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Homework 1

Problem 1.1

$y'' + y' = \sin 20x$

$y_1 = \cos x + \sin x$

$y_2 = \cos 20x + \sin x$

$y_3 = \cos x + \sin 20x$

$y_1 = \cos x + \sin x$	$y_2 = \cos 20x + \sin x$	$y_3 = \cos x + \sin 20x$
$y_1' = -\sin x + \cos x$	$y_2' = -20\sin 20x + \cos x$	$y_3' = -\sin x + 20\cos 20x$
$y_1'' = -\cos x - \sin x$	$y_2'' = -400\cos 20x - \sin x$	$y_3'' = -\cos x - 400\sin 20x$

$LHS = y_1'' + y_1'$

$= -\cos x - \sin x + (-\sin x + \cos x)$

$= -2\sin x \neq \sin 20x$

$LHS \neq RHS$

$y_1 = \cos x + \sin x$ is NOT a solution.

$LHS = y_2'' + y_2'$

$= -400\cos 20x - \sin x + (-20\sin 20x + \cos x)$

$= -400\cos 20x - \sin x - 20\sin 20x + \cos x \neq \sin 20x$

$LHS \neq RHS$

$y_2 = \cos 20x + \sin x$ is NOT a solution.

$LHS = y_3'' + y_3'$

$= -\cos x - 400\sin 20x + (-\sin x + 20\cos 20x)$

$= -\cos x - 400\sin 20x - \sin x + 20\cos 20x \neq \sin 20x$

$LHS \neq RHS$

$y_3 = \cos x + \sin 20x$ is NOT a solution.

Problem 1.2

$y' + 9x^8 y = 0$

$y(x) = c \cdot e^{-x^3}$

$y(0) = 1984$

$y' + 9x^8 y = 0$

$y' = -9x^8 y$

$y' = -9x^8 \cdot c \cdot e^{-x^3}$

$y' + 9x^8 y = 0$

$-9x^8 \cdot c \cdot e^{-x^3} + 9x^8 \cdot c \cdot e^{-x^3} = 0$

$9x^3 \cdot e^{-x^3}(-c + c) = 0$

$-c + c = 0$

$y(x)$ satisfies given DE.

$y(0) = 1984 \Rightarrow \quad c \cdot e^{-0^3} = 1984$

$c \cdot e^{-0} = 1984$

$c \cdot 1 = 1984$

$c = 1984$

$y(x) = 1984 \cdot e^{-x^3}$

Problem 1.3

$$\begin{cases} y'\sin x + y\cos x = 0 \\ y(\frac{\pi}{4}) = 1 \end{cases}$$

$(y\sin x)' = y'\sin x + y\cos x$

$(y\sin x)' = 0$

$\int (y\sin x)' = \int 0\, dx$

$y\sin x = C$

$y(x) = \frac{C}{\sin x}$

$y(\frac{\pi}{4}) = 1 \Rightarrow \quad 1 = \frac{C}{\sin\frac{\pi}{4}}$

$1 = \frac{C}{\frac{1}{\sqrt{2}}}$

$C = \frac{1}{\sqrt{2}}$

$y(x) = \frac{\frac{1}{\sqrt{2}}}{\sin x}$

$y(x) = \frac{1}{\sqrt{2}\sin x}$

Problem 1.4

$$\begin{cases} xy' - y = 2020x^2 \\ y(1) = 1 \end{cases}$$

$xy' - y = 2020x^2$

$y' - \frac{1}{x}y = 2020x$

$P(x) = -\frac{1}{x}$

$Q(x) = 2020x$

$\rho(x) = e^{\int P(x)dx}$

$= e^{\int -\frac{1}{x}dx}$

$= e^{-\ln x}$

$= \frac{1}{e^{\ln x}}$

$= \frac{1}{x}$

$\frac{1}{x}(y' - \frac{1}{x}y = 2020x)$

$\frac{1}{x}y' - \frac{1}{x^2}y = 2020$

$(\frac{1}{x}y)' = \frac{1}{x}y' - \frac{1}{x^2}y$

$(\frac{1}{x}y)' = 2020$

$\int (\frac{1}{x}y)' = \int 2020\, dx$

$\frac{1}{x}y = 2020x + C$

$y(1) = 1 \Rightarrow \quad \frac{1}{1}\cdot 1 = 2020\cdot 1 + C$

$1 = 2020 + C$

$C = -2019$

$\frac{1}{x}y = 2020x - 2019$

$y(x) = x(2020x - 2019)$