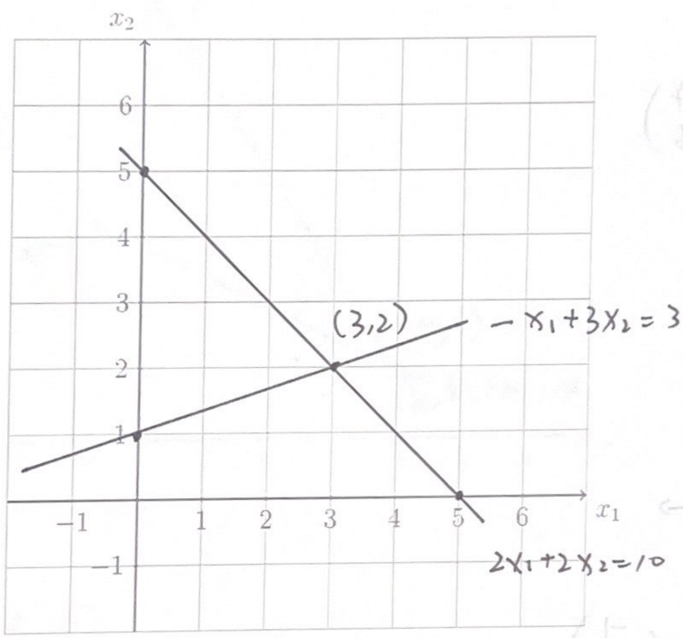
1. Consider the following system of linear equations

$$2x_1 + 2x_2 = 10$$
$$-x_1 + 3x_2 = 3$$

(a) Verify that $x_1 = 3$ and $x_2 = 2$ is a solution to the above system of equations.

(b) Verify the above solution geometrically by plotting (on a plot similar to that shown below) the two lines and showing that their intersection is the point $(3, 2)$.



(a) $x_1=3 \quad x_2=2$

$$\begin{cases} 2x_1 + 2x_2 = 10 \\ -x_1 + 3x_2 = 3 \end{cases}$$

$2\times3 + 2\times2 = 10$

$-3+3\times2 = -3+6=3.$

After substitute $x_1 \cdot x_2$ back into the equation, we have the same result

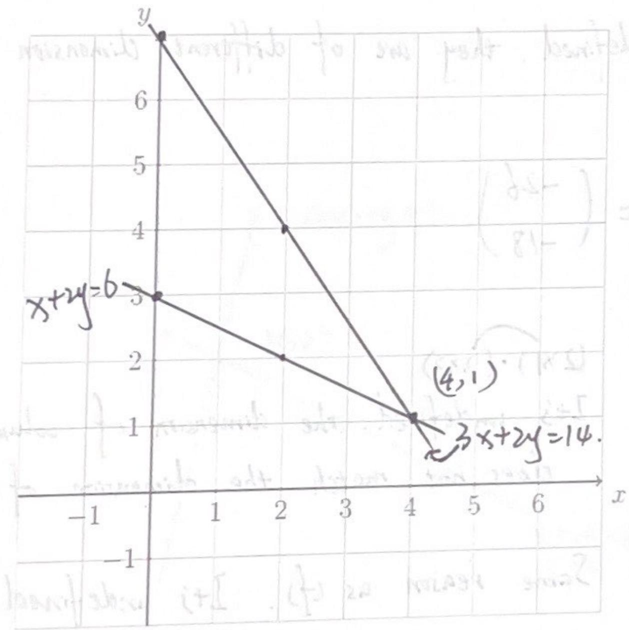
$\Rightarrow$ They are solutions to the system of equation

2. Consider the following system of linear equations

$$3x + 2y = 14$$
$$x + 2y = 6$$

(a) Solve for x and y (show your work).

