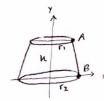
cross-sectional area:

$$A(y) = \pi \Gamma^{2}$$

$$= \pi \left(\frac{r_{1} + r_{2}}{h} y + r_{2} \right)^{2}$$



Torricelli's draining

$$\begin{cases} \frac{A(y)dy}{dt} = -k\sqrt{y} \\ y(t=0) = y_0 \end{cases}$$

$$\frac{A(y)dy}{\sqrt{y}} = -kdt$$

$$\int_{h}^{0} \frac{A(y)dy}{\sqrt{y}} = \int_{0}^{T_{1}} -k dt$$

$$\int_{0}^{0} \pi \left(\frac{r_{1} + r_{2}}{N} y + r_{2} \right)^{2} dy = \int_{0}^{T_{1}} -k dt$$

$$\int_{0}^{0} \pi \left(\left(\frac{r_{1} + r_{2}}{n} y \right)^{2} + r_{2}^{2} + 2r_{2} \left(\frac{r_{1} + r_{2}}{n} \right) y \right) dy = \int_{0}^{T_{1}} -k dt$$

$$\int_{0}^{0} \pi y^{2} \frac{\left(\frac{r_{1}+r_{2}}{n}\right)^{2} + \pi r_{2}^{2} + 2\pi r_{2} \left(\frac{r_{1}+r_{2}}{n}\right) y}{\sqrt{y}} dy = \int_{0}^{T_{1}} -k dt$$

$$\int_{1}^{0} \pi y^{3/2} \left(\frac{r_{1} t r_{2}}{u} \right)^{2} + \pi r_{2}^{2} \sqrt{y} + 2 \pi r_{2} \sqrt{y} \left(\frac{r_{1} t r_{2}}{u} \right) dy = \int_{0}^{T_{1}} -k dt$$

$$\left[\pi\left(\frac{r_{1}+r_{2}}{n}\right)^{2} \stackrel{?}{=} y^{\frac{5}{2}} + \pi r_{2}^{2}\left(2\sqrt{y}\right) + 2\pi r_{2}\left(\frac{r_{1}+r_{2}}{n}\right) \stackrel{?}{=} y^{\frac{3}{2}}\right]_{N}^{0} = -kt \left|_{0}^{T_{1}}\right|_{0}^{T_{2}}$$

$$0 - \left(\pi \left(\frac{r_1 + r_2}{n}\right)^2 - \frac{2}{5} h^{\frac{5}{2}} + \pi r_2^2 2\sqrt{n} + 2\pi r_2 \left(\frac{r_1 + r_2}{n}\right) - \frac{2}{5} h^{\frac{3}{2}} = -T_1 k$$

$$-\pi \left(\frac{r_{1}+r_{2}}{n}\right)^{2} = \frac{1}{5} \ln^{5/2} - \pi r_{2}^{2} 2\sqrt{n} - 2\pi r_{2} \left(\frac{r_{1}+r_{2}}{n}\right)^{\frac{2}{3}} \ln^{3/2} = -T_{1} k$$

$$\pi \left(\frac{r_1 + r_2}{r_1} \right)^2 = \frac{1}{5} \pi \left(\frac{r_1 + r_2}{r_1} \right)^2 h^{\frac{5}{2}} + 2\pi r_2 \sqrt{h} + \frac{4}{5} \pi r_2 h^{\frac{3}{2}} \left(\frac{r_1 + r_2}{h} \right)$$

cross-sectional area:

$$A(y) = \pi r^{2}$$

$$= \pi \left(\left(\frac{r_{2} - r_{1}}{n} \right) y + r_{1} \right)^{2}$$



Torriceili's draining

$$\begin{cases} \frac{A(y)dy}{at} = -\kappa\sqrt{y} \\ y(t=0) = y0 \end{cases}$$

$$\int_{h}^{0} \pi \left(\frac{r_2 - r_1}{h} y + r_1 \right)^2 dy = \int_{0}^{T_2} -k dt$$

$$\int_{0}^{0} \pi \left(\left(\frac{r_{2} - r_{1}}{n} y \right)^{2} + r_{1}^{2} + 2r_{1} y \left(\frac{r_{2} - r_{1}}{n} \right) \right) dy = \int_{0}^{T_{2}} -k dt$$

