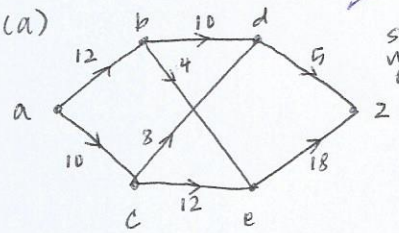
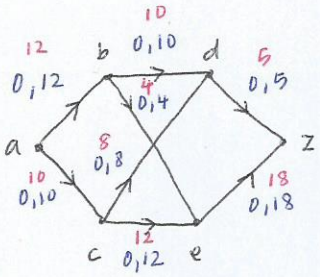
Chapter 4.3

Reina Li
110909646

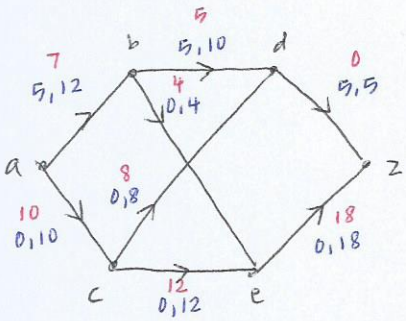
2. (a)



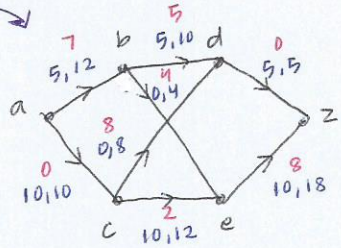
start with flow=0



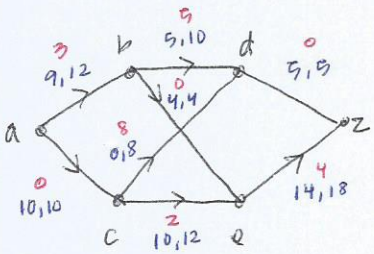
augmented path: a → b → d → z
min. residual = 5
flow = 5



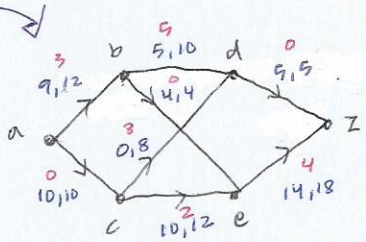
augmented path: a → c → e → z
min. residual = 10
flow = 10



aug. path: a → b → e → z
min. residual: 4
flow: 4



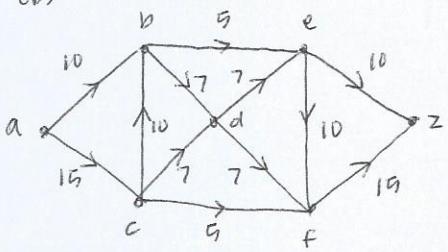
augmented path: a → c → d → z
min. residual: 0
flow: 0



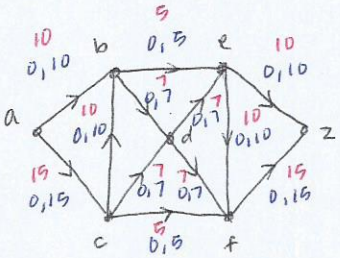
max. flow = 5 + 10 + 4
= 19

min. cut = 19
{a, b, d}

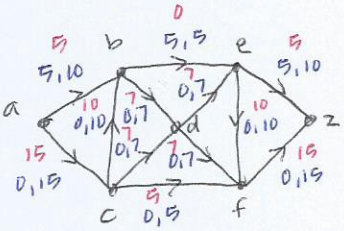
(b)



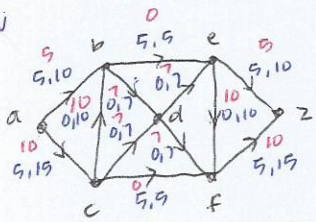
start with flow = 0



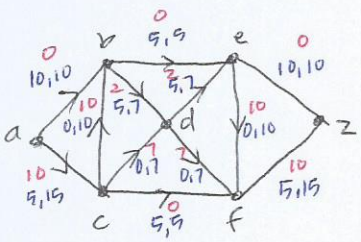
aug. path: a → b → e → z
min. residual: 5
flow = 5



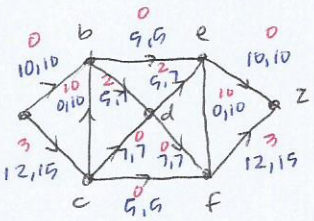
aug. path: a → c → f → z
min. residual: 5
flow: 5



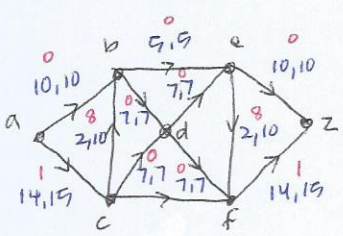
aug. path: a → b → d → e → z
min. residual: 5
flow: 5



aug. path: a → c → d → f → z
min. res: 7
flow: 7



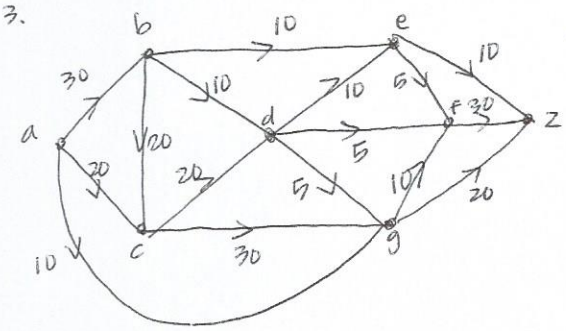
aug. path: a → c → b → d → e → f → z
min. resid: 2
flow: 2



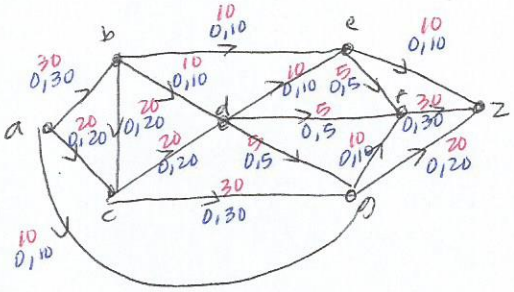
max. flow = 5 + 5 + 5 + 7 + 2
= 24

min. cut = 24
{a, b, c}

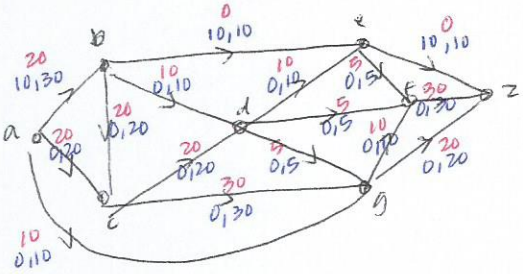
3.



