

On Measurement: The Relativity of Information Frames

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Abstract

1 Introduction

In this paper we begin with a motivational intuition. That of a principle of relativity. We then thought the paper endeavor to make that intuition mathematically precise. First by introducing the tools we will use, sheaf theory contextuality and markov kernels. Then we use these tools to give a precise meaning to this relativity principle.

Finally we show that this principle generates a space that has the properties of a Hilbert space. Using Gleason's theorem [?] we retrieve the born rule. We give a natural explanation for the pointer basis and give as a theorem Wigner friend's consistency, natural Markoviality of physical systems and the arrow of time.

We conclude with a ontological intepretation of this principle and suggestions for further reasearch directions. We conclude that, if this principle is accepted as a valid restriction for reality, the Copehangen intepretation axioms are derived instead of postulated. Giving a potential solution to the measurement problem.

We also compare it to other explanations such as GRW and Penrose collapse. Also relates to its direct cousin, Relation Quantum Mechanics. This is not a intepretation but a new framework.

Acceptance of this principle depends upon accepting the Relativity of information frames as physically fundamental. Further work is needed to see its consequences, in principle it does not disagree with quantum mechanics and provides a clean resolution to some of its puzzling features. No experiment to derive prove its truth is known to the authors of this paper at the current formulation.

2 Motivation - The Relativity of Information Frames

Consider two physical observers, Alice and Bob, each equipped with a clock and a ruler. To infer a particle's momentum, they make two position measurements and record the elapsed time.

However,

- if they agree on the spatial separation, they must disagree on the elapsed time;

- if they agree on the elapsed time, the measured spatial separation must differ.

Their interactions with the world differ — and so does what each can resolve as an event.

What Alice calls “particle at position x at time t ” is determined by her interaction channels and detection thresholds.

Thus there is no global, frame-independent σ -algebra of events. Every physical system carries its own information frame: a σ -algebra of distinguishable outcomes accessible through its interactions.

Einstein taught that coordinate descriptions are relative while causal order is invariant. We extend this principle.

Relativity of Information Frames (RIF)

Nature does not favor one perspective over another. Nature is the same no matter the frame of information.

Measurement is not the revelation of a pre-existing global state; it is the joint refinement (and, when necessary, coarse-graining) of information frames when systems interact. From this symmetry, quantum state update, pointer bases, and even causal geometry follow as consequences.

3 Background

3.1 Measure Theory

The complete introduction to the richness of measure theory probability theory is not in the scope of this work, we refer to [?] for that, we will at least the concept of probability space. We hope the work is understandable with only this crude introduction but familiarity with the subject is advised.

σ -algebras

Probability Measure

Probability Spaces

Throughout this paper, we will often not be using specific probability measures, working only with the sample space and the σ -algebras.

Measurable Functions

Pushforward Measure

Markov Kernels

We will also need the definition of *Markov Kernels*, which give us to talk about how different probability spaces interact.

Definition 3.1 (Markov Kernels). Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Markov Kernel* is a function:

$$K : \Omega_1 \times \mathcal{F}_2 \rightarrow [0, 1]$$

Where we have:

- For every fixed $\omega \in \Omega_1$

$K(\omega, \cdot)$ is a probability measure in \mathcal{F}_2

- For every fixed $A \in \mathcal{F}_2$

$K(\cdot, A)$ is a measurable function $\Omega_1 \rightarrow ([0, 1], \mathcal{B}([0, 1]))$

The idea is that Markov Kernels define probability measures on the target space that respect the structure of the source space. It is often written $K : \Omega_1 \rightarrow \mathcal{P}(\Omega_2)$ to say, a Kernel that defines probabilities on Ω_2 from Ω_1 .

Markov Kernels allow us to define probability measures on the target space from measures on the source.

