Chapter 10.1

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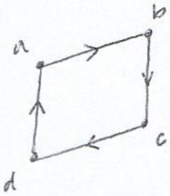
1. Find a kernel in the following graphs or show why none can exist:

(a)



Kernel : {a, c} or {b, d}

Both are true:

1. There's no edge btw. any two vertices in the kernel.

No edge btw. b and d
No edge btw. a and c

2. There is an edge from every nonkernel vertex to some kernel vertex.

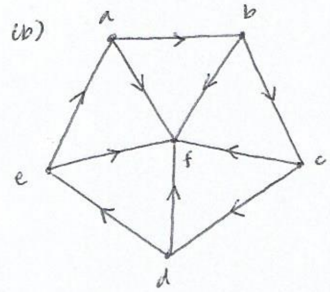
Kernel: {a, c}
Nonkernel: {b, d}:

Edge btw.
a (kernel) → b (nonkernel)
b (nonkernel) → c (kernel)
c (kernel) → d (nonkernel)
d (nonkernel) → a (kernel)

Kernel: {b, d}
Nonkernel: {a, c}

Edge btw.
a (nonkernel) → b (kernel)
b (kernel) → c (nonkernel)
c (nonkernel) → d (kernel)
d (kernel) → a (nonkernel)

(b)



Kernel : {f}

Both are true:

1. There is no edge btw. any two vertices in the kernel.
f is the only vertex in the kernel

2. There is an edge from every nonkernel vertex to some kernel vertex.

a (nonkernel) → f (kernel)
b (nonkernel) → f (kernel)
c (nonkernel) → f (kernel)
d (nonkernel) → f (kernel)
e (nonkernel) → f (kernel)

3. Repeat Example 2 with 2, 3, or 7 cents added each time. Find the set of positions in the kernel.

winning vertices: 4, 9, 16, 25, 36, "over 40"
losing vertices:

Table of Grundy Numbers

vertex x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
g(x)	1	1	2	0	0	1	1	2	2	0	1	0	0	2	1	1

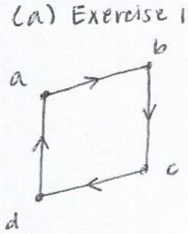
vertex x	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
g(x)	0	0	2	1	1	0	2	2	1	0	0	0	2	1	1

vertex x	31	32	33	34	35	36	37	38	39	40	over 40
g(x)	0	0	2	1	1	0	2	2	1	1	0

Kernel set: {3, 4, 9, 11, 12, 16, 17, 21, 25, 26, 27, 31, 32, 36}

8. Find the Grundy function for graphs or games in:

(a) Exercise 1(a)



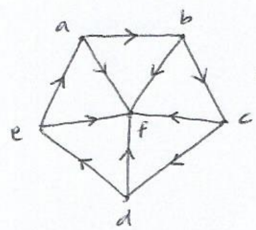
If {ac} is kernel set:

vertex x	a	b	c	d
g(x)	0	1	0	1

If {bd} is kernel set:

vertex x	a	b	c	d
g(x)	1	0	1	0

(b) Exercise 1(b)



If vertex kernel f has grundy value of 0, then a has g(x) = 1, e has g(x) of 2, d has g(x) = 1, c has g(x) = 2, b has g(x) = 1 but there is no Grundy function because you cannot have a Grundy function around an odd-length circuit.

12. (a) Show that if $f(x) = k$ for a vertex x in the progressively finite graph G, then k is the length of the longest path starting at x in G.

If your level is , then your longest path is 0.

Assume your level number is k. Then your longest path is k.
All of the successors are at level k, but one has to be at level k−1, which one has to be at level k−2, and so on.

(otherwise you could be at level k−1)

15. Show that both the level numbers and the Grundy function in a progressively finite graph G constitute proper colorings of G.

For level numbers, you go to a lower level number. You cannot go to another edge with the same level number.
By definition, Grundy number is the smallest number that used by any successor.
So, by default, you cannot be adjacent to the same level number or the same Grundy value.

Chapter 10.2

1. Find the Grundy number of the initial position and make the first move in a winning strategy for the following Nim games:

(a)
I I
I I I
I I I
I I I I I

	2^3	2^2	2^1	2^0
2	0	0	1	0
3	0	0	1	1
3	0	0	1	1
5	0	1	0	1
	0	1	1	1 = 7

Grundy number = 7

row 4 has 1 in the leftmost column that has an odd # of 1s

So, the first move is decrease the size of 4 th pile by 3.

(b)
I
I I I
I I I I I
I I I I I I I

	2^3	2^2	2^1	2^0
1	0	0	0	1
3	0	0	1	1
5	0	1	0	1
7	0	1	1	1
	0	0	0	0 = 0

Grundy number = 0

All of the columns have an even # of 1s.