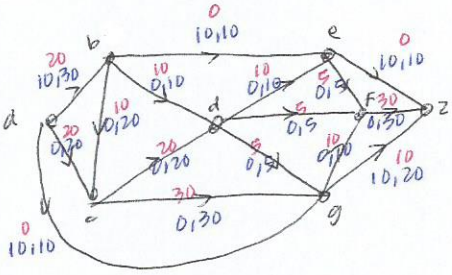
start with flow=0



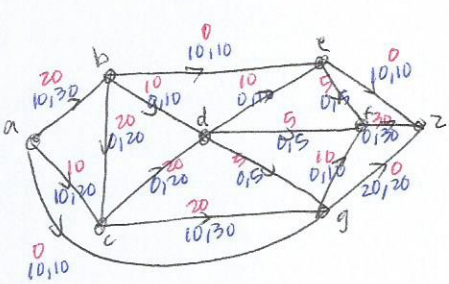
aug path: a→b→e→z
min. residual: 10
flow: 10



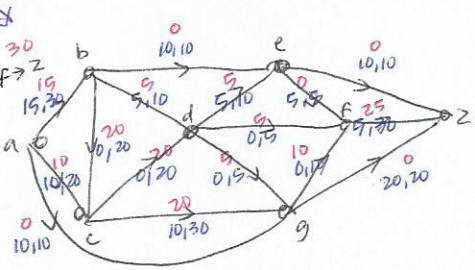
aug path: a→g→z
min. res: 10
flow: 10



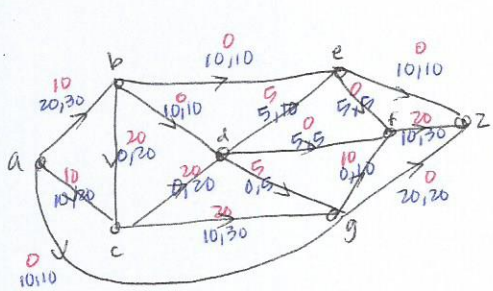
aug path: a→c→g→z
min. res: 10
flow: 10



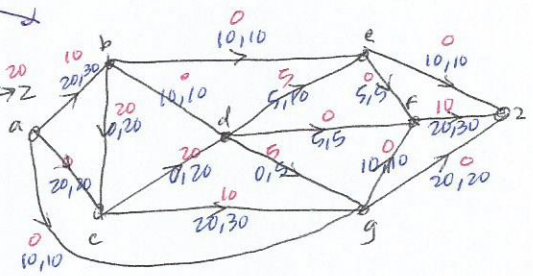
aug path: a→b→d→e→f→z
min res: 5
Flow: 5



aug path: a→b→d→f→z
min res: 5
flow: 5

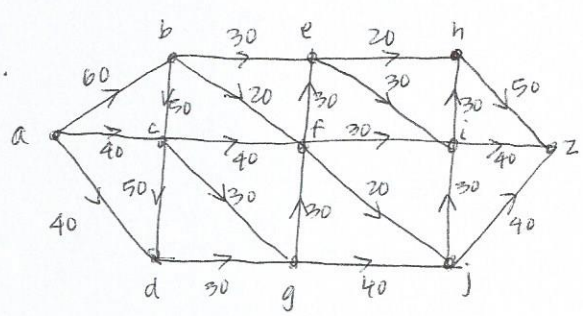


aug path: a→c→g→f→z
min. res.: 10
flow: 10

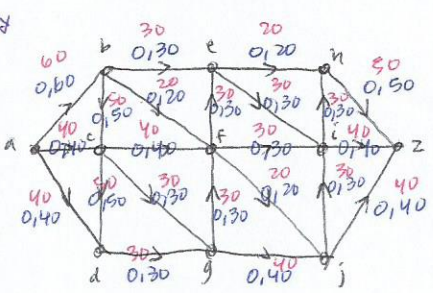


max. flow = 10+10+10+5+5+10
= 50
min. cut = 50
{f, z}

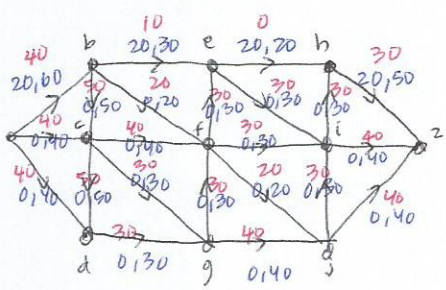
6.



