1. During the dry season, a forest is characterized as having a high risk of fire (h), medium risk of fire (m), or low risk of fire (l). If there is a high risk of fire, then the forest will be in state h, m, or l tomorrow with respective probabilities 0.8, 0.15, and 0.05. If there is a medium risk of fire, then the forest will be in state h, m, or l tomorrow with respective probabilities 0.4, 0.5, and 0.1. If there is a low risk of fire, then the forest will be in states h, m, or l tomorrow with respective probabilities 0.2, 0.4, and 0.4.
   a. What proportion of days during the dry season is the forest at a high risk for fire?
   b. On high risk days, the local fire department spends \$1000 per day monitoring the forest and putting out small fires as they occur. They spend \$500 for similar efforts during medium risk days, and spend nothing on low risk days. What is the average cost per day of the dry season for the fire department?

The process is in:
- state 0 if there is a high risk of fire
- state 1 if there is a medium risk of fire
- state 2 if there is a low risk of fire

The transition probabilities P_{ij} $(i, j = 0,1,2)$:

P_{00} = probability that there is high risk of fire, the forest is at a high risk of fire tomorrow = 0.8

P_{01} = probability that there is high risk of fire, the forest is at a medium risk of fire tomorrow = 0.15

P_{02} = probability that there is high risk of fire, the forest is at a low risk of fire tomorrow = 0.05

P_{10} = probability that there is medium risk of fire, the forest is at a high risk of fire tomorrow = 0.4

P_{11} = probability that there is medium risk of fire, the forest is at a medium risk of fire tomorrow = 0.5

P_{10} = probability that there is medium risk of fire, the forest is at a low risk of fire tomorrow = 0.1

P_{20} = probability that there is low risk of fire, the forest is at a high risk of fire tomorrow = 0.2

P_{21} = probability that there is low risk of fire, the forest is at a medium risk of fire tomorrow = 0.4

P_{22} = probability that there is low risk of fire, the forest is at a low risk of fire tomorrow = 0.4

This is a Markov chain with three states 0, 1, 2 with transition probability matrix **P**:

$$\mathbf{P} = \begin{Vmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{Vmatrix} = \begin{Vmatrix} 0.8 & 0.15 & 0.05 \\ 0.4 & 0.5 & 0.1 \\ 0.2 & 0.4 & 0.4 \end{Vmatrix}$$

The long-run proportions π_i:

$$\pi_0 = 0.8\pi_0 + 0.4\pi_1 + 0.2\pi_2$$

$$\pi_1 = 0.15\pi_0 + 0.5\pi_1 + 0.4\pi_2$$

$$\pi_2 = 0.05\pi_0 + 0.1\pi_1 + 0.4\pi_2$$

$$\pi_0 + \pi_1 + \pi_2 = 1$$

π_0 = long-run proportion of having high risk of fire

$$\pi_0 = \frac{26}{41}$$

$\pi_1 =$ long-run proportion of having medium risk of fire

$$\pi_1 = \frac{11}{41}$$

$\pi_2 =$ long-run proportion of having low risk of fire

$$\pi_2 = \frac{4}{41}$$

A: The proportion of days during the dry season that the forest is at high risk for fire is $\frac{26}{41}$

$$\begin{aligned} average\ cost &= \pi_0 * 1000 + \pi_1 * 500 + \pi_2 * 0 \\ &= \left(\frac{26}{41}\right) * 1000 + \left(\frac{11}{41}\right) * 500 + \left(\frac{4}{41}\right) * 0 \\ &= \frac{26 * 1000 + 11 * 500}{41} \\ &= \frac{26000 + 5500}{41} \\ &= \frac{31500}{41} \\ &\approx \$768.29 \text{ per day} \end{aligned}$$

A: The average cost per day of the dry season for the fire department is \$768.29

2. Suppose that, during a heat wave, whether or not it will be extremely hot outside tomorrow (above 100F) depends on past weather conditions only through the last two days. Specifically, suppose that if it was extremely hot yesterday and today, then it will be extremely hot tomorrow with probability 0.6; if it was extremely hot yesterday but not today, then it will be extremely hot tomorrow with probability 0.1; if it was extremely hot today but not yesterday, then it will be extremely hot tomorrow with probability 0.2; and if it was not extremely hot either yesterday or today, then it will be extremely hot tomorrow with probability 0.05. During the heat wave, for what proportion of days is it extremely hot outside?

