On Measurement: The Relativity of Information Frames

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Abstract

1 Introduction

In this paper we begin with a motivational intuition. That of a principle of relativity. We then throught the paper endevor to make that intuition mathematically precise. First by introducing the tools we will use, sheaf theory contextuality and markov kernels. Then we use these tools to give a precise meaning to this relativity principle.

Finally we show that this principle generates a space that has the properties of a Hilbert space. Using Gleanson's theorem [?] we retrieve the born rule. We give a natural explanation for the pointer basis and give as a theorem Wigner friend's consistency, natural Markoviality of physical systems and the arrow of time.

We conclude with a ontological intrepetation of this principle and suggestions for further reasearch directions. We conclude that, if this principle is accepted as a valid restriction for reality, the Copehangen intrepetation axioms are derived instead of postulated. Giving a potential solution to the measurement problem.

We also compare it to other explanations such as GRW and Penrose collapse. Also relates to its direct cousin, Relation Quantum Mechanics. This is not a intrepretation but a new framework.

Acceptance of this principle depends upon accepting the Relativity of information frames as physically fundamental. Further work is needed to see its consequences, in principle it does not disagree with quantum mechanics and provides a clean resolution to some of its puzzling features. No experiment to derive prove its truth is known to the authors of this paper at the current formulation.

2 Motivation - The Relativity of Information Frames

Consider two physical observers, Alice and Bob, each equipped with a clock and a ruler. To infer a particle's momentum, they make two position measurements and record the elapsed time.

However,

- if they agree on the spatial separation, they must disagree on the elapsed time;

- if they agree on the elapsed time, the measured spatial separation must differ.

Their interactions with the world differ — and so does what each can resolve as an event.

What Alice calls "particle at position x at time t" is determined by her interaction channels and detection thresholds.

Thus there is no global, frame-independent σ-algebra of events. Every physical system carries its own information frame: a σ-algebra of distinguishable outcomes accessible through its interactions.

Einstein taught that coordinate descriptions are relative while causal order is invariant. We extend this principle.

Relativity of Information Frames (RIF)
Nature does not favor one perspective over another. Nature is the same no matter the observer frame of information.

Measurement is not the revelation of a pre-existing global state; it is the joint refinement (and, when necessary, coarse-graining) of information frames when systems interact. From this symmetry, quantum state update, pointer bases, and even causal geometry follow as consequences.

3 Background

3.1 Measure Theory

The complete introduction to the richness of measure theory probability theory is not in the scope of this work, we refer to [**?**] for that, we will at least the concept of probability space. We hope the work is understandable with only this crude introduction but familiarity with the subject is advised.

σ-algebras

Probability Measure

Probability Spaces

Throughout this paper, we will often not be using specific probability measures, often working only with the sample space and the σ-algebras.

Measurable Functions

Pushfoward Measure

Markov Kernels

We will also need the definition of *Markov Kernels*, which give us to talk about how different probability spaces interact.

Definition 3.1 (Markov Kernels)**.** Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Markov Kernel* is a function:

$$K : \Omega_1 \times \mathcal{F}_2 \to [0, 1]$$

Where we have:

- For every fixed $\omega \in \Omega_1$
$$K(\omega, \cdot) \text{ is a probability measure in } \mathcal{F}_2$$

- For every fixed $A \in \mathcal{F}_2$
$$K(\cdot, A) \text{ is a measurable function } \Omega_1 \to \big([0,1], \mathcal{B}([0,1])\big)$$

One important use of Markov Kernels we will need is its pushfoward measure:

Definition 3.2 (Markov Pushfoward Measure)**.** Given the measurable spaces and Kernel on the definition 3 4. Let μ be a probability measure on Ω_1. The *pushfoward measure* given by the Kernel

$$(\mu K)(A) := \int_{\Omega_1} K(\omega, A)\, \mu(dx), \qquad A \in \mathcal{F}_2$$

And it is a probability measure on $\mathcal{F}_2$.