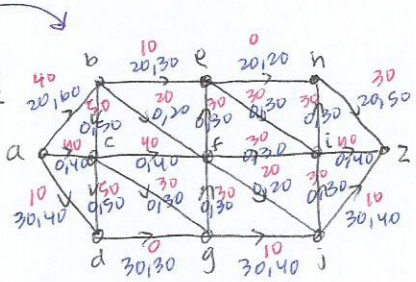
start with flow=0



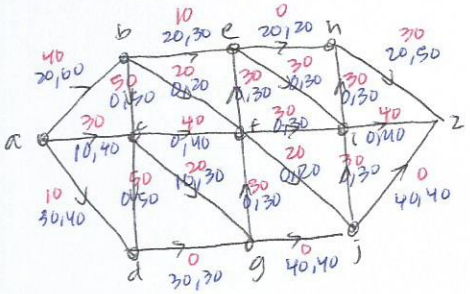
aug path: a→b→e→h→z
min. res: 20
flow: 20



aug path: a→d→g→j→z
min. res: 30
flow: 30



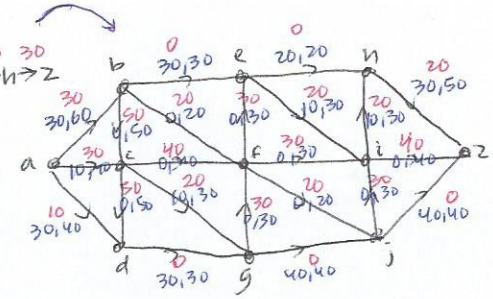
aug path: a→c→g→j→z
min. res: 10
flow: 10



aug. path: a→b→e→i→h→z (40 10 30 30 30)

min. res: 10

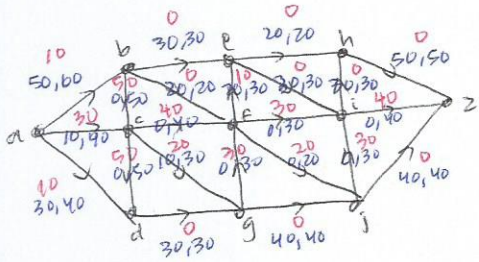
flow: 10



aug. path: a→b→f→e→i→h→z (30 20 30 20 20 20)

min res: 20

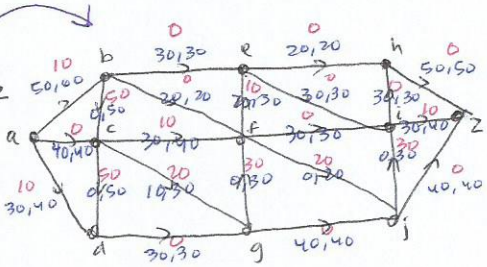
flow: 20



aug. path: a→c→f→i→z (30 40 30 40)

min. res: 30

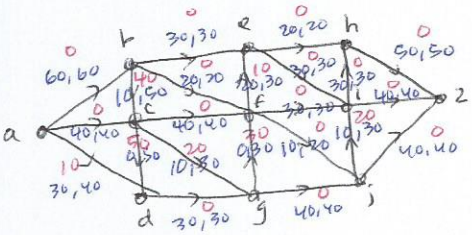
flow: 30



aug. path: a→b→c→f→j→i→z (10 50 10 20 30 10)

min res: 10

flow: 10



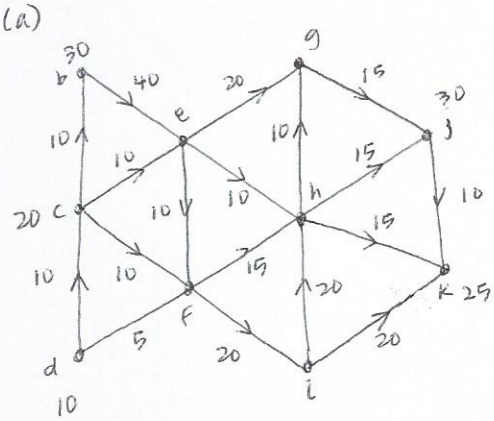
max. flow = 20 + 30 + 10 + 10 + 20 + 30 + 10

= 130

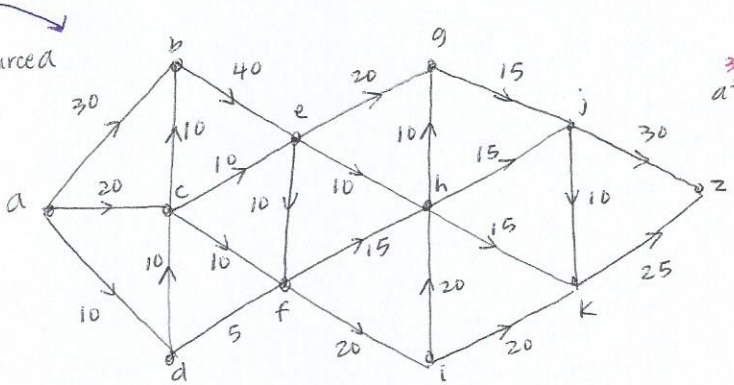
Yes, there is a flow meeting the demands in the figure.

The edges coming into z has a flow value of 130, and are saturated.

8. (a)

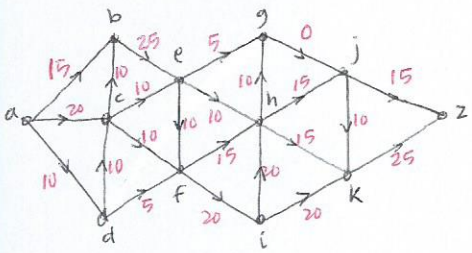


add source a and sink z



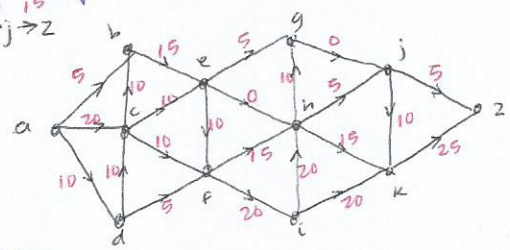
a→b→e→g→j→z (30 40 20 15 30)

flow = 15



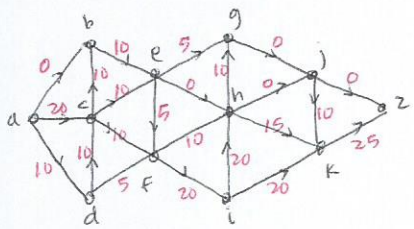
a→b→e→h→j→z (15 25 10 15 15)

