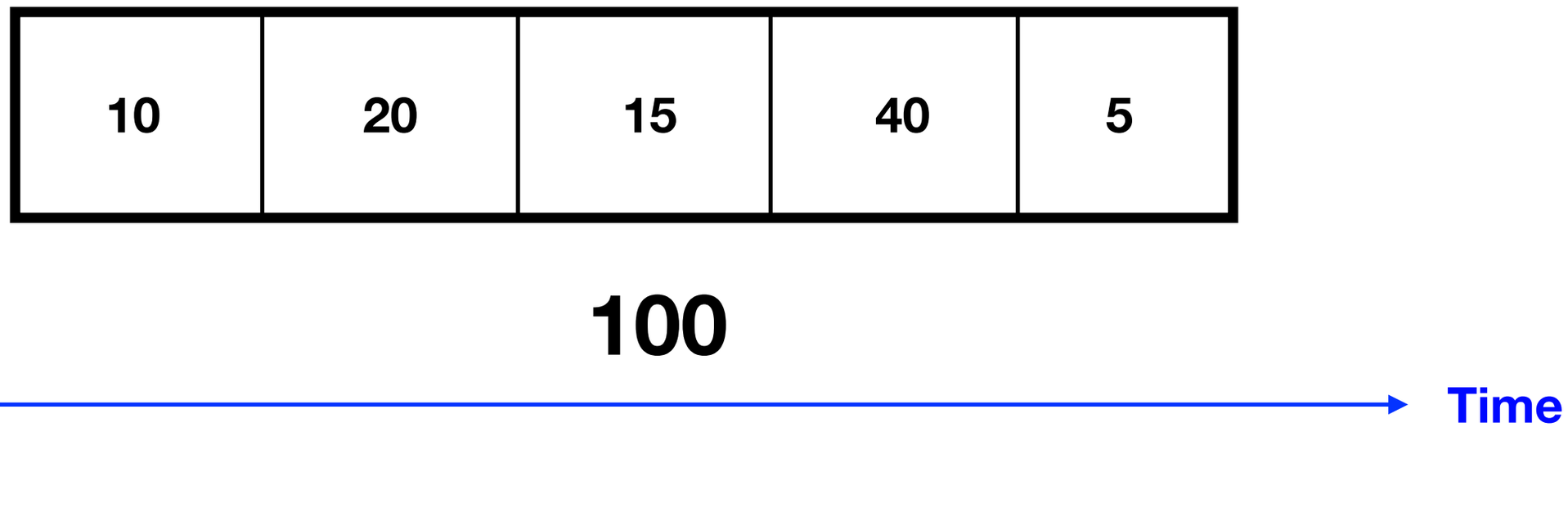
Order book trading algorithms: algorithmic execution and market-making

What is an execution algorithm?

- Need to buy/sell large order
  - Break large **parent** order in smaller **child** orders
  - Place child orders with the goal to minimize a large order's impact on the market and result in price improvement

- How to break parent order into child orders (what size of child orders)?
- At what time to execute child orders?
- Should child orders be executed via limit or market orders?

