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Anti-Sway Control of Container Cranes as a Flexible Cable System

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***Abstract*—In this paper, a new approach for the anti-sway control of container cranes using coupled partial and ordinary differential equations is investigated. The dynamics of the hoisting rope is derived in the form of an axially moving cable equation, while the dynamics of the actuator is given in the form of an ODE. The control objective is to suppress the transverse vibrations of the load via a domain control. A control law based upon the Lyapunov's second method is derived. It is revealed that a time-varying control force and a suitable passive damping can successfully suppress the transverse vibrations. The exponential stability of the closed loop system is proved.**

I. Introduction

The container crane that transports containers from a container ship to trucks, and vice versa, is a key equipment in automated container terminals. In this paper, the sway control problem of the suspended container in a crane from the aspect of suppressing the vibration of the load hanging at the end of a rope is investigated. Since fast loading and unloading of the containers to/from a container ship is most crucial, the time-optimal control has been widely pursued as a plausible solution. But, as the structure of the crane gets larger and the higher precision is required, the time optimal control based upon an ordinary differential model does not give the answer to the problem due to modeling uncertainty and disturbance.

Fig. 1 depicts the loading and unloading in the terminal. The ccontainers are first transferred from the ship to waiting trucks or to AGVs (Automated Guided Vehicles) by the crane. The truck then carries the container to an open storage area, where another crane stacks the container to a pre-assigned place. The bottleneck of this cycle lies in the transfer of the containers from the ship to the truck. Therefore, minimizing this transfer time will bring about a large cost saving. Since a large swing of the load during the transfer is also dangerous, the problem is to transfer containers to the desired place as quickly as possible while minimizing the swing of the container during transfer as well as at the end of a transfer.

The position control problem, for oscillatory systems, that pursues the minimal residual vibration while achieving the minimal control time has been investigated for over a decade. However, when the maneuvering time is minimized, large transient amplitudes and steady state oscillations may occur. A particular control objective for the systems with elastic modes often requires limited vibrations both during and after the maneuver. Residual vibrations should be minimized in order to achieve precise motions of a flexible mechanical system. In most cases, the residual vibration at the end of a movement is the most undesirable phenomenon that limits the overall performance of the system. The effective use of flexible systems can be achieved only when such vibration can be properly handled. As a result, there is an active research interest in finding methods that will eliminate vibration for a variety of mechanical and structural systems.

Briefly reviewing the results on crane controls, Sakawa and Shindo [14] derived an optimal motion law that minimizes the swing of the load. In Auernig and Troger [2] a time optimal control law was derived by using an extension of the Pontryagin maximum principle. Various acceleration strategies for achieving swing-free movements are found in Starr [17] and Strip [18]. The input shaping technique has been applied to reduce the residual vibrations in crane systems (Singhose and Towell, [15]; Singhose *et al.*, [16]). Alternatively, with regard to the closed-loop approaches, state-feedback strategies on linearized systems were implemented in (Mustafa and Ebeid, [10]; Moustafa, [11]).

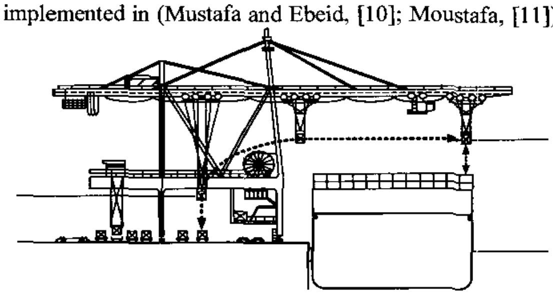

Fig. 1. Container Crane System.

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Recently, infinite dimensional approaches are actively investigated: Boustany and d'Andrea-Novel, [3]; d'Andrea-Novel *et al.*, [4]; d'Andrea-Novel and Coron, [5]; Rahn *et al.*, [13]).

To reduce vibrations of the considered system, we propose a different approach. The sway dynamics is modeled as a hybrid partial differential equation (PDE) and ordinary differential equation (ODE) system of a laterally moving cable. The flexibility of the cable, which moves to the horizontal direction, is given in a partial differential equation, while the dynamics of the actuator is given in an ODE.