flow = 10



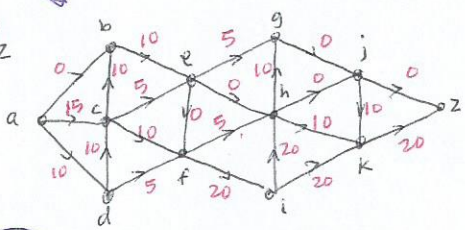
a→b→e→f→h→j→z (5 15 10 15 5 5)

flow = 5



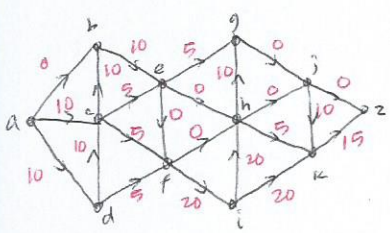
a→c→e→f→h→k→z (20 10 5 10 15 25)

flow = 5



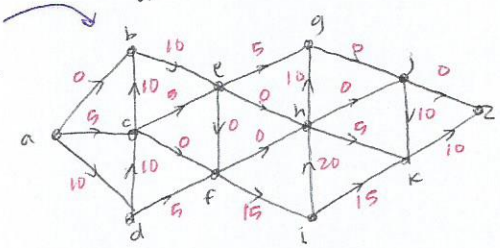
a→c→f→h→k→z (15 10 5 10 20)

flow = 5



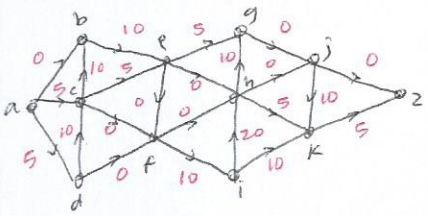
a→c→f→i→k→z (10 5 20 20 15)

flow = 5



a→d→f→i→k→z (10 5 15 15 10)

flow = 5

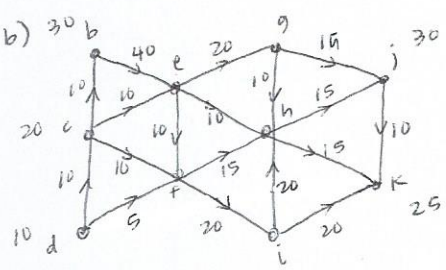


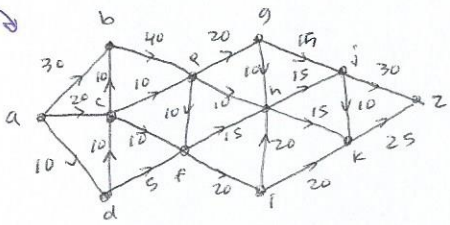
There is no flow satisfying the demands.

max. flow = 15 + 10 + 5 + 5 + 5 + 5 = 50

we wanted 55.

(b)

