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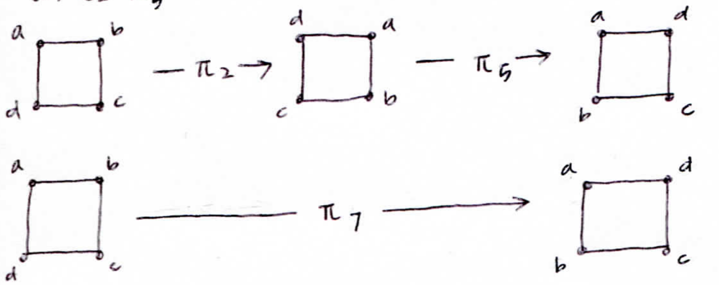
Chapter 9.1

5. For the symmetries of the square listed in Figure 9.2, give the associated permutation of 2-colorings, as in (1), for

(c) $\pi_5 = \begin{pmatrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_1 & C_3 & C_2 & C_5 & C_4 & C_6 & C_9 & C_8 & C_7 & C_{11} & C_{10} & C_{15} & C_{14} & C_{13} & C_{12} & C_{16} \end{pmatrix}$

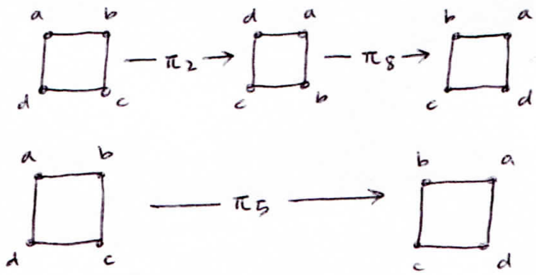
10. Find the symmetry of the square equal to the following products:

(b) $\pi_2 \cdot \pi_5$

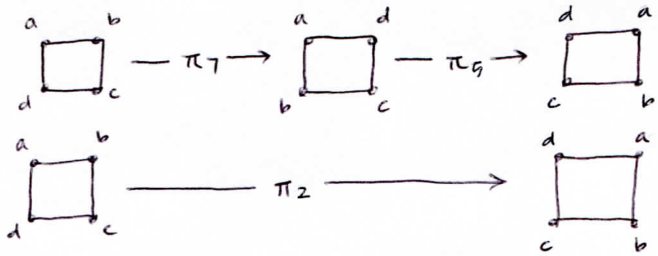


$\pi_2 \cdot \pi_5 = \pi_7$

(c) $\pi_7 \cdot (\pi_2 \cdot \pi_8)$



$\pi_2 \cdot \pi_8 = \pi_5$



$\pi_7 \cdot (\pi_2 \cdot \pi_8) = \pi_2$

Chapter 9.2

3. Fifteen balls are put in a triangular array as shown. How many different arrays can be made using balls of three colors if the array is free to rotate?

	$\Psi(\pi)$
$0°$	3^{15}
$120°$	3^5
$240°$	3^5

$\frac{1}{3}(3^{15}+3^5+3^5)$

By Burnside's theorem: $\frac{1}{3}[\Psi(0°)+\Psi(120°)+\Psi(240°)]$

$= \frac{1}{3}(3^{15}+3^5+3^5)$

4. How many different ways are there to 2-color the 64 squares of an 8×8 chessboard that rotates freely?

	$\Psi(\pi)$
$0°$	2^{64}
$90°$	2^{16}
$180°$	2^{16}
$270°$	2^{32}

$\frac{1}{4}(2^{64}+2^{16}+2^{16}+2^{32})$

By Burnside's theorem: $\frac{1}{4}[\Psi(0°)+\Psi(90°)+\Psi(180°)+\Psi(270°)]$

$= \frac{1}{4}(2^{64}+2^{16}+2^{16}+2^{32})$

10. How many ways are there to 3-color the n-bands of a baton if adjacent bands must have different colors?

	$\Psi(\pi)$
$0°$	3
$180°$	2^{n-1}

$\frac{1}{2}(3+2^{n-1})$

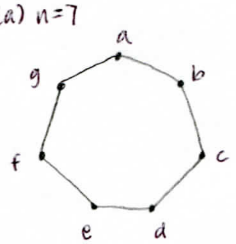
By Burnside's theorem: $\frac{1}{2}[\Psi(\pi_1)+\Psi(\pi_2)]$

