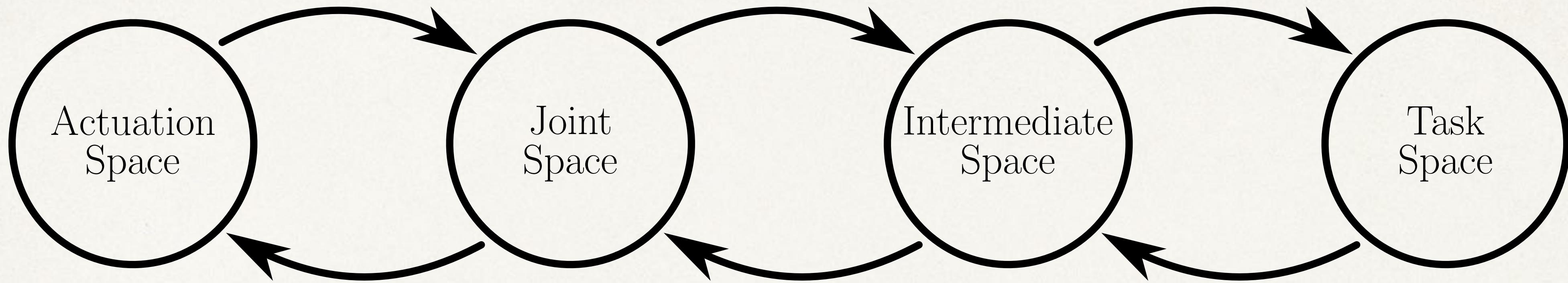


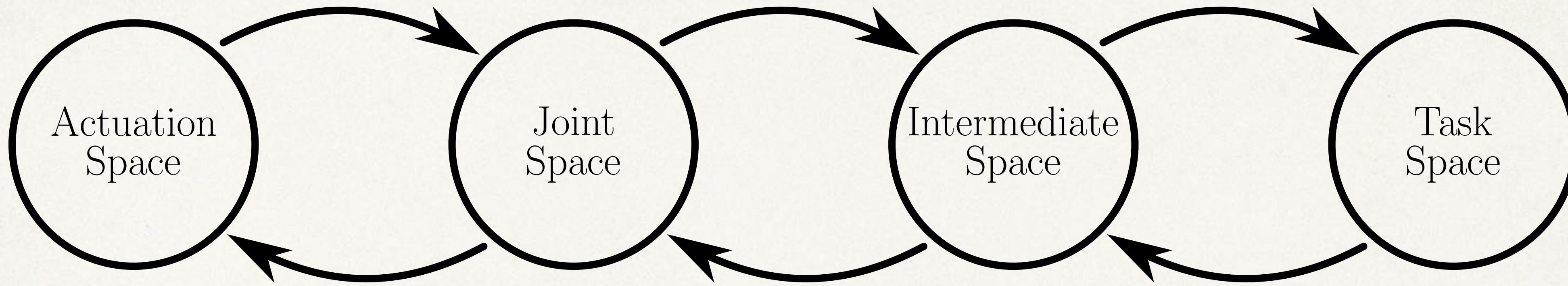
Clarke Transform: A Lingua Franca for Continuum Robotics

Benchmarking in Soft Robotics: Standardizing Data Collection and Evaluation for Actuation, Sensing, and Control

Reinhard M. Grassmann

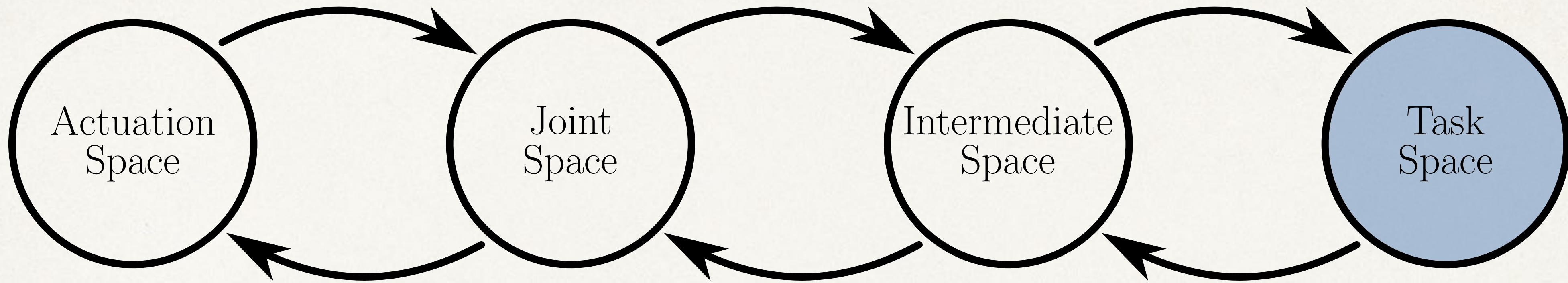


Robotics is the art of transformations between spaces

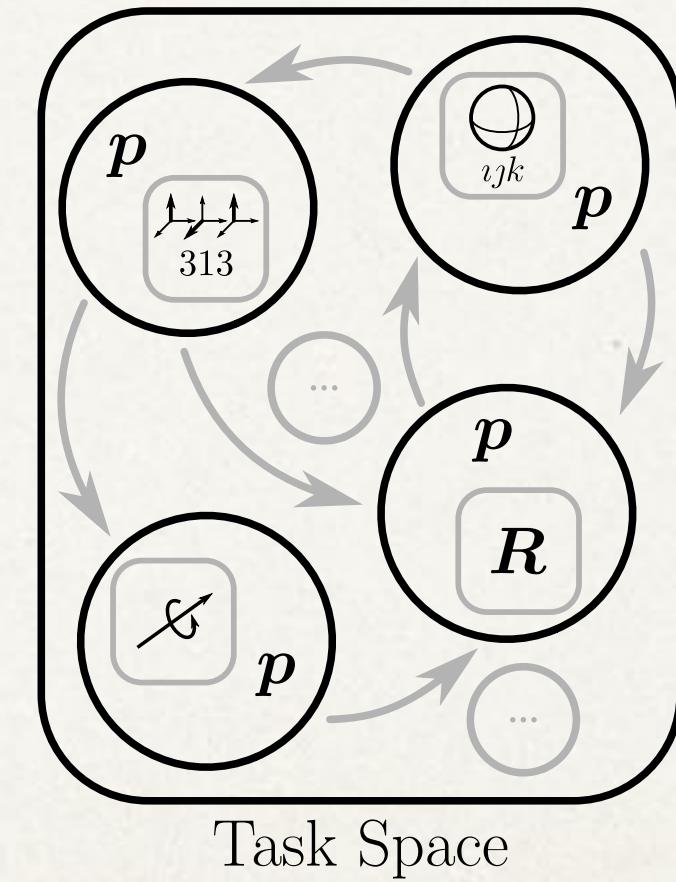


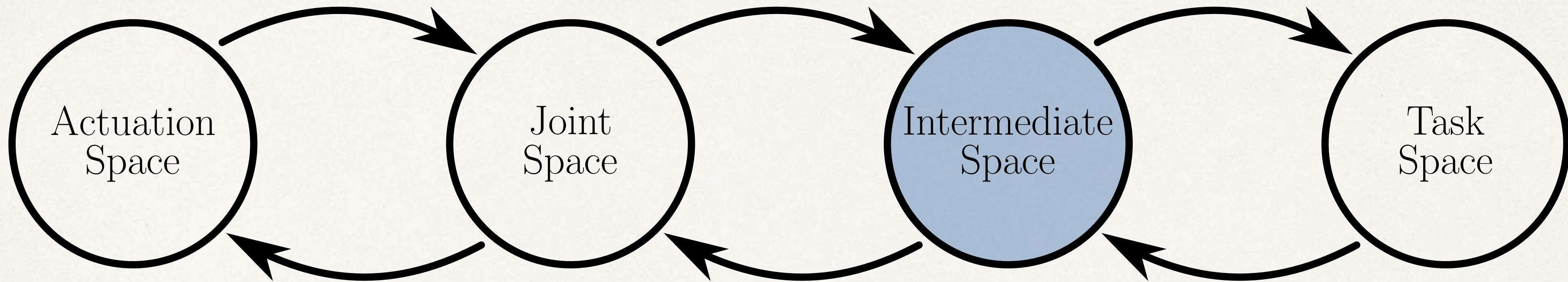
Robotics is the art of transformations between spaces

At the same time, each of the spaces has its own pitfalls.

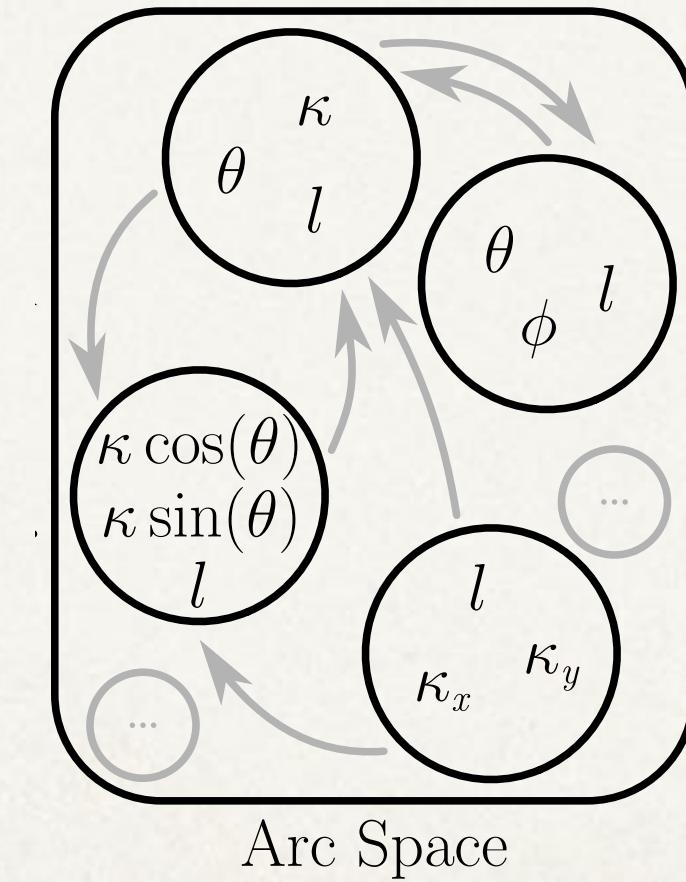


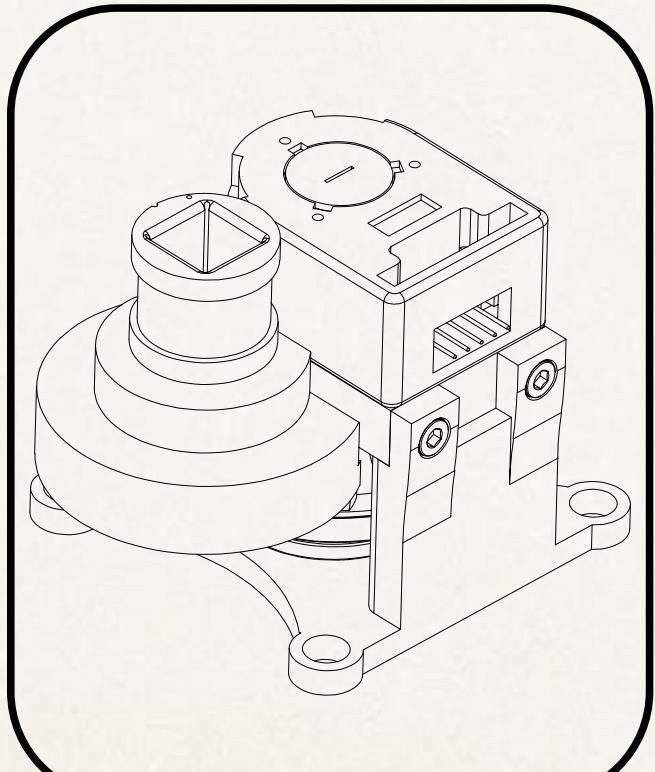
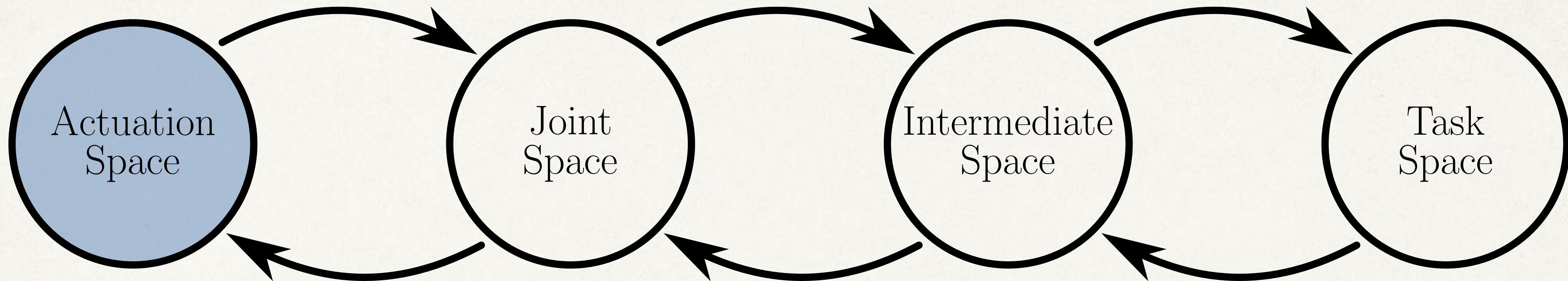
- ❖ Euler angles
 - ❖ 12 sets, singularities, ...
- ❖ unit quaternions
 - ❖ double coverage, unit length
- ❖ rotation matrix
 - ❖ 9 values, 6 constraints





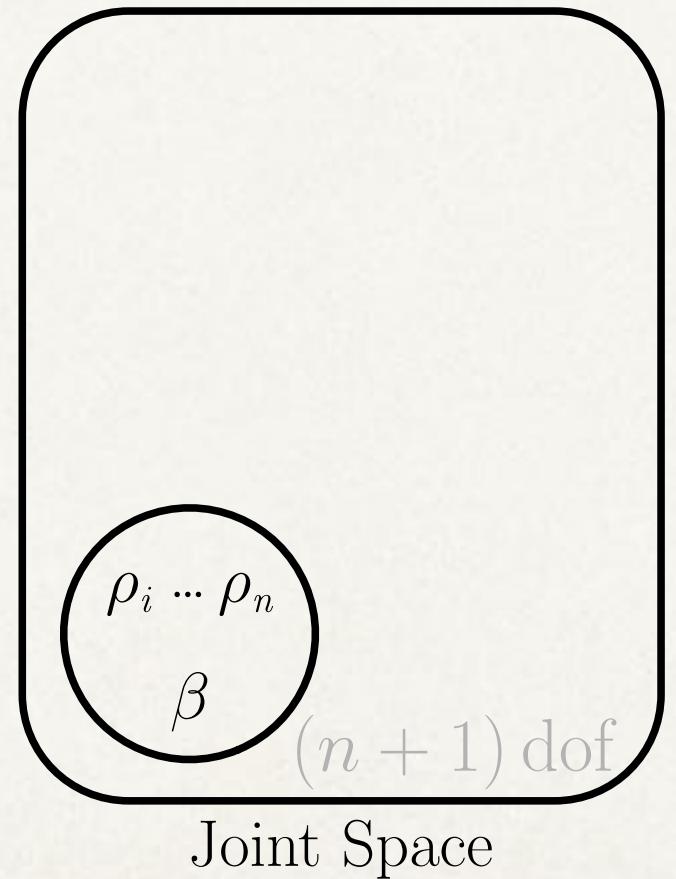
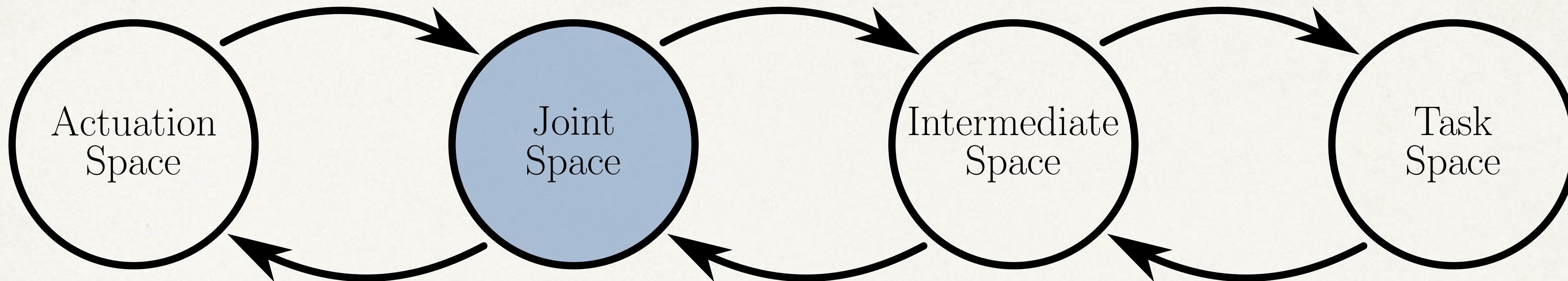
- ❖ curvature
- ❖ coordinate singularities, ...
- ❖ arc parameters
- ❖ linear combination
- ❖ non-linear combination
- ❖ intermediate space