Contributions of this paper are as follows. First, a new model of the container crane that includes the actuator dynamics is presented. In all other works in the literature, the acceleration of the trolley is the control input. But, in this paper, a separate actuator that can be provided an independent control input besides the acceleration of the trolley is considered. Second, the derived control law requires the velocity of actuating spot. Therefore, by measuring the velocity of the point of attachment of the cable to control device enables the implementation of the control law. Finally, the exponential stability of the closed loop system is established.

II. Equations of Motion

Fig. 2 shows a pattern of the traveling distance of the trolley as time passes. The pattern consists of three zones: The first zone is an accelerating zone, the second is a maximum speed traveling zone that will be fully discussed in this paper, and the last is a decelerating zone. In Fig. 2, an acceleration of 1 m/s^2, a constant speed of $3\,m/s$, and deceleration of -1 m/s^2 have been considered.

Fig. 3 shows a schematic diagram of the plant for analyzing the dynamics and deriving a pointwise control law. The actuator is located below the trolley rail, where a control force $F_c(t)$ is applied. Note that the lower boundary (the

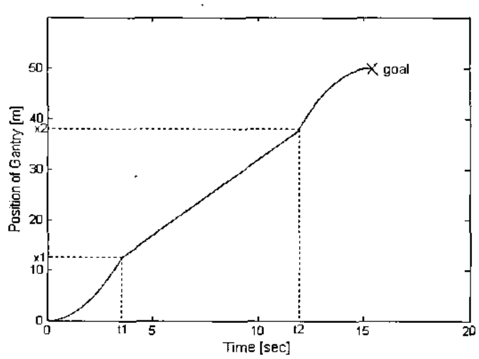

Fig. 2. Position diagram of the trolley with three zones: acceleration, maximum speed traveling, and deceleration.

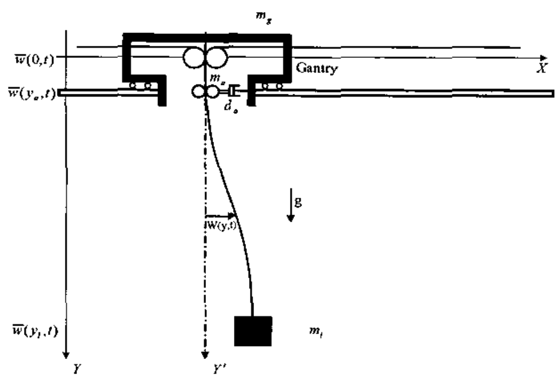

Fig. 3. A schematic diagram for the payload movement.

payload) is free to move.

Let t be the time, X be the displacement of the trolley in the horizontal direction, Y be the vertical direction, y be the spatial coordinate along the longitude of motion of the rope, t_1 be the time at which the maximum speed traveling starts, t_2 be the time at which the maximum speed traveling ends, v be the maximum speed of the trolley, $w(y,t)$ be the transversal displacement of the rope at spatial coordinate y and time t, and $P(y,t)$ be the tension caused by the payload and the weight of the rope. y_a is the position at which the actuator is located, and l is the length of the rope. The following variables are introduced:

$$\begin{cases} \overline{w}(y,t) = x_1 + v\cdot(t-t_1) + w(y,t), \\ \overline{w}_t(y,t) = v + w_t(y,t), \\ \overline{w}_{tt} = (x_1 + v\cdot(t-t_1) + w)_{tt} = w_{tt}, \end{cases} \tag{1}$$

$$\begin{cases} \overline{w}_y = w_y, \\ \overline{w}_{yy} = w_{yy}. \end{cases} \tag{2}$$

Now, to derive the equations of motion, the following Hamilton's principle (McIver, 1973) is utilized:

$$\int_{T_1}^{T_2} (\delta T - \delta V + \delta W)\,dt = 0\,, \tag{3}$$

where T is the kinetic energy, V is the strain energy, W is the work done by the control force. The tension $P(y,t)$ is given by

$$P(y,t) = \{m_l + \rho(l-y)\}g\,, \tag{4}$$

where m_l is the mass of the payload, ρ is the mass of the rope per unit length, and g is the gravitational acceleration.

