

### Questions: Strengths and Weaknesses

For each of the following laws of indices, find the questions that are worked out correctly and those that are not. Spot the errors!

#### 1. Law 1 (Multiplication): $a^m \times a^n = a^{m+n}$

Simplify the following expressions:

a.  $2^3 \times 2^2 = 2^{3+2} = 2^5$

c.  $3^1 \times 3^3 = 3^{1+3} = 3^4$

e.  $5^4 \times 5^{-2} = 5^{4+(-2)} = 5^2$

b.  $x^5 \times x^4 = x^{5 \times 4} = x^{20}$

d.  $y^2 \times y^7 = y^{2 \times 7} = y^{14}$

f.  $z^6 \times z^0 = z^{6+0} = z^6$

#### 2. Law 2 (Division): $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

Simplify the following expressions:

a.  $2^5 \div 2^2 = 2^{5 \div 2} = 2^{2.5}$

c.  $3^4 \div 3^1 = 3^{4-1} = 3^3$

e.  $5^2 \div 5^4 = 5^{4-2} = 5^2$

b.  $x^7 \div x^3 = x^{7-3} = x^4$

d.  $y^9 \div y^7 = y^{9-7} = y^2$

f.  $z^6 \div z^6 = z^{6-6} = z^0$  (or 1, assuming  $z \neq 0$ )

#### 3. Law 3 (Power): $(a^m)^n = a^{mn}$

Simplify the following expressions:

a.  $(2^3)^2 = 2^{3 \times 2} = 2^6$

d.  $(y^2)^5 = y^{2+5} = y^7$

f.  $(z^0)^7 = z^{0 \times 7} = z^0$  (or 1, assuming  $z \neq 0$ )

b.  $(x^5)^3 = x^{5+3} = x^8$

e.  $(5^4)^{-1} = 5^{4 \times -1} = 5^{-4}$  (or  $\frac{1}{5^4}$ )

c.  $(3^1)^4 = 3^{1 \times 4} = 3^4$

#### 4. Law 4 (Negative Power): $a^{-n} = \frac{1}{a^n}$

Rewrite the following expressions using positive indices or as fractions:

a.  $2^{-3} = -2^3 = -8$

c.  $3^{-1} = \frac{1}{3^{-1}}$

e.  $\frac{1}{5^{-4}} = 5^4$

b.  $x^{-5} = \frac{1}{x^5}$

d.  $y^{-2} = \frac{1}{y^2}$

f.  $\frac{1}{z^{-6}} = z^6$

#### 5. Law 5 (Zero Power): $a^0 = 1$ if $a \neq 0$

Evaluate the following expressions (assume variables are non-zero):

a.  $7^0 = 1$

c.  $(100)^0 = 1$

e.  $(ab)^0 = 1$

b.  $x^0 = 0$

d.  $(-5)^0 = -1$

f.  $(\frac{2}{3})^0 = 1$

#### 6. Law 6 (Distributivity): $(ab)^n = a^n b^n$

Expand the following expressions:

a.  $(2x)^3 = 2x^3$

d.  $(5z)^1 = 5^1 z^1 = 5z$

f.  $(2k)^{-2} = 2^{-2} k^{-2} = \frac{1}{2^2} \frac{1}{k^2} = \frac{1}{4k^2}$

b.  $(3y)^2 = 3^2 y^2 = 9y^2$

e.  $(4p)^0 = 1$  (as any non-zero base to the power of 0 is 1)

c.  $(ab)^5 = ab^5$

## Answers

### 1. Law 1 (Multiplication): $a^m \times a^n = a^{m+n}$

- a.  $2^3 \times 2^2 = 2^{3+2} = 2^5$ . **Correct.**
- b.  $x^5 \times x^4 = x^{5 \times 4} = x^{20}$ . **Incorrect.** The exponents should be added:  $x^5 \times x^4 = x^{5+4} = x^9$ . The mistake was  $x^{5 \times 4}$ .
- c.  $3^1 \times 3^3 = 3^{1+3} = 3^4$ . **Correct.**
- d.  $y^2 \times y^7 = y^{2 \times 7} = y^{14}$ . **Incorrect.** The exponents should be added:  $y^2 \times y^7 = y^{2+7} = y^9$ . The mistake was  $y^{2 \times 7}$ .
- e.  $5^4 \times 5^{-2} = 5^{4+(-2)} = 5^2$ . **Correct.**
- f.  $z^6 \times z^0 = z^{6+0} = z^6$ . **Correct.**

