# On Measurement: The Relativity of Information Frames

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**Abstract**

## 1 Introduction

In this paper we begin with a motivational intuition. That of a principle of relativity. We then throught the paper endevor to make that intuition mathematically precise. First by introducing the tools we will use, sheaf theory contextuality and markov kernels. Then we use these tools to give a precise meaning to this relativity principle.

Finally we show that this principle generates a space that has the properties of a Hilbert space. Using Gleanson's theorem [**?**] we retrieve the born rule. We give a natural explanation for the pointer basis and give as a theorem Wigner friend's consistency, natural Markoviality of physical systems and the arrow of time.

We conclude with a ontological intrepetation of this principle and suggestions for further reasearch directions. We conclude that, if this principle is accepted as a valid restriction for reality, the Copehangen intrepetation axioms are derived instead of postulated. Giving a potential solution to the measurement problem.

We also compare it to other explanations such as GRW and Penrose collapse. Also relates to its direct cousin, Relation Quantum Mechanics. This is not a intrepretation but a new framework.

Acceptance of this principle depends upon accepting the Relativity of information frames as physically fundamental. Further work is needed to see its consequences, in principle it does not disagree with quantum mechanics and provides a clean resolution to some of its puzzling features. No experiment to derive prove its truth is known to the authors of this paper at the current formulation.

## 2 Motivation - The Relativity of Information Frames

Consider two physical observers, Alice and Bob, each equipped with a clock and a ruler. To infer a particle's momentum, they make two position measurements and record the elapsed time.

However,

- if they agree on the spatial separation, they must disagree on the elapsed time;

- if they agree on the elapsed time, the measured spatial separation must differ.

Their interactions with the world differ — and so does what each can resolve as an event.

What Alice calls "particle at position $x$ at time $t$" is determined by her interaction channels and detection thresholds.

Thus there is no global, frame-independent $\sigma$-algebra of events. Every physical system carries its own information frame: a $\sigma$-algebra of distinguishable outcomes accessible through its interactions.

Einstein taught that coordinate descriptions are relative while causal order is invariant. We extend this principle.

**Relativity of Information Frames (RIF)**

Nature does not favor one perspective over another. Nature is the same no matter the frame of information.

Measurement is not the revelation of a pre-existing global state; it is the joint refinement (and, when necessary, coarse-graining) of information frames when systems interact. From this symmetry, quantum state update, pointer bases, and even causal geometry follow as consequences.

# 3 Background

## 3.1 Measure Theory

The complete introduction to the richness of measure theory probability theory is not in the scope of this work, we refer to [1] for that, we will at least the concept of probability space. We hope the work is understandable with only this crude introduction but familiarity with the subject is advised.

### Probability Spaces

In measure theoretical probability, a probability space consists of a triplet $(\Omega, \mathcal{F}, \mu$. Each piece represents a core element of a probality model.

### The Sample Space $\Omega$

The sample space $\Omega$ represents all events that can happen. It is often not defined to be something in particular. Simply a space where we can draw samples $\omega \in \Omega$ from.

It can be seen as particular realizations of an experiment, or trials of a coin toss or observations of a particular population model.

### The $\sigma$-algebra $\mathcal{F}$

A $\sigma$-algebra represents the set of events the model view as possible. It can be seen as what can happen in the probability model. Events are are sets composed of samples $\omega \in \Omega$.

The events in a probability space must obey certain rules. The rules,

### The Probability Measure $\mu$

Throughout this paper, we will often not be using specific probability measures, working only with the sample space and the $\sigma$-algebras.

**Measurable Functions**

**Pushfoward Measure**

**Markov Kernels**

We will also need the definition of *Markov Kernels*, which give us to talk about how different probability spaces interact.

**Definition 3.1** (Markov Kernels)**.** Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Markov Kernel* is a function:

$$K : \Omega_1 \times \mathcal{F}_2 \to [0, 1]$$

Where we have:

- For every fixed $\omega \in \Omega_1$
$$K(\omega, \cdot) \text{ is a probability measure in } \mathcal{F}_2$$
- For every fixed $A \in \mathcal{F}_2$
$$K(\cdot, A) \text{ is a measurable function } \Omega_1 \to ([0, 1], \mathcal{B}([0, 1]))$$

The idea is that Markov Kernels define probability measures on the target space that respect the structure of the source space. It is often written $K : \Omega_1 \to \mathcal{P}(\Omega_2)$ to say, a Kernel that defines probabilities on $\Omega_2$ from $\Omega_1$.

Markov Kernels allow us to define probability measures on the target space from measures on the source.

