Applied Wave Optics: Waveguide Coupling

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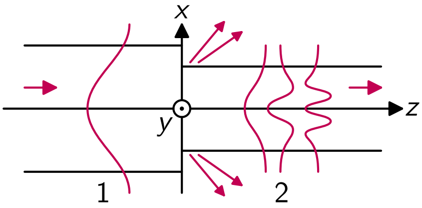
Version date: December 16, 2025

All sources and vector graphics are available on the GitHub repository `https://github.com/reincas/AWO_course`.

Axial Waveguide Coupling

The simplest approach to transfer an optical signal from one waveguide 1 to another waveguide 2 with different geometry and/or material properties is butt-coupling of both waveguides:



Mode expansion of the incident field at the end face of waveguide 2 reveals that even a single mode from waveguide 1 is distributed over the whole mode spectrum of waveguide 2 in general. This inevitably leads to losses via radiation modes.

Note: For the coupling efficiency between a certain pair of modes it doesn't matter which of the two waveguides has the larger core.

Taper Coupling

We demonstrate a more efficient coupling approach using a **simplified case** for the coupling of two single mode film waveguides. The power P_1 carried in waveguide 1 is coupled to power $P_2 = \eta P_1$ in waveguide 2 with the coupling efficiency

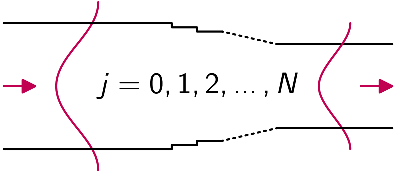
$$\eta = \frac{\left| \int n_2 \, E_1 E_2^* \, dx \right|^2}{\int n_2 \, E_1 E_1^* \, dx \cdot \int n_2 \, E_2 E_2^* \, dx}$$

For the two modes we assume a Gaussian field distribution $E_j = E_{m,j} \, \mathrm{e}^{-x^2/\omega_j^2}$. When we ignore the integration over the evanescent field in the cladding, index distributions $n_2(x)$ become constants, which cancel out. We get a **symmetric** expression for the efficiency:

$$\eta = \frac{\left| \int \exp\left[-x^2 \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \right) \right] dx \right|^2}{\int \exp\left(\frac{-2x^2}{\omega_1^2} \right) dx \cdot \int \exp\left(\frac{-2x^2}{\omega_2^2} \right) dx} = \frac{2\omega_1 \omega_2}{\omega_1^2 + \omega_2^2} = \left[\frac{1}{2} \left(\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} \right) \right]^{-1}$$

Taper Coupling (cont.)

Instead of a single large step, we now build a coupling section made of a number of smaller steps consisting of waveguide segments 0 to N. The geometry of the intermediate steps is designed such that we have $\omega_j = \omega_0 \exp(-j\Delta\omega)$ with $\Delta\omega = \ln(\omega_0/\omega_N)/N$.



The coupling efficiency for the small step between two consecutive segments is

$$\eta_{j,j+1} = \left[\frac{1}{2}\left(\mathrm{e}^{\Delta\omega} + \mathrm{e}^{-\Delta\omega}\right)\right]^{-1} = \cosh^{-1}(\Delta\omega)$$

Taper Coupling (cont.)

When we skip one intermediate segment, the coupling efficiency for the larger step to the segment after the next is

$$\eta_{j,j+2} = \left[\frac{1}{2}\left(\mathrm{e}^{2\Delta\omega} + \mathrm{e}^{-2\Delta\omega}\right)\right]^{-1} = \cosh^{-1}(2\Delta\omega)$$

It turns out that the efficiency of two smaller steps is larger than that of one larger step

$$\frac{\eta_{j,j+1}\cdot\eta_{j+1,j+2}}{\eta_{j,j+2}} = \frac{\cosh(2\Delta\omega)}{\cosh^2(\Delta\omega)} > 1$$

and infinitely small steps result in a lossless transition between the two waveguide modes:

$$\lim_{N\to\infty}\eta_{0,N} = \lim_{N\to\infty}\frac{P_N}{P_0} = \lim_{N\to\infty}\frac{1}{\cosh^N\left(\frac{\ln(\omega_0/\omega_N)}{N}\right)} = 1$$

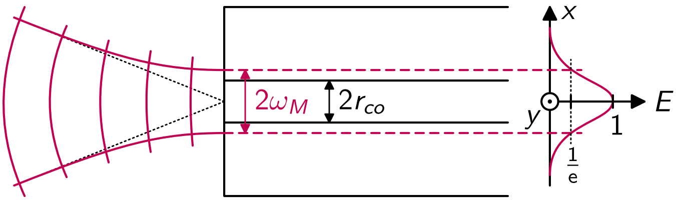
Note: Such an **adiabatic taper** is lossless for one pair of modes only.

Laser Coupling

The fundamental mode of a laser with free space resonator usually has a Gaussian beam profile with the magnitude

$$E_0(x, y) = E_m\, \mathrm{e}^{-\frac{x^2+y^2}{\omega_M^2}}$$

and the so called beam radius ω_M defined by the field reduction to $1/\mathrm{e} \approx 37\%$. This profile fits very well to the fundamental mode profile of quadratic or circular step-index waveguides.

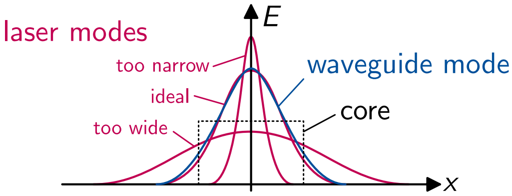


Laser Coupling (cont.)

