

On Measurement: The Relativity of Information Frames

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Abstract

1 Introduction

In this paper we begin with a motivational intuition. That of a principle of relativity. We then thought the paper endeavor to make that intuition mathematically precise. First by introducing the tools we will use, sheaf theory contextuality and markov kernels. Then we use these tools to give a precise meaning to this relativity principle.

Finally we show that this principle generates a space that has the properties of a Hilbert space. Using Gleason's theorem [?] we retrieve the born rule. We give a natural explanation for the pointer basis and give as a theorem Wigner friend's consistency, natural Markoviality of physical systems and the arrow of time.

We conclude with a ontological intepretation of this principle and suggestions for further reasearch directions. We conclude that, if this principle is accepted as a valid restriction for reality, the Copehangen intepretation axioms are derived instead of postulated. Giving a potential solution to the measurement problem.

We also compare it to other explanations such as GRW and Penrose collapse. Also relates to its direct cousin, Relation Quantum Mechanics. This is not a intepretation but a new framework.

Acceptance of this principle depends upon accepting the Relativity of information frames as physically fundamental. Further work is needed to see its consequences, in principle it does not disagree with quantum mechanics and provides a clean resolution to some of its puzzling features. No experiment to derive prove its truth is known to the authors of this paper at the current formulation.

2 Motivation - The Relativity of Information Frames

Consider two physical observers, Alice and Bob, each equipped with a clock and a ruler. To infer a particle's momentum, they make two position measurements and record the elapsed time.

However,

- if they agree on the spatial separation, they must disagree on the elapsed time;

- if they agree on the elapsed time, the measured spatial separation must differ.

Their interactions with the world differ — and so does what each can resolve as an event.

What Alice calls “particle at position x at time t ” is determined by her interaction channels and detection thresholds.

Thus there is no global, frame-independent σ -algebra of events. Every physical system carries its own information frame: a σ -algebra of distinguishable outcomes accessible through its interactions.

Einstein taught that coordinate descriptions are relative while causal order is invariant. We extend this principle.

Relativity of Information Frames (RIF)

Nature does not favor one perspective over another. Nature is the same no matter the observer frame of reference. When agreements cannot be reached, nature coarse grains distinctions to find a agreement.

Measurement is not the revelation of a pre-existing global state; it is the joint refinement (and, when necessary, coarse-graining) of information frames when systems interact. From this symmetry, quantum state update, pointer bases, and even causal geometry follow as consequences.

3 Required concepts

3.1 Required definitions

Labels and Contexts

The first important concept the theory relies upon is that of contextuality. All our definitions here are translated from the [?] contextuality in sheaf-theory.

First we look at the definition of a measurement label. A measurement label intuitively represents what one can tell apart, that is what questions a system can ask. It can be seen as the fundamental degrees of freedom of a given model. Which leads us to the definition:

Definition 3.1 (Measurement labels). A measurement label is an abstract symbol m that identifies a physical distinction we may attempt to extract from the system. Together with its outcome space $(\Omega_m, \mathcal{F}_m)$. That is:

$$m \rightarrow (\Omega_m, \mathcal{F}_m)$$

The set of all measurement labels the model considers primitive is called \mathcal{M} .

Next we talk about contexts. Contexts are given by the subset of labels or degrees of freedom a given observer cares about something he is interacting with. It can be seen as the fundamental set of questions he can ask about the part of the model he interacts with.

Definition 3.2 (Context). A context $C \subseteq \mathcal{M}$ is a finite collection of measurement labels that are jointly meaningful. To each context we associate a measurable space:

$$(\Omega_C, \mathcal{F}_C) := \left(\prod_{m \in C} \Omega_m, \bigotimes_{m \in C} \mathcal{F}_m \right)$$

Where $\mathcal{F}_1 \otimes \mathcal{F}_2 = \sigma(\{F_1 \times F_2 : F_1 \in \mathcal{F}_1, F_2 \in \mathcal{F}_2\})$

Within a context we also define the projections:

Definition 3.3 (Canonical Context Projections). The projections for a context C are defined measurable functions, or random variable:

$$\pi_{C \rightarrow \{m\}} : \Omega_C \rightarrow \Omega_m$$

With:

$$\pi_{C \rightarrow \{m\}}(\omega) = \omega_m \quad \forall \omega \in C$$

That is, the projections map to the context corresponding to the label m within the context C .

The projection can be seen as the *perspective* of C on m , in general.

For context projection we will also need its inverse definition.

Definition 3.4 (Cylinder Embedding). For every outcome $\omega_1 \in \Omega_m$ we embed it in the context space $(\Omega_C, \mathcal{F}_C)$. This gives the definition:

$$\pi_{C \rightarrow \{m\}}^{-1}(A) := \{\omega \in \Omega_C : x_m \in A\}$$

Since π is a measurable function by definition $\pi_{C \rightarrow \{m\}}^{-1}(A) \in \mathcal{F}_C$.

