On Measurement: The Relativity of Information Frames

Gabriel Masarin

g.masarin@proton.me

Abstract

We introduce a structural framework based on a symmetry principle we call the *Relativity of Information Frames* (RIF). The framework treats information frames as fundamental objects and constrains how their event structures may be jointly realized during interaction. Using measure theory and contextuality, we formalize interactions as embeddings into a joint measurable space and show that enforcing perspectival symmetry induces a canonical coarse-graining of events. From a single RIF axiom, we derive a distinguished σ-algebra - the *pointer algebra* - and prove that it is the maximally informative algebra admitting non-contextual probability measures. Any consistent empirical family admits a global probability measure on the pointer algebra, whereas any strict extension generically reintroduces contextuality.

We then apply the framework to quantum theory. Without attempting a full reconstruction we show how standard quantum structure are naturally represented: observables arise as perspectival maps, probability emerges from coarse graining, and the born rule follows from a martingale argument. Wigner-type scenarios are shown to be consistent via the tower law, and unitary evolution is intrepreted as a manifestation of gauge freedom in the choice of embeddings. We further discuss how the induced coarse-graining introduces an intrinsic arrow of time and outline possible connections to causal structure and geometry. The results are structural rather than dynamical and provide a unified perspective on contextuality, probability, and measurement.

1 Introduction

This work develops a structural framework motivated by a simple but persistent tension in the foundations of physics: the apparent incompatibility between globally consistent descriptions of physical systems and the perspectival nature of information obtained through interaction. While the framework did not initially arise from a single ghiding principle, its development ultimately crystallized around a symmetry statement, which we call the **Relativity of Information Frames** (RIF).

At its core, RIF asserts that no informational perspective should be privileged over another in the description of physical events. This principle is not imposed dynamically, but structurally: it constrains which event structures can be jointly realized when multiple information frames interact. The framework developed here formalizes this idea using measure theory and contextuality, treating information frames as the fundamental ontic objects and interactions as operations that enforce perspectival consistency.

Part I of this paper establishes the formal framework. We introduce information frames, their embeddings into joint measurable spaces, and a notion of structural contextuality that captures when full informational structures cannot be jointly preserved. From a single RIF axiom, we derive a canonical coarse-graining of the joint event structure and prove that the resulting σ-algebra is, in a precise sense, the maximally informative non-contextual algebra compatible with the interaction. We refer to this algebra as the **pointer algebra**. We further show that, while probability measures need not exist on the full joint structure, any consistent empirical family admits a global probability measure on the pointer algebra, and that any strict extension of this algebra reintroduces contextuality.

Part II applies the framework to quantum theory. Without attempting a full reconstruction, we show how standard quantum structures are naturally represented within RIF. Observables arise as perspectival maps, conditioning corresponds to interaction, and probability assingnments emerge from coarse-graining rather than being postulated. We derive the Born rule operationally via a martingale argument, establish the consistency of Wigner-type scenarios using the tower law, and interpret unitary evolution as a manifestation of gauge freedom in the choice of embeddings. We also outline how familiar gauge symmetries may be viewed as relations between information frames.

Beyond quantum mechanics, we explore several conceptual consequences of the framework. That enforced coarse-graining introduces an intrinsic irreversibility, providing a structural arrow of time. Interactions define a relational graph of frames that suggests a notion of causal locality, and we briefly discuss how the degree of coarse-graining may be related to geometric and gravitational considerations.

The results presented here are structural rather than dynamical. No specific dynamics are assumed, and no new empirical predictions are claimed. Instead, the framework offers a unified perspective on contextuality, probability, and measuremen, clarifying the origin of quantum features that are often treated as axiomatic. We conclude by comparing RIF to existing approaches, including decoherence-based accounts, and by outlining directions for further development.

2 Motivation - The Relativity of Information Frames

Consider two physical observers, Alice and Bob, each equipped with a clock and a ruler. To infer a particle's momentum, they make two position measurements and record the elapsed time.

However,

- If they agree on the spatial separation, they must disagree on the elapsed time;
- If they agree on the elapsed time, the measured spatial separation must differ.

Their interactions with the world differ — and so does what each can resolve as an event.

What Alice calls "particle at position x at time t" is determined by her interaction channels and detection thresholds.

Thus there is no global, frame-independent σ-algebra of events. Every physical system carries its own information frame: a σ-algebra of distinguishable outcomes accessible through its interactions.

Einstein taught that coordinate descriptions are relative while causal order is invariant. We extend this principle.

Relativity of Information Frames (RIF)

Nature does not privilege one information frame over another. What is physical is what all information frames can agree upon.

Measurement is not the revelation of a pre-existing global state; it is the joint refinement (and, when necessary, coarse-graining) of information frames when systems interact. From this symmetry, quantum state update, pointer bases, and even causal geometry follow as consequences.

Part I
Structural Framework

3 Background

3.1 Measure Theory

A full account of measure theory and probability theory is outside the scope of this paper; for a standard reference, see [1]. In this section we introduce only the notion of a probability space and the measure-theoretic concepts that will be explicitly used later. The exposition is intentionally brief and self-contained, though some familiarity with the subject is helpful.

Probability Spaces

In measure-theoretic probability, a probability space is given by a triple:

$$(\Omega, \mathcal{F}, \mu)$$

where each component encodes a distinct element of a probality model.

The Sample Space Ω

The sample space Ω is the set of all possible outcomes of an experiment. It's elements $\omega \in \Omega$ represent individual realizations or trials. In general, Ω is endowed with additional structure beyond serving as the underlying space from which outcomes are drawn.

Depending on the context, elements of Ω may correspond to coin toss outcomes, experiment runs, or realizations of an abstract physical system.

The σ-algebra $\mathcal{F}$

The σ-algebra $\mathcal{F}$ specifies which subsets of Ω are considered **events**. Events are thus sets of outcomes $\omega \in \Omega$ to which probabilities may be assigned.

Definition 3.1 (σ-algebra)**.** A σ-algebra $\mathcal{F}$ is a collection of subsets of Ω such that:

1. $, \Omega \in \mathcal{F}$

2. Closed under complemetns. $F \in \mathcal{F} \to F^c \in \mathcal{F}$

3. Closed under countable unions.

$$\{F_i\}_{i\in\mathbb{N}} \in \mathcal{F} \to \bigcup_{i\in\mathbb{N}} F_i \in \mathcal{F}$$

The σ-algebra encodes the events that are meaningfully distinguishable within the model and therefore forms the central structural component of a probability space.

The Probability Measure μ

A probability measure assigns probabilities to events in $\mathcal{F}$.

Definition 3.2 (Probability Measures)**.** A probability measure is a function $\mu : \mathcal{F} \to [0, 1]$ satisfying:

1. Total probabilities:
$$\mu(\emptyset) = 0 \quad \mu(\Omega) = 1$$

2. For any countable collection of pairwise disjoint sets $\{F_i\}_{i\in\mathbb{N}}$, with $F_i \cap F_j = \emptyset$ for all $i \neq j$.
$$\mu\left(\bigcup_{i\in\mathbb{N}} F_i\right) = \sum_{i\in\mathbb{N}} \mu(F_i)$$

In this work, probability measures will be intrepreted as states on a measurable space. Encoding how probabilities are distributed over the events in $\mathcal{F}$.

Measurable Functions

A **measurable function**, often refered to in probability theory as a **random variable**, is a function between measurable spaces

$$f : \Omega_1 \to \Omega_2$$

such that:

$$\forall A \in \mathcal{F}_2 \qquad f^{-1}(A) \in \mathcal{F}_1$$

That is, the preimage of every event in $\mathcal{F}_2$ is a event in $\mathcal{F}_1$.

Pushfoward Measure

Given a measurable function

$$f : \Omega_1 \to \Omega_2$$

And a probability measure μ in $(\Omega_1, \mathcal{F}_1)$, we can define a probability measure on $(\Omega_2, \mathcal{F}_2)$ by

$$(f_*\mu)(A) := \mu\big(f^{-1}(A)\big), \qquad A \in \mathcal{F}_2$$

The measure $f_*\mu$ is called the **pushfoward** of μ along f.

