}), & t > T \end{cases} \tag{14}$$

where a, b, c and T are positive constants.

4. ADAPTIVE ANTI-SWAY CONTROL

4.1 Coupled Uncertainties

The overhead crane in a decoupled control affine form (5) consists of two subsystems, $\mathbf{p}$- dynamics and θ- dynamics having lumped uncertainties $\mathbf{\Omega_p} = (\Omega_{p1}, \Omega_{p2})^T = (\Omega_x, \Omega_l)^T$ and Ω_θ, respectively. In this subsection, it is shown that the lumped uncertainties $\mathbf{\Omega_p}$ and

Ω_θ can be compensated by estimating only $\mathbf{\Omega_p}$ by resorting to the fact that Ω_{p1} and Ω_θ are coupled with each other.

Property 1. Consider an overhead crane including uncertainties as in (5). If the total system uncertainty including parameter variations, wind disturbance, and actuator nonlinearity are decoupled into $\mathbf{\Omega_p}$ and Ω_θ for each subsystem, then they satisfy the followings:

$$\alpha\Omega_\theta = -\bar{B}_1\Omega_{p1} = -\beta\cos(\theta)\Omega_x \tag{15}$$

4.2 Fuzzy uncertainty observer

In this paper, we design two FUOs $\hat{\Omega}_x$ and $\hat{\Omega}_l$ to estimate the lumped uncertainty Ω_x and Ω_l in (5). Moreover, parameter adaptation laws are also provided so that the estimate is guaranteed to monitor and represent well the whole uncertainties existing in the overhead crane as well as the asymptotic stability is preserved. First, to construct an FUO for the estimate $\hat{\Omega}_i$ for $i = x,\ l$, the following assumption is required.

Assumption 4. Let the input vector $\mathbf{x}$ for the fuzzy logic system (2) belong to a compact set $M_\mathbf{x}$. The optimal parameter vector ϕ_i of (2) is defined as follows:

$$\phi_i^* = \arg\min_{\phi_i \in M_{\phi_i}}\left(\sup_{\mathbf{x}\in M_\mathbf{x}}\left|\Omega_i(\mathbf{x},\Upsilon) - \hat{\Omega}_i(\mathbf{x},\Upsilon \mid \hat{\phi}_i)\right|\right)$$

which exists in a convex region $M_{\phi_i} = \{\phi_i \mid \|\phi_i\| \le m_{\phi_i}\}$ where m_{ϕ_i} is a design parameter.

Then, the fuzzy logic system (2), which is a zero-order TS fuzzy system, is used as an FUO for the estimate $\hat{\Omega}_i$ of the uncertainty Ω_i as follows:

$$\hat{\Omega}_i \triangleq \hat{\phi}_i^T\xi(\mathbf{x},\mathbf{x}^d) \tag{16}$$

Remark 1. In (16), to avoid algebraic loop in the implementation of FUO as discussed in [13], $\mathbf{x}^d = (x^d, \dot{x}^d, \ddot{x}^d, l^d, \dot{l}^d, \ddot{l}^d)$ is used instead of control input Υ, since the control input is a function of the state $\mathbf{x}$ and reference trajectories $\mathbf{x}^d$.

Also, as in [13], the following observation dynamics is used:

$$\dot{\mu}_i = -\sigma\mu_i + p_i(i, \dot{i}, \hat{\phi}_i) \tag{17}$$

where $p_i(i,\dot{i},\hat{\phi}_i) \triangleq F_i + G_i\Upsilon + \hat{\Omega}_i(\mathbf{x},\mathbf{x}^d \mid \hat{\phi}_i) + \sigma_i\dot{i}$, $\sigma_i > 0$ is an observer design parameter, and an observation error variable is defined as $\varsigma_i \triangleq \dot{i} - \mu_i$. Moreover, observation error dynamics is expressed as

$$\dot{\varsigma}_i = -\sigma\varsigma_i + \Omega_i(\mathbf{x},\mathbf{x}^d) - \hat{\Omega}_i(\mathbf{x},\mathbf{x}^d \mid \hat{\phi}_i) \tag{18}$$