The other piece that will be important for this theory is:

Theorem 3.3 (DPI for KL-Divergence in Markov Kernels)**.** *Every Markov Kernel safisfies the* data processing inequality *for KL-Divergence. Let μ, ν be probability measures on* $(\Omega_1, \mathcal{F}_1)$ *and* $K : \Omega_1 \to \Omega_2$ *be a Markov Kernel. Then:*

$$D_{KL}(\mu K || \nu K) \leq D_{KL}(\mu || \nu)$$

Meaning we only lose or retain KL-distinguishbility by applying Markov Kernels.

Proof. This theorem is prove in [**?**]. □

Deterministic Markov Kernels

These are a special class of Markov Kernels that can act as transport structure from one probability space to another.

Definition 3.4 (Deterministic Markov Kernels)**.** Let $(\Omega_1, \mathcal{F}_1)$ and $(\Omega_2, \mathcal{F}_2)$ be two measurable spaces. A *Deterministic Markov Kernel* is induced by a measurable function $f : \Omega_1 \to \Omega_2$:

$$K_f : \Omega_1 \times \mathcal{F}_2 \to [0,1]$$

That is given for $\omega_1 \in \Omega_1$ and $F_2 \in \mathcal{F}_2$

$$K_f(\omega_1, F_2) := \mathbf{1}_{f(\omega_1) \in F_2}$$

The function f can be seen as the transport from one probability space into another. The Kernel then, induces a probability measure in the target space.

Embeddings

As we have seen, we can use a measurable function $f : \Omega_1 \to \Omega_2$ to define a Markov Kernel. The Markov Kernel then allows us to transport probabilities to the new space.

There are a few special measurable functions we will be interested in. The first defines a full *isomorphism* of spaces:

Definition 3.5 (Measurable Isomorphism)**.** A measurable function $T : \Omega_1 \to \Omega_2$ that is *bijective* and whose inverse $T^{-1} : \Omega_2 \to \Omega_1$ is also measurable defines a *isomorphism* of probability spaces:

$$\big(\Omega_1, \mathcal{F}_1\big) \cong \big(\Omega_2, \mathcal{F}_2\big)$$

Naturally such bijections form a group:

Definition 3.6 (Measurable Space Automorphisms)**.**

$$\mathrm{Aut}_{\Omega,\mathcal{F}} := \big\{ T : \Omega \to \Omega \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable } \big\}$$

These maps preserve the full structure of the measurable space, for our work we will need a class that still preserves structure but can embed the measurable space into a larger one.

Definition 3.7 (Measurable Embedding)**.** A measurable embedding $i : \Omega_1 \to \Omega_2$ is a *injective* measurable function with a measurable inverse.

These embeddings preserve the σ-algebra of the source space entirely in the target space.

Quocient of σ-algebras

We now look at the construction of the *Quocient* measurable space. For the rest of this section we work on the probability space $\big(\Omega, \mathcal{F}\big)$.

First we must have a equivalence relation on Ω, points in Ω we cannot distinguish.

3.2 Contextuality

The first important concept the theory relies upon is that of contextuality. All our definitions here are translated from the [**?**] contextuality in sheaf-theory. They have been adapted to a measure theory framework.

Labels and Contexts

First we look at the definition of a measurement label. A measurement label intuitively represents what one can tell apart, that is what questions a system can ask. It can be seen as the fundamental degrees of freedom of a given model.

Measurement Labels

Definition 3.8 (Measurement labels)**.** A measurement label is an abstract symbol m that identifies a physical distinction we may attempt to extract from the system. Together with its outcome space $(\Omega_m, \mathcal{F}_m)$. That is:

$$m \to (\Omega_m, \mathcal{F}_m)$$

The set of all measurement labels the model considers primitive is called $\mathcal{M}$.