The process is in:

state 0 if it was extremely hot today and extremely hot yesterday
state 1 if it was extremely hot today and but not yesterday
state 2 if it was extremely hot yesterday but not today
state 3 if it was neither extremely hot today nor extremely hot yesterday

This is a Markov chain with four states 0, 1, 2, 3 with transition probability matrix **P**:

$$\mathbf{P} = \begin{Vmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{Vmatrix} = \begin{Vmatrix} 0.6 & 0 & 0.4 & 0 \\ 0.2 & 0 & 0.8 & 0 \\ 0 & 0.1 & 0 & 0.9 \\ 0 & 0.05 & 0 & 0.95 \end{Vmatrix}$$

The long-run proportions π_i:

$$\pi_0 = 0.6\pi_0 + 0.2\pi_1$$

$$\pi_1 = 0.1\pi_2 + 0.05\pi_3$$

$$\pi_2 = 0.4\pi_0 + 0.8\pi_1$$

$$\pi_3 = 0.9\pi_2 + 0.95\pi_3$$

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

$$\pi_0 = \frac{1}{41}$$

$$\pi_1 = \frac{2}{41}$$

$$\pi_2 = \frac{2}{41}$$

$$\pi_3 = \frac{36}{41}$$

$\pi_0 + \pi_1 =$ long-run proportion of it being extremely hot outside

$$\pi_0 + \pi_1 = \frac{1}{41} + \frac{2}{41} = \frac{3}{41}$$

<u>A:</u> The proportion of days during the heat wave that it is extremely hot outside is $\frac{3}{41}$

3. A gram of radioactive material emits α-particles according to a Poisson process with rate λ per second. Suppose that two particles arrive during the first second. Find the probability that
   a. Both arrived in the first 0.15 seconds,
   b. At least one arrived in the first 0.15 seconds.

Let N(t) be the number of particles that arrive in the interval [0,t] seconds

$$P\{N(t)=n\}=e^{-\lambda t}\frac{(\lambda t)^n}{n!}, for\ all\ n=0,1,2,\ldots$$

$$P\{Both\ particles\ arrived\ in\ the\ first\ 0.15\ seconds\}=$$

$$=P\{N(0.15\ seconds)=2 \mid N(1\ second)=2\}$$

$$=\frac{P\{N(0.15)=2,\ N(1)=2\}}{P\{N(1)=2\}}$$

$$=\frac{P\{N(0.15)=2,\ N(1)-N(0.15)=0\}}{P\{N(1)=2\}}$$

$$=\frac{P\{N(0.15)=2\}*P\{N(0.85)=0\}}{P\{N(1)=2\}}$$

$$P\{N(0.15)=2\}=e^{-\lambda(0.15)}\frac{(0.15\lambda)^2}{2!}$$

$$P\{N(0.85)=0\}=e^{-\lambda(0.85)}\frac{(0.85\lambda)^0}{0!}$$

$$P\{N(1)=2\}=e^{-\lambda}\frac{\lambda^2}{2!}$$

$$=\frac{\left[e^{-\lambda(0.15)}\frac{(0.15\lambda)^2}{2!}\right]*\left[e^{-\lambda(0.85)}\frac{(0.85\lambda)^0}{0!}\right]}{e^{-\lambda}\frac{\lambda^2}{2!}}$$

$$=\frac{e^{-0.15\lambda}*(0.15\lambda)^2}{2!}*e^{-0.85\lambda}*\frac{2!}{e^{-\lambda}*\lambda^2}$$