(b) Verify your solution geometrically by plotting the two lines and their intersection.



$$\begin{cases} 3x+2y=14 & ① \\ x+2y=6 & ② \end{cases}$$

$\Rightarrow$ ① − ② $\quad 2x = 8$

$x = 4$

$\Rightarrow \quad y = 1$

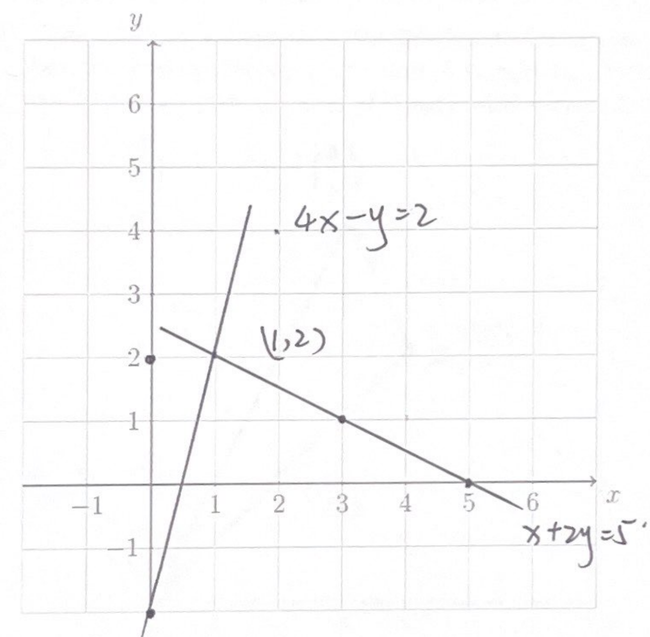
$$\begin{cases} x=4 \\ y=1 \end{cases}$$

3. Consider the following system of linear equations

$$4x - y = 2$$
$$x + 2y = 5$$

(a) Solve for x and y (show your work).

(b) Verify your solution geometrically by plotting the two lines and their intersection.



$$\begin{cases} 4x - y = 2 & ① \\ x + 2y = 5 & ② \end{cases}$$

$2 \times ① + ②$

$\Rightarrow \quad 9x = 9$

$x = 1$

$\Rightarrow y = 2$

$$\begin{cases} x = 1 \\ y = 2 \end{cases}$$

4. Consider the following system of linear equations.

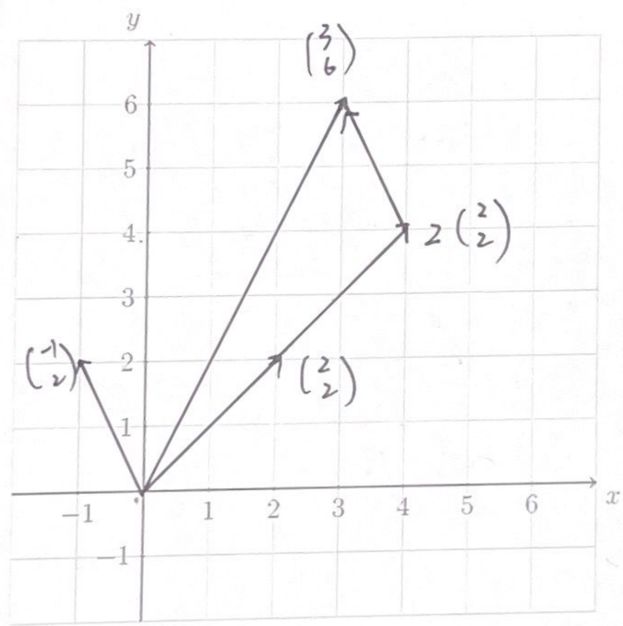
$$2x - y = 3$$
$$2x + 2y = 6$$

(a) Solve for x and y (show your work).

(b) Express the linear equations in vector-form as a sum of two scaled vectors.

(c) Draw the two vectors from the left-hand side of your solution to part (b).

(d) Draw the two vectors from the left-hand side of your solution to part (b) but this time scaled by your solution to part (a). Using the parallelogram construction, draw the sum of these scaled vectors to verify your solution.



