Problem 2.1

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Homework2

$$y' = (xy' + y)\, y^{1985}$$

$$y' = xy^{1985} y' + y^{1986}$$

$$y' - xy^{1985} y' = y^{1986}$$

$$y'(1 - xy^{1985}) = y^{1986}$$

$$y' = \frac{y^{1986}}{1 - xy^{1985}}$$

$$\frac{dy}{dx} = \frac{y^{1986}}{1 - xy^{1985}}$$

$$y^{1986}\, dx = (1 - xy^{1985})\, dy$$

$$y^{1986}\, dx - (1 - xy^{1985})\, dy = 0$$

$$M(x,y) = y^{1986}$$
$$N(x,y) = -1 + xy^{1985}$$

$$\frac{\partial M}{\partial y} = 1986\, y^{1985}$$

$$\frac{\partial N}{\partial x} = y^{1985}$$

NOT EXACT. $M_y \neq N_x$

Check if $\frac{M_y - N_x}{N} = f(x)$ is a function of purely x, or $\frac{M_y - N_x}{M} = g(y)$ is a function of purely y

$\frac{1986y^{1985} - y^{1985}}{-1 + xy^{1985}}$ is not a function of purely x

$\frac{1986y^{1985} - y^{1985}}{y^{1986}} = \frac{y^{1985}(1986-1)}{y^{1986}} = \frac{1985}{y}$ is a function of purely y

$$\mu(y) = e^{-\int g(y)\, dy}$$
$$= e^{-\int \frac{1985}{y}\, dy}$$
$$= e^{-1985 \ln y}$$
$$= y^{-1985}$$
$$= \frac{1}{y^{1985}}$$

$$\frac{1}{y^{1985}} \left(y^{1986}\, dx - (1 - xy^{1985})\, dy = 0 \right)$$

$$y\, dx - \left(\frac{1}{y^{1985}} - x \right) dy = 0$$

$$M(x,y) = y$$
$$N(x,y) = -\frac{1}{y^{1985}} + x$$

$$\frac{\partial M}{\partial y} = 1$$

$$\frac{\partial N}{\partial x} = 1$$

$M_y = N_x$ EXACT DE.

$$F(x,y) = \int N\, dy = \int -\frac{1}{y^{1985}} + x \; dy = \frac{1}{1984y^{1984}} + xy + g(x)$$

$$\frac{\partial F}{\partial x} = M(x,y)$$

$$\frac{\partial F}{\partial x} = y + g'(x)$$

$$y + g'(x) = y$$

$$g'(x) = 0$$

$$\int g'(x) = \int 0 \, dx$$

$$g(x) = C$$

$$F(x,y) = \frac{1}{1984y^{1984}} + xy + g(x)$$

$$= \frac{1}{1984y^{1984}} + xy + C$$

$$\frac{1}{1984y^{1984}} + xy = C$$

Problem 2.2

$$xy'' + 3y' = 1984x^{1982}$$

$$u = y'$$

$$u' = y''$$

$$xu' + 3u = 1984x^{1982}$$

$$u' + \frac{3}{x}u = 1984x^{1981}$$

$$P(x) = \frac{3}{x}$$

$$Q(x) = 1984x^{1981}$$

$$\rho(x) = e^{\int P(x)dx} = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$