$$=\frac{e^{-0.15\lambda}e^{-0.85\lambda}}{e^{-\lambda}}*\frac{2!}{2!}*\frac{(0.15)^2\lambda^2}{\lambda^2}$$

$$=\frac{e^{-\lambda}}{e^{-\lambda}}*\frac{2!}{2!}*\frac{(0.15)^2\lambda^2}{\lambda^2}$$

$$=(0.15)^2$$

$$=0.0225$$

<u>A:</u> The probability that both arrived in the first 0.15 seconds is 0.0225

$$P\{At\ least\ one\ particle\ arrived\ in\ the\ first\ 0.15\ seconds\}=$$

$= P\{N(0.15\ seconds) \geq 1 \mid N(1\ second) = 2\}$

$= 1 - P\{neither\ of\ the\ two\ particles\ arrived\ in\ the\ first\ 15\ seconds\}$

$= 1 - P\{N(0.15) = 0 \mid N(1) = 2\}$

$$= 1 - \frac{P\{N(0.15) = 0,\ N(1) = 2\}}{P\{N(1) = 2\}}$$

$$= 1 - \frac{P\{N(0.15) = 0,\ N(1) - N(0.15) = 2\}}{P\{N(1) = 2\}}$$

$$= 1 - \frac{P\{N(0.15) = 0\} * P\{N(0.85) = 2\}}{P\{N(1) = 2\}}$$

$$P\{N(0.15) = 0\} = e^{-\lambda(0.15)} \frac{(0.15\lambda)^0}{0!}$$

$$P\{N(0.85) = 2\} = e^{-\lambda(0.85)} \frac{(0.85\lambda)^2}{2!}$$

$$P\{N(1) = 2\} = e^{-\lambda} \frac{\lambda^2}{2!}$$

$$= 1 - \frac{[e^{-\lambda(0.15)} \frac{(0.15\lambda)^0}{0!}] * [e^{-\lambda(0.85)} \frac{(0.85\lambda)^2}{2!}]}{e^{-\lambda} \frac{\lambda^2}{2!}}$$

$$= 1 - e^{-0.15\lambda} * \frac{e^{-0.85\lambda} * (0.85\lambda)^2}{2!} * \frac{2!}{e^{-\lambda} * \lambda^2}$$

$$= 1 - \frac{e^{-0.15\lambda} e^{-0.85\lambda}}{e^{-\lambda}} * \frac{2!}{2!} * \frac{(0.85)^2 \lambda^2}{\lambda^2}$$

$$= 1 - \frac{e^{-\lambda}}{e^{-\lambda}} * \frac{2!}{2!} * \frac{(0.85)^2 \lambda^2}{\lambda^2}$$

$= 1 - (0.85)^2$

$= 1 - 0.7225$

$= 0.2775$

A: The probability that at least one arrived in the first 0.15 seconds is 0.2775

4. Now suppose that the gram of radioactive material emits α-particles according to a Poisson process $\{N(t): t \geq 0\}$ at the specific rate of λ=3.2 particles per second. This exercise will walk you through an algorithm for generating samples of the arrival times of the particles in the first two seconds.
   a. What is the distribution of N(2), the number of particles that will arrive in the first two seconds? You don't need to explain why your answer is correct – just name the distribution.
   b. Next, we need to generate a sample of N(2), which can be done by implementing Hastings-Metropolis MCMC. Choose a starting point for $X_0 \in \{0,1,2,\ldots\}$, and select a proposed move Y for X_1. A good choice might be a symmetric random walk on $\{0,1,2,\ldots\}$ (but your need to be careful at the endpoint 0). For each sample of the proposed move, you will need to implement an accept/reject mechanism following the Hastings-Metropolis formula. Use this procedure (you should manually sample coin flips and uniform samples from [0,1] as needed – feel free to determine ways to do the latter by computer, or simply pretend!) to generate X_1, X_2, X_3, and X_4, and set N(2)=X_4 as your sample for N(2). Your submission should involve you walking through each step of the algorithm as your generate the samples, and accept/reject, etc.
      Note that the choice of n=4 was arbitrary, in practice we would do this by computer and generate X_n for $n \gg 4$ as our sample for N(2). If you are inclined to solve this exercise by writing code (which would allow you to set N(2)=X_n for n large) you are certainly encouraged to do that, but in that case please submit a copy of your code.
   c. Suppose N(2)=k was the sample you generated in 4(b) above (if you were unable to complete exercise 4(b), feel free to pick some reasonable value for N(2) at this stage). Now generate samples of the arrival times for the k emitted α-particles in the first 2 seconds. As above, your submission should involve you walking through the details of the procedure you are implementing to generate the samples.