Actuation Space

- ✿ flow and effort of the system
- ✿ mostly position controlled
- ✿ a few torque controlled

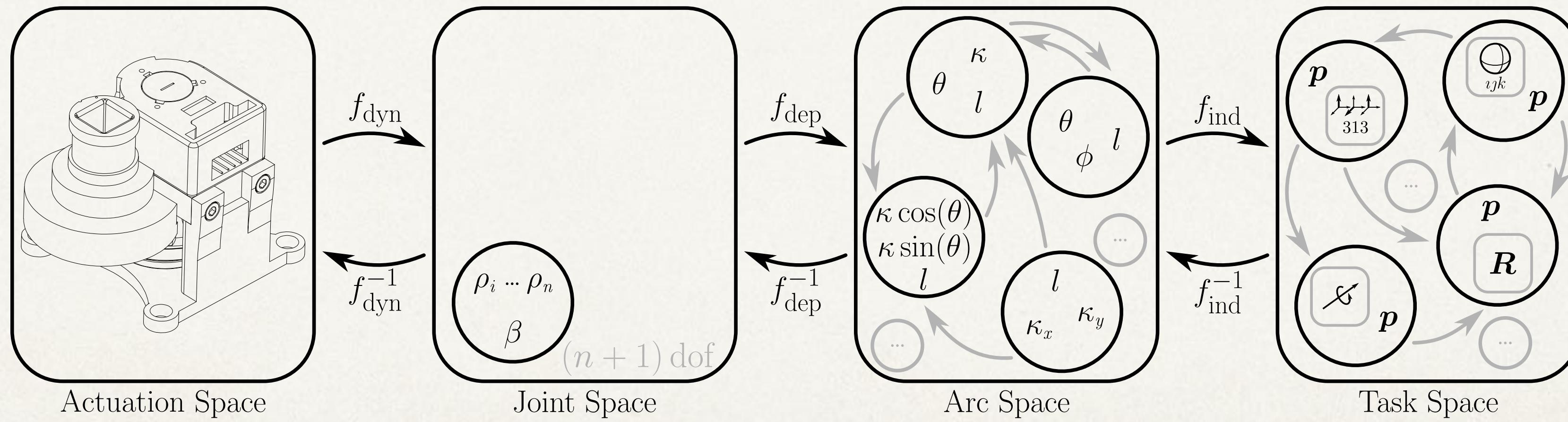
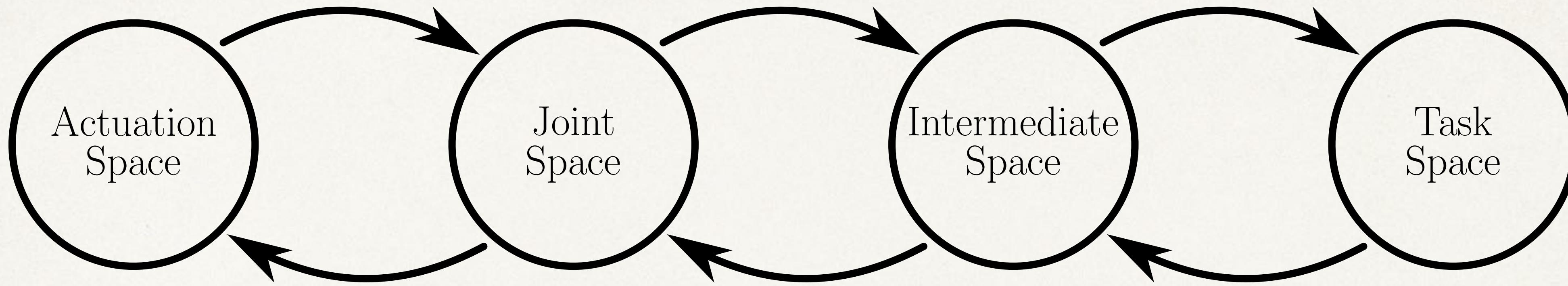


- ⊕ improved state representations
- ⊕ limited to 3 and 4 joints
- ⊕ missing manifold
- ⊕ overlooked constraints

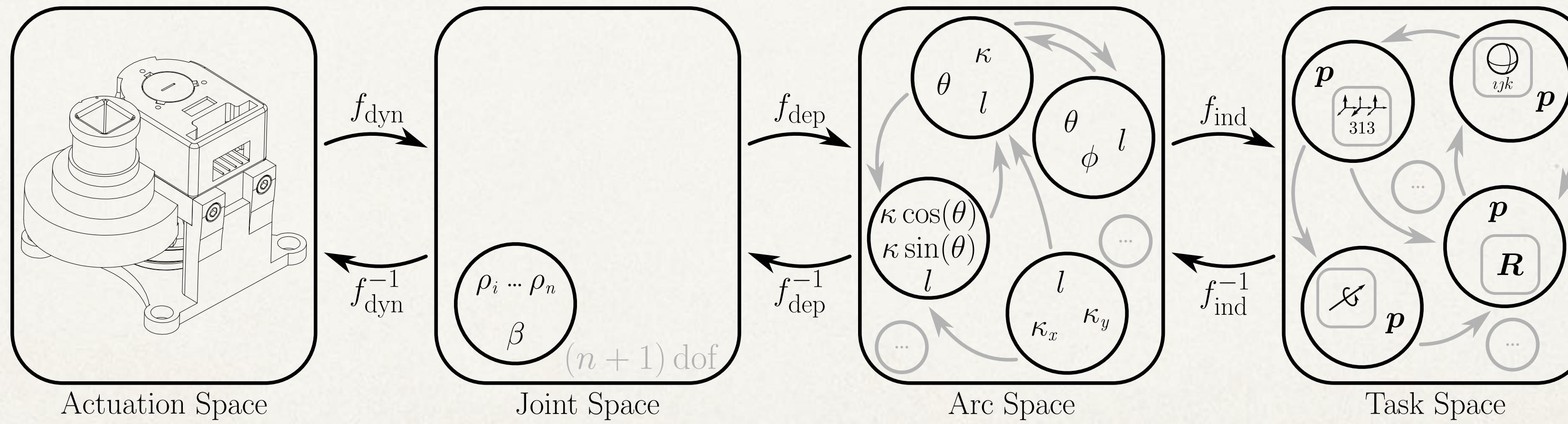
[Della Santina et al., RA-L 2020] “On an Improved State Parametrization for Soft Robots with Piecewise Constant Curvature and Its Use in Model-Based Control”

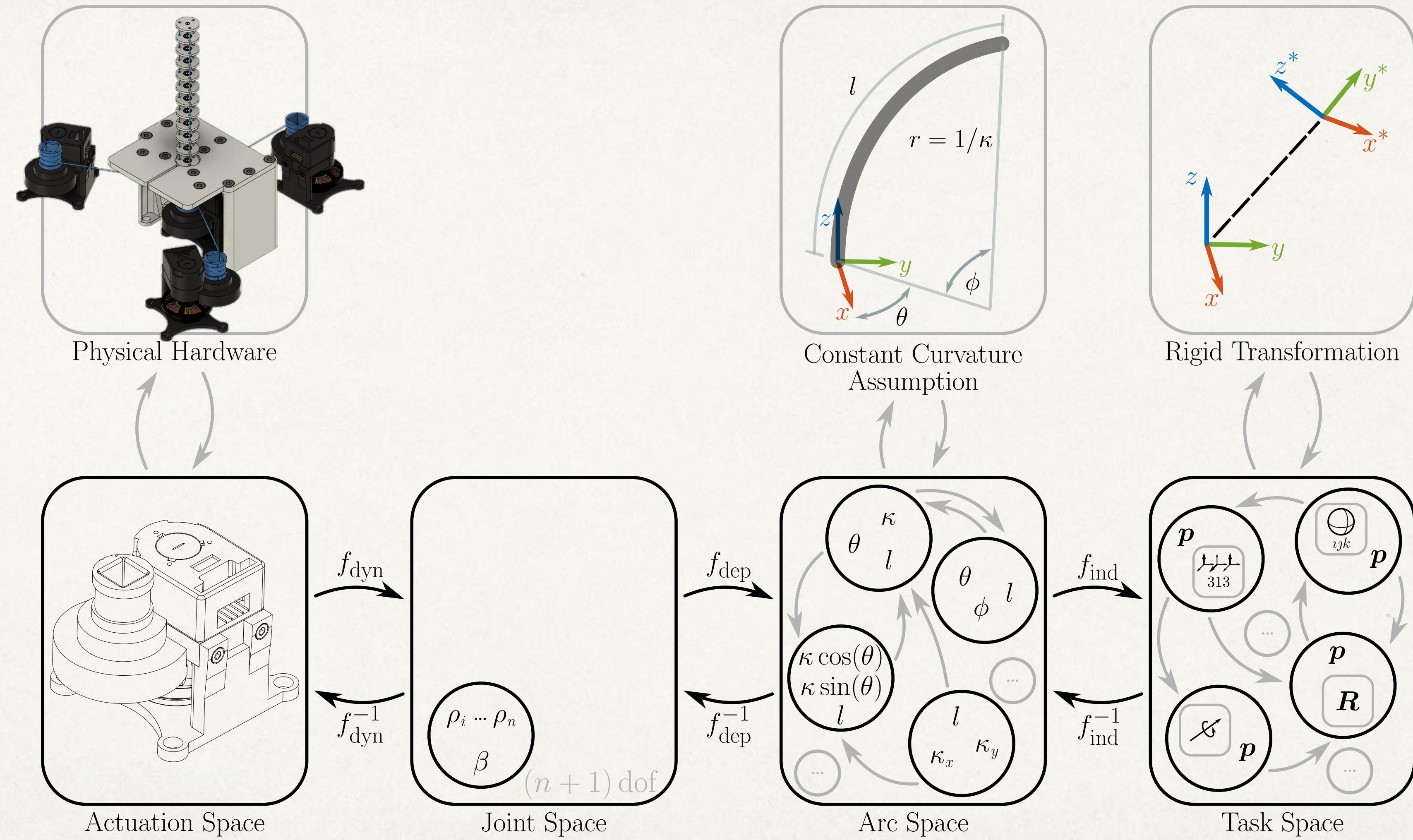
[Allen et al., RoboSoft 2020] “Closed-Form Non-Singular Constant-Curvature Continuum Manipulator Kinematics”

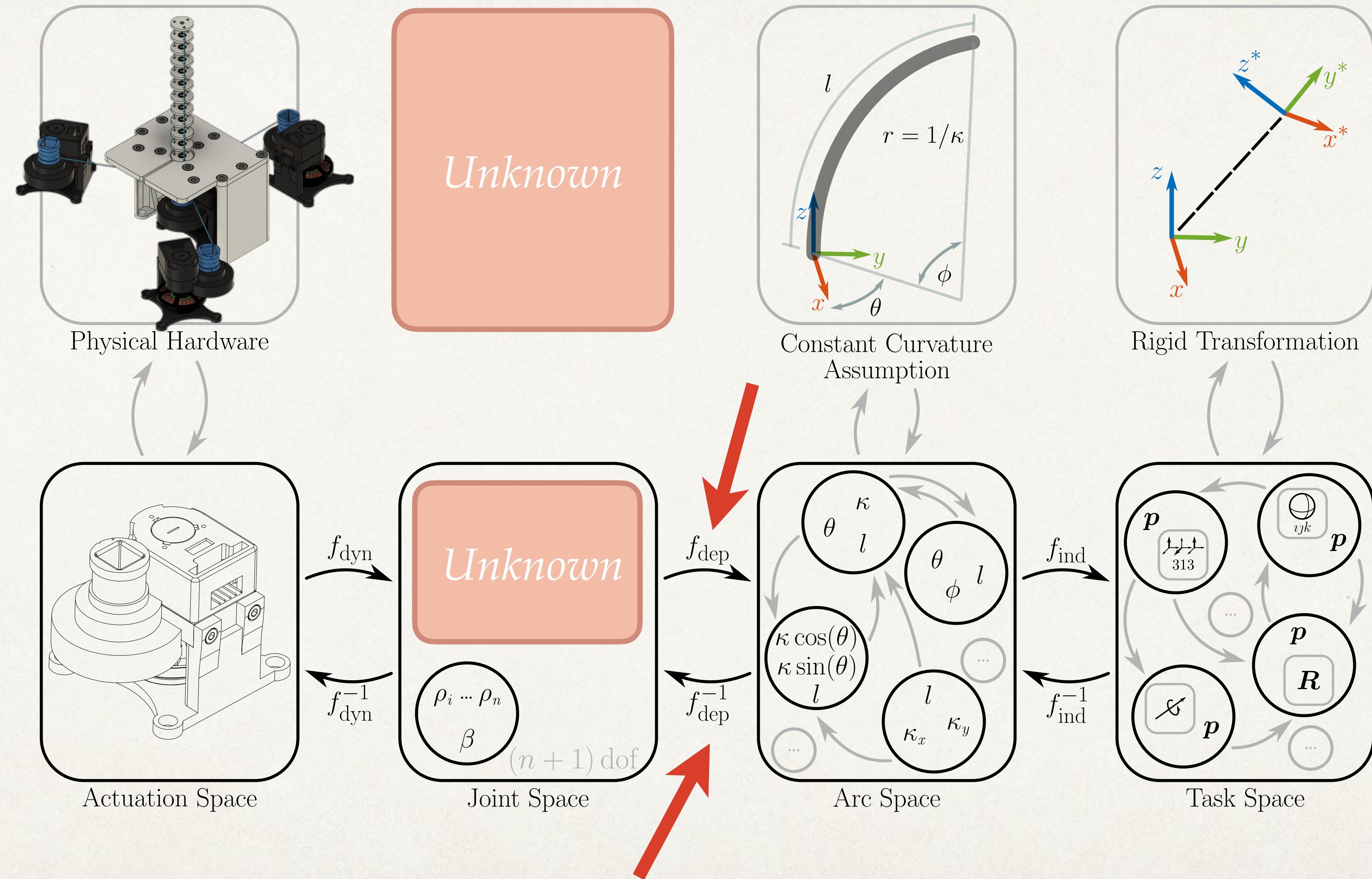
[Dian et al., Access 2022] “A Novel Disturbance-Rejection Control Framework for Cable-Driven Continuum Robots With Improved State Parameterization”