**Definition 3.2** (Markov Pushfoward Measure)**.** Given the measurable spaces and Kernel on the Definition 3 4. Let $\mu$ be a probability measure on $\Omega_1$. The *pushfoward measure* given by the Kernel

$$(\mu K)(A) := \int_{\Omega_1} K(\omega, A)\, \mu(dx), \qquad A \in \mathcal{F}_2$$

And it is a probability measure on $\mathcal{F}_2$.

And finally, we also need to look at the definition of a *effective $\sigma$-algebra* of a Markov Kernel.

**Definition 3.3** (Effective $\sigma$-algebra of K)**.** For a Kernel $K : (\Omega_1, \mathcal{F}_1) \to (\Omega_2, \mathcal{F}_2)$. The effective sigma algebra of $K$ is given by:

$$\mathcal{F}_K := \sigma \{\omega \to K(\omega, A) \mid A \in \mathcal{F}\}$$

That is, the algebra of all random variables that $K$ generates. It can also be seen as the algebra of events of $\Omega_1$ that remain distinguishable after passing through $K$.

**Composition of Markov Kernels**

Markov Kernels compose, in particular if $K_1 : \Omega_1 \to \mathcal{P}(\Omega_2)$ and $K_2 : \Omega_2 \to \mathcal{P}(\Omega_3)$ are Markov Kernels. Then their composite is given for $A \in \mathcal{F}_3$:

$$(K_2 \circ K_1)(x, A) := \int_{\Omega_2} K_2(y, A) dK_1(x, \cdot)$$

And naturally defines probability measures on $\Omega_3$.

### Deterministic Markov Kernels

These are a special class of Markov Kernels that can act as transport structure from one probability space to another.

**Definition 3.4** (Deterministic Markov Kernels). Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Deterministic Markov Kernel* is induced by a measurable function $f : \Omega_1 \to \Omega_2$:

$$K_f : \Omega_1 \times \mathcal{F}_2 \to [0, 1]$$

That is given for $\omega_1 \in \Omega_1$ and $F_2 \in \mathcal{F}_2$

$$K_f(\omega_1, F_2) := \mathbf{1}_{f(\omega_1) \in F_2}$$

The function $f$ can be seen as the transport from one probability space into another. The Kernel then, allows us to pushfoward probability measures from it.

### Embeddings

As we have seen, we can use a measurable function $f : \Omega_1 \to \Omega_2$ to define a Markov Kernel. The Markov Kernel then allows us to transport probabilities to the new space.

There are a few special measurable functions we will be interested in. The first defines a full *isomorphism* of spaces:

**Definition 3.5** (Measurable Isomorphism). A measurable function $T : \Omega_1 \to \Omega_2$ that is *bijective* and whose inverse $T^{-1} : \Omega_2 \to \Omega_1$ is also measurable defines a *isomorphism* of probability spaces:

$$(\Omega_1, \mathcal{F}_1) \cong \big(\Omega_2, \mathcal{F}_2\big)$$

Naturally such bijections form a group:

**Definition 3.6** (Measurable Space Automorphisms).

$$\mathrm{Aut}_{\Omega,\mathcal{F}} := \big\{T : \Omega \to \Omega \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable } \big\}$$

These maps preserve the full structure of the measurable space, for our work we will need a class that still preserves structure but can embed the measurable space into a larger one.

**Definition 3.7** (Measurable Embedding). A measurable embedding $\iota : \Omega_1 \to \Omega_2$ is a *injective* measurable function with a measurable inverse. It can also be seen as a local homomorphism.

These embeddings preserve the $\sigma$-algebra of the source space entirely in the target space.

## 3.2 Contextuality

The first important concept the theory relies upon is that of contextuality. All our definitions here are translated from the [**?**] contextuality in sheaf-theory. They have been adapted to a measure theory framework.

### Labels and Contexts

First we look at the definition of a measurement label. A measurement label intuitively represents what one can tell apart, that is what questions a system can ask. It can be seen as the fundamental degrees of freedom of a given model.

### Measurement Labels

**Definition 3.8** (Measurement labels)**.** A measurement label is an abstract symbol $m$ that identifies a physical distinction we may attempt to extract from the system. Together with its outcome space $(\Omega_m, \mathcal{F}_m)$. That is:

$$m \to (\Omega_m, \mathcal{F}_m)$$

The set of all measurement labels the model considers primitive is called $\mathcal{M}$.