Definition 3.2 (Markov Pushforward Measure). Given the measurable spaces and Kernel on the Definition 3.4. Let μ be a probability measure on Ω_1 . The *pushforward measure* given by the Kernel

$$(\mu K)(A) := \int_{\Omega_1} K(\omega, A) \mu(dx), \quad A \in \mathcal{F}_2$$

And it is a probability measure on \mathcal{F}_2 .

And finally, we also need to look at the definition of a *effective σ -algebra* of a Markov Kernel.

Definition 3.3 (Effective σ -algebra of K). For a Kernel $K : (\Omega_1, \mathcal{F}_1) \rightarrow (\Omega_2, \mathcal{F}_2)$. The effective sigma algebra of K is given by:

$$\mathcal{F}_K := \sigma \{ \omega \rightarrow K(\omega, A) \mid A \in \mathcal{F}_2 \}$$

That is, the algebra of all random variables that K generates. It can also be seen as the algebra of events of Ω_1 that remain distinguishable after passing through K .

Composition of Markov Kernels

Markov Kernels compose, in particular if $K_1 : \Omega_1 \rightarrow \mathcal{P}(\Omega_2)$ and $K_2 : \Omega_2 \rightarrow \mathcal{P}(\Omega_3)$ are Markov Kernels. Then their composite is given for $A \in \mathcal{F}_3$:

$$(K_2 \circ K_1)(x, A) := \int_{\Omega_2} K_2(y, A) dK_1(x, \cdot)$$

And naturally defines probability measures on Ω_3 .

Deterministic Markov Kernels

These are a special class of Markov Kernels that can act as transport structure from one probability space to another.

Definition 3.4 (Deterministic Markov Kernels). Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Deterministic Markov Kernel* is induced by a measurable function $f : \Omega_1 \rightarrow \Omega_2$:

$$K_f : \Omega_1 \times \mathcal{F}_2 \rightarrow [0, 1]$$

That is given for $\omega_1 \in \Omega_1$ and $F_2 \in \mathcal{F}_2$

$$K_f(\omega_1, F_2) := \mathbf{1}_{f(\omega_1) \in F_2}$$

The function f can be seen as the transport from one probability space into another. The Kernel then, allows us to pushforward probability measures from it.

Embeddings

As we have seen, we can use a measurable function $f : \Omega_1 \rightarrow \Omega_2$ to define a Markov Kernel. The Markov Kernel then allows us to transport probabilities to the new space.

There are a few special measurable functions we will be interested in. The first defines a full *isomorphism* of spaces:

Definition 3.5 (Measurable Isomorphism). A measurable function $T : \Omega_1 \rightarrow \Omega_2$ that is *bijective* and whose inverse $T^{-1} : \Omega_2 \rightarrow \Omega_1$ is also measurable defines a *isomorphism* of probability spaces:

$$(\Omega_1, \mathcal{F}_1) \cong (\Omega_2, \mathcal{F}_2)$$

Naturally such bijections form a group:

Definition 3.6 (Measurable Space Automorphisms).

$$\text{Aut}_{\Omega, \mathcal{F}} := \{T : \Omega \rightarrow \Omega \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable} \}$$

These maps preserve the full structure of the measurable space, for our work we will need a class that still preserves structure but can embed the measurable space into a larger one.

Definition 3.7 (Measurable Embedding). A measurable embedding $\iota : \Omega_1 \rightarrow \Omega_2$ is a *injective* measurable function with a measurable inverse. It can also be seen as a local homomorphism.

These embeddings preserve the σ -algebra of the source space entirely in the target space.

3.2 Contextuality

The first important concept the theory relies upon is that of contextuality. All our definitions here are translated from the [?] contextuality in sheaf-theory. They have been adapted to a measure theory framework.

Labels and Contexts

First we look at the definition of a measurement label. A measurement label intuitively represents what one can tell apart, that is what questions a system can ask. It can be seen as the fundamental degrees of freedom of a given model.