Then naturally, our global space, where all measurament labels exist is then:

Definition 3.9 (Global Space)**.** The global space, the space of all degrees of freedom and all their distinctions is

$$(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}}) = (\prod_{m\in\mathcal{M}} \Omega_m \ , \ \bigotimes_{m\in\mathcal{M}} \mathcal{F}_m)$$

We note that, we do not define a particular probability measure on this space, that is because what we are interested in at the moment is the structure of the space, not a specfic measure on it.

Contexts

Next we talk about contexts. Contexts are given by the subset of labels or degrees of freedom a given observer cares about something he is interacting with. It can be seen as the fundamental set of questions he can ask about the part of the model he interacts with.

Definition 3.10 (Context)**.** A context $C \subseteq \mathcal{M}$ is a finite collection of measurement labels that are jointly meaningful. To each context we associate a measurable space:

$$(\Omega_C, \mathcal{F}_C) := \left(\prod_{m\in C} \Omega_m \ , \ \bigotimes_{m\in C} \mathcal{F}_m \right)$$

Where $\mathcal{F}_1 \otimes \mathcal{F}_2 = \sigma\big(\{F_1 \times F_2 : F_1 \in \mathcal{F}_1, F_2 \in \mathcal{F}_2\}\big)$

Within a context we also define the projections:

Definition 3.11 (Canonical Context Projections)**.** The projections for a context C are defined as *measurable functions* from a context to one of its label spaces.

$$\pi_{C\to\{m\}} : \Omega_C \to \Omega_m$$

With:

$$\pi_{C\to\{m\}}\big(\omega\big) = \omega_m \quad \forall \omega \in C$$

That is, the projections map to the context corresponding to the label m within the context C. The extension to a subcontext $D \subseteq C$ is naturally $\pi_{C\to D}$.

Intuitively projection can be seen as the *perspective* C has on m. For context projection we will also need its inverse definition.

Definition 3.12 (Cylinder Embedding)**.** For every outcome $\omega_1 \in \Omega_m$ we embed it in the context space $(\Omega_C, \mathcal{F}_C)$. This gives the definition:

$$\pi^{-1}_{C\to\{m\}}(A) := \{\omega \in \Omega_C : \ x_m \in A\}$$

Since π is a measurable function by definition $\pi^{-1}_{C\to\{m\}}(A) \in \mathcal{F}_C$.

We importantly note that, a context require π. They tell the context where its events come from. In principle, a context does not hold *information that comes from nowhere.*

Empirical Model

We will now introduce the first concept that requires the use of specific probability measures that is the definition of an *empirical model*.

Intuitively can be seen as a particular realization of the model, or a particular realization of a *perspective* on the underlying world. We can also think of it as a particular family of *coodinates* in the probability spaces of the contexts.

Definition 3.13 (Empirical model)**.** An *empirical model* is a family $\{e_C\}_{C\in\mathcal{M}}$ of probability measures on $(\Omega_C, \mathcal{F}_C)$. For all $C, C' \in \mathcal{M}$ and all $D \in C \cap C'$ we have:

$$(\pi_{C\to D})_* e_C \;=\; (\pi_{C'\to D})_* e_{C'}$$

This condition means that on overlaps, the probability measures must agree. They come from the same underlying labels.

Contextuality

The empirical families allows us to define what will be the driving feature of our framework. It is the defintion of *Contextuality*. When *perspectives* only completly exist on the context they came from.

Definition 3.14 (Contextuality)**.** The family $\{e_E\}_{E\in\mathcal{M}}$ is called contextual in $\mathcal{M}$ if no probability measure μ on the global space $\mathcal{M}$ exists satisfying:

$$(\pi_{C\to E})_* \mu \;= e_E \quad \forall E \subset \mathcal{M}$$

They are called non-contextual, if such probability measure exists.

Intuitively, it means that in that shared space, the questions still make perfect sense together if they are non-contextual. We know exactly where they came from.