$$\int_{0}^{0} \pi y^{2} \frac{\left(\frac{r_{2}-r_{1}}{n}\right)^{2} + \pi r_{1}^{2} + 2\pi r_{1} y \left(\frac{r_{2}-r_{1}}{n}\right)}{\sqrt{y}} dy = \int_{0}^{T_{2}} -k dt$$

$$\int_{0}^{0} \pi y^{3/2} \left(\frac{r_{2}-r_{1}}{n}\right)^{2} + \pi r_{1}^{2} \frac{1}{\sqrt{y}} + 2\pi r_{1} \sqrt{y} \left(\frac{r_{2}-r_{1}}{n}\right) dy = \int_{0}^{T_{2}} -k dt$$

$$\left[\pi \left(\frac{r_{2}-r_{1}}{n}\right)^{2} \frac{2}{5} y^{5/2} + \pi r_{1}^{2} 2\sqrt{y} + 2\pi r_{1} \left(\frac{r_{2}-r_{1}}{n}\right) \frac{2}{3} y^{3/2}\right]_{0}^{0} = -kt \left|_{0}^{T_{2}} -k t\right|_{0}^{T_{2}}$$

$$0 - \left(\pi \left(\frac{r_{2}-r_{1}}{n}\right)^{2} \frac{2}{5} h^{\frac{5}{2}} + \pi r_{1}^{2} 2\sqrt{h} + 2\pi r_{1} \left(\frac{r_{2}-r_{1}}{n}\right) \frac{2}{3} h^{3/2}\right) = -T_{2}k$$

$$-\pi \left(\frac{r_{2}-r_{1}}{n}\right)^{2} \frac{2}{5} h^{\frac{5}{2}} - \pi r_{1}^{2} 2\sqrt{h} - 2\pi r_{1} \left(\frac{r_{2}-r_{1}}{n}\right) \frac{2}{3} h^{3/2} = -T_{2}k$$

$$T_{2} = \frac{2}{5} \pi \left(\frac{r_{2}-r_{1}}{n}\right)^{2} h^{5/2} + 2\pi r_{1}^{2} \sqrt{h} + \frac{4}{3} \pi r_{1} h^{3/2} \left(\frac{r_{2}-r_{1}}{n}\right)$$

If
$$r_1 = 0$$
:

$$T_1 = \frac{2}{5}\pi \left(\frac{r_1 + r_2}{n}\right)^2 n^{5/2} + 2\pi r_2^2 \sqrt{n} + \frac{4}{3}\pi r_2 n^{3/2} \left(\frac{r_1 + r_2}{n}\right)$$

$$= \frac{2}{5}\pi \left(\frac{r_2}{n}\right)^2 n^{5/2} + 2\pi r_2^2 \sqrt{n} + \frac{4}{3}\pi r_2 n^{3/2} \left(\frac{r_2}{n}\right)$$

$$= \frac{2}{5}\pi r_2^2 \sqrt{n} + 2\pi r_2^2 \sqrt{n} + \frac{4}{3}\pi r_2^2 \sqrt{n}$$

$$= \frac{5\nu}{19}\pi r_2^2 \sqrt{n}$$

$$T_{2} = \frac{1}{5} \pi \left(\frac{r_{2} - r_{1}}{h} \right)^{2} h^{\frac{9}{2}} + 2\pi r_{1}^{2} \sqrt{h} + \frac{4}{5} \pi r_{1} h^{\frac{3}{2}} \left(\frac{r_{2} - r_{1}}{h} \right)$$

