

# Design of Gain Scheduling Anti-Sway Crane Controller Using Genetic Fuzzy System

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**Abstract**—The objective of the paper is to describe the genetic fuzzy-based method of designing the gain scheduling control system, which is the effective and wide used adaptive technique based on varying controller parameters according to the exogenous variables. The problem is addressed in the paper to find the minimum set of linear controllers scheduled by fuzzy interpolation scheme. The proposed method is based on the pole placement approach and interval mathematic used to define the desired region of poles around the operating point at which the controller is designed. The genetic algorithm is used to explore of searching space by competition between the fuzzy gain scheduling systems differing in number of fuzzy partitions distributed within the intervals of scheduling variables, and fine tuning the membership functions. The proposed approach is addressed in the paper to the anti-sway crane control problem.

## I. INTRODUCTION

The design process of a gain scheduling control system consists in selecting such a suitable set of operating points at which the linear controllers are determined that interpolation control scheme ensures the expected control quality within the known range of nonlinear system parameters changes, when those parameters vary in relation to some exogenous variables, called scheduling variables. The paper is addressed to the problem of selecting the minimum set of controllers for gain scheduling system. This problem arises together with increase of number of scheduling variables required to identify the current operating point at which the controller gains are interpolated. The paper describes the anti-sway crane control system based on the gain scheduling control scheme designed using the genetic fuzzy system (GFS) and pole placement method (PPM).

The straightforward solution of the problem of gain scheduling control system designing can be based on dividing the range of key system variables into equal intervals and determining the linear controllers at the midpoint of each region [10]. The iterative method applied to time-discrete gain scheduling system with one exogenous variable is presented in [8]. The proposed method utilizes interval mathematic and

PPM to define the desired poles region of linear closed-loop control system in form of interval vector of desired coefficients of characteristic equation. The method consists in incrementing the value of scheduling variable starting from the nominal poles of the considered region until the Diophantine equation with interval vector of desired coefficients of characteristic equation is satisfied. This iterative technique was applied in [12] to design the anti-sway crane control system, where the fuzzy control scheme was based on the two scheduling variables, rope length and mass of a payload used for gains interpolation. In [7] the genetic algorithm (GA) is used to optimize the number of intervals of simple example of gain scheduling control system based on the proportional controller and one exogenous variable. In [6] the GA was applied to tune the fuzzy PID controller gains and membership functions specified for three scheduling variables. The optimal gains of the PID controllers are tuned based on the objective function to minimize the overshoot and settle time used to quantify the control system performances.

The solution of anti-sway crane control problem are frequently proposed in scientific works based on the soft computing techniques, fuzzy logic, artificial neural network and GA. The fuzzy gain scheduling approach is presented e.g. in [3, 13]. The gain scheduling system design technique based on the clustering method is delivered in [11] for the anti-sway tower crane control system. The time-optimal control with using GA was proposed for unconstrained optimal crane control in [4]. The real-coded GA was used to find the desired initial co-states of the system with no-constraints. The objective function was formulated as the minimum cost co-states calculated based on the ability to move the system to the desired state after a given amount of time. In [5] the GA was used to determine the coefficients of an optimal feedforward control law used to suppress the load sway of a shipboard crane due to the ship rolling. In [9] the anti-sway crane control problem was solved by using the neural controller trained by GA. The unconventional method of chromosomes tree-encoding in GA applied in crane control system was presented in [2]. The control algorithm was presented in form rules

separately for swing increasing and dumping, based on heuristic knowledge about the crane laboratory model. The parameters of controller were modified by GA during experiments carried out on the laboratory stand.

The paper is organized as follows. The section II presents the fuzzy interpolation scheme proposed for anti-sway crane control system, and PPM-based approach to design a set of controllers at specified operating points. The section III delivers the GA-based strategy used to design the fuzzy gain scheduling system. The section IV presents the examples of results of anti-sway crane gain-scheduling control system designed for assumed intervals of gain scheduling variables. The section V delivers the short review of the paper and conclusions.

