

Algorithm Analysis

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Task 6.3.1

In the examination of behavior of function $f(n) = 10^{12}n^3 + 10^6n^2 + n + 1$ against polynomial $g(n) = n^4$, we have a theoretical analysis complemented by limit calculations both with paper and pencil and using SymPy to derive the following conclusions:

1. **Is $10^{12}n^3 + 10^6n^2 + n + 1$ not equivalent to $\Theta(n^4)$?**
Yes. $10^{12}n^3 + 10^6n^2 + n + 1$ is not $\Theta(n^4)$ because the highest order term, n^3 , says what the growth rate of the function is. As n approaches infinity, the n^4 term would grow faster than the n^3 term. $10^{12}n^3 + 10^6n^2 + n + 1$ is $\Theta(n^3)$, not $\Theta(n^4)$. It cannot be bounded above and below by n^4 for a large n .
2. **Is $10^{12}n^3 + 10^6n^2 + n + 1$ equivalent to $o(n^4)$?**
Yes. $10^{12}n^3 + 10^6n^2 + n + 1$ is $o(n^4)$ because as n grows, the function $10^{12}n^3 + 10^6n^2 + n + 1$ increase at a slower rate than a function proportional to n^4 . Thus, it can be bound above by $C \cdot n^4$ for some constant C and sufficiently large n .
3. **Is $10^{12}n^3 + 10^6n^2 + n + 1$ equivalent to $\omega(n^4)$?**
No. $10^{12}n^3 + 10^6n^2 + n + 1$ is not $\omega(n^4)$ because it does not grow faster than any function proportional to n^4 . For $f(n)$ to be $\omega(g(n))$, $f(n)$ must eventually exceed $C \cdot g(n)$ for any constant C , which is not the case here as n^3 is the dominant term in $f(n)$, and it grows slower than n^4 .

6.3.2

Application Case:

- 250GB hard disk with 160GB of files.
- CPU with one core, 4GHz clock speed, and 64-bit registers.
- Capable of 4×10^9 8-character comparisons per second.

Search Time Analysis

1. 1KB Signature:

The time needed to search for a 1KB signature case is 5 seconds. This is a worst case scenario, which means it is unrealistic and a Boyer-Moore calculation will be better.

2. Database of 1 Million 1KB Signatures:

For a Database with 1 million 1KB signatures, 57.87 days will approximately be a worst case scenario. This is where the signature searching is independent and linear. The Boyer-Moore algorithm is going to be faster than this with sublinear performance.

3. Improving Efficiency:

To further improve the efficiency of malware/virus detection through string matching, we utilize parallel processing by distributing the search workload across multiple CPU cores or even different machines; implementing more advanced string matching algorithms such as Aho-Corasick, which is designed for searching multiple patterns simultaneously, can significantly reduce the time complexity. Optimizing the algorithm for preprocessing of the virus signatures and employing heuristic methods to skip unlikely sections of data can also enhance performance.

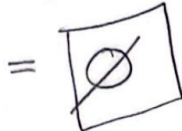
Screenshots of Hand Written Calculations

1. $\forall c > 0, \exists n \text{ s.t. } 10^{12} \cdot \underbrace{n^3}_{n^4} + 10^6 \cdot n^2 + n + 1 > c \cdot n^4$

Since $n^4 > n^3$, the polynomial cannot be $O(n^4)$.
 n^4 grows faster than n^3 . The function does not reach or exceed the growth of n^4 !

2. $\lim_{n \rightarrow \infty} \left(\frac{10^{12} \cdot n^3 + 10^6 \cdot n^2 + n + 1}{n^4} \right)$

$= \lim_{n \rightarrow \infty} \left(\frac{10^{12}}{n} \right) + \lim_{n \rightarrow \infty} \left(\frac{1,000,000}{n^2} \right) + \lim_{n \rightarrow \infty} \left(n^{-3} \right) + \lim_{n \rightarrow \infty} \left(n^{-4} \right) = 0 + 0 + 0 + 0$



This means the polynomial grows slower than n^4 ; exactly what $o(n^4)$ describes!
 The polynomial is $o(n^4)$!

3. For the polynomial to be $\omega(n^4)$ it must be infinite.
 The limit is not ∞ !

Figure 1: Hand written calculations for task 6.3.1

1. $\frac{160 \cdot 10^9}{8} \div (4 \cdot 10^9)_s = 5_s \text{ for } 1 \text{ KO sig.}$

2. $\frac{5 \cdot 10^6}{60} = 57.87_s \text{ for } 1,000,000 \text{ 1KO sig.}$
 $\frac{60}{24}$

Figure 2: Hand written calculations for task 6.3.2

References

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