Measurement Labels

Definition 3.8 (Measurement labels). A measurement label is an abstract symbol m that identifies a physical distinction we may attempt to extract from the system. Together with its outcome space

$(\Omega_m, \mathcal{F}_m)$. That is:

$$m \rightarrow (\Omega_m, \mathcal{F}_m)$$

The set of all measurement labels the model considers primitive is called \mathcal{M} .

Then naturally, our global space, where all measurement labels exist is then:

Definition 3.9 (Global Space). The global space, the space of all degrees of freedom and all their distinctions is

$$(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}}) = \left(\prod_{m \in \mathcal{M}} \Omega_m, \bigotimes_{m \in \mathcal{M}} \mathcal{F}_m \right)$$

Where $\mathcal{F}_1 \otimes \mathcal{F}_2 = \sigma(\{F_1 \times F_2 : F_1 \in \mathcal{F}_1, F_2 \in \mathcal{F}_2\})$. We note that, we do not define a particular probability measure on this space, that is because what we are interested in at the moment is the structure of the space, not a specific measure on it.

Contexts

Next we talk about contexts. Contexts are given by the subset of labels or degrees of freedom a given observer cares about something he is interacting with. It can be seen as the fundamental set of questions he can ask about the part of the model he interacts with.

Definition 3.10 (Context). A context $C \subseteq \mathcal{M}$ is a finite collection of measurement labels that are jointly meaningful. To each context we associate a measurable space:

$$(\Omega_C, \mathcal{F}_C) := \left(\prod_{m \in C} \Omega_m, \bigotimes_{m \in C} \mathcal{F}_m \right)$$

Within a context we also define the projections:

Definition 3.11 (Canonical Context Projections). The projections for a context C are defined as *measurable functions* from a context to one of its label spaces.

$$\pi_{C \rightarrow \{m\}} : \Omega_C \rightarrow \Omega_m$$

With:

$$\pi_{C \rightarrow \{m\}}(\omega) = \omega_m \quad \forall \omega \in C$$

That is, the projections map to the context corresponding to the label m within the context C . The extension to a subcontext $D \subseteq C$ is naturally $\pi_{C \rightarrow D}$.

Intuitively projection can be seen as the *perspective* C has on m . For context projection we will also need its inverse definition.

Definition 3.12 (Cylinder Embedding). For every outcome $\omega_1 \in \Omega_m$ we embed it in the context space $(\Omega_C, \mathcal{F}_C)$. This gives the definition:

$$\pi_{C \rightarrow \{m\}}^{-1}(A) := \{\omega \in \Omega_C : x_m \in A\}$$

Since π is a measurable function by definition $\pi_{C \rightarrow \{m\}}^{-1}(A) \in \mathcal{F}_C$.

We importantly note that, a context require π . They tell the context where its events come from. In principle, a context does not hold *information that comes from nowhere*.

Empirical Model

We will now introduce the first concept that requires the use of specific probability measures that is the definition of an *empirical model*.

Intuitively can be seen as a particular realization of the model, or a particular realization of a *perspective* on the underlying world. We can also think of it as a particular family of *coordinates* in the probability spaces of the contexts.

Definition 3.13 (Empirical model). An *empirical model* is a family $\{e_C\}_{C \in \mathcal{M}}$ of probability measures on $(\Omega_C, \mathcal{F}_C)$. For all $C, C' \in \mathcal{M}$ and all $D \in C \cap C'$ we have:

$$(\pi_{C \rightarrow D})_* e_C = (\pi_{C' \rightarrow D})_* e_{C'}$$

This condition means that on overlaps, the probability measures must agree. They come from the same underlying labels.

Contextuality

The empirical families allows us to define what will be the driving feature of our framework. It is the definition of *Contextuality*. When *perspectives* only completely exist on the context they came from.

