

# The Relativity of Information Frames

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## Abstract

We introduce a structural framework based on a symmetry principle we call the *Relativity of Information Frames* (RIF). The framework treats information frames as fundamental objects and constrains how their event structures may be jointly realized during interaction. Using measure theory and contextuality, we formalize interactions as embeddings into a joint measurable space and show that enforcing perspectival symmetry induces a canonical coarse-graining of events. From a single RIF axiom, we derive a distinguished  $\sigma$ -algebra - the *pointer algebra* - and prove that it is the maximally informative algebra admitting non-contextual probability measures. Any consistent empirical family admits a global probability measure on the pointer algebra, whereas any strict extension generically reintroduces contextuality.

We then apply the framework to quantum theory. Without attempting a full reconstruction we show how standard quantum structure are naturally represented: observables arise as perspectival maps, probability emerges from coarse graining, and the born rule follows from a martingale argument. Wigner-type scenarios are shown to be consistent, and unitary evolution is interpreted as a manifestation of gauge freedom in the choice of embeddings. We further discuss how the induced coarse-graining introduces an intrinsic arrow of time and outline possible connections to causal structure and geometry. The results are structural rather than dynamical and provide a unified perspective on contextuality, probability, and measurement.

## 1 Introduction

This work develops a structural framework motivated by a simple but persistent tension in the foundations of physics: the apparent incompatibility between globally consistent descriptions of physical systems and the perspectival nature of information obtained through interaction. While the framework did not initially arise from a single guiding principle, its development ultimately crystallized around a symmetry statement, which we call the **Relativity of Information Frames** (RIF).

At its core, RIF asserts that no informational perspective should be privileged over another in the description of physical events. This principle is not imposed dynamically, but structurally: it constrains which event structures can be jointly realized when multiple information frames interact. The framework developed here formalizes this idea using measure theory and contextuality, treating information frames as the fundamental ontic objects and interactions as operations that enforce perspectival consistency.

Part I of this paper establishes the formal framework. We introduce information frames, their embeddings into joint measurable spaces, and a notion of structural contextuality that captures

when full informational structures cannot be jointly preserved. From a single RIF axiom, we derive a canonical coarse-graining of the joint event structure and prove that the resulting  $\sigma$ -algebra is, in a precise sense, the maximally informative non-contextual algebra compatible with the interaction. We refer to this algebra as the **pointer algebra**. We further show that, while probability measures need not exist on the full joint structure, any consistent empirical family admits a global probability measure on the pointer algebra, and that any strict extension of this algebra reintroduces contextuality.

Part II applies the framework to quantum theory. Without attempting a full reconstruction, we show how standard quantum structures are naturally represented within RIF. Quantum logic naturally fits this theory, and standard results showing how the Hilbert space relates to a non-distributive orthomodular lattice then give their relation. Observables arise as perspectival maps, conditioning corresponds to interaction, and probability assignments emerge from coarse-graining rather than being postulated. We derive the Born rule operationally via a martingale argument, establish the consistency of Wigner-type scenarios, and interpret unitary evolution as a manifestation of gauge freedom in the choice of embeddings. We also outline how familiar gauge symmetries may be viewed as relations between information frames.

Beyond quantum mechanics, we explore several conceptual consequences of the framework. That enforced coarse-graining introduces an intrinsic irreversibility, providing a structural arrow of time. Interactions define a relational graph of frames that suggests a notion of causal locality, and we briefly discuss how the degree of coarse-graining may be related to geometric and gravitational considerations.

The results presented here are structural rather than dynamical. No specific dynamics are assumed, and no new empirical predictions are claimed. Instead, the framework offers a unified perspective on contextuality, probability, and measurement, clarifying the origin of quantum features that are often treated as axiomatic. We conclude by comparing RIF to existing approaches, including decoherence-based accounts, and by outlining directions for further development.

## 2 Motivation - The Relativity of Information Frames

Consider two physical observers, Alice and Bob, each equipped with a clock and a ruler. To infer a particle's momentum, they make two position measurements and record the elapsed time.

However,

- If they agree on the spatial separation, they must disagree on the elapsed time;
- If they agree on the elapsed time, the measured spatial separation must differ.

Their interactions with the world differ — and so does what each can resolve as an event.

What Alice calls “particle at position  $x$  at time  $t$ ” is determined by her interaction channels and detection thresholds.

Thus there is no global, frame-independent  $\sigma$ -algebra of events. Every physical system carries its own information frame: a  $\sigma$ -algebra of distinguishable outcomes accessible through its interactions.

Einstein taught that coordinate descriptions are relative while causal order is invariant. We extend this principle.

### Relativity of Information Frames (RIF)

Nature does not privilege one information frame over another. What is physical is what all information frames can agree upon.

Measurement is not the revelation of a pre-existing global state; it is the joint refinement (and, when necessary, coarse-graining) of information frames when systems interact. From this symmetry, quantum state update, pointer bases, and even causal geometry follow as consequences.

## Part I

# Structural Framework

### 3 Background

#### 3.1 Measure Theory

A full account of measure theory and probability theory is outside the scope of this paper; for a standard reference, see [1]. In this section we introduce only the notion of a probability space and the measure-theoretic concepts that will be explicitly used later. The exposition is intentionally brief and self-contained, though some familiarity with the subject is helpful.

#### Probability Spaces

In measure-theoretic probability, a probability space is given by a triple:

$$(\Omega, \mathcal{F}, \mu)$$

where each component encodes a distinct element of a probability model.

#### The Sample Space $\Omega$

The sample space  $\Omega$  is the set of all possible outcomes of an experiment. Its elements  $\omega \in \Omega$  represent individual realizations or trials. In general,  $\Omega$  is endowed with additional structure beyond serving as the underlying space from which outcomes are drawn.

Depending on the context, elements of  $\Omega$  may correspond to coin toss outcomes, experiment runs, or realizations of an abstract physical system.

#### The $\sigma$ -algebra $\mathcal{F}$

The  $\sigma$ -algebra  $\mathcal{F}$  specifies which subsets of  $\Omega$  are considered **events**. Events are thus sets of outcomes  $\omega \in \Omega$  to which probabilities may be assigned.

**Definition 3.1** ( $\sigma$ -algebra). A  $\sigma$ -algebra  $\mathcal{F}$  is a collection of subsets of  $\Omega$  such that:

1.  $, \Omega \in \mathcal{F}$
2. Closed under complements.  $F \in \mathcal{F} \rightarrow F^c \in \mathcal{F}$

3. Closed under countable unions.

$$\{F_i\}_{i \in \mathbb{N}} \in \mathcal{F} \rightarrow \bigcup_{i \in \mathbb{N}} F_i \in \mathcal{F}$$

The  $\sigma$ -algebra encodes the events that are meaningfully distinguishable within the model and therefore forms the central structural component of a probability space.

### The Probability Measure $\mu$

A probability measure assigns probabilities to events in  $\mathcal{F}$ .

**Definition 3.2** (Probability Measures). A probability measure is a function  $\mu : \mathcal{F} \rightarrow [0, 1]$  satisfying:

1. Total probabilities:

$$\mu(\emptyset) = 0 \quad \mu(\Omega) = 1$$

2. For any countable collection of pairwise disjoint sets  $\{F_i\}_{i \in \mathbb{N}}$ , with  $F_i \cap F_j = \emptyset$  for all  $i \neq j$ .

$$\mu \left( \bigcup_{i \in \mathbb{N}} F_i \right) = \sum_{i \in \mathbb{N}} \mu(F_i)$$

In this work, probability measures will be interpreted as states on a measurable space. Encoding how probabilities are distributed over the events in  $\mathcal{F}$ .

### Measurable Functions

A **measurable function**, often referred to in probability theory as a **random variable**, is a function between measurable spaces

$$f : \Omega_1 \rightarrow \Omega_2$$

such that:

$$\forall A \in \mathcal{F}_2 \quad f^{-1}(A) \in \mathcal{F}_1$$

That is, the preimage of every event in  $\mathcal{F}_2$  is an event in  $\mathcal{F}_1$ .

### Pushforward Measure

Given a measurable function

$$f : \Omega_1 \rightarrow \Omega_2$$

And a probability measure  $\mu$  in  $(\Omega_1, \mathcal{F}_1)$ , we can define a probability measure on  $(\Omega_2, \mathcal{F}_2)$  by

$$(f_*\mu)(A) := \mu(f^{-1}(A)), \quad A \in \mathcal{F}_2$$

The measure  $f_*\mu$  is called the **pushforward** of  $\mu$  along  $f$ .

The pushforward measure represents the probability distribution induced on  $\Omega_2$  by the map  $f$  when the underlying space is described by the measure  $\mu$ .

## Embeddings

We will be particularly interested in certain classes of measurable maps that preserve the structure of measurable spaces.

**Definition 3.3** (Measurable Isomorphism). A measurable function  $T : \Omega_1 \rightarrow \Omega_2$  is called a **measurable isomorphism** if it is **bijective** and its inverse  $T^{-1} : \Omega_2 \rightarrow \Omega_1$  is also measurable. In this case, the measurable spaces are **isomorphic**, denoted:

$$(\Omega_1, \mathcal{F}_1) \cong (\Omega_2, \mathcal{F}_2)$$

Such maps preserve the full measurable structure.

**Definition 3.4** (Measurable Space Automorphisms). For a measurable space  $(\Omega, \mathcal{F})$ , the group of measurable automorphisms is

$$\text{Aut}(\Omega, \mathcal{F}) := \{T : \Omega \rightarrow \Omega \mid T \text{ is bijective, } T \text{ and } T^{-1} \text{ are measurable.}\}$$

While measurable isomorphisms preserve the entire structure of a space, our work requires maps that allow a measurable space to be faithfully represented within a larger one.

**Definition 3.5** (Measurable Embedding). A **measurable embedding** is a injective measurable function

$$\iota : \Omega_1 \rightarrow \Omega_2$$

such that the inverse map

$$\iota^{-1} : \iota(\Omega_1) \rightarrow \Omega_1$$

is measurable, where  $\iota(\Omega_1)$  is equipped with the sub- $\sigma$ -algebra induced from  $\mathcal{F}_2$ .

In this case,  $\iota$  identifies  $(\Omega_1, \mathcal{F}_1)$  with a measurable subspace of  $(\Omega_2, \mathcal{F}_2)$ , preserving the  $\sigma$ -algebra of the source space.

**Definition 3.6** (Embedding-induced  $\sigma$ -algebra). Let  $\iota : \Omega_1 \rightarrow \Omega_2$  be a measurable embedding. The *embedding-induced  $\sigma$ -algebra* of  $\iota$  is the sub- $\sigma$ -algebra on  $\iota(\Omega_1) \subset \Omega_2$  induced from  $\mathcal{F}_2$ , defined by

$$\mathcal{F}^\iota := \mathcal{F}_2 \upharpoonright_{\iota(\Omega_1)} = \{A \cap \iota(\Omega_1) \mid A \in \mathcal{F}_2\}.$$

The same sub- $\sigma$ -algebra used in the definition above.

## 3.2 Contextuality

A central concept underlying the theory developed in this work is that of **contextuality**. The definitions presented here are adapted from the sheaf-theoretic formulation of contextuality introduced in [2], to a measure-theoretic framework.

### Labels and Contexts

We begin by introducing the notion of a measurement label. Intuitively, a measurement label represents a physical distinction that can be probed in a system. Measurement labels encode the primitive degrees of freedom of the model.