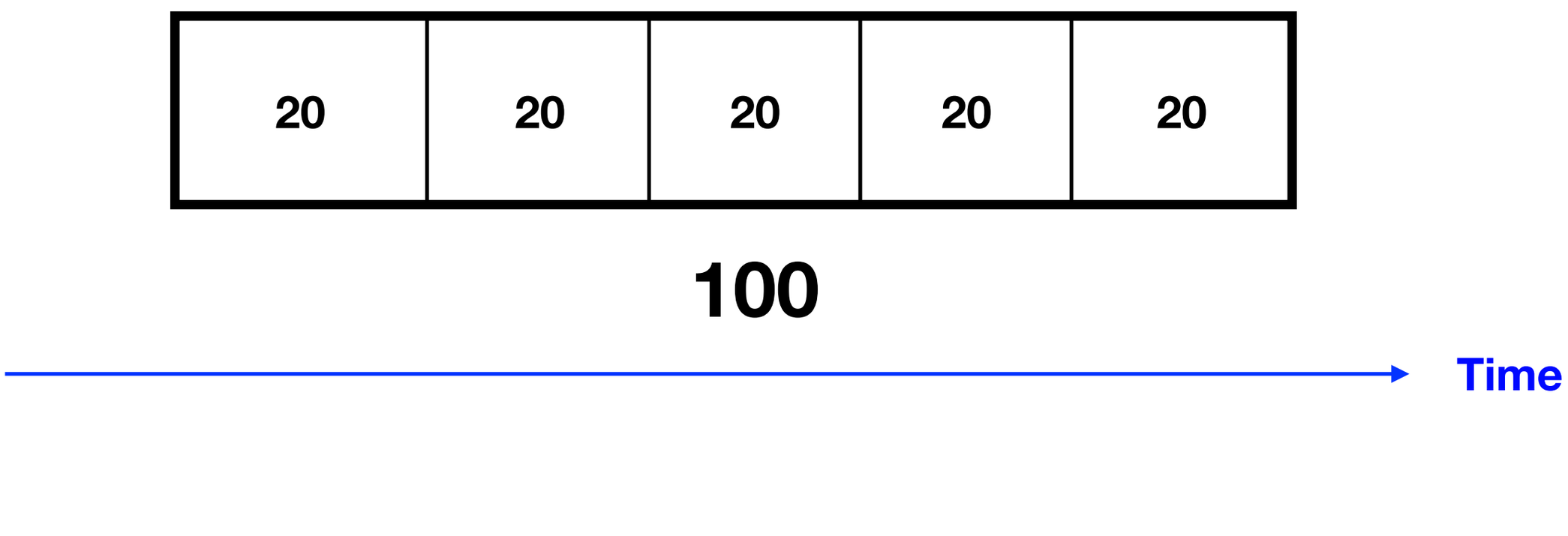


Static vs dynamic execution algorithms

- **Static:**
  - Schedule of placement of all child orders known proper to the first order placement
    - TWAP, VWAP, etc.
- **Dynamic:**
  - Child order placement is determined during execution
  - POV, reinforcement-learning based algorithms