The pushfoward measure represents the probability distribution induced on Ω_2 by the map f when the underlying space is described by the measure μ.

Embeddings

We will be particulary interested in certain classes of measurable maps that preserve the structure of measurable spaces.

Definition 3.3 (Measurable Isomorphism)**.** A measurable function $T : \Omega_1 \to \Omega_2$ is called a **measurable isomorphism** if it is **bijective** and it's inverse $T^{-1} : \Omega_2 \to \Omega_1$ is also measurable. In this case, the measurable spaces are **isomorphic**, denoted:

$$(\Omega_1, \mathcal{F}_1) \cong \big(\Omega_2, \mathcal{F}_2\big)$$

Such maps preserve the full measurable structure.

Definition 3.4 (Measurable Space Automorphisms)**.** For a measurable space $(\Omega, \mathcal{F})$, the group of measurable automorphisms is

$$\mathrm{Aut}\,(\Omega, \mathcal{F}) := \big\{T : \Omega \to \Omega \ \mid\ T \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable.}\big\}$$

While measurable isomorphisms preserve the entire structure of a space, our work requires maps that allow a measurable space to be faithfully represented within a larger one.

Definition 3.5 (Measurable Embedding)**.** A **measurable embedding** is a injective measurable function

$$\iota : \Omega_1 \to \Omega_2$$

such that the inverse map

$$\iota^{-1} : \iota(\Omega_1) \to \Omega_1$$

is measurable, where $\iota(\Omega_1)$ is equipped with the sub-σ-algebra induced from $\mathcal{F}_2$.

In this case, ι identifies $(\Omega_1, \mathcal{F}_1)$ with a measurable subspace of $(\Omega_2, \mathcal{F}_2)$, preserving the σ-algebra of the source space.

Definition 3.6 (Embedding-induced σ-algebra)**.** Let $\iota : \Omega_1 \to \Omega_2$ be a measurable embedding. The *embedding-induced σ-algebra* of ι is the sub-σ-algebra on $\iota(\Omega_1) \subset \Omega_2$ induced from $\mathcal{F}_2$, defined by

$$\mathcal{F}^{\iota} \ :=\ \mathcal{F}_2 \restriction_{\iota(\Omega_1)} =\ \{\, A \cap \iota(\Omega_1) \mid A \in \mathcal{F}_2 \,\}.$$

The same sub-σ-algebra used in the definition above.

3.2 Contextuality

A central concept underlying the theory developed in this work is that of **contextuality**. The definitions presented here are adapted from the sheaf-theoretic formulation of contextuality introduced in [**?**], to a measure-theoretic framework.

Labels and Contexts

We begin by introducing the notion of a measurement label. Intuitively, a measurement label represents a physical distinction that can be probed in a system. Measurement labels encode the primitive degrees of freedom of the model.

Measurement Labels

Definition 3.7 (Measurement labels)**.** A **measurement label** is an abstract symbol m associated with an measurable space $(\Omega_m, \mathcal{F}_m)$, representing the possible outcomes of measuring m. We write

$$m \longmapsto (\Omega_m, \mathcal{F}_m)$$

The collection of all measurement labels cosidered by a model considers is denoted $\mathcal{M}$.

Global Space

Definition 3.8 (Global measurable space)**.** The **global space**, representing the joint space of all measurement labels, is the product measurable space

$$(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}}) := \left(\prod_{m \in \mathcal{M}} \Omega_m \, , \bigotimes_{m \in \mathcal{M}} \mathcal{F}_m \right)$$

Here $\bigotimes_{m \in \mathcal{M}} \mathcal{F}_m$ denotes the product σ-algebra generated by cylinder sets. For two measurable spaces $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ the product σ-algebra is given by

$$\mathcal{F}_1 \otimes \mathcal{F}_2 := \sigma\left(\{ F_1 \times F_2 : F_1 \in \mathcal{F}_1, F_2 \in \mathcal{F}_2 \} \right)$$

We emphasize that no probability measure is specified on the global space at this stage. Our interest here lies in the measurable structure itself, independently of any particular choice of global state.

Contexts

We now introduce the notion of a context. A context represents a collection of measurement labels that are jointly accessible to an observer, or equivalently, a set of degrees of freedom that can be meaningfully considered together. Intuitively, a context specifies the degrees an observer may simultaneously interact with on the system.

Definition 3.9 (Context)**.** A **context** is a finite subset $C \subseteq \mathcal{M}$ of measurement labels. To each context we associate a measurable space

$$(\Omega_C, \mathcal{F}_C) := \left(\prod_{m \in C} \Omega_m \, , \bigotimes_{m \in C} \mathcal{F}_m \right)$$

Canonical Context Projections

Within a context, we define canonical projection maps onto the outcome spaces of individual measurament labels.

Definition 3.10 (Canonical context projections)**.** For a context C and a label $m \in C$, the canonical projection is the measurable function

$$\pi_{C \to \{m\}} : \Omega_C \to \Omega_m$$

defined by

$$\pi_{C\to\{m\}}(\omega) = \omega_m \qquad \forall\omega \in \Omega_C$$

More generally, for any subcontext $D \subseteq C$, we define the projection

$$\pi_{C\to D} : \Omega_C \to \Omega_D$$

by restriction to the coordinates indexed by D.

Intuitively, the projection $\pi_{C\to\{m\}}$ the **perspective** C has on the measurament label m. The inverse images of projections define canonical measurable subsets of a context space.

Definition 3.11 (Cylinder Sets)**.** Let C be a context and $m \in C$. For any $A \in \mathcal{F}_m$, the corresponding **cylinder set** in Ω_C is defined by

$$\pi^{-1}_{C\to\{m\}}(A) := \{\omega \in \Omega_C \mid \omega_m \in A\}$$

Since π is a measurable function, all cylinder sets belong to $\mathcal{F}_C$.

Contexts do not introduce independent information: all events in a context arise as pullbacks of events associated with its measurement labels via the canonical projections. In this sense, a context contains no information that does not originate from its constituent labels.

Empirical Model

We now introduce the first concept that explicitly involves probability measures: the notion of an **empirical model**.

Intuitively, an empirical model represents a particular realization of the system as accessed through different contexts. Equivalently, it may be viewed as a family of probability distributions describing the observable statistics associated with each context, subject to consistency on overlaps.

Definition 3.12 (Empirical model)**.** An **empirical model** is a family

$$\{\mu_C\}_{C\in\mathcal{C}}$$

of probability measures, where for each context C,

$$\mu_C \text{ is a probability measure on } (\Omega_C, \mathcal{F}_C)$$

These measures are required to satisfy the following *compatibility condition*: For all $C, C' \in \mathcal{C}$ and all subcontexts $D \subseteq C \cap C'$,

$$(\pi_{C\to D})_*\, \mu_C \;=\; (\pi_{C'\to D})_*\, \mu_{C'}$$

This condition expresses the requirement that the probability distributions assigned to different contexts agree on their common measurement labels, and hence represent consistent marginals of a single underlying empirical situation.

Contextuality

Empirical models allow us to define a central notion driving the framework developed in this work, that of **contextuality**. Informally, contextuality captures the failure of different observational perspectives to arise as consistent restrictions of a single global description.

Definition 3.13 (Contextuality)**.** Let $\{\mu_C\}_{C\in\mathcal{C}}$ be an empirical model. The empirical model is said to be **contextual** if there exists no probability measure μ on the global measurable space $(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}})$ such that

$$(\pi_{\mathcal{M}\to C})_* \mu \;=\; \mu_C \qquad \forall C \subset \mathcal{C}$$

If such a probability measure exists, the empirical model is called **non-contextual**.

Intuitively, non-contextuality means that the probabilistic data obtained from all contexts can be understood as arising from a single joint probability distribution on the global space, with each context revealing only a partial perspective of that global state.

Contextuality, by contrast, indicates that no such global probability measure exists: although each context admits a well-defined probabilistic description, these descriptions cannot be combined into a single coherent global model.

The existence of contextual empirical models is not merely a formal possibility. It is well established that there are experimental scenarios for which no non-contextual global description exists; a detailed analysis can be found in [**?**]. The notion introduced here corresponds to probabilistic contextuality in the sense of [**?**], expressed in measure-theoretic language.