We also need the definition of an *empirical model*. It can be seen as the family of valid *perspectives* on a context.

Definition 3.5 (Empirical model). An *empirical model* is a family $\{e_C\}_{C \in \mathcal{M}}$ of probability measures on $(\Omega_C, \mathcal{F}_C)$. For all $C, C' \in \mathcal{M}$ and all $D \in C \cap C'$ we have:

$$(\pi_{C \rightarrow D})_* e_C = (\pi_{C' \rightarrow D})_* e_{C'}$$

That is the pushforward measures overlap, a probability measure based on the projection.

Using empirical families allows us to define what will be the driving feature of our framework. It is the definition of *Contextuality*. When perspectives depend upon the context they came from.

Definition 3.6 (Contextuality). The family $\{e_E\}_{E \in \mathcal{M}}$ is called contextual in \mathcal{M} if no probability measure μ on the global space \mathcal{M} exists satisfying:

$$(\pi_{C \rightarrow E})_* \mu = e_E \quad \forall E \subset \mathcal{M}$$

They are called non-contextual, if such probability measure exists.

Intuitively, it means that in that shared space, the questions still make perfect sense together when they are non-contextual. We can have a perspective where all contextual perspectives are perfectly valid in the global space. If they are contextual then there is no perspective on the global space that agrees with all perspectives of the contexts of that space.

The existence of contextuality is something shown in [?]. For our work here, it is useful to see contextuality in the following way.

Definition 3.7 (Relative Contextuality). If we have a series of contexts C_1, C_2, \dots . We can consider perspectives on each of them e_{C_n} . Then they are called *relatively contextual* if in the joint space $\mathcal{M} = C_1 \cup C_2 \dots$ they are contextual.

We then add a relabeling of definition fitting *RIF*.

Definition 3.8 (Perspective). Given a probability measure e on C , its σ -algebra $\sigma(e)$, the set of events it can distinguish, is a *perspective* on C .

These are the pieces from sheaf-theory contextuality besides empirical models, which will be introduced later, which we require.

Markov Kernels

We will also need the definition of *Markov Kernels*, which allows us to define how different measurable spaces interact.

Definition 3.9 (Markov Kernels). Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Markov Kernel* is a function:

$$K : \Omega_1 \times \mathcal{F}_2 \rightarrow [0, 1]$$

Where we have:

- For every fixed $\omega \in \Omega_1$

$K(\omega, \cdot)$ is a probability measure in \mathcal{F}_2

- For every fixed $A \in \mathcal{F}_2$

$K(\cdot, A)$ is a measurable function $\Omega_1 \rightarrow \Omega_2$

One important use of Markov Kernels we will need is the pushforward measure:

Definition 3.10 (Markov Pushforward Measure). Given the measurable spaces and Kernel on the definition 3.4. Let μ be a probability measure on Ω_1 . The *pushforward measure* given by the Kernel

$$(\mu K)(A) := \int_{\Omega_1} K(\omega, A) \mu(dx), \quad A \in \mathcal{F}_2$$

And it is a probability measure on \mathcal{F}_2 .

The other piece that will be important for this theory is:

Theorem 3.11 (DPI for KL-Divergence in Markov Kernels). *Every Markov Kernel satisfies the data processing inequality for KL-Divergence. Let μ, ν be probability measures on $(\Omega_1, \mathcal{F}_1)$ and $K : \Omega_1 \rightarrow \Omega_2$ be a Markov Kernel. Then:*

$$D_{KL}(\mu K || \nu K) \leq D_{KL}(\mu || \nu)$$

Meaning we only lose or retain KL-distinguishability by applying Markov Kernels.

Proof. This theorem is prove in [?].

□

3.2 Information Frames

Information frames are the only ontic objects in RIF. Intuitively a information frame in RIF represents a system and its perspective, what distinctions it can make about the model, or interaction, we are describing.

Definition 3.12 (Information Frame). Given a context C and a perspective $\sigma(e)$ its information frame is:

$$\mathcal{I}_{C,e} = (\Omega_C, \sigma(e), e)$$

Naturally $\sigma(e)$ is a sub- σ -algebra of the context total σ -algebra.

We will often omit the probability measure e in our constructions.

We note that, we will be using the projections on information frames. This is because, you can always project it to the information frame with $\mathbb{E}(\pi_{C \rightarrow \{m\}} \mid \mathcal{F})$.

Also, when we are talking about \mathcal{F}_m of a label $m \in C$ for a information frame, we are potentially talking about a sub σ -algebra of \mathcal{F}_m .

For convinience of notation, when the frames perspective is not important, we may omit it.