Definition 3.14 (Contextuality). The family $\{e_E\}_{E \in \mathcal{M}}$ is called contextual in \mathcal{M} if no probability measure μ on the global space \mathcal{M} exists satisfying:

$$(\pi_{C \rightarrow E})_* \mu = e_E \quad \forall E \subset \mathcal{M}$$

They are called non-contextual, if such probability measure exists.

Intuitively, it means that in that shared space, the questions still make perfect sense together if they are non-contextual. We know exactly where they came from.

If they are contextual then there is no way to pick a coordinate, or probability measure on the global space that agrees with all probabilities the contexts of that space found.

The core feature we will need here is that, there exists experiments or real situations where the global space is contextual this fact can be seen in depth in [?]. This particular definition, is of probabilistic contextuality as defined on [?].

4 Structural Contextuality

To get a better understanding on RIF we need to look at other formulations of contextuality. We want to understand contextuality without particular probability distributions.

For this, we adapt the sheaf like condition, than just the failure of probabilities to have the correct marginals, this in particular matches the *strong contextuality* defined in [?]. First we define a *Information Frame*.

Definition 4.1 (Information Frame). A information frame is a probability space over a context $C \subseteq \mathcal{M}$ with a particular event algebra $\mathcal{F} \subseteq \bigotimes_{m \in C} \mathcal{F}_m$ corresponding to the set of distinctions that frame can see about the labels m .

$$\mathcal{I}_{C,\mathcal{F}} := \left(\prod_{m \in C} \Omega_m, \mathcal{F} \right)$$

A information frame could be seen as a perspective on the world. A view of what it can in principle see about a system. A probability measure there represents a particular state of the world. For notation convinience we will use $\mathcal{I}_1 = \mathcal{I}_{C_1, \mathcal{F}_1}$

In general, Information Frames are defined on the support of some probability measure, meaning we get rid of events that a context does not see as possible.

Definition 4.2 (Structural Contextuality). Given a family of contexts \mathcal{C} of \mathcal{M} and corresponding information frames $\mathcal{I}_{C, \mathcal{F}_C}$ and a global space (Ω, \mathcal{F}) .

The family is said to be *structurally non-contextual* if there is a family of corresponding *embeddings* and some sub- σ -algebra $\mathcal{G} \subseteq \mathcal{F}$ such that:

$$\iota_{C \in \mathcal{C}} : \mathcal{I}_{C, \mathcal{F}_C} \rightarrow (\Omega, \mathcal{G})$$

Such that for every pair C, C' and every event $E \in \mathcal{F}_C \cap \mathcal{F}_{C'}$, that is events on shared labels, we have:

$$\iota_C(E) = \iota_{C'}(E)$$

If no such embedding exists then the family is called *structurally contextual*.

The existence of *strong contextuality* as seen in [?] guarantees structural contextuality also exists, in particular the results in [?] show that, if we try to keep the full σ -algebra event structure, structural contextuality will eventually happen. This shows contextuality is essentially inevitable in sufficiently complex logics.

5 The Relativity Of Information Frames

5.1 Interaction

Now we work on making precise the meaning of the *Relativity of Information Frames*, RIF, for short. To do so, the central objects of study will be *Information Frames* as in definition 4.1. Information frames are the only ontic objects in this theory, everything works on their interactions.

We now turn our attention to making the definition of interaction precise. The first step is to define the *Joint Frame*. A information frame where the interaction takes place. Before this we will note that when we write:

$$\iota(\mathcal{F}_C) := \mathcal{G}$$

That is, there is some sub- σ -algebra \mathcal{G} on the global space where that embedding exists. We also note that there can be many such embeddings. These differences are not of particular interest to us, they are related by the automorphism symmetry defined in Definition 3.7, so we can treat them as equivalent.

Definition 5.1 (Joint Frame). Given a family of information frames $\{\mathcal{J}_i\}_{i \in I}$ we can construct their joint frame as:

$$\mathcal{J}_I := (\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I}) := \left(\prod_{m \in \{\bigcup_{i \in I} C_i \subseteq \mathcal{M}\}} \Omega_m, \bigotimes_{i \in I} \iota_i(\mathcal{F}_i) \right)$$

The space that hold all distinctions of all information frames.