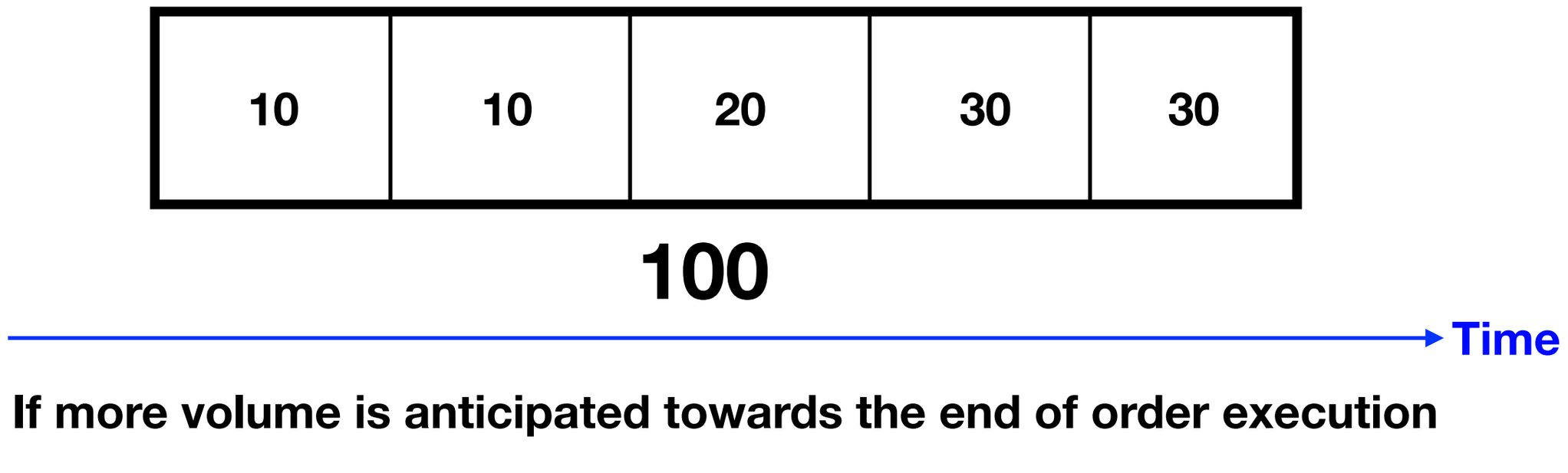
TWAP

- **Time Weighted Average Price**
- $$P_{TWAP} = \frac{\sum_j P_j T_j}{\sum_j T_j}$$
- Static



VWAP

- **Volume Weighted Average Price**
- $$V_{VWAP} = \frac{\sum_j P_j V_j}{\sum_j V_j}$$
- Use predicted volume profile (e.g., based on historical volumes) - place large child orders when more volume is anticipated20



If more volume is anticipated towards the end of order execution

Micro order placement

- Each of the child orders in both TWAP and VWAP is being placed via a combination of limit/market orders

- Simplest micro order placement?

POV

- **Percent of volume**
- Adjusting child order volumes to participate at a given rate
- $$\textbf{POV} = \frac{\textbf{child order volume}}{\textbf{traded market volume}}$$

Problem Statement

- Limit orders - low cost, high cost variance
- Market orders - high cost, low cost variance

- Executing more close to the beginning of parent order arrival - low volatility risk
- Executing more close to the end of parent order arrival - high volatility risk

- Need to optimize for both expected cost and cost variance
- $$\min E(X) + \lambda V(X)$$