By Assumption 4 and the universal approximation capability of the FUO in (16), the lumped uncertainty Ω_i can be described as

$$\Omega_i = \hat{\Omega}_i(\mathbf{x},\mathbf{x}^d \mid \phi_i^*) + \varepsilon_i(\mathbf{x},\mathbf{x}^d) \tag{19}$$

where $\varepsilon_i(\mathbf{x},\mathbf{x}^d)$ denotes an uncertainty estimation error satisfying $|\varepsilon_i(\mathbf{x},\mathbf{x}^d)| \le \bar{\varepsilon}_i$ and its upper bound $\bar{\varepsilon}_i$ can be arbitrarily shrunken by increasing the number of the fuzzy rules. Then, (18) becomes

$$\dot{\varsigma}_i = -\sigma\varsigma_i + \hat{\Omega}_i(\mathbf{x},\mathbf{x}^d \mid \hat{\phi}_i^*) - \hat{\Omega}_i(\mathbf{x},\mathbf{x}^d \mid \hat{\phi}_i) + \varepsilon_i(\mathbf{x},\mathbf{x}^d) \tag{20}$$

Remark 2. If the bounded and adjustable parameter vector $\hat{\phi}_i$ is tuned by

$$\dot{\hat{\phi}}_i = \rho_i\varsigma_i\xi(\mathbf{x},\mathbf{x}^d) \tag{21}$$

where $\rho_i > 0$ is a observer design parameter, then the uncertainty observation error ς_i is uniformly ultimately bounded within a region of which size can be kept arbitrarily small (see Theorem 1 in [13]). Therefore, if the observation error ς_i converges to zero, $\hat{\Omega}_i$ converges to the close neighborhood of the actual uncertainty Ω_i.

Remark 3. The parameter adaptation law (21) can be modified by the projection algorithm to ensure the boundedness $\|\phi_i\|$ given in Assumption 4 as follows [13]:

$$\dot{\hat{\phi}} = \rho\varsigma\xi(\mathbf{x},\mathbf{x}^d) - I_\phi\rho\left(\varsigma\hat{\phi}^T\xi(\mathbf{x},\mathbf{x}^d)\hat{\phi}\right)/\|\phi_i\|^2$$

where

$$I_\phi = \begin{cases} 0, & if\ \left(\|\phi_i\| < M_{\phi_i}\right)\ or\ \left(\|\phi_i\| = M_{\phi_i}\ and\ \varsigma_i\hat{\phi}_i^T\xi(\mathbf{x},\mathbf{x}^d) \le 0\right) \\ 1, & if\ \|\phi_i\| = M_{\phi_i}\ and\ \varsigma_i\hat{\phi}_i^T\xi(\mathbf{x},\mathbf{x}^d) > 0 \end{cases}$$

5. ADAPTIVE ANTI-SWAY CONTROL

Using the two FUOs in (16) and FNC in (13), adaptive anti-sway control law is proposed as

$$\Upsilon^{AFNC} = \Upsilon^C - \hat{\mathbf{\Omega}}_\mathbf{p} \tag{22}$$

where $\hat{\mathbf{\Omega}}_\mathbf{p} = (\hat{\Omega}_x, \hat{\Omega}_l)^T$. Now, using the property in (15) and (19) and AFNC in (22), the closed-loop dynamics becomes

$$\begin{aligned} &\ddot{\mathbf{e}} + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} = \Upsilon^C + \tilde{\mathbf{\Omega}}_\mathbf{p} + \boldsymbol{\varepsilon} \\ &\ddot{\theta} + k_\theta\dot{\theta} + k_g g\sin(\theta) = \text{-}\mathbf{D}(\text{-}\boldsymbol{\chi}_\mathbf{p} + \Upsilon^{FNC})\text{-}D_1(\tilde{\Omega}_x + \varepsilon_x) \end{aligned} \tag{23}$$

where $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_l)^T$ such that $\|\boldsymbol{\varepsilon}(\mathbf{x},\mathbf{x}^d)\| \le \bar{\boldsymbol{\varepsilon}} = (\bar{\varepsilon}_x, \bar{\varepsilon}_l)^T$. Moreover, by defining $\mathbf{\Xi} = (\mathbf{e}, \dot{\mathbf{e}}, \varsigma_x, \varsigma_l)^T$ and using (20) and (23), the augmented closed-loop trajectory tracking error dynamics are given as

$$\dot{\mathbf{\Xi}} = \mathbf{\Pi}\mathbf{\Xi} + \mathbf{\Gamma}_1\mathbf{\Phi}_\mathbf{p}^T\xi(\mathbf{x},\mathbf{x}^d) + \mathbf{\Gamma}_1\boldsymbol{\varepsilon} + \mathbf{\Gamma}_0\Upsilon^C \tag{24}$$

where all matrix functions are given in Appendix B.