The kinetic energy of the crane system including the trolley, actuator, and payload is given by

$$T = \frac{1}{2}\rho\int_0^{y_a^-} \overline{w}_t^2 dy + \frac{1}{2}\rho\int_{y_a^+}^{y_l} \overline{w}_t^2 dy + \frac{1}{2}m_g\left[\frac{D\overline{w}(0,t)}{Dt}\right]^2$$

$$+\frac{1}{2}m_a\left[\frac{D\overline{w}(y_a,t)}{Dt}\right]^2+\frac{1}{2}m_l\left[\frac{D\overline{w}(y_l,t)}{Dt}\right]^2, \tag{5}$$

where $(\cdot)_t=\partial(\cdot)/\partial t$ and $(\cdot)_x=\partial(\cdot)/\partial x$ denote the partial derivatives, and $\frac{Dw(y,t)}{Dt}=\frac{\partial w(y,t)}{\partial t}+\frac{\partial w(y,t)}{\partial y}\frac{dy}{dt}$ is the material derivative. In (5) m_g is the mass of the trolley and m_a is the mass of the actuator. The potential energy is

$$V=\int_0^{y_a^-}P(y,t)\varepsilon_y dy+\int_{y_a^+}^{l}P(y,t)\varepsilon_y dy$$
$$=\frac{1}{2}\int_0^{y_a^-}P(y,t)\overline{w}_y^2dy+\frac{1}{2}\int_{y_a^+}^{y_l}P(y,t)\overline{w}_y^2dy. \tag{6}$$

The first term in (6) is due to the cable tension and ε_y is the strain. If the infinitesimal distance dy is replaced by the infinitesimal length ds, the strain ε_y can be approximated as (Wickert, 1992):

$$\varepsilon_y\cong\frac{1}{2}\overline{w}_y^2. \tag{7}$$

The virtual work done by the control force and damping force is

$$W=F_c(t)\overline{w}(y_a,t)-d_a\overline{w}_t(y_a,t)\overline{w}(y_a,t). \tag{8}$$

The variations of (5) and (6) are

$$\delta T=\rho\int_0^{y_a^-}\overline{w}_t\delta\overline{w}_t dy+\rho\int_{y_a^+}^{y_l}\overline{w}_t\delta\overline{w}_t dy$$
$$+m_a\left[\frac{D\overline{w}(y_a,t)}{Dt}\right]\delta\left[\frac{D\overline{w}(y_a,t)}{Dt}\right]$$
$$+m_l\left[\frac{D\overline{w}(y_l,t)}{Dt}\right]\delta\left[\frac{D\overline{w}(y_l,t)}{Dt}\right], \tag{9}$$

$$\delta V=\frac{1}{2}\int_0^{y_a^-}\overline{w}_y^2\delta P(y,t)dy+\int_0^{y_a^-}P(y,t)\overline{w}_y\delta\overline{w}_y dy$$
$$+\frac{1}{2}\int_{y_a^+}^{y_l}\overline{w}_y^2\delta P(y,t)dy+\int_{y_a^+}^{y_l}P(y,t)\overline{w}_y\delta\overline{w}_y dy. \tag{10}$$

Also, the variations of the work is

$$\delta W=F_c(t)\delta\overline{w}(y_a,t)-d_a\overline{w}_t(y_a,t)\delta\overline{w}(y_a,t), \tag{11}$$

where $F_c(t)$ is the control force.

The substitution of (9)-(11) into (3) yields the governing equation as follows:

$$\rho\overline{w}_{tt}-\{P(y,t)\overline{w}_y\}_y=0,\ y\neq y_a. \tag{12}$$

The boundary conditions are

$$\overline{w}(0,t)=X+v\cdot(t-t_1),\ \overline{w}_t(0,t)=v,$$
$$m_l\overline{w}_{tt}(y_l,t)+P(y_l,t)\overline{w}_y(y_l,t)=0, \tag{13}$$

and the internal condition is

$$F_c(t)=m_a\overline{w}_{tt}(y_a,t)+d_a\overline{w}_t(y_a,t)$$
$$+P(y_a^-,t)\overline{w}_y(y_a^-,t)-P(y_a^+,t)\overline{w}_y(y_a^+,t). \tag{14}$$

Note that (12) is a partial differential equation representing the transverse motion, while (13)-(14) are ordinary differential equations relating the motion at the lower boundary (the lower end of the rope) and the control force.