A: $N(2)\ has\ a\ Poisson\ distribution\ with\ parameter\ \lambda t\ =\ 3.2(2) = 6.4$

$$P\{N(t) = n\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!}, for\ all\ n = 0,1,2,\ldots$$

$$P\{N(2) = n\} = e^{-6.4}\frac{(6.4)^n}{n!}, for\ all\ n = 0,1,2,\ldots$$

I solved this exercise using Mathematica, so I set N(2)=X_n for n=100.

Attached in the following pages are screenshots of my code.

The code also includes the output.

A: $N(2) = X_{100} = 7$

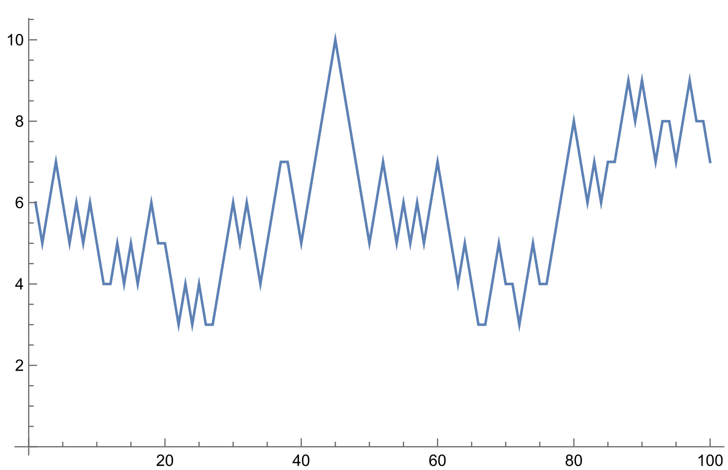
For your reference, I also attached the Mathematica Notebook file to the Assignment on Blackboard.

Summary:

x0=6

x1= 6	x21= 4	x41= 6	x61= 6	x81= 7
x2= 5	x22= 3	x42= 7	x62= 5	x82= 6
x3= 6	x23= 4	x43= 8	x63= 4	x83= 7
x4= 7	x24= 3	x44= 9	x64= 5	x84= 6
x5= 6	x25= 4	x45= 10	x65= 4	x85= 7
x6= 5	x26= 3	x46= 9	x66= 3	x86= 7
x7= 6	x27= 3	x47= 8	x67= 3	x87= 8
x8= 5	x28= 4	x48= 7	x68= 4	x88= 9
x9= 6	x29= 5	x49= 6	x69= 5	x89= 8
x10= 5	x30= 6	x50= 5	x70= 4	x90= 9
x11= 4	x31= 5	x51= 6	x71= 4	x91= 8
x12= 4	x32= 6	x52= 7	x72= 3	x92= 7
x13= 5	x33= 5	x53= 6	x73= 4	x93= 8
x14= 4	x34= 4	x54= 5	x74= 5	x94= 8
x15= 5	x35= 5	x55= 6	x75= 4	x95= 7
x16= 4	x36= 6	x56= 5	x76= 4	x96= 8
x17= 5	x37= 7	x57= 6	x77= 5	x97= 9
x18= 6	x38= 7	x58= 5	x78= 6	x98= 9
x19= 5	x39= 6	x59= 6	x79= 7	x99= 8
x20= 5	x40= 5	x60= 7	x80= 8	x100= 7

Plot from created with Mathematica:



(I made comments to my code when generating X_1.)

```
In[363]:= Clear["Global`*"]
          (* initial declarations *)
          xcurrent = 0;
          xnew = 0;
          Y = 0;

          (* P(N(2)=n)=e^-6.4 * 6.4^n/n! *)
          probN2[n_] := e^-6.4 * 6.4^n/n!;

In[368]:= (* Choose starting point X0 *)
          xcurrent = 6;
          (* Make list of all x values *)
          x = {};

In[373]:= (* Propose move by a symmetric random walk *)
          (* By flipping a coin *)
          (* If Heads, x0+1 *)
          (* If Tails, x0-1 *)
          a = RandomChoice[{0.5, 0.5} → {"Heads", "Tails"}];
          Which[a == "Heads", Y = xcurrent + 1, a == "Tails", Y = xcurrent - 1];
          (* Print what we got as confirmation *)
          Print["We flipped ", a, ", so the proposed move is Y=", Y]

          We flipped Heads, so the proposed move is Y=7

In[376]:= (* Accept/Reject phase *)
          (* Compute acceptance probability *)
          alpha = Min[(probN2[Y] * 1/2)/(probN2[xcurrent] * 1/2), 1];

In[377]:= (* Generate U~Unif[0,1] *)
          U = RandomVariate[UniformDistribution[{0, 1}]];

In[378]:= (* If U<alpha, x1=Y *)
          (* If U≥alpha, x1=x0 *)
          (* Print what we got for alpha and U for confirmation *)
          Print["U=", U, " and alpha=", alpha]
          If[U < alpha, xnew = Y, xnew = xcurrent];
          Print["x1=", xnew]
          (* Add the new x value to the end of the list *)
          AppendTo[x, xnew];

          U=0.965853 and alpha=0.914286

          x1=6
```

(As for generating $X_2, \ldots, X_n$, where n=100, I used a loop, created from the steps of generating X_1.)

```
In[382]:= (* Repeat for n large, so I set n=100 *)
(* Make counter for loop *)
n = 2;
While[n ≤ 100,
 (* Now we are working with x1, so set that as the current x value *)
 xcurrent = xnew;
 a = RandomChoice[{0.5, 0.5} → {"Heads", "Tails"}];
 Which[a == "Heads", Y = xcurrent + 1, a == "Tails", Y = xcurrent - 1];
 alpha = Min[(probN2[Y] * 1/2)/(probN2[xcurrent] * 1/2), 1];
 U = RandomVariate[UniformDistribution[{0, 1}]];
 If[U < alpha, xnew = Y, xnew = xcurrent];
 Print["We flipped ", a, ", so the proposed move is Y=", Y];
 Print["U=", U, " and alpha=", alpha];
 Print["x", n, "=", xnew];
 AppendTo[x, xnew];
 n++;
]
```