Optimal Execution of Portfolio Transactions*

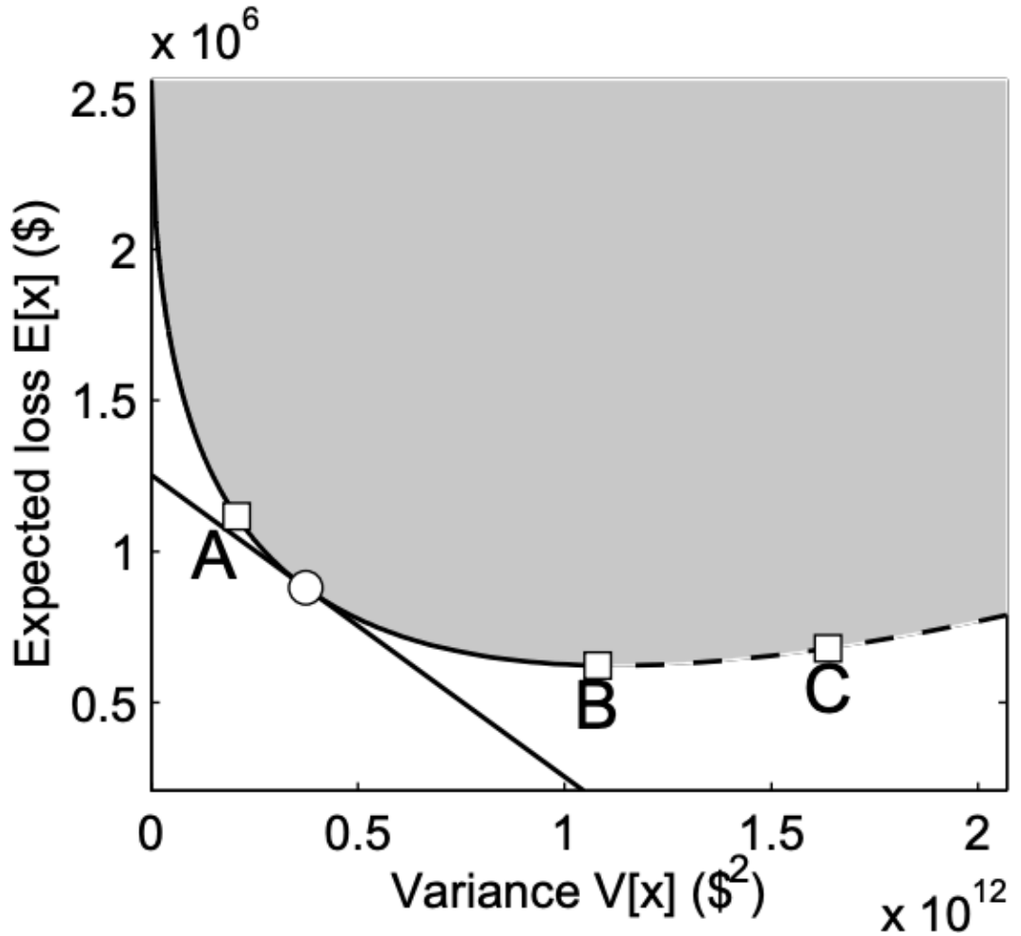
Robert Almgren† and Neil Chriss‡

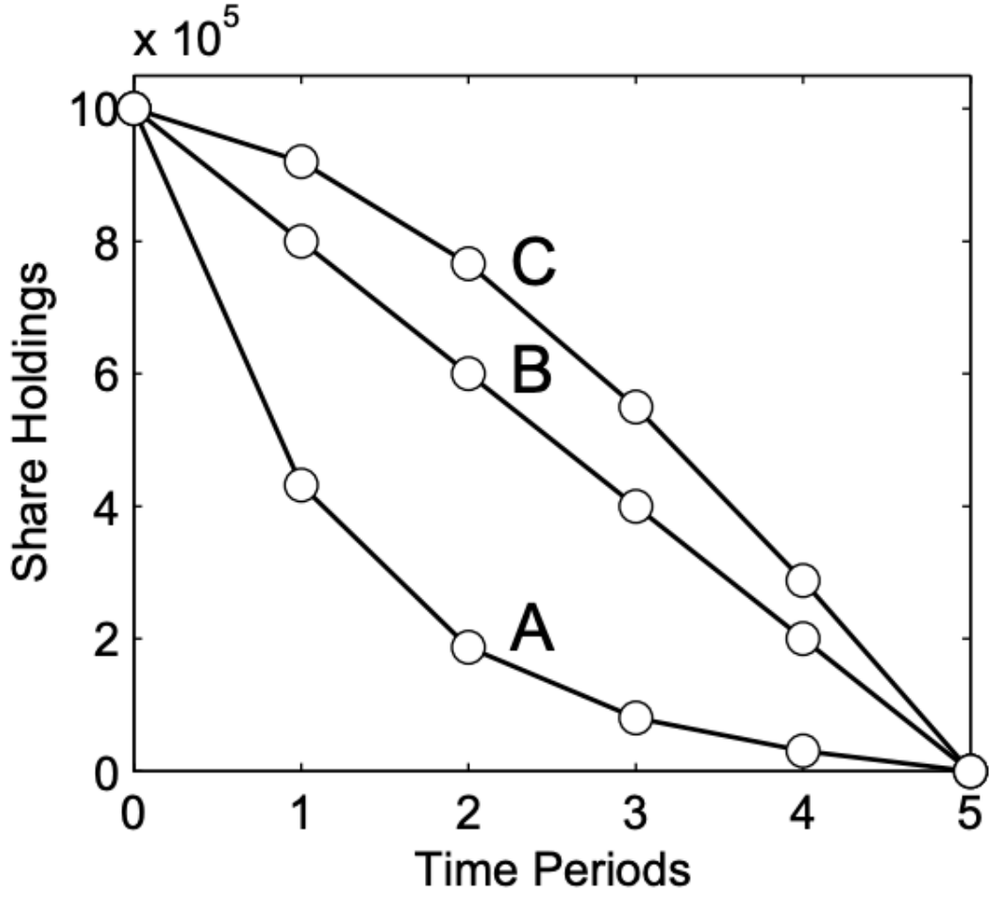
December 2000

- Optimize for cost and cost variance with respect to time of execution

Structure of the frontier

- λ - risk aversion parameter
- $\lambda = 2 \times 10^{-16}$ reduce volatility risk
- $\lambda = 0$ neutral to volatility risk
- $\lambda = -2 \times 10^{-7}$ exposure to volatility risk





Reinforcement Learning-based execution

- Optimize with respect to a given objective function (cost, risk adjusted cost, etc.)