3.3 Admissible Maps: Invertable Deterministic Channels.

Definition 3.13 (Admissible Map). An admissible map is a Markov Kernel that acts on information frames $K : \mathcal{I}_{C,e} \rightarrow (\Omega, \mathcal{F})$ and preserve perspectives. They are invertable deterministic channels, they do not change what distinctions a system makes.

Formally, let $(\Omega_C \times \Omega_Z, \mathcal{F}_C \otimes \mathcal{F}_Z)$ be such space. Then there exists a injective measurable $f : \Omega_C \rightarrow \Omega_Z$ whose inverse f^{-1} is also measurable in its natural domain $f(\Omega_C)$ for which:

$$K(\omega, \cdot) = \delta_{(\omega, f(\omega))} \quad \forall \omega \in \Omega_C$$

And they preserve KL -Divergence exactly.

It is work noting that here the codomain may be larger. All we require is that there is a subspace that can hold all distinctions consistently.

Intuitively, all perspectives that generate the same set of distinctions are treated as just relabeling of that perspective. Such maps in fact, define a symmetry when mapping to the same space.

Definition 3.14 (The Symmetry Of Perspectives). When mapping from $\mathcal{I}_{C,e} \rightarrow \mathcal{I}_{C,e}$ the admissible maps form a symmetry group. In classic probability and information theory this group is called the *automorphism group of the probability space*. It is called $\text{Aut}(\Omega_C, \sigma(e), e)$ and defined as:

$$\{T : \Omega_C \rightarrow \Omega_C \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable, } e(T^{-1}(A)) = e(A) \quad \forall A \in \sigma(e)\}$$

Effectively, all probability measures consistent with this symmetry are considered the same for us.

While throught the rest of the paper we will be working only with admissible maps, we will not be restricting it only to automorphisms. We may define the following:

Definition 3.15 (Effective σ -algebra of K). For $K : \mathcal{I}_{C,e} \rightarrow (\Omega, \mathcal{F})$. We define the effective σ -algebra of K , denoted $\sigma(K)$ as:

$$\sigma(K) := \sigma\{K(\cdot, A) : A \in \mathcal{F}_2\}$$

That is, the distinctions that survive through K . The effective σ -algebra of K represents the perspective of $\mathcal{I}_{C,e}$ on the target space.

4 The Joint Frames

We now define a new concept, that of *joint frames*. During this chapter we will work with the join of only two frames. But this can be naturally extended to multiple joins.

Given two frames \mathcal{I}_{C_1,e_1} and \mathcal{I}_{C_2,e_2} , we can join them together into a single probability space:

$$\mathcal{J}_{\text{total}} := (\Omega_{C_1} \times \Omega_{C_2}, \mathcal{F}_{C_1} \otimes \mathcal{F}_{C_2})$$

This space has all the distinctions that both contexts could possibly make. All potential distinctions that the *labels*, or degrees of freedom, allow.

But RIF makes a additional restriction. We must have all *perspectives* preserved. Failure to do so would mean we are privileging some perspective over another. Therefore when joining two information frames, it must happen through admissible maps.

For convinience we will use $\mathcal{I}_1 = \mathcal{I}_{C_1,e_1}$ and $\mathcal{I}_2 = \mathcal{I}_{C_2,e_2}$ and .

Definition 4.1 (Candidate Joint Frames). A *Candidate Joint Frame* is a measurable space, \mathcal{J} reachable through admissible maps from each frame. That is, there exists $K_1 : \mathcal{I}_1 \rightarrow \mathcal{J}_{\text{total}}$ and a $K_2 : \mathcal{I}_2 \rightarrow \mathcal{J}_{\text{total}}$ with:

$$\mathcal{J} := (\Omega_{C_1} \times \Omega_{C_2}, \sigma(\sigma(K_1) \cup \sigma(K_2)))$$

The measurable space that keeps all distinctions, all the information, of both frames.

Each candidate frame can be viewed as a particular realization of a symmetry that tells valid ways to view a joint system from the perspective of their component frames.

4.1 The Admissible σ -algebras

We have an important class of σ -algebras that represent valid ways both frames could in principle view each other. That is, the class of all σ -algebras reachable by some admissible interaction.