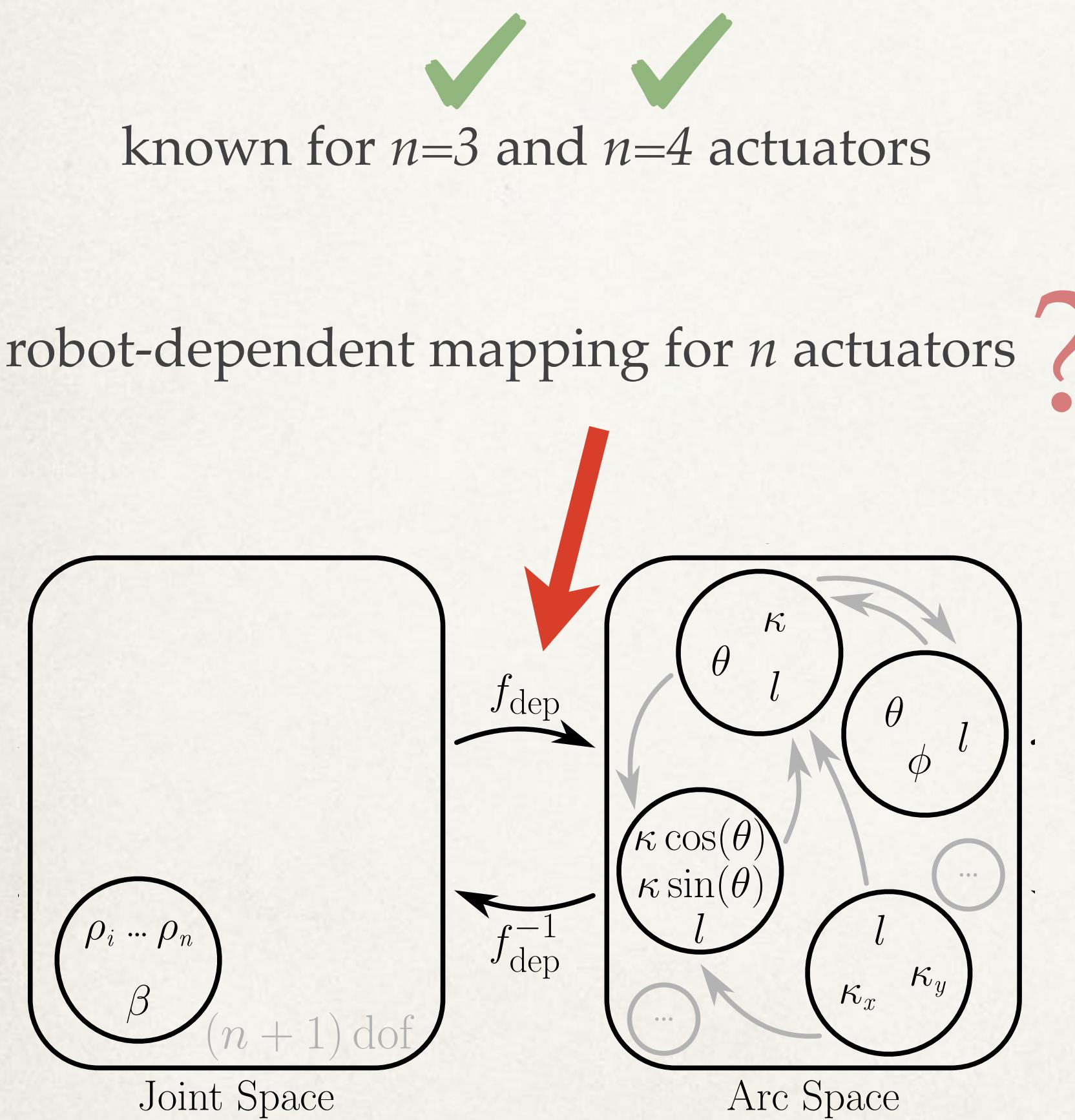
We need to master this







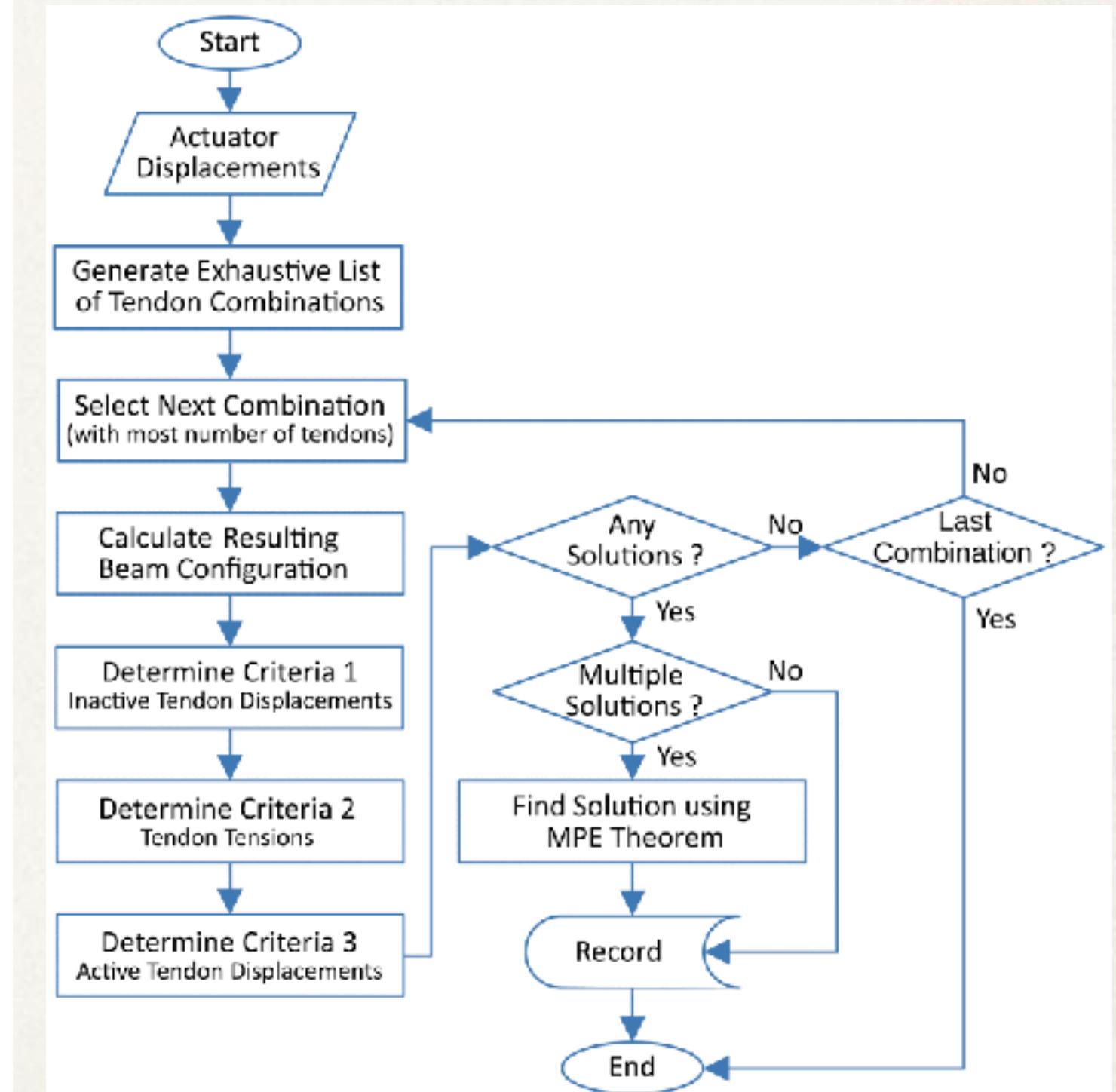
Ripple Effects



```

input : Actuator displacements ( $[\Delta L_{act}]$ ), tendon locations ( $[\alpha_{act}]$ ,  $[R_{act}]$ ) and geometry and mechanical properties of beam and tendons
output: Active/Slack set of tendons and the resulting beam configuration ( $\theta$ ,  $\phi$ ,  $L_c$ )
Algorithm SolveForwardKinematics ( $[\Delta L_{act}]$ ,  $[\alpha_{act}]$ ,  $[R_{act}]$ ,  $[properties]$ )
21
22    $n \leftarrow \text{size}([\alpha_{act}], 1)$ ;
23   Generate exhaustive list of  $n$ - to 1-tendon combinations;
24   foreach  $j \in n$  do
25     foreach  $i \in \text{number of } j\text{-tendon combinations}$  do
26        $\theta_0 \leftarrow 0$ ;
27        $\phi_0 \leftarrow 0$ ;
28        $L_{c0} \leftarrow L_c$ ;
29       [ $\text{comb}$ ]  $\leftarrow i\text{-th } j\text{-tendon combinations}$ ;
30        $[\theta, \phi, L_c] = \text{FindBeamConfig}([\text{comb}], [\theta_0, \phi_0, L_{c0}], [\Delta L_{act}], [\alpha_{act}], [R_{act}])$ ;
31       Find resulting tendon displacements  $[\Delta L_i]_{res}$  using Equation (2);
32        $S_1 \leftarrow \text{Any}([\Delta L_i]_{res} \text{ out of } [\text{comb}] \text{ larger than the corresponding } [\Delta L_i])$ ;
33       Find resulting  $[\text{comb}]$  tendon tensions  $[F_i]_{res}$  using Equation (3);
34        $S_2 \leftarrow \text{Not}(\text{Any}([F_i]_{res} < 0))$ ;
35       Find resulting  $[\text{comb}]$  tendon stretches  $[\delta_i]_{res}$  using Equation (4);
36       Find resulting  $[\text{comb}]$  actuator displacements using  $[\Delta L_{ai}]_{res} = [\Delta L_i]_{res} + [\delta_i]_{res}$ ;
37        $S_3 \leftarrow \text{Any}([\Delta L_{ai}]_{res} \text{ of } [\text{comb}] \text{ differs from the corresponding } [\Delta L_{ai}])$ ;
38       Compute the beam potential energy using Equation (5);
39       Record status parameters of  $S_1$ ,  $S_2$ , and  $S_3$ , as well as the potential energy of this combination;
40       [ $\text{sol}_{index}$ ],  $[\text{sol}_{comb}] \leftarrow \text{Find}(\text{combinations with all } S_1, S_2, \text{ and } S_3 \text{ equal to 1})$ ;
41       if  $\text{size}([\text{sol}_{index}], 1) = 1$  then
42          $solution \leftarrow [\text{sol}_{comb}]$ ;
43         return  $solution$  and  $[\theta, \phi, L_c]$ ;
44       else if  $\text{size}([\text{sol}_{index}], 1) > 1$  then
45          $solution \leftarrow [\text{sol}_{comb}]$  element with the least amount of potential energy);
46         return  $solution$  and  $[\theta, \phi, L_c]$ ;
47       else
48         Proceed with the next list of  $(j - 1)$ -tendon combinations;
49       return
50
51 Procedure FindBeamConfig ( $[\text{comb}]$ ,  $[\theta_0, \phi_0, L_{c0}]$ ,  $[\Delta L_{act}]$ ,  $[\alpha_{act}]$ )
52
53   Set  $[\Delta L_{ai}]$  based on  $[\text{comb}]$  and  $[\Delta L_{act}]$ ;
54   Set  $[\alpha_i]$  based on  $[\text{comb}]$  and  $[\alpha_{act}]$ ;
55   Set  $[R_i]$  based on  $[\text{comb}]$  and  $[R_{act}]$ ;
56   Derive tendon displacements  $[\Delta L_i]$  using Equation (2) ;
57   Derive tendon tensions  $[F_i]$  using Equation (3);
58   Derive tendon stretches  $[\delta_i]$  using Equation (4);
59   Define equations  $[Eq_i] = [\Delta I_{ai}] - [\Delta I_i] - [E_i]$ ;
60   Find beam configuration  $[\theta, \phi, L_c]$  by numerically solving the set of Equations  $[Eq_i] = 0$  based on the initial condition  $[\theta_0, \phi_0, L_{c0}]$ ;
61   return  $[\theta, \phi, L_c]$ ;
62

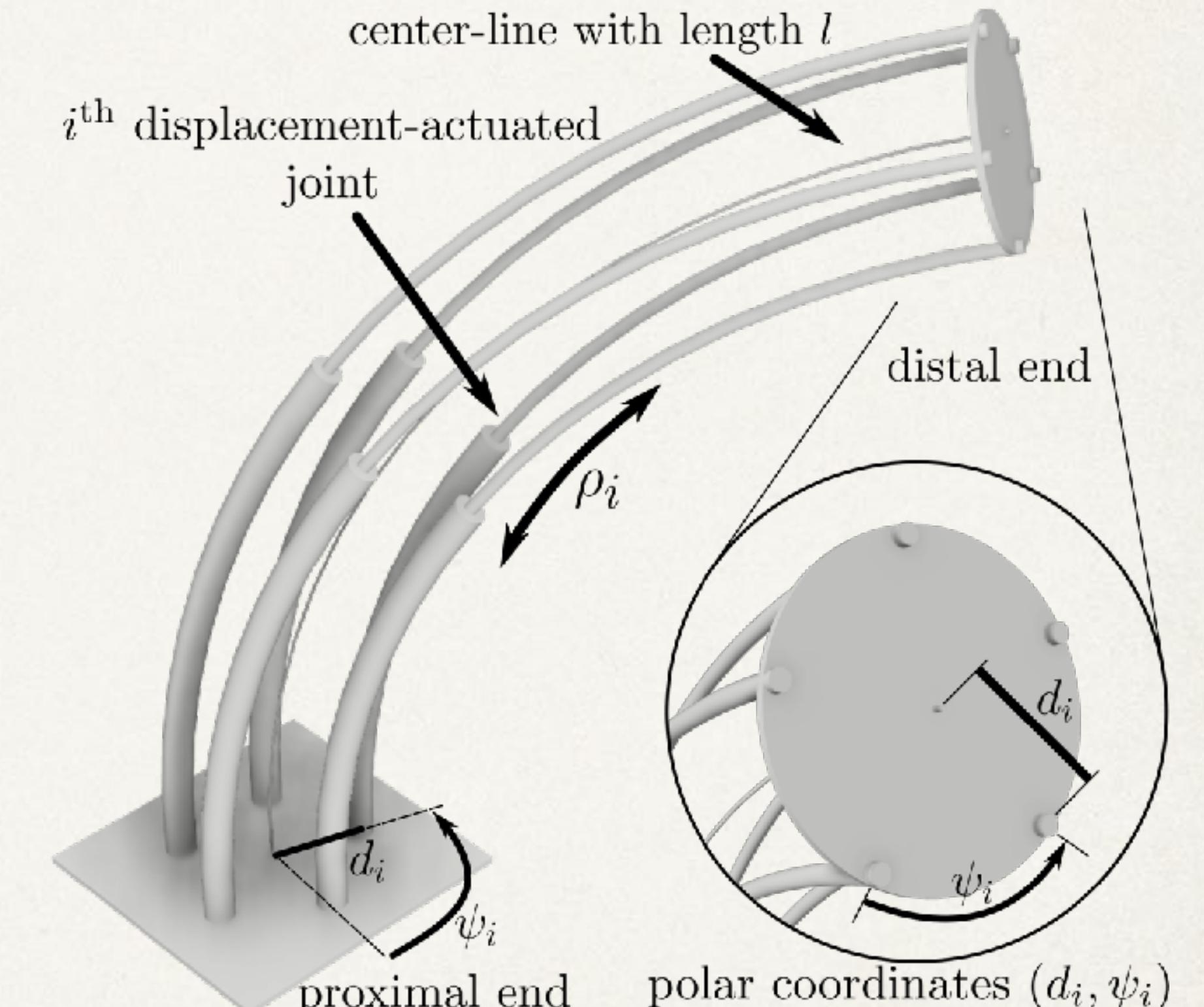
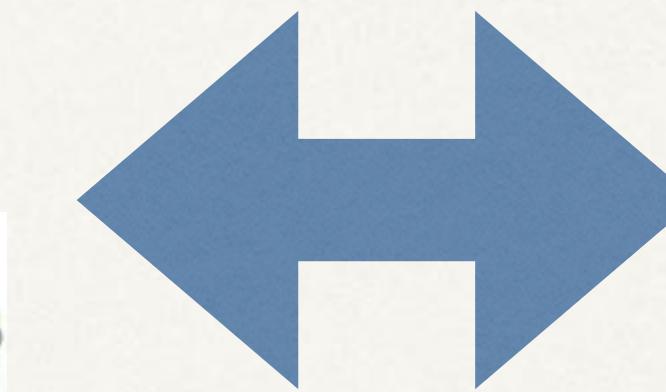
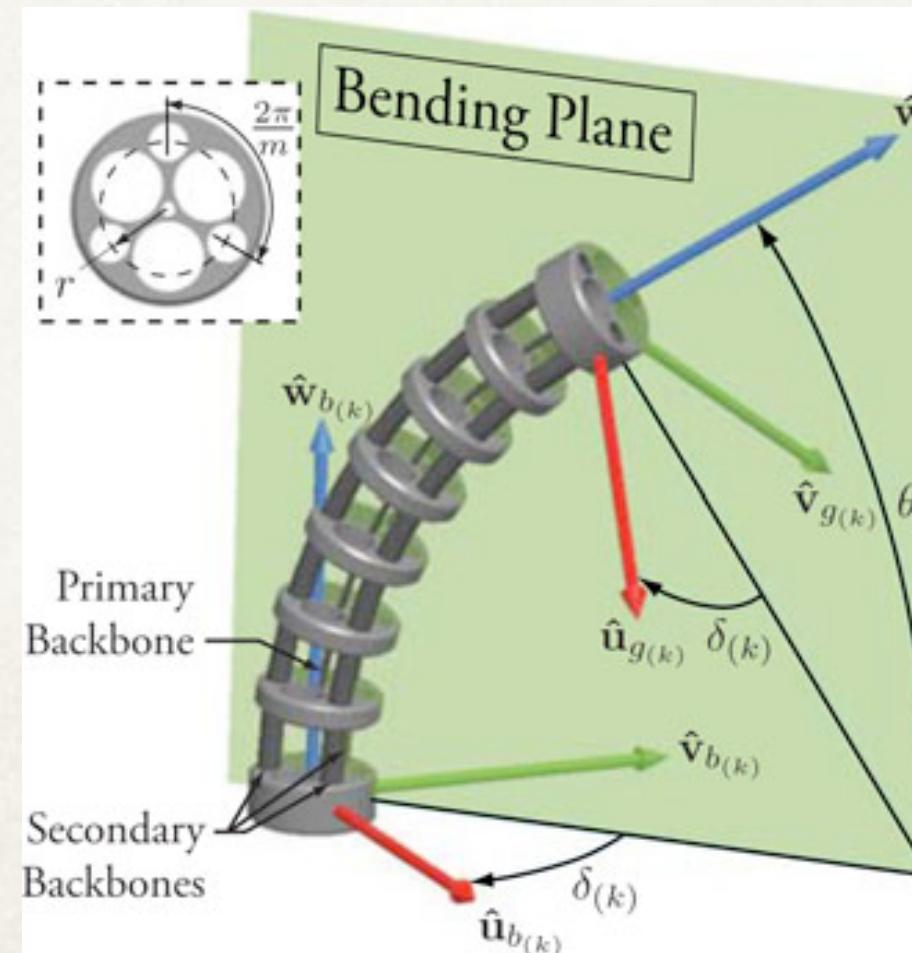
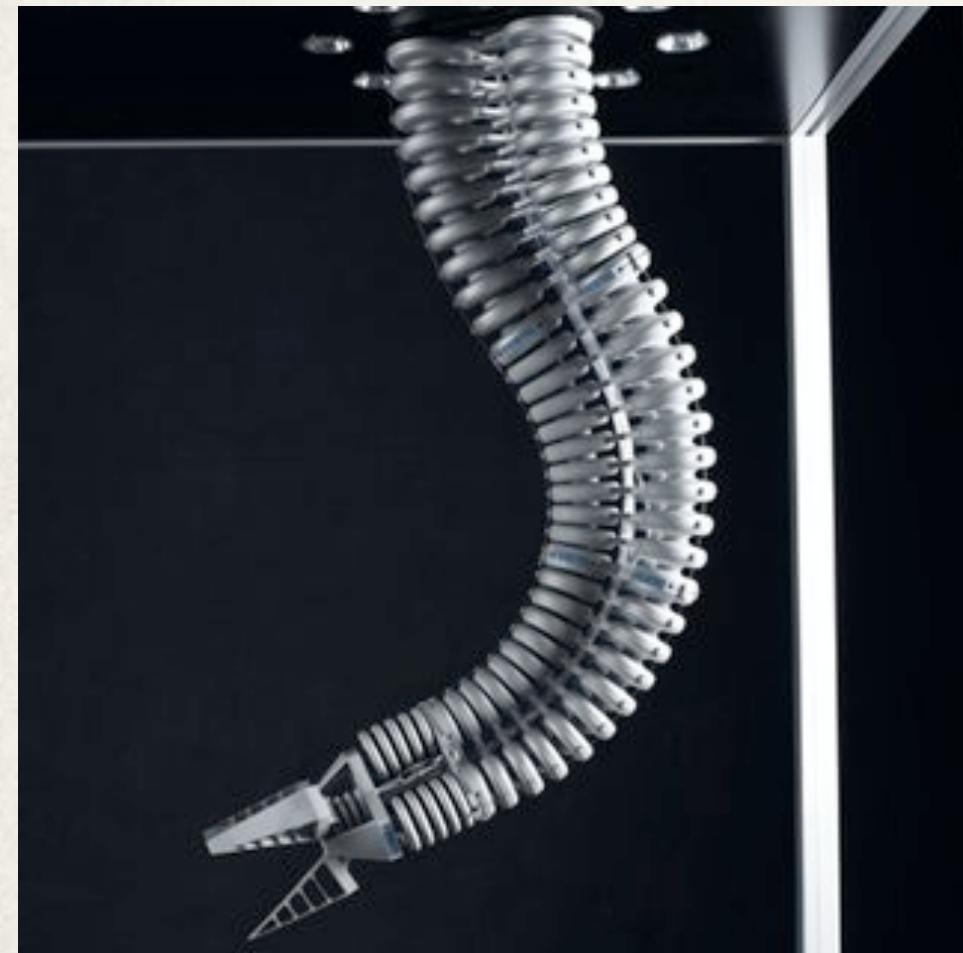
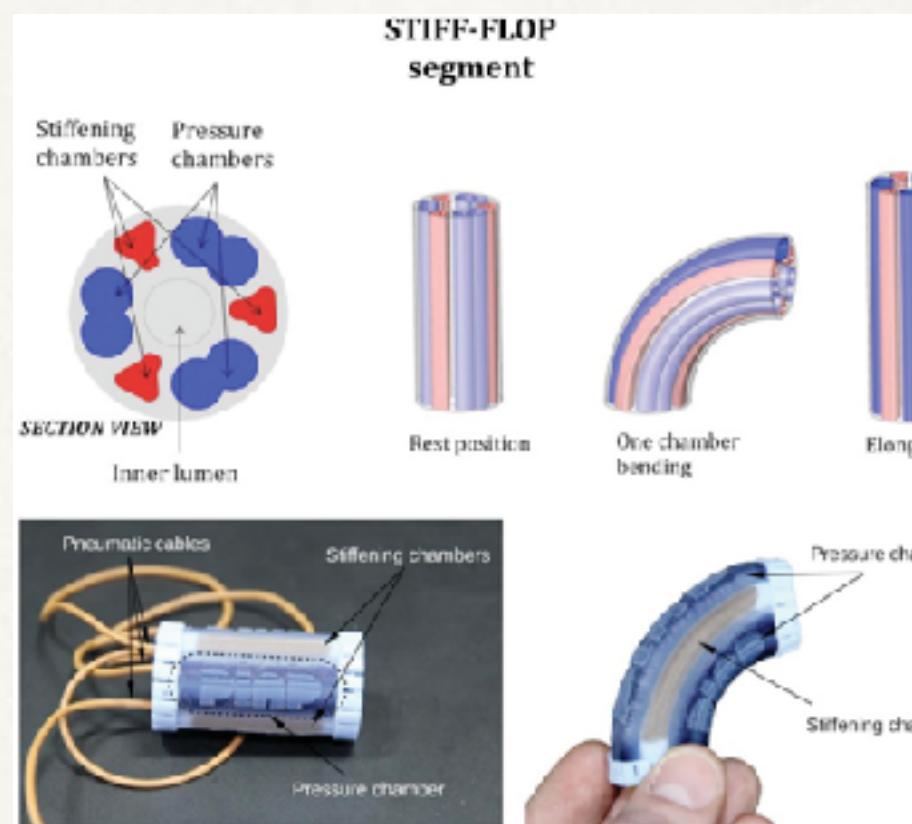
```