## II. FUZZY GAIN SCHEDULING APPROACH TO ANTI-SWAY CRANE CONTROL

### A. The fuzzy control scheme

A nonlinear crane dynamic system (Fig. 1) can be simplified to the linear model with time-varying parameters, and presented in the form of two continuous transmittances (1) and (2) derived from Lagrange's second law type equation, where masses of a crane and a payload, rope length and sway angle are, respectively, denoted  $M, m, l$ , and  $\alpha$ .

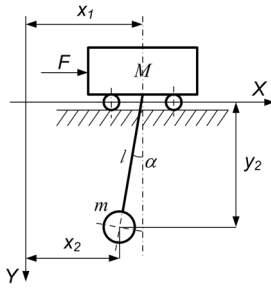


Figure 1. The two-mass model of a crane

$$G_1(s) = \frac{\alpha(s)}{U(s)} = \frac{-\frac{1}{Ml}}{s^2 + \left(1 + \frac{m}{M}\right)\frac{g}{l}}. \quad (1)$$

$$G_2(s) = \frac{X(s)}{\alpha(s)} = \frac{-ls^2 - g}{s^2}. \quad (2)$$

where:

$g = 9,81[m/s^2]$  - acceleration of gravity.

$\omega_n = \sqrt{\left(1 + \frac{m}{M}\right)\frac{g}{l}}$  - the natural pulsation of the payload sway.

The gain scheduling control system (Fig. 2) is created based on the fuzzy interpolation scheme using fuzzy rule-based system (FRBS) type of Takagi-Sugeno-Kang (TSK) with rule base (RB) representing a set of linear proportional-derivative (PD) controllers used in the feedbacks from crane position  $x$  and sway angle of a payload  $\alpha$ .

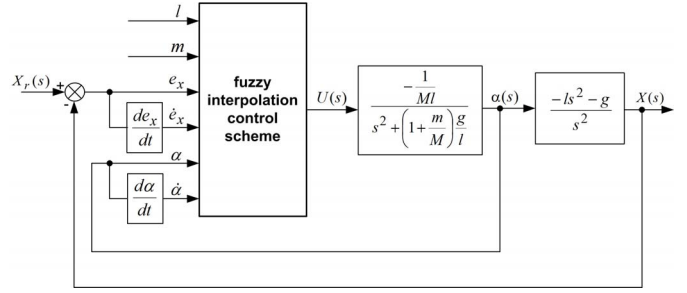


Figure 2. The anti-sway crane control system with fuzzy gain scheduling control scheme

The rope length  $l$  and mass of a payload  $m$  are the scheduling variables, while the crane position error  $e_x = x_r - x$ , sway angle of a payload  $\alpha$  and their time derivatives are the input variables used to calculate the output signal  $u$  according the function specified in the conclusion of rules:

$R_k$ : IF  $l$  is  $MF_i(l)$  and  $m$  is  $MF_j(m)$

$$\text{THEN } u_k = \begin{bmatrix} e_x \\ \dot{e}_x \\ \alpha \\ \dot{\alpha} \end{bmatrix}^T \cdot \begin{bmatrix} K_{k1} \\ K_{k2} \\ K_{k3} \\ K_{k4} \end{bmatrix}. \quad (3)$$

where:

$l, m$  - the scheduling variables,

$MF(l), MF(m)$  - the membership functions (MFs) distributed within the domains of scheduling variables, represented in antecedent of fuzzy rule by linguistic terms,

$K_{k1}, K_{k2}, K_{k3}, K_{k4}$  - the gains of linear controller specified for a given fuzzy region of operating points,

$k = 1, 2, \dots, N$  - the number of fuzzy rule in FRBS.

The output function of the fuzzy controller is determined as the weighted average of all rules output (4), taking into consideration the weights of rules calculated as a product of membership grades of the crisp input values of rope length and mass of a payload (5).

$$u = \sum_{k=1}^N w_k \cdot u_k \cdot \left( \sum_{k=1}^N w_k \right)^{-1}. \quad (4)$$

$$w_k = \mu_{MF_i}(l) \cdot \mu_{MF_j}(m). \quad (5)$$

### B. Pole placement approach to determine the minimum set of operating points

It was assumed that scheduling variables are fuzzified using triangular-shaped MFs, and neighboring MFs overlap uniformly (Fig. 3). Hence, the each rule of FRBS represents the fuzzy region of operating points limited by scheduling variables intervals. The midpoints of those regions are located at the centre points  $\{b_{li}, b_{mj}\}$  of triangular MFs. Thus, the

fuzzy control scheme is based on the  $N$  linear controllers, specified in rules conclusions, determined at midpoints of those fuzzy regions.