III. Feedback Control Law

The purpose of control is to dissipate the vibration of the container. A Lyapunov function candidate $V(t)$ for the considered system is introduced as follows:

$$V(t)=\frac{1}{2}\rho\int_0^{y_a^-}\overline{w}_t^2dy+\frac{1}{2}\rho\int_{y_a^+}^{y_l}\overline{w}_t^2dy+\frac{1}{2}\int_0^{y_a^-}P(y,t)\overline{w}_y^2dy$$
$$+\frac{1}{2}\int_{y_a^+}^{y_l}P(y,t)\overline{w}_y^2dy+\frac{1}{2}m_g\left[\frac{D\overline{w}(0,t)}{Dt}\right]^2$$
$$+\frac{1}{2}m_a\left[\frac{D\overline{w}(y_a,t)}{Dt}\right]^2+\frac{1}{2}m_l\left[\frac{D\overline{w}(y_l,t)}{Dt}\right]^2. \tag{15}$$

Using (15), $\overline{V}(t)$ is introduced as follows:

$$V(t)\le\overline{V}(t)=\frac{1}{2}\rho\int_0^{y_a^-}\overline{w}_t^2dy+\frac{1}{2}\rho\int_{y_a^+}^{y_l}\overline{w}_t^2dy$$
$$+\frac{1}{2}\int_0^{y_a^-}P_{\max}\overline{w}_y^2dy+\frac{1}{2}\int_{y_a^+}^{y_l}P_{\max}\overline{w}_y^2dy+\frac{1}{2}m_g\left[\frac{D\overline{w}(0,t)}{Dt}\right]^2$$
$$+\frac{1}{2}m_a\left[\frac{D\overline{w}(y_a,t)}{Dt}\right]^2+\frac{1}{2}m_l\left[\frac{D\overline{w}(y_l,t)}{Dt}\right]^2, \tag{16}$$

where the tension $P(y,t)$ is replaced by $P_{\max}=(m_l+\rho l)g$. This is because the mass of the load is sufficiently bigger than the mass of the cable, for instance, $P_{\min}=(43500)\times g\le[m_l+\rho(y-l)]g\le P_{\max}=(43500+5\times10)\times g$).

Now, the total derivative (or the material derivative) of (16) is evaluated. The time derivative of $\overline{V}(y,t)$ becomes

$$\frac{d}{dt}\overline{V}(y,t)=\frac{\partial}{\partial t}\overline{V}(y,t)+\frac{\partial}{\partial t}\overline{V}(y,t)\frac{\partial y}{\partial t}, \tag{17}$$

where $\frac{\partial y}{\partial t}=\tilde{v}$, $\tilde{v}$ is the speed of hoisting. But, in this paper only the part $\tilde{v}=0$ is considered. So (17) is

$$\frac{d}{dt}\overline{V}(y,t)=\frac{\partial}{\partial t}\overline{V}(y,t). \tag{18}$$

From (12)-(14), the total derivative of (16) becomes

$$\frac{d}{dt}\overline{V}(y,t)=\rho\int_0^{y_a^-}\overline{w}_t\overline{w}_{tt}dy+\rho\int_{y_a^+}^{y_l}\overline{w}_t\overline{w}_{tt}dy$$
$$+m_a\left[\frac{D\overline{w}(y_a,t)}{Dt}\right]\left[\frac{D\overline{w}^2(y_a,t)}{Dt^2}\right]+P_{\max}\int_0^{y_a^-}\overline{w}_y\overline{w}_{yt}dy$$
$$+m_l\left[\frac{D\overline{w}(y_l,t)}{Dt}\right]\left[\frac{D^2\overline{w}(y_l,t)}{Dt^2}\right]+P_{\max}\int_{y_a^+}^{y_l}\overline{w}_y\overline{w}_{yt}dy$$
$$=P_{\max}\overline{w}_y(y_a^-,t)\overline{w}_t(y_a^-,t)-P_{\max}\overline{w}_y(y_a^+,t)\overline{w}_t(y_a^+,t)$$
$$+\overline{w}_t(y_a,t)\{F_c(t)-d_a\overline{w}_t(y_a,t)\}$$

$$+\overline{w}_t(y_a,t)\left\{-P_{\max}\overline{w}_y(y_a^-,t)+P_{\max}\overline{w}_y(y_a^+,t)\right\}. \quad (19)$$