$$= \frac{\frac{2}{5}\pi \left(\frac{r_2}{n}\right)^2 h^{\frac{9}{12}}}{k}$$

```
problem 4.2
 y'' - y' - 2y = 0
 TS: Y = explix)
      y'=rexplix)
      y'' = r^2 \exp(rx)
 r2explix) - rexplix) - 2 explix) = 0
  exp(rx)(r^2-r-2)=0
       r 2 -r -2 =0
      Lr-2) (r+1)=0
       r, = 2
       r2 = -1
 GS: Y = C1 EXP(2x) + C2 EXP(-x)
 Check: y = CIEXP(2X) + C2 exp(-X)
         Y' = 2 CIEXP(2x) - Czexpi-x)
         y" = 4 C1 exp (2x) + czexp (-x)
 4"-4'-24= 4 CI exp(2x) + (2 exp(-x) - (2 CI exp(2x) - CZ exp(-x)) - 2 (CI exp(2x) + CZ exp(-x))
             = 4 ciexp(2x) + Cz exp(-x) - 2 ciexp(2x) + Cz exp(-x) - 2 Ciexp(2x) - 2 Cz exp(-x)
             = 0
  GS: Y(x) = C1 Exp(2x) + C2 exp(-x)
```

Problem 4.3
$$y'' + 2y' + 2y = 0$$

$$TS: y = exp(rx)$$

$$y'' = y exp(rx)$$

$$y'' = r^{2} exp(rx) + 2y exp(rx) + 2 exp(rx) = 0$$

$$exp(rx) (r^{2} + 2y + 2) = 0$$

$$r^{3} + 2y + 2 = 0$$

$$r = -b \pm \sqrt{b^{2} + 4ac}$$

$$= -2 \pm \sqrt{4 + 4(2)}$$

$$= -2 \pm \sqrt{4}$$

$$= -2 \pm \frac{1}{2}i$$

$$= -1 \pm i$$

$$Y_{1,2} = exp((-1 \pm i)x)$$

$$GS: y = C_{1} exp((-1 \pm i)x) + C_{2} exp((-1 - i)x)$$

$$y = exp(-x) (C_{1} exp(ix) + C_{2} exp(-ix))$$

$$exp(\pm ix) = cosx \pm isinx$$

$$y = exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

$$= exp(-x) (C_{1} (cosx + isinx) + C_{2} (cosx - isinx))$$

GS: Y(x)= exp(-x) (c1 cosx + c2 sinx)

```
Problem 4.4
y" + 2y' + y = 0
TS: Y= explix)
     y =xexplix)
     y"= r2 exp(rx)
 r2 exp(rx) + 2r exp(rx) + exp(rx) =0
    explix) ( r2+2r+1)=0
          r2+2r+1=0
          (r+1)(r+1)=0
          r112 = -1
   Y = exp(-x)
   12 = x expl-x)
 check:
   Y = exp(-x)
                                                  12 = x exp(-x)
  11) = - exp(-x)
                                                  42) = -xexp(-x)
  Y." = exp(-x)
                                                  42" = x exp (-x)
y"+2y'+y = exp(-x)+2(-exp(-x))+exp(-x)
                                               y"+24'+y = x exp(-x) + 2(-x exp(-x)) + x exp(-x)
         = exp(-x) - 2 exp(-x) + exp(-x)
                                                         = x exp(-x) - 2 x exp(-x) + x exp(-x)
         = 0
                                                         = 0
```

GS: YLX) = C1 EXP(-X) + C2 X EXP(-X)

```
problem 4.5
  y" + y " + y + y = 0
 TS: Y= expinx)
      y'= rexp(x)
      y"= r2 exp inx)
      YIII = r3 expirx)
  r3 expirx) + r2 expirx) + r expirx) + expirx) =0
    explox) (r3+r2+r+1)=0
         r3+r2+r+1=0
       r2 (r+1) tr+1 =0
        (r2+1)(r+1)=0
       r= ±i r=-1
    11,2,3 = +1,-1,-1
   Y1 = exp(ix)
   42 = exp (-ix)
   43 = exp(-x)
 65: y(x) = C1 exp(ix) + C2 exp(-ix) + C3 exp(-x)
     exp(±ix) = cosx ± isinx
     CIEXPLIX) = CILCOSX + isinx)
     (zexpl-ix) = (z (cosx -isinx)
    Y= C1 (cosx + isinx) + C2 (cosx - isinx) + C3 expl-x)
       = (c1+c2) cosx + (ic1-ic2) sinx + c3 expl-x)
    Let G = CITC2
          C2 = iC1 - iC2
   95: y(x)= c1 cosx + c2 sinx + C3 exp(-x)
```