The coupling efficiency from the field E_0 of the laser beam to the field E_1 of the fundamental waveguide mode is given by the respective normalised weight factor from the mode decomposition procedure:

$$\eta = \frac{\left|\int n\, E_0 E_1^* dxdy\right|^2}{\int n\, E_0 E_0^* dxdy \cdot \int n\, E_1 E_1^* dxdy}$$

The better the overlap of E_0 and E_1, the higher is the efficiency. For the optimum $E_0 = E_1$ it reaches 100%. Note also its symmetry. For the efficiency it does not matter whether the laser or the waveguide mode is the wider one:



Laser Coupling (cont.)

The radius of a Gaussian beam as function of the distance z from its focus point is

$$\omega(z) = \omega_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

with the Rayleigh length $z_R = \pi\omega_0^2/\lambda$. This equation can be used to determine the focus diameter ω_0 when a laser beam with diameter $\omega = \omega_L$ is focussed on the end face of a waveguide using a lens with focal length $z = f$. The coupling losses to the fundamental mode of a step-index fibre are minimised, when $2\omega_0$ is equal to the mode field diameter of the fibre. For a weakly-guiding step-index fibre this condition is

$$MFD = 2r_{co}\left(0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^6}\right) \stackrel{!}{=} 2\omega_0$$

with the V-number

$$V = \frac{2\pi r_{co}}{\lambda} NA$$

Lateral Coupling

Two waveguide cores 1 and 2 in close proximity with overlapping individual mode fields

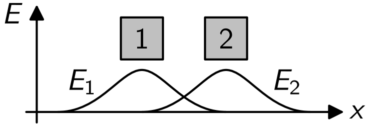
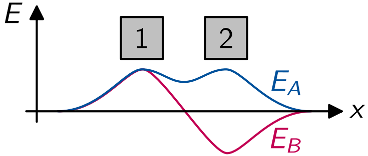


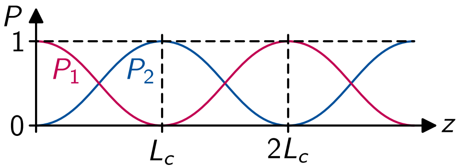
exhibit a common symmetric fundamental mode A and antisymmetric mode B:



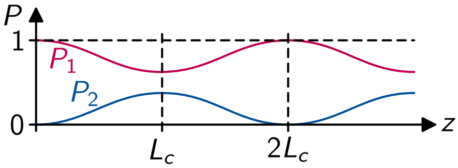
If we feed light in core 1 only, we excite the wave $\frac{1}{2}(E_A + E_B)$, which is almost identical to E_1, while $\frac{1}{2}(E_A - E_B)$ describes a wave travelling in core 2 only, resembling E_2.

Lateral Coupling (cont.)

Due to the different phase velocity of E_A and E_B, light coupled into one core (E_1) oscillates between both cores along the structure in z direction. At a distance called the **coupling length** L_c, the wave has completely crossed over to the second core (E_2):

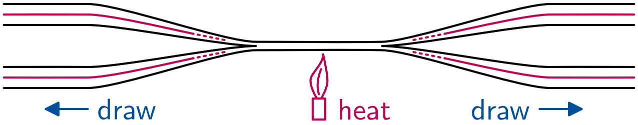


Such a device with identical cores is called a symmetric **directional coupler**. It can be fabricated for any coupling ratio depending on its length. In an asymmetric coupler the wave is oscillating as well, but not coupling completely:

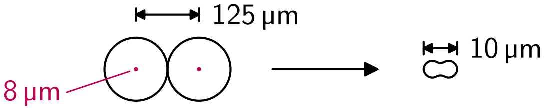


Fibre Coupler

For the fabrication of fibre couplers, two optical fibres are aligned in parallel, touching each other. The fibres are heated in the centre and pulled axially:



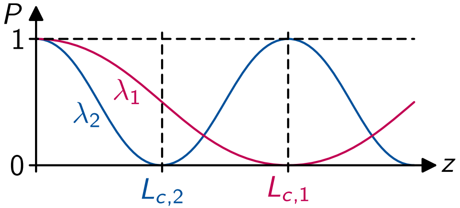
In this process the fibres melt together and form tapers with a thin bone-shaped cross-section in the centre:



In the taper section, the fibre core is reduced so much that the cladding takes over the role of the guiding core.

Wavelength Coupler

The coupling length depends on the overlap between the two individual core modes. For larger wavelengths $\lambda_2 > \lambda_1$ the mode diameter of a waveguide increases. Therefore, the coupling becomes stronger and the coupling length decreases: $L_{c,2} < L_{c,1}$.

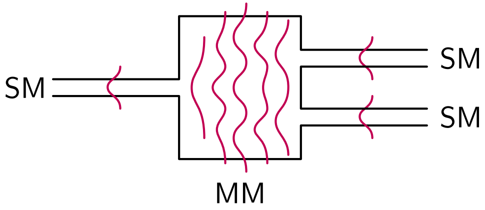


A coupler with $L_{c,1} = 2L_{c,2}$ is called a **wavelength division multiplexer** (WDM). It is used to multiplex two signals with different wavelengths into a single fibre or to demultiplex two signals from one fibre into two.

Multimode Interference Coupler

Directional couplers with short coupling length require a very small gap between the two cores for a strong coupling. While this is not critical for fibre couplers, it is a problem in planar integrated optics where it demands for extreme lithography resolution.

A better solution in integrated optics are therefore multimode interference (MMI) couplers. They use the known propagation characteristics of an optical signal from a single-mode waveguide coupled into a multimode section:



Note: Even though the mode coefficients are fixed inside the MM-section, the field is changing due to the different mode velocities.