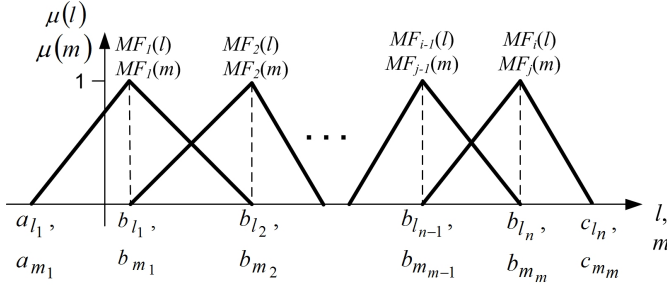


Figure 3. Distribution of MFs within the rope length  $l$  and mass of a payload  $m$  scheduling variables

Diophantine equation formulated for fourth-order characteristic equation of closed-loop control system represented by a single  $k$  rule (3) can be presented as follows:

$$\mathbf{S} \cdot \mathbf{K}_k = \mathbf{P}_k \quad (6)$$

where the  $\mathbf{S}$  is an eliminant matrix (Sylvester matrix) consisting of object parameters (7), the vector  $\mathbf{K}_k$  consists of controller gains (8), and the  $\mathbf{P}$  is a vector of desired coefficients of characteristic equation.

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & \frac{1}{M} & 0 & \frac{1}{Ml} \\ \left(1 + \frac{m}{M}\right) \frac{g}{l} & \frac{1}{M} & 0 & \frac{1}{Ml} & 0 \\ 0 & 0 & \frac{g}{Ml} & 0 & 0 \\ \left(1 + \frac{m}{M}\right) \frac{g}{l} & \frac{g}{Ml} & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

$$\mathbf{K}_k = \begin{bmatrix} 1 \\ K_{k1} \\ K_{k2} \\ K_{k3} \\ K_{k4} \end{bmatrix} \quad (8)$$

The vector of gains  $\mathbf{K}_k$  specified at a nominal operating point of a fuzzy region  $k$  is determined for 4 the same poles equal the natural pulsation of a system, based on the system equation (6) with the vector of desired coefficients defined as:

$$\mathbf{P}_k = \begin{bmatrix} 4\omega_n \\ 6\omega_n^2 \\ 4\omega_n^3 \\ \omega_n^4 \end{bmatrix} \quad (9)$$

while the region of acceptable poles changes, specified for assumed tolerance of closed-loop control performances is

formulated by interval vector of characteristic polynomial coefficients (10).

$$\mathbf{P}_k = \begin{bmatrix} p_{k1 \min}, & p_{k1 \max} \\ p_{k2 \min}, & p_{k2 \max} \\ p_{k3 \min}, & p_{k3 \max} \\ p_{k4 \min}, & p_{k4 \max} \end{bmatrix} = \begin{bmatrix} 4\omega_n \left( \xi \pm \sqrt{1 - \xi^2} \right) \\ 6\omega_n^2 \left( \xi \pm \sqrt{1 - \xi^2} \right)^2 \\ 4\omega_n^3 \left( \xi \pm \sqrt{1 - \xi^2} \right)^3 \\ \omega_n^4 \left( \xi \pm \sqrt{1 - \xi^2} \right)^4 \end{bmatrix} \quad (10)$$

Hence, the objective of GA-based optimization of the FRBS is to find minimum set of MFs specified for scheduling variables intervals  $l=[l_{\min}, l_{\max}]$  and  $m=[m_{\min}, m_{\max}]$ . Assuming that the neighboring MFs overlap uniformly (Fig. 3), the most hazardous operating points are the bounds of considered intervals of gain scheduling variables ( $l_{\min}, l_{\max}, m_{\min}, m_{\max}$ ), and crossover points at which the neighboring MFs overlap. Hence, the fuzzy gain scheduling system satisfies expected requirements when the condition (11) is satisfied for all most hazardous operating points.