## Measurement Labels

**Definition 3.7** (Measurement labels). A **measurement label** is an abstract symbol  $m$  associated with a measurable space  $(\Omega_m, \mathcal{F}_m)$ , representing the possible outcomes of measuring  $m$ . We write

$$m \longmapsto (\Omega_m, \mathcal{F}_m)$$

The collection of all measurement labels considered by a model is denoted  $\mathcal{M}$ .

## Global Space

**Definition 3.8** (Global measurable space). The **global space**, representing the joint space of all measurement labels, is the product measurable space

$$(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}}) := \left( \prod_{m \in \mathcal{M}} \Omega_m, \bigotimes_{m \in \mathcal{M}} \mathcal{F}_m \right)$$

Here  $\bigotimes_{m \in \mathcal{M}} \mathcal{F}_m$  denotes the product  $\sigma$ -algebra generated by cylinder sets. For two measurable spaces  $(\Omega_1, \mathcal{F}_1)$  and  $(\Omega_2, \mathcal{F}_2)$  the product  $\sigma$ -algebra is given by

$$\mathcal{F}_1 \otimes \mathcal{F}_2 := \sigma(\{F_1 \times F_2 : F_1 \in \mathcal{F}_1, F_2 \in \mathcal{F}_2\})$$

We emphasize that no probability measure is specified on the global space at this stage. Our interest here lies in the measurable structure itself, independently of any particular choice of global state.

## Contexts

We now introduce the notion of a context. A context represents a collection of measurement labels that are jointly accessible to an observer, or equivalently, a set of degrees of freedom that can be meaningfully considered together. Intuitively, a context specifies the degrees an observer may simultaneously interact with on the system.

**Definition 3.9** (Context). A **context** is a finite subset  $C \subseteq \mathcal{M}$  of measurement labels. To each context we associate a measurable space

$$(\Omega_C, \mathcal{F}_C) := \left( \prod_{m \in C} \Omega_m, \bigotimes_{m \in C} \mathcal{F}_m \right)$$

## Canonical Context Projections

Within a context, we define canonical projection maps onto the outcome spaces of individual measurement labels.

**Definition 3.10** (Canonical context projections). For a context  $C$  and a label  $m \in C$ , the canonical projection is the measurable function

$$\pi_{C \rightarrow \{m\}} : \Omega_C \rightarrow \Omega_m$$

defined by

$$\pi_{C \rightarrow \{m\}}(\omega) = \omega_m \quad \forall \omega \in \Omega_C$$

More generally, for any subcontext  $D \subseteq C$ , we define the projection

$$\pi_{C \rightarrow D} : \Omega_C \rightarrow \Omega_D$$

by restriction to the coordinates indexed by  $D$ .

Intuitively, the projection  $\pi_{C \rightarrow \{m\}}$  the **perspective**  $C$  has on the measurement label  $m$ . The inverse images of projections define canonical measurable subsets of a context space.

**Definition 3.11** (Cylinder Sets). Let  $C$  be a context and  $m \in C$ . For any  $A \in \mathcal{F}_m$ , the corresponding **cylinder set** in  $\Omega_C$  is defined by

$$\pi_{C \rightarrow \{m\}}^{-1}(A) := \{\omega \in \Omega_C \mid \omega_m \in A\}$$

Since  $\pi$  is a measurable function, all cylinder sets belong to  $\mathcal{F}_C$ .

Contexts do not introduce independent information: all events in a context arise as pullbacks of events associated with its measurement labels via the canonical projections. In this sense, a context contains no information that does not originate from its constituent labels.

### Empirical Model

We now introduce the first concept that explicitly involves probability measures: the notion of an **empirical model**.

Intuitively, an empirical model represents a particular realization of the system as accessed through different contexts. Equivalently, it may be viewed as a family of probability distributions describing the observable statistics associated with each context, subject to consistency on overlaps.

**Definition 3.12** (Empirical model). An **empirical model** is a family

$$\{\mu_C\}_{C \in \mathcal{C}}$$

of probability measures, where for each context  $C$ ,

$$\mu_C \text{ is a probability measure on } (\Omega_C, \mathcal{F}_C)$$

These measures are required to satisfy the following *compatibility condition*: For all  $C, C' \in \mathcal{C}$  and all subcontexts  $D \subseteq C \cap C'$ ,

$$(\pi_{C \rightarrow D})_* \mu_C = (\pi_{C' \rightarrow D})_* \mu_{C'}$$

This condition expresses the requirement that the probability distributions assigned to different contexts agree on their common measurement labels, and hence represent consistent marginals of a single underlying empirical situation.

## Contextuality

Empirical models allow us to define a central notion driving the framework developed in this work, that of **contextuality**. Informally, contextuality captures the failure of different observational perspectives to arise as consistent restrictions of a single global description.

**Definition 3.13** (Contextuality). Let  $\{\mu_C\}_{C \in \mathcal{C}}$  be an empirical model. The empirical model is said to be **contextual** if there exists no probability measure  $\mu$  on the global measurable space  $(\Omega_{\mathcal{M}}, \mathcal{F}_{\mathcal{M}})$  such that

$$(\pi_{\mathcal{M} \rightarrow C})_* \mu = \mu_C \quad \forall C \subset \mathcal{C}$$

If such a probability measure exists, the empirical model is called **non-contextual**.

Intuitively, non-contextuality means that the probabilistic data obtained from all contexts can be understood as arising from a single joint probability distribution on the global space, with each context revealing only a partial perspective of that global state.

Contextuality, by contrast, indicates that no such global probability measure exists: although each context admits a well-defined probabilistic description, these descriptions cannot be combined into a single coherent global model.

The existence of contextual empirical models is not merely a formal possibility. It is well established that there are experimental scenarios for which no non-contextual global description exists; a detailed analysis can be found in [2]. The notion introduced here corresponds to probabilistic contextuality in the sense of [2], expressed in measure-theoretic language.

## Standing Assumptions

All measurable spaces considered in this work are assumed to be standard Borel spaces. Probability measures are taken to be Borel probability measures satisfying the usual regularity conditions required for the constructions used below.

## 4 Structural Contextuality

To better understand RIF, we consider formulations of contextuality that do not rely on the specification of particular probability distributions. Instead, we focus on the underlying structural constraints imposed by measurable spaces themselves.

This perspective is inspired by the sheaf-theoretic approach to contextuality, but goes beyond probabilistic inconsistency. In particular, it aligns with the notion of **strong contextuality** as defined in [2], where obstruction arises already at the level of compatible local descriptions.

### 4.1 Information Frames

We begin by introducing the notion of an **information frame**.

**Definition 4.1** (Information Frame). Let  $C \subseteq \mathcal{M}$  be a context. An **information frame** over  $C$  is a measurable space of the form

$$\mathcal{I}_{C, \mathcal{F}} := \left( \prod_{m \in C} \Omega_m, \mathcal{F} \right)$$

Where  $\mathcal{F}$  is a  $\sigma$ -algebra satisfying

$$\mathcal{F} \subseteq \bigotimes_{m \in C} \mathcal{F}_m$$

The  $\sigma$ -algebra  $\mathcal{F}$  represents the set of distinctions that the frame is able to make about the measurement labels in  $C$ .

An information frame may be interpreted as a perspective on the system: it specifies what can, in principle, be distinguished within the context  $C$ . Probability measures on  $\mathcal{I}_{C,\mathcal{F}}$  then represent particular states compatible with that perspective.

We note that when a probability measure is specified on an information frame, events are understood operationally up to null sets. That is, events that differ only on a set of measure zero are identified as representing the same distinction from the perspective of that frame.

For notational convenience, when no ambiguity arises we write

$$\mathcal{I}_i = \mathcal{I}_{C_i, \mathcal{F}_i}$$

## 4.2 Structural Contextuality

### Shared Events

First we must establish when two events in different contexts carry **shared** meaning.

**Definition 4.2** (Shared Event). Let  $C, C' \in \mathcal{C}$  be contexts with nonempty intersection

$$L := C \cap C' \neq \emptyset$$

An event  $E_L \in \mathcal{F}_L$  is called a **shared event** of  $C$  and  $C'$ .

The corresponding events in the context spaces  $(\Omega_C, \mathcal{F}_C)$  and  $(\Omega_{C'}, \mathcal{F}_{C'})$  are given by the cylinder sets

$$E_C := \pi_{C \rightarrow L}^{-1}(E_L), \quad E_{C'} := \pi_{C' \rightarrow L}^{-1}(E_L)$$

### Embeddings and Event images

We also note that, since all spaces considered are standard Borel, any embedding used in this work is understood to be a Borel embedding. In particular, the image of an embedding is a measurable subset of the target space. Consequently, for a embedding

$$\iota : (\Omega_1, \mathcal{F}_1) \rightarrow (\Omega_2, \mathcal{F}_2)$$

and any event  $E \in \mathcal{F}_1$ , we have

$$\iota(E) \in \mathcal{F}_2$$

In fact, for the purposes of this work, general  $\sigma$ -algebra homomorphism suffice. Under the standing assumptions, any measurable embedding induces an injective  $\sigma$ -algebra homomorphism. We therefore freely move between these equivalent perspectives when convenient.

### Structural Contextuality

We can now make precise a definition of contextuality that does not rely on probability measures.

**Definition 4.3** (Structural Contextuality). Given a family of contexts  $\mathcal{C}$  over  $\mathcal{M}$  with corresponding information frames  $\mathcal{I}_{C,\mathcal{F}_C}$  and a global space  $(\Omega, \mathcal{F})$ .

The family is said to be **structuraly contextual** if there exists no family of **embeddings** into a common sub- $\sigma$ -algebra  $\mathcal{G} \subseteq \mathcal{F}$ ,

$$\iota_{C \in \mathcal{C}} : \mathcal{I}_{C,\mathcal{F}_C} \rightarrow (\Omega, \mathcal{G}) \quad C \in \mathcal{C}$$

Such that for every pair  $C, C'$  with  $L = C \cap C'$  and every **shared event**  $E_L \in \mathcal{F}_L$ ,

$$\iota_C(\pi_{C \rightarrow L}^{-1}(E_L)) = \iota_{C'}(\pi_{C' \rightarrow L}^{-1}(E_L))$$

If such a family of embeddings exists, the family of frames is called **structurally non-contextual**.

Intuitvely, structural contextuality relies critcially on the failure of existence of *any* compatible embedding. That is, the events structure of the contexts are incompatible to the extent that no faithful realization can identify shared events globally while preserving the full informational content of each context.

### Relationship with strong contextuality

While this notion is distinct from probabilistic contextuality, it is closely related. When event algebras fail to embed compatibly, any attempt to assign a single global probability measure respecting all contextual distinctions necessarily requires coarse-graining, and may fail entirely.

The existence of **strong contextuality** in the sense of [2] guarantees that structural contextuality occurs for suitably fine event structures. In particular the Kochen–Specker results [3] show that retaining the full  $\sigma$ -algebra structure of measurement contexts eventually obstructs any global realization. This highlights the structural inevitability of contextuality in sufficiently rich logics.

As a illustration in the product space, this can be viewed as the failure of cylinder sets to capture the full complexity of the contextual event structures. Consider, for example, two embeddings

$$\iota_1(x) = (x, f(x)) \quad \iota_2(y) = (g(y), y)$$

Here the embedding is fixed on the degrees of freedom controlled by each context, while the remaining components are unconstrained. Structural contextuality does not arise from a lack of freedom in choosing the functions  $f$  and  $g$ , but from the requirement that they simultaneously preserve the full event structures of the contexts. When shared events are refined incompatibly across contexts, no choice of  $f$  and  $g$  can reconcile all induced events in a single global-algebra.