Then naturally, our global space, where all measurament labels exist is then:

**Definition 3.9** (Global Space)**.** The global space, the space of all degrees of freedom and all their distinctions is

$$(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}}) = \left( \prod_{m \in \mathcal{M}} \Omega_m \, , \bigotimes_{m \in \mathcal{M}} \mathcal{F}_m \right)$$

Where $\mathcal{F}_1 \otimes \mathcal{F}_2 = \sigma\left(\{F_1 \times F_2 : F_1 \in \mathcal{F}_1, F_2 \in \mathcal{F}_2\}\right)$. We note that, we do not define a particular probability measure on this space, that is because what we are interested in at the moment is the structure of the space, not a specfic measure on it.

### Contexts

Next we talk about contexts. Contexts are given by the subset of labels or degrees of freedom a given observer cares about something he is interacting with. It can be seen as the fundamental set of questions he can ask about the part of the model he interacts with.

**Definition 3.10** (Context)**.** A context $C \subseteq \mathcal{M}$ is a finite collection of measurement labels that are jointly meaningful. To each context we associate a measurable space:

$$(\Omega_C, \mathcal{F}_C) := \left( \prod_{m \in C} \Omega_m \, , \bigotimes_{m \in C} \mathcal{F}_m \right)$$

Within a context we also define the projections:

**Definition 3.11** (Canonical Context Projections)**.** The projections for a context $C$ are defined as *measurable functions* from a context to one of its label spaces.

$$\pi_{C \to \{m\}} : \Omega_C \to \Omega_m$$

With:

$$\pi_{C \to \{m\}}(\omega) = \omega_m \quad \forall \omega \in C$$

That is, the projections map to the context corresponding to the label $m$ within the context $C$. The extension to a subcontext $D \subseteq C$ is naturally $\pi_{C \to D}$.

Intuitively projection can be seen as the *perspective* $C$ has on $m$. For context projection we will also need its inverse definition.

**Definition 3.12** (Cylinder Embedding)**.** For every outcome $\omega_1 \in \Omega_m$ we embed it in the context space $(\Omega_C, \mathcal{F}_C)$. This gives the definition:

$$\pi^{-1}_{C \to \{m\}}(A) := \{\omega \in \Omega_C : \; x_m \in A\}$$

Since $\pi$ is a measurable function by definition $\pi^{-1}_{C \to \{m\}}(A) \in \mathcal{F}_C$.

We importantly note that, a context require $\pi$. They tell the context where its events come from. In principle, a context does not hold *information that comes from nowhere.*

### Empirical Model

We will now introduce the first concept that requires the use of specific probability measures that is the definition of an *empirical model.*

Intuitively can be seen as a particular realization of the model, or a particular realization of a *perspective* on the underlying world. We can also think of it as a particular family of *coodinates* in the probability spaces of the contexts.

**Definition 3.13** (Empirical model)**.** An *empirical model* is a family $\{e_C\}_{C \in \mathcal{M}}$ of probability measures on $(\Omega_C, \mathcal{F}_C)$. For all $C, C' \in \mathcal{M}$ and all $D \in C \cap C'$ we have:

$$(\pi_{C \to D})_* \, e_C \;=\; (\pi_{C' \to D})_* \, e_{C'}$$

This condition means that on overlaps, the probability measures must agree. They come from the same underlying labels.

### Contextuality

The empirical families allows us to define what will be the driving feature of our framework. It is the defintion of *Contextuality.* When *perspectives* only completly exist on the context they came from.

**Definition 3.14** (Contextuality)**.** The family $\{e_E\}_{E \in \mathcal{M}}$ is called contextual in $\mathcal{M}$ if no probability measure $\mu$ on the global space $\mathcal{M}$ exists satisfying:

$$(\pi_{C \to E})_* \, \mu \;= e_E \quad \forall E \subset \mathcal{M}$$

They are called non-contextual, if such probability measure exists.

Intuitively, it means that in that shared space, the questions still make perfect sense together if they are non-contextual. We know exactly where they came from.

If they are contextual then there is no way to pick a coordinate, or probability measure on the global space that agrees with all probabilities the contexts of that space found.

The core feature we will need here is that, there exists experiments or real situations where the global space is contextual this fact can be seen in depth in [**?**]. This particular definition, is of probabilistic contextuality as defined on [**?**].

# 4 Structural Contextuality

To get a better understanding on RIF we need to look at other formulations of contextuality. We want to understand contextuality without particular probability distributions.

For this, we adapt the sheaf like condition, than just the failure of probabilities to have the correct marginals, this in particular matches the *strong contextuality* defined in [**?**]. First we define a *Information Frame.*

**Definition 4.1** (Information Frame)**.** A information frame is a probability space over a context $C \subseteq \mathcal{M}$ with a particular event algebra $\mathcal{F} \subseteq \bigotimes_{m \in C} \mathcal{F}_m$ corresponding to the set of distinctions that frame can see about the labels $m$.

$$\mathscr{I}_{C,\mathcal{F}} := \left( \prod_{m \in C} \Omega_m, \mathcal{F} \right)$$

A information frame could be seen as a perspective on the world. A view of what it can in principle see about a system. A probability measure there represents a particular state of the world. For notation convinience we will use $\mathscr{I}_1 = \mathscr{I}_{C1,\mathcal{F}_1}$

In general, Information Frames are defined on the support of some probability measure, meaning we get rid of events that a context does not see as possible.