$$\mathbf{S}_h \cdot \sum_{k=1}^N (w_k \cdot \mathbf{K}_k) \cdot \left( \sum_{k=1}^N w_k \right) \in \bar{\mathbf{P}}, \quad h = 1, 2, (m+1) \cdot (n+1). \quad (11)$$

where the  $\mathbf{S}_h$  is the eliminant matrix consisting of system parameters at hazardous operating point,  $N = n \cdot m$  is the number of rules in RB depending on the number of MFs distributed within the scheduling variables intervals (Fig. 3), and  $\bar{\mathbf{P}}$  is the vector consisting of average values of two smallest right bounds and two highest left bounds of interval vectors (10) specified for the nominal operating points of fuzzy rules used to interpolate the gains (rules with the weight  $w_k > 0$ ):

$$\mathbf{P}_k = \begin{bmatrix} \bar{p}_{1 \min}, & \bar{p}_{1 \max} \\ \bar{p}_{2 \min}, & \bar{p}_{2 \max} \\ \bar{p}_{3 \min}, & \bar{p}_{3 \max} \\ \bar{p}_{4 \min}, & \bar{p}_{4 \max} \end{bmatrix} \quad (12)$$

Thus, the fitness function is formulated as follows:

$$Fitness = \left( \sum_{h=1}^H \sum \left[ \left( \bar{\mathbf{P}}_{\max} - \mathbf{S}_h \cdot \frac{\sum_{k=1}^N w_k \cdot \mathbf{K}_k}{\sum_{k=1}^N w_k} \right) \cdot \sigma_1 \right] \right)^{-1} \quad (13)$$

where:

$H = (n+1) \cdot (m+1)$  - the total number of hazardous operating points,  $n$  and  $m$  is the number of MFs specified for rope length and mass of a payload input variables respectively,  $\sigma_1$  - is the penalty factor taking value  $\sigma_1 = 1$  or  $\sigma_1 < -1$  if the condition (11) is, respectively, satisfied or failed.

### III. GENETIC FUZZY APPROACH TO DESIGN GAIN SCHEDULING SYSTEM

The GA-based strategy was based on the Pittsburgh approach [1] used to optimize the FRBS, hence the single proposition of fuzzy gain scheduling system is real-valued encoded chromosome composed of two vectors (14) with MFs parameters (Fig. 3).

$$l: [a_{l1}, b_{l1}, b_{l2}, \dots, b_{li}, \dots, b_{ln}, c_{ln}]$$

$$m: [a_{m1}, b_{m1}, b_{m2}, \dots, b_{mj}, \dots, b_{mm}, c_{mm}] \quad (14)$$

The objective of the evolutionary searching method is explore the possible solution space consisting of the individuals which differ in the size rule base (the number of linear controllers). Thus, the classic reproduction methods were modified in order to prevent trapping local optimum by converging to the population of individuals with the same rules number. In order to meet those requirements we propose the three-stage reproduction strategy (Fig. 4) consisting of two different types of mutation operations and arithmetical crossover which ensure exploration of searching space of fuzzy models differing in rule base size.

