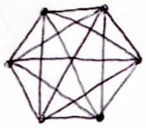
Chapter 5

9. Write down the chromatic polynomials of

(i) the complete graph K_6



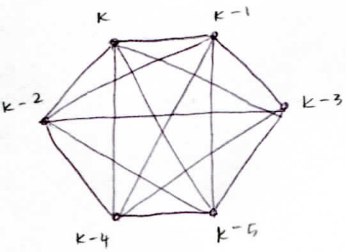
K_6

$n=6$

$m=15$

chromatic polynomial form:

$k^6-15k^5+ak^4-bk^3+ck^2-dk$



$k(k-1)(k-2)(k-3)(k-4)(k-5)$

$P_{K_6}(k) = k(k-1)(k-2)(k-3)(k-4)(k-5)$

Let $k=7$

K_6 can be colored $7(7-1)(7-2)(7-3)(7-4)(7-5)$ ways.

$= 7(6)(5)(4)(3)(2)$

$= 5040$ ways.

9.(ii) the complete bipartite graph $K_{1,5}$

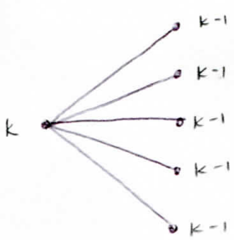
$K_{1,5}$

$n=6$

$m=5$

chromatic polynomial form:

$k^6-5k^5+ak^4-bk^3+ck^2-dk$



$k(k-1)^5$

$P_{K_{1,5}}(k) = k(k-1)^5$

Let $k=7$

$K_{1,5}$ can be colored $7(7-1)^5$ ways.

$= 7(6)^5$

$= 54432$ ways

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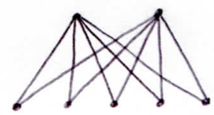
12. Find the chromatic polynomials of

(i) the complete bipartite graph $K_{2,5}$



$K_{2,5}$

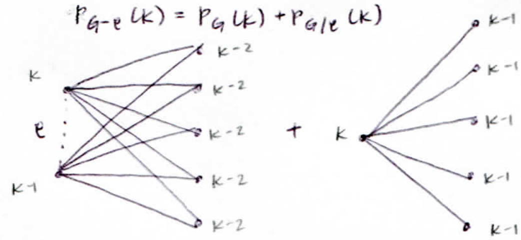
$n=7$

$m=10$

chromatic polynomial form:

$k^7-10k^6+ak^5-bk^4+ck^3-dk^2+ek$

$P_{G-e}(k) = P_G(k) + P_{G/e}(k)$



$k(k-1)(k-2)^5 \quad + \quad k(k-1)^5$

$P_{K_{2,5}}(k) = k(k-1)(k-2)^5 + k(k-1)^5$

12.(ii) the cycle graph C_5

C_5

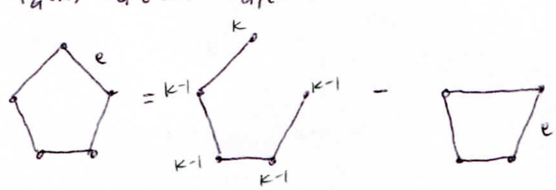
$n=5$

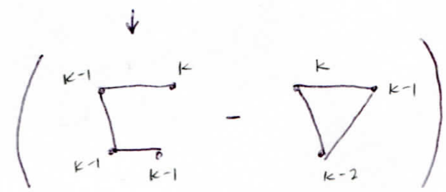
$m=5$

chromatic polynomial form:

$k^5-5k^4+ak^3-bk^2+ck$

$P_G(k) = P_{G-e}(k) - P_{G/e}(k)$

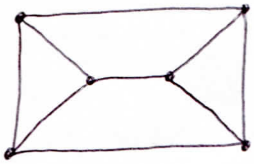




$k(k-1)^4 \quad - \quad (k(k-1)^3 \quad - \quad k(k-1)(k-2))$

$P_{C_5}(k) = k(k-1)^4 - (k(k-1)^3 - k(k-1)(k-2))$

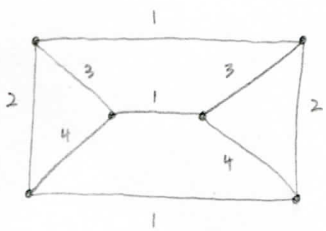
25. Find the chromatic index of the graph.



Theorem states: if G is a simple graph with largest degree Δ, then $\Delta \le \chi'(G) \le \Delta+1$

$\Delta = 3$, so

$3 \le \chi'(G) \le 4$



$\chi'(G) = 4$

31. Prove that if G is a cubic Hamiltonian graph, then $\chi'(G) = 3$.

cubic graph $\Rightarrow$ deg $= 3$
Hamiltonian graph $\Rightarrow$ has a Hamilton circuit.

every 2 edges contribute to the circuit

cubic graph, so each vertex has degree 3

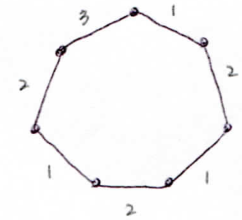
circuit with chords that are mutually not touching

We use 2 colors to edge color the circuit.
3 colors for the circuit with chords.

The circuit is even length because we cannot have a vertex going no where. So, there is an even # of vertices with odd degree 3.

28. Compare the lower and upper bounds for the chromatic index given by Vizing's theorem with the correct value, for:

(i) the cycle graph C_7

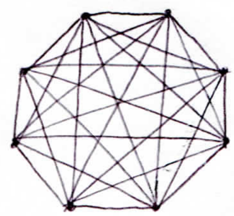


$\Delta = 2$ so

$2 \le \chi'(G) \le 3$

$\chi'(G) = 3$

(ii) the complete graph K_8



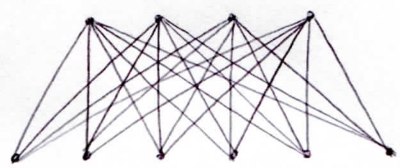
$\Delta = 7$ so

$7 \le \chi'(G) \le 8$

Theorem states for K_n, $\chi'(K_n) = n$ if n is odd and $= n-1$ if n is even.

$n = 8$, so $\chi'(G) = 7$

(iii) the complete bipartite graph $K_{4,6}$



$\Delta = 6$ so

$6 \le \chi'(G) \le 7$

$\chi'(G) = 6$