$$\left(x \frac{d}{dx} - 1\right)^{2020}, y(x) = 0$$

$$\left(\exp(t)\frac{d}{\exp(t)dt}-1\right)^{2020}y=0$$

$$\left(\frac{d}{dt} - 1\right)^{2020} y = 0$$

$$(r-1)^{2020}=0$$

There are 2020 identical real roots.

$$y_{c(t)} = (c_{1} + c_{2}t + c_{3}t^{2} + ... + c_{2020}t^{2019}) \exp(t)$$

$$= (\sum_{k=0}^{2020} c_{k}t^{k-1}) \exp(t)$$

$$yp = At^{2020} \exp(t)$$

$$= \left(\frac{d}{dt} - 1\right)^{2019} \left(\frac{d}{dt} - 1\right) \left(At^{2020} \exp(t)\right)$$

$$= \left(\frac{d}{dt} - 1\right)^{2019} \left(\frac{d}{dt} \left(At^{2020} \exp(t)\right) - At^{2020} \exp(t)\right)$$

=
$$\left(\frac{d}{dt}-1\right)^{2019}$$
 (2020 At 2019 exp(t))

=
$$\left(\frac{d}{dt} - 1\right)^{2018} \left(\frac{d}{dt} - 1\right) \left(2020 \text{ At}^{2019} \exp(t)\right)$$

$$\left(\frac{d}{at}-1\right)^{2020}\left(At^{2020}\exp(t)\right)=\exp(t)$$

$$A(2020!) \exp(t) = \exp(t)$$

$$\Rightarrow A = \bot$$

GS:
$$Y(t) = \exp(t) \left(C_1 + C_2 t + C_3 t^2 + ... + C_2 t^{2019} \right) + \frac{1}{2020} t^{2020} \exp(t)$$

 $Y(x) = x \left(C_1 + C_2 \ln x + C_3 \ln^2 x + ... + C_{2020} \left(\ln x \right)^{2019} \right) + \frac{1}{2020} \left(\ln x \right)^{2020} X$

```
Problem 4.2
      y "+ y" + y" + y = 1 + cosx + sinx + e x + e x
 y 111 + y"+ y' + y = 0
 13+12+1+1=0
 r=(r+1)+(r+1)=0
 (r2+1) (r+1)=0
 r= ±i, -1
 YC = C1 exp(-x) + C2 exp(ix) + C3 exp(-ix)
 YP A+ x (BCOSX + CSINX) + D COSX + ESINX + FX EXP(-X) + G exp(x)
 Yp = x (-Beinx + Ccosx) + BCOSX + Csinx - DEMX + ECOSX - FX exp(-x) + Fexp(-x) + Gexp(x)
 YP = x (-BCOSX-CSINX) - BSINX+CCOSX - BSINX+CCOSX - DCOSX - ESINX + FX EXP(-X) - FEXP(-X) - FEXP(-X) + GEXP(X)
 Yp = x (Bsinx - Ccosx) - Bcosx - Csinx - Bcosx - Csinx - Bcosx - Csinx + Dsinx - Gcosx - Fxexp(-x) + Fexp(-x) + Fexp(-x)
A+ x (BCOSX+ CSINX) + DCOSX+ ESINX + FXEXPL-X) + GEXP(X) + X (-BSINX+ CCOSX) + BCOSX + CSINX - DSINX+ ECOSX - FXEXP(-X) +
FEXPL-X) + GEXPLX) + X (-BCOSX - CSINX) - BSINX + CLOSA - BSINX + CLOSA - DCOSA - ESINX + FX EXPL-X) - FEXPL-X) - FEXPL-X) -
Gexplx) + x (BSINX - CCOSX) -BCOSX - CSINX - BCOSX - CSINX - BCOSX - CSINX + DSINX - ECOSX - FX exp(-x) + Fexp(-x) +
Fexp(-x) + Fexp(-x) + Gexp(x) = 1 + cosx + sinx +e-x +ex
   A=1
```