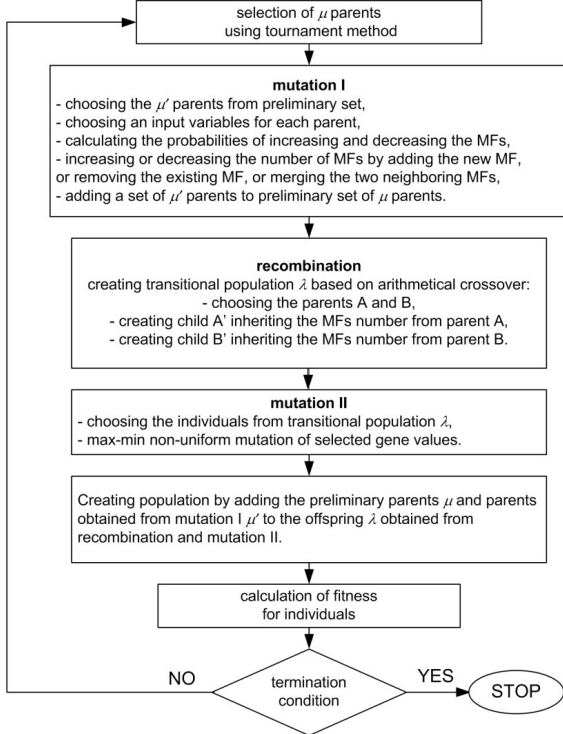


Figure 4. The general flowchart of GA strategy

The first genetic operation (Mutation I) consists in adding to a set of parents  $\mu$  a small group of individuals  $\mu'$  differing

from current population in the number of rules, that prevents of premature convergence (domination of population by the same or similar solutions). The operation is based on the mutation performed by adding or removing the selected MF of randomly chosen parent, and adding the new individual to the current group of parents. The number of MFs of considered individual is decreased or increased based on, respectively,  $p_D$  and  $p_I$  probabilities, which are calculated based on  $\bar{n}$  average,  $n_{\min}$  minimum, and  $n_{\max}$  maximum number of MFs for a given input variable:

$$p_D = \frac{\bar{n} - n_{\min}}{n_{\max} - n_{\min}}, \quad p_I = 1 - p_D. \quad (15)$$

This operation leads to obtain a new set of parents ( $\mu + \mu'$ ), which are next used to produce the offspring by recombination and mutation. During the recombination process the arithmetical crossover is conducted on the two randomly chosen individuals A and B to obtain the two child A' and B' using, respectively the formulas (16) and (17). The offspring A' and B' inherit the number of MFs and rules from, respectively, parent A and B.

$$A': g_j^{(A')} = N(0,1) \cdot g_j^{(A)} + (1 - N(0,1)) \cdot g_j^{(B)}. \quad (16)$$

$$B': g_j^{(B')} = N(0,1) \cdot g_j^{(B)} + (1 - N(0,1)) \cdot g_j^{(A)}. \quad (17)$$

where:

$g_j^{(A)}, g_j^{(B)}, g_j^{(A')}, g_j^{(B')}$  - the genes of chromosomes, and the  $j$  is a locus of a gene in the chromosome,  $N(0,1)$  - normally distributed random scalar.

The last operation (Mutation II), conducted on the transitional population, is the max-min non-uniform mutation used to change the selected gene of randomly chosen individual according to the one method selected from formulas (18) and (19).

$$g'_j = g_j + N(0,1) \cdot \sigma_2 \cdot (g_{j+1} - g_j). \quad (18)$$

$$g'_j = g_j - N(0,1) \cdot \sigma_2 \cdot (g_j - g_{j-1}). \quad (19)$$

where the factor  $\sigma_2$  determines the strength of the mutation, which is adjusted based on the GA performance expressed as a ratio between the best fitness of first and last generation.

The selection is performed using tournament method on the population composed of parents and reproduced offspring.

### IV. SIMULATION RESULTS

The proposed evolutionary strategy was applied to design the fuzzy gain scheduling scheme for anti-sway crane control system. The expected quality of control system was