**Definition 4.2** (Structural Contextuality)**.** Given a family of contexts $\mathcal{C}$ of $\mathcal{M}$ and corresponding information frames $\mathscr{I}_{C,\mathcal{F}_C}$ and a global space $(\Omega, \mathcal{F})$.

The family is said to be *structuraly non-contextual* if there is a family of corresponding *embeddings* and some sub-$\sigma$-algebra $\mathcal{G} \subseteq \mathcal{F}$ such that:

$$\iota_{C \in \mathcal{C}} : \mathscr{I}_{C,\mathcal{F}_C} \to (\Omega, \mathcal{G})$$

Such that for every pair $C, C'$ and every event $E \in \mathcal{F}_C \cap \mathcal{F}_{C'}$, that is events on shared labels, we have:

$$\iota_C(E) = \iota_{C'}(E)$$

If no such embedding exists then the family is called *structuraly contextual.*

The existence of *strong contextuality* as seem in [**?**] guarantees structural contextuality also exists, in particular the results in [**?**] show that, if we try to keep the full $\sigma$-algebra event structure, structural contextuality will eventually happen. This shows contextuality is essentially inevitable in sufficiently complex logics.

# 5 The Relativity Of Information Frames

## 5.1 Interaction

Now we work on making precise the meaning of the *Relativity of Information Frames*, RIF, for short. To do so, the central objects of study will be *Information Frames* as in defintion 4 .1. Information frames are the only ontic objects in this theory, everything works on their interactions.

We now turn our attetion to making the definition of interaction precise. The first step is to define the *Joint Frame*. A information frame where the interaction takes place. Before this we will note that when we write:

$$\iota(\mathcal{F}_C) := \mathcal{G}$$

That is, there is some sub-$\sigma$-algebra $\mathcal{G}$ on the global space where that embedding exits. We also note that there can be many such embeddings. These differences are not of particular interest to us, they are related by the automorphism symmetry defined in Definition 3.7, so we can treat them as equivalent.

**Definition 5.1** (Joint Frame)**.** Given a family of information frames $\{\mathscr{I}_i\}_{i\in I}$ we can construct their joint frame as:

$$\mathcal{J}_I := (\Omega_{\mathcal{J}_I},\ \mathcal{F}_{\mathcal{J}_I}) := \left( \prod_{m\in\{\bigcup_{i\in I} C_i \subseteq \mathcal{M}\}} \Omega_m,\ \bigotimes_{i\in I} \iota_i\left(\mathcal{F}_i\right) \right)$$

The space that hold all distinctions of all information frames.

Such joint frame always exists, but it might be contextual. Structural contextuality tells us directly when it is even possible to make a non-contextual joint frame.

One important thing to note however, is that once we generate such a joint space, the embeddings are no longer nescessarly embeddings. That is because in contextual cases, the event in the intersection $E$ must choose one of the embeddings. This is critical for our relativity principle.

### 5.2 The Structure of the contexts

To make our realitivity principle precise we need a way to compare the structure of contexts in the joint frame.The problem is once we fix a joint frame $\mathcal{J}_I$ through some family of embeddings $\iota_{i\in I}$ we still can't properly compare the contexts as they do not live in the global space.

A way to do this is to consider the following map

**Definition 5.2** (Local Projections)**.** Given a family of embeddings $\iota_i$ that generated a particular frame $\mathcal{J}_I$. The local projections $e_i$ are

$$e_i := e_{C_i} := (\iota_i \circ \pi_{\mathcal{J}_I \to C_i}) : \Omega_{\mathcal{J}_I} \to \Omega_{\mathcal{J}_I}$$

These $e_i$ represent the full event structure of the frame $\mathscr{I}_i$ in the joint frame.

### 5.3 The Symmetry Of Information

#### Privilege

We are finally ready to introduce the concept of *The Relativity of Information Frames*. We begin with the definition of *Privilege* in a *Joint Frame*.

**Definition 5.3** (Privilege)**.** We say a joint frame $\mathcal{J}_{C_1\cup C_2}$ privileges $C_1$ over $C_2$ if for some shared event $E \in C_1 \cap C_2$ if, for its local projections $e_1, e_2$ we have:

$$e_1(E) = e_1(e_2(E)) \quad \text{and} \quad e_2(E) \neq e_2(e_1(E))$$

The idea is that one context is favored over another if its perspective is closer to that of the joint frame than another context. It can be thought of as preserving the its structure more than that of another frame.