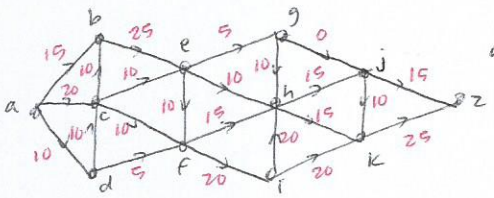




30 40 20 15 30

a → b → e → g → j → z

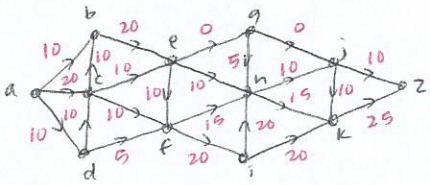
flow = 15



15 25 5 10 15 15

a → b → e → g → h → j → z

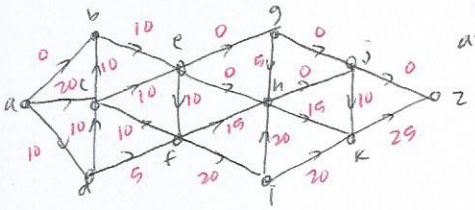
flow = 5



10 20 10 10 10

a → b → e → h → j → z

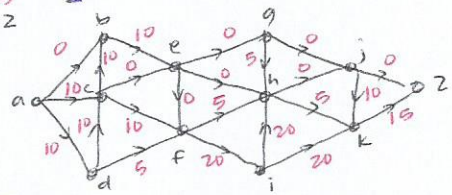
flow = 10



20 10 10 15 15 25

a → c → e → f → h → k → z

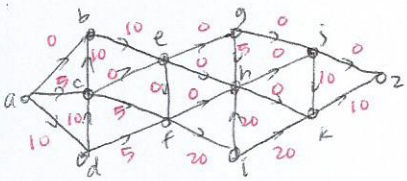
flow = 10



10 10 5 5 15

a → c → f → h → k → z

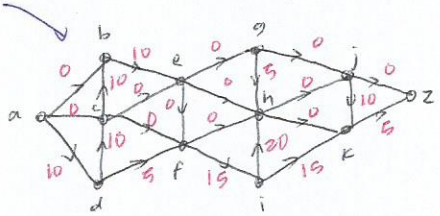
flow = 5



5 5 20 20 10

a → c → f → i → k → z

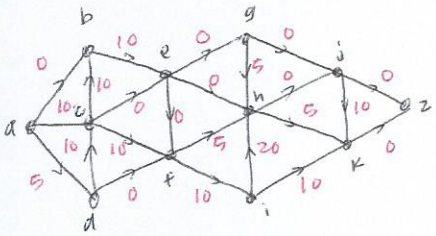
flow = 5



10 5 15 15 5

a → d → f → i → k → z

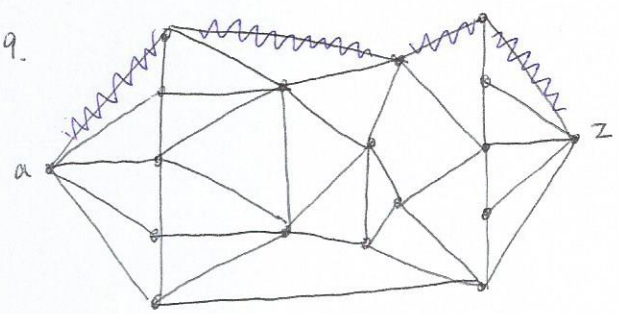
flow = 5

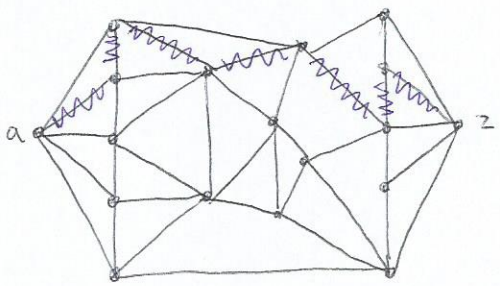


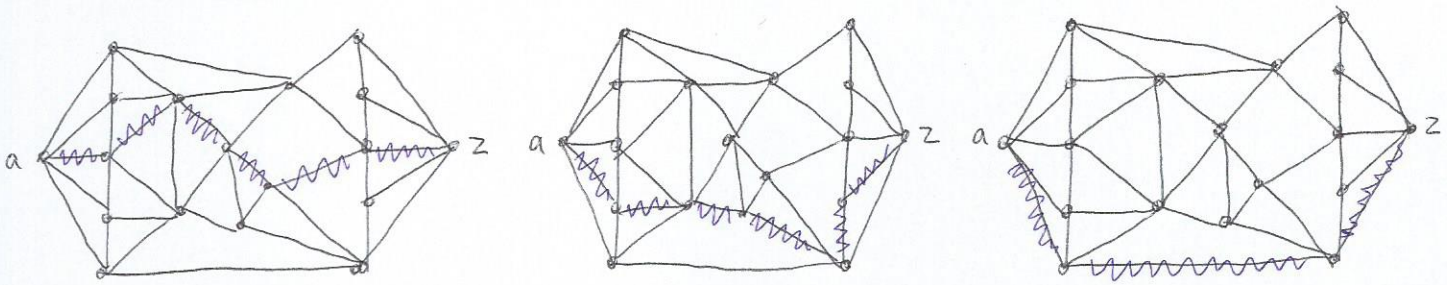
max.
flow = 15 + 5 + 10 + 10 + 5 + 5 + 5

This flow satisfies the demands.

9.

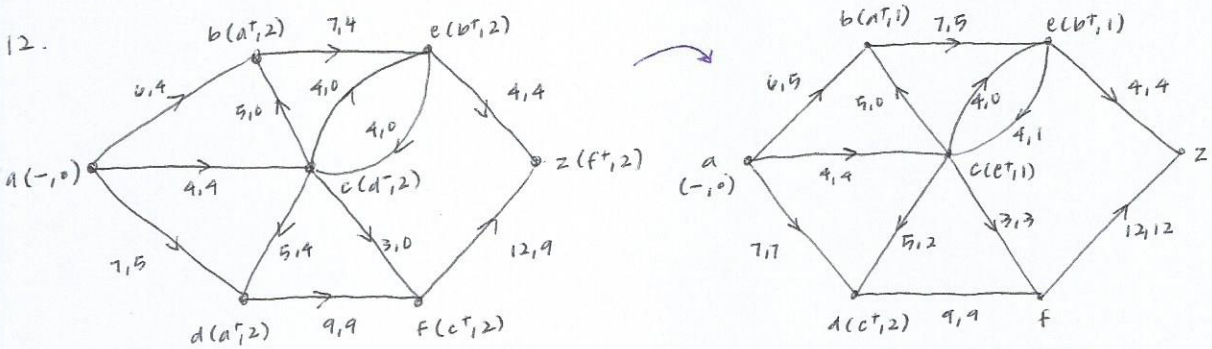


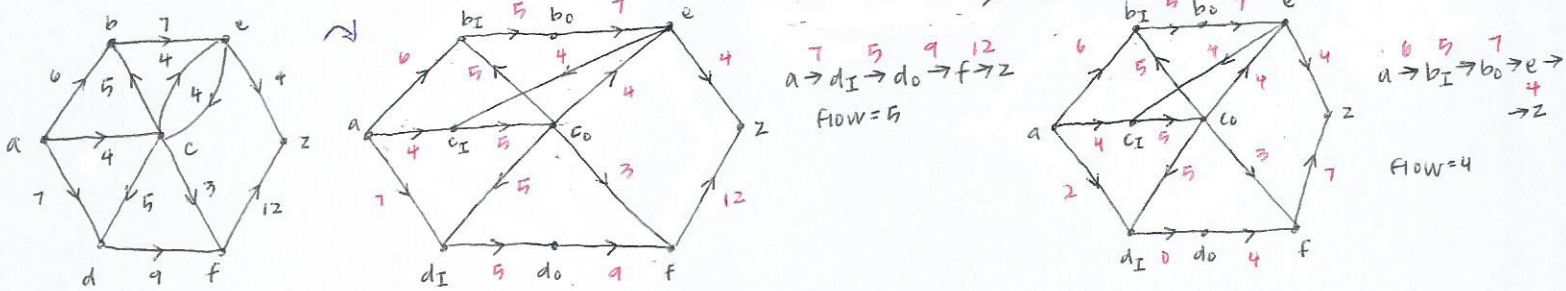


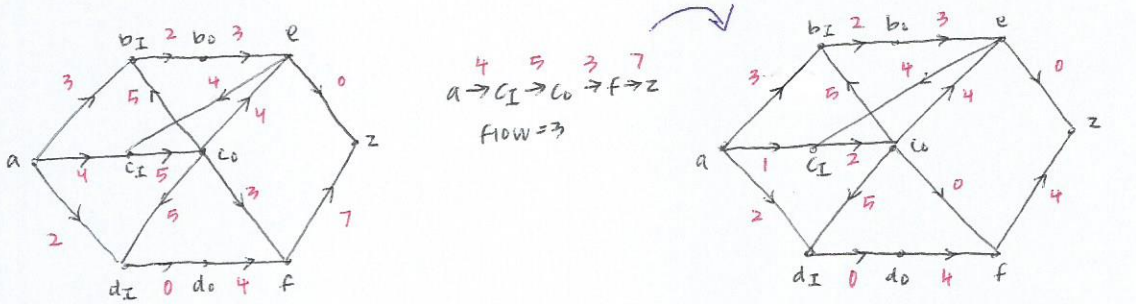


There are 5 messengers that can be sent. (There are 5 edge-disjoint paths.)