$$x^3(u' + \frac{3}{x}u = 1984x^{1981})$$

$$x^3u' + 3x^2u = 1984x^{1984}$$

$$(x^3u)' = 1984x^{1984}$$

$$\int (x^3u)' = \int 1984x^{1984}\,dx$$

$$x^3u = \frac{1984x^{1985}}{1985} + c$$

$$u = \frac{1984x^{1982}}{1985} + \frac{c}{x^3}$$

$$y' = \frac{1984x^{1982}}{1985} + \frac{c}{x^3}$$

$$\int y' = \int \frac{1984x^{1982}}{1985} + \frac{c}{x^3}\,dx$$

$$y = \frac{1984}{1985 \cdot 1983}x^{1983} - \frac{c}{2x^2} + c_1$$

$$y = \frac{1984}{3936255}x^{1983} - \frac{c}{2x^2} + c_1$$

Problem 2.3

$$xy' - y = x^{2017}y^2$$

$$y' - \frac{1}{x}y = x^{2016}y^2$$

$$P(x) = -\frac{1}{x}$$

$$Q(x) = x^{2016}$$

$$n = 2$$

B-sub: $v = y^{1-n}$

$$v = y^{1-2}$$

$$v = y^{-1}$$

$$\frac{1}{1-n}v' + P(x)v = Q(x)$$

$$\frac{1}{1-2}v' - \frac{1}{x}v = x^{2016}$$

$$-v' - \frac{1}{x}v = x^{2016}$$

$$v' + \frac{1}{x}v = -x^{2016}$$

$$P(x) = \frac{1}{x}$$

$$Q(x) = -x^{2016}$$

$$\rho(x) = e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

$$x\left(v' + \frac{1}{x}v = -x^{2016}\right)$$

$$xv' + v = -x^{2017}$$

$$(xv)' = -x^{2017}$$

$$\int (xv)' = \int -x^{2017}dx$$

$$xv = -\frac{x^{2018}}{2018} + c$$

$$v = -\frac{x^{2017}}{2018} + \frac{c}{x}$$

$$y^{-1} = -\frac{x^{2017}}{2018} + \frac{c}{x}$$

$$\frac{1}{y} = \frac{-x^{2018} + 2018c}{2018x}$$

$$y = -\frac{2018x}{x^{2018} - 2018c}$$

Problem 2.4

$$xy' + y(1+\ln(xy)) = 0$$

$$xy' = -y(1+\ln(xy))$$

$$y' = -\frac{y(1+\ln(xy))}{x}$$

$$\frac{dy}{dx} = -\frac{y(1+\ln(xy))}{x}$$

$$-y(1+\ln(xy))\,dx = -x\,dy$$

$$y(1+\ln(xy))\,dx + x\,dy = 0$$

$$M(x,y) = y(1+\ln(xy))$$

$$N(x,y) = x$$

Check if exact.

$$\frac{\partial M}{\partial y} = \ln(xy) + 2$$

$$\frac{\partial N}{\partial x} = 1$$

NOT EXACT. $M_y \neq N_x$

Check if $\frac{M_y - N_x}{N} = f(x)$ is a function of purely x or, $\frac{M_y - N_x}{M} = g(y)$ is a function of purely y

$\frac{\ln(xy)+2-1}{x} = \frac{\ln(xy)+1}{x}$ is not a function of purely x $\quad$ $\frac{\ln(xy)+2-1}{y(1+\ln(xy))} = \frac{\ln(xy)+1}{y(1+\ln(xy))} = \frac{1}{y}$ is a function of purely y

$$\rho(y) = e^{-\int g(y)\,dy}$$

$$= e^{-\int \frac{\ln(xy)+1}{y(1+\ln(xy))}\,dy}$$

$$= e^{-\int \frac{1}{y}\,dy}$$

$$= e^{-\ln y}$$

$$= y^{-1}$$

$$= \frac{1}{y}$$

$$\frac{1}{y}\left(y(1+\ln(xy))\,dx + x\,dy = 0\right)$$

$$(1+\ln(xy))\,dx + \frac{x}{y}\,dy = 0$$

$$M(x,y) = 1+\ln(xy)$$

$$N(x,y) = \frac{x}{y}$$

$$\frac{\partial M}{\partial y} = \frac{1}{y}$$

$$\frac{\partial N}{\partial x} = \frac{1}{y}$$

$M_y = N_x$ Exact DE.

Exact DE method.

$$F(x,y) = \int N\,dy = \int \frac{x}{y}\,dy = x\ln y + g(x)$$

$$\frac{\partial F}{\partial x} = M(x,y)$$

$$\frac{\partial F}{\partial x} = \ln y + g'(x)$$

$$\ln y + g'(x) = 1 + \ln(xy)$$

$$\ln y + g'(x) = 1 + \ln x + \ln y$$

$$g'(x) = 1 + \ln x$$

$$\int g'(x) = \int 1 + \ln x \, dx$$

$$g(x) = x + x\ln x - x + C$$

$$g(x) = x\ln x + C$$

$$F(x,y) = x\ln y + g(x)$$

$$= x\ln y + x\ln x + C$$

$$x\ln y + x\ln x = C$$