Standing Assumptions

All measurable spaces considered in this work are assumed to be standard Borel spaces. Probability measures are taken to be Borel probability measures satisfying the usual regularity conditions required for the constructions used below.

4 Structural Contextuality

To better understand RIF, we consider formulations of contextuality that do not rely on the specification of particular probability distributions. Instead, we focus on the underlying structural constraints imposed by measurable spaces themselves.

This perspective is inspired by the sheaf-theoretic approach to contextuality, but goes beyond probabilistic inconsistency. In particular, it aligns with1 the notion of **strong contextuality** as defined in [**?**], where obstruction arises already at the level of compatible local descriptions.

4.1 Information Frames

We begin by introducing the notion of an **information frame**.

Definition 4.1 (Information Frame)**.** Let $C \subseteq \mathcal{M}$ be a context. An **information frame** over C is a measurable space of the form

$$\mathscr{I}_{C,\mathcal{F}} := \left(\prod_{m\in C} \Omega_m, \mathcal{F}\right)$$

Where $\mathcal{F}$ is a σ-algebra satisfying

$$\mathcal{F} \subseteq \bigotimes_{m \in C} \mathcal{F}_m$$

The σ-algebra $\mathcal{F}$ represents the set of distinctions that the frame is able to make about the measurement labels in C.

An information frame may be interpreted as a perspective on the system: it specifies what can, in principle, be distinguished within the context C. Probability measures on $\mathscr{I}_{C,\mathcal{F}}$ then represent particular states compatible with that perspective.

We note that when a probability measure is specified on an information frame, events are understood operationally up to null sets. That is, events that differ only on a set of measure zero are identified as representing the same distinction from the perspective of that frame.

For notational convenience, when no ambiguity arises we write

$$\mathscr{I}_i \;=\; \mathscr{I}_{C_i,\mathcal{F}_i}$$

4.2 Structural Contextuality

Shared Events

First we must establish when two events in different contexts carry **shared** meaning.

Definition 4.2 (Shared Event)**.** Let $C, C' \in \mathcal{C}$ be contexts with nonempty intersection

$$L := C \cap C' \neq \emptyset$$

An event $E_L \in \mathcal{F}_L$ is called a **shared event** of C and C'.

The corresponding events in the context spaces $(\Omega_C, \mathcal{F}_C)$ and $(\Omega_{C'}, \mathcal{F}_{C'})$ are given by the cylinder sets

$$E_C := \pi^{-1}_{C \to L}(E_L), \qquad E_{C'} := \pi^{-1}_{C' \to L}(E_L)$$

Embeddings and Event images

We also note that, since all spaces considered are standard Borel, any embedding used in this work is understood to be a Borel embedding. In particular, the image of an embedding is a measurable subset of the target space. Consequently, for a embedding

$$\iota : (\Omega_1, \mathcal{F}_1) \to (\Omega_2, \mathcal{F}_2)$$

and any event $E \in \mathcal{F}_1$, we have

$$\iota(E) \in \mathcal{F}_2$$

In fact, for the purposes of this work, general σ-algebra homomorphism suffice. Under the standing assumptions, any measurable embedding induces an injective σ-algebra homomorphism. We therefore freely move between these equivalent perspectives when convenient.

Structural Contextuality

We can now make precise a definition of contextuality that does not rely on probability measures.

Definition 4.3 (Structural Contextuality)**.** Given a family of contexts $\mathcal{C}$ over $\mathcal{M}$ with corresponding information frames $\mathscr{I}_{C,\mathcal{F}_C}$ and a global space $(\Omega, \mathcal{F})$.

The family is said to be **structuraly contextual** if there exists no family of **embeddings** into a common sub-σ-algebra $\mathcal{G} \subseteq \mathcal{F}$,

$$\iota_{C\in\mathcal{C}} : \mathscr{I}_{C,\mathcal{F}_C} \to (\Omega, \mathcal{G}) \qquad C \in \mathcal{C}$$

Such that for every pair C, C' with $L = C \cap C'$ and every **shared event** $E_L \in \mathcal{F}_L$,

$$\iota_C(\pi^{-1}_{C\to L}(E_L)) = \iota_{C'}(\pi^{-1}_{C'\to L}(E_L))$$

If such a family of embeddings exists, the family of frames is called **structurally non-contextual**.

Intuitvely, structural contextuality relies criticaly on the failure of existence of *any* compatible embedding. That is, the events structure of the contexts are incompatible to the extent that no faithful realization can identify shared events globally while preserving the full informational content of each context.

Relationship with strong contextuality

While this notion is distinct from probabilistic contextuality, it is closely related. When event algebras fail to embed compatibly, any attempt to assign a single global probability measure respecting all contextual distinctions necessarily requires coarse-graining, and may fail entirely.

The existence of **strong contextuality** in the sense of [**?**] guarantees that structural contextuality occurs for suitably fine event structures. In particular the Kochen–Specker results [**?**] show that retaining the full σ-algebra structure of measurement contexts eventually obstructs any global realization. This highlights the structural inevitability of contextuality in sufficiently rich logics.

As a illustration in the product space, this can be viewed as the failure of cylinder sets to capture the full complexity of the contextual event structures. Consider, for example, two embeddings

$$\iota_1(x) = (x, f(x)) \qquad \iota_2(y) = (g(y), y)$$

Here the embedding is fixed on the degrees of freedom controlled by each context, while the remaining components are unconstrained. Structural contextuality does not arise from a lack of freedom in choosing the functions f and g, but from the requirement that they simultaneously preserve the full event structures of the contexts. When shared events are refined incompatibly across contexts, no choice of f and g can reconcile all induced events in a single global-algebra.

5 The Relativity Of Information Frames

5.1 Interaction

We now make precise the meaning of the **Relativity of Information Frames (RIF)**. The central objects of the theory are **Information Frames** from definition 4.1, which are taken as the only ontic objects. All physical content arises from their interactions.

We define interaction structurally, without reference to dynamics or time, as the co-realization of multiple information frames within a common joint frame.

Before proceeding, we note that when writting

$$\iota_C(\mathcal{F}_C) \subseteq \mathcal{G}$$

we implicitly refer to a sub-σ-algebra $\mathcal{G}$ of the codomain induced by the embedding ι_C. Different choices of embeddings are related by measurable automorphisms of the codomain and are therefore treated as **gauge-equivalent**.

Definition 5.1 (Joint Frame)**.** Let $\{\mathscr{I}_i\}_{i\in I}$ be a family of information frames, with $\mathscr{I}_i = (\Omega_{C_i}, \mathcal{F}_i)$. Define the joint label set

$$\mathcal{C} := \bigcup_{i\in I} C_i$$

The **joint frame** is the measurable space

$$\mathcal{J}_I := (\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I})$$

where

$$\Omega_{\mathcal{J}_I} := \prod_{m\in\mathcal{C}} \Omega_m, \qquad \mathcal{F}_{\mathcal{J}_I} := \sigma\left(\bigcup_{i\in I} \iota_i(\mathcal{F}_i)\right)$$

The joint frame represents the space capable of expressing all distinctions accessible to the interacting frames. While such frame is always definable at the level of measurable structure, structural contextuality may obstruct the existence of a σ-algebra in which all shared events are consistently identified.

Once a joint realization is fixed, the embeddings $\{\iota_i\}$ are not allowed to vary within that realization. Different realizations related by automorphisms are treated as gauge-equivalent, but embeddings are fixed within each gauge choice.

5.2 Local Perspectives

To make the relativity principle precise, we require a way to compare contexts within a fixed joint realization. Although a joint frame $\mathcal{J}_I$ is constructed via embeddings $\{\iota_i\}_{i\in I}$, the contextual frames themselves do not directly live in the joint space.

This comparison is achieved through the following maps.

Definition 5.2 (Local Perspective Maps)**.** Let $\mathcal{J}_I = (\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I})$ be a joint frame generated by the embeddings

$$\iota_i : \Omega_{C_i} \hookrightarrow \Omega_{\mathcal{J}_I}$$

The **local perspective map** associated with frame $\mathscr{I}_i$ is the measurable map

$$e_i \ := \ \iota_i \circ \pi_{\mathcal{J}_I \to C_i} : \ \Omega_{\mathcal{J}_I} \to \Omega_{\mathcal{J}_I}$$

Note: Once more, under the standing assumptions adopted throughout this work, each local perspective map e_i is measurable and therefore induces a endomorphism of the joint σ-algebra via pullback. By abuse of notation, we use the same symbol e_i to denote both the measurable map on Ω_{J_I} and its induced action on events.

