Dipshitmaths

O-LEVEL
MATHEMATICS
PROBLEM SET-WEEK 0

1. Simplify $\frac{4p^2r}{3} \div \frac{2r^3}{p}$

$$\frac{4p^2r}{3} \div \frac{2r^3}{p}$$
$$= \frac{4p^2r}{3} \times \frac{p}{2r^3}$$
$$= \frac{\cancel{4}2p^2\cancel{r}}{3} \times \frac{p}{\cancel{2}\cancel{r^3}r^2}$$
$$= \frac{2p^3}{3r^2}$$

2. $a = \frac{3b+4c}{5-b}$

   (a) Evaluate a when $b = 6$ and $c = -2$

$$a = \frac{3b+4c}{5-b}$$
$$= \frac{3(6)+4(-2)}{5-(6)}$$
$$= \frac{18-8}{-1}$$
$$= -10$$

   (b) Express b in terms of a and c

$$a = \frac{3b+4c}{5-b}$$
$$a(5-b) = 3b+4c$$
$$5a - ab = 3b + 4c$$
$$5a - 4c = 3b + ab$$
$$5a - 4c = b(3+a)$$
$$b = \frac{a+3}{5a-4c}$$

3. (a) Express $9 - 7x + x^2$ in the form $p + (q + x)^2$

$$p + (q + x)^2 = 9 - 7x + x^2$$

$$p + (q + x)(q + x) = p + q^2 + 2qx + x^2 = 9 - 7x + x^2$$

Solving for each terms,

$$2qx = -7x$$

$$q = -7/2 = -3.5$$

$$p + q^2 = 9$$

$$p = 9 - q^2 = 9 - (3.5)^2 = 9 - 12.25 = -3.25$$

(b) Write down the coordinates of the minimum point of the graph of $9 - 7x + x^2$

Minimum point occurs when first derivative/tangent of the graph equals to zero

$$y = 9 - 7x + x^2$$

$$\frac{dy}{dx} = -7 + 2x = 0$$

$$x = 3.5$$

$$y = 9 - 7(3.5) + (3.5)^2 = 9 - 24.5 + 12.25 = -3.25$$

4. Solve $\frac{1}{x-3} + \frac{6}{x-1} = 2$

$$\frac{1}{x-3} + \frac{6}{x-1}$$

$$= \frac{x-1}{(x-3)(x-1)} + \frac{6(x-3)}{(x-3)(x-1)}$$

$$= \frac{(x-1) + 6(x-3)}{(x-3)(x-1)}$$

$$= \frac{x - 1 + 6x - 18}{x^2 - 4x + 3}$$

$$= \frac{7x - 19}{x^2 - 4x + 3}$$

We know that

$$\frac{7x - 19}{x^2 - 4x + 3} = 2$$

$$7x - 19 = 2(x^2 - 4x + 3) = 2x^2 - 8x + 6$$

$$0 = 2x^2 - 8x - 7x + 6 + 19 = 2x^2 - 15x + 25$$

Factorizing,

$$2x^2 - 15x + 21 = (2x - 5)(x - 5) = 0$$

Thus we have two solutions $2x - 5 = 0$ and $x - 5 = 0$

$$2x_1 - 5 = 0$$

$$x_1 = 5/2 = 2.5$$

$$x_2 - 5 = 0$$

$$x_2 = 5$$