## 5 The Relativity Of Information Frames

### 5.1 Interaction

We now make precise the meaning of the **Relativity of Information Frames (RIF)**. The central objects of the theory are **Information Frames** from definition 4.1, which are taken as the only ontic objects. All physical content arises from their interactions.

We define interaction structurally, without reference to dynamics or time, as the co-realization of multiple information frames within a common joint frame.

Before proceeding, we note that when writing

$$\iota_C(\mathcal{F}_C) \subseteq \mathcal{G}$$

we implicitly refer to a sub- $\sigma$ -algebra  $\mathcal{G}$  of the codomain induced by the embedding  $\iota_C$ . Different choices of embeddings are related by measurable automorphisms of the codomain and are therefore treated as **gauge-equivalent**.

**Definition 5.1** (Joint Frame). Let  $\{\mathcal{I}_i\}_{i \in I}$  be a family of information frames, with  $\mathcal{I}_i = (\Omega_{C_i}, \mathcal{F}_i)$ . Define the joint label set

$$\mathcal{C} := \bigcup_{i \in I} C_i$$

The **joint frame** is the measurable space

$$\mathcal{J}_I := (\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I})$$

where

$$\Omega_{\mathcal{J}_I} := \prod_{m \in \mathcal{C}} \Omega_m, \quad \mathcal{F}_{\mathcal{J}_I} := \sigma \left( \bigcup_{i \in I} \iota_i(\mathcal{F}_i) \right)$$

The joint frame represents the space capable of expressing all distinctions accessible to the interacting frames. While such frame is always definable at the level of measurable structure, structural contextuality may obstruct the existence of a  $\sigma$ -algebra in which all shared events are consistently identified.

Once a joint realization is fixed, the embeddings  $\{\iota_i\}$  are not allowed to vary within that realization. Different realizations related by automorphisms are treated as gauge-equivalent, but embeddings are fixed within each gauge choice.

## 5.2 Local Perspectives

To make the relativity principle precise, we require a way to compare contexts within a fixed joint realization. Although a joint frame  $\mathcal{J}_I$  is constructed via embeddings  $\{\iota_i\}_{i \in I}$ , the contextual frames themselves do not directly live in the joint space.

This comparison is achieved through the following maps.

**Definition 5.2** (Local Perspective Maps). Let  $\mathcal{J}_I = (\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I})$  be a joint frame generated by the embeddings

$$\iota_i : \Omega_{C_i} \hookrightarrow \Omega_{\mathcal{J}_I}$$

The **local perspective map** associated with frame  $\mathcal{I}_i$  is the measurable map

$$e_i := \iota_i \circ \pi_{\mathcal{J}_I \rightarrow C_i} : \Omega_{\mathcal{J}_I} \rightarrow \Omega_{C_i}$$

**Note:** Once more, under the standing assumptions adopted throughout this work, each local perspective map  $e_i$  is measurable and therefore induces a endomorphism of the joint  $\sigma$ -algebra via pullback. By abuse of notation, we use the same symbol  $e_i$  to denote both the measurable map on  $\Omega_{\mathcal{J}_I}$  and its induced action on events.

The maps  $e_i$  represent the full event structure of the information frame  $\mathcal{I}_i$  as embedded in the joint frame, intuitively the perspective of that frame on the interaction.

**Proposition 5.3** ( $e_i$  are idempotent). *The local perspective maps  $e_i$  are idempotent measurable endomorphisms of  $(\Omega_{\mathcal{J}_I}, \mathcal{F}_{\mathcal{J}_I})$ .*

*Proof.* By definition  $e_i = \iota_i \circ \pi_{\mathcal{J}_I \rightarrow C_i}$ . Since  $\pi_{\mathcal{J}_I \rightarrow C_i} \circ \iota_i = \text{id}_{\Omega_{C_i}}$  we have  $e_i \circ e_i = e_i$  □

**Remark 5.4.** *The maps  $e_i$  act as coarse-grainings of the joint event structure: two events are identified whenever they induce the same event on the contextual frame  $C_i$ .*

### 5.3 The Symmetry Of Information

#### Privilege

We are now ready to introduce the central concept underlying the **Relativity of Information Frames** the concept of privilege.

**Definition 5.5** (Privilege). We say that a joint frame  $\mathcal{J}_I$  privileges  $\mathcal{I}_i$  over  $\mathcal{I}_j$  for a shared event  $E \in \iota_i(\mathcal{F}_i) \cap \iota_j(\mathcal{F}_j)$  if, for the corresponding local perspective maps  $e_i, e_j$  we have

$$e_j(E) \neq e_j(e_i(E))$$

Equality is understood at the level of events in the joint  $\sigma$ -algebra (or up to null set equivalences under the standing assumptions).

Intuitively, a joint frame privileges  $\mathcal{I}_i$  over  $\mathcal{I}_j$  whenever the perspective of  $\mathcal{I}_i$  alters what  $\mathcal{I}_j$  sees. That is, conditioning on  $\mathcal{I}_i$ 's interpretation of an event changes  $\mathcal{I}_j$ 's interpretation.

We note that privilege is witnessed by a failure of commutation of the local perspective maps on the event  $E$ :

$$e_i \circ e_j(E) \neq e_j \circ e_i(E)$$

Finally, privilege is not an ordering relation: it may occur in both directions for the same event.

#### The Relativity of Information Frames

We can now state the relativity of information frames precisely.

**Axiom 1 (The Relativity Of Information Frames).** *After interaction, an event is physically admissible if and only if it does not privilege one information frame over another. Equivalently, in the physically admissible joint frame, no information frame is privileged over another for any event in its  $\sigma$ -algebra.*

More explicitly for any pair of frames  $i, j$  of the joint frame and any shared event  $E$  we have:

$$e_i(E) = e_i(e_j(E)) \quad \text{and} \quad e_j(E) = e_j(e_i(E))$$

## The Frame Pointer Algebra

Next, with axiom 1 in mind we consider the set of physical events for a given information frame  $\mathcal{J}_i$  during an interaction:

$$\mathcal{F}_i^{\text{phys}} := \{E \in \iota_i(\mathcal{F}_i) \mid e_j \circ e_i(E) = e_j(E) \quad \forall j : E \in \iota_j(\mathcal{F}_j)\}$$

That is, the events that do not privilege  $\mathcal{J}_i$  over any other frame.

**Proposition 5.6.** *The sets in  $\mathcal{F}_i^{\text{phys}}$  form a  $\sigma$ -algebra.*

*Proof.* If  $E \in \mathcal{F}_i^{\text{phys}}$  then, for every  $j$ :

$$e_j(e_i(E^c)) = e_j(e_i(E)^c) = e_j(e_i(E))^c = e_j(E)^c = e_j(E^c)$$

So  $E^c \in \mathcal{F}_i^{\text{phys}}$ .

And let  $E_n$  be a countable collection of RIF-valid events, that is:

$$e_j(e_i(E_n)) = e_j(E_n) \quad \forall n, j$$

Since:

$$e_j \left( \bigcup_n E_n \right) = \bigcup_n e_j(E_n)$$

Given any  $j \in I$  we have:

$$e_j \left( e_i \left( \bigcup_n E_n \right) \right) = e_j \left( \bigcup_n e_i(E_n) \right) = \bigcup_n e_j \circ e_i(E_n) = \bigcup_n e_j(E_n) = e_j \left( \bigcup_n E_n \right)$$

□

We give therefore the special name:

**Definition 5.7** (Frame Pointer Algebra). The algebra of physically admissible events for a frame is called the **frame pointer algebra**.

$$\mathcal{F}_i^{\text{ptr}} = \mathcal{F}_i^{\text{phys}}$$

## The Pointer Algebra

The frame pointer algebras in definition 5.7 represent the physical events from the context of each information frame. However, the events that can happen during a interaction are the events that satisfy RIF in general for the contexts of each interaction.

**Definition 5.8** (Pointer Algebra). The **pointer algebra** for the joint frame  $\mathcal{F}_{\mathcal{J}_I}^* \subseteq F_{\mathcal{J}_I}$  is the algebra:

$$\mathcal{F}_{\mathcal{J}_I}^* := \sigma \left( \bigcup_{i \in I} \mathcal{F}_i^{\text{ptr}} \right)$$

When the joint frame is obvious we may simply refer to the pointer algebra as  $\mathcal{F}^*$ .

## Interpretation

Local perspective maps do not represent physical operations performed in time, but rather encode how a given information frame identifies events within a joint description.

A privilege occurs precisely when two frames cannot consistently identify the same event without reference to an ordering of perspectives, signaling the absence of a joint description for that event.

Frame pointer algebras therefore collect those events that admit a stable interpretation from the perspective of a given frame during an interaction. The pointer algebra of the joint frame is generated by all such non-privileging events, and represents the maximal event structure that can be jointly realized without privileging any information frame.

This algebra captures the emergent classical structure associated with the interaction.

## Coarse-Graining and the Emergence of Probability.

**Remark 5.9.** *In this framework, probability is not taken as a primitive notion. Instead, probabilistic structure arises as a consequence of coarse-graining enforced by axiom 1.*

When an interaction removes distinctions that cannot be jointly maintained across information frames, multiple incompatible bookkeeping events are identified as a single physically admissible event in the pointer algebra. From the perspective of an individual frame, these identified events are indistinguishable, yet no further structural information remains available to discriminate between them.

Any assignment of weights to physically admissible events that is stable under further coarse-graining and compatible with the structure of the pointer algebra must therefore take the form of a probability measure. In this sense, probabilities encode the residual information accessible to a frame after incompatible distinctions have been eliminated, rather than reflecting intrinsic randomness or ignorance of an underlying reality.

## 5.4 Relation to Contextuality

Structural contextuality makes precise the close relationship between the Relativity of Information Frames and contextuality in the usual sense. In particular, if a family of information frames is structurally non-contextual in the sense of definition 4.3, then no privileged events arise and the pointer algebra coincides with the full  $\sigma$ -algebra of the joint frame.

**Proposition 5.10** (Pointer algebra relation to structural contextuality). *If  $\{\mathcal{I}_i\}_{i \in I}$  is a family of information frames and  $\{\iota_i\}_{i \in I}$  their corresponding embeddings. Then the following hold for the pointer algebra of the joint frame*

- if the family is structurally non-contextual  $\mathcal{F}_{\mathcal{J}_I}^* = \mathcal{F}_{\mathcal{J}_I}$
- if the family is structurally contextual  $\mathcal{F}_{\mathcal{J}_I}^* \subsetneq \mathcal{F}_{\mathcal{J}_I}$

*Proof.* The proof follows directly from the definitions. □

While this observation already captures a nontrivial physical mechanism - namely, the elimination of incompatible distinctions - we now establish a stronger and more informative result. Specifically, we show that the pointer algebra is, in a precise sense, the **maximally informative non-contextual algebra** compatible with the given joint structure.

Structural contextuality alone is not sufficient for our purposes, as it detects contextuality only when one attempts to preserve the full informational structure of each frame. A similar limitation applies to standard probabilistic contextuality. However, probability measures provide additional flexibility, allowing one to distinguish between different  $\sigma$ -algebras on the joint frame without modifying the underlying event structure.

We therefore fix a consistent empirical model  $\{\mathcal{I}_i\}_{i \in I}$  as in definition 3.12. For each context we consider the associated information frame  $\mathcal{I}_i$ , with event algebras representing the support of the corresponding probability measures, as described previously. Using a choice of embeddings  $\{\iota_i\}_{i \in I}$ , we construct the joint frame  $\mathcal{J}_I$ .