[Dalvand et al., Access 2022]

current state-of-the-art approach

Abstraction — DACR



Displacement-Actuated Continuum Robot

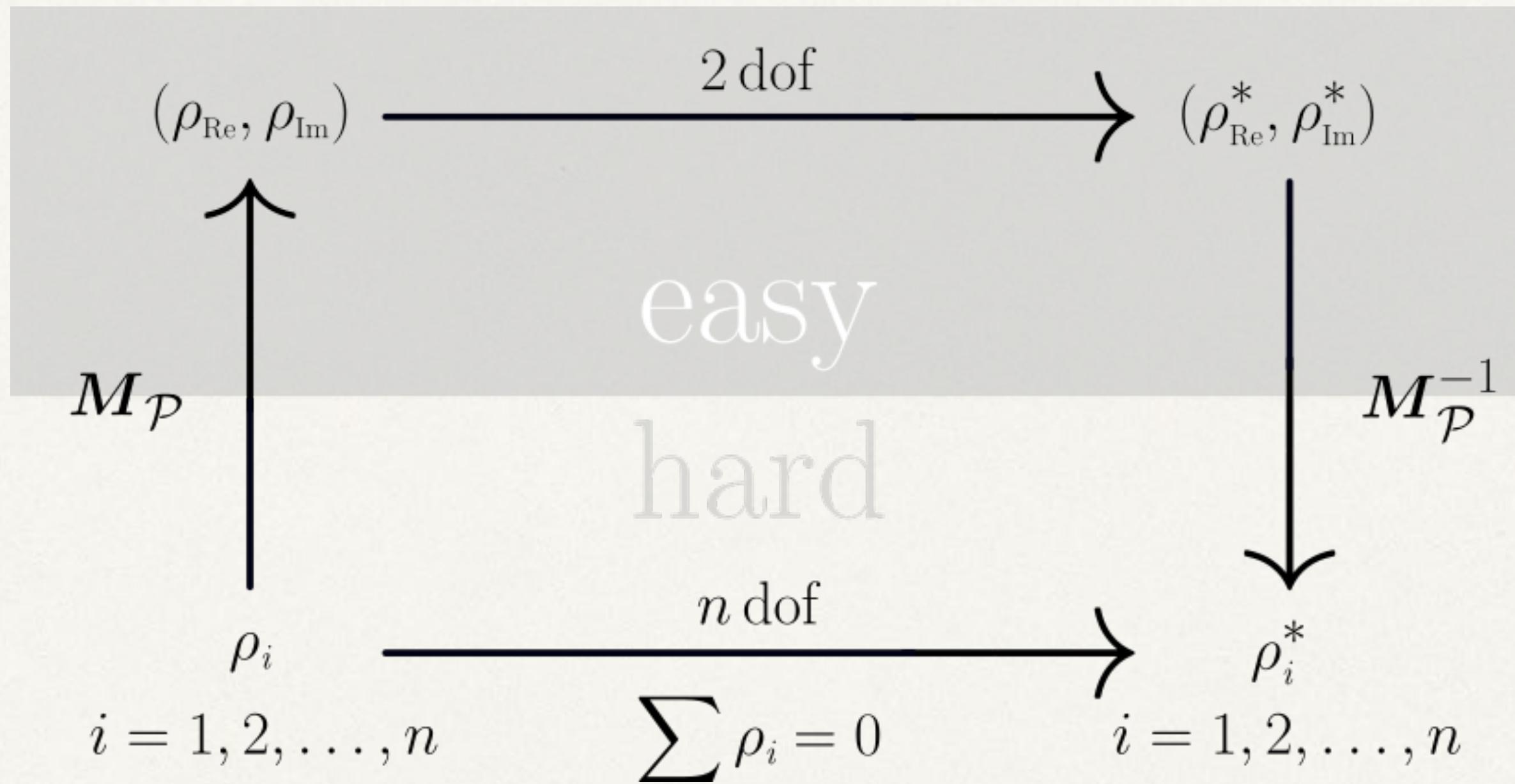
[Grassmann et al., arXiv (under review)] “Clarke Transform — A Fundamental Tool for Continuum Robotics”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Displacement-Actuated Continuum Robot: A Joint Space Abstraction”

Clarke Transform and Clarke Coordinates

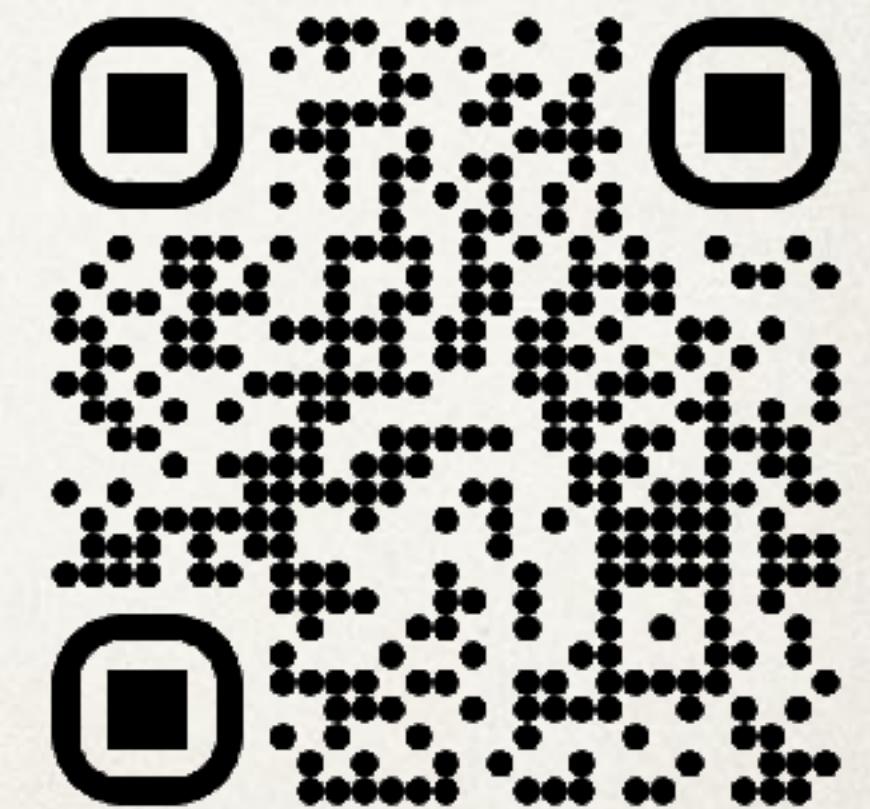
Forward

$$\bar{\rho} = M_{\mathcal{P}} \rho \in \mathbb{R}^2$$



Backward

$$\rho = M_{\mathcal{P}}^{-1} \bar{\rho} \in \mathbb{R}^n$$

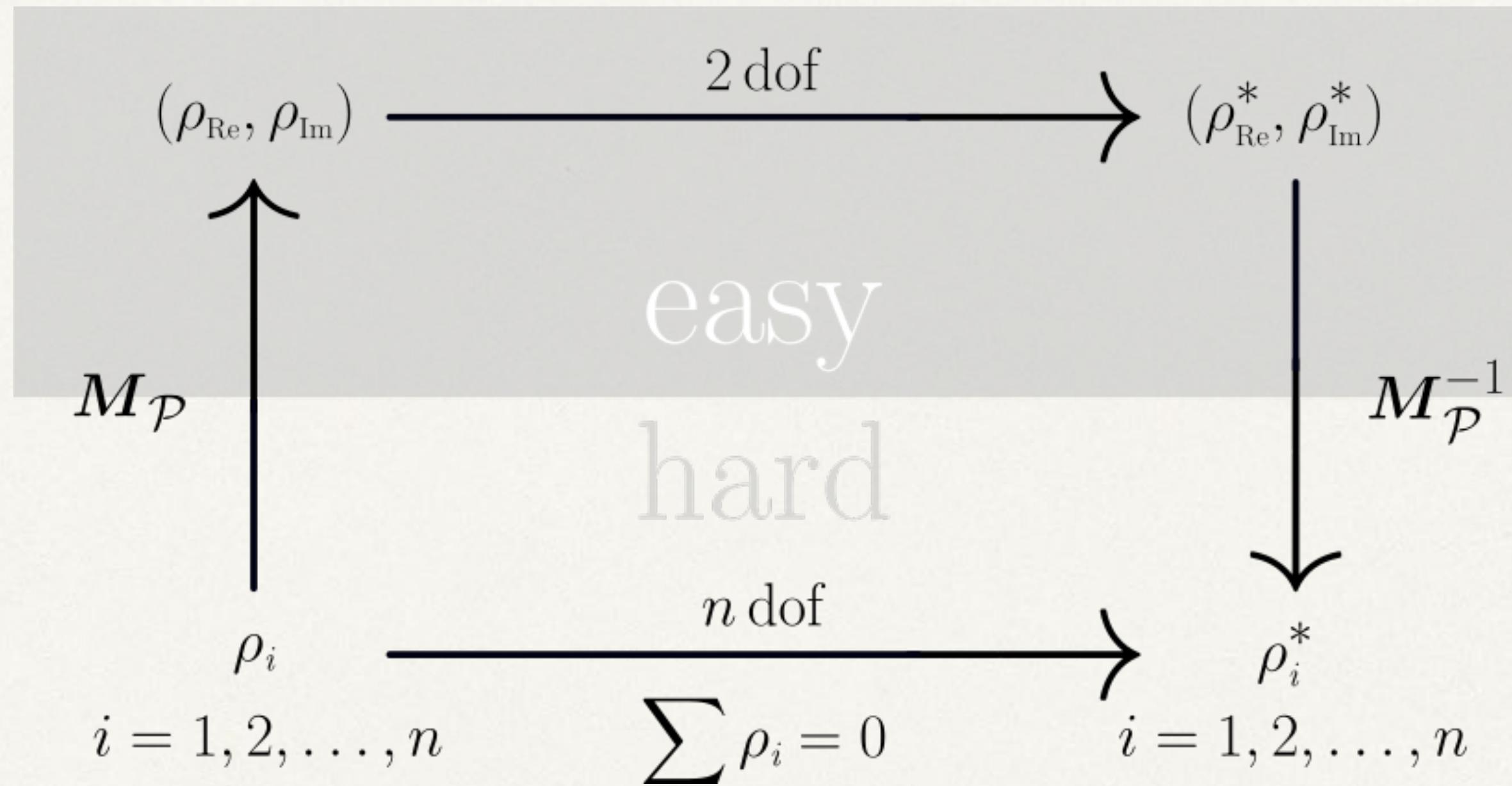


Clarke Transform
(manuscript on arXiv)

Clarke Transform and Clarke Coordinates

Forward

$$\bar{\rho} = M_{\mathcal{P}} \rho \in \mathbb{R}^2$$



Backward

$$\rho = M_{\mathcal{P}}^{-1} \bar{\rho} \in \mathbb{R}^n$$



Clarke Transform provide solution that are exact, closed-formed, and interpretable.