The displacement of the rope is continuous at $y = y_a$:

$$\overline{w}(y_a^-,t)=\overline{w}(y_a,t)=\overline{w}(y_a^+,t). \quad (20)$$

The time derivative of (20) yields

$$\overline{w}_t(y_a^-,t)=\frac{D\overline{w}(y_a,t)}{Dt}=\overline{w}_t(y_a^+,t). \quad (21)$$

By using (21), the following feedback control law will make (19) negative semi-definite.

$$F_c(t)=-K\overline{w}_t(y_a,t), \quad (22)$$

where $0<K<y_l$ is a constant real number. All above developments are summarized in the following theorem.

Theorem 1: Consider the container crane system (12)-(14). Then, the closed-loop system with the control law (22) is uniformly asymptotically stable.

IV. Exponential Stability

In this section, the exponential stability of the transversal motion of the container crane with feedback control law (22) is further investigated. In order to analyze this, the state space Λ is introduced as follows:

$$\Lambda=\left\{\left(\overline{w},\dot{\overline{w}},\overline{w}_t(y_a),\overline{w}_t(y_l)\right)^T\middle|\,\overline{w}\in H_{0,l}^1,\ \dot{\overline{w}}\in \mathrm{L}^2,\overline{w}_t(y_a),\overline{w}_t(y_l)\in R\right\}, \quad (23)$$

where L^2 and $H_{0,l}^k$ are defined as

$$\mathrm{L}^2:=\left\{f:[0,L]\to R\,\middle|\,\int_0^L f^2dy<\infty\right\}, \quad (24)$$

$$H_{o,l}^k:=\left\{f\in\mathrm{L}^2\,\middle|\,f',f'',\cdots,f^{(k)}\in\mathrm{L}^2, and\ f(0)=0\right\}. \quad (25)$$

Equations (12)-(14) can be written in the state space form as follows:

$$\dot{z}=Az,\ z(0)\in\Lambda, \quad (26)$$

where $z=\left(\overline{w},\dot{\overline{w}},\overline{w}_t(y_a),\overline{w}_t(y_l)\right)^T\in\Lambda$, the operator $A:\Lambda\to\Lambda$ is a linear operator defined as

$$Az=\begin{bmatrix}\overline{w}_t\\ \frac{1}{\rho}\left\{P(y,t)\overline{w}_y\right\}_y\\ \frac{F_c(t)-d_a\overline{w}_t(y_a)-P(y_a^-)\overline{w}_y(y_a^-)+P(y_a^+)\overline{w}_y(y_a^+)}{m_a}\\ -\frac{P(y_l,t)\overline{w}_y(y_l,t)}{m_l}\end{bmatrix}, \quad (27)$$

where $\dot{\overline{w}}=\overline{w}_t$ and $\ddot{\overline{w}}=\overline{w}_{tt}$ have been utilized. The domain $D(A)$ of the linear operator A for the system with the feedback control law is

$$D(A):=\left\{\left(\overline{w},\dot{\overline{w}},\overline{w}_t(y_a),\overline{w}_t(y_l)\right)^T\middle|\,\overline{w}\in H_{0,l}^2,\dot{w}\in H_{0,l}^1,\ \overline{w}_t(y_a),\overline{w}_t(y_l)\in R,F_c(t)=-K\overline{w}_t(y_a,t)\right\}. \quad (28)$$

Lemma 1: The operator defined in (27) generates a C_0-semigroup of contraction. That is, the transverse dynamics of the system (12) with piecewise control law (22) is dissipative.