$$A = 1$$

$$D + B + E + C + C - D - B - B - E + Bx + Cx^{2} - Bx - Cx = 1$$

$$-2B + 2C = 1$$

$$E + C - D - B - B - E - C - C - C + D + Cx - Bx - Cx + Bx = 1$$

$$-2B - 2C = 1$$

$$B = -\frac{1}{2}$$

$$C = 0$$

$$Fx - Fx + F + Fx - F - F - Fx + F + F + F = 1$$

$$2F = 1$$

$$F = \frac{1}{2}$$

$$4G = 1$$

$$G = \frac{1}{4}$$

GS: C1 exp(-x) + C2 exp(ix) + C3 exp(-ix) +1+ x (-1/2 cosx) + 1/2 x exp(-x) + 1/4 exp(x)

```
Problem 6.3
              x2y" + xy + 192y = cosciq inx) + sin(19 inx)
    4(X) = X r
    y'(x) = Y x Y-1 .
     y"(x) = r(r-1) x"-2
                    x2 r(r-1) x -2 + x r x -1 + 192 x = 0
                            x ( ( 2 - r ) + x + x + x + 192 = 0
                           xr (r2-r+r+192)=0
                                xr(r2+192)=0
                                      r = ± 19 i
     Yc = CIEXPLIAIX) + CZEXP (-1aix)
        YP = x (Acos(1911x) + BSIN(1911x))
        \gamma \dot{p} = x \left( -A \sin(19 \ln x) \left( \frac{19}{x} \right) + B \cos(19 \ln x) \left( \frac{19}{x} \right) \right) + \left( A \cos(19 \ln x) + B \sin(19 \ln x) \right)
                                X (- 19 A sin (1911x) + 19 B cos (1911x)) + (A cos (1911x) + B sin (1911x))
       y_p^{\mu} = x \left( x \left( -\frac{19}{4} \cos \left( \frac{19}{10} \ln x \right) \left( \frac{\frac{19}{x}}{x} \right) \right) + \left( \frac{19}{4} \sin \left( \frac{19}{10} \ln x \right) \right) + \left( \frac{19}{x} \cos \left( \frac{19}{x} \right) \right) + \left( \frac{19}{10} \cos \left( \frac{19}{x} \right) \right) + \left( \frac{19}{x} \cos \left(
                                 \left(\frac{-iq A \sin(iq in x)}{x} + iq \frac{B \cos(iq in x)}{x}\right) + \left(\frac{-A \sin(iq in x)}{x}\right) + B \cos(iq in x)\left(\frac{iq}{x}\right)
                     = \times \left( -19^{2} \text{ Accs (19 In x)} + (19 \text{ A sin (19 In x)}) + -19^{2} \text{B sin (19 In x)} + 19 \text{ B cos (19 In x)} \right) + \times 2
                        ( -19 A sincia inx) + 19 BCOS (19 Inx) + (-19 A SINCIA INX) + 19 BCOS (19 INX)
                               -192 Acos (1911) + 19 Asin (1911) - 192 BSin (1911x) + 19 Bcos (1911x) - 38 Asin (1911x) + 38 Bcos (1911x)
                            -192Acosciainx) -19Asinciainx) -192Bsinciainx) +57Bcosciainx)
   x2y11 + xy1+192y = x E192Acoscia inx)-19Asincia inx)-192Bsinciainx) + 57Bcosciainx)) + x (-19Asinciainx)+19Bcosciainx))+
                                                             x (Acosciainx) + Beinciainx)) + 192x (Acosciainx) + Beinciainx))
                                                           = -192 x A cos (19 lnx) - 19 x A sin (19 lnx) - 192 x B sin (19 lnx) + 57 x B cos (19 lnx) - 19 x A sin (19 lnx) +
                                                            19xBcosliginx) + xAcosliginx) + xBsiniiginx) + 192xAcosliginx) + 192xBsiniiginx)
                                                            = -38 x A sin(19 10x) + x Bsin(19 10x) + 76 x Bcos(1910x) + x Acos(1910x)
                                                                    -38A+B=1
                                                                        76B + A = 1
                                                                     A = \frac{-25}{903}
B = \frac{39}{903}
                               y(x) = C1 exp(19ix) + C2 exp(-19ix) + x (-25/963 cos (19 10x) + 39/963 sin (19 10x))
```

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Problem 6.4

$$y'' + y = tan(19x)$$
 $y''' + y = tan(19x)$
 $y''' + y = 0$
 $r^2 + 1 = 0$
 $r = 1i$
 $y_1 = cos(x)$
 $y_2 = sin(x)$
 $y_1'' = -sin(x)$
 $y_1'' = -sin(x)$
 $y_2'' = cos(x)$
 $y_1'' + y_2' + y_2' = 0$
 $y_1'' + y_2' + y_2' = f(x)$
 $y_2'' + y_2' + y_2' = f(x)$
 $y_2'' + y_2'' + y_$

$$y_P(x) = \int_{-\infty}^{\infty} K(x,t) + (t) dt$$

$$= \int_{-\infty}^{\infty} \sin(x-t) \tan(|at|) dt$$

$$= -\cos x \ln |\sec x + \tan x|$$