We begin by defining the following induced measure on the joint frame:

$$\tilde{\mu}_i(E) := \mu_i(\iota_i^{-1}(E)), \quad E \in \iota_i(\mathcal{F}_i) \quad (1)$$

Which represents the pushforward of  $\mu_i$  to the embedded event algebra. Restricting this measure to the frame pointer algebra yields

$$\tilde{\mu}_i^{\text{ptr}} := \tilde{\mu}_i \upharpoonright \mathcal{F}_i^{\text{ptr}} \quad (2)$$

In order to glue these restricted measures into a single global measure on the pointer algebra  $\mathcal{F}^*$ , it is necessary that they agree on overlaps:

$$\tilde{\mu}_i^{\text{ptr}}(E) = \tilde{\mu}_j^{\text{ptr}}(E) \quad \forall E \in \mathcal{F}_i^{\text{ptr}} \cap \mathcal{F}_j^{\text{ptr}}. \quad (3)$$

We then state the following lemma:

**Lemma 5.11** (Pointer Overlap Consistency). *If  $E \in \mathcal{F}_i^{\text{ptr}} \cap \mathcal{F}_j^{\text{ptr}}$ , then  $E$  is a **shared event** whose identification is order-independent, hence any empirical family assigns it the same weight.*

*Sketch.* By construction, events in  $\mathcal{F}_i^{\text{ptr}} \cap \mathcal{F}_j^{\text{ptr}}$  admit a frame-independent identification in the joint structure. Such events correspond to shared events whose embeddings agree up to  $\sigma$ -algebra homomorphisms, and hence empirical consistency on overlaps implies the equality of their assigned weights.  $\square$

From this lemma, it is obvious that eq. (3) holds, which allows us to state the desired result

**Proposition 5.12** (Existence of a non-contextual measure). *If  $\tilde{\mu}_i^{\text{ptr}}$  agree on overlaps, there exists a probability measure  $\mu^*$  on  $(\Omega_{\mathcal{J}}, \mathcal{F}^*)$  extending them.*

*Sketch.* We define a pre-measure  $\mu_0$  for  $\mathcal{F}^*$ , since the frame pointer algebras are the generators of  $\mathcal{F}^*$  we can define, whenever  $E \in \mathcal{F}_i^{\text{ptr}}$

$$\mu_0(E) = \tilde{\mu}_i^{\text{ptr}}(E)$$

Since we have eq. (3), this is well defined. Since  $\mu_0(\Omega_{\mathcal{J}}) = 1$  and  $\mu_0 \geq 0$  we can use the *Carathéodory Extension Theorem*[4, 5] to get a measure  $\mu^*$  on  $\mathcal{F}^*$  with the required overlap consistency.  $\square$

This result shows that, starting from a empirical model which may or may not be contextual on the constructed joint frame, the potential coarse graining to the pointer algebra directly admits a consistent probability measure with that empirical model.

In what follows, since we had fixed a empirical model and constructed the joint frame and pointer algebra relative to the  $\sigma$ -algebraic structure induced by the that model. Maximality here is understood relative to this joint frame.

**Proposition 5.13** (Maximality of the pointer algebra). *Any strict extension  $\mathcal{F}^* \subsetneq \mathcal{G} \subseteq \mathcal{F}_{\mathcal{J}_I}$  fails to admit a consistent global measure extending the fixed empirical family.*

*Sketch.* Since  $\mathcal{G}$  must have a event  $E$  that is not **RIF admissible** we have for some pair  $i, j$ :

$$E \in \iota_i(\mathcal{F}_i) \cap \iota_j(\mathcal{F}_j) \quad \text{and} \quad e_j(e_i(E)) \neq e_j(E)$$

Since the events are in the support from the initial assumptions and they are distinct events we must have:

$$\tilde{\mu}_j(e_j(e_i(E))) \neq \tilde{\mu}_j(e_j(E))$$

Then suppose we can build a global measure. But as we have seen, consistency would require

$$\tilde{\mu}_j(e_j(e_i(E))) = \tilde{\mu}_j(e_j(E))$$

So  $\mathcal{G}$  cannot admit a consistent global measure. □

These results imply a strong correlation between contextuality and the RIF pointer algebra.

**Theorem 5.14** (The Pointer Algebra is the Maximally Informative Non-Contextual Algebra). *For a joint frame built from a family of information frames the pointer algebra is the largest  $\sigma$ -algebra that admits consistent probability measures.*

### Remarks

We note that this result is in some sense informal, there is a real limiation on how contextuality is defined to talk about a maximally informative **non-contextual** algebra, the construction is far more natural in the **RIF** definition, but the arguments strongly show the link between the concepts.

It is important to note that if one fixes the joint frame first, and look at what empirical families can be built on it. Non-contextuality of the pointer algebra becomes relative to this joint frame. With this understanding, the pointer algebra has two key properites:

- Any consistent empirical family admits a global probability measure on the pointer algebra.
- Any strict extension of the pointer algebra admits a locally consistent probability assignment that do not glue to a global measure.

## Part II

# Physical Structure and Consequences

## 6 Quantum Mechanics

In this section we clarify the connection between the RIF framework and quantum theory. Our goal is not a full formal reconstruction of quantum mechanics, but to show that the event structures arising

naturally in RIF coincide with the algebraic structures underlying quantum theory. In particular, we show that the collection of contextual event algebras forms an orthomodular lattice, and that standard Hilbert space realizations arise under the usual structural assumptions. Consequences such as the Born rule and Wigner-type consistency conditions will then be seen to follow naturally from RIF invariance.

## 6.1 The Hilbert Space Realization

We will not attempt a full Hilbert space reconstruction in this work. Instead, we situate the RIF event structure within the well-established framework of quantum logic, highlighting where RIF provides a natural physical interpretation of the underlying assumptions.

### The Orthomodular Lattice of Events

Let  $(\mathcal{F}_i)_{i \in I}$  denote the Boolean  $\sigma$ -algebras associated to each information frame (context), and let the usual embeddings  $(\iota_i)_{i \in I}$  into a joint frame, as definition 5.1, be given.

From the usual quantum logic perspective, the events on the joint frame algebra form a lattice.

$$\mathcal{L} := \mathcal{F}_{\mathcal{J}_I} := \sigma \left( \bigcup_{i \in I} \iota_i(\mathcal{F}_i) \right)$$

Naturally  $\mathcal{L}$  is a lattice with its operations

- complement  $E \rightarrow E^c$ ,
- meet:  $E \wedge F = E \cap F$ ,
- join:  $E \vee F = \text{cl}_{\mathcal{L}}(E \cup F)$

**Proposition 6.1** ( $\mathcal{L}$  is a orthomodular lattice). *The event structure  $\mathcal{L}$  is an orthomodular lattice. Each contextual algebra  $\iota_i(\mathcal{F}_i)$  embeds as a maximal Boolean subalgebra of  $\mathcal{L}$ .*

*Sketch.* Each  $\mathcal{F}_i$  is a Boolean algebra and therefore an orthocomplemented distributive lattice. The embeddings  $\iota_i$  preserve complements and finite meets, ensuring that  $\mathcal{L}$  is orthocomplemented.

Distributivity fails in  $\mathcal{L}$  whenever events arising from incompatible frames cannot be jointly refined, which is precisely the manifestation of contextuality in the RIF framework. However, the consistency of partial refinements between compatible events ensures that the orthomodular law holds. Thus  $\mathcal{L}$  is an orthomodular, but generally non-distributive lattice.  $\square$

Under standard assumptions, the same we established in this framework, classical results in quantum logic [6, 7] imply that  $\mathcal{L}$  admits a representation as the projection lattice of a Hilbert space:

$$\mathcal{L} \cong \text{Proj}(\mathcal{H})$$

where  $\mathcal{H}$  is a Hilbert space over  $\mathbb{R}$ ,  $\mathbb{C}$  or  $\mathbb{H}$ .

## 6.2 Unitary Evolution and the Gauge Symmetries in RIF

### Relabeling Gauge of the Joint Frame - Unitary Action

The joint frame construction  $\mathcal{J}_I$  introduces an intrinsic gauge freedom. Since  $\Omega_{\mathcal{J}_I}$  is a product space over labels  $\mathcal{C}$ , different measurable relabelings of the joint sample space may induce the same relational event structure. This redundancy is not an additional assumption, but is present by definition.

Let  $\text{Aut}(\mathcal{J}_I)$  denote the group of measurable bijections  $T : \Omega_{\mathcal{J}_I} \rightarrow \Omega_{\mathcal{J}_I}$  preserving the joint  $\sigma$ -algebra  $\mathcal{F}_{\mathcal{J}_I}$ . Fixing a reference frame  $i \in I$ , we define the relabeling gauge group relative to  $i$  by

$$G_i := \{T \in \text{Aut}(\mathcal{J}_I) : T^{-1}(\iota_i(\mathcal{F}_i)) = \iota_i(\mathcal{F}_i)\}.$$

Elements of  $G_i$  correspond to relabelings of the joint description that leave invariant the event structure accessible to frame  $i$ , while re-identifying how other contexts are embedded.

Two joint-frame realizations are said to be gauge-equivalent relative to  $i$  when they are related by an element of  $G_i$ . The physical content of the RIF construction is thus identified with structures invariant under this relabeling gauge.

### One parameter gauge slices and the emergence of a time parameter

Upon translation to a Hilbert space realization  $\mathcal{F}_{\mathcal{J}_I} \cong \text{Proj}(\mathcal{H})$ , the relabeling gauge is represented by the familiar projective **unitary** (and antiunitary) symmetries of the Hilbert description.

In particular, the fact that admissible re-descriptions in RIF must:

- be invertible,
- preserve probability measure,
- preserve conditional structure.

This structure is represented in the Hilbert space description by unitary operators  $U$  satisfying

$$U^\dagger U = I$$

Once we make a few *additional structural choices*:

- descriptions can be related continuously,
- coarse-graining changes continuously under small interactions,
- inference is stable under infinitesimal re-descriptions.

Under these choices one may select a *strongly continuous one-parameter* subgroup of the gauge. Then by Stone's theorem [8], the strongly continuous one-parameter group admits a self-adjoint generator.

$$U(\lambda) = e^{-i\lambda G}$$

The time  $t$  is a choice of  $\lambda$  used to track a chosen gauge slice, and  $G$  labels the motion along a gauge orbit, not the dynamical evolution of the system.

### Predictiveness: why gauge unitarity still yields sharp sequential probabilities

Consider a bookkeeping joint frame  $\mathcal{J} := (\Omega, \mathcal{F})$  representing all interactions over time. Let  $\mathcal{F}_t^*$  represent the pointer algebra at the stage where we label the slice by  $t$ , for an eventual pointer event  $A \in \mathcal{F}_{t_2}^*$  we define the probability assignment at time  $t_1$ :

$$M(t_1; A) := \mathbb{E} [\mathbf{1}_A | \mathcal{F}_{t_1}^*]$$

We note that no particular probability measure is fixed in the definition of  $\mathbb{E}$ . This is intentional, as gauge transport acts on the event structure independently of the chosen measure. The emergence and transport of specific probability measures via pushforward are discussed in section 6.5.

Consistency across stages is then a martingale:

$$\mathbb{E}[M(t_1; A) | \mathcal{F}_{t_2}^*] = M(t_2; A), \quad (t_1 \leq t_2)$$

So the propagation between  $t_2$  and  $t_1$ , which in the Hilbert space representation is

$$U(t_2 - t_1)$$

it acts as an *identification map* induced by the chosen gauge slices. The only non-trivial update occurs at  $t_2$ , where conditioning onto the later pointer algebra is performed. Gauge unitarity provides the coherent transport structure required for this conditioning to be well-defined across descriptions, without representing dynamical evolution. We note that predictive consistency is measure-covariant, it is preserved under arbitrary choices of probability measures compatible with a given information frame.