Such joint frame always exists, but it might be contextual. Structural contextuality tells us directly when it is even possible to make a non-contextual joint frame.

One important thing to note however, is that once we generate such a joint space, the embeddings are no longer necessarily embeddings. That is because in contextual cases, the event in the intersection E must choose one of the embeddings. This is critical for our relativity principle.

5.2 The Structure of the contexts

To make our relativity principle precise we need a way to compare the structure of contexts in the joint frame. The problem is once we fix a joint frame \mathcal{J}_I through some family of embeddings $\iota_{i \in I}$ we still can't properly compare the contexts as they do not live in the global space.

A way to do this is to consider the following map

Definition 5.2 (Local Projections). Given a family of embeddings ι_i that generated a particular frame \mathcal{J}_I . The local projections e_i are

$$e_i := e_{C_i} := (\iota_i \circ \pi_{\mathcal{J}_I \rightarrow C_i}) : \Omega_{\mathcal{J}_I} \rightarrow \Omega_{\mathcal{J}_I}$$

These e_i represent the full event structure of the frame \mathcal{J}_i in the joint frame.

5.3 The Symmetry Of Information

Privilege

We are finally ready to introduce the concept of *The Relativity of Information Frames*. We begin with the definition of *Privilege* in a *Joint Frame*.

Definition 5.3 (Privilege). We say a joint frame $\mathcal{J}_{C_1 \cup C_2}$ privileges C_1 over C_2 if for some shared event $E \in C_1 \cap C_2$ if, for its local projections e_1, e_2 we have:

$$e_1(E) = e_1(e_2(E)) \quad \text{and} \quad e_2(E) \neq e_2(e_1(E))$$

Intuitively, when the joint frame was built, when there is contextuality the joint frame had to pick one ι over another for each event they disagreed upon. That is favoring on that concept over that event.

In particular, its worth noting that the local projections do not commute in the contextual case:

$$e_1 \circ e_2(E) \neq e_2 \circ e_1(E)$$

The Relativity of Information Frames

We can now state the relativity of information frames precisely.

Axiom 1 (The Relativity Of Information Frames). *After interaction, a physically admissible frame J_{phys} must not privilege any interacting information frame over another on any event.*

More explicitly for any pair of frames i, j of the joint frame and any shared event E we have:

$$e_i(E) = e_i(e_j(E)) \quad \text{and} \quad e_j(E) = e_j(e_i(E))$$

Equivalently, the event is in the intersection of fixed points:

$$E \in \text{Fix}(e_i) \cap \text{Fix}(e_j)$$

The above condition shows that:

$$E \in \text{Fix}(e_i) \cap \text{Fix}(e_j) \quad \forall E \in \mathcal{F}_{\text{phys}} \quad \forall i, j$$

We define a important object using the definition above, with \mathcal{F}_I being the σ -algebra of the full joint frame:

Definition 5.4 (Pointer algebra). The pointer algebra of a joint frame is given by:

$$\mathcal{F}_{\text{ptr}} := \bigcap_i \text{Fix}(e_i) = \{E \in \mathcal{F}_I : e_i(E) = E \forall i\}$$

It is clear from the definition that:

$$\mathcal{F}_{\text{phys}} \subseteq \mathcal{F}_{\text{ptr}}$$

In fact, since its obviously true that for all $E \in \mathcal{F}_{\text{ptr}}$

$$e_i(E) = e_i(e_j(E)) \quad \text{and} \quad e_j(E) = e_j(e_i(E))$$

This intersection is the largest algebra that is physically admissible for the interaction.

5.4 Symmetry Breaking: Collapse

Now our study in contextuality already revealed that, the naive embedding joint frame, that attempts to preserve all distinctions in all information frames, may yield a contextual joint frame.