The maps e_i represent the full event structure of the information frame $\mathscr{I}_i$ as embedded in the joint frame, intuitively the perspective of that frame on the interaction.

Proposition 5.3 (e_i are idempotent)**.** *The local perspective maps e_i are idempotent measurable endomorphisms of $(\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I})$.*

Proof. By definition $e_i = \iota_i \circ \pi_{\mathcal{J}_I \to C_i}$. Since $\pi_{\mathcal{J}_I \to C_i} \circ \iota_i = \mathrm{id}_{\Omega_{C_i}}$ we have $e_i \circ e_i = e_i$ □

Remark 5.4. *The maps e_i act as coarse-grainings of the joint event structure: two events are identified whenever they induce the same event on the contextual frame C_i.*

5.3 The Symmetry Of Information

Privilege

We are now ready to introduce the central concept underlying the **Relativity of Information Frames** the concept of privilege.

Definition 5.5 (Privilege)**.** We say that a joint frame $\mathcal{J}_I$ privileges $\mathscr{I}_i$ over $\mathscr{I}_j$ for a shared event $E \in \iota_i(\mathcal{F}_i) \cap \iota_j(\mathcal{F}_j)$ if, for the corresponding local perspective maps e_i, e_j we have

$$e_j(E) \neq e_j(e_i(E))$$

Equality is understood at the level of events in the joint σ-algebra (or up to null set equivalences under the standing assumptions).

Intuitively, a joint frame privileges $\mathscr{I}_i$ over $\mathscr{I}_j$ whenever the perspective of $\mathscr{I}_i$ alters what $\mathscr{I}_j$ sees. That is, conditioning on $\mathscr{I}_i$'s interpretation of an event changes $\mathscr{I}_j$'s interpretation.

We note that privilege is witnessed by a failure of commutation of the local perspective maps on the event E:

$$e_i \circ e_j(E) \neq e_j \circ e_i(E)$$

Finally, privilege is not an ordering relation: it may occur in both directions for the same event.

The Relativity of Information Frames

We can now state the relativity of information frames precisely.

Axiom 1 (The Relativity Of Information Frames). *After interaction, an event is physically admissible if and only if it does not privilege one information frame over another. Equivalently, in the physically admissible joint frame, no information frame is privileged over another for any event in its σ-algebra.*

More explicitly for any pair of frames i, j of the joint frame and any shared event E we have:

$$e_i(E) = e_i(e_j(E)) \quad \text{and} \quad e_j(E) = e_j(e_i(E))$$

The Frame Pointer Algebra

Next, with axiom 1 in mind we consider the set of physical events for a given information frame $\mathscr{I}_i$ during an interaction:

$$\mathcal{F}_i^{\text{phys}} := \{E \in \iota_i(\mathcal{F}_i) \mid e_j \circ e_i(E) = e_j(E) \quad \forall j :\ E \in \iota_j(\mathcal{F}_j)\}$$

That is, the events that do not privilege $\mathscr{I}_i$ over any other frame.

Proposition 5.6. *The sets in $\mathcal{F}_i^{phys}$ form a σ-algebra.*

Proof. If $E \in \mathcal{F}_i^{\text{phys}}$ then, for every j:

$$e_j(e_i(E^c)) = e_j(e_i(E)^c) = e_j(e_i(E))^c = e_j(E)^c = e_j(E^c)$$

So $E^c \in \mathcal{F}_i^{\text{phys}}$.

And let E_n be a countable collection of RIF-valid events, that is:

$$e_j(e_i(E_n)) = e_j(E_n) \quad \forall n, j$$

Since:

$$e_j\left(\bigcup_n E_n\right) = \bigcup_n e_j(E_n)$$

Given any $j \in I$ we have:

$$e_j\left(e_i\left(\bigcup_n E_n\right)\right) = e_j\left(\bigcup_n e_i(E_n)\right) = \bigcup_n e_j \circ e_i(E_n) = \bigcup_n e_j(E_n) = e_j\left(\bigcup_n E_n\right)$$

□

We give therefore the special name:

Definition 5.7 (Frame Pointer Algebra)**.** The algebra of physically admissible events for a frame is called the **frame pointer algebra**.

$$\mathcal{F}_i^{\text{ptr}} = \mathcal{F}_i^{\text{phys}}$$

The Pointer Algebra

The frame pointer algebras in definition 5.7 represent the physical events from the context of each information frame. However, the events that can happen during a interaction are the events that satisfy rif in general for the contexts of each interaction.

Definition 5.8 (Pointer Algebra)**.** The **pointer algebra** for the joint frame $\mathcal{F}^*_{\mathcal{J}_I} \subseteq F_{\mathcal{J}_I}$ is the algebra:

$$F^*_{\mathcal{J}_I} := \sigma\left(\bigcup_{i \in I} \mathcal{F}_i^{\text{ptr}}\right)$$

When the joint frame is obvious we may simply refer to the pointer algebra as $\mathcal{F}^*$.

Interpretation

Local perspective maps do not represent physical operations performed in time, but rather encode how a given information frame identifies events within a joint description.

A privilege occurs precisely when two frames cannot consistently identify the same event without reference to an ordering of perspectives, signaling the absence of a joint description for that event.

Frame pointer algebras therefore collect those events that admit a stable interpretation from the perspective of a given frame during an interaction. The pointer algebra of the joint frame is generated by all such non-privileging events, and represents the maximal event structure that can be jointly realized without privileging any information frame.

This algebra captures the emergent classical structure associated with the interaction.

Coarse-Graining and the Emergence of Probability.

Remark 5.9. *In this framework, probability is not taken as a primitive notion. Instead, probabilistic structure arises as a consequence of coarse-graining enforced by axiom 1.*

When an interaction removes distinctions that cannot be jointly maintained across information frames, multiple incompatible bookkeeping events are identified as a single physically admissible event in the pointer algebra. From the perspective of an individual frame, these identified events are indistinguishable, yet no further structural information remains available to discriminate between them.

Any assignment of weights to physically admissible events that is stable under further coarse-graining and compatible with the structure of the pointer algebra must therefore take the form of a probability measure. In this sense, probabilities encode the residual information accessible to a frame after incompatible distinctions have been eliminated, rather than reflecting intrinsic randomness or ignorance of an underlying reality.

5.4 Relation to Contextuality

Structural contextuality makes precise the close relationship between the Relativity of Information Frames and contextuality in the usual sense. In particular, if a family of information frames is structurally non-contextual in the sense of definition 4.3, then no privileged events arise and the pointer algebra coincides with the full σ-algebra of the joint frame.

Proposition 5.10 (Pointer algebra relation to structural contextuality)**.** *If $\{\mathscr{I}_i\}_{i\in I}$ is a family of information frames as and $\{\iota_i\}_{i\in I}$ their corresponding embeddings. Then the following hold for the* ***pointer algebra of the joint frame***

- *if the family is structurally non-contextual $\mathcal{F}^*_{\mathcal{J}_I} = \mathcal{F}_{\mathcal{J}_I}$*
- *if the family is structurally contextual $\mathcal{F}^*_{\mathcal{J}_I} \subsetneq \mathcal{F}_{\mathcal{J}_I}$*

Proof. The proof follows directly from the definitions. $\square$

While this observation already captures a nontrivial physical mechanism - namely, the elimination of incompatible distinctions - we now establish a stronger and more informative result. Specifically, we show that the pointer algebra is, in a precise sense, the **maximally informative non-contextual algebra** compatible with the given joint structure.

Structural contextuality alone is not sufficient for our purposes, as it detects contextuality only when one attempts to preserve the full informational structure of each frame. A similar limitation applies to standard probabilistic contextuality. However, probability measures provide additional flexibility, allowing one to distinguish between different σ-algebras on the joint frame without modifying the underlying event structure.