If they are contextual then there is no way to pick a coordinate, or probability measure on the global space that agrees with all probabilities the contexts of that space found.

The core feature we will need here is that, there exists experiments or real situations where the global space is contextual this fact can be seen in depth in [**?**].

These are the pieces from sheaf-theory contextuality, adapted to measure theory which we require.

4 Structural Contextuality

To define RIF precisely we need to modify contextuality. As it stands, contextuality relies on a specific *empirical model* realization. It can be thought of specific coordinate assigment on the probability spaces.

We wish to define contextuality from only the structure of the probability spaces. That is, their σ-algebras.

5 The Relativity Of Information Frames

With the required concepts now in place, we are able to make RIF's definition precise.

5.1 Information Frames

Information frames are the only ontic objects in RIF. Intuitively a information frame in RIF represents a system and its perspective, what distinctions it can make about the model, or interaction, we are describing.

Definition 5.1 (Information Frame)**.** Given a context C and a perspective $\sigma(e)$ its information frame is:

$$\mathscr{I}_{C,e} = \big(\Omega_C, \sigma(e), e\big)$$

Naturally $\sigma(e)$ is a sub-σ-algebra of the context total σ-algebra.

We will often omit the probability measure e in our constructions.

We note that, we will be using the projections on information frames. This is because, you can always project it to the information frame with $\mathbb{E}\big(\pi_{C\to\{m\}} \mid \mathcal{F}\big)$.

Also, when we are talking about $\mathcal{F}_m$ of a label $m \in C$ for a information frame, we are potentially talking about a sub σ-algebra of $\mathcal{F}_m$.

For convinience of notation, when the frames perspective is not important, we may omit it.

5.2 Admissible Maps: Invertable Deterministic Channels.

Definition 5.2 (Admissible Map)**.** An admissible map is a Markov Kernel that acts on information frames $K : \mathscr{I}_{C,e} \to (\Omega, \mathcal{F})$ and preserve perspectives. They are invertable deterministic channels, they do not change what distinctions a system makes.

Formally, let $\big(\Omega_C \times \Omega_Z, \mathcal{F}_C \otimes \mathcal{F}_Z\big)$ be such space. Then there exists a injective measurable $f : \Omega_C \to \Omega_Z$ whose inverse f^{-1} is also measurable in its natural domain $f(\Omega_C)$ for which:

$$K(\omega, \cdot) = \delta_{(\omega, f(\omega))} \quad \forall \omega \in \Omega_C$$

And they preserve $KL-$Divergence exactly.

It is work noting that here the codomain may be larger. All we require is that there is a subspace that can hold all distinctions consistently.

Intuitively, all perspectives that generate the same set of distinctions are treated as just relabeling of that perspective. Such maps in fact, define a symmetry when mapping to the same space.

Definition 5.3 (The Symmetry Of Perspectives)**.** When mapping from $\mathscr{I}_{C,e} \to \mathscr{I}_{C,e}$ the admissible maps form a symmetry group. In classic probability and information theory this group is called the *automorphism group of the probability space.* It is called $\mathrm{Aut}(\Omega_C, \sigma(e), e)$ and defined as:

$$\big\{T : \Omega_C \to \Omega_C \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable }, e(T^{-1}(A)) = e(A) \quad \forall A \in \sigma(e)\big\}$$

Effectively, all probability measures consistent with this symmetry are considered the same for us.

While throught the rest of the paper we will be working only with admissible maps, we will not be restricting it only to automorphisms. We may define the following:

Definition 5.4 (Effective σ-algebra of K)**.** For $K : \mathscr{I}_{C,e} \to (\Omega, \mathcal{F})$. We define the effective σ-algebra of K, denoted $\sigma\big(K\big)$ as:

$$\sigma(K) := \sigma\big\{K(., A) : A \in \mathcal{F}_2\big\}$$

That is, the distinctions that survive through K. The effective σ-algebra of K represents the perspective of $\mathscr{I}_{C,e}$ on the target space.