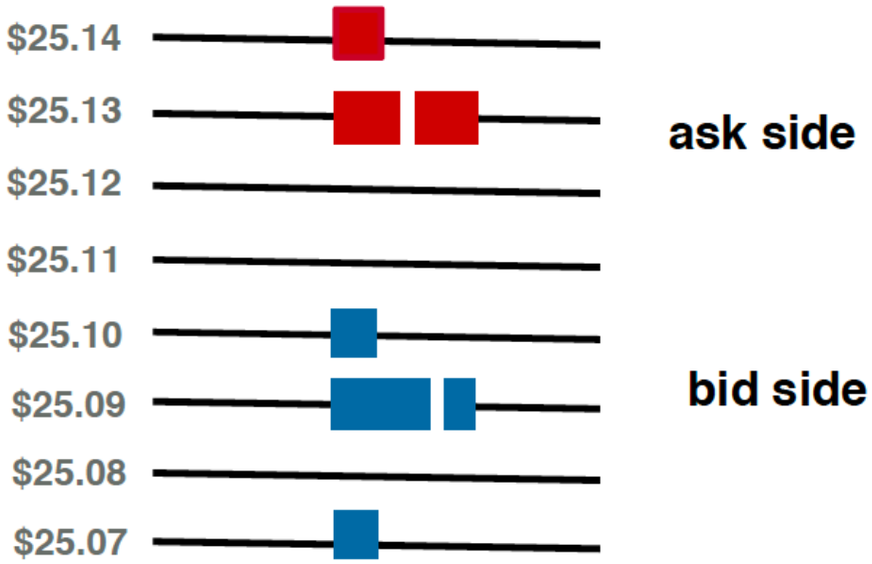
- Can optimize with respect to choice of **limit/market orders** and **time of execution/size of child orders**

- Dynamically place child orders to the market via limit or market orders based **on signals**

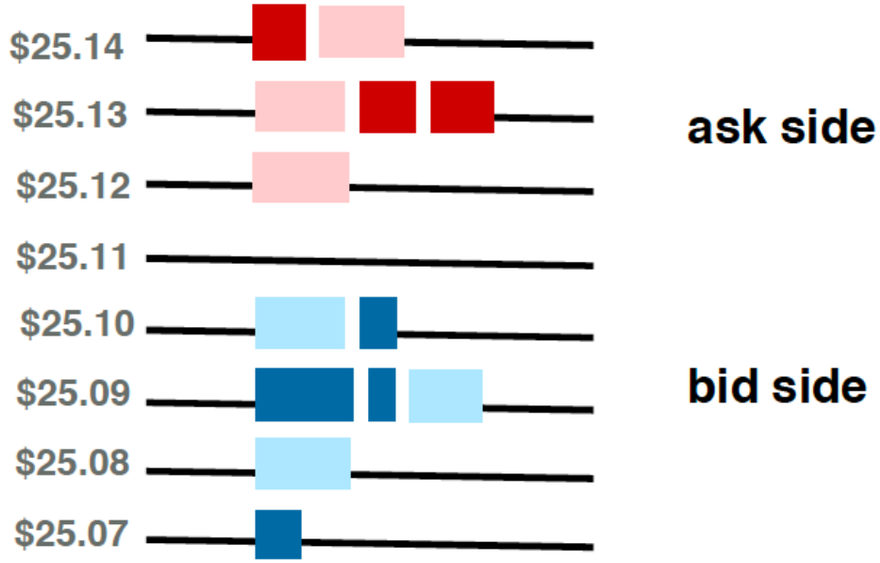
Market Making

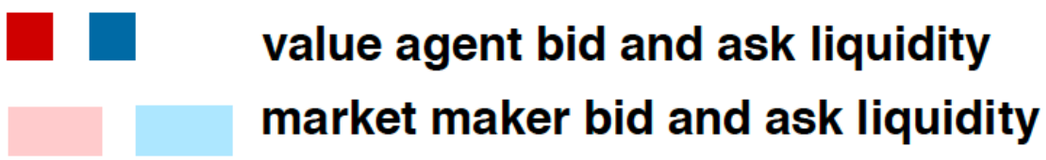
- Market makers post limit orders on both sides of limit order book subject to regulatory volume and spread constraints
- Need to make markets in ‘thin’ liquidity