12.





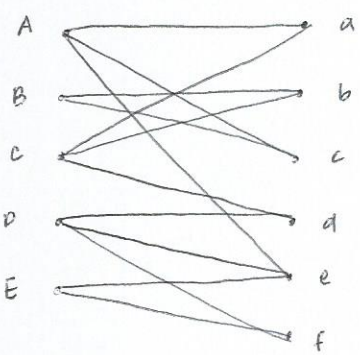


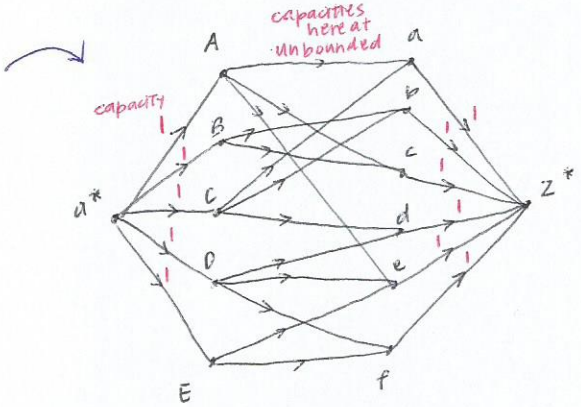
max flow = 5 + 4 + 3
= 12

21. So vertex a is the source and will not have any incoming edges. Vertex a will only have outgoing edges. (Skip 2a) So the next vertex we scan must be an outgoing edge of vertex a. We repeat Step 2b for all the outgoing edges of vertex a. This will form a tree rooted at vertex a.

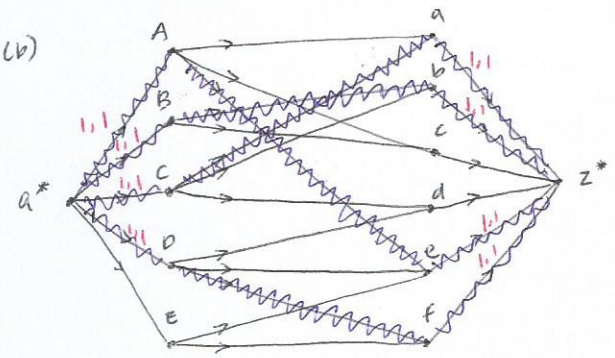
Chapter 4.4

2. (a)

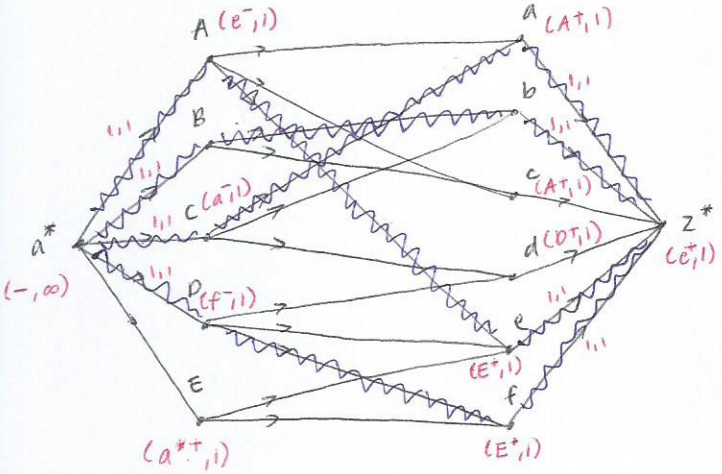




(b)

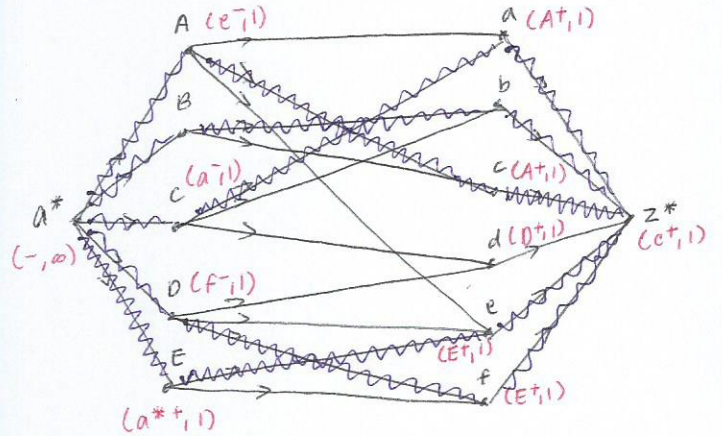


committee E does have a representative to send because the only members in committee E are members e and f, and they are already representing another committee.



order of labeling vertices:

a*	(−,∞)
E	(a*⁺,1)
e	(E⁺,1)
f	(E⁺,1)
A	(e⁻,1)
D	(f⁻,1)
a	(A⁺,1)
c	(A⁺,1)
d	(D⁺,1)
C	(a⁻,1)
z*	(c⁺,1)

