# HW-03 Part TWO

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# 1  3.3.50

## 1.1  Part a: Evaluating the Proposition

We are tasked with evaluating the veracity of the proposition:

"For every object $x$, there exists an object $y$ such that if $x \neq y$, then $x$ and $y$ have different colors."

Given the objects are confined to the colors blue, grey, or black, and for each object, there are at least two objects of a different color, the proposition holds as **True**.

## 1.2  Part b: Logical Formalism

To formally represent the statement in question, we can employ predicate logic. The logical expression becomes:

$$\forall x \, (\exists y \, (x \neq y \rightarrow \sim (\text{TheSameColor}(x, y))))$$

Here, $\text{TheSameColor}(x, y)$ denotes "$x$ has the same color as $y$".

## 1.3  Part c: Negation in Logical Notation

The objective is to negate the logical statement initially presented in Part b. Starting with the negation of the original logical expression, we have:

$$\sim (\forall x(\exists y(x \neq y \rightarrow \sim \text{TheSameColor}(x,y)))) \equiv \exists x(\sim (\exists y(x \neq y \rightarrow \sim \text{TheSameColor}(x,y)))) \quad ...(1)$$

$$\equiv \exists x(\forall y(\sim (x \neq y \rightarrow \sim \text{TheSameColor}(x,y)))) \quad ...(2)$$

$$\equiv \exists x(\forall y(x \neq y \wedge \sim (\sim \text{TheSameColor}(x,y)))) \quad ...(3)$$

$$\equiv \exists x(\forall y(\sim (x \neq y \wedge \text{TheSameColor}(x,y)))) \quad ...(4)$$

We see **(1)** & **(2)** are Negation statements. **(3)** $\sim (P \rightarrow Q) \equiv P \wedge \sim Q$. Finally, **(4)** is Negation Law twice. Therefore, the negation of the statement in logical form is $\exists x(\forall y(x \neq y \wedge \text{TheSameColor}(x,y)))$.

# 2    Logical Argument and Reordered Premises 3.4.32

The original argument presents the following premises:

1. When I work a logic example without grumbling, you may be sure it is one I understood.

2. The arguments in these examples are not arranged in regular order like the ones I am used to.

3. No easy example makes my headache.

4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.

5. I never grumble at an example unless it gives me a headache.

 The conclusion drawn is: "These examples are not easy."

## 2.1    Objective

The objective is to reorder the premises in the above argument to demonstrate the conclusion is valid.

## 2.2   Rewriting the Premises

We rewrite the premises in an if-then form:

1. If I can't understand any logic example, I grumble.

2. If an argument belongs to these examples, then the arguments are not arranged in regular order like the ones I am used to.

3. If I get a headache, then the example is not easy.

4. If the arguments are not arranged in regular order like the ones I am used to, then I can't understand.

5. If I grumble at any example, then it gives me a headache.

## 2.3   Reordered Premises

Upon reordering, the premises are as follows:

2. If an argument belongs to these examples, then the arguments are not arranged in regular order like the ones I am used to.

4. If the arguments are not arranged in regular order like the ones I am used to, then I can't understand.

1. If I can't understand any logic example, I grumble.

5. If I grumble at any example, then it gives me a headache.

3. If I get a headache, then the example is not easy.

## 2.4   Conclusion

By logically chaining premises 2 and 3, we arrive at the same conclusion: "These examples are not easy." Therefore, the reordered premises validate the conclusion.