Proof: The transversal energy of the container crane is introduced as follows:

$$E(t)=\langle z,z\rangle_\Lambda=\|z(t)\|_\Lambda^2=\frac{1}{2}\int_0^{y_l}\left(\rho\overline{w}_t^2+P(y,t)\overline{w}_y^2\right)dy+\frac{1}{2}\left\{m_a\overline{w}_t^2(y_a,t)+m_l\overline{w}_t^2(y_l,t)\right\}, \quad (29)$$

$$E(t)\le\widetilde{E}(t)=\frac{1}{2}\int_0^{y_l}\left(\rho\overline{w}_t^2+P(y,t)\overline{w}_y^2\right)dy+\frac{1}{2}\left\{m_a\overline{w}_t^2(y_a,t)+m_l\overline{w}_t^2(y_l,t)\right\}+\frac{1}{2}\int_{y_a^-}^{y_a^+}\left(\rho\overline{w}_t^2+P(y,t)\overline{w}_y^2\right)dy. \quad (30)$$

The time derivative of (30) becomes

$$\dot{\widetilde{E}}=\widetilde{E}_t=\int_0^{y_l}\left\{\rho\overline{w}_t\overline{w}_{tt}+P(y,t)\overline{w}_y\overline{w}_{yt}\right\}dy+\int_{y_a^-}^{y_a^+}\left\{\rho\overline{w}_t\overline{w}_{tt}+P(y,t)\overline{w}_y\overline{w}_{yt}\right\}dy+m_a\overline{w}_t(y_a,t)\overline{w}_{tt}(y_a,t)+m_l\overline{w}_t(y_l,t)\overline{w}_{tt}(y_l,t). \quad (31)$$

The substitution of (12)-(14) into (31) yields

$$\begin{aligned}\dot{\widetilde{E}}\Big|_{(12)-(14)}&=\int_0^{y_l}\left\{P(y,t)\overline{w}_t\overline{w}_{yy}+P(y,t)\overline{w}_y\overline{w}_{yt}\right\}dy\\&+\int_{y_a^-}^{y_a^+}\left\{P(y,t)\overline{w}_t\overline{w}_{yy}+P(y,t)\overline{w}_y\overline{w}_{yt}\right\}dy\\&+\overline{w}_t(y_a,t)\left\{F_c(t)-d_a\overline{w}_t(y_a,t)\right\}\\&+\overline{w}_t(y_a,t)\left\{-P_{\max}\overline{w}_y(y_a^-,t)+P_{\max}\overline{w}_y(y_a^+,t)\right\}\\&-P(y_l,t)\overline{w}_y(y_l,t)\overline{w}_t(y_l,t).\end{aligned} \quad (32)$$

Again, the substitution of (22) into (32) yields

$$\dot{\widetilde{E}}\Big|_{(12)-(14),\ (22)}=-(K+d_a)\overline{w}_t^2(y_a,t)\le 0. \quad (33)$$

Hence, the closed-loop operator with (22) is dissipative on Λ. Therefore, a C_0 semigroup $S(t)$ of contraction on Λ is generated, where $S(t)$ is a bounded operator on Λ for $t\ge 0$.

Theorem 2: The crane system (12) under control law (22) is exponentially stable. That is, there exist constants $\mu>0$ and $M>0$ such that

$$\widetilde{E}(t)\le Me^{-\mu t},t\ge 0. \quad (34)$$

Proof: To prove that the system decays exponentially to zero, the following positive definite function, by following the approach in [8], is introduced.