The outputs from the loop (contains $X_2, \ldots X_n$, where n=100):

```
We flipped Tails, so the proposed move is Y=5

U=0.278459 and alpha=0.9375

x2=5

We flipped Heads, so the proposed move is Y=6

U=0.882011 and alpha=1

x3=6

We flipped Heads, so the proposed move is Y=7

U=0.301856 and alpha=0.914286

x4=7

We flipped Tails, so the proposed move is Y=6

U=0.914818 and alpha=1

x5=6

We flipped Tails, so the proposed move is Y=5

U=0.714249 and alpha=0.9375

x6=5

We flipped Heads, so the proposed move is Y=6

U=0.0450906 and alpha=1

x7=6

We flipped Tails, so the proposed move is Y=5

U=0.726902 and alpha=0.9375

x8=5

We flipped Heads, so the proposed move is Y=6

U=0.352474 and alpha=1

x9=6

We flipped Tails, so the proposed move is Y=5

U=0.628926 and alpha=0.9375

x10=5

We flipped Tails, so the proposed move is Y=4

U=0.296511 and alpha=0.78125

x11=4
```

```
We flipped Tails, so the proposed move is Y=3

U=0.689516 and alpha=0.625

x12=4

We flipped Heads, so the proposed move is Y=5

U=0.498155 and alpha=1

x13=5

We flipped Tails, so the proposed move is Y=4

U=0.0874171 and alpha=0.78125

x14=4

We flipped Heads, so the proposed move is Y=5

U=0.205951 and alpha=1

x15=5

We flipped Tails, so the proposed move is Y=4

U=0.143836 and alpha=0.78125

x16=4

We flipped Heads, so the proposed move is Y=5

U=0.979369 and alpha=1

x17=5

We flipped Heads, so the proposed move is Y=6

U=0.211252 and alpha=1

x18=6

We flipped Tails, so the proposed move is Y=5

U=0.0263513 and alpha=0.9375

x19=5

We flipped Tails, so the proposed move is Y=4

U=0.90133 and alpha=0.78125

x20=5

We flipped Tails, so the proposed move is Y=4

U=0.482767 and alpha=0.78125

x21=4
```

```
We flipped Tails, so the proposed move is Y=3

U=0.197139 and alpha=0.625

x22=3

We flipped Heads, so the proposed move is Y=4

U=0.143657 and alpha=1

x23=4

We flipped Tails, so the proposed move is Y=3

U=0.290908 and alpha=0.625

x24=3

We flipped Heads, so the proposed move is Y=4

U=0.477937 and alpha=1

x25=4

We flipped Tails, so the proposed move is Y=3

U=0.262954 and alpha=0.625

x26=3

We flipped Tails, so the proposed move is Y=2

U=0.692732 and alpha=0.46875

x27=3

We flipped Heads, so the proposed move is Y=4

U=0.405781 and alpha=1

x28=4

We flipped Heads, so the proposed move is Y=5

U=0.589027 and alpha=1

x29=5

We flipped Heads, so the proposed move is Y=6

U=0.757279 and alpha=1

x30=6

We flipped Tails, so the proposed move is Y=5

U=0.0940006 and alpha=0.9375

x31=5
```

```
We flipped Heads, so the proposed move is Y=6

U=0.37807 and alpha=1

x32=6

We flipped Tails, so the proposed move is Y=5

U=0.446968 and alpha=0.9375

x33=5

We flipped Tails, so the proposed move is Y=4

U=0.105504 and alpha=0.78125

x34=4

We flipped Heads, so the proposed move is Y=5

U=0.820777 and alpha=1

x35=5

We flipped Heads, so the proposed move is Y=6

U=0.100767 and alpha=1

x36=6

We flipped Heads, so the proposed move is Y=7

U=0.063883 and alpha=0.914286

x37=7

We flipped Heads, so the proposed move is Y=8

U=0.988398 and alpha=0.8

x38=7

We flipped Tails, so the proposed move is Y=6

U=0.309623 and alpha=1

x39=6

We flipped Tails, so the proposed move is Y=5

U=0.850492 and alpha=0.9375

x40=5

We flipped Heads, so the proposed move is Y=6

U=0.0483342 and alpha=1

x41=6
```

```
We flipped Heads, so the proposed move is Y=7
U=0.743992 and alpha=0.914286
x42=7
We flipped Heads, so the proposed move is Y=8
U=0.123955 and alpha=0.8
x43=8
We flipped Heads, so the proposed move is Y=9
U=0.157245 and alpha=0.711111
x44=9
We flipped Heads, so the proposed move is Y=10
U=0.366232 and alpha=0.64
x45=10
We flipped Tails, so the proposed move is Y=9
U=0.107916 and alpha=1
x46=9
We flipped Tails, so the proposed move is Y=8
U=0.349199 and alpha=1
x47=8
We flipped Tails, so the proposed move is Y=7
U=0.728106 and alpha=1
x48=7
We flipped Tails, so the proposed move is Y=6
U=0.64148 and alpha=1
x49=6
We flipped Tails, so the proposed move is Y=5
U=0.0136421 and alpha=0.9375
x50=5
We flipped Heads, so the proposed move is Y=6
U=0.748748 and alpha=1
x51=6
```

```
We flipped Heads, so the proposed move is Y=7
U=0.749329 and alpha=0.914286
x52=7
We flipped Tails, so the proposed move is Y=6
U=0.719548 and alpha=1
x53=6
We flipped Tails, so the proposed move is Y=5
U=0.661698 and alpha=0.9375
x54=5
We flipped Heads, so the proposed move is Y=6
U=0.33827 and alpha=1
x55=6
We flipped Tails, so the proposed move is Y=5
U=0.266494 and alpha=0.9375
x56=5
We flipped Heads, so the proposed move is Y=6
U=0.523008 and alpha=1
x57=6
We flipped Tails, so the proposed move is Y=5
U=0.299359 and alpha=0.9375
x58=5
We flipped Heads, so the proposed move is Y=6
U=0.565947 and alpha=1
x59=6
We flipped Heads, so the proposed move is Y=7
U=0.707763 and alpha=0.914286
x60=7
We flipped Tails, so the proposed move is Y=6
U=0.229713 and alpha=1
x61=6
```

```
We flipped Tails, so the proposed move is Y=5
U=0.373637 and alpha=0.9375
x62=5
We flipped Tails, so the proposed move is Y=4
U=0.703999 and alpha=0.78125
x63=4
We flipped Heads, so the proposed move is Y=5
U=0.912872 and alpha=1
x64=5
We flipped Tails, so the proposed move is Y=4
U=0.493835 and alpha=0.78125
x65=4
We flipped Tails, so the proposed move is Y=3
U=0.223689 and alpha=0.625
x66=3
We flipped Tails, so the proposed move is Y=2
U=0.68977 and alpha=0.46875
x67=3
We flipped Heads, so the proposed move is Y=4
U=0.158316 and alpha=1
x68=4
We flipped Heads, so the proposed move is Y=5
U=0.751635 and alpha=1
x69=5
We flipped Tails, so the proposed move is Y=4
U=0.462525 and alpha=0.78125
x70=4
We flipped Tails, so the proposed move is Y=3
U=0.881561 and alpha=0.625
x71=4
```

```
We flipped Tails, so the proposed move is Y=3
U=0.614993 and alpha=0.625
x72=3
We flipped Heads, so the proposed move is Y=4
U=0.457444 and alpha=1
x73=4
We flipped Heads, so the proposed move is Y=5
U=0.795992 and alpha=1
x74=5
We flipped Tails, so the proposed move is Y=4
U=0.373312 and alpha=0.78125
x75=4
We flipped Tails, so the proposed move is Y=3
U=0.702587 and alpha=0.625
x76=4
We flipped Heads, so the proposed move is Y=5
U=0.646043 and alpha=1
x77=5
We flipped Heads, so the proposed move is Y=6
U=0.272935 and alpha=1
x78=6
We flipped Heads, so the proposed move is Y=7
U=0.279177 and alpha=0.914286
x79=7
We flipped Heads, so the proposed move is Y=8
U=0.0823189 and alpha=0.8
x80=8
We flipped Tails, so the proposed move is Y=7
U=0.0361107 and alpha=1
x81=7
```

