HW-04 Part One

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1 4.2.22: Parity of $x^2 + x$ for Odd x

Objective: The purpose of this examination is to delve into the arithmetic properties of integers, specifically odd integers, and to ascertain the veracity of the following conjecture:

If x is an odd integer, then $x^2 + x$ is even?

Theorem:

Let x be an arbitrary odd integer.

Proof:

We begin our proof by using a basic property of integer arithmetic: the product of any two odd integers is odd (Property 3). From this property, it follows x^2 is an odd integer, assuming x itself is odd.

Next, we note x is given as an odd integer for this proof.

Using another integer arithmetic property—namely the sum of two odd integers is even (Property 2)—we can safely conclude $x^2 + x$ is even.

Thus, we confirm our initial statement is true: for any odd integer x, $x^2 + x$ is indeed even.

2 4.2.28: Rational Solutions for $\frac{ax+b}{cx+d} = 1$

Objective: Given integers a, b, c, and d with the constraint $a \neq c$, must x be rational? If so, express x as a ratio of two integers when it satisfies the equation:

$$\frac{ax + b}{cx + d} = 1$$

Theorem:

In the context of the equation above, x must be a ratio of two integers.

Proof:

We start by applying cross-multiplication to the equation, resulting in:

$$ax + b = cx + d$$

Further simplification yields:

$$ax - cx = d - b$$

$$x(a - c) = d - b$$

$$x = \frac{d - b}{a - c}, \quad a \neq c$$

Let $n = d - b$ and $m = a - c$. Both n and m are integers, as the difference between any two integers is also an integer. Additionally, $m \neq 0$ because $a \neq c$.

Therefore, $x = \frac{n}{m}$ for integers n and m where $m \neq 0$.

This confirms x is a ratio of two integers, satisfying our initial objective and proving our theorem.

Conclusion:

Based on the proof, x is expressible as a ratio of two integers $\frac{n}{m}$ where $m \neq 0$. This makes x a rational number.