Clarke Transform
(manuscript on arXiv)

Robot-Dependent Mapping

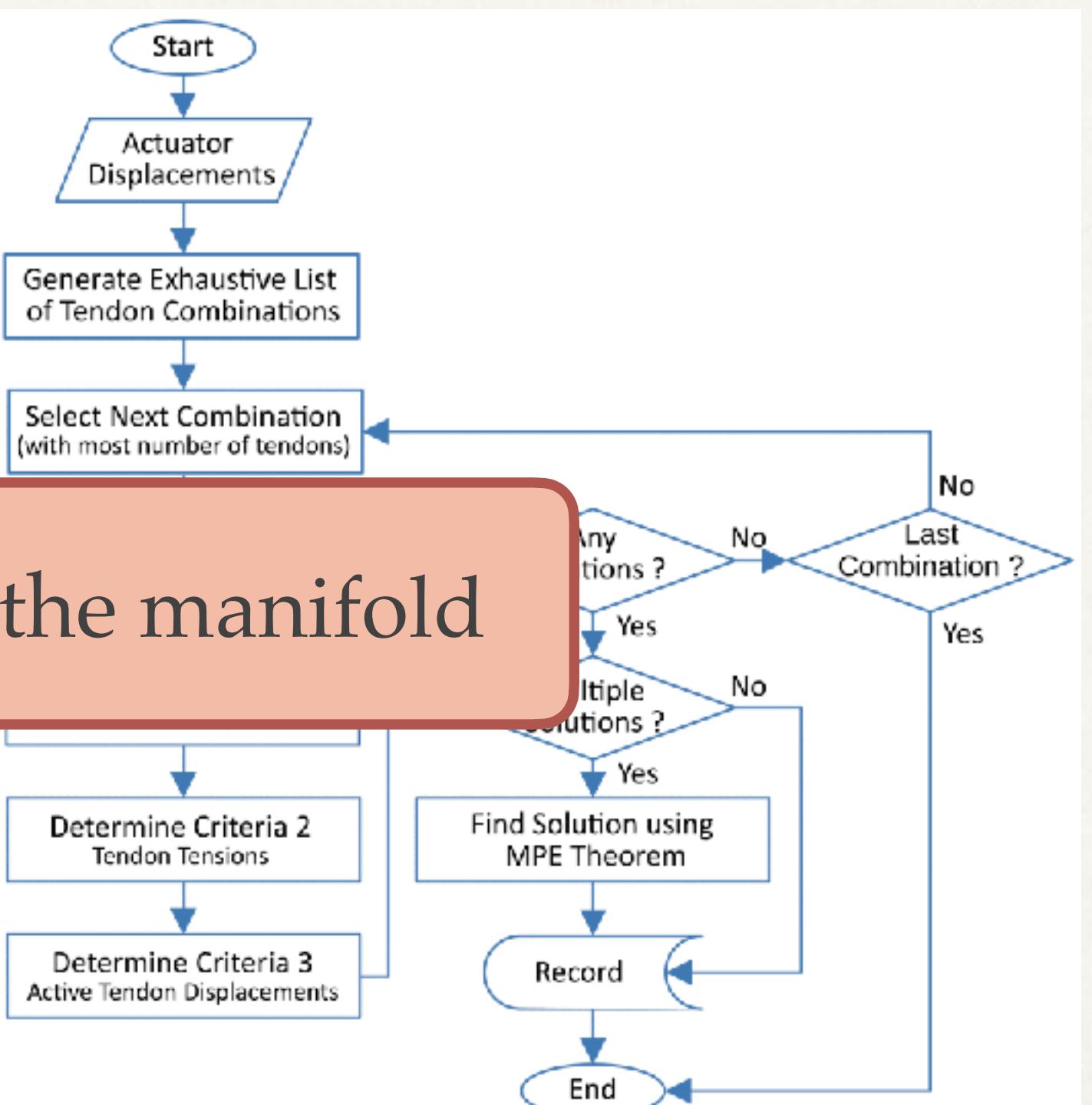
```

input : Actuator displacements ( $[\Delta L_{act}]$ ), tendon locations ( $[\alpha_{act}]$ ,  $[R_{act}]$ ) and geometry and mechanical properties of beam and tendons
output: Active/Slack set of tendons and the resulting beam configuration ( $[\theta, \phi, L_c]$ )

Algorithm SolveForwardKinematics ( $[\Delta L_{act}]$ ,  $[\alpha_{act}]$ ,  $[R_{act}]$ ,  $[properties]$ )
  22    $n \leftarrow \text{size}([\alpha_{act}], 1)$ ;
  23   Generate exhaustive list of  $n$ - to 1-tendon combinations;
  24   foreach  $j \in n$  do
  25     foreach  $i \in \text{number of } j\text{-tendon combinations}$  do
  26        $\theta_0 \leftarrow 0$ ;
  27        $\phi_0 \leftarrow 0$ ;
  28        $L_{c0} \leftarrow L_c$ ;
  29        $[\text{comb}] \leftarrow i\text{-th } j\text{-tendon combinations}$ ;
  30        $[\theta, \phi, L_c] = \text{FindBeamConfig}([\text{comb}], [\theta_0, \phi_0, L_{c0}], [\Delta L_{act}], [\alpha_{act}], [R_{act}])$ ;
  31       Find resulting tendon displacements  $[\Delta L_t]_{res}$  using Equation (2);
  32        $S_1 \leftarrow \text{Any}([\Delta L_t]_{res} \text{ out of } [\text{comb}] \text{ larger than the corresponding } [\Delta L_t])$ ;
  33       Find resulting  $[\text{comb}]$  tendon tensions  $[F_t]_{res}$  using Equation (3);
  34        $S_2 \leftarrow \text{Not}(\text{Any}([F_t]_{res} < 0))$ ;
  35       Find resulting  $[\text{comb}]$  tendon stretches  $[\beta_t]_{res}$  using Equation (4);
  36       Find resulting  $[\text{comb}]$  actuator displacements using  $[\Delta L_{act}]_{res} = [\Delta L_t]_{res} + [\beta_t]_{res}$ ;
  37        $S_3 \leftarrow \text{Any}([\Delta L_{act}]_{res} \text{ of } [\text{comb}] \text{ differs from the corresponding } [\Delta L_{act}])$ ;
  38       Compute the beam potential energy using Equation (5);
  39       Record status parameters of  $S_1$ ,  $S_2$  and  $S_3$ , as well as the potential energy of this combination;
  40   end
  41    $[\text{sol}_index], [\text{sol}_{comb}] \leftarrow \text{FindIndex}([\text{sol}_index], [\text{sol}_{comb}])$ 
  42   if  $\text{size}([\text{sol}_index], 1) = 1$  then
  43      $\text{solution} \leftarrow [\text{sol}_{comb}]$ ;
  44   return  $\text{solution}$  and  $[\theta, \phi, L_c]$ 
  45 else if  $\text{size}([\text{sol}_index], 1) > 1$  then
  46    $\text{solution} \leftarrow \text{Find}([\text{sol}_{comb}])$ ;
  47   return  $\text{solution}$  and  $[\theta, \phi, L_c]$ 
  48 else
  49   Proceed with the next list of  $(j - 1)$ -tendon combinations,
  50 end
  51 return

Procedure FindBeamConfig ( $[\text{comb}]$ ,  $[\theta_0, \phi_0, L_{c0}]$ ,  $[\Delta L_{act}]$ ,  $[\alpha_{act}]$ )
  52   Set  $[\Delta L_{act}]$  based on  $[\text{comb}]$  and  $[\Delta L_{act}]$ ;
  53   Set  $[\alpha_t]$  based on  $[\text{comb}]$  and  $[\alpha_{act}]$ ;
  54   Set  $[R_t]$  based on  $[\text{comb}]$  and  $[R_{act}]$ ;
  55   Derive tendon displacements  $[\Delta L_t]$  using Equation (2);
  56   Derive tendon tensions  $[F_t]$  using Equation (3);
  57   Derive tendon stretches  $[\beta_t]$  using Equation (4);
  58   Define equations  $[E_{\theta}] = [\Delta L_{act}] - [\Delta L_t] - [\beta_t]$ ;
  59   Find beam configuration  $[\theta, \phi, L_c]$  by numerically solving the set of Equations  $[E_{\theta}] = 0$  based on the initial condition  $[\theta_0, \phi_0, L_{c0}]$ ;
  60   return  $[\theta, \phi, L_c]$ 

```



without utilizing the manifold

[Dalvand et al., Access 2022]

[Dalvand et al., Access 2022]

Robot-Dependent Mapping

```

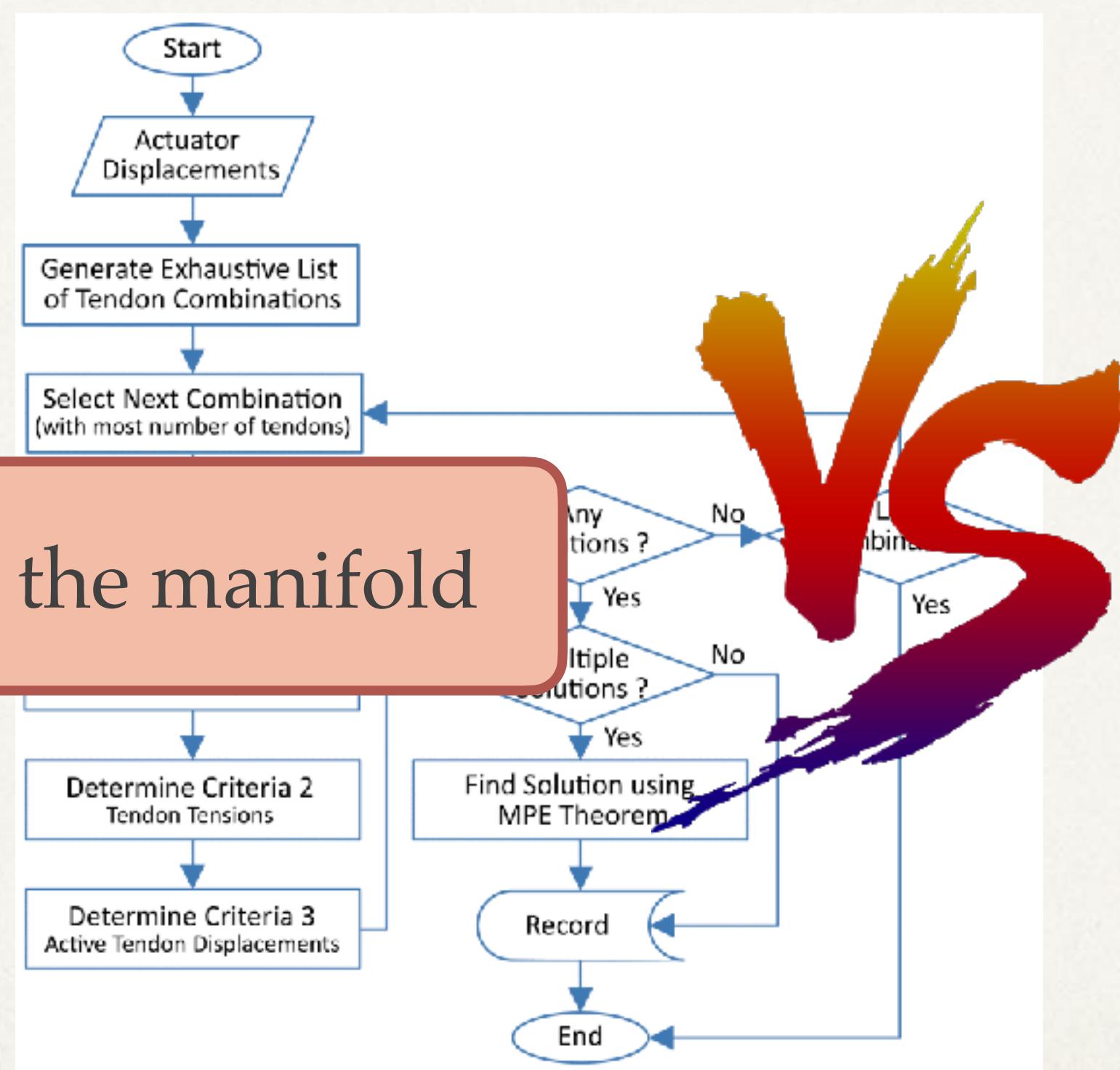
input : Actuator displacements ( $[\Delta L_{act}]$ ), tendon locations ( $[\alpha_{act}]$ ,  $[R_{act}]$ ) and geometry and mechanical properties of beam and tendons
output: Active/Slack set of tendons and the resulting beam configuration ( $[\theta, \phi, L_c]$ )

Algorithm SolveForwardKinematics ( $[\Delta L_{act}]$ ,  $[\alpha_{act}]$ ,  $[R_{act}]$ ,  $[properties]$ )
  for  $j \in n$  do
    foreach  $i \in \text{number of } j\text{-tendon combinations}$  do
       $\theta_0 \leftarrow 0;$ 
       $\phi_0 \leftarrow 0;$ 
       $L_{c0} \leftarrow L_c;$ 
       $[\text{comb}] \leftarrow i\text{-th } j\text{-tendon combinations};$ 
       $[\theta, \phi, L_c] = \text{FindBeamConfig}([\text{comb}], [\theta_0, \phi_0, L_{c0}], [\Delta L_{act}], [\alpha_{act}], [R_{act}]);$ 
      Find resulting tendon displacements  $[\Delta L_t]_{res}$  using Equation (2);
       $S_1 \leftarrow \text{Any}([\Delta L_t]_{res} \text{ out of } [\text{comb}] \text{ larger than the corresponding } [\Delta L_t]);$ 
      Find resulting  $[\text{comb}]$  tendon tensions  $[F_t]_{res}$  using Equation (3);
       $S_2 \leftarrow \text{Not}(\text{Any}([F_t]_{res} < 0));$ 
      Find resulting  $[\text{comb}]$  tendon stretches  $[\beta_t]_{res}$  using Equation (4);
      Find resulting  $[\text{comb}]$  actuator displacements using  $[\Delta L_{act}]_{res} = [\Delta L_t]_{res} + [\beta_t]_{res};$ 
       $S_3 \leftarrow \text{Any}([\Delta L_{act}]_{res} \text{ of } [\text{comb}] \text{ differs from the corresponding } [\Delta L_{act}]);$ 
      Compute the beam potential energy using Equation (5);
      Record status parameters of  $S_1$ ,  $S_2$  and  $S_3$  as well as the potential energy of this combination;
       $[\text{sol}_{index}], [\text{sol}_{comb}] \leftarrow \text{FindSolution}([\text{comb}], [\theta, \phi, L_c]);$ 
      if  $\text{size}([\text{sol}_{index}], 1) = 1$  then
         $\text{solution} \leftarrow [\text{sol}_{comb}];$ 
        return  $\text{solution}$  and  $[\theta, \phi, L_c];$ 
      else if  $\text{size}([\text{sol}_{index}], 1) > 1$  then
         $\text{solution} \leftarrow \text{Find}([\text{sol}_{index}], [\text{sol}_{comb}]);$ 
        return  $\text{solution}$  and  $[\theta, \phi, L_c];$ 
      else
        Proceed with the next list of  $(j-1)$ -tendon combinations;
  return

Procedure FindBeamConfig ( $[\text{comb}]$ ,  $[\theta_0, \phi_0, L_{c0}]$ ,  $[\Delta L_{act}]$ ,  $[\alpha_{act}]$ )
  Set  $[\Delta L_{act}]$  based on  $[\text{comb}]$  and  $[\Delta L_{act}]$ ;
  Set  $[\alpha_i]$  based on  $[\text{comb}]$  and  $[\alpha_{act}]$ ;
  Set  $[R_i]$  based on  $[\text{comb}]$  and  $[R_{act}]$ ;
  Derive tendon displacements  $[\Delta L_t]$  using Equation (2);
  Derive tendon tensions  $[F_t]$  using Equation (3);
  Derive tendon stretches  $[\beta_t]$  using Equation (4);
  Define equations  $[E_{\theta}] = [\Delta L_{act}] - [\Delta L_t] - [\beta_t];$ 
  Find beam configuration  $[\theta, \phi, L_c]$  by numerically solving the set of Equations  $[E_{\theta}] = 0$  based on the initial condition  $[\theta_0, \phi_0, L_{c0}];$ 
  return  $[\theta, \phi, L_c];$ 