Definition 4.2 (Admissible Joint Frames). The family of σ -algebras that are considered admissible for the joint frame is defined as follows:

$$\mathcal{F}_{\mathcal{I}_1 \otimes \mathcal{I}_2} := \{ \sigma : \exists K_1 : \mathcal{I}_1 \rightarrow \mathcal{J}_{\text{total}}, \exists K_2 : \mathcal{I}_2 \rightarrow \mathcal{J}_{\text{total}}, \text{st: } \sigma = (\sigma(K_1) \cup \sigma(K_2)) \}$$

Theorem 4.3. *The family $\mathcal{F}_{\mathcal{I}_1 \otimes \mathcal{I}_2}$ is not necessarily closed under meets. That is, there is $\sigma_1, \sigma_2 \in \mathcal{F}_{\mathcal{I}_1 \otimes \mathcal{I}_2}$ such that $\sigma_1 \cap \sigma_2 \notin \mathcal{F}_{\mathcal{I}_1 \otimes \mathcal{I}_2}$*

Proof. Here is enough to provide a single case where the meet fails. By [?] we know that we can construct a model where non-contextuality fails. If we translate that to our language this means there there is context C_1 and C_2 such that the global space is contextual.

Our situation maps cleanly to the contextuality case, if (Ω, \mathcal{F}) is the global space. If a admissible map existed then we can get $f_1 : \Omega_{C_1} \rightarrow \Omega$ and $f_2 : \Omega_{C_2} \rightarrow \Omega$ we have:

$$e_{C_i}^* := e_{C_i} \circ f_i^{-1}$$

Which is a probability measure on the sub- σ -algebra $f_i(\mathcal{F}_{C_i})$.

But here since $f_i(\mathcal{F}_{C_i}) \subseteq \sigma_1 \cap \sigma_2$ we must have a global measure μ on (Ω, \mathcal{F}) , induced by $\sigma_1 \cap \sigma_2$ and the marginals e_{C_i} with:

$$\mu|_{f_i(\mathcal{F}_i)} = e_{C_i}^*$$

This is possible because empirical models must agree on the intersections.

Therefore the existence of contextual models shows that meets do not always remain in this family. \square

4.2 The Pointer Frame

We define a special σ -algebra using the family $\mathcal{F}_{\bigotimes_{i \in I} \mathcal{I}_i}$. Intuitively we want a sigma algebra that contains only distinctions all frames can agree upon.

Definition 4.4 (The Pointer σ -algebra). For a family of *admissible joint frames* $\mathcal{F}_{\bigotimes_{i \in I} \mathcal{I}_i}$ its pointer σ -algebra is:

$$\mathcal{F}_{ptr} = \bigcap_{i \in I} \sigma_i$$

Theorem 4.5 (The Pointer Frame is non-contextual). *If we consider the joint frame given by $\mathcal{I}_{ptr} = (\Pi_{i \in I} \Omega_{C_i}, \mathcal{F}_{ptr})$ then there is a probability measure μ on it for which:*

$$\mu|_{\sigma(e_{C_i})} = e_{C_i}$$

Proof. For simplicity we consider the case of two frames.

Let $E \in \mathcal{F}_{ptr}$ and let e_{C_1} and e_{C_2} in be the perspective measure on the frames. Then since $E \in \sigma(K_1) \cap \sigma(K_2)$ for some $K_i : \mathcal{I}_i \rightarrow (\Omega_{C_1} \times \Omega_{C_2}, \mathcal{F}_{C_1} \otimes \mathcal{F}_{C_2})$ we have:

$$E \in \sigma(K_1) \cap \sigma(K_2) \rightarrow E \in \mathcal{F}_{C_1} \cap \mathcal{F}_{C_2}$$

Since for a empirical model and $E \subseteq C_1 \cap C_2$ we must have:

$$(\pi_{C_1 \rightarrow E})_* e_{C_1} = (\pi_{C_2 \rightarrow E})_* e_{C_2}$$

The probability marginals agree on E . Since this is true for every $E \in \mathcal{F}_{ptr}$ we can define:

$$\mu(E) := e_{C_1}(\pi_{C_1 \cup C_2 \rightarrow C_1}(E))$$

Then we automatically have:

$$\mu(E) = e_{C_2}(\pi_{C_1 \cup C_2 \rightarrow C_2}(E))$$

Therefore the frame is non-contextual. \square

This is the largest non-contextual sub-algebra touching the family of admissible frames. Any larger frame would have at least one element that some frame cannot detect and agree upon.

Another way to construct the pointer algebra is through quotient of admissible maps:

Definition 4.6 (The Quotient Pointer Space).

5 The Relativity of Information Frames

With the definitions in place we are finally able to make the relativity of information frames mathematically precise.

5.1 The Symmetry Of Information

5.2 Symmetry Breaking: Collapse

6 Hilbert Space

6.1 The Lattice of Admissible Joint Frames

6.2 Orthogonality

6.3 Gleason Theorem

7 Born Rule Martingale

8 No Global Wavefunction

9 Arrow of Time

10 Markovianity

11 Wigner's Friend Consistency

12 Intrepetation and Comparisons

12.1 Spin Example

12.2 The Ontology of Information Frames

12.3 Comparisons - Collapse Models

12.4 Comparisons - Intrepetations

13 Conclusion

References

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