We therefore fix a consistent empirical model $\{\mathscr{I}_i\}_{i\in I}$ as in definition 3.12. For each context we consider the associated information frame $\mathscr{I}_i$, with event algebras representing the support of the corresponding probability measures, as described previously. Using a choice of embeddings $\{\iota_i\}_{i\in I}$, we construct the joint frame $\mathcal{J}_I$.

We begin by defining the following induced measure on the joint frame:

$$\tilde{\mu_i}(E) := \mu_i(\iota_i^{-1}(E)), \qquad E \in \iota_i(\mathcal{F}_i) \tag{1}$$

Which represents the ppushfoward of μ_i to the embedded event algebra. Restricting this measure to the frame pointer algebra yields

$$\tilde{\mu}_i^{\text{ptr}} := \tilde{\mu}_i \upharpoonright \mathcal{F}_i^{\text{ptr}} \tag{2}$$

In order to glue these restricted measures into a single global measure on the pointer algebra $\mathcal{F}^*$, it is necessary that they agree on overlaps:

$$\tilde{\mu}_i^{\text{ptr}}(E) = \tilde{\mu}_j^{\text{ptr}}(E) \qquad \forall E \in \mathcal{F}_i^{\text{ptr}} \cap \mathcal{F}_j^{\text{ptr}}. \tag{3}$$

We then state the following lemma:

Lemma 5.11 (Pointer Overlap Consistency)**.** *If $E \in \mathcal{F}_i^{\text{ptr}} \cap \mathcal{F}_j^{\text{ptr}}$, then E is a* ***shared event*** *whose identification is order-independent, hence any empirical family assigns it the same weight.*

Sketch. By construction, events in $\mathcal{F}_i^{\text{ptr}} \cap \mathcal{F}_j^{\text{ptr}}$ admit a frame-independent identification in the joint structure. Such events correspond to shared events whose embeddings agree up to σ-algebra homomorphisms, and hence empirical consistency on overlaps implies the equality of their assigned weights. □

From this lemma, it is obvious that eq. (3) holds, which allows us to state the desired result

Proposition 5.12 (Existence of a non-contextual measure)**.** *If $\tilde{\mu}_i^{\text{ptr}}$ agree on overlaps, there exists a probability measure μ^* on $(\Omega_{\mathcal{J}}, \mathcal{F}^*)$ extending them.*

Sketch. We define a pre-measure μ_0 for $\mathcal{F}^*$, since the frame pointer algebras are the generators of $\mathcal{F}^*$ we can define, whenever $E \in \mathcal{F}_i^{\text{ptr}}$

$$\mu_0(E) = \tilde{\mu}_i^{\text{ptr}}(E)$$

Since we have eq. (3), this is well defined. Since $\mu_0(\Omega_{\mathcal{J}}) = 1$ and $\mu_0 \geq 0$ we can use the *Caratheodory Extension Theorem* **??** to get a measure μ^* on $\mathcal{F}^*$ with the required overlap consistency. □

This result shows that, starting from a empirical model which may or may not be contextual on the constructed joint frame, the potential coarse graining to the pointer algebra directly admits a consistent probability measure with that empirical model.

In what follows, since we had fixed a empirical model and constructed the joint frame and pointer algebra relative to the σ-algebraic structure induced by the that model. Maximality here is understood relative to this joint frame.

Proposition 5.13 (Maximality of the pointer algebra)**.** *Any strict extension $\mathcal{F}^* \subsetneq \mathcal{G} \subseteq \mathcal{F}_{\mathcal{J}_I}$ fails to admit a consistent global measure extending the fixed empirical family.*

Sketch. Since $\mathcal{G}$ must have a event E that is not **RIF admissible** we have for some pair i, j:

$$E \in \iota_i(\mathcal{F}_i) \cap \iota_j(\mathcal{F}_j) \quad \text{and} \quad e_j(e_i(E)) \neq e_j(E)$$

Since the events are in the support from the initial assumptions and they are distinct events we must have:

$$\tilde{\mu}_j(e_j(e_i(E))) \neq \tilde{\mu}_j(e_j(E))$$

Then suppose we can build a global measure. But as we have seen, consistency would require

$$\tilde{\mu}_j(e_j(e_i(E))) = \tilde{\mu}_j(e_j(E))$$

So $\mathcal{G}$ cannot admit a consistent global measure. □

These results imply a strong correlation between contextuality and the RIF pointer algebra.

Theorem 5.14 (The Pointer Algebra is the Maximally Informative Non-Contextual Algebra)**.** *For a joint frame built from a family of information frames the pointer algebra is the largest σ-algebra that admits consistent probability measures.*

Remarks

We note that this result is in some sense informal, there is a real limiation on how contextuality is defined to talk about a maximally informative **non-contextual** algebra, the construction is far more natural in the **RIF** definition, but the arguments strongly show the link between the concepts.

It is important to note that if one fixes the joint frame first, and look at what empirical families can be built on it. Non-contextuality of the pointer algebra becomes relative to this joint frame. With this understanding, the pointer algebra has two key properites:

- Any consistent empirical family admits a global probability measure on the pointer algebra.
- Any strict extension of the pointer algebra admits a locally consistent probability assignment that do not glue to a global measure.

Part II
Physical Structure and Consequences

6 Quantum Mechanics

In this section we clarify the connection between the RIF framework and quantum theory. Our goal is not a full formal reconstruction of quantum meachanics, but to show that the event structures arising naturally in RIF coincide with the algebraic structures underlying quantum theory. In particular, we show that the collection of contextual event algebras forms an orthomodular lattice, and that standard Hilbert space realizations arise under the usual structural assumptions. Consequences such as the Born rule and Wigner-type consistency conditions will then be seen to follow naturally from RIF invariance.

6.1 The Hilbert Space Realization

We will not attempt a full Hilbert space reconstruction in this work. Instead, we situate the RIF event structure within the well-established framework of quantum logic, highlighting where RIF provides a natural physical interpretation of the underlying assumptions.

The Orthomodular Lattice of Events

Let $(\mathcal{F}_i)_{i\in I}$ denote the Boolean σ-algebras associated to each information frame (context), and let the usual embeddings $(\iota_i)_{i\in I}$ into a joint frame, as definition 5.1, be given.

From the usual quantum logic perspective, the events on the joint frame algebra form a lattice.

$$\mathcal{L} := \mathcal{F}_{\mathcal{J}_I} := \sigma\left(\bigcup_{i\in I} \iota_i(\mathcal{F}_i)\right)$$

Naturally $\mathcal{L}$ is a lattice with its operations

- complement $E \to E^c$,
- meet: $E \wedge F = E \cap F$,
- join: $E \vee F = \mathrm{cl}_{\mathcal{L}}(E \cup F)$

Proposition 6.1 ($\mathcal{L}$ is a orthomodular lattice)**.** *The event structure $\mathcal{L}$ is an orthomodular lattice. Each contextual algebra $\iota_i(\mathcal{F}_i)$ embeds as a maximal Boolean subalgebra of $\mathcal{L}$.*

Sketch. Each $\mathcal{F}_i$ is a Boolean algebra and therefore an orthocomplemented distributive lattice. The embeddings ι_i preserve complements and finite meets, ensuring that $\mathcal{L}$ is orthocomplemented.

Distributivity fails in $\mathcal{L}$ whenever events arising from incompatible frames cannot be jointly refined, which is precisely the manifestation of contextuality in the RIF framework. However, the consistency of partial refinements between compatible events ensures that the orthomodular law holds. Thus $\mathcal{L}$ is an orthomodular, but generally non-distributive lattice. □

Under standard assumptions, the same we established in this framework, classical results in quantum logic [**?**] imply that $\mathcal{L}$ admits a representation as the projection lattice of a Hilbert space:

$$\mathcal{L} \cong \mathsf{Proj}(\mathcal{H})$$

where $\mathcal{H}$ is a Hilbert space over $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$.

Relabeling Gauge of the Joint Frame - Unitary Action

The joint frame construction $\mathcal{J}_I$ introduces an intrinsic gauge freedom. Since $\Omega_{\mathcal{J}_I}$ is a product space over labels $\mathcal{C}$, different measurable relabelings of the joint sample space may induce the same relational event structure. This redundancy is not an additional assumption, but is present by definition.