### Retrodiction and the appearance of dynamical evolution

After an experiment is performed and an event  $A \in \mathcal{F}_{t_2}^*$  is registered, earlier descriptions are updated by conditioning on  $A$ . For events  $B \in \mathcal{F}_{t_1}^*$  with  $t_1 \leq t_2$  we have

$$\mathbb{E} [\mathbf{1}_B | A],$$

with conditional expectation taken with respect to the later pointer algebra.

The apparent predictiveness of unitary evolution arises from the fact that gauge transport and conditionalization commute in a controlled way. This allows coherent *forward prediction* and *backward retrodiction* across descriptions. When retrodictive updates are interpreted as physical processes occurring between  $t_1$  and  $t_2$ , this coherence is often mistaken for underlying dynamical evolution, even though no such evolution is represented in the RIF framework.

### Complex Structure and Gauge Freedom

The Hilbert space realization of the joint-frame event structure carries, by construction, a representation of the intrinsic relabeling gauge of the RIF framework. This gauge is implemented in the Hilbert description as a projective unitary symmetry acting on  $\mathcal{H}$ .

Supporting a nontrivial and continuous projective unitary action places strong constraints on the underlying scalar field. Real Hilbert spaces do not admit a sufficiently rich phase structure to

represent generic gauge transformations, while quaternionic Hilbert spaces introduce additional constraints on the localization and composition of such symmetries.

By contrast, complex Hilbert spaces provide the minimal setting in which continuous projective unitary representations, local gauge freedom, and consistent composition of independent subsystems coexist. From the RIF perspective, the appearance of complex structure is therefore not an independent postulate, but a natural consequence of representing the intrinsic joint-frame gauge in a linear space.

### Structural Remark: Internal Gauge Symmetry

The relabeling gauge inherent in the joint-frame construction is represented, upon a Hilbert-space realization of the framework, as a projective unitary symmetry acting on  $\mathcal{H}$ . At this level, the gauge symmetry is generically very large, reflecting the freedom in identifying joint descriptions related by relabeling of events.

A further restriction on this relabeling gauge arises from the presence of local perspective maps and the requirement that no information frame be privileged, as formalized by axiom 1. While  $\text{Aut}(\mathcal{J}_I)$  represents the full descriptive redundancy of the joint frame, not all such relabelings are compatible with perspectival symmetry.

In particular, admissible relabelings must preserve the equivalence of descriptions induced by local perspective maps, in the sense that no information frame can detect a preferred identification of joint events. This requirement selects a distinguished subgroup of the relabeling gauge, consisting of transformations that are relationally invisible across all frames.

Upon Hilbert-space realization, this perspectively admissible gauge is represented as a restricted subgroup of the projective unitary symmetry.

We emphasize that no derivation of a specific gauge group is claimed here. Rather, this discussion is intended to indicate that familiar gauge symmetries, including those of the Standard Model, are compatible with—and may be viewed as particular reductions of—the intrinsic relabeling gauge symmetry present in the RIF framework, once restricted to physically admissible transformations.

## 6.3 Observables, Measurements and Operators

### Measurement Device

In the RIF framework, a measurement device is represented by a pure information frame  $\mathcal{J}_{\text{mes}} := (\Omega_{\text{mes}}, \mathcal{F}_{\text{mes}})$ . The event algebra  $\mathcal{F}_{\text{mes}}$  encodes the distinctions that the device is capable of registering. No additional structure is assumed.

### Observables

An observable associated with an information frame  $\mathcal{J}_i$  is a measurable function

$$O_i : \Omega_i \rightarrow \mathcal{O}$$

Where  $\mathcal{O}$  is an outcome space, such as  $\mathbb{R}$  or a discrete set. Through the embedding  $\iota_i$ , each observable induces a corresponding random variable on the joint frame, defined on the embedded subspace by

$$\tilde{O}_i := O_i \circ \iota_i^{-1}$$

The  $\sigma$ -algebra generated by  $\tilde{O}_i$  represents the collection of events distinguishable by the observable  $O_i$ . Two observables are said to be **compatible** if their induced  $\sigma$ -algebras generate a jointly Boolean algebra in the joint frame.

### Observable-Induced Operators

To see the relation with operator non-commutativity, it is convenient to consider the coarse-graining maps induced by observables. Given an event  $E$  in the joint frame and an observable  $O_i$ , we define the observable-induced map

$$e_i^{O_i}(E) := \iota_i(O_i^{-1}(O_i(\pi_i(E))))$$

which represents the coarse-graining of  $E$  according to the distinctions accessible to  $O_i$ . For observables associated with different frames, these maps need not commute:

$$e_i^{O_i} \circ e_j^{O_j} \neq e_j^{O_j} \circ e_i^{O_i}$$

reflecting the incompatibility of the corresponding observables.

### Measurement

A measurement of an observable  $O_i$  corresponds to a full interaction and conditioning the joint description on an event in the frame pointer algebra  $\mathcal{F}_i^{\text{ptr}}$ . Only events belonging to this algebra are physically admissible outcomes of the measurement. The coarse-graining implicit in the pointer algebra identifies multiple fine-grained events as a single measurement outcome, thereby inducing a probabilistic description as per remark 5.9.

Repeated measurements of the same observable correspond to an already resolved conditioning on the same pointer event and therefore yield stable outcomes. In contrast, an interaction with an incompatible information frame can be seen as a new interaction that induces a new coarse-graining, reintroducing distinctions that were previously suppressed. In this sense, measurement outcomes are not destroyed but rendered frame-relative by subsequent incompatible interactions.

### Relation to POVMs

When the RIF framework is represented on a Hilbert space, the coarse-grained event structure associated with a measurement naturally gives rise to positive operator-valued measures. Each pointer event  $E \in \mathcal{F}_i^{\text{ptr}}$  corresponds to an equivalence class of fine-grained events, and the probability assigned to such an event by the induced global measure may be represented as  $\text{Tr}(\rho, E_i)$  for a positive operator  $E_i$ .

The non-projective nature of these operators reflects the fact that pointer events are defined by coarse-graining rather than by sharp partitions of the joint space. In this sense, POVMs arise as faithful representations of frame-relative measurements within the RIF framework. Projective measurements then represent situations where there is no coarse-graining in the representation.

## 6.4 Stern-Gerlach in RIF

We end with an informal analysis of the Stern-Gerlach test [9] using the RIF framework. We will model the standard sequence:

1. measure spin along  $z$  (device A),
2. measure spin along  $x$  (device B),
3. then measure along  $z$  again (device A).

## Setup

We begin with modeling the three information frames, the system  $\mathcal{I}_S$ , and the two devices  $\mathcal{I}_A, \mathcal{I}_B$ . After interaction, their frame pointer algebras are represented by:

$$\mathcal{F}_A^{\text{ptr}} = \sigma(\{Z+, Z-\}) \quad \text{and} \quad \mathcal{F}_B^{\text{ptr}} = \sigma(\{X+, X-\})$$

### Interaction 1: System and device A

The first interaction constructs the joint frame  $\mathcal{J}_{SA}$  using embeddings  $\iota_S, \iota_A$ , then impose RIF and restrict to the pointer algebra  $\mathcal{F}_{SA}^*$ .

The outcomes A can see from the measurement is a event in is frame pointer algebra:

$$E_A \in \mathcal{F}_A^{\text{ptr}} \subseteq \mathcal{F}_{SA}^*$$

From A's perspective, repeated measurements in the same interaction context is stable:

$$Z+ \text{ then } Z+ \quad \text{or} \quad Z- \text{ then } Z-$$

Assume for this sequence that the measurement resulted in  $Z+$ .

### Interaction 2: System and device B

Now B interacts with the composite frame  $\mathcal{J}_{SA}$ , producing a new joint frame  $\mathcal{J}_{SAB}$  with its own pointer restriction  $\mathcal{F}_{SAB}^*$ . Since the  $Z$  and  $X$  distinctions are incompatible in the RIF sense, the second interaction induces a further coarse-graining of the admissible event structure.

The  $x$ -measurement is not "reading a pre-existing  $x$  value"; the interaction is creating a new coarse-grained joint structure and selecting a event within it. That means, conditioning on the earlier  $Z+$  event, B sees:

$$\mathbb{P}(X+ | Z+) = \mathbb{P}(X- | Z+) = \frac{1}{2}$$

Again, after the interaction repeated  $x$ -measurements are stable from B's point of view, assume that the resulting interaction yielded  $X+$ .

### Interaction 3: System and device A again

Now A is interacting, not with the original  $\mathcal{J}_{SA}$  but with the new composite that has undergone an incompatible interaction with B.

The event labeled  $Z+$  after the first interaction does not correspond to the same admissible event after the incompatible interaction with B, since the underlying pointer algebra has changed

$$\mathbb{P}(Z+ | X+) = \mathbb{P}(Z- | X+) = \frac{1}{2}$$

This does not represent a physical disturbance propagating from  $B$  to  $A$ , but a change in the admissible joint description induced by an incompatible interaction.

## Conclusion

This models the sequences of tests as a series of interaction in RIF. We note that this analysis is not a final objective truth of what is happening, it is simply a interpretation of the mathematics in a easy to reason standard.

Similar interpretations could be done with a joint frame built by all involved interactions directly and seeing measurement as a sampling that shifts the events. Or how quantum logic might normally view such structure.

The main result here, is that interaction is relational, in the RIF framework, a globally defined event structure is not merely unnecessary but ill-defined as a physical notion. Any attempt to treat a global description as ontic would privilege descriptions that would violate axiom 1. Admissible events are therefore defined only relative to interactions, and no single frame provides a complete description of what occurs.

## 6.5 Born Rule Martingale

In this section we explore how the Born rule appears in this framework. We do not claim, with the current tools, to recover the quadratic nature of the born rule or its usual form. That is left to a reconstruction of the Hilbert space.

### Setup

First we consider a particular context's frame  $\mathcal{I}_i$ . We define a probability measure representing that frame  $\mu_0$ .

In the joint frame we can look at the pushfoward of the contextual measure  $\mu_0$  to the joint  $\sigma$ -algebra.

$$\mu := \mu_0 \circ \pi_i = \mu_0(\pi_i(E)) \quad E \in \mathcal{F}_{\mathcal{J}}$$

We note that this probability measure does not, necessarily, represent all contexts of the joint frame. In fact, in contextual cases, that is not possible. This is simply the probability of the context  $i$  transported to the global frame.

### The Collapse Filtration

Since the pointer algebra  $\mathcal{F}^*$  is a sub- $\sigma$  of  $\mathcal{F}_{\mathcal{J}}$ , we may consider any decreasing family of  $\sigma$ -algebras.

$$\mathcal{F}_{\mathcal{J}} =: F_0 \supseteq F_1 \supseteq F_2 \supseteq \dots \mathcal{F}^*$$

representing successive coarse-grainings of the joint description.

While the collapse happens directly to the pointer algebra upon interaction, these filtrations represent partial descriptions, this sequence can be interpreted heuristically as successive partial information updates - analogous to **weak or partial measurements**.

## The Collapse Martingale

We can now use  $\mu$  as a probability measure to define, for any event  $A \in \mathcal{F}_{\mathcal{J}}$  we define:

$$M_n := \mathbb{E}_{\mu} [\mathbf{1}_A \mid \mathcal{F}_n]$$

As seen [1] we know this is a Martingale on the reverse filtration, and by Doob convergence theorem we have:

$$M_n \rightarrow \mathbb{E}_{\mu} [\mathbf{1}_A \mid F_{\text{ptr}}] \quad (\text{a.s.})$$

Thus the conditional probability of  $A$  given the pointer algebra coincides with the coarse-grained probability  $\mu(A)$  in that algebra. Collapse therefore corresponds, in measure-theoretic terms, to conditioning on the pointer  $\sigma$ -algebra.