$= \frac{1}{2}(3+2^{n-1})$

Chapter 9.3

4. How many different n-bead necklaces (cyclicly distinct) can be made from three colors of beads when :

(a) $n=7$



7 symmetries

$0°$	7 1-cycles	x_1^7
$\frac{360°}{7}$	(abcdefg)	x_7
$\frac{720°}{7}$	(acegbdf)	x_7
$\frac{1080°}{7}$	(adgcfbe)	x_7
$\frac{1440°}{7}$	(aebfcgd)	x_7
$\frac{1800°}{7}$	(afdbgec)	x_7
$\frac{2160°}{7}$	(agfedcb)	x_7

$P_G = \frac{1}{7}(x_1^7 + 6x^7)$

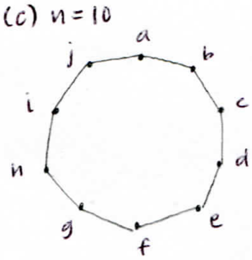
The number of different m-colored necklaces of seven beads is :

$\frac{1}{7}(m^7 + 6m)$

$m = 3$

$\frac{1}{7}(3^7 + 6(3)) = \frac{1}{7}(2205) = 315$

(c) $n=10$



10 symmetries

$0°$	10 1-cycles	x_1^{10}
$36°$	(abcdefghij)	x_{10}
$72°$	(acegi)(bdfhj)	x_5^2
$108°$	(adgjcfibeh)	x_{10}
$144°$	(aeicg)(bfjdh)	x_5^2
$180°$	(af)(bg)(ch)(di)(ej)	x_2^5
$216°$	(agcie)(bhdjf)	x_5^2
$252°$	(ahebifcjgd)	x_{10}
$288°$	(aigec)(bjhfd)	x_5^2
$324°$	(ajihgfedcb)	x_{10}

$P_G = \frac{1}{10}(x_1^{10} + 4x_{10} + 4x_5^2 + x_2^5)$

The number of different m-colored necklaces of ten beads is

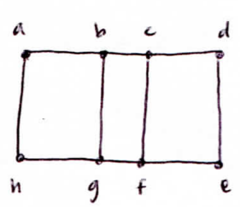
$\frac{1}{10}(m^{10} + 4m + 4m^2 + m^5)$

$m = 3$

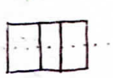
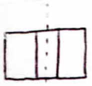
$\frac{1}{10}(3^{10} + 4(3) + 4(3)^2 + 3^5) = \frac{1}{10}(59340) = 5934$

5. Find the number of different m-colorings of the vertices of the following floating figures.

(d)



4 symmetries

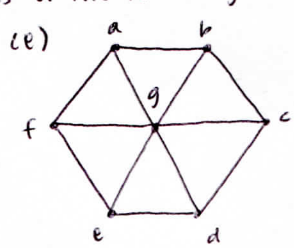
$0°$	x_1^3
$180°$	x_2^4
	x_2^4
	x_2^4

$P_G = \frac{1}{4}(x_1^8 + 3x_2^4)$

The number of m-colorings is :

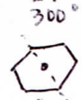
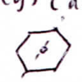
$\frac{1}{4}(m^8 + 3m^4)$

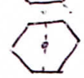
(e)



12 symmetries

$0°$	x_1^7	
$60°$	(g)(abcdef)	$x_1 x_6$
$120°$	(g)(ace)(bdf)	$x_1 x_3^2$
$180°$	(g)(ad)(be)(cf)	$x_1 x_2^3$
$240°$	(g)(aec)(bfd)	$x_1 x_3^2$
$300°$	(g)(afedcb)	$x_1 x_6$

 and  and  $x_1^3 x_2^2$ each

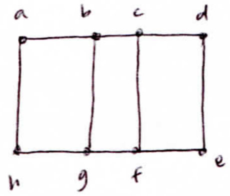
 and  and  $x_1 x_2^3$ each

The number of m-colorings is :

$\frac{1}{12}(m^7 + 2m^2 + 2m^3 + 4m^4 + 3m^5)$

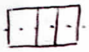
7. How many ways are there to m-color the edges of the floating figures in Exercise 5?