Intuitively, as the joint frame is built, if there is contextuality the joint frame had to pick one $\iota$ over another for each event they disagreed upon. This manifests as favoring.

Another way to see this is that, the $\iota$ of a privileged frame, is no longer a true embedding on the joint frame with its full sigma algebra, it now forgets some distinctions.

In particular, its worth noting that the local projections do not commute in the contextual case:

$$e_1 \circ e_2(E) \neq e_2 \circ e_1(E)$$

**The Relativity of Information Frames**

We can now state the relativity of information frames precisely.

**Axiom 1 (The Relativity Of Information Frames).** *After interaction, a physically admissible frame $J_{phys}$ must not privilege any interacting information frame over another on any event.*

More explicitly for any pair of frames $i, j$ of the joint frame and any shared event $E$ we have:

$$e_i(E) = e_i(e_j(E)) \quad \text{and} \quad e_j(E) = e_j(e_i(E))$$

Equivalently, the event is in the intersection of fixed points:

$$E \in \mathrm{Fix}(e_i) \cap \mathrm{Fix}(e_j)$$

The above condition shows that:

$$E \in \mathrm{Fix}(e_i) \cap \mathrm{Fix}(e_j) \quad \forall E \in \mathcal{F}_{\mathrm{phys}} \quad \forall i, j$$

We define a important object using the definition above, with $\mathcal{F}_I$ being the $\sigma$-algebra of the full joint frame:

**Definition 5.4** (Pointer algebra)**.** The pointer algebra of a joint frame is given by:

$$\mathcal{F}_{\mathrm{ptr}} := \bigcap_i \mathrm{Fix}(e_i) = \{E \in \mathcal{F}_I : \; e_i(E) = E \; \forall i\}$$

It is clear from the definition that:

$$\mathcal{F}_{\mathrm{phys}} \subseteq \mathcal{F}_{\mathrm{ptr}}$$

In fact, since its obviously true that for all $E \in \mathcal{F}_{\mathrm{ptr}}$

$$e_i(E) = e_i(e_j(E)) \quad \text{and} \quad e_j(E) = e_j(e_i(E))$$

This intersection is the largest algebra that is physically admissible for the interaction.

### 5.4 Symmetry Breaking: Collapse

Now our study in contextuality already revealed that, the naive embedding joint frame, that attempts to preserve all distinctions in all information frames, may yield a contextual joint frame.

**Theorem 5.5** (Maximality of the pointer algebra)**.** *The pointer algera $\mathcal{F}_{ptr}$ is the largest $\sigma$-algebra that is physically admissible under Axiom 1.*

*Proof.* Such joint frame cannot obey Axiom 1. Because structural contextuality shows that for any joint frame $\mathcal{J}_I$ that is built from a contextual family there is some event $E \in \mathcal{F}_I$ and some pair $i, j$ for which

$$e_i(E) \neq e_j(E)$$

Therefore for at least some particular $e_i$ we must have:

$$\mathcal{F}_{\text{ptr}} \subseteq \text{Fix}(e_i)$$

And for any $\mathcal{F}_{\text{ptr}} \subsetneq \mathcal{G} \subseteq \mathcal{F}_I$ we must have:

$$E \in \mathcal{G} \setminus \mathcal{F}_{\text{ptr}} \quad \exists k \in I \to e_k(e_k(E)) \neq e_k(E)$$

But by construction of the joint frame, the event $e_k(E)$ must have come from some context. So we must have $\iota_j(E) = e_k(E)$. And this means:

$$e_j(E) = e_j(e_k(E)) \quad \text{and} \quad e_k(e_j(E)) = e_k(e_k(E)) \neq e_k(E)$$

□

And finally we have:

**Theorem 5.6** (Collapse Theorem)**.** *Let $\mathcal{F}_I$ be the contextual joint $\sigma$-algebra of the joint frame generated by the interaction of the contextual families $\mathcal{F}_I$. Let $\mathcal{F}_{textptr}$ be its pointer algebra.*

*Then under Axiom 1 we have $\mathcal{F}_{phys} = \mathcal{F}_{ptr}$.*

*Proof.* For every event $E \notin \mathcal{F}_{\text{ptr}}$ by Theorem 5.5 we know it exhibits privilege for some pair of contexts. Therefore such events are physically forbidden by RIF. Every event in $\mathcal{F}_{\text{ptr}}$ is physically admissible. So we have:

$$\mathcal{F}_{\text{phys}} = \mathcal{F}_{\text{ptr}}$$

□

While we do not have a dynamical picture of collapse in this work, one may picture collapse as the effect of repeatedly applying all contextual projections. Each projection shaves off parts of an event it cannot stabilize, only the events that survive all projections belong to the pointer algebra.

