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AI6123 – Time Series Analysis Assignment

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Question 1: Let:

$$X_t = \begin{cases} Y_t & if\ t\ is\ even \\ Y_{t+1} & if\ t\ is\ odd \end{cases}$$

where Y_t is a stationary time series. Is X_t stationary?

> *Answer*: A stationary time series is one whose statistical properties such as mean, variance, and autocorrelation are constant over time [1]. Since Y_t is stationary, this indicates that the mean, variance, and autocorrelation do not change over time. However, by adding 1 to Y_t when *t* is odd, we are introducing a systematic change to the mean of X_t at even odd time index. This means that the *mean* is not constant over time. Since the mean is not constant over time, then the resulting series $\boldsymbol{X_t}$ **must be non-stationary.**

Question 2: Suppose that:

$$X_t = (1 + 2t)S_t + Z_t$$

where $S_t = S_{t-12}$. Suggest a transform for X_t so that the transformed series is stationary.

> *Answer:* A stationary time series is one whose statistical properties, such as mean, variance, and autocorrelation, do not change over time [1]. Given $S_t = S_{t-12}$, this implies that S_t is a seasonal component with a period of 12, representing monthly data with yearly seasonality. On the other hand, Z_t represents the noise or error component. This seasonal component introduces non-stationarity into the series. We can also see that the time series X_t has a time-dependent coefficient $1 + 2t$ in front of S_t. This means that the magnitude of the seasonal component increases over time, which introduces non-stationarity.
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> In order to make X_t stationary, we need to remove both the seasonal component S_t and the increasing trend. A common way to address the increasing trend is to apply a transformation function that reduces the magnitude of values as they increase; for example, a **logarithmic transformation**. Aside from this, since the trend is linear, we can also consider using differencing to remove this if transformation is not sufficient.
>
> After the increasing trend has been removed, we can apply seasonal differencing to remove the seasonal component S_t. The transformed series W_t can be represented as:
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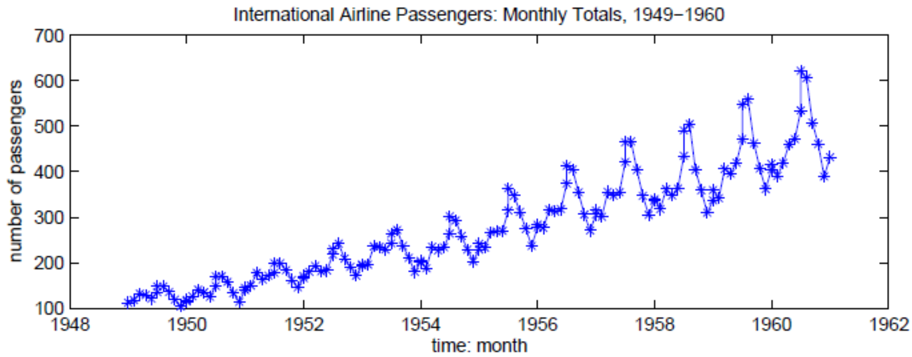
> $$W_t = transform(X_t) - transform(X_{t-12})$$

Question 3: Based don the time plot of International Airline Passengers, answer the following questions.

(a) Is it stationary? Justify it
(b) What kind of time series components do the data contain?
(c) Suggest a transformation so that it may equalize the seasonal variation.



Answers:

(a) No, **the series is non-stationary**. This plot clearly shows both an increasing trend and seasonal fluctuations, which indicates that the mean and variance are changing over time.
(b) The time series components present are:
   1. **Trend**. A clear upward trend indicates an increase in passenger numbers over time.
   2. **Seasonality**. There are regular fluctuations within each year which likely correspond to seasonal patterns. Furthermore, we can see a clear non-uniform seasonal variation in the seasonal patterns.
   3. **Random/Irregular Variations** (error). Likely present but cannot be fully determined just by visual inspection. Statistical analysis is required.
(c) To adjust the seasonal variation, we need to use a **logarithmic or Box Cox transformation** to stabilize increasing variance due to an upward trend and then apply **seasonal differencing** where we subtract observations from prior seasons (e.g., subtracting each January's value from the previous January's value) which can help stabilize variance related to seasonality.

Reference

[1] A. Aue, "1.2: Stationary Time Series," in Time Series Analysis, University of California, Davis, 2022. [Online]