6 The Joint Frames

We now define a new concept, that of *joint frames.* During this chapter we will work with the join of only two frames. But this can be naturally extended to multiple joins.

Given two frames $\mathscr{I}_{C_1,e_1}$ and $\mathscr{I}_{C_2,e_2}$, we can join them together into a single probability space:

$$\mathcal{J}_{\text{total}} := \big(\Omega_{C_1} \times \Omega_{C_2}, \mathcal{F}_{C_1} \otimes \mathcal{F}_{C_2}\big)$$

This space has all the distinctions that both contexts could possibly make. All potential distinctions that the *labels*, or degrees of freedom, allow.

But RIF makes a additional restriction. We must have all *perspectives* preserved. Failure to do so would mean we are priviliging some perspective over another. Therefore when joining two information frames, it must happen through admissible maps.

For convinience we will use $\mathscr{I}_1 = \mathscr{I}_{C_1,e_1}$ and $\mathscr{I}_2 = \mathscr{I}_{C_2,e_2}$ and .

Definition 6.1 (Candidate Joint Frames)**.** A *Candidate Joint Frame* is a measurable space, $\mathcal{J}$ reacheable through admissible maps from each frame. That is, there exists $K_1 : \mathscr{I}_1 \to \mathcal{J}_{\text{total}}$ and a $K_2 : \mathscr{I}_2 \to \mathcal{J}_{\text{total}}$ with:

$$\mathcal{J} := \big(\Omega_{C_1} \times \Omega_{C_2},\ \sigma\big(\sigma(K_1) \cup \sigma(K_2)\big)\big)$$

The measurable space that keeps all distinctions, all the information, of both frames.

Each candidate frame can be viewed as a particular realization of a symmetry that tells valid ways to view a joint system from the perspective of their component frames.

6.1 The Admissible σ-algebras

We have an important class of σ-algebras that represent valid ways both frames could in principle view each other. That is, the class of all σ-algebras reacheable by some admissible interaction.

Definition 6.2 (Admissible Joint Frames)**.** The family of σ-algebras that are considered admissible for the joint frame is defined as follows:

$$\mathscr{F}_{\mathscr{I}_1 \otimes \mathscr{I}_2} := \big\{\sigma :\ \exists K_1 : \mathscr{I}_1 \to \mathcal{J}_{\text{total}}, \exists K_2 : \mathscr{I}_2 \to \mathcal{J}_{\text{total}}, \text{ st: } \sigma = \big(\sigma(K_1) \cup \sigma(K_2)\big)\big\}$$

Theorem 6.3. *The family $\mathscr{F}_{\mathscr{I}_1 \otimes \mathscr{I}_2}$ is not nescessarely closed under meets. That is, there is $\sigma_1, \sigma_2 \in \mathscr{F}_{\mathscr{I}_1 \otimes \mathscr{I}_2}$ such that $\sigma_1 \cap \sigma_2 \notin \mathscr{F}_{\mathscr{I}_1 \otimes \mathscr{I}_2}$*

Proof. Here is enought to provide a single case where the meet fails. By [**?**] we know that we can construct a model where non-contextuality fails. If we translate that to our language this means there there is context C_1 and C_2 such that the global space is contextual.

Our situation maps cleanly to the contextuality case, if $(\Omega, \mathcal{F})$ is the global space. If a admissible map existed then we can get $f_1 : \Omega_{C_1} \to \Omega$ and $f_1 : \Omega_{C_2} \to \Omega$ we have:

$$e^*_{C_i} := e_{C_i} \circ f_i^{-1}$$

Which is a probability measure on the sub-σ-algebra $f_i(\mathcal{F}_{C_i})$.

But here since $f_i(\mathcal{F}_{C_i}) \subseteq \sigma_1 \cap \sigma_2$ we must have a global measure μ on $(\Omega, \mathcal{F})$, induced by $\sigma_1 \cap \sigma_2$ and the marginals e_{C_i} with:

$$\mu \mid_{f_i(\mathcal{F}_i)} = e^*_{C_i}$$

This is possible because empirical models must agree on the intersections.

