UNIVARIATE OLS, SQUARED ERROR

Model

$\hat{y}_i = \alpha x_i + \beta$

$x_i = \cdot$	$\alpha = \cdot$
$y_i = \cdot$	$\beta = \cdot$

$X = [\cdot$

$y = [\cdot$

Cost

$$J(\alpha, \beta) = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} (\alpha x_i + \beta - y_i)^2$$

∇

$$\frac{dJ}{d\alpha} = \frac{1}{N} \sum_{i=1}^{N} 2(\alpha x_i + \beta - y_i) \cdot x_i$$

$$= \frac{1}{N} \sum_{i=1}^{N} 2x_i (\alpha x_i + \beta - y_i)$$

$$\frac{dJ}{d\beta} = \frac{1}{N} \sum_{i=1}^{N} 2(\alpha x_i + \beta - y_i)$$

Psuedocode

```
x = [ ... ]
y = [ ... ]
R = 0.001
α = random(), β = random()
for i in iterations:
    cost = 1/N sum(αx + β - y)^2
    α -= R · 2/N sum[(αx + β - y) x]
    β -= R · 2/N sum(αx_i + β - y)
```

MULTIVARIATE OLS, SQUARED ERROR

Model

$\hat{y}_i = \theta x_i$

$\theta = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$ $\quad x_i = [\cdot \ \cdot \ \cdot]$

$y_i = \cdot$

$X = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$ $\quad y = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$

Cost

$$J(\theta) = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} (\theta x_i - y_i)^2$$

∇

$$\frac{dJ}{d\theta} = \frac{1}{N} \sum_{i=1}^{N} 2(\theta x_i - y_i) x_i$$

$$= \frac{1}{N} 2(X\theta - Y)X^T$$

ASIDE

$0 = \frac{2}{N}(\theta X - Y)X^T$

$0 = \theta X^T X - X^T y$

$(X^T X)^{-1} X^T Y = \theta$

pseudocode

$X = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$ $\quad y = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$ $\quad R = 0.001$

α = random(), β = random()

for i in iterations:

cost = $\frac{1}{N}$ sum$(X\theta - Y)^2$

θ -= $\frac{2}{N}$ sum$((X\theta - Y)X^T) \cdot R$