Remark 4. $\mathbf{\Pi}$ is stable matrix as stated in Appendix B and there exists a positive definite matrix $\mathbf{P}$ such that

$$\mathbf{\Pi}^T\mathbf{P} + \mathbf{P}\mathbf{\Xi} = -\mathbf{Q}$$

for an arbitrary positive definite matrix $\mathbf{Q}$.

As a consequence, the main result of this paper is summarized as the following statement

Theorem 1. Consider the 2D overhead crane with lumped uncertainties (5). If adaptive anti-sway trajectory tracking control law is given as (22) and (7), then, the trajectory tracking error dynamics is asymptotically stable while the sway angle is uniformly bounded and finally converges to zero after trolley reaches the destination.

Proof First, a Lyaounov function candidate for (24) is considered as

$$V_{\mathbf{e}} = \frac{1}{2}\mathbf{\Xi}^T\mathbf{P}\mathbf{\Xi} + \frac{1}{2\gamma_x}\tilde{\phi}_x^T\tilde{\phi}_x + \frac{1}{2\gamma_l}\tilde{\phi}_l^T\tilde{\phi}_l \tag{25}$$

Then, using (24), the time derivative of the Lyapunov function candidate becomes

$$\begin{aligned} \dot{V}_{\mathbf{e}} &= \mathbf{\Xi}^T\mathbf{P\Pi\Xi} + \mathbf{\Xi}^T\mathbf{P\Gamma}_1\mathbf{\Phi}_{\mathbf{p}}^T\xi + \mathbf{\Xi}^T\mathbf{P\Gamma}_1\boldsymbol{\varepsilon} + \mathbf{\Xi}^T\mathbf{P\Gamma}_0\Upsilon^C \\ &\qquad + \gamma_x^{-1}\tilde{\phi}_x^T\dot{\tilde{\phi}}_x + \gamma_l^{-1}\tilde{\phi}_l^T\dot{\tilde{\phi}}_l \\ &= \mathbf{\Xi}^T\mathbf{P\Pi\Xi} + \mathbf{\Xi}^T\mathbf{P\Gamma}_1\boldsymbol{\varepsilon} + \mathbf{\Xi}^T\mathbf{P\Gamma}_0\Upsilon^C \\ &\qquad + \tilde{\phi}_x^T(\mathbf{\Xi}^T\mathbf{P\Gamma}_1^1\xi + \gamma_x^{-1}\dot{\tilde{\phi}}_x) + \tilde{\phi}_l^T(\mathbf{\Xi}^T\mathbf{P\Gamma}_1^2\xi + \gamma_l^{-1}\dot{\tilde{\phi}}_l) \end{aligned} \tag{26}$$

By choosing fuzzy rule tuning methods as

$$\begin{cases} \dot{\tilde{\phi}}_x = -\gamma_x\mathbf{\Xi}^T\mathbf{P\Gamma}_1^1\xi \quad or \quad \dot{\hat{\phi}}_x = \gamma_x\mathbf{\Xi}^T\mathbf{P\Gamma}_1^1\xi \\ \dot{\tilde{\phi}}_l = -\gamma_l\mathbf{\Xi}^T\mathbf{P\Gamma}_1^2\xi \quad or \quad \dot{\hat{\phi}}_l = \gamma_l\mathbf{\Xi}^T\mathbf{P\Gamma}_1^2\xi \end{cases} \tag{27}$$

and using $\mathbf{\Lambda} = \mathbf{\Gamma}_1\boldsymbol{\varepsilon} + \mathbf{\Gamma}_0\Upsilon^C$, one can get

$$\dot{V}_{\mathbf{e}} \le -\frac{1}{2}\mathbf{\Xi}^T\mathbf{Q\Xi} + \mathbf{\Xi}^T\mathbf{P\Lambda} \tag{28}$$