- **“Liquidity makers”**: value agents
- **“Liquidity takers”** : retails agents



- **“Liquidity makers”:** value agents, market maker (MM)
- **“Liquidity takers”** : retails agents





Market Making

- Market makers make money on spread
- Risks:
  - Adverse selection - market makers must post “binding” quotes and face the risk to be picked off by more informed traders who see an opportunity to buy low/ sell high
  - Inventory risk

Market Making : simple model

- Parametrize market maker by spread (*S*) and depth (*D*)
- Stylized market marker policy dictated by the regulatory requirements:
    - place certain volume an spread *S* and depth *D* every *t* seconds
- Can this market maker provide enough liquidity?
- Can this market maker be profitable?

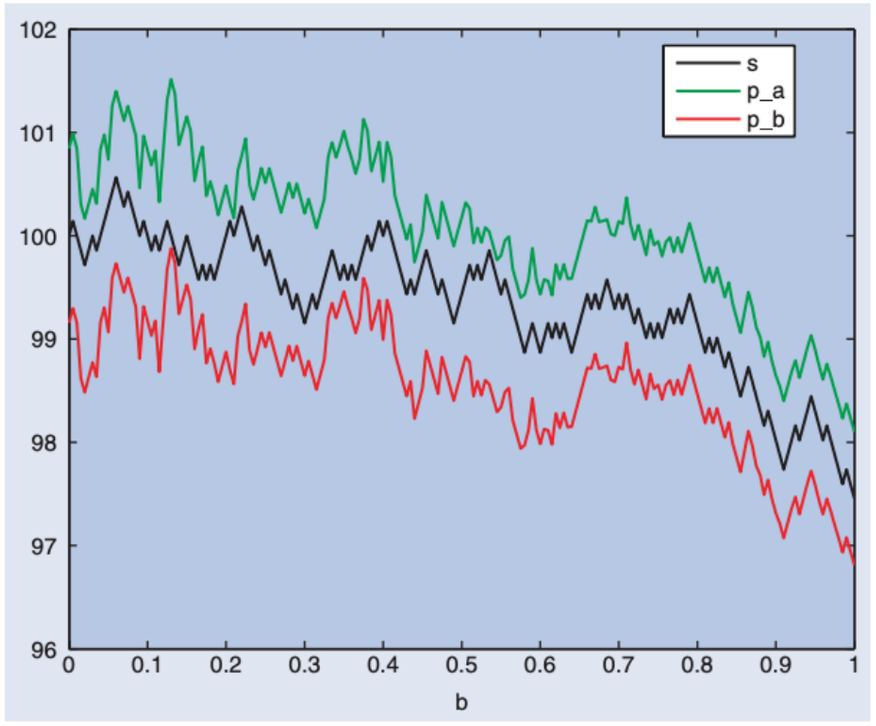
High-frequency trading in a limit order book

MARCO AVELLANEDA and SASHA STOIKOV*

Mathematics, New York University, 251 Mercer Street, New York, NY 10012, USA

A-S Market Maker

- Comparison of symmetric market maker that places around **midprice** to inventory (A-S) market maker that places around **reservation price**
- t = 0.2 - more likely to buy stock
- Account for inventory risk