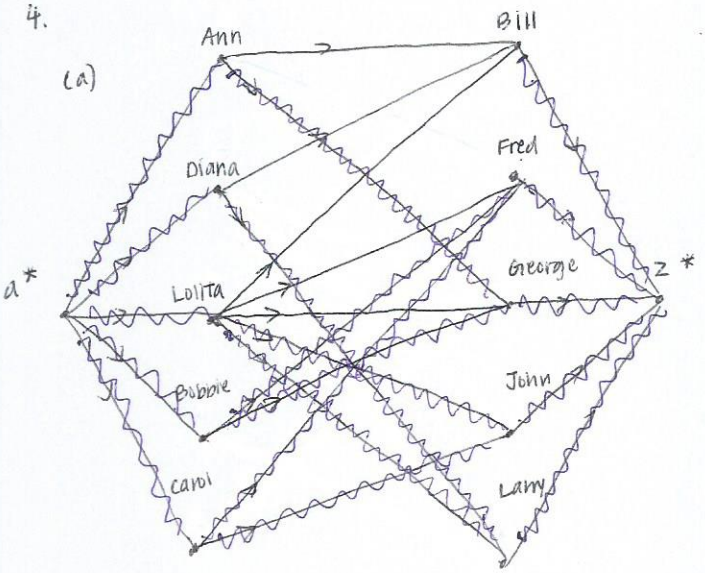


new matching:

A − c
B − b
C − a
D − f
E − e

4.

(a)



1) Larry only has 2 girls that like him, and since we want each boy to be paired twice, Larry will pair with
2) Lolita and Diana.
John also only has 2 girls that like him, so John will pair with Lolita and Carol.
3) We only want each girl to be paired twice. Lolita has now been paired twice, so she cannot pair with any other boy now.
4) Now that no other boy can pair with Lolita, there are only 2 other girls that like George. So, George will pair with Ann and Bobbie.
5) No other boy can pair with Lolita, so only 2 other girls like Fred. Fred will pair with Bobbie and Carol.
6) Bobbie and Carol have both been paired twice now, so no other boy can pair with them now.
7) Bill will pair with the 2 girls that like him: Ann and Diana.
8) Now, each boy and each girl has been paired twice.

final pairings:

check each boy paired twice:

Larry - Lolita, Diana
John - Lolita, Carol
George - Ann, Bobbie
Fred - Bobbie, Carol
Bill - Ann, Diana

Check each girl paired twice:

Ann - George, Bill
Diana - Larry, Bill
Lolita - Larry, John
Bobbie - George, Fred
Carol - John, Fred

5. No, not all the Ph.D.s will get a job.
Some of the colleges want to hire seven Ph.D.s, but will not hire more than one Ph.D. from any university.
There are only six universities though.

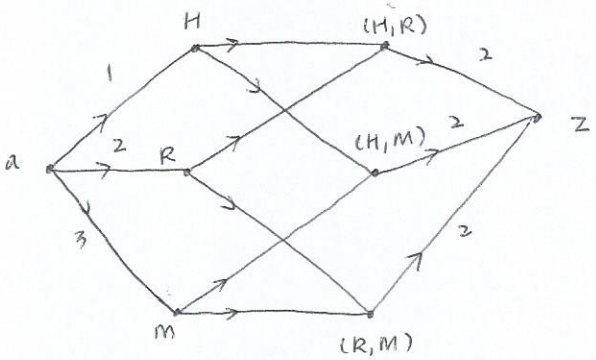
8. Assuming that Vikings win all remaining games:

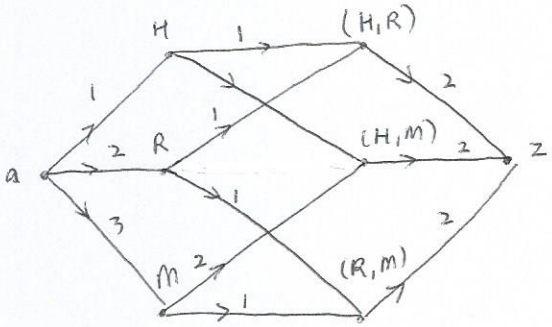
Vikings:
22 + 6
= 28
wins

Huns:
28 - 27
= 1 possible win in remaining games

Romans:
28 - 26
= 2 possible wins in remaining games

Mongols:
28 - 25
= 3 possible wins in remaining games



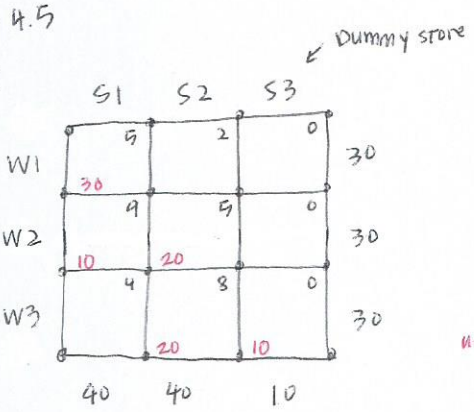


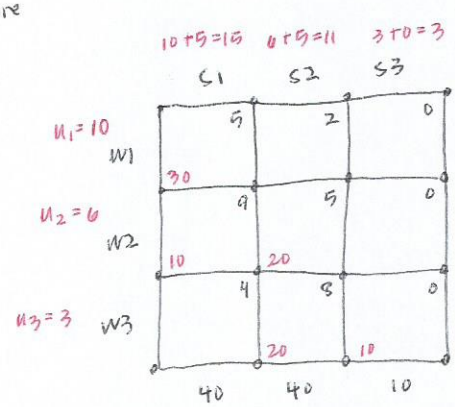
IF Huns win against Romans 1 time, (Huns win 1 game)
Romans win against Huns 1 time,
and against Mongols 1 time, (Romans win 2 games)
Mongols win against Huns 2 times
and against Romans 1 time, (Mongols win 3 games)

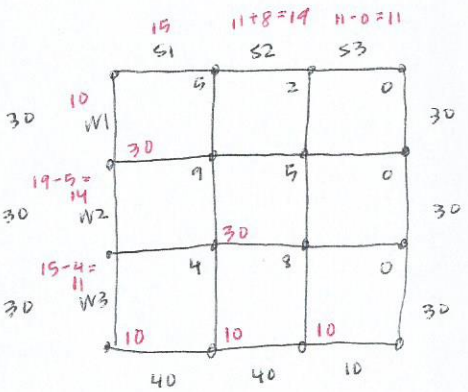
then Vikings will be champion (sole).