By completing the square, after adding and subtracting $(a/4)\mathbf{\Xi}^T\mathbf{P\Xi} + (1/a)\mathbf{\Lambda}^T\mathbf{P\Lambda}$, (28) becomes

$$\begin{aligned} \dot{V}_{\mathbf{e}} &\le -\frac{1}{2}\mathbf{\Xi}^T\mathbf{Q\Xi} + \frac{a}{4}\mathbf{\Xi}^T\mathbf{P\Xi} + \frac{1}{a}\mathbf{\Lambda}^T\mathbf{P\Lambda} \\ &\le -\frac{1}{4}\lambda_{\min}(\mathbf{Q})\|\mathbf{\Xi}\|^2 + \frac{\lambda_{\max}(\mathbf{P})^2}{\lambda_{\min}(\mathbf{Q})}\|\mathbf{\Lambda}\|^2 \\ &\le -\frac{1}{4}\lambda_{\min}(\mathbf{Q})\|\mathbf{\Xi}\|^2 + \frac{\lambda_{\max}(\mathbf{P})^2}{\lambda_{\min}(\mathbf{Q})}\left(2\|\boldsymbol{\varepsilon}\|^2 + \|\Upsilon^C\|^2\right) \end{aligned} \tag{29}$$

where $a = \lambda_{\min}(\mathbf{Q})/\lambda_{\max}(\mathbf{P})^2$; $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote maximum and minimum eigenvalues, respectively.

Thus, $\dot{V}_{\mathbf{e}}$ is negative outside a compact set $\left(\|\mathbf{\Xi}\| \ge 2\lambda_{\max}(\mathbf{P})/\lambda_{\min}(\mathbf{Q})(2\|\boldsymbol{\varepsilon}\|^2 + \|\Upsilon^C\|^2)\right)$ and, the augmented error $\mathbf{\Xi}$ is uniformly ultimately bounded. Moreover, using Barbalat's Lemma [14], $\|\mathbf{\Xi}(t)\| \to 0$ as $t \to \infty$

Second, let another Lyapunv function candidate be given by

$$V_\theta = \frac{1}{2}\mathbf{\Theta}^T\mathbf{P}_\theta\mathbf{\Theta} + 2k_g g(1-\cos\theta) + \frac{1}{2\gamma_x}\tilde{\phi}_x^T\tilde{\phi}_x \tag{30}$$

where $\mathbf{\Theta} = (\theta, \dot{\theta})^T$ and $\mathbf{P}_\theta = [k_\theta^2\ k_\theta; k_\theta\ 2]$. By using (23) and assumption 1, the time derivative of (25) becomes

$$\begin{aligned} \dot{V}_\theta &= -k_\theta\dot{\theta}^2 - k_\theta k_g g\sin\theta\cdot\theta - \mathbf{D}(-\boldsymbol{\chi}_{\mathbf{p}} + \Upsilon^{FNC}) + D_1\chi_\theta\varepsilon_x \\ &\qquad - \tilde{\phi}_x^T D_1\chi_\theta\xi + \gamma_x^{-1}\tilde{\phi}_x^T\dot{\tilde{\phi}}_x \\ &\le -k_\theta\dot{\theta}^2 - k_\theta k_g g\theta^2 + D_1\varepsilon_x - \tilde{\phi}_x^T(D_1\chi_\theta\xi - \gamma_x^{-1}\dot{\tilde{\phi}}_x) \end{aligned} \tag{31}$$

By choosing the tuning method for $\hat{\Omega}_x$ as

$$\dot{\tilde{\phi}}_x = -\gamma_x D_1\chi_\theta\xi \quad or \quad \dot{\hat{\phi}}_x = \gamma_x D_1\chi_\theta\xi \tag{32}$$

(31) becomes

$$\dot{V}_\theta \le -\lambda\|\mathbf{\Theta}\| + |D_1\chi_\theta|\,\overline{\varepsilon}_x \tag{33}$$

where $\lambda = \min\{k_\theta k_g g, k_\theta\}$, that is, $\lambda = k_\theta$ if $k_g g > 1$. Therefore, $\dot{V}_\theta$ is negative outside a compact set $\left(\|\mathbf{\Theta}\| \ge +(|D_1\chi_\theta|/\lambda)\overline{\varepsilon}_x\right)$, which leads the asymptotic stability of sway dynamics. This complete the proof. ■

Remark 5. From (27) and (32), it is inferred that the tuning method for $\hat{\Omega}_x$ can be chosen as $\gamma_x\psi(e_x, \dot{e}_x, \theta, \dot{\theta}, \sigma_x)\xi$, where $\psi(\cdot)$ is a affine function of it arguments as $\mathbf{\Xi}^T\mathbf{P\Gamma}_1^1 - D_1\chi_\theta$.