## Interpretation

In this sense, the Born rule emerges as the statement that the observed probabilities are those of the conditional measure obtained by coarse-graining to the pointer algebra.

The stochastic character of measurement outcomes is therefore not fundamental but a reflection of information loss under coarse-graining.

The martingale formulation expresses the stability of these conditional probabilities under repeated measurement, as required by empirical repeatability.

## 6.6 Wigner's Friend Consistency

We will now turn our attention to looking at how Wigner's Friend paradox looks like in this framework.

### Setup

For the Wigner friend scenario we actually have two interactions. First the interaction of the friend, which we will map to the joint frame  $\mathcal{J}_{\text{friend}}$  which represents the interaction between the system and the friend.

Once that interaction goes through, the joint frame has the physically admissible algebra, the pointer algebra  $\mathcal{F}_{\text{friend}}^*$ . Then Wigner comes and interacts with that joint frame, forming a new joint frame  $\mathcal{J}_{\text{Wigner}}$ , which again has its coarse-grained pointer algebra  $\mathcal{F}_{\text{Wigner}}^*$ .

We use the fact that, these algebras can all be seen as filtrations of each other to show that, Wigner cannot assign inconsistent probabilities to the events he can observe.

### The Sequence of Filtrations

When the friend interacts with the system, he generates the joint frame  $\mathcal{J}_{\text{friend}}$ . When Wigner interacts with that frame, a new joint frame must be built. That joint frame starts by lifting the algebras through the embeddings  $\iota_{\text{friend}}$  and  $\iota_{\text{Wigner}}$ .

The nature of embeddings mean we can consider this all a sequence of filtrations on the same space, in particular we consider the **frame pointer algebra** as in definition 5.7 for what can

physically happen in the joint frame.

$$\mathcal{F}_{\text{Wigner}} \supseteq \iota_{\text{friend}}(\mathcal{F}_{\text{friend}}^*) \supseteq \mathcal{F}_{\text{Wigner}}^{\text{ptr}} \cap \mathcal{F}_{\text{friend}}^{\text{ptr}}$$

With that relation in place, we can look at the same style of probability assignments we had in the born rule.

### 6.6.1 The Probabilities

Now let  $A \in \mathcal{F}_{\text{friend}}^{\text{ptr}} \cap \mathcal{F}_{\text{Wigner}}^{\text{ptr}}$  be an event that both frames can meaningfully talk about. We can determine the friend's probability for event  $A$  as we did for the born rule martingale:

$$\mathbb{E}_{\mu_f} [\mathbf{1}_A | \mathcal{F}_{\text{friend}}^{\text{ptr}}]$$

Now for Wigner, we have the same:

$$\mathbb{E}_{\mu_w} [\mathbf{1}_A | \mathcal{F}_{\text{Wigner}}^{\text{ptr}}]$$

But this trivially yields:

$$\mathbb{E}_{\mu_w} [\mathbf{1}_A | \mathcal{F}_{\text{Wigner}}^{\text{ptr}}] = \mathbf{1}_A = \mathbb{E}_{\mu_f} [\mathbf{1}_A | \mathcal{F}_{\text{friend}}^{\text{ptr}}]$$

Since admissible events already belong to the relevant pointer algebras, conditioning acts trivially, and probabilities coincide. Thus, when the friend's description is updated to the Wigner's pointer frame it is exactly Wigner's own description. Wigner and the friend cannot assign inconsistent probabilities to any event that both can meaningfully discuss.

## 7 Dynamics

In this framework, dynamics is not introduced as a primitive law such as a Hamiltonian flow or differential equation. Instead, dynamical behavior arises from the structure of interactions between information frames.

Each interaction between contexts induces:

- The creation of a joint frame,
- a corresponding coarse-grained pointer algebra,
- and the induced probability measures on the interacting frames.

From the perspective of a fixed frame, the time evolution of its description of the world is thus given by a sequence of coarse-grained  $\sigma$ -algebras generated by successive interactions. This defined a filtration on its *frame pointer algebras*:

$$\mathcal{F}_{\text{frame}} =: \mathcal{F}_0 \supseteq \mathcal{F}_1^{\text{ptr}} \supseteq \mathcal{F}_2^{\text{ptr}} \supseteq \dots \supseteq \mathcal{F}_n^{\text{ptr}} \supseteq \dots$$

This perspective makes several dynamical features appear naturally:

## 1. Arrow of Time.

Each contextual interaction corresponds to additional coarse-graining, the evolution of  $\sigma$ -algebras is monotone and information-losing. This monotonicity defines a natural, frame-relative direction of time.

## 2. Locality Graph.

Interactions occur only between specific frames. The pattern of which frames interact defines an emergent graph structure, which plays the role of space locality.

## 3. Maximum Speed of Influence

In contextual scenarios, incompatibility forces coarse-graining. The maximal number of interaction steps before contextuality appears bounds how influence can propagate along the locality graph.

## 4. No Signaling.

Because interactions only merge  $\sigma$ -algebras along edges of the locality graph, and the physically admissible events remain consistent across frames, no frame can influence another without a mediated interaction.

## 5. Gravity.

Interactions force coarse-graining of admissible event algebras in order to maintain physical consistency. Frames that are already highly coarse-grained are comparatively stable under further interaction, while less coarse-grained frames must adapt their descriptions more strongly. This persistent asymmetry under interaction induces a directional bias in how descriptions evolve, defining a gravitational-like structure at the level of information flow.

None of these dynamical features require any additional axioms. They follow from Axiom 1 and the definitions governing interactions.

### 7.1 Arrow of Time

In this framework, time is not an external parameter. Instead, the ordering of interactions between information frames induces a canonical direction: each interaction generates a bookkeeping joint frame in which the initial frame algebra is embedded. By axiom 1, this interaction forces a coarse-grained description of the frame in the corresponding *frame pointer algebra*. This monotone loss of distinguishability defines a natural arrow of time for a given information frame.

#### Interaction-induced evolution of $\sigma$ -algebras

To begin, we first fix an information frame  $\mathcal{I}_0$ , which proceeds to interact successively with other frames  $\mathcal{I}_1, \mathcal{I}_2$ , and so on.

At each step, a joint frame is created to host the interaction, which then converges into a physically realizable joint frame with a corresponding frame pointer algebra, as defined in definition 5.7,

$$\mathcal{F}_n^{\text{ptr}}$$

That is, the frame pointer algebra on the  $n$ -th joint frame. Since each such pointer algebra is constructed by embedding the pointer algebra from the previous step, we may track the evolution of  $\mathcal{I}_0$ 's description along successive interactions.

We identify the effective  $\sigma$ -algebra of  $\mathcal{I}_0$  after  $n$  interactions with the subalgebra of events in  $\mathcal{F}_0$  that remain physically admissible in the pointer algebra  $\mathcal{F}_n^{\text{ptr}}$ , and define

$$\mathcal{F}_n := \mathcal{F}_n^{\text{ptr}}$$

Which represents the effective  $\sigma$ -algebra of  $\mathcal{I}_0$  after  $n$  interactions. This gives a natural sequence:

$$\mathcal{F}_0 \supseteq \mathcal{F}_1 \supseteq \mathcal{F}_2 \supseteq \dots$$

Each interaction removes distinctions incompatible with the new join, producing strictly coarser  $\sigma$ -algebras. There is no mechanism within the framework to restore the lost distinctions. The resulting monotone coarse-graining defines a natural, frame-relative direction of time.

## 7.2 Locality Graph

In this framework, **locality** is not introduced as a geometric primitive. Instead, it is defined as a combinatorial structure recording which frames have interacted, and therefore share a joint pointer algebra.

### The Locality Graph

Let  $\{\mathcal{I}_i\}_{i \in I}$  be a family of relevant information frames. We define an undirected graph

$$G = (V, E)$$

With vertex set  $V = I$  and edge set  $E \subseteq I \times I$  given by

$$(i, j) \in E \iff \text{frames } \mathcal{I}_i \text{ and } \mathcal{I}_j \text{ have interacted.}$$

An edge thus records the existence of a joint frame in which the interaction between  $\mathcal{I}_i$  and  $\mathcal{I}_j$  has generated a shared pointer algebra.

### Locality as constraint on admissible events

If frames  $\mathcal{I}_i$  and  $\mathcal{I}_j$  have interacted, then their local perspective maps  $e_i$  and  $e_j$  (cf. definition 5.2) act on a common joint frame and therefore constrain the same pointer algebra. If no such interaction has occurred, their perspectives do not act on a shared admissible event structure.

As consequence:

- Frames in the same connected component of  $G$  may influence one another through constraints propagated via a shared pointer algebra.
- Frames in different connected components remain strictly independent; no constraints or influence propagate between them.

This reproduces the operational notion of locality, influence propagates only along paths in the interaction graph.

### Dynamics respects the locality graph

When a new interaction occurs between frames  $\mathcal{I}_i$  and  $\mathcal{I}_j$ , the locality graph  $G$  is updated by:

1. adding an edge  $(i, j)$  to  $G$ ;
2. constructing the joint frame associated with the connected component of  $G$  containing  $i$  and  $j$ .

As a consequence, each connected component of  $G$  behaves as a local region: interactions affect only the  $\sigma$ -algebra generated by frames within that component. No update acts on frames outside the affected component.

Locality therefore manifests in two complementary forms:

- **Graph locality:** interactions are represented by edges in  $G$ , and influence propagates only along paths in the graph.
- **Probabilistic locality:** as a consequence, probability assignments and conditional expectations factor across disconnected components of  $G$ .

### Interpretation

This construction provides a purely structural notion of spacetime locality:

- Information frames play the role of relational "positions",
- interactions define adjacency relations analogous to lightlike contact,
- paths in the locality graph  $G$  are the only channels through which constraints and influence can propagate.

At this stage, no geometric or metric structure is assumed: Geometry, if present, must arise as additional structure placed on top of the locality graph.

We emphasize that both "locality" and "time" in this framework are structural notions. They arise from the pattern and ordering of interactions between frames, rather than from a pre-assumed spacetime background. Any system describable in terms of interacting information frames therefore admits a well-defined locality graph and a induced temporal order, even in the absence of an underlying geometric spacetime.

### 7.3 Maximum Speed of Influence

In this framework, influence propagates only through interactions between information frames, which are represented by edges in the locality graph  $G$ . Any propagation of constraints must therefore occur along paths in  $G$ .

However, contextuality imposes an additional structural limitation. In sufficiently rich families of interacting information frames sharing degrees of freedom, joint non-contextual descriptions cannot be maintained indefinitely [2]. When contextual incompatibility arises, the joint description must coarse-grain to a physically admissible pointer algebra.

### Contextuality-induced coarse-graining

While contextuality need not arise in every interaction sequence, in generic interaction patterns involving mixed degrees of freedom it inevitably appears after a finite number of steps. When this occurs, at least one participating frame undergoes a strict coarse-graining of its admissible  $\sigma$ -algebra.

Such an event marks the end of a propagation epoch: beyond this point, further interactions cannot transmit fine-grained distinctions without degradation.

### Propagation depth for a fixed frame

Fix an information frame  $\mathcal{I}_i$ , and consider a path in the locality graph

$$i = i_0 \rightarrow i_1 \rightarrow \cdots \rightarrow i_n.$$

Let  $\{\mathcal{F}_k\}$  denote the effective  $\sigma$ -algebras of  $\mathcal{I}_i$  induced by successive interactions along this path. Define the propagation depth of  $\mathcal{I}_i$  as

$$v_i := \sup\{n \mid \mathcal{F}_k = \mathcal{F}_0 \text{ for all } k \leq n\},$$

that is, the largest number of sequential interactions along which the frame's admissible event structure remains unchanged.