Therefore the existence of contextual models shows that meets do not always remain in this family. □

6.2 The Pointer Frame

We define a special σ-algebra using the family $\mathscr{F}_{\bigotimes_{i\in I}\mathscr{I}_i}$. Intuitively we want a sigma algebra that contains only distinctions all frames can agree upon.

Definition 6.4 (The Pointer σ-algebra)**.** For a family of *admissible joint frames* $\mathscr{F}_{\bigotimes_{i\in I}\mathscr{I}_i}$ its pointer σ-algebra is:

$$\mathcal{F}_{ptr} = \bigcap_{i\in I} \sigma_i$$

Theorem 6.5 (The Pointer Frame is non-contextual)**.** *If we consider the joint frame given by* $\mathscr{I}_{ptr} = \left(\Pi_{i\in I}\Omega_{C_i}, \mathcal{F}_{ptr}\right)$ *then there is a probability measure* μ *on it for which:*

$$\mu \mid_{\sigma(e_{C_i})} = e_{C_i}$$

Proof. For simplicity we consider the case of two frames.

Let $E \in \mathcal{F}_{ptr}$ and let e_{C_1} and e_{C_2} in be the perspective measure on the frames. Then since $E \in \sigma(K1) \cap \sigma(K2)$ for some $K_i : \mathscr{I}_i \to \left(\Omega_{C_1} \times \Omega_{C_2}, \mathcal{F}_{C_1} \otimes F_{C_2}\right)$ we have:

$$E \in \sigma(K_1) \cap \sigma(K_2) \to E \in \mathcal{F}_{C_1} \cap \mathcal{F}_{C_2}$$

Since for a empirical model and $E \subseteq C_1 \cap C_2$ we must have:

$$\left(\pi_{C_1 \to E}\right)_* e_{C_1} \;=\; \left(\pi_{C_2 \to E}\right)_* e_{C_2}$$

The probability marginals agree on E. Since this is true for every $E \in \mathcal{F}_{ptr}$ we can define:

$$\mu(E) := e_{C_1}\left(\pi_{C_1 \cup C_2 \to C_1}(E)\right)$$

Then we automatically have:

$$\mu(E) = e_{C_2}\left(\pi_{C_1 \cup C_2 \to C_2}(E)\right)$$

Therefore the frame is non-contextual. □

This is the largest non-contextual sub-algebra touching the family of admissible frames. Any larger frame would have at least one element that some frame cannot detect and agree upon.

Another way to construct the pointer algebra is through quocient of admissible maps:

Definition 6.6 (The Quocient Pointer Space)**.**

7 The Relativity of Information Frames

With the definitions in place we are finally able to make the relativity of information frames mathemathically precise.

7.1 The Symmetry Of Information

7.2 Symmetry Breaking: Collapse

8 Hilbert Space

8.1 The Lattice of Admissible Joint Frames

8.2 Orthogonality

8.3 Gleanson Theorem

9 Born Rule Martingale

10 No Global Wavefunction

11 Arrow of Time

12 Markovianity

13 Wigner's Friend Consistency

14 Intrepetation and Comparisons

14.1 Spin Example

14.2 The Ontology of Information Frames

14.3 Comparisons - Collapse Models

14.4 Comparisons - Intrepetations

15 Conclusion

References

[1] D. Williams, *Probability with Martingales*, Cambridge University Press, 1991.

[2] D. Pollard, *A User's Guide to Measure-Theoretic Probability*, Cambridge University Press, 2002.

[3] O. Kallenberg, *Foundations of Modern Probability*, Springer, 2002.

[4] I. Csiszár, *Information-type measures of difference of probability distributions and indirect observation*, Studia Sci. Math. Hungarica, 1967.