6. SIMULATIONS

To evaluate the performance of the proposed control law, two cases are considered: 1) parameter uncertainty, wind disturbance, and actuator nonlinearity exist. 2) hoisting velocity is increased about 300%. Figure 2 and 3 show the results of case 1 and 2, respectively.

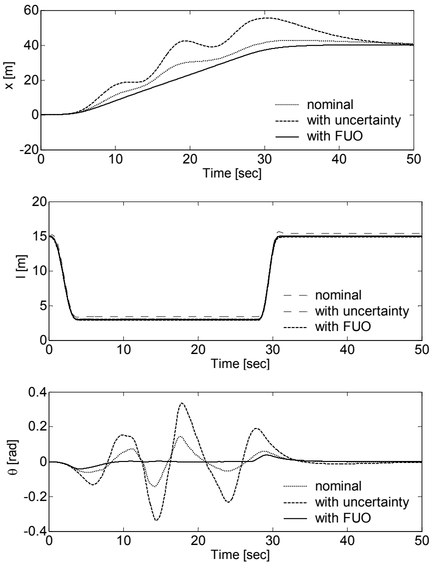

Figure 2: performance evaluation for uncertainty (case 1)

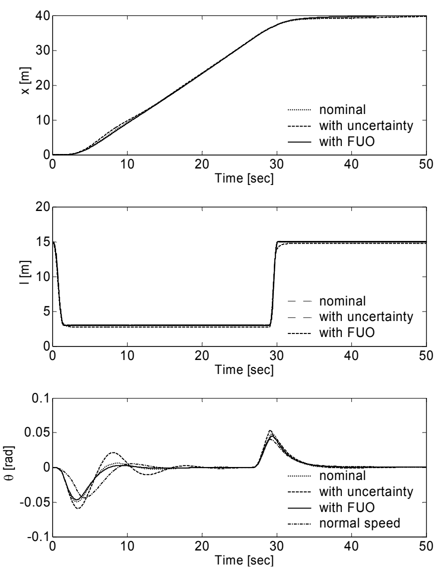

Figure 3: performance evaluation for high-speed hoisting motion (case 2)

The nominal parameter values employed in simulations are given as follows: $M = 1000$ kg, $m = 500kg$, $m_l = 500$ kg, $g = 9.8$ m/s^2, and the hoisting rope length l varies from 3 to 15 m. The design parameters for controllers and observers are given as $k_x = 0.3$, $k_l = 5$, $k_\theta = 5$, $k_g = 1/3$ $\sigma_x = \sigma_l = 100$, $\gamma_x = \gamma_l = 20000$. The design parameters in (14) for FNC are given as $a = 0$, $b = 10$, $c = 0.5$, and $T = 5$, respectively. For the premise parts of the two FUOs in (16), seven Gaussian functions and two sigmoid functions are used as in [13]

From figure 2, it is shown that the proposed control law guarantees robust anti-sway performance against load weight variations, wind disturbance, and actuator dead- zone. In addition, robust anti-sway performance against high-speed hoisting motion is confirmed from the results illustrated in figure 3.

7. CONCLUSION

In this study, we proposed adaptive fuzzy nonlinear control law for the anti-sway trajectory tracking control of uncertain overhead cranes with high-speed hoisting. From the stability analysis and numerical simulation results, it was shown that the proposed control law guarantees robust anti-sway and tracking performance against parameter variations, wind disturbance, and actuator nonlinearity.