So what survives is:

$$E \cap \bigcap_{i \in I} \text{Fix}(e_i)$$

We end with the definition of the joint pointer frame.

**Definition 5.7** (Joint Pointer Frame)**.**

$$\mathcal{J}_I^{\text{ptr}} := (\Omega_I, \mathcal{F}_{\text{ptr}})$$

### 5.5 Relation to Contextuality

What we aim to do here is to show that $\mathcal{F}_{\text{ptr}}$ is non-contextual. While every larger sub-sigma algebra is contextual.

But we note that, the structural definition we used before is not possible here. It requires spaces that allow for full embeddings, which we know does not fit the pointer frame. We provide very rough proof sketches as not to crowd the paper, but it should be enough to see the correlation.

So we turn our attention to probabilistic contextuality. We have the following theorem:

**Theorem 5.8** (The Joint Pointer Frame is noncontextual)**.** *Let $\{p_i\}_{i\in I}$ be a empirical family of probabilties on the contexts. There exists a probability measure $\mu$ in the pointer frame generated by the information frames corresponding to that empirical family with:*

$$(\pi_{J\to C_i})_*\mu = p_i$$

*Proof sketch.* First pick the information frames with the support of each $p_i$ for its sigma algebra. Then construct the joint frame with embeddings. Because the event structure is picked only to agree with all embeddings we can restrict the embedding to common events.

Then the probabilties will pass along to those events. □

For larger families the pointer frames, if a empirical familiy is contextual it remains contextual in all algebras larger than the pointer.

**Theorem 5.9** (Algebras larger than the pointer are contextual)**.** *With the same setup as the previous theorem a algebra $\mathcal{F}_{ptr} \subsetneq \mathcal{G}$ there is no probability measure that has the correct partials.*

*Proof sketch.* Here, since in $\mathcal{G}$ we still have privilege we know there are two frame's contexts $C_j$ and $C_k$ and shared event $E$ such that:

$$e_j(E) \neq e_j(e_k(E)) \quad \text{while} \quad e_k(E) = e_k(e_j(E))$$

And since there is consistency of empirical families on shared events no statistics on the joint frame will be able to reproduce how the statistics of this event $E$ behave for both contexts. □

### 5.6 The Markov View

A important thing to note is that, once a joint frame has been built, and it has picked a algebra. The initial embeddings are no longer nescessarely embeddings in that frame.

In fact, for the ones that are not privileged the kernel generated by the original embedding is no longer injective. It is a strict coarse graining. This will allow us to give precise numerics and dynamics by implementing information geometry once we are free to assume states (particular probability distributions). But this is not in the scope of this paper.

## 6 Quantum Mechanics

Here we endevor to trace the parallels with standard quantum mechanics. Where each ingredient fits in the picture of information frames we have built and how are they related.

## 6.1 Observables, Measurements and Operators

### Measurement Device

We begin by defining the measurement devices. In this theory a measurement device is given by a pure information frame $\mathscr{I}_{\text{mes}}$. That is a measurement device is represented by $(\Omega_{\text{mes}}, \mathcal{F}_{\text{mes}})$, it can be seen as what the measurement device sees about the world, the events it is capable of recognizing.

### Observables

An observable, for a particular measurement device $i$, is a random variable $O_i : \Omega_i \to \mathcal{O}$. Where $\mathcal{O}$ is the outcome space, it can be $\mathbb{R}$ or other such spaces.

Each observable through the original embeddings, induces a global random variable on the joint context, which we will denote with $(\Omega, \mathcal{F})$. To define the global random variables we need the partial inverse:

$$\pi_i : \iota_i(\Omega_i) \to \Omega_i$$

Then the representative of the observable in the joint frame is:

$$\widetilde{O}_i := O_i \circ \pi_i : \Omega \to \mathcal{O}$$

The $\sigma$-algebra generated by $\widetilde{O}_i$ represents the set of events distinguishable by the observable $O_i$. We say observables are *compatible* if their algebras remain jointly Boolean.

### Observable Operators

To see how incompatible observables do not commute, we can look at a strategy similar to what we did we the full algebras, it also the representation that matches operators more directly. Start with a event $E$ in the global space, for observables of different contexts $O_i$ and $O_j$ we can define:

$$e_i^{O_i} := \iota_i(O_i^{-1}(O_i(\pi_i(E)))) \quad \text{and} \quad e_j^{O_j} := \iota_j(O_j^{-1}(O_j(\pi_j(E))))$$

If they are incompatible, we will have:

$$e_i^{O_i} \circ e_j^{O_j} \neq e_j^{O_j} \circ e_i^{O_i}$$

Similarly to what we saw for local projections.

