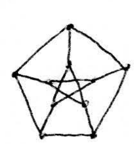
Reina Li
110909646

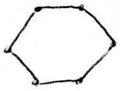
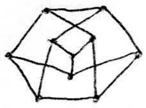
Chapter 1

15. If G is a simple graph with at least two vertices, prove that G must contain two or more vertices of the same degree.

Let G have n vertices. If G has n vertices and the degrees of the n vertices are different, then the max. degree is $(n-1)$. The possible degrees will be $0, 1, 2, \dots, n-1$. That is n different values. But if a vertex has degree 0, then the max. degree is $(n-2)$. We cannot have a vertex with 0 because it must be connected to no other vertices, nor can we have a vertex of degree $(n-1)$ where it is connected to all other vertices.

16. Which graphs are subgraphs?

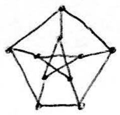
 is a subgraph of 

 and  are subgraphs of 

29. If G has n vertices and is regular of degree r, how many edges has G? Use your answer to check the number of edges in the Petersen graph.

$$e = \frac{rn}{2}$$

Petersen graph:

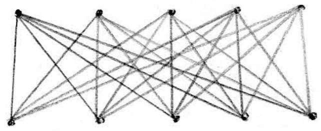


$r = 3$
$n = 10$
$e = 15$

$$e = \frac{rn}{2} = \frac{3 \cdot 10}{2} = \frac{30}{2} = 15$$

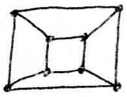
31. Give an example (if it exists) of each of the following:

(i) a bipartite graph that is regular of degree 5



$K_{5,5}$

(ii) a bipartite Platonic graph



cube (bipartite because graph can be 2-colored)

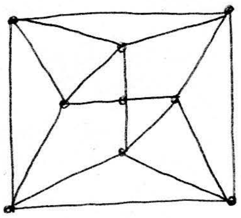
(iii) a complete graph that is a wheel

does not exist

(iv) a cubic graph with 11 vertices

does not exist, one vertice will have degree 2

(v) a graph (other than K_5, $K_{4,4}$, or Q_4) that is regular of degree 4.

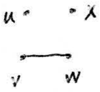


Chapter 2

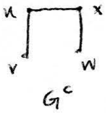
3. Write down the girths of:

(i) $K_9 = 3$
(ii) $K_{5,7} = 4$
(iii) $C_8 = 8$
(iv) $W_8 = 3$
(vi) the Petersen graph $= 5$
(vii) the graph of the dodecahedron $= 5$

5. Prove that a simple graph G and its complement cannot both be disconnected.



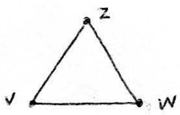
G



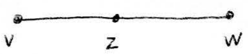
G^c

In G, G is disconnected. There is no edge between wx in G, so there is an edge between wx in G^c. There is an edge between vw in G, and no edge between them in G^c. If we have a vertex u that is not connected to vertices v, w, x in G, then there will be a path in G^c. G^c is connected.

9. (i) If $d(v,w) \geq 2$, show that there exists a vertex z such that $d(v,z) + d(z,w) = d(v,w)$.



The path P_{vw} is the shortest path between vertices v and w.



If we consider vertex z on path P_{vw}, then $d(v,w) = d(v,z) + d(z,w)$.

Applied Comb.
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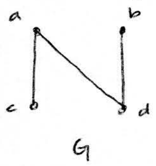
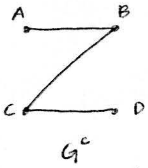
33. (a) Find a graph that is isomorphic to its own complement.



G



G^c

$n = 4$
$m = 3$

$\deg(a) = 2$
$\deg(b) = 1$
$\deg(c) = 1$
$\deg(d) = 2$

$\deg(A) = 1$
$\deg(B) = 2$
$\deg(C) = 2$
$\deg(D) = 1$

(b) Show that any self-complementary graph must have either $4k$ or $4k+1$ vertices, for some integer k.

The # of edges in G is equal to the # of edges in G^c.

$$m_G = m_{G^c}$$

$$m_{K_n} = \frac{n(n-1)}{2}$$

2 consecutive numbers cannot be even, so $m_G = m_{G^c} = \frac{n(n-1)}{4}$

So, 4 has to divide n or $n-1$.