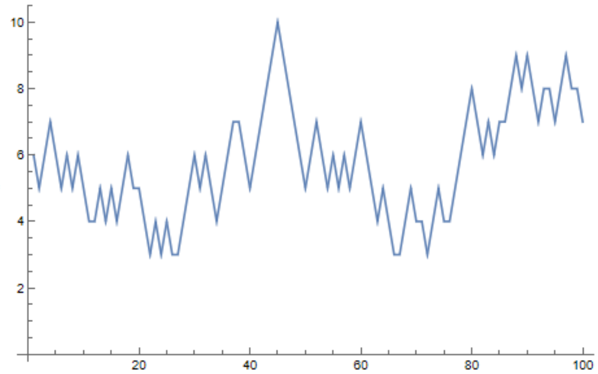
```
We flipped Tails, so the proposed move is Y=6
U=0.144304 and alpha=1
x82=6
We flipped Heads, so the proposed move is Y=7
U=0.163201 and alpha=0.914286
x83=7
We flipped Tails, so the proposed move is Y=6
U=0.983559 and alpha=1
x84=6
We flipped Heads, so the proposed move is Y=7
U=0.003886 and alpha=0.914286
x85=7
We flipped Heads, so the proposed move is Y=8
U=0.804052 and alpha=0.8
x86=7
We flipped Heads, so the proposed move is Y=8
U=0.563504 and alpha=0.8
x87=8
We flipped Heads, so the proposed move is Y=9
U=0.360285 and alpha=0.711111
x88=9
We flipped Tails, so the proposed move is Y=8
U=0.269859 and alpha=1
x89=8
We flipped Heads, so the proposed move is Y=9
U=0.0627054 and alpha=0.711111
x90=9
We flipped Tails, so the proposed move is Y=8
U=0.697844 and alpha=1
x91=8
```

```
We flipped Tails, so the proposed move is Y=7
U=0.376022 and alpha=1
x92=7
We flipped Heads, so the proposed move is Y=8
U=0.789224 and alpha=0.8
x93=8
We flipped Heads, so the proposed move is Y=9
U=0.809083 and alpha=0.711111
x94=8
We flipped Tails, so the proposed move is Y=7
U=0.293868 and alpha=1
x95=7
We flipped Heads, so the proposed move is Y=8
U=0.774925 and alpha=0.8
x96=8
We flipped Heads, so the proposed move is Y=9
U=0.359478 and alpha=0.711111
x97=9
We flipped Tails, so the proposed move is Y=8
U=0.229677 and alpha=1
x98=8
We flipped Heads, so the proposed move is Y=9
U=0.896603 and alpha=0.711111
x99=8
We flipped Tails, so the proposed move is Y=7
U=0.213041 and alpha=1
x100=7
```