$$\eta(t)=t\widetilde{E}(t)+\int_0^{y_l} y\sqrt{\rho P_{\max}}\left(2\overline{w}_y\overline{w}_t\right)dy + m_a\overline{w}_t^2(y_a,t)+m_l\overline{w}_t^2(y_l,t)\,,t\geq 0\,, \tag{35}$$

where $E(t)$ is defined in (29). The following terms of (35) satisfies the following inequalities:

$$2\sqrt{\rho P_{\max}}\int_0^{y_l} y\overline{w}_y\overline{w}_t dy + m_a\overline{w}_t^2(y_a,t)+m_l\overline{w}_t^2(y_l,t) \leq \int_0^{y_l} P_{\max}\overline{w}_y^2 dy+\int_0^{y_l}\rho\overline{w}_t^2 dy + m_a\overline{w}_t^2(y_a,t)+m_l\overline{w}_t^2(y_l,t)\leq 2\widetilde{E}(t)\leq C\widetilde{E}(t)\,, \tag{36}$$

where $P_{\max}=[m_l+\rho l]g$, $C>2$ is a constant. Hence, the following holds:

$$0\leq (t-C)\widetilde{E}(t)\leq \eta(t)\leq (t+C)\widetilde{E}(t)\,, \tag{37}$$

for $t>C$ sufficiently large. With the use of (12), the differentiation of (35) with respect to time yields:

$$\begin{aligned}
\dot{\eta}(t)\big|_{(12)} &= \widetilde{E}(t)+t\dot{\widetilde{E}}(t) + 2\sqrt{\rho P_{\max}}\int_0^{y_l}\left(y\overline{w}_t\overline{w}_{yt}+y\overline{w}_y\overline{w}_{tt}\right)dy \\
&\quad +2m_a\overline{w}_t(y_a,t)\overline{w}_{tt}(y_a,t)+2m_l\overline{w}_t(y_l,t)\overline{w}_{tt}(y_l,t) \\
&= \widetilde{E}(t)+t\dot{\widetilde{E}}(t) + 2\sqrt{\rho P_{\max}}\int_0^{y_l}\left(y\overline{w}_t\overline{w}_{yt}+yP_y(y,t)\overline{w}_y^2+yP(y,t)\overline{w}_y\overline{w}_{yy}\right)dy \\
&\quad +2m_a\overline{w}_t(y_a,t)\overline{w}_{tt}(y_a,t)+2m_l\overline{w}_t(y_l,t)\overline{w}_{tt}(y_l,t) \\
&= \widetilde{E}(t)+t\dot{\widetilde{E}}(t)+2\sqrt{\rho P_{\max}}\int_0^{y_l} y\overline{w}_t\overline{w}_{yt}dy + 2\sqrt{\rho P_{\max}}\int_0^{y_l} yP_y(y,t)\overline{w}_y^2 dy \\
&\quad + 2\sqrt{\rho P_{\max}}\int_0^{y_l} yP(y,t)\overline{w}_y\overline{w}_{yy}dy +2m_a\overline{w}_t(y_a,t)\overline{w}_{tt}(y_a,t)+2m_l\overline{w}_t(y_l,t)\overline{w}_{tt}(y_l,t)\,.
\end{aligned} \tag{38}$$

Also, the following terms of (38) satisfy the following equalities.

$$2\sqrt{\rho P_{\max}}\int_0^{y_l} y\overline{w}_t\overline{w}_{yt}dy = \sqrt{\rho P_{\max}}\left\{y_l\overline{w}_t^2(y_l,t)-\int_0^{y_l}\overline{w}_t^2\,dy\right\}, \tag{39}$$

$$2\sqrt{\rho P_{\max}}\int_0^{y_l}\left\{yP_y(y,t)\overline{w}_y^2+yP(y,t)\overline{w}_y\overline{w}_{yy}\right\}dy = \sqrt{\rho P_{\max}}\left\{y_lP(y_l,t)\overline{w}_y^2(y_l,t)-\int_0^{y_l}P(y,t)\overline{w}_y^2 dy\right\} + \sqrt{\rho P_{\max}}\int_0^{y_l} yP_y(y,t)\overline{w}_y^2 dy\,. \tag{40}$$

Finally, (38) is expressed as follows:

$$\begin{aligned}
\dot{\eta}(t) &= \widetilde{E}(t)+t\dot{\widetilde{E}}(t)+\sqrt{\rho P_{\max}}\left\{y_l\overline{w}_t^2(y_l,t)-\int_0^{y_l}\overline{w}_t^2 dy\right\} \\
&\quad +\sqrt{\rho P_{\max}}\left\{y_lP(y_l,t)\overline{w}_y^2(y_l,t)-\int_0^{y_l}P(y,t)\overline{w}_y^2 dy\right\} \\
&\quad +\sqrt{\rho P_{\max}}\int_0^{y_l} yP_y(y,t)\overline{w}_y^2 dy \\
&\quad +2m_a\overline{w}_t(y_a,t)\overline{w}_{tt}(y_a,t)+2m_l\overline{w}_t(y_l,t)\overline{w}_{tt}(y_l,t)\,.
\end{aligned} \tag{41}$$