Let $\mathrm{Aut}(\mathcal{J}_I)$ denote the group of measurable bijections $T : \Omega_{\mathcal{J}_I} \to \Omega_{\mathcal{J}_I}$ preserving the joint σ-algebra $\mathcal{F}_{\mathcal{J}_I}$. Fixing a reference frame $i \in I$, we define the relabeling gauge group relative to i by

$$G_i \;:=\; \big\{T \in \mathrm{Aut}(\mathcal{J}_I) \;:\; T^{-1}(\iota_i(\mathcal{F}_i)) = \iota_i(\mathcal{F}_i)\big\}.$$

Elements of G_i correspond to relabelings of the joint description that leave invariant the event structure accessible to frame i, while re-identifying how other contexts are embedded.

Two joint-frame realizations are said to be gauge-equivalent relative to i when they are related by an element of G_i. The physical content of the RIF construction is thus identified with structures invariant under this relabeling gauge.

Upon translation to a Hilbert space realization $\mathcal{F}_{\mathcal{J}_I} \cong \mathsf{Proj}(\mathcal{H})$, this relabeling gauge is represented by the familiar projective **unitary** (and antiunitary) symmetries of the Hilbert description. Parametrizations of subgroups of G_i may be introduced for convenience, but are not fundamental at the level of RIF.

Complex Structure and Gauge Freedom

The Hilbert space realization of the joint-frame event structure carries, by construction, a representation of the intrinsic relabeling gauge of the RIF framework. This gauge is implemented in the Hilbert description as a projective unitary symmetry acting on $\mathcal{H}$.

Supporting a nontrivial and continuous projective unitary action places strong constraints on the underlying scalar field. Real Hilbert spaces do not admit a sufficiently rich phase structure to represent generic gauge transformations, while quaternionic Hilbert spaces introduce additional constraints on the localization and composition of such symmetries.

By contrast, complex Hilbert spaces provide the minimal setting in which continuous projective unitary representations, local gauge freedom, and consistent composition of independent subsystems coexist. From the RIF perspective, the appearance of complex structure is therefore not an independent postulate, but a natural consequence of representing the intrinsic joint-frame gauge in a linear space.

A Structural Remark on Internal Gauge Symmetry

The relabeling gauge inherent in the joint-frame construction is represented, upon a Hilbert-space realization of the framework, as a projective unitary symmetry acting on $\mathcal{H}$. At this level, the gauge symmetry is generically very large, reflecting the freedom in identifying joint descriptions related by relabeling of events.

A further restriction on this relabeling gauge arises from the presence of local perspective maps and the requirement that no information frame be privileged, as formalized by axiom 1. While $\mathrm{Aut}(\mathcal{J}_I)$ represents the full descriptive redundancy of the joint frame, not all such relabelings are compatible with perspectival symmetry.

In particular, admissible relabelings must preserve the equivalence of descriptions induced by local perspective maps, in the sense that no information frame can detect a preferred identification of joint events. This requirement selects a distinguished subgroup of the relabeling gauge, consisting of transformations that are relationally invisible across all frames.

Upon Hilbert-space realization, this perspectivally admissible gauge is represented as a restricted subgroup of the projective unitary symmetry.

We emphasize that no derivation of a specific gauge group is claimed here. Rather, this discussion is intended to indicate that familiar gauge symmetries, including those of the Standard Model, are compatible with—and may be viewed as particular reductions of—the intrinsic relabeling gauge symmetry present in the RIF framework, once restricted to physically admissible transformations.

6.2 Observables, Measurements and Operators

Measurement Device

In the RIF framework, a measurement device is represented by a pure information frame $\mathscr{I}_{\text{mes}} := (\Omega_{\text{mes}}, \mathcal{F}_{\text{mes}})$. The event algebra $\mathcal{F}_{\text{mes}}$ encodes the distinctions that the device is capable of registering. No additional structure is assumed.

Observables

An observable associated with an information frame $\mathscr{I}_i$ is a measurable function

$$O_i : \Omega_i \to \mathcal{O}$$

Where $\mathcal{O}$ is an outcome space, such as $\mathbb{R}$ or a discrete set. Through the embedding ι_i, each observable induces a corresponding random variable on the joint frame, defined on the embedded subspace by

$$\widetilde{O}_i := O_i \circ \iota_i^{-1}$$

The σ-algebra generated by $\widetilde{O}_i$ represents the collection of events distinguishable by the observable O_i. Two observables are sait to be **compatible** if their induced σ-algebras generate a jointly Boolean algebra in the joint frame.

Observable-Induced Operators

To see the relation with operator non-commutativity, it is convenient to consider the coarse-graining maps induced by observables. Given an event E in the joint frame and an observable O_i, we define the observable-induced map

$$e_i^{O_i}(E) := \iota_i\big(O_i^{-1}\big(O_i(\pi_i(E))\big)\big)$$

which represents the coarse-graining of E according to the distinctions accessible to O_i. For observables associated with different frames, these maps need not commute:

$$e_i^{O_i} \circ e_j^{O_j} \neq e_j^{O_j} \circ e_i^{O_i}$$

reflecting the incompatibility of the corresponding observables.

Measurement

A measurement of an observable O_i corresponds to a full interaction and conditioning the joint description on an event in the frame pointer algebra $\mathcal{F}_i^{\mathrm{ptr}}$. Only events belonging to this algebra are physically admissible outcomes of the measurement. The coarse-graining implicit in the pointer algebra identifies multiple fine-grained events as a single measurement outcome, thereby inducing a probabilistic description as per remark 5.9.

Repeated measurements of the same observable correspond to an already resolved conditioning on the same pointer event and therefore yield stable outcomes. In contrast, an interaction with an incompatible information frame can be seen as a new interaction that induces a new coarse-graining, reintroducing distinctions that were previously suppressed. In this sense, measurement outcomes are not destroyed but rendered frame-relative by subsequent incompatible interactions.

Relation to POVMs

When the RIF framework is represented on a Hilbert space, the coarse-grained event structure associated with a measurement naturally gives rise to positive operator-valued measures. Each pointer event $E \in \mathcal{F}_i^{\mathrm{ptr}}$ corresponds to an equivalence class of fine-grained events, and the probability assigned to such an event by the induced global measure may be represented as $\mathrm{Tr}(\rho, E_i)$ for a positive operator E_i.

The non-projective nature of these operators reflects the fact that pointer events are defined by coarse-graining rather than by sharp partitions of the joint space. In this sense, POVMs arise as faithful representations of frame-relative measurements within the RIF framework. Projective measurements then represent situations where there is no coarse-graining in the representation.

6.3 Stern-Gerlach in RIF

We end with a informal analysis of the Stern-Gerlach test [?] using the RIF framework. We will model the standard sequence:

1. measure spin along z (device A),
2. measure spin along x (device B),
3. then measure along z again (device A).

Setup

We begin with modeling the three information frames, the system $\mathscr{I}_S$, and the two devices $\mathscr{I}_A, \mathscr{I}_B$. After interaction, their frame pointer algebras are represented by:

$$\mathcal{F}_A^{\mathrm{ptr}} = \sigma(\{Z+, Z-\}) \quad \text{and} \quad \mathcal{F}_B^{\mathrm{ptr}} = \sigma(\{X+, X-\})$$

Interaction 1: System and device A

The first interaction constructs the joint frame $\mathcal{J}_{SA}$ using embeddings ι_S, ι_A, then impose RIF and restrict to the pointer algebra $\mathcal{F}_{SA}^*$.

The outcomes A can see from the measurement is a event in is frame pointer algebra:

$$E_A \in \mathcal{F}_A^{\mathrm{ptr}} \subseteq F_{SA}^*$$

From A's perspective, repeated measurements in the same interaction context is stable:

$$Z+ \text{ then } Z+ \quad \text{or} \quad Z- \text{ then } Z-$$

Assume for this sequence that the measurement resulted in $Z+$.

Interaction 2: System and device B

Now B interacts with the composite frame $\mathcal{J}_{SA}$, producing a new joint frame $\mathcal{J}_{SAB}$ with its own pointer restriction $\mathcal{F}_{SAB}^*$. Since the Z and X distinctions are incompatible in the RIF sense, the second interaction induces a further coarse-graining of the admissible event structure.