formulated as the tolerance of maximum overshoot  $\chi \leq 0.02$  of crane position for unit-step input function. Hence, the set of fuzzy rules (linear controllers of scheduling scheme) were determined for interval vectors of desired characteristic equation coefficients (10) for dimensionless dumping coefficient  $\xi = 0.747$ . The population size was 48 individuals, and the crossover and mutation probability was  $p_c=0.875$  and  $p_m=0.09$ . The termination condition was the number of iteration set on 500. In the figures from 5 to 8 the results of two experiments are presented in the form of MFs tuned by GA. The objective of searching process was to design the fuzzy gain scheduling system for scheduling variables intervals:  $l=[1, 6]$  [m] and  $m=[10, 500]$  [kg] in experiment 1, and  $l=[1, 12]$  [m] and  $m=[10, 1000]$  [kg] in experiment 2. The first experiment resulted in FRBS with 12 rules (linear controllers). The rope length and mass of a payload were split into 4 and 3 fuzzy partitions respectively, corresponded to the triangular MFs presented in the figure 5. During the experiment 2, the input variables intervals  $l=[1, 12]$  [m] and  $m=[10, 1000]$  [kg] were, respectively, divided into 6 and 4 MFs (Fig. 6), that led to obtain 24 linear controllers specified in RB. The figure 7 depicts the fitness variation in both experiments, while the figure 8 illustrates the evolution of individuals with a given number of rules during the first 100 generations of experiment 2 through comparison of RB size of 12 individuals selected in each iteration. The best solution, fuzzy system with 24 linear controllers, is the winner of competition between the individuals which differ in RB size, even though during some generations the population is dominated by other solutions, and in some iterations the FRBS with 24 rules completely died out to be next restored.

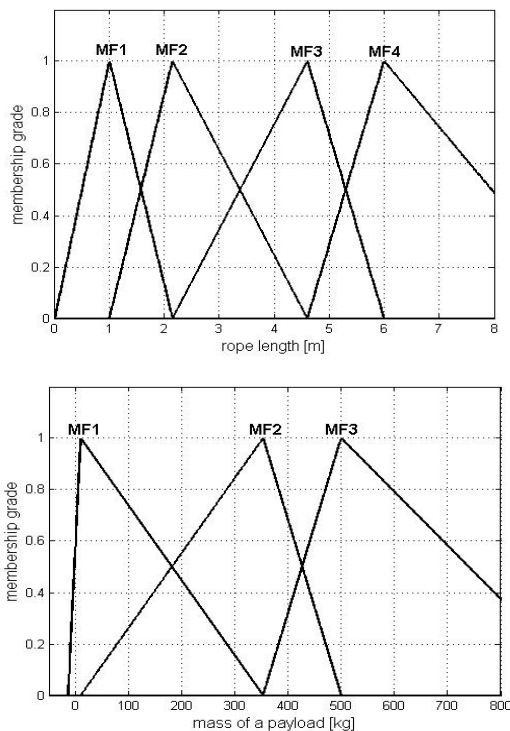


Figure 5. The MFs tuned by GA for rope length and mass of a payload intervals  $l=[1, 6]$  [m] and  $m=[10, 500]$  [kg]

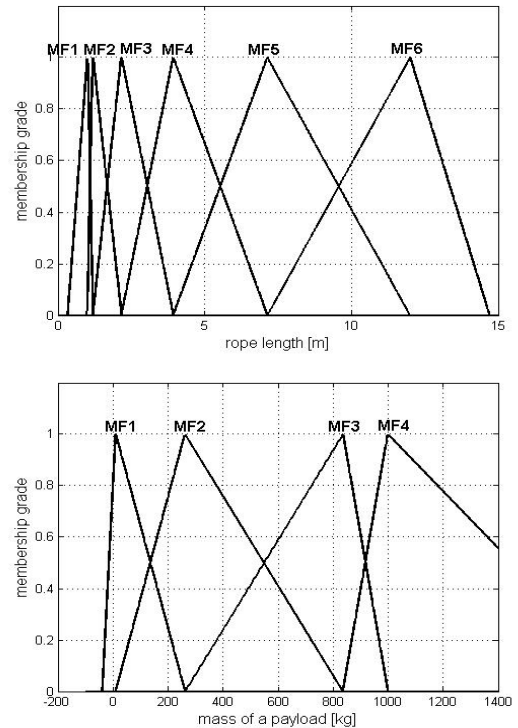


Figure 6. The MFs tuned by GA for rope length and mass of a payload intervals  $l=[1, 12]$  [m] and  $m=[10, 1000]$  [kg]