By construction,  $v_i$  is finite in generic contextual scenarios.

### Interpretation

This bound should not be interpreted as the emergence of a specific relativistic speed. Rather, it is a structural consequence of contextuality: in sufficiently complex systems of interacting information frames, repeated interactions necessarily lead to incompatibilities that force coarse-graining.

The resulting bound limits how far influence can propagate along the locality graph without loss of distinguishability. Although different frames may exhibit different local bounds, propagation without degradation is always finite in physically relevant systems.

## 7.4 No Signaling

No signaling is an immediate structural consequence of the locality graph and axiom 1. Since interactions are encoded as edges in the locality graph  $G$ , and the RIF update rules apply only on joint frames associated with interacting frames, no frame can influence another without a path of interactions connecting them.

### Local independence

Let  $\mathcal{I}_i$  and  $\mathcal{I}_j$  be two information frames. If they have never interacted, then they lie in different connected components of the locality graph  $G$ . In this case, no local perspective map  $e_i$  acts on any admissible  $\sigma$ -algebra accessible to  $\mathcal{I}_j$ .

Consequently, any interaction step not involving  $\mathcal{I}_j$  leaves its effective  $\sigma$ -algebra unchanged:

$$\mathcal{F}_j^{n+1} = \mathcal{F}_j^n.$$

A frame's admissible event structure can be altered only through interactions in which it directly participates.

### Causal separation

More generally, let  $\mathcal{I}_k$  be a frame participating in a new interaction within the connected component of  $\mathcal{I}_i$ , with  $\mathcal{I}_j$  lying in a different connected component. Since there is no path in  $G$  connecting  $k$  to  $j$ , the local perspective map  $e_k$  acts only on joint frames disjoint from those accessible to  $\mathcal{I}_j$ . As a result, the interaction has no effect on  $\mathcal{F}_j$ .

It follows that:

- the effects of interactions propagate only along paths in the locality graph;
- only frames belonging to the same connected component of  $G$  can influence one another.

This defines a purely structural notion of causal separation.

### Interpretation

No signaling in this framework is a direct corollary of the interaction structure: information cannot be transmitted between frames that are not connected by a sequence of interactions. No additional dynamical or probabilistic assumptions are required.

## 7.5 Asymmetric Coarse-Graining and Gravitational Structure

At this stage of the framework, no geometric notion of spacetime has been introduced. All structure arises from interactions between information frames, the resulting pointer algebras, and the cumulative effects of coarse-graining. Nevertheless, the framework admits a natural structural notion corresponding to gravitational behavior.

### Asymmetry under interaction

Interactions between information frames induce coarse-graining of their admissible  $\sigma$ -algebras in order to maintain physical consistency. When two frames interact, the induced joint pointer algebra may require unequal coarse-graining of the respective descriptions.

We say that a frame  $\mathcal{I}_i$  is *interaction-stable* if, for every interaction with another frame  $\mathcal{I}_j$ , the induced coarse-graining map leaves  $\mathcal{F}_i^{\text{ptr}}$  invariant, while  $\mathcal{F}_j^{\text{ptr}}$  undergoes strict coarse-graining in the joint frame. Such frames have already resolved a large amount of contextuality and therefore exhibit minimal further loss of distinguishability under interaction.

Repeated interactions of this kind induce a systematic asymmetry: neighboring frames are forced to adapt their admissible event structures toward that of  $\mathcal{I}_i$ , while  $\mathcal{I}_i$  remains comparatively unchanged. This persistent asymmetry defines a gravitational structure within the interaction network.

### Stability versus magnitude

In realistic scenarios, no frame remains strictly invariant under all interactions. Nevertheless, the framework does not require a quantitative measure of coarse-graining to identify gravitational

structure. The ordering induced by asymmetric loss of distinguishability is sufficient: frames whose admissible algebras are coarser than those of their neighbors act as stable attractors under interaction, forcing surrounding descriptions to adapt preferentially toward them.

### Geometric interpretation (heuristic)

If probability assignments on pointer algebras are represented in an information-geometric space—such as a Fisher–Rao metric space—this asymmetry manifests as a directional distortion of probabilistic transport. Highly coarse-grained descriptions correspond to stable regions toward which nearby probabilistic descriptions are deflected, producing effective curvature in the information-geometric representation.

No claim is made here that this mechanism reproduces gravitational dynamics or spacetime geometry. The purpose of this interpretation is solely to indicate how gravitational structure, if emergent within the RIF framework, would admit a geometric representation.

## 8 Interpretation and Comparisons

Here we situate the RIF framework within the broader literature on quantum measurement and interpretation, and discuss its ontological commitments and explanatory structure.

### 8.1 The Ontology of Information Frames

An information frame represents a contextual organization of events, rather than a physical entity or epistemic state. Frames are defined relationally, through their capacity to be embedded into other frames and to participate in consistency constraints imposed by the RIF axiom.

In RIF, states do not evolve and probabilities are not fundamental; both emerge as contextual summaries constrained by consistency under interactions between frames. Even spacetime structure arises as an effective description of these interactions.

Objectivity in RIF is secured not by appeal to a privileged description, but by the invariants that persist across incompatible frames. The absence of a global description does not undermine realism; rather, it shifts realism from descriptive objects to relational structure. What is real is not what any one frame asserts, but what no consistent interaction between frames can eliminate.

### 8.2 No Privileged Frame, No Global Description

In RIF, the assumption that there exists a single global description capable of consistently accounting for all event distinctions is not merely unmotivated, but structurally forbidden. The RIF axiom constrains which joint descriptions may exist, and in general does not permit a global event space in which all contextual distinctions can be simultaneously embedded.

This structural restriction manifests, in operational terms, as a failure of assumptions commonly used to motivate global hidden-variable models. In particular, the preparation independence condition employed in Bell-type arguments does not hold in RIF: systems prepared independently do not retain independent descriptions once interaction is taken into account. Their joint description is not defined prior to interaction, but is constituted through it.

From this perspective, Bell-type nonlocality does not signal superluminal influence or dynamical violation of locality. Rather, it reflects the impossibility of assigning a global factorized description

to events whose joint structure is only defined relationally. Any attempt to impose such a global description encounters precisely the inconsistencies highlighted by Bell’s theorem.

While this failure is often phrased in terms of locality, RIF reveals a more fundamental origin. The gauge nature of time evolution implies that interaction itself constrains which events can meaningfully occur. Once interaction takes place, later distinctions are not independent of earlier contextual structure, not due to causal influence propagating in spacetime, but because no privileged, interaction-independent description exists.

Importantly, experimental violations of Bell inequalities do not contradict RIF; rather, they confirm the impossibility of a global, interaction-independent description of events. RIF does not satisfy the assumptions required to derive Bell inequalities, and therefore lies outside their domain of applicability while remaining fully consistent with experimental results.

### 8.3 Predictiveness Without Fundamental Dynamics

Throughout this work, we have seen how a single axiom restricting which descriptions may be jointly defined gives rise to a rich and highly constrained structure. The predictive power of RIF does not primarily lie in the generation of numerical time evolutions, but in the limitation it places on which joint event structures are admissible. Predictions arise from the requirement that descriptions remain mutually consistent under interaction, rather than from the propagation of states in time.

This perspective clarifies why RIF remains empirically predictive despite lacking fundamental dynamics. What changes between interactions is not an underlying physical state, but the relational structure through which events are jointly describable. Apparent temporal evolution and probability updating emerge from successive interactions and consistency constraints, rather than from an ontic time-parameterized process. Predictiveness in RIF is therefore structural: it consists in ruling out incompatible descriptions and correlations, leaving only those consistent with the framework’s relational constraints.

### 8.4 Comparisons with Quantum Interpretations

The RIF framework is not an interpretation of quantum mechanics in the traditional sense. It does not begin with the formalism of quantum theory and assign meaning to its elements. Instead, RIF imposes a structural restriction on what can be jointly described as physically real, from which much of the standard quantum formalism emerges as an effective description.

As a consequence, most of the structures predicted by RIF are fully aligned with quantum mechanics in its established domain of applicability. Where RIF diverges, it does so primarily outside the conventional scope of quantum theory, notably in regimes involving chaotic dynamics and emergent gravitational structure. Within the quantum domain, RIF is empirically equivalent to standard quantum mechanics while differing in its ontological commitments.

Nevertheless, by grounding quantum phenomena in relational constraints rather than fundamental dynamics or global states, RIF induces a distinct perspective on the nature of the quantum world. It is these interpretative consequences—arising from the framework rather than imposed upon it—that we examine in the following comparisons.

#### 8.4.1 Comparison table

In table 1 we give a overall view of how RIF relates to some popular intrepretations.

Table 1: Schematic comparison of interpretations. Entries indicate structural roles rather than evaluative judgments.

Aspect	Copenhagen	Many Worlds	QBism	Relational QM	RIF
Primitive postulates	Classical–quantum cut; measurement primitives	Universal state (global wavefunction); universal unitary	Agent-centered coherence/normative consistency; personal probabilities	Relational facts between systems; no absolute state	Joint describability restriction (RIF axiom); frame non-privilege
Global description (fundamental)	Not defined / not required	Posited (global state)	Not meaningful (agent-relative)	No absolute state, but inter-system relational description	Forbidden (no privileged global frame)
Measurement (fundamental status)	Primitive update / collapse at cut	No collapse; branching within global unitary	Action + belief update for an agent	Event is relative to interacting systems	Joint description formation + conditioning; not a primitive dynamical law
Observer (fundamental role)	Essential in practice (classical description)	Not fundamental (emergent from global state)	Fundamental (agent and experiences central)	Not fundamental as “observer,” but systems define relations	None (frames are relational, not agents)
Dynamics (fundamental status)	Mixed (unitary + collapse rules)	Fundamental unitary dynamics	No ontic dynamics (normative updating)	Typically unitary for closed systems; relational updating	Unitary as gauge; fundamental constraints are interaction-based
Probability	Born rule (rule for outcomes)	Branch weights (Born measure as typicality)	Subjective degrees of belief (normative)	Relational probabilities	Emergent from contextual coarse-graining
Joint descriptions	Classical joint descriptions privileged at cut	Global joint description always available (in principle)	Joint description is agent-relative	Joint facts only relative to systems	Joint description generally requires coarse-graining

## 8.5 Comparisons with Objective Collapse Models

The RIF framework is not compatible with objective collapse models. This incompatibility is structural rather than empirical. In RIF, time is not a fundamental parameter but an emergent ordering arising from interaction and conditioning. Any model that assigns collapse a fixed rate, mass scale, or spacetime-localized trigger therefore presupposes precisely the kind of privileged temporal or frame-dependent structure that RIF excludes.

In objective collapse theories, collapse is treated as a physical dynamical process occurring in time. In contrast, RIF does not regard collapse as a physical mechanism at all. Events that do not belong to the pointer algebra are not physically admissible descriptions in the first place; collapse corresponds to the formation of a joint description followed by conditioning, not to an additional dynamical law modifying unitary evolution.

From this perspective, RIF also clarifies a persistent difficulty faced by collapse models: the assignment of a collapse time. Any such assignment necessarily privileges a particular frame or foliation, reintroducing a global structure incompatible with frame non-privilege. As a result, collapse models must either tolerate hidden signaling structures or impose additional constraints to suppress them, whereas in RIF no such tension arises, since collapse is not a time-localized physical event.