The x-measurement is not "reading a pre-existing x value"; the interaction is creating a new coarse-grained joint structure and selecting a event within it. That means, conditioning on the earlier $Z+$ event, B sees:

$$\mathbb{P}(X+ \mid Z+) = \mathbb{P}(X- \mid Z+) = \frac{1}{2}$$

Again, after the interaction repeated x-measurements are stable from B's point of view, assume that the resulting interaction yielded $X+$.

Interaction 3: System and device A again

Now A is interacting, not with the original $\mathcal{J}_{SA}$ but with the new composite that has undergone an incompatible interaction with B.

The event labeled $Z+$ after the first interaction does not correspond to the same admissible event after the incompatible interaction with B, since the underlying pointer algebra has changed

$$\mathbb{P}(Z+ \mid X+) = \mathbb{P}(Z- \mid X+) = \frac{1}{2}$$

This does not represent a physical disturbance propagating from B to A, but a change in the admissible joint description induced by an incompatible interaction.

Conclusion

This models the sequences of tests as a series of interaction in RIF. We note that this analysis is not a final objective truth of what is happening, it is simply a intrepretation of the mathematics in a easy to reason standard.

Similar intrepretations could be done with a joint frame built by all involved interactions directly and seeing measurement as a sampling that shifts the events. Or how quantum logic might normally view such structure.

The main result here, is that interaction is relational, in the RIF framework, a globally defined event structure is not merely unnecessary but ill-defined as a physical notion. Any attempt to treat a global description as ontic would privilege descriptions that would violate axiom 1. Admissible events are therefore defined only relative to interactions, and no single frame provides a complete description of what occurs.

6.4 Born Rule Martingale

In this section we explore how the Born rule appears in this framework. We do not claim, with the current tools, to recover the quadratic nature of the born rule or its usual form. That is left to a reconstruction of the Hilbert space.

Setup

First we consider a particular context's frame $\mathscr{I}_i$. We define a probability measure representing that frame μ_0.

In the joint frame we can look at the pushfoward of the contextual measure μ_0 to the joint σ-algebra.

$$\mu := \mu_0 \circ \iota_i^{-1} = \mu_0(\iota_i^{-1}(E)) \quad E \in \mathcal{F}_{\mathcal{J}}$$

We note that this probability measure does not, nescessarly, represent all contexts of the joint frame. In fact, in contextual cases, that is not possible. This is simply the probability of the context i transported to the global frame.

The Collapse Filtration

Since the pointer algebra $\mathcal{F}^*$ is a sub-σ of $\mathcal{F}_{\mathcal{J}}$, we may consider any decreasing family of σ-algebras.

$$\mathcal{F}_{\mathcal{J}} =: F_0 \supseteq F_1 \supseteq F_2 \supseteq ...\mathcal{F}^*$$

representing successive coarse-grainings of the joint description.

While the collapse happens directly to the pointer algebra upon interaction, these filtrations represent partial descriptions, this sequence can be interpreted heuristically as successive partial information updates - analogous to **weak or partial measurements**.

The Collapse Martingale

We can now use μ as a probability measure to define, for any event $A \in \mathcal{F}_{\mathcal{J}}$ we define:

$$M_n := \mathbb{E}_\mu \left[\mathbf{1}_A \mid \mathcal{F}_n \right]$$

As seen [1] we know this is a Martingale on the reverse filtration, and by Doob convergence theorem we have:

$$M_n \to \mathbb{E}_\mu \left[\mathbf{1}_A \mid F_{\text{ptr}} \right] \quad \text{(a.s.)}$$

Thus the conditional probability of A given the pointer algebra coincides with the coarse-grained probability $\mu(A)$ in that algebra. Collapse therefore corresponds, in measure-theoretic terms, to conditioning on the pointer σ-algebra.

Intrepretation

In this sense, the Born rule emerges as the statement that the observed probabilities are those of the conditional measure obtained by coarse-graining to the pointer algebra.

The stochastic character of measurement outcomes is therefore not fundamental but a reflection of information loss under coarse-graining.

The martingale formulation expresses the stability of these conditional probabilities under repeated measurement, as required by empirical repeatability.

6.5 Wigner's Friend Consistency

We will now turn our attention to looking at how Wigner's Friend paradox looks like in this framework.

Setup

For the Wigner friend scenario we acutually have two interactions. First the interaction of the friend, which we will map to the joint frame $\mathcal{J}_{\text{friend}}$. Which represents the interaction between the system and the friend.

Once that interaction goes through the system has collapsed to the pointer frame $\mathcal{J}^{\text{ptr}}_{\text{friend}}$. Then Wigner comes and interacts with that joint frame, forming a new frame $\mathcal{J}_{\text{Wigner}}$, which again must collapse to $\mathcal{J}^{\text{ptr}}_{\text{Wigner}}$.

We use the fact that, these algebras can all be seen as filtrations of each other to show that, Wigner cannot assign inconsistant probabilities to the events it can observe.

The Sequence of Filtrations

When the friend interacts with the system, he generates the joint frame $\mathcal{J}^{\text{ptr}}_{\text{friend}}$. When Wigner interacts with that frame, a new joint frame must be built. That joint frame starts by lifting the algebras through the embeddings ι_{friend} and ι_{Wigner}.

The nature of embeddings mean we can consider this all a sequence of filtrations on the same space. Namely:

$$\mathcal{F} \supseteq \mathcal{F}^{\text{ptr}}_{\text{friend}} \supseteq F^{\text{ptr}}_{\text{Wigner}}$$

With these filtrations in place, we can look at the same style of probability assignments we had in the born rule.

6.5.1 The Probabilities

Now let $A \in \mathcal{F}^{\text{ptr}}_{\text{Wigner}}$ be a event that exists in the final pointe algebra. That is, a event that all systems involved can talk about.

We can determine the friend's probaility for event A as we did for the born rule martingale:

$$M_{\text{friend}} := \mathbb{E}_{\mu}\left[\mathbf{1}_A \mid \mathcal{F}^{\text{ptr}}_{\text{friend}}\right]$$

Now for Wigner, we have the same:

$$M_{\text{Wigner}} := \mathbb{E}_{\mu}\left[\mathbf{1}_A \mid \mathcal{F}^{\text{ptr}}_{\text{Wigner}}\right]$$

But since we have

$$F_{\text{Wigner}}^{\text{ptr}} \subseteq \mathcal{F}_{\text{friend}}^{\text{ptr}} \subseteq \mathcal{F}$$

The tower law for martingales gives:

$$\mathbb{E}_{\mu}\left[M_{\text{friend}} \mid \mathcal{F}_{\text{Wigner}}^{\text{ptr}}\right] = \mathbb{E}_{\mu}\left[\mathbb{E}_{\mu}\left[\mathbf{1}_{A} \mid \mathcal{F}_{\text{friend}}^{\text{ptr}}\right] \mid \mathcal{F}_{\text{Wigner}}^{\text{ptr}}\right] = \mathbb{E}_{\mu}\left[\mathbf{1}_{A} \mid \mathcal{F}_{\text{Wigner}}^{\text{ptr}}\right]$$

Thus, when the friend's description is updated to the Wigner's pointer frame is exactly Wigner's own description.

7 Dynamics

In this version of the framework, dynamics is not introduced as a primitive law like a Hamiltonian flow or differential equation. Instead, dynamics arises from the structure of interactions between information frames.

Each interaction between contexts induces:

- A creation of a joint frame,
- followed by collapse to its pointer algebra,
- and a pushfoward updated of the preparation measure.

Thus, from a fixed frame point of view, the time evolution of it's description of the world is simply a sequence of coarse-grained σ-algebras obtained from sucessive interations, effectively a filtration:

$$\mathcal{F}_{\text{frame}} =: \mathcal{F}_0 \supseteq \mathcal{F}_1 \supseteq \mathcal{F}_2 \supseteq \dots \supseteq \mathcal{F}_n \supseteq \dots$$

This point of view makes several dynamical properties appear naturally:

1. Arrow of Time

   Because each contextual interaction corresponds to additional coarse-graining, the evolution of σ-algebras is monotone (information losing). This monotonicity defines a natural direction of time.