(I print all the generated x values and plot it.)

```
In[384]:= (* Print all the x values *)
          x
Out[384]= {6, 5, 6, 7, 6, 5, 6, 5, 6, 5, 4, 4, 5, 4, 5, 4, 5, 6, 5, 5, 4, 3, 4, 3, 4, 3, 3, 4, 5, 6, 5, 6, 5, 4, 5, 6, 7, 7, 6, 5, 6, 7, 8, 9, 10, 9, 8, 7, 6, 5,
           6, 7, 6, 5, 6, 5, 6, 5, 6, 7, 6, 5, 4, 5, 4, 3, 3, 4, 5, 4, 4, 3, 4, 5, 4, 4, 5, 6, 7, 8, 7, 6, 7, 6, 7, 7, 8, 9, 8, 9, 8, 7, 8, 8, 7, 8, 9, 8, 8, 7}

In[386]:= (* Plot *)
          ListLinePlot[x]
Out[386]=
```



I solved this exercise using Mathematica.

N(2)=k

k=7 (from part (b) above)

N(2)=7

Attached in the following pages are screenshots of my code.

The code also includes the output.

A:

0.203671 seconds

0.568199 seconds

0.718515 seconds

1.30699 seconds

1.79866 seconds

1.85194 seconds

1.96382 seconds

For your reference, I also attached the Mathematica Notebook file to the Assignment on Blackboard.

```
(* N(2)=k=7 *)
(* Generate samples of the arrival times for k emitted α particles in the first 2 seconds *)
U = RandomVariate[UniformDistribution[{0, 2}], 7]

Out[1]= {1.96382, 1.30699, 0.568199, 1.85194, 0.203671, 1.79866, 0.718515}

In[2]:= (* Sort the samples *)
Sort[U]

Out[2]= {0.203671, 0.568199, 0.718515, 1.30699, 1.79866, 1.85194, 1.96382}

In[3]:= (* Print them in order *)
Print[Sort[U]]

{0.203671, 0.568199, 0.718515, 1.30699, 1.79866, 1.85194, 1.96382}
```