```

without utilizing the manifold



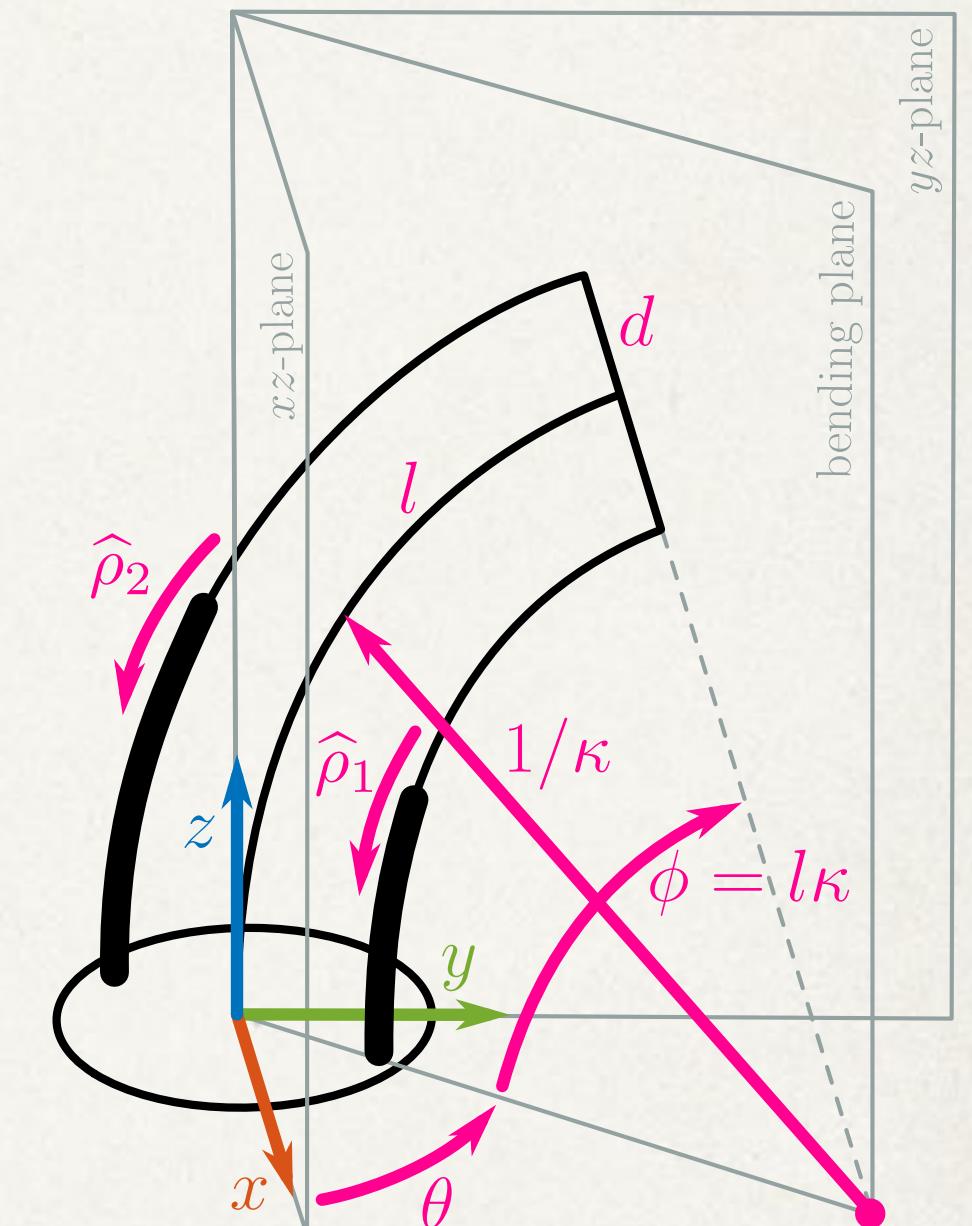
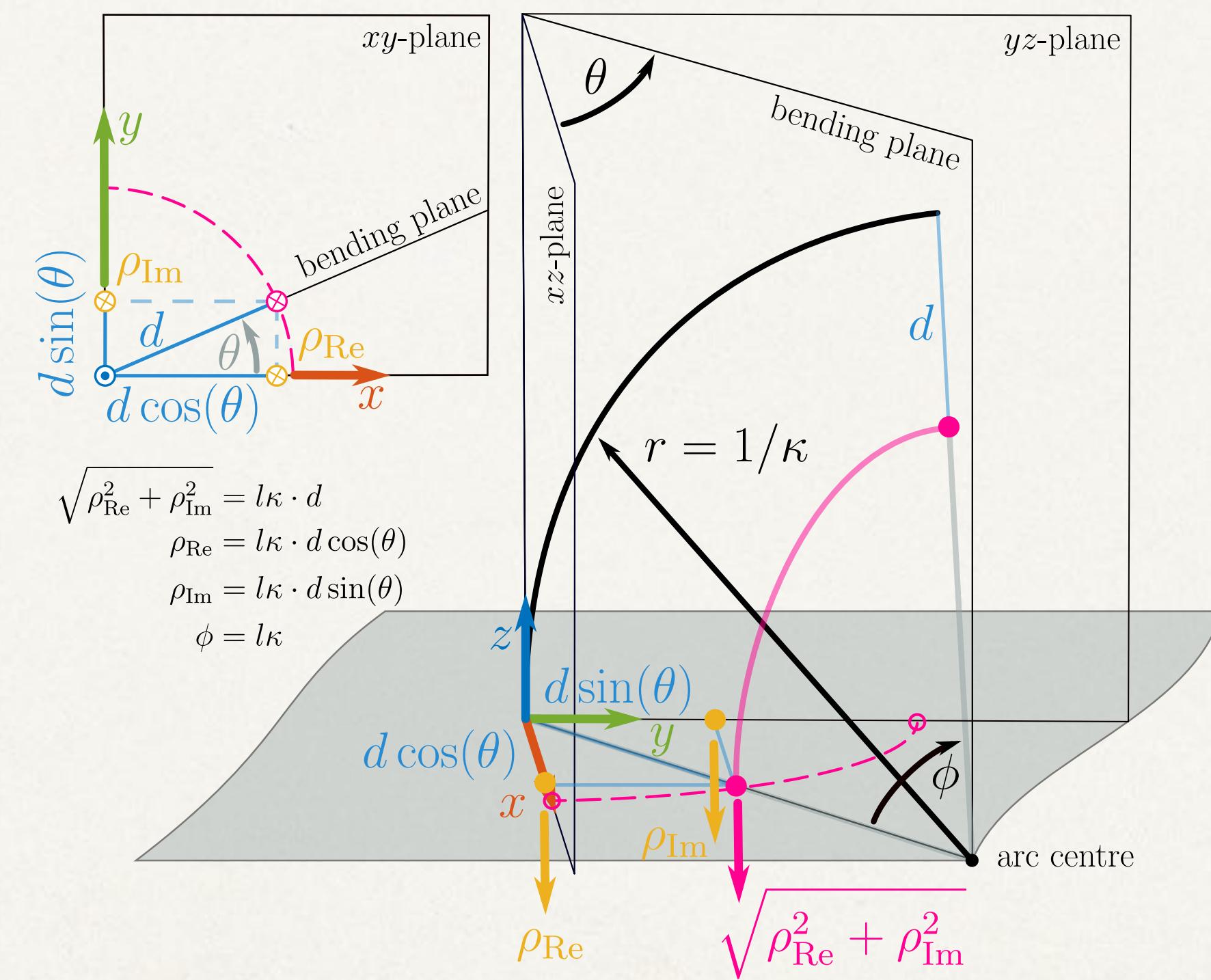
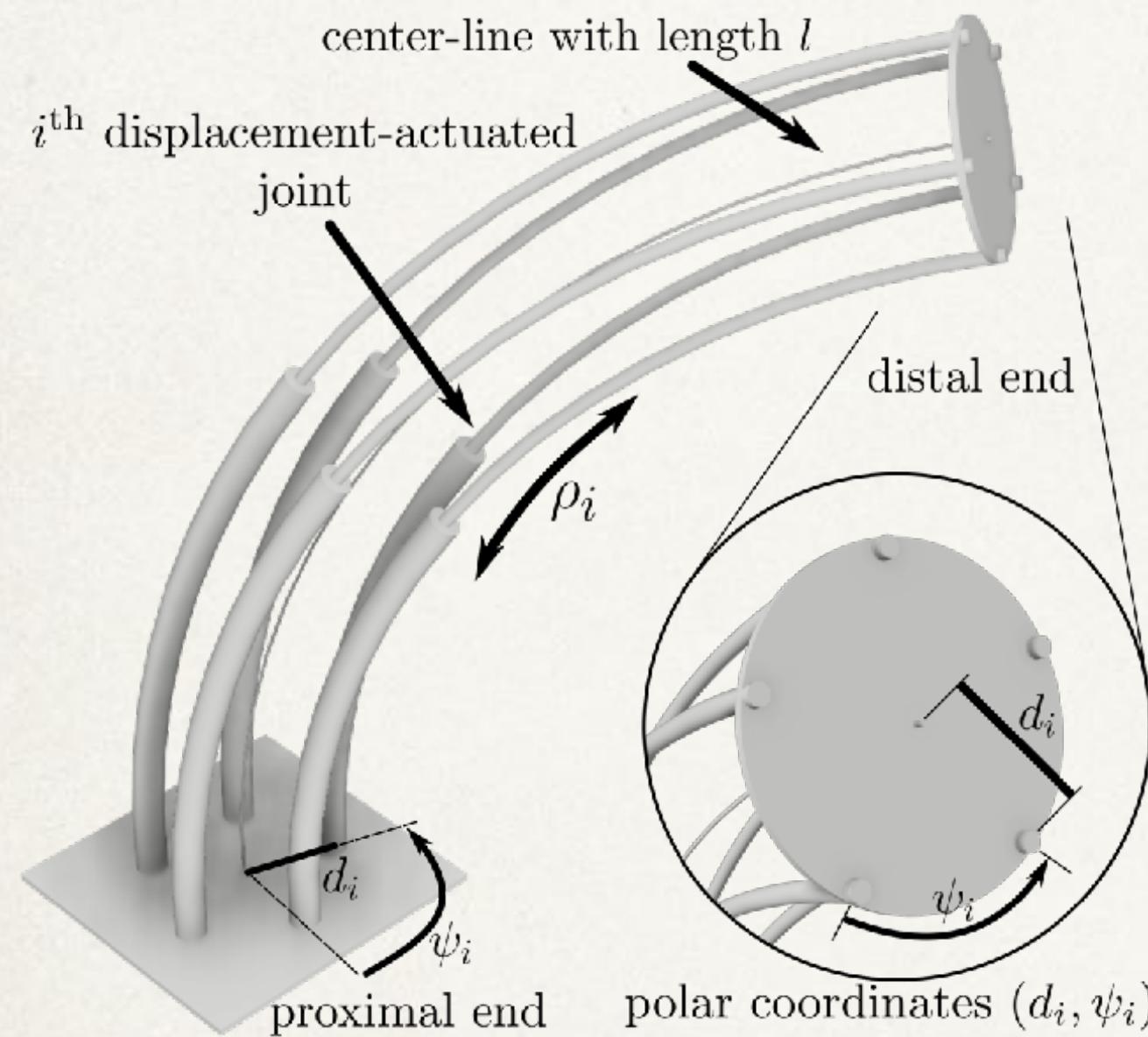
[Dalvand et al., Access 2022]

[Dalvand et al., Access 2022]

$$\begin{bmatrix} \kappa \cos(\theta) \\ \kappa \sin(\theta) \end{bmatrix} = \underbrace{\frac{1}{l}}_{\text{removes } l} \underbrace{\widehat{M}_{\mathcal{P}}}_{\text{removes } \psi_i} \underbrace{\text{diag}(1/d_i) \rho}_{\text{removes } d_i}$$

utilizing the manifold

Interpretable



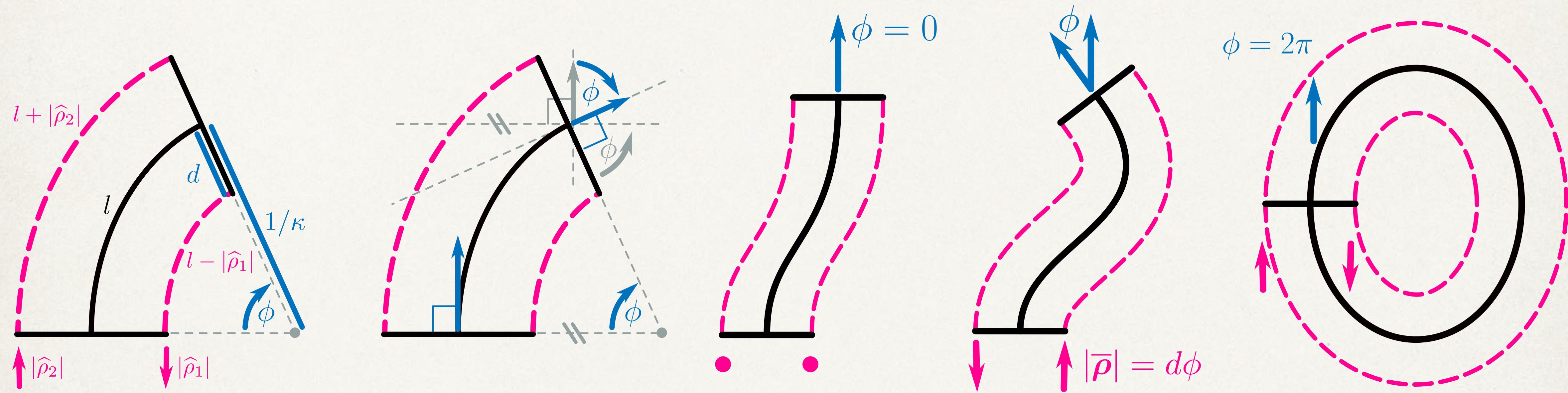
[Grassmann et al., arXiv (under review)] “Clarke Transform — A Fundamental Tool for Continuum Robotics”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Using Clarke Transform to Create a Framework on the Manifold”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Clarke Coordinates Are Generalized Improved State Parametrization for Continuum Robots”

[Grassmann & Burgner-Kahrs, arXiv (under review)] “Displacement-Actuated Continuum Robots: A Joint Space Abstraction”