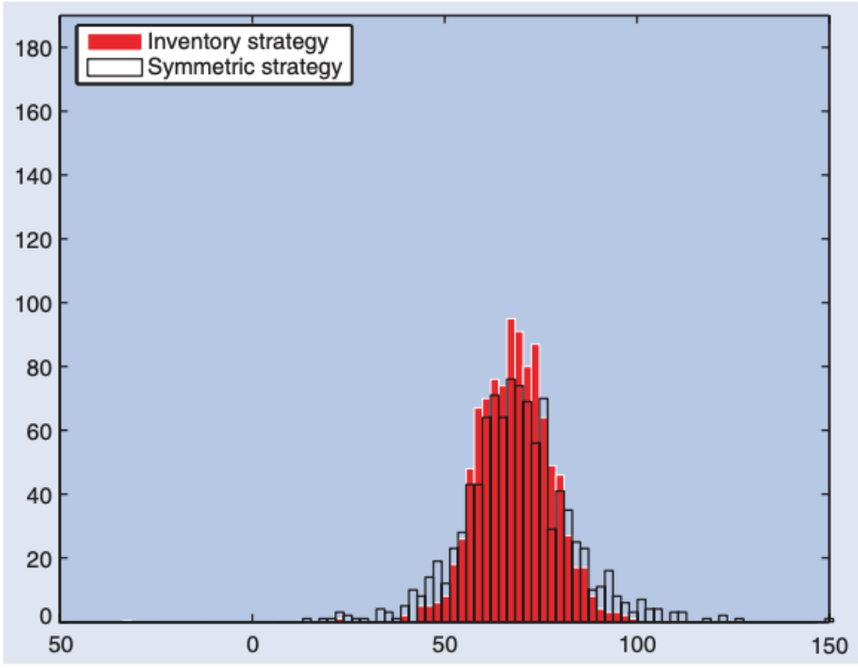


Figure 3. $\gamma = 0.01$.

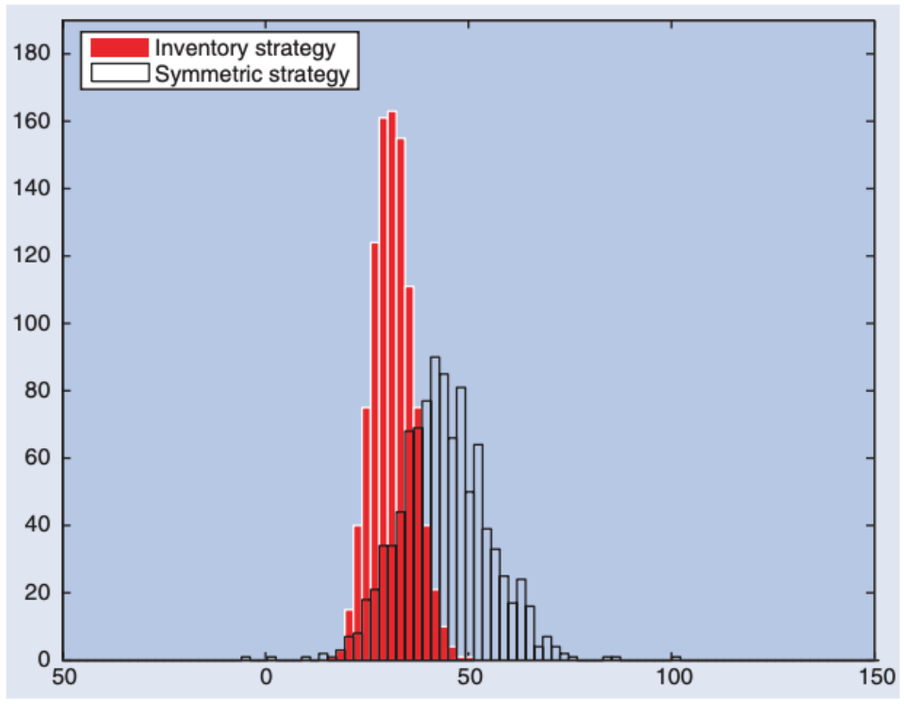


Figure 4. $\gamma = 1$.

RL Market Maker

- Act upon **market signals**
- Optimize for PnL/risk-adjusted PnL
- Account for inventory risk
- Way to account for adverse selection

HW and how to write a good proposal