2. Locality Graph

   Interactions occur only between specific frames. The pattern of which frame couples to which determines a graph structure, which plays the role of space locality.

3. Maximum Speed of Influence

   In contextual scenarios, incompatibility forces coarse-graining. The maximal number of interaction steps before contextuality appears bounds how "fast" influence can propagate along the locality graph.

4. No Signaling

   Because interactions only merge σ-algebras along edges of the locality graph, and collapse happens locally on the joint frame, no frame can influence another without a mediated interaction.

5. Gravity

   The coarse graining imballance

None of these dynamical features require any additional axioms. They follow from Axiom 1 and the definitions of how interactions are built.

7.1 Arrow of Time

In this framework, time is not an external parameter. Instead we use the ordering of interactions between information frames to induce a canonical direction: each interaction generates a joint frame where the initial frame algebra is embedded, collapse forces a coarse graining of its algebra, this monotone loss of distinguishability defines a natural arrow of time for that frame.

Interaction-induced evolution of σ-algebras

To begin, we first fix a information frame $\mathscr{I}_0$. It then proceeds to interact with several frames $\mathscr{I}_{j_1}, \mathscr{I}_{j_2}$ and so on.

At each step, a joint frame is first created to host the interaction, Axiom 1 and the collapse theorem then converge into a physically realizable joint frame with a sigma algebra

$$\mathcal{F}^{\mathrm{ptr}}_{\mathcal{J}_n}$$

That is, the pointer frame on the n-th joint frame. Since this is always constructed by embedding the original σ-algebra $\mathcal{F}_0$, we can track its evolution along the interactions.

We can look at the events of $\mathcal{F}_0$ that survive in the pointer frame $\mathcal{F}^{\mathrm{ptr}}_{\mathcal{J}_1}$ as the σ-algebra of $\mathscr{I}_0$ at step 1. That is:

$$\mathcal{F}_1 := \mathcal{F}^{\mathrm{ptr}}_{\mathcal{J}_1} \cap \iota_1(\mathcal{F}_0)$$

This gives a natural sequence:

$$\mathcal{F}_0 \supseteq \mathcal{F}_1 \supseteq \mathcal{F}_1 \supseteq ...$$

Each step removes events that are incompatible with the new join. The future σ-algebras representations of $\mathcal{F}_0$ are strictly coarser algebras. There is no way physical way in the framework to restore the lost distinctions.

7.2 Locality Graph

In this theory, **locality** is not a geometric primitive yet. It is the combinatorial structure that records which frames have interacted and must therefore share a joint pointer algebra.

The Locality Graph

Let $\{\mathscr{I}_i\}_{i\in I}$ be a family of relevant information frames. We define the graph:

$$G = (V, E)$$

$$(i,j) \in E \quad \iff \quad \text{frames } \mathscr{I}_i \text{ and } \mathscr{I}_j \text{ have interacted.}$$

And at every point, an edge represents there is a joint frame where the collapse has taken place between $(i, j) \in E$.

Locality as constraint on the pointer basis

If frames i and j have interacted, then their local projections e_i and e_j both act on the same joint frame. Namely the pointer basis there must satisfy:

$$\mathcal{F}_{\text{ptr}} \subseteq \text{Fix}(e_i) \cap \text{Fix}(e_j)$$

And if they have not interacted their projetions do not constrain the same σ-algebra, which means:

- Frames in the same subgraph in G influence one another through constraints in the same pointer algebra
- Frames in different subgraphs remain completely independent. No constraints propagate between them.

This reproduces the operational notion of locality, influence propagates only along paths in the interaction graph.

Dynamics respects the locality graph

When a new interaction occurs between frames i and j we:

1. add an edge (i, j) to G;
2. build the joint frame of all contexts in the connected component containing i and j
3. collapse using all local projections in that component.

Therefore every subgraph of G behaves as local regions, every interaction affects only the σ-algebra generated by its own components.

Locality therefore appears as:

- graph locality where edges are interactions
- probabilistic locality conditional expectations factor across components of the sub graph

Intrepretation

This provides a structural notion of spacetime locality:

- Information frames as "positions".
- Interactions as "lightlike" adjacency.
- Paths in G are the only channels through which constraints can propagate.

There is no geometry at this stage, geometry would be additional structure placed on top of the locality graph.

We emphasize that "locality" and "time" in this theory are entirely structural notions. They arise from the patterns of interactions between frames, not from any pre-assumed geometric spacetime. Any system that can be expressed through interacting information frames possesses a well-defined locality graph and a induced temporal order, even when no traditional spacetime background is specified.

7.3 Maximum Speed

In this framework, influence can propagate only through interactions between frames, and such interactions are encoded by edges in the locality graph G.

However contextuality imposes an additional constraint: after a sufficiently long chain of interactions on mixed degrees of freedom, contextual incompatibility nescessarely appears. Forcing a coarse-graining of the joint σ-algebra. This defines a **time step**, the length of the chain in that time step is always bounded by contextuality.

Contextuality forces coarse-graining

A fundamental structural fact of contextuality ([**?**]) is that for a sufficiently large family of information frames on shared degrees of freedom, their combined σ-algebra on the joint frame cannot remain non-contextual.

While this is not a formal statement, as we could stay replaying interactions on non-contextual frames, in realistic scenarios contextuality will show up. At such step, the pointer algebra will nescessarly coarse grain for some frame. That frame then sees the length of the chain over the time step as the first interaction step.

Over several interactions, such chains which are always finite, have varying length. But any such interaction will have a finite upper bound.

Maximum speed for frame i

Once we fix a frame i we see its interaction path in G as:

$$i = i_0 \to i_1 \to ... \to i_n = j$$

In the step i_n is when the first coarse graining happens. We can then define:

$$v_i := \sup n : \mathcal{F}_n = \mathcal{F}_0$$

That is, the largest number of sequential interactions along which the σ-algebra of i remains untouched.

Intrepetation

This pehnomenon should not be interpreted as the emergence of a specific relativistic sped such as a "speed of light". It is a structural consequence of contextuality: in any sufficiently rich system of interacting information frames, repeated interactions inevitably produce contextual incompatibilities, which force coarse-graining and thereby prevent influence from propagating indefinitely without degradation.

The resulting bound is a maximum interaction speed inherent to the information structure itself. Although different contexts may have different local bounds, a common finite upper bound always exists for any fixed system.

7.4 No Signaling

No signaling is an immediate structural consequence of the locality graph and Axiom 1. Because interactions are encoded as edges in the graph, and Axiom 1 applies only on the joint frames of the interacting frames, no frame can influence another without a path of interaction connecting them.

Local Independence

Let i and j represent two frames. If they have never interacted, they lie on different subgraphs of G. Then no local projection e_i of i acts on any algebra observed by j.

Thus any mutual time step between then leaves $\mathcal{F}_j$ untouched, that is:

$$\mathcal{F}_j^{n+1} = \mathcal{F}_j^n$$

When that time step operates on i, quite simply that is, a frame cannot alter the algebra of a frame unless they interact.

Causal Separation

If i and j lie in different subgraphs, let e_k bet the local projection of a new interaction on the subgraph of i. Since there is no path from k to j the projection e_k has no overlap with the algebras j can see. Therefore, e_k has no effect on $\mathcal{F}_j$.

Therefore

- Effects of interactions propagate only along paths in the locality graph.
- Only frames lying in the same subgraph can influence each other.

This is a purely structural notion of causal separation.

Intrepretation

No signaling in this framework is almost tautological, information cannot be transmitted between frames that have not interacted.

7.5 Gravity

How gravity is the imbalance between algebra

8 Intrepetation and Comparisons

Here we explore the theory in the context of the broader literature on measurement and study some of its features, including the implications it has on ontology. We begin with a informal example of the framework applied to the spin of a electron. To illustrate how this framework maps to standard Quantum Mechanics.

8.1 The Ontology of Information Frames

8.2 Comparisons - Collapse Models

8.3 Comparisons - Intrepetations

9 Discussion

9.1 Geometry and Dynamics

9.2 Reconstruction Programs

9.3 Chaos

9.4 Implications for gravity

10 Conclusion

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