Applicable Beyond Constant-Curvature



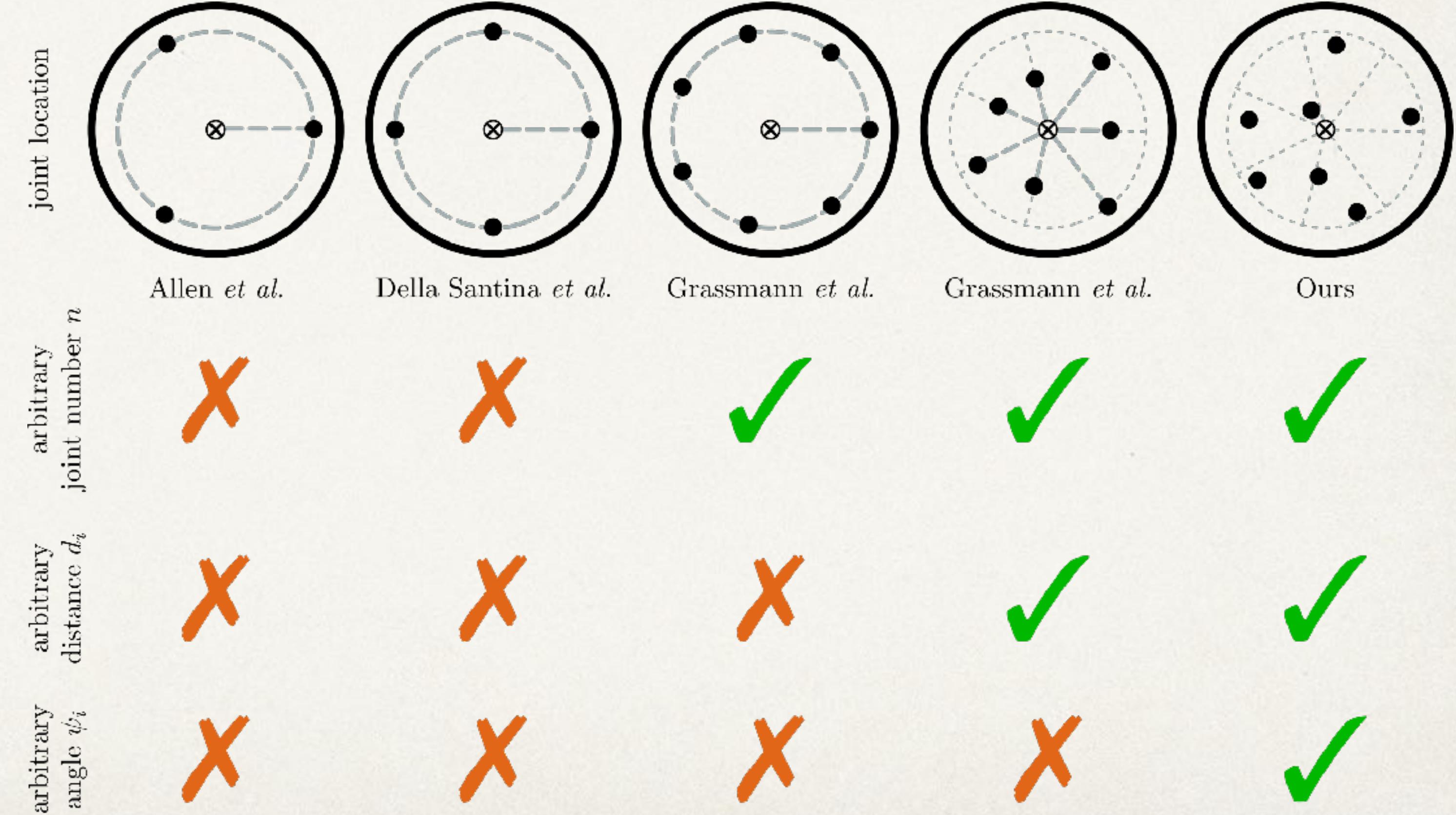
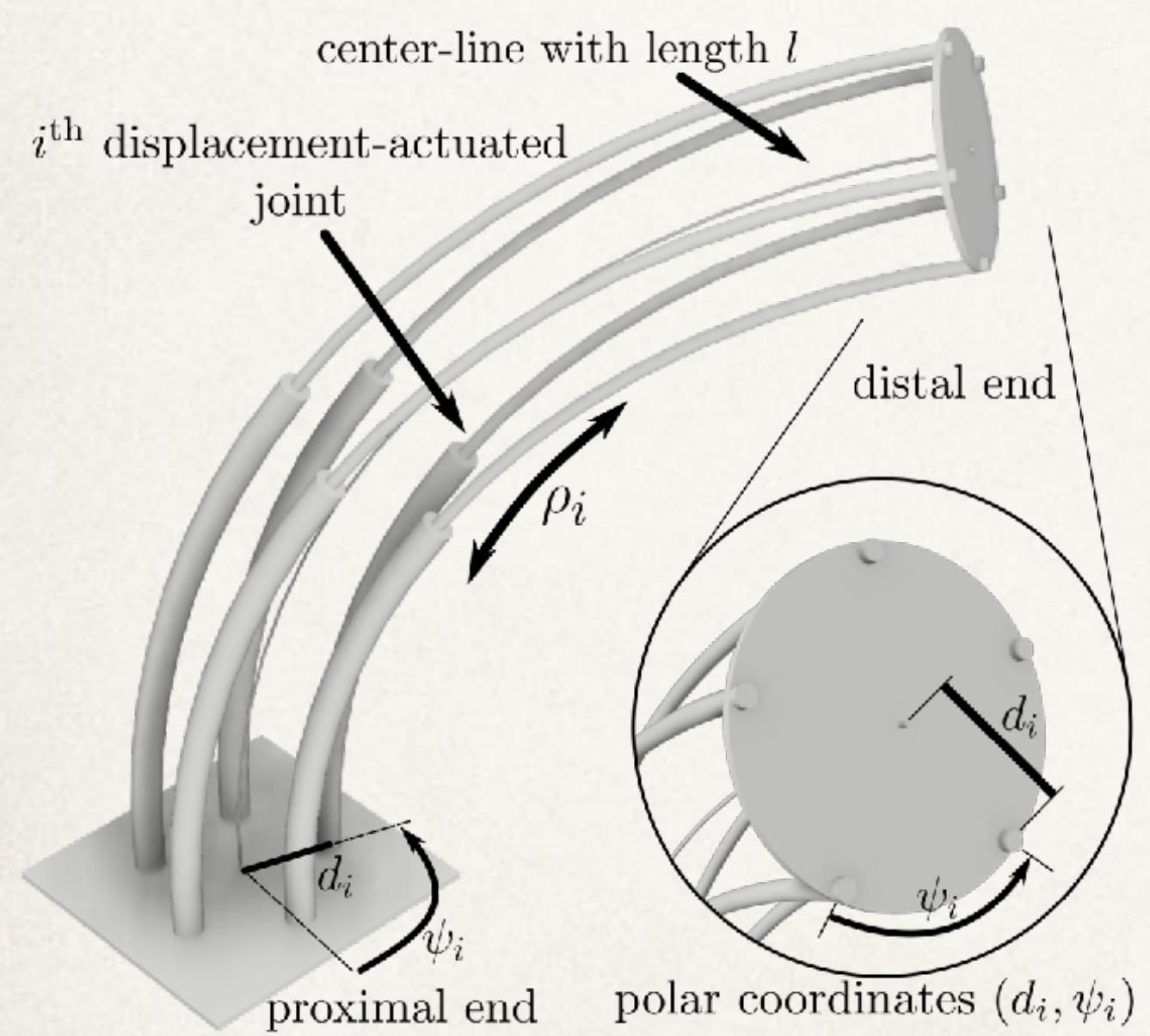
$$\begin{bmatrix} d\phi \cos(\theta) \\ d\phi \sin(\theta) \end{bmatrix} = \bar{\rho} = M_{\mathcal{P}} \rho$$

Improved State Parameterizations

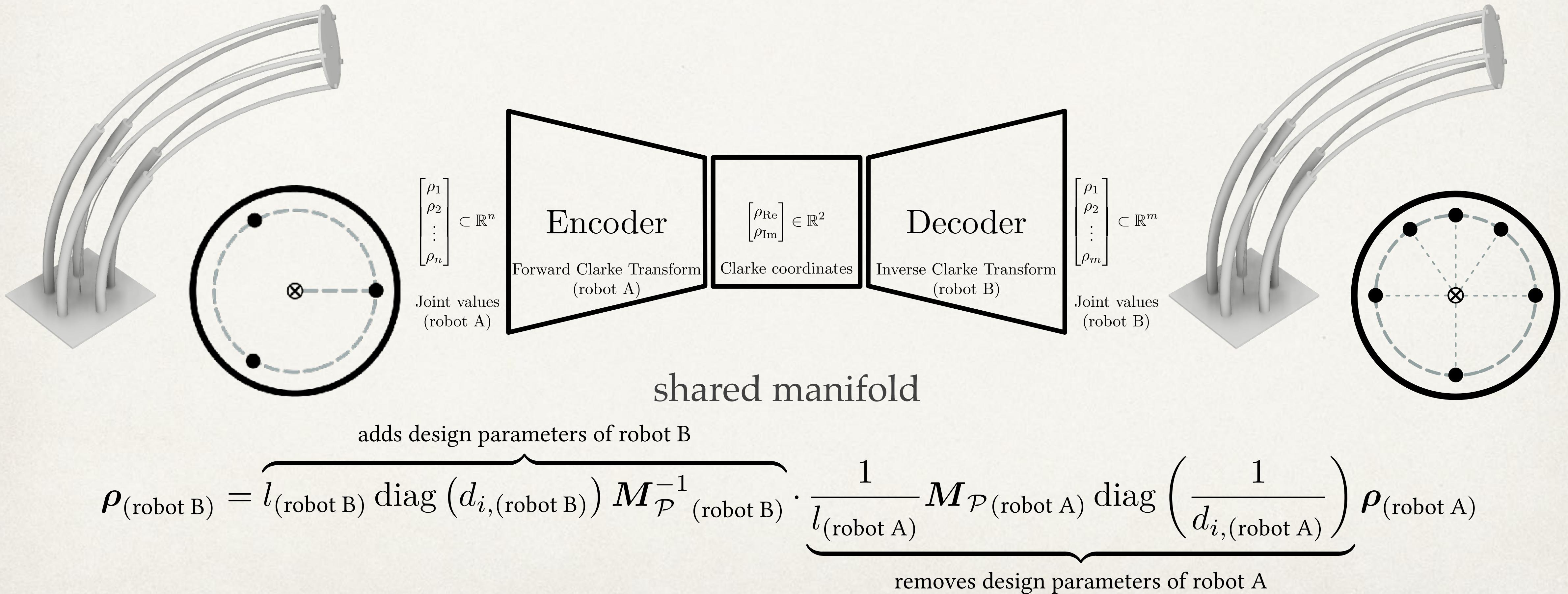
Reference	n	Parameterization w.r.t. joint values	Parameterization w.r.t. Clarke Coordinates
Della Santina <i>et al.</i>	4	$\Delta_x = \frac{l_3 - l_1}{2}$	$\Delta_x = \rho_{\text{Re}}$
Allen <i>et al.</i>	3	$u = \frac{l_2 - l_3}{\sqrt{3}d}$	$v = \frac{(l_1 + l_2 + l_3)/3 - l_1}{d}$
Allen <i>et al.</i>	4	$u = \frac{l_2 - l_4}{d}$	$v = \frac{l_3 - l_1}{d}$
Dian <i>et al.</i>	3	$\Delta x = \frac{l_2 + l_3 - 2l_1}{3}$	$\Delta y = \frac{l_3 - l_2}{\sqrt{3}}$
Grassmann <i>et al.</i>	n	$[\rho_{\text{Re}}, \rho_{\text{Im}}]^\top = \mathbf{M}_{\mathcal{P}} \boldsymbol{\rho}$	ρ_{Re}
			ρ_{Im}

- [Della Santina et al., RA-L 2020] “On an Improved State Parametrization for Soft Robots with Piecewise Constant Curvature and Its Use in Model-Based Control”
- [Allen et al., RoboSoft 2020] “Closed-Form Non-Singular Constant-Curvature Continuum Manipulator Kinematics”
- [Dian et al., Access 2022] “A Novel Disturbance-Rejection Control Framework for Cable-Driven Continuum Robots With Improved State Parameterization”
- [Grassmann & Burgner-Kahrs, arXiv (under review)] “Clarke Coordinates Are Generalized Improved State Parametrization for Continuum Robots”