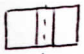
(d)



0° x_1^{10}

180° x_2^5

 $x_1^4 \quad x_2^3$ ← they are fixed

 $x_1^2 \quad x_2^4$ ← they are fixed

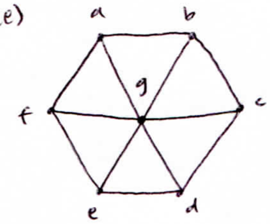
$P_G = \frac{1}{4}(x_1^{10} + x_2^5 + x_1^4 x_2^3 + x_1^2 x_2^4)$

The number of m-colorings is:

$\frac{1}{4}(m^{10} + m^5 + m^4 m^3 + m^2 m^4)$

$= \frac{1}{4}(m^{10} + m^5 + m^7 + m^6)$

(e)



0° x_1^{12}

60° $x_6 x_2^3$

120° $x_3^2 x_2^3$

180° $x_2^3 x_2^3$

240° $x_3^2 x_2^3$

300° $x_6 x_2^3$

$P_G = \frac{1}{12}(x_1^{12} + 2(x_6 x_2^3) + 2(x_3^2 x_2^3) + x_2^3 x_2^3)$

 and  and  $x_1^2 x_2^5$ each

 and  and  $x_1^2 x_2^5$ each

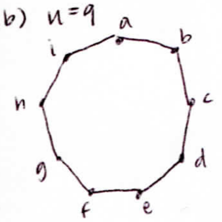
The number of m-colorings is:

$\frac{1}{12}(m^{12} + 2m^4 + 2m^5 + m^6 + 6m^7)$

Chapter 9.4

2. Find an expression for the pattern inventory for black-white, n-bead necklaces (rotations only) and find the number of necklaces with three white beads and the rest black:

(b) $n=9$



0° 9 1-cycles x_1^9

40° (abcdefghi) x_9

80° (acegibdfh) x_9

120° (adg)(beh)(cfi) x_3^3

160° (aeidhcgbf) x_9

200° (afbgchdie) x_9

240° (agd)(bhe)(cif) x_3^3

280° (ahfdbigec) x_9

320° (aihgfedcb) x_9

$P_G = \frac{1}{9}(x_1^9 + 6x_9 + 2x_3^3)$

$\frac{1}{9}((b+w)^9 + 6(b^9+w^9) + 2(b^3+w^3)^3)$

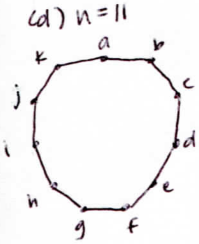
(c) $n=10$

From Chapter 9.3 #4c:

$P_G = \frac{1}{10}(x_1^{10} + 4x_{10} + 4x_5^2 + x_2^5)$

$\frac{1}{10}((b+w)^{10} + 4(b^{10}+w^{10}) + 4(b^5+w^5)^2 + (b^2+w^2)^5)$

(d) $n=11$



11 symmetries

0° 11 1-cycles x_1^{11}

$\frac{360°}{11}$ (abcdefghijk) x_{11}

$\frac{720°}{11}$ (acegikbdfhj) x_{11}

$\frac{1080°}{11}$ (adgjbehkcfi) x_{11}

$\frac{1440°}{11}$ (aeibfjcgkdh) x_{11}

$\frac{1800°}{11}$ (afkejdichbg) x_{11}

$\frac{2160°}{11}$ (agbhcidjekf) x_{11}

$\frac{2520°}{11}$ (ahdkgcjfbie) x_{11}

$\frac{2880°}{11}$ (aifckhebjgd) x_{11}

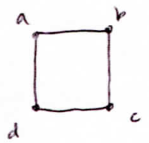
$\frac{3240°}{11}$ (ajhfdbkigec) x_{11}

$\frac{3600°}{11}$ (akjihgfedcb) x_{11}

$P_G = \frac{1}{11}(x_1^{11} + 10x_{11})$

$\frac{1}{11}((b+w)^{11} + 10(b^{11}+w^{11}))$

3. Find the pattern inventory for black, white, and red corner colorings of a floating square.



0°	4 1-cycles	x_1^4
90°	(abcd)	x_4
180°	(ac)(bd)	x_2^2
270°	(adcb)	x_4

$$P_G = \frac{1}{4}\left(x_1^4 + 2x_4 + x_2^2\right)$$

$$\frac{1}{4}\left((b+w+r)^4 + 2(b^4+w^4+r^4) + (b^2+w^2+r^2)^2\right)$$

$$= \frac{1}{4}\left(b^4 + 4b^3w + 4b^3r + 6b^2w^2 + 6b^2r^2 + 12b^2wr + 4bw^3 + 4br^3 + 12bwr^2 + 12bw^2r + w^4 + r^4 + 4wr^3 + 6w^2r^2 + \right.$$