### 2. Law 2 (Division): $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

- a.  $2^5 \div 2^2 = 2^{5 \div 2} = 2^{2.5}$ . **Incorrect.** The exponents should be subtracted:  $2^5 \div 2^2 = 2^{5-2} = 2^3$ . The mistake was  $2^{5 \div 2}$ .
- b.  $x^7 \div x^3 = x^{7-3} = x^4$ . **Correct.**
- c.  $3^4 \div 3^1 = 3^{4-1} = 3^3$ . **Correct.**
- d.  $y^9 \div y^7 = y^{9-7} = y^2$ . **Correct.**
- e.  $5^2 \div 5^4 = 5^{4-2} = 5^2$ . **Incorrect.** The exponents should be subtracted in the correct order:  $5^2 \div 5^4 = 5^{2-4} = 5^{-2}$  (or  $\frac{1}{5^2}$ ). The mistake was  $5^{4-2}$ .
- f.  $z^6 \div z^6 = z^{6-6} = z^0$  (or 1, assuming  $z \neq 0$ ). **Correct.**

### 3. Law 3 (Power): $(a^m)^n = a^{mn}$

- a.  $(2^3)^2 = 2^{3 \times 2} = 2^6$ . **Correct.**
- b.  $(x^5)^3 = x^{5+3} = x^8$ . **Incorrect.** The exponents should be multiplied:  $(x^5)^3 = x^{5 \times 3} = x^{15}$ . The mistake was  $x^{5+3}$ .
- c.  $(3^1)^4 = 3^{1 \times 4} = 3^4$ . **Correct.**
- d.  $(y^2)^5 = y^{2+5} = y^7$ . **Incorrect.** The exponents should be multiplied:  $(y^2)^5 = y^{2 \times 5} = y^{10}$ . The mistake was  $y^{2+5}$ .
- e.  $(5^4)^{-1} = 5^{4 \times -1} = 5^{-4}$  (or  $\frac{1}{5^4}$ ). **Correct.**
- f.  $(z^0)^7 = z^{0 \times 7} = z^0$  (or 1, assuming  $z \neq 0$ ). **Correct.**

### 4. Law 4 (Negative Power): $a^{-n} = \frac{1}{a^n}$

- a.  $2^{-3} = -2^3 = -8$ . **Incorrect.** A negative exponent means the reciprocal of the base to the positive exponent:  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ . The mistake was  $2^{-3} = -2^3$ .
- b.  $x^{-5} = \frac{1}{x^5}$ . **Correct.**
- c.  $3^{-1} = \frac{1}{3^{-1}}$ . **Incorrect.** A negative exponent means the reciprocal of the base to the positive exponent:  $3^{-1} = \frac{1}{3^1} = \frac{1}{3}$ . The mistake was  $3^{-1} = \frac{1}{3^{-1}}$ .
- d.  $y^{-2} = \frac{1}{y^2}$ . **Correct.**

e.  $\frac{1}{5^{-4}} = 5^4$ . **Correct.**

f.  $\frac{1}{z^{-6}} = z^6$ . **Correct.**

**5. Law 5 (Zero Power):**  $a^0 = 1$  if  $a \neq 0$

a.  $7^0 = 1$ . **Correct.**

b.  $x^0 = 0$ . **Incorrect.** Any non-zero base to the power of 0 is 1:  $x^0 = 1$ . The mistake was  $x^0 = 0$ .

c.  $(100)^0 = 1$ . **Correct.**

d.  $(-5)^0 = -1$ . **Incorrect.** Any non-zero base to the power of 0 is 1:  $(-5)^0 = 1$ . The mistake was  $(-5)^0 = -1$ .

e.  $(ab)^0 = 1$ . **Correct.**

f.  $(\frac{2}{3})^0 = 1$ . **Correct.**

**6. Law 6 (Distributivity):**  $(ab)^n = a^n b^n$

a.  $(2x)^3 = 2x^3$ . **Incorrect.** The exponent should be distributed to both factors:  $(2x)^3 = 2^3 x^3 = 8x^3$ . The mistake was  $(2x)^3 = 2x^3$ .

b.  $(3y)^2 = 3^2 y^2 = 9y^2$ . **Correct.**

c.  $(ab)^5 = ab^5$ . **Incorrect.** The exponent should be distributed to both factors:  $(ab)^5 = a^5 b^5$ . The mistake was  $(ab)^5 = ab^5$ .

d.  $(5z)^1 = 5^1 z^1 = 5z$ . **Correct.**

e.  $(4p)^0 = 1$  (as any non-zero base to the power of 0 is 1). **Correct.**

f.  $(2k)^{-2} = 2^{-2} k^{-2} = \frac{1}{2^2} \frac{1}{k^2} = \frac{1}{4k^2}$ . **Correct.**