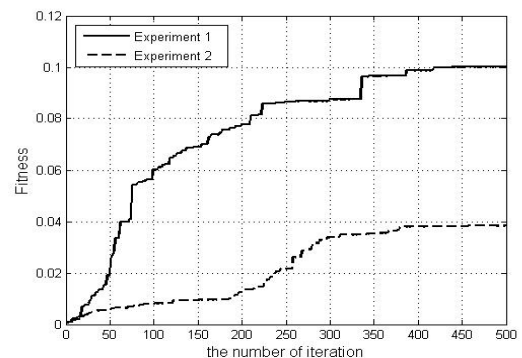


Figure 7. The evolution of fitness

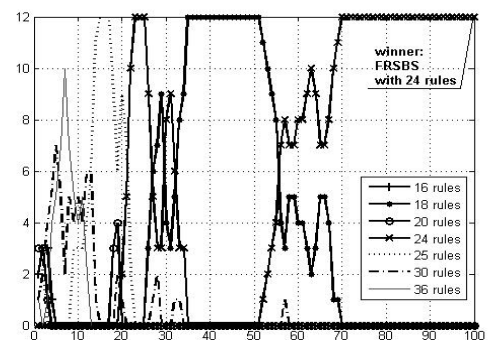


Figure 8. The number of individuals after selection with a given number of rules

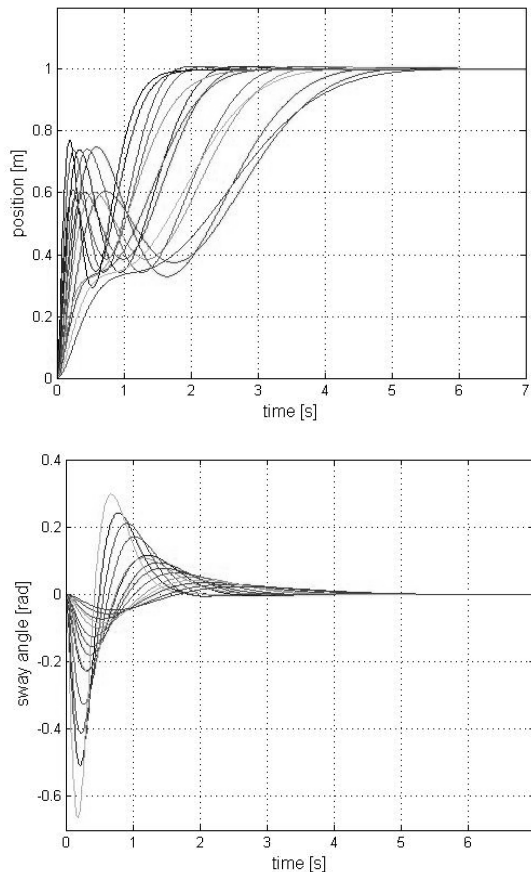


Figure 9. The performances of gain scheduling control system designed by GA for key system variables intervals  $l=[1, 12]$  [m] and  $m=[10, 1000]$  [kg], presented in form of crane position and sway angle of a payload

The performances of closed-loop control system with fuzzy interpolation mechanism based on 24 linear controllers is presented in the figure 9 in the form of crane position and sway angle of a payload curves, obtained at operating points associated with crossover points of overlapping MFs (Fig. 6). The performances prove that fuzzy gain scheduling system was successfully designed by GA leading to obtain the adaptive control system which ensures the expected quality of control in the specified interval of parameters variation  $l=[1, 12]$  [m] and  $m=[10, 1000]$  [kg].

## V. CONCLUSIONS

The paper describes the method of designing the fuzzy gain scheduling system using the GA, PPM and interval mathematics. The objective of elaborated method is to minimize the number of linear controllers and tune their gains to obtain the nonlinear interpolation control scheme ensuring the desired control quality within the all range of system parameters changes. A set of linear controllers are designed at operating points corresponded to the midpoints of fuzzy regions represented by fuzzy rules in RB. The proposed evolutionary methods was based on the Pittsburgh approach to explore the searching space consisting of solutions (fuzzy gain scheduling controllers) which differ in RB size. Hence, the

searching strategy was based on the genetic operators that ensure wide exploration of searching space, competition between solutions with different number of fuzzy partitions distributed within the scheduling variables intervals, and fine tuning the MFs.

The proposed method was addressed to the anti-sway crane control problem, and proved in experiments conducted for different intervals of scheduling variables.

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