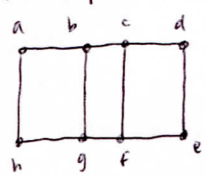
$$\left. 4w^3r + 2b^4 + 2w^4 + 2r^4 + 2b^2w^2 + 2b^2r^2 + b^4 + w^4 + 2w^2r^2 + r^4\right)$$

$$= \frac{1}{4}\left(4b^4 + 4b^3w + 4b^3r + 8b^2w^2 + 8b^2r^2 + 12b^2wr + 4bw^3 + 4br^3 + 12bwr^2 + 12bw^2r + 4w^4 + 4r^4 + 4wr^3 + 8w^2r^2 + 4w^3r\right)$$

$$= b^4 + b^3w + b^3r + 2b^2w^2 + 2b^2r^2 + 3b^2wr + bw^3 + br^3 + 3bwr^2 + 3bw^2r + w^4 + r^4 + wr^3 + 2w^2r^2 + w^3r$$

7. Find an expression for the pattern inventory for edge 2-colorings of the floating figures in Exercise 5 in Section 9.3.

(d)

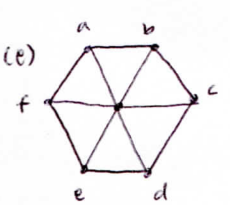


From Chapter 9.3 #7d:

$$P_G = \frac{1}{4}\left(x_1^{10} + x_2^5 + x_1^4 x_2^3 + x_1^2 x_2^4\right)$$

$$\frac{1}{4}\left((b+w)^{10} + (b^2+w^2)^5 + (b+w)^4(b^2+w^2)^3 + (b+w)^2(b^2+w^2)^4\right)$$

(e)



From Chapter 9.3 #7e:

$$P_G = \frac{1}{12}\left(x_1^{12} + 2(x_6 x_2^3) + 2(x_3^2 x_2^3) + x_2^3 x_2^3\right)$$

$$\frac{1}{12}\left((b+w)^{12} + 2(b^6+w^6)(b^2+w^2)^3 + 2(b^3+w^3)^2(b^2+w^2)^3 + (b^2+w^2)^3(b^2+w^2)^3\right.$$

9. Find an expressing for the pattern inventory for face 2-colorings of:

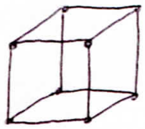
(a) A floating tetrahedron

0°	x_1^4		
120°	$x_2 x_1^3$	(4 times)	one vertex fixed
240°	$x_2 x_1^3$	(4 times)	one vertex fixed
180°	x_4	(3 times)	opp. edges

$$P_G = \frac{1}{12}\left(x_1^4 + 4x_2x_1^3 + 4x_2x_1^3 + x_4\right)$$

$$\frac{1}{12}\left((b+w)^4 + 4(b^2+w^2)(b+w)^3 + 4(b^2+w^2)(b+w)^3 + (b^4+w^4)\right)$$

(b) A floating cube



$0°$	x_1^6		
$90°$	$x_1^2 x_4$	(3 times)	
$180°$	$x_1^2 x_4$	(3 times)	
$270°$	$x_1^2 x_4$	(3 times)	
$180°$	x_6	(opp. edges)	(6 times)
$120°$	x_6	(opp. corners)	(4 times)
$240°$	x_6	(opp. corners)	(4 times)

$$P_G = \frac{1}{24}\left(x_1^6 + 3x_1^2x_4 + 3x_1^2x_4 + 3x_1^2x_4 + 6x_6 + 4x_6 + 4x_6 \right)$$

$$\frac{1}{24}\left((b+w)^6 + 3(b+w)^2(b^4+w^4) + 3(b+w)^2(b^4+w^4) + 3(b+w)^2(b^4+w^4) + 6(b^6+w^6) + 4(b^6+w^6) + 4(b^6+w^6)\right)\Big)$$