Theorem 5.5 (Maximality of the pointer algebra). *The pointer algebra \mathcal{F}_{ptr} is the largest σ -algebra that is physically admissible under Axiom 1.*

Proof. Such joint frame cannot obey Axiom 1. Because structural contextuality shows that for any joint frame \mathcal{J}_I that is built from a contextual family there is some event $E \in \mathcal{F}_I$ and some pair i, j for which

$$e_i(E) \neq e_j(E)$$

Therefore for at least some particular e_i we must have:

$$\mathcal{F}_{\text{ptr}} \subseteq \text{Fix}(e_i)$$

And for any $\mathcal{F}_{\text{ptr}} \subsetneq \mathcal{G} \subseteq \mathcal{F}_I$ we must have:

$$E \in \mathcal{G} \setminus \mathcal{F}_{\text{ptr}} \quad \exists k \in I \rightarrow e_k(e_k(E)) \neq e_k(E)$$

But by construction of the joint frame, the event $e_k(E)$ must have come from some context. So we must have $\iota_j(E) = e_k(E)$. And this means:

$$e_j(E) = e_j(e_k(E)) \quad \text{and} \quad e_k(e_j(E)) = e_k(e_k(E)) \neq e_k(E)$$

□

And finally we have:

Theorem 5.6 (Collapse Theorem). *Let \mathcal{F}_I be the contextual joint σ -algebra of the joint frame generated by the interaction of the contextual families \mathcal{F}_I . Let $\mathcal{F}_{\text{extptr}}$ be its pointer algebra.*

Then under Axiom 1 we have $\mathcal{F}_{\text{phys}} = \mathcal{F}_{\text{ptr}}$.

Proof. For every event $E \notin \mathcal{F}_{\text{ptr}}$ by Theorem 5.5 we know it exhibits privilege for some pair of contexts. Therefore such events are physically forbidden by RIF. Every event in \mathcal{F}_{ptr} is physically admissible. So we have:

$$\mathcal{F}_{\text{phys}} = \mathcal{F}_{\text{ptr}}$$

□

5.5 The Markov View

A important thing to note is that, once a joint frame has been built, and it has picked a algebra. The initial embeddings are no longer necessarily embeddings in that frame.

In fact, for the ones that are not privileged the kernel generated by the original embedding is no longer injective. It is a strict coarse graining. This will allow us to give precise numerics and dynamics by implementing information geometry once we are free to assume states (particular probability distributions). But this is not in the scope of this paper.

- 6 Born Rule Martingale**
- 7 Arrow of Time**
- 8 Locality Graph**
- 9 Maximum Speed**
- 10 Markovianity**
- 11 Wigner's Friend Consistency**
- 12 Intrepetation and Comparisons**
 - 12.1 Spin Example**
 - 12.2 The Ontology of Information Frames**
 - 12.3 Comparisons - Collapse Models**
 - 12.4 Comparisons - Intrepetations**
- 13 Discussion**

- 13.1 The Hilbert Space Reconstruction**

It is well known that the contextual space, here the joint frame, form a boolean lattice that recovers most of the properties of a Hilbert Space.

This view gives us a additional tool, mainly the automorphisms or the idea that, when a event must collapse down to a coarse grained version it picks up a phase, representing all the distinctions it could have held that map down to the same event. That might allow us to get the complex field.

The rest of the reconstruction should already have precedent with contextuality frameworks.

- 13.2 The Fisher metric view**

By taking the Markov view in section 5.5 we can explore this framework from the point of view of the fisher metric. When contextuality happens, the ricci curvature of the fisher metric blows up. That should trigger a back action on the tensor to adjust for the pointer basis.

Known derivations of the schrodinger equation using the quantum fisher metric can be explained and made canonical with this framework.

Furthermore, one can define a action principle of interactions. Using information geometry, we believe that can give a new way to model physical systems.

14 Conclusion

References

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