### General Operators

Other operators, those not associated with observables such as time evolution, stochastic maps, or Hilbert-space operators without direct observational intrepetation appear only as a transformation built from the basic local projections of Definition 5.2 and their compositions and mixtures.

In general, in the Hilbert space representation, the contextual transformations are represented by completely positive maps associated with the corresponding measurement.

#### Measurement

A measurement of an observable $O_i$ corresponds to evaluating its lifted form $\widetilde{O}_i : \Omega \to \mathcal{O}$. However, the collapse theorem restricts the physically admissible events to those lying in the pointer $\sigma$-algebra $\mathcal{F}_{\text{ptr}}$.

Consequently, only those values of $\widetilde{O}_i$ whose events are compatible with $\mathcal{F}_{\text{ptr}}$ can occur as outcomes. In this sense, measurement reduces to evaluating the observable on the pointer algebra. The observable possible values are exactly those that survive the collpase into $\mathcal{F}_{\text{ptr}}$.

### 6.2 Born Rule Martingale

In this section we explore how the Born rule appears in this framework. We do not claim, with the current tools, to recover the quadratic nature of the born rule or its usual form. That is left to a reconstruction of the Hilbert space.

#### Setup

First we consider a particular context's frame $\mathscr{I}_i$. We define a probability measure representing that frame $\mu_0$. Th

In the joint frame we can look at the pushfoward measure given by the original embedding.

$$\mu := \mu_0 \pi_i = \mu_0(\pi_i(E)) \quad E \in \mathcal{F}_{\mathcal{J}}$$

We note that this probability measure does not, nescessarly, represent all contexts of the joint frame. In fact, in contextual cases, that is not possible. This is simply the probability of the context $i$ transported to the global frame.

In fact, this pushfoward probability is the probability distribution given by the quantum trace rule for the observable of that context. To see this, simply consider the observable model we had before and note:

$$\mu(\widetilde{O}_i^{-1}(B)) = \mu_0(O_i^{-1}(B))$$

In the Hilbert space, this looks like:

$$tr(\rho E_B) = \rho(E_b)$$

#### The Collapse Filtration

The collpase theorem, in particular for each partial $\mathcal{F}_{\text{ptr}} \subsetneq \mathcal{G} \subseteq \mathcal{F}_{\mathcal{J}}$, is a sub-$\sigma$ of $\mathcal{F}_{\mathcal{J}}$.

So we can define the reverse filtration converging to the pointer basis.

$$\mathcal{F}_{\mathcal{J}} =: F_0 \supseteq F_1 \supseteq F_2 \supseteq ...\mathcal{F}_{\text{ptr}}$$

This filtration can be seen as the joint space being coarse grained to the pointer basis.

#### The Collapse Martingale

We can now use $\mu$ as a probability measure to define, for any event $A \in \mathcal{F}_{\mathcal{J}}$ we define:

$$M_n := \mathbb{E}_\mu \left[ \mathbf{1}_A \mid \mathcal{F}_n \right]$$

As seen [1] we know this is a Martingale on the reverse filtration, and by Doob convergence theorem we have:

$$M_n \to \mathbb{E}_\mu \left[\mathbf{1}_A \mid F_{\text{ptr}}\right] \quad \text{(a.s.)}$$

Which means that the probability of event $A$ in the pointer filtration is exactly $\mu(A)$.

### 6.3 Wigner's Friend Consistency

We will now turn our attention to looking at how Wigner's Friend paradox looks like in this framework.

#### Setup

For the Wigner friend scenario we acutually have two interactions. First the interaction of the friend, which we will map to the joint frame $\mathcal{J}_{\text{friend}}$. Which represents the interaction between the system and the friend.

Once that interaction goes through the system has collapsed to the pointer frame $\mathcal{J}^{\text{ptr}}_{\text{friend}}$. Then Wigner comes and interacts with that joint frame, forming a new frame $\mathcal{J}_{\text{Wigner}}$, which again must collapse to $\mathcal{J}^{\text{ptr}}_{\text{Wigner}}$.

We use the fact that, these algebras can all be seen as filtrations of each other to show that, Wigner cannot assign inconsistant probabilities to the events it can observe.