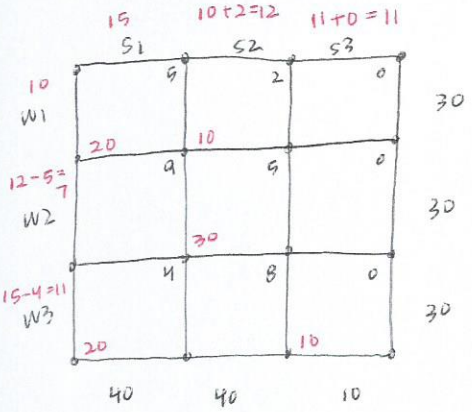
Chapter 4.5

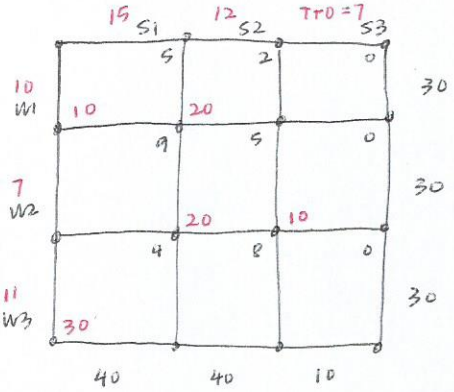
4. (a)



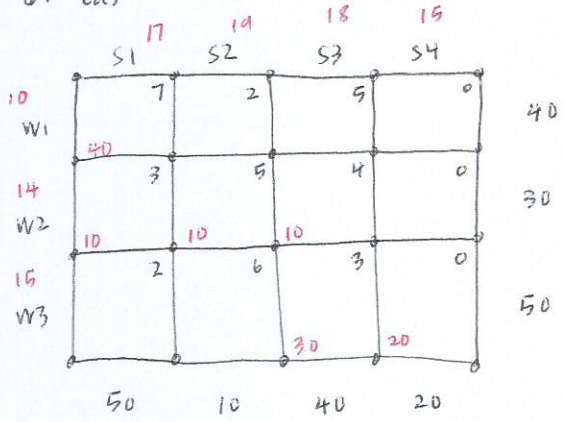




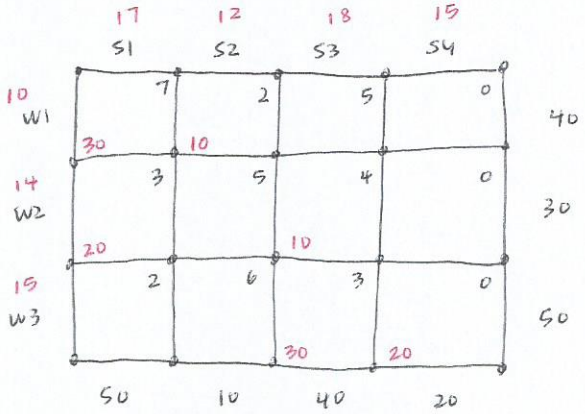




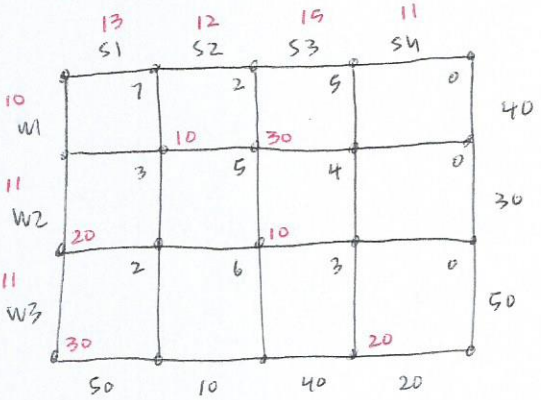
6. (a)



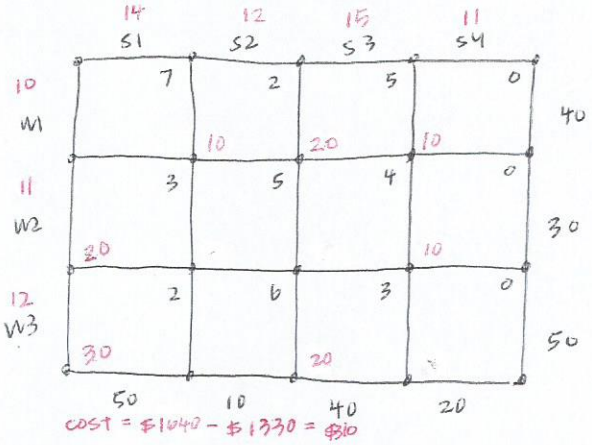
Transportation costs = $2060 − $1570 = $490



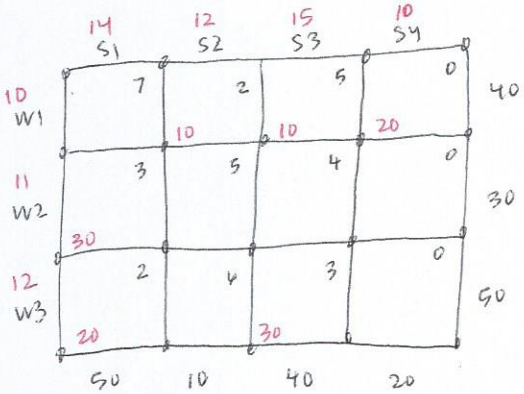
cost = $1990 − $1570 = $420



cost = $1590 − $1280 = $310



cost = $1640 − $1330 = $310



cost = $1620 − $1330 = $290