The substitution of (22) into (41) yields:

$$\begin{aligned}
\dot{\eta}(t)\big|_{(12)-(14),\,(22)} &= \widetilde{E}(t)+t\dot{\widetilde{E}}(t) +\sqrt{\rho P_{\max}}\left\{y_l\overline{w}_t^2(y_l,t)-\int_0^{y_l}\overline{w}_t^2 dy\right\} \\
&\quad +\sqrt{\rho P_{\max}}\left\{y_lP(y_l,t)\overline{w}_y^2(y_l,t)-\int_0^{y_l}P(y,t)\overline{w}_y^2 dy\right\} \\
&\quad +\sqrt{\rho P_{\max}}\int_0^{y_l} yP_y(y,t)\overline{w}_y^2 dy \\
&\quad +2\overline{w}_t(y_a,t)\{F_c(t)-d_a\overline{w}_t(y_a,t)\} \\
&\quad +2\overline{w}_t(y_a,t)\{-P(y_a^-,t)\overline{w}_y(y_a^-,t)+P(y_a^+,t)\overline{w}_y(y_a^+,t)\} \\
&\quad -2\overline{w}_t(y_l,t)\{P(y_l,t)\overline{w}_y(y_l,t)\},
\end{aligned} \tag{42}$$

where $\dot{\widetilde{E}}(t)\leq 0$. Thus, noting that $E(t)$, $\overline{w}_t(y_a,t)$, $\overline{w}_y(y_a^-,t)$, $\overline{w}_y(y_a^+,t)$, $\overline{w}_t(y_l,t)$, $\overline{w}_y(y_l,t)$ are bounded, (42) is negative for a sufficiently large time Ω. That is, when $t>\Omega$, (42) satisfies the following inequality.

$$\dot{\eta}(t)\leq 0\,. \tag{43}$$

From (37) and (43), the following holds:

$$E(t)\leq\frac{\eta(t)}{t-C}\,,t>\Omega\,. \tag{44}$$

Thus, from (29), (44), and the semi-group property of the solution, the following inequality is obtained.

$$\int_0^{\infty}E^2(t)dt=\int_0^{\Omega}E^2(t)dt+\int_{\Omega}^{\infty}E^2(t)dt \leq \int_0^{\Omega}\|S(t)z(0)\|_{\Lambda}^4 dt+\int_{\Omega}^{\infty}\frac{\eta(t)^2}{(t-C)^2}dt<\infty\,, \tag{45}$$

where $z(0)\in D(A)$. (45) implies that

$$\int_0^{\infty}\|S(t)z(0)\|_{\Lambda}^4 dt<\infty\,. \tag{46}$$

Then, by the semigroup theorem [12], there exist constants $\mu'>0$ and $M'>0$ such that $\|S(t)\|_{\Lambda}\leq M'e^{-\mu' t}$. That is,

$$\|z(t)\|_{\Lambda}\leq M'\|z(0)\|_{\Lambda}e^{-\mu' t}\,. \tag{47}$$

From (29),

$$E(t)=\|z(t)\|_{\Lambda}^2\leq (M')^2e^{-2\mu' t}\|z(0)\|_{\Lambda}^2\leq Me^{-\mu t}\,, \tag{48}$$

where $M=\|z(0)\|_{\Lambda}^2(M')^2$ and $\mu=2\mu'$. Therefore, the theorem is proved.

V. CONCLUSION

In this paper, we considered the container crane system as a flexible cable. The derived control law requires the velocity of actuating spot. Therefore, by measuring the velocity of the point of attachment of the cable to control device enables the implementation of the control law. The boundary control law was derived in such a way that total energy of the container crane dissipates exponentially.

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