(c)
I I
I I I I
I I I I
I I I I I I

	2^3	2^2	2^1	2^0
2	0	0	1	0
4	0	1	0	0
4	0	1	0	0
6	0	1	1	0
	0	1	0	0 = 4

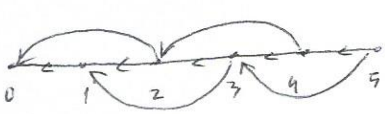
Grundy number = 4

row 2, 3, 4 in the leftmost column that has an odd # of 1s

So, the first move is decrease the size of the 2nd, 3rd, and 4th piles by 4.

2. Suppose that no more than two sticks can be removed at a time from any pile. Repeat the games in Exercise 1 with this additional condition.

(a)



g(x)

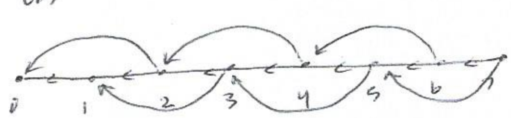
0	1	2	0	1	2

sticks	g(x)	2^1	2^0
2	2	1	0
3	0	0	0
3	0	0	0
5	2	1	0
		0	0

Grundy # = 2+0+0+2=4

All the columns have an even # of 1s.

(b)



g(x)

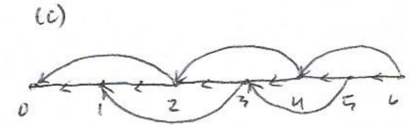
0	1	2	0	1	2	0	1

sticks	g(x)	2^1	2^0
1	1	0	1
3	0	0	0
5	2	1	0
7	1	0	1
		1	0

Grundy # = 1+0+2+1=4

Take 2 away from pile of 5.

(c)



g(x)

0	1	2	0	1	2	0

sticks	g(x)	2^1	2^0
2	2	1	0
4	1	0	1
4	1	0	1
6	0	0	0
		1	0

Grundy # 2+1+1+0=4

Take 2 away from pile of 2.

3. Suppose that no more than i sticks can be removed at a time from the i th pile. Repeat Exercise 1 with this additional condition.

(a)

			2^1	2^0
I I	mod 2	= 0	0	0
I I I	mod 3	= 0	0	0
I I I	mod 4	= 3	1	1
I I I I I	mod 5	= 0	0	0
			1	1

Grundy # = 2 + 1 = 3

Remove 3 from pile 3.

(b)

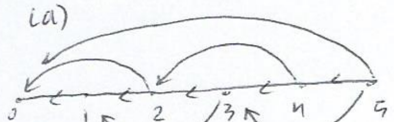
			2^1	2^0
I	mod 2	= 1	0	1
I I I	mod 3	= 0	0	0
I I I I I	mod 4	= 1	0	1
I I I I I I I	mod 5	= 2	1	0
			1	0

Grundy # = 2

Remove 2 from pile 4.

4. Suppose that only one or two or five sticks can be removed from each pile. Repeat the games in Exercise 1 with this constraint.

(a)



g(x)

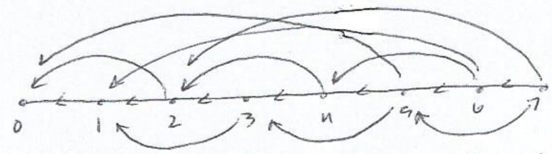
0 1 2 0 1 2

sticks	g(x)	2^1	2^0
2	2	1	0
3	0	0	0
3	0	0	0
5	2	1	0
		0	0

Grundy # = 2 + 0 + 0 + 2 = 4

All columns have even # of 1s.

(b)



g(x)

0 1 2 0 1 2 0 1

sticks	g(x)	2^1	2^0
1	1	0	1
3	0	0	0
5	2	1	0
7	1	0	1
		1	0

Grundy # = 1 + 0 + 2 + 1 = 4

Remove 2 from pile 3.

6. For the Nim game in Exercise 1 (c), find moves that yield positions with Grundy number equal to:

(a) 1

	2^2	2^1	2^0
I I	0	1	0
I I I I	1	0	0
I I I I	1	0	0
I I I I I I	1	1	0
	1	0	0 = 4

(we want 0 0 1 = 1)

Remove 3 from pile 2, OR

Remove 3 from pile 3, OR

Remove 5 from pile 4.

(b) 2

	2^2	2^1	2^0
I I	0	1	0
I I I I	1	0	0
I I I I	1	0	0
I I I I I I	1	1	0
	1	0	0 = 4

(we want 0 1 0 = 2)

Remove 2 from pile 2, OR

Remove 2 from pile 3, OR

Remove 4 from pile 4.