#### The Sequence of Filtrations

When the friend interacts with the system, he generates the joint frame $\mathcal{J}^{\text{ptr}}_{\text{friend}}$. When Wigner interacts with that frame, a new joint frame must be built. That joint frame starts by lifting the algebras through the embeddings $\iota_{\text{friend}}$ and $\iota_{\text{Wigner}}$.

The nature of embeddings mean we can consider this all a sequence of filtrations on the same space. Namely:

$$\mathcal{F} \supseteq \mathcal{F}^{\text{ptr}}_{\text{friend}} \supseteq F^{\text{ptr}}_{\text{Wigner}}$$

With these filtrations in place, we can look at the same style of probability assignments we had in the born rule.

#### 6.3.1 The Probabilities

Now let $A \in \mathcal{F}^{\text{ptr}}_{\text{Wigner}}$ be a event that exists in the final pointe algebra. That is, a event that all systems involved can talk about.

We can determine the friend's probaility for event $A$ as we did for the born rule martingale:

$$M_{\text{friend}} := \mathbb{E}_\mu \left[\mathbf{1}_A \mid \mathcal{F}^{\text{ptr}}_{\text{friend}}\right]$$

Now for Wigner, we have the same:

$$M_{\text{Wigner}} := \mathbb{E}_\mu \left[\mathbf{1}_A \mid \mathcal{F}^{\text{ptr}}_{\text{Wigner}}\right]$$

But since we have

$$F^{\text{ptr}}_{\text{Wigner}} \subseteq \mathcal{F}^{\text{ptr}}_{\text{friend}} \subseteq \mathcal{F}$$

The tower law for martingales gives:

$$\mathbb{E}_\mu \left[M_{\text{friend}} \mid \mathcal{F}^{\text{ptr}}_{\text{Wigner}}\right] = \mathbb{E}_\mu \left[\mathbb{E}_\mu \left[\mathbf{1}_A \mid \mathcal{F}^{\text{ptr}}_{\text{friend}}\right] \mid \mathcal{F}^{\text{ptr}}_{\text{Wigner}}\right] = \mathbb{E}_\mu \left[\mathbf{1}_A \mid \mathcal{F}^{\text{ptr}}_{\text{Wigner}}\right]$$

Thus, when the friend's description is updated to the Wigner's pointer frame is exactly Wigner's own description.

## 6.4 The Hilbert Space Realization

We will not attempt a full Hilbert space reconstruction in this paper. But we note the relations to previous work that fits this picture and where some structure might help narrow some things.

### The Orthomodular Lattice

We have Boolean $\sigma$-algebras corresponding to each context $\mathcal{F}_i$. We have their embeddings $\iota_i$ into the joint frame.

We can then define on the joint frame:

$$\mathcal{L} := \text{ the closure of } \bigcup_i \iota_i(\mathcal{F}_i)$$

Events here have:

- events complement $E \to E^c$,
- events meet $E \wedge F = E \cap F$,
- events join $E \vee F = \mathrm{cl}_{\mathcal{L}}(E \cup F)$,

# 7 Dynamics

## 7.1 Arrow of Time

## 7.2 Locality Graph

## 7.3 Maximum Speed

## 7.4 Markovianity

# 8 Intrepetation and Comparisons

## 8.1 Spin Example

## 8.2 The Ontology of Information Frames

## 8.3 Comparisons - Collapse Models

## 8.4 Comparisons - Intrepetations

# 9 Discussion

## 9.1 The Hilbert Space Reconstruction

It is well known that the contextual space, here the joint frame, form a boolean lattice that recovers most of the properties of a Hilbert Space.

This view gives us a additional tool, mainly the automorphisms or the idea that, when a event must collpase down to a coarse grained version it picks up a phase, representing all the distinctions it could have held that map down to the same event. That might allow us to get the complex field.

The rest of the reconstruction should already have precedent with contextuality frameworks.

## 9.2 The Fisher metric view

By taking the Markov view in section 5.5 we can explore this framework from the point of view of the fisher metric. When contextuality happens, the ricci curvature of the fisher metric blows up. That should trigger a back action on the tensor to adjust for the pointer basis.

Known derivations of the schrodiger equation using the quantum fisher metric can be explained and made canonical with this framework.

Furthermore, one can define a action principle of interactions. Using information geometry, we believe that can give a new way to model physical systems.

# 10 Conclusion

# References

[1] D. Williams, *Probability with Martingales*, Cambridge University Press, 1991.

[2] D. Pollard, *A User's Guide to Measure-Theoretic Probability*, Cambridge University Press, 2002.

[3] O. Kallenberg, *Foundations of Modern Probability*, Springer, 2002.

[4] I. Csiszár, *Information-type measures of difference of probability distributions and indirect observation*, Studia Sci. Math. Hungarica, 1967.