Arbitrary Joint Locations

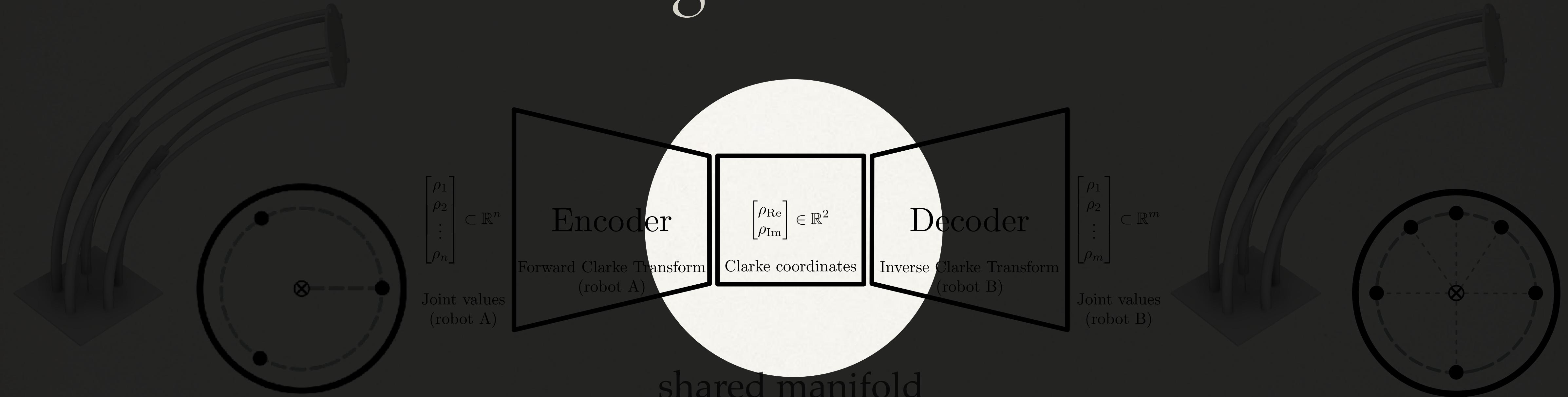


Re-using Previous Approaches



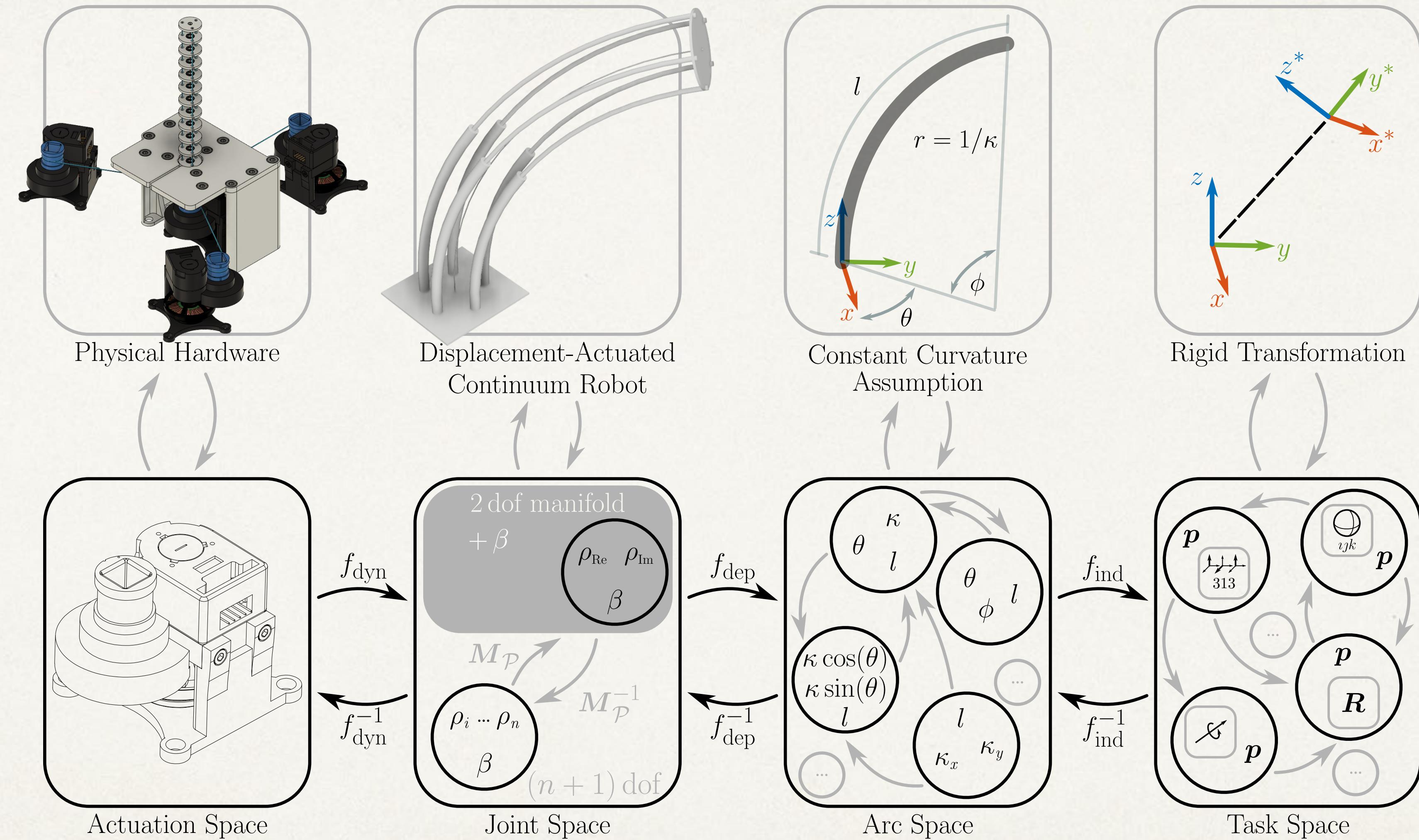
Re-using Previous Approaches

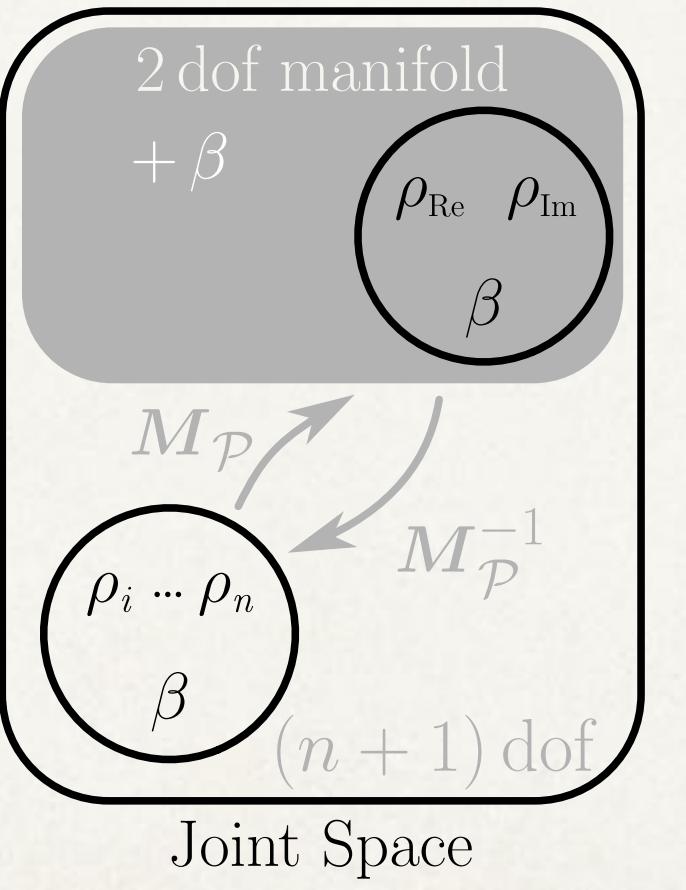
Lingua Franca

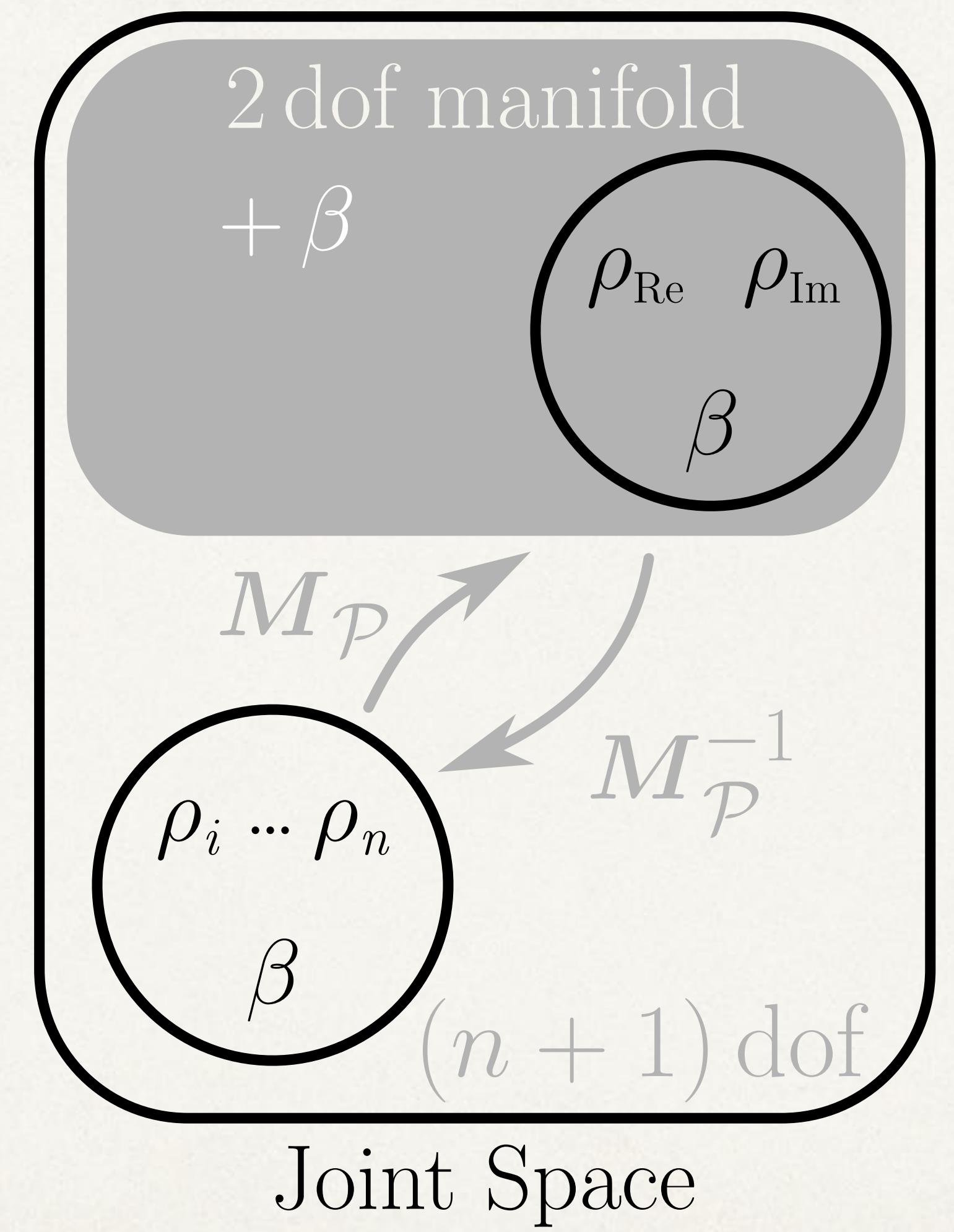


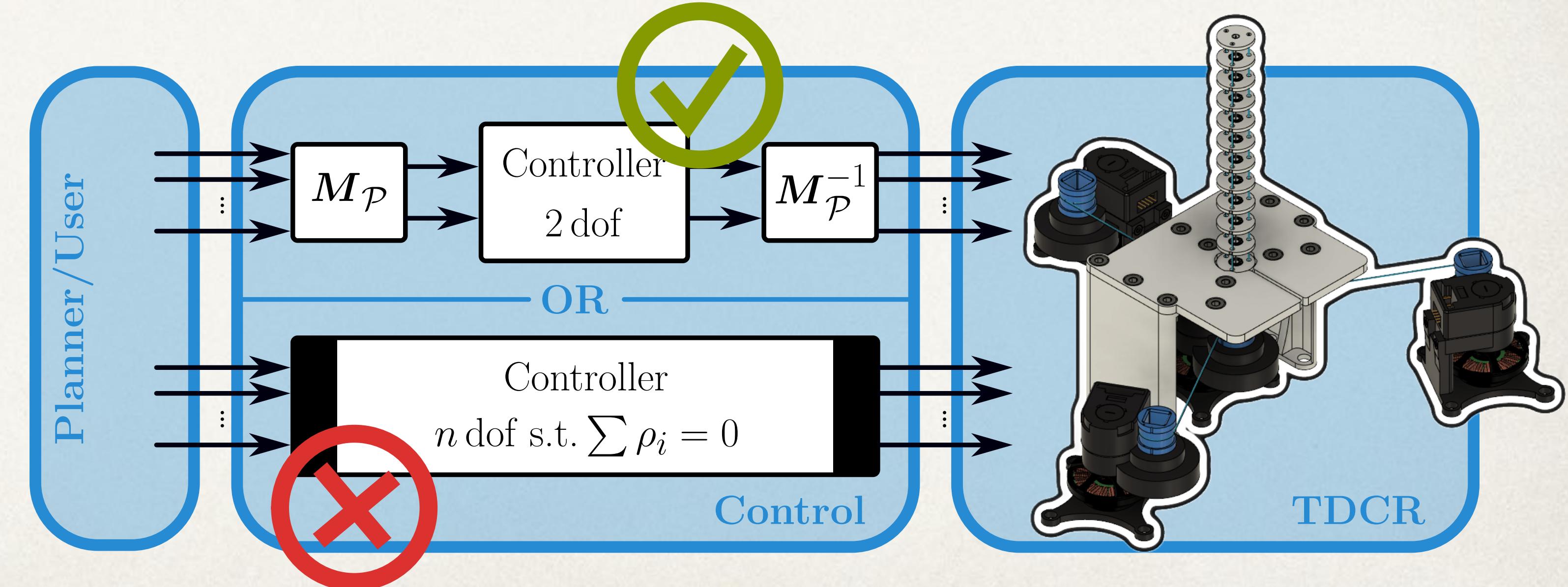
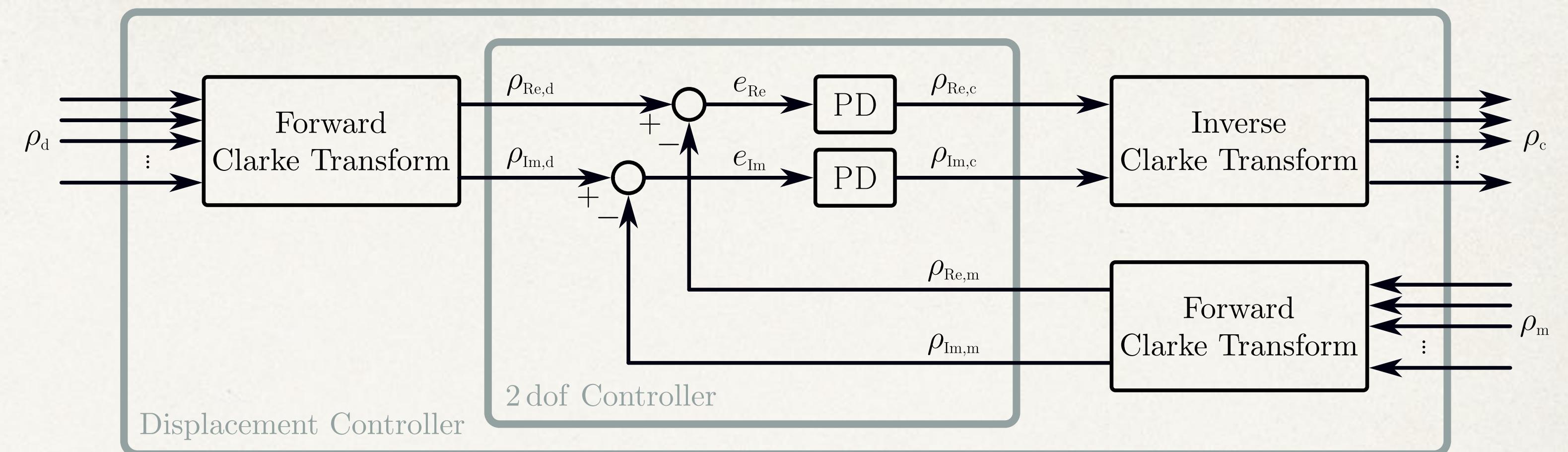
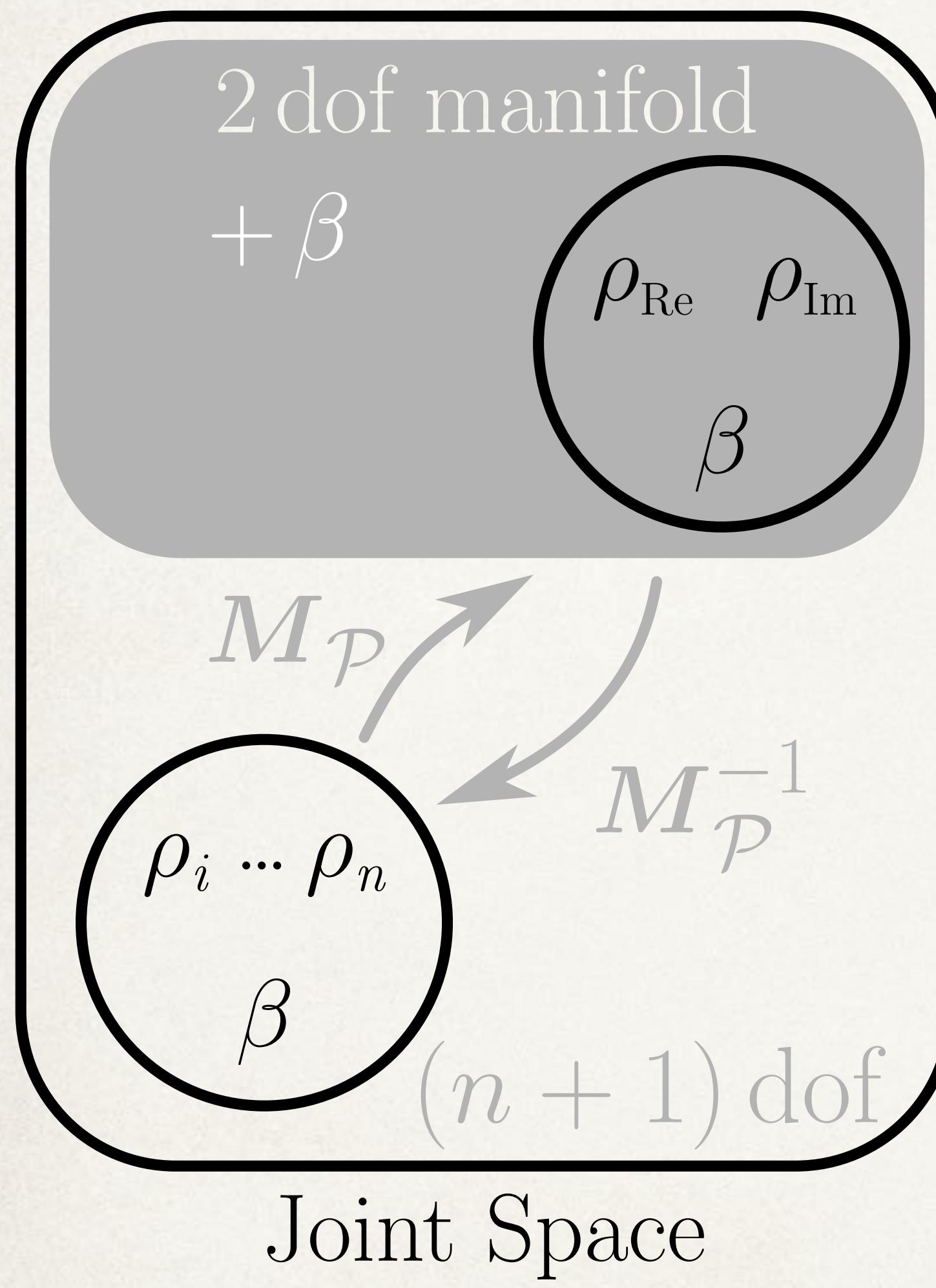
$\rho_{(\text{robot B})} l_{(\text{robot B})} \text{diag}(\omega_{i,(\text{robot B})}) M_{\mathcal{P}_{(\text{robot B})}}^{-1}$ adds design parameters of robot B

$M_{\mathcal{P}_{(\text{robot A})}}^{-1} l_{(\text{robot A})} \text{diag}(d_{i,(\text{robot A})}) \rho_{(\text{robot A})}$ removes design parameters of robot A

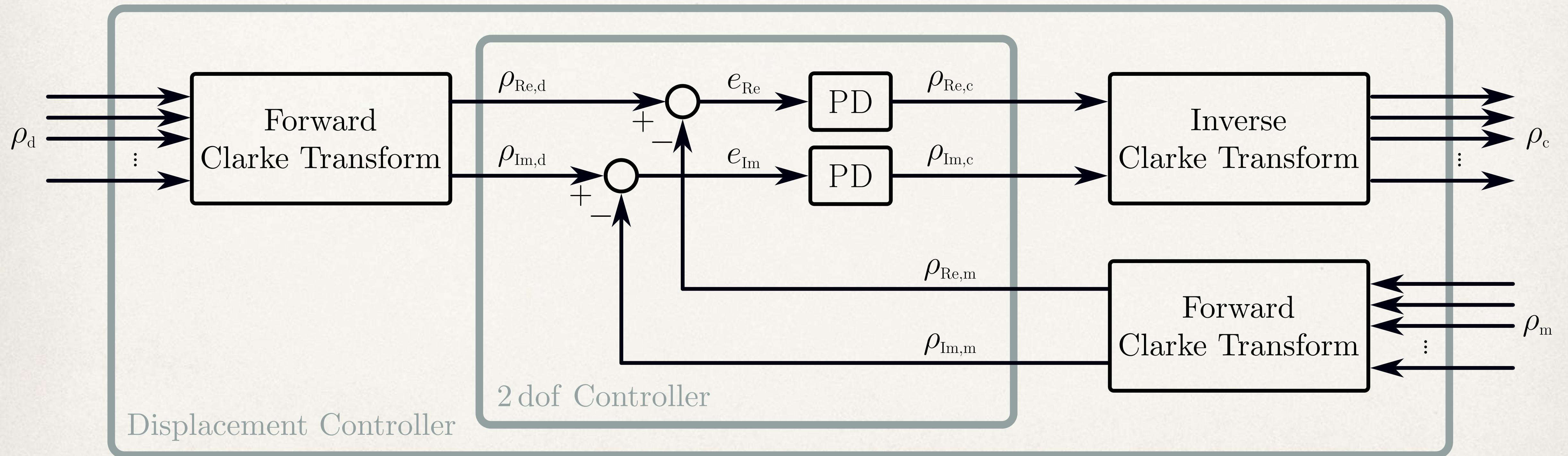