## 8.6 Implications for Decoherence

Standard decoherence theory proceeds by specifying a system–environment Hamiltonian and tracing over environmental degrees of freedom, yielding the suppression of interference in a preferred basis. Within decoherence theory, the emergence of a pointer basis is often presented as a dynamical consequence of environmental interaction. From the RIF perspective, however, this ordering is reversed. The choice of Hamiltonian already fixes the invariant structure of the interaction, and with it the pointer algebra. Decoherence does not select the pointer basis; it reveals the basis that remains invariant under interaction. The demotion of unitary evolution to gauge makes this structural role explicit.

Lindblad dynamics can be understood analogously. Rather than describing a fundamental physical process that destroys coherence, Lindblad operators characterize the effect of noise on predictive models within a given context. The environment functions as a noise generator, and the Lindblad equation tracks how increasing noise degrades the ability of a contextual description to maintain sharp probabilistic predictions. As noise increases, fewer features of the description remain stable across interaction, until only the invariant event structure—the pointer algebra—survives.

From this viewpoint, decoherence is not the physical mechanism by which classical outcomes are produced, but a measure of how predictive a given description remains under environmental noise. When decoherence is weak, detailed probabilistic predictions are possible; when decoherence is strong, predictive power is lost, and only coarse, invariant events remain admissible. The appearance of classicality reflects the survival of these invariant structures, not the elimination of superpositions as physical entities.

This interpretation is directly mirrored in classical modeling. Consider a classical description of a projectile fired toward a wall. A detailed dynamical model predicts the projectile’s trajectory, and weak probes placed along the path refine the probability distribution for the point of impact. As long as environmental noise is limited, the model remains predictive. If strong noise is introduced—for

example through an uncontrolled magnetic field—the detailed predictions degrade. The probability distribution for the point of impact broadens, yet the invariant fact that the projectile will strike the wall within a finite region remains. What survives is not the detailed model, but the stable event structure.

Decoherence operates in precisely this way. Under high environmental noise, detailed quantum descriptions lose predictive sharpness, and the only admissible events are those associated with the pointer algebra. This does not signal a physical collapse induced by the environment, but the exhaustion of the descriptive power of a given context. In classical settings this distinction is rarely emphasized, because it is evident that one is dealing with a model. In the quantum setting, decoherence has often been misinterpreted as providing more than this: not merely a description of noise-induced loss of predictability, but an explanation of outcome selection. RIF makes clear that decoherence accomplishes the former, not the latter.

## 8.7 Classical Limits, Chaos, and Emergent Forces

Classical behavior in RIF arises in regimes where information frames are highly compatible, allowing joint descriptions to remain stable across interaction. In such regimes, coarse-graining is weak and successive interactions preserve a rich event structure, yielding the appearance of deterministic dynamics. The classical limit is therefore not defined by the absence of contextuality, but by its effective suppression through compatibility.

These regimes form a continuum: from highly compatible interactions supporting stable classical descriptions, through intermediate regimes where small incompatibilities are amplified, to strongly coarse-grained regimes in which only invariant structures remain predictive.

Chaos occupies an intermediate and revealing role. In standard formulations, chaos is characterized by sensitivity to initial conditions within a given model. From the RIF perspective, this sensitivity reflects a deeper instability: the rapid amplification of small incompatibilities between contextual descriptions. As interactions accumulate, joint descriptions require increasingly aggressive coarse-graining in order to remain consistent, causing detailed event distinctions to become unstable.

In this sense, chaotic systems are those for which the maintenance of fine-grained joint descriptions is structurally fragile. What appears as exponential divergence of trajectories is the effective manifestation of repeated contextual mismatches that cannot be jointly resolved without loss of information. Chaos thus marks the boundary between regimes where detailed classical descriptions remain predictive and regimes where only coarse, invariant structures survive.

This amplification of coarse-graining has direct consequences for emergent forces. As contextual mismatches accumulate, certain structural features dominate joint descriptions, biasing the effective organization of events. These biases appear, at the descriptive level, as force-like tendencies guiding coarse-grained behavior. Gravity provides a prominent example of such an emergent structure as seen in section 7.5, where persistent asymmetry under interaction gives rise to stable, directionally biased descriptions.

## 9 Discussion

### 9.1 Geometry and Dynamics

The framework developed in this work defines dynamical behavior structurally, through constraints on joint describability and interaction, rather than through fundamental time evolution. Nevertheless, the structure uncovered suggests that a geometric formulation may be both natural and fruitful. In particular, an information-geometric approach, in which compatibility between information frames is parametrized by an appropriate metric—such as a Fisher–Rao—offers a promising direction.

Within such a formulation, contextual incompatibility between frames would be reflected as curvature in the information geometry. Interactions that require coarse-graining could then be interpreted as processes that deform the effective metric in order to stabilize relations between descriptions. Dynamics, in this setting, would not be fundamental but emergent, arising from extremal principles defined on the information-geometric structure.

A fully explicit construction of RIF dynamics remains an open problem. Developing such a formulation is an important next step and may provide a unified language for extending the framework beyond its present scope.

### 9.2 Reconstruction Programs

As discussed in section 6.1, RIF naturally lends itself to reconstruction programs in quantum foundations. Starting from a single restriction on joint describability , the framework provides new tools for addressing long-standing questions, including the emergence of Hilbert space structure, the preference for the complex field  $\mathbb{C}$  over alternatives, and the origin of unitary transformations.

Unlike traditional reconstruction approaches, which often assume significant portions of the quantum formalism from the outset, RIF constrains structure indirectly through relational and consistency requirements. This suggests that several features usually taken as axiomatic may instead be derivable as necessary conditions for maintaining non-privileged, interaction-consistent descriptions.

### 9.3 Chaos

The reinterpretation of chaos offered by RIF points toward a number of open research directions. Rather than treating chaos solely as sensitivity to initial conditions within a fixed model, RIF associates chaotic behavior with the instability of fine-grained joint descriptions under repeated interaction and coarse-graining.

A more formal study of this relationship may clarify the role of contextuality in classical and semiclassical systems, and could provide new tools for analyzing chaotic regimes without relying exclusively on trajectory-based descriptions. Such an investigation would likely benefit from an explicit geometric and dynamical formulation of the framework, as discussed above.

### 9.4 Implications for gravity

RIF suggests that gravitational phenomena may be understood as emergent effects arising from imbalances in contextual interactions. In regimes where incompatibility between descriptions accumulates, coarse-graining becomes increasingly asymmetric, biasing the structure of joint descriptions. At the effective level, this bias may manifest as curvature or force-like behavior.

When combined with the connection between chaos and contextual incompatibility, this perspective raises the possibility that phenomena commonly attributed to dark matter could, in part, reflect unaccounted-for curvature induced by highly chaotic systems. While speculative, this viewpoint motivates further investigation into the relationship between information loss, contextuality, and effective geometry.

As an illustrative consistency check, the observational data from [10], [11] and [12] summarized in table 2 shows how these principles might qualitatively appear in certain astrophysical scenarios. No empirical prediction is claimed: the present framework does not yet support numerical modeling, and the listed systems are included solely to illustrate the structural correlations suggested by RIF.

Table 2: Illustrative consistency check between dynamical regime and classical gravitational reconstruction. Quoted dark-matter ratios are observationally inferred under standard assumptions and are not RIF-derived quantities. The table highlights a structural requirement implied by RIF rather than a phenomenological model.

System	$M_\star (M_\odot)$	Dynamical regime	Inferred DM ratio	RIF consistency
NGC 1052–DF2 / DF4	$\sim 10^8$	Cold, weakly coupled	$\sim 0$	Stable pointer algebra; negligible reconstruction bias
Antlia II	$\sim 10^6$	Tidal, non-equilibrium	Moderate	Partial loss of joint describability; biased mass inference
Segue 1	$\sim 10^3$	Extreme tidal heating	$\sim 10^{3:1}$	Severe contextual instability; maximal reconstruction bias

Within this framework, black holes may be interpreted as systems whose admissible event structure collapses to an extremely coarse, effectively trivial sigma algebra. Such a perspective may offer new insights into the informational aspects of quantum gravity, without presupposing a fundamental spacetime background.

## 9.5 Cosmological constant

In standard general relativity, the cosmological constant  $\Lambda$  appears as a fixed background parameter, commonly interpreted as vacuum energy or large-scale spacetime curvature. In RIF, by contrast, spacetime itself is not fundamental; large-scale geometry is descriptive, emerging from accumulated interactions across the cosmic network.

From this perspective,  $\Lambda$  can be interpreted as a measure of the aggregate effect of asymmetric coarse-graining at cosmological scales. The universe is dominated by highly coarse-grained, interaction-stable structures, and their cumulative imbalance produces an effective expansion tendency in geometric descriptions. It is this tendency that  $\Lambda$  parametrizes.

Because coarse-graining in RIF is historical and cumulative, the effective value of  $\Lambda$  need not be strictly constant. As interactions proceed and contextual incompatibilities are resolved at different rates across regions of the universe, the net asymmetry may evolve slowly over cosmic time. This suggests a natural avenue for exploring departures from strict constancy without introducing new fundamental fields or background structure.

## 10 Conclusion

In this work we introduced the **Relativity of Information Frames(RIF)** as a structural principle governing the joint description of interacting systems. Information frames were treated as the primary ontic objects, with physical admissibility determined not by dynamics or collapse postulates, but by the absence of privileged perspectives under interaction.

Using measure-theoretic tools and contextuality theory, we showed that enforcing RIF leads naturally to a distinguished coarse-grained event structure—the **pointer algebra**- which is maximal among  $\sigma$ -algebras admitting consistent probability measures across frames. This result reframes the appearance of probabilistic behavior not as a fundamental feature of nature, but as an emergent consequence of necessary coarse-graining imposed by incompatible informational perspectives.

The framework clarifies the relationship between contextuality, probability, and measurement. Events are not assumed to possess global meaning independent of interaction; rather, joint descriptions arise only through compatible embeddings of frame-specific event structures. When such compatibility fails, probability measures appear as bookkeeping devices encoding unavoidable loss of distinction. In this sense, probabilistic collapse is not a physical process in time, but a structural feature of interaction.

Within this setting, unitary evolution emerges as a gauge symmetry associated with relational re-descriptions, rather than as a fundamental dynamical law. Temporal ordering and irreversibility arise only once coarse-graining is imposed, suggesting that time itself is an emergent notion tied to interaction and information loss. This perspective allows standard quantum phenomena—including incompatible measurements, Born-rule statistics, and Wigner-type scenarios—to be consistently represented without invoking observer-dependent collapse or hidden ontic states.

We emphasize that the results presented here are **structural rather than dynamical**. No specific Hamiltonians, equations of motion, or quantitative predictions are derived. Instead, the contribution of this work lies in isolating a minimal and physically natural principle-non-privilege of informational perspectives-and exploring its consequences at the level of event structure and probability.

The framework suggests several directions for future research. These include a full reconstruction of Hilbert-space quantum theory from RIF principles, a systematic study of emergent temporal and causal structures, and a deeper investigation into how classicality, chaos, and effective forces arise from compatibility regimes of information frames. More broadly, RIF offers a unifying language in which quantum measurement, contextuality, and probabilistic reasoning appear as aspects of a single structural constraint on physical description.

Whether RIF ultimately serves as a foundation for quantum theory or as a clarifying reformulation of its conceptual core, it provides a concrete framework in which long-standing interpretational tensions can be addressed without adding ad hoc dynamics or ontological assumptions. The extent to which this perspective can be extended or experimentally constrained remains an open and promising question.

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