(a) $\begin{cases} 2x - y = 3 & ① \\ 2x + 2y = 6 & ② \end{cases}$

(b) $\binom{2}{2}x + \binom{-1}{2}y = \binom{3}{6}$

② − ①

$\Rightarrow 3y = 3$

$y = 1$

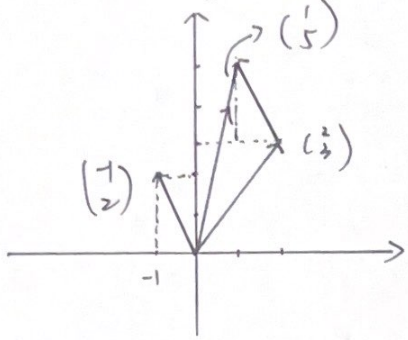
$x = 2$

$\Rightarrow \begin{cases} x = 2 \\ y = 1. \end{cases}$

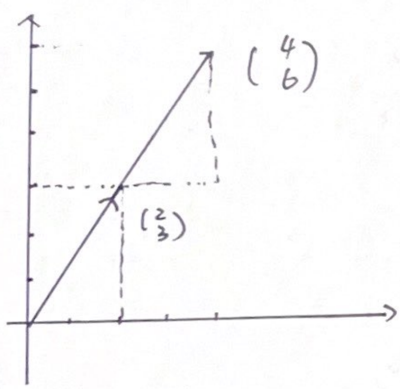
5.

(a) $A+B = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

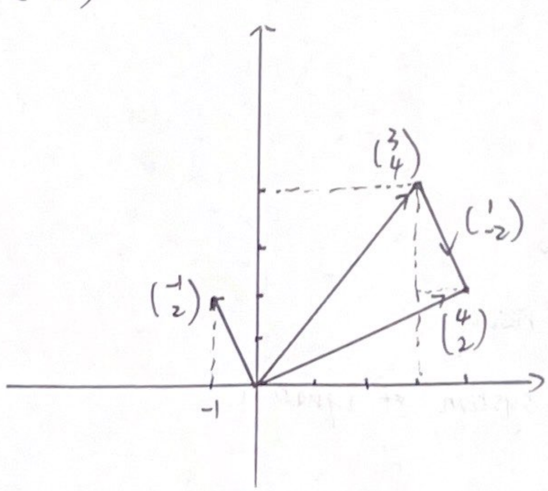


(b) $2B = 2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$



(c) $C - A = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 2 \end{pmatrix}$



6.

(a) $3A = 3 \cdot \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 9 \\ -15 \end{pmatrix}$

(b) $\frac{1}{2}B = \frac{1}{2} \cdot \begin{pmatrix} -2 & 4 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}$

(c) $B - C = \begin{pmatrix} -2 & 4 \\ 4 & 6 \end{pmatrix} - \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 6 \\ 1 & 7 \end{pmatrix}$

(d) $C + A$. It's ~~not~~ undefined, they are of different dimension in terms of adding.

(e) $BA = \begin{pmatrix} -2 & 4 \\ 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -26 \\ -18 \end{pmatrix}$

(f) $AC = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix}$ $(2\times1)\cdot(2\times2)$

It's undefined. the dimension of columns in the first matrix does not match the dimension of rows in the second matrix

(g) AD $(2\times1)\cdot(3\times2)$ Same reason as (f). It's undefined.

(h) $DA = \begin{pmatrix} 3 & 2 \\ -1 & 4 \\ 6 & -3 \end{pmatrix} \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 9-10 \\ -3-20 \\ 18+15 \end{pmatrix} = \begin{pmatrix} -1 \\ -23 \\ 33 \end{pmatrix}$

(i) $BD = \begin{pmatrix} -2 & 4 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ -1 & 4 \\ 6 & -3 \end{pmatrix}$ $(2\times2)\cdot(3\times2)$ It's undefined. Same reason as (f)

(j) $DC = \begin{pmatrix} 3 & 2 \\ -1 & 4 \\ 6 & -3 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 3+6 & -6-2 \\ -1+12 & 2-4 \\ 6-9 & -12+3 \end{pmatrix} = \begin{pmatrix} 9 & -8 \\ 11 & -2 \\ -3 & -9 \end{pmatrix}$