APPENDIX A

All matrix functions in (5) are given as follows:

$$\mathbf{F_p} = \frac{\bar{\mathbf{W}}}{\det(\bar{\mathbf{M}})},\quad \mathbf{G_p} = \frac{\mathbf{P}}{\det(\bar{\mathbf{M}})},\quad \mathbf{\Sigma_p} = \frac{\bar{\mathbf{S}}}{\det(\bar{\mathbf{M}})},$$

$$F_\theta = \frac{\bar{\mathbf{V}}}{\alpha\det(\bar{\mathbf{M}})},\quad \mathbf{G}_\theta = \frac{\mathbf{Q}}{\alpha\det(\bar{\mathbf{M}})},\quad \Sigma_\theta = \frac{\mathbf{R}}{\alpha\det(\bar{\mathbf{M}})},$$

with

$$\bar{\mathbf{W}} = -\bar{\mathbf{M}}^A\mathbf{W},\quad \mathbf{P} = \bar{\mathbf{M}}^A,\quad \bar{\mathbf{S}} = -\bar{\mathbf{M}}^A\bar{\mathbf{d}},$$

$$\bar{\mathbf{V}} = \bar{\mathbf{B}}\bar{\mathbf{M}}^A\mathbf{W} - V\cdot\det(\bar{\mathbf{M}}),\ \mathbf{Q} = -\bar{\mathbf{B}}\bar{\mathbf{M}}^A,\ \mathbf{R} = \bar{\mathbf{B}}\bar{\mathbf{M}}^A\bar{\mathbf{d}},$$

where

$$\bar{\mathbf{M}}^A = \begin{bmatrix} \kappa & -m\sin(\theta) \\ -m\sin(\theta) & \gamma - \dfrac{\beta^2\cos^2(\theta)}{\alpha} \end{bmatrix},$$

$$\mathbf{W} = \begin{bmatrix} 2\left(m - \dfrac{\beta^2}{\alpha}\right)\cos(\theta)\dot{l}\dot{\theta} - \beta\sin(\theta)\dot{\theta}^2 + \dfrac{\eta\beta\cos(\theta)\sin(\theta)}{\alpha} \\ -\beta\dot{\theta}^2 + (\eta/l)\cos(\theta) \end{bmatrix},$$

$$\bar{\mathbf{d}} = \begin{bmatrix} \mu_x\dot{x} - d - \Delta v_x \\ \mu_l\dot{l} + \Delta v_l \end{bmatrix},\quad \Upsilon = \begin{bmatrix} \upsilon_x \\ \upsilon_l \end{bmatrix},$$

$$\bar{\mathbf{B}} = \begin{bmatrix} \bar{B}_1 & \bar{B}_2 \end{bmatrix} = \begin{bmatrix} \beta\cos(\theta) & 0 \end{bmatrix},\quad V = 2\beta\dot{l}\dot{\theta} - \eta\sin(\theta),$$

$$\det(\bar{\mathbf{M}}) = \gamma\kappa - \frac{\beta^2\kappa\cos^2(\theta)}{\alpha} - \frac{\beta^4}{\alpha^2}\sin^2(\theta),$$

The lumped uncertainties are defined as

$$\mathbf{\Omega_p}(\mathbf{x},\Upsilon) \triangleq \mathbf{\Delta F_p} + \mathbf{\Delta G_p}\Upsilon + \mathbf{\Sigma_p}$$

$$\Omega_\theta(\mathbf{x},\Upsilon) \triangleq \Delta F_\theta + \mathbf{\Delta G}_\theta\Upsilon + \Sigma_\theta$$

APPENDIX B

All matrix functions in (24) are given as follows:

$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{A} & \mathbf{O}_{24}^T \\ \mathbf{O}_{24} & \mathbf{\Sigma} \end{bmatrix},\ \mathbf{\Gamma}_1 = \begin{bmatrix} \mathbf{\Gamma}_1^1 \\ \mathbf{\Gamma}_1^2 \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \mathbf{I}_2 \end{bmatrix},\ \mathbf{\Gamma}_0 = \begin{bmatrix} \mathbf{H} \\ \mathbf{O}_2 \end{bmatrix}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{O}_2 & \mathbf{I}_2 \\ -\mathbf{K}_p & -\mathbf{K}_d \end{bmatrix},\ \mathbf{\Sigma} = \begin{bmatrix} -\sigma_x & 0 \\ 0 & -\sigma_l \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} \mathbf{O}_2 \\ \mathbf{I}_2 \end{bmatrix},$$

and $\mathbf{I}_2 \in \mathbf{R}^{2\times 2}$, $\mathbf{O}_2 \in \mathbf{R}^{2\times 2}$, $\mathbf{O}_{24} \in \mathbf{R}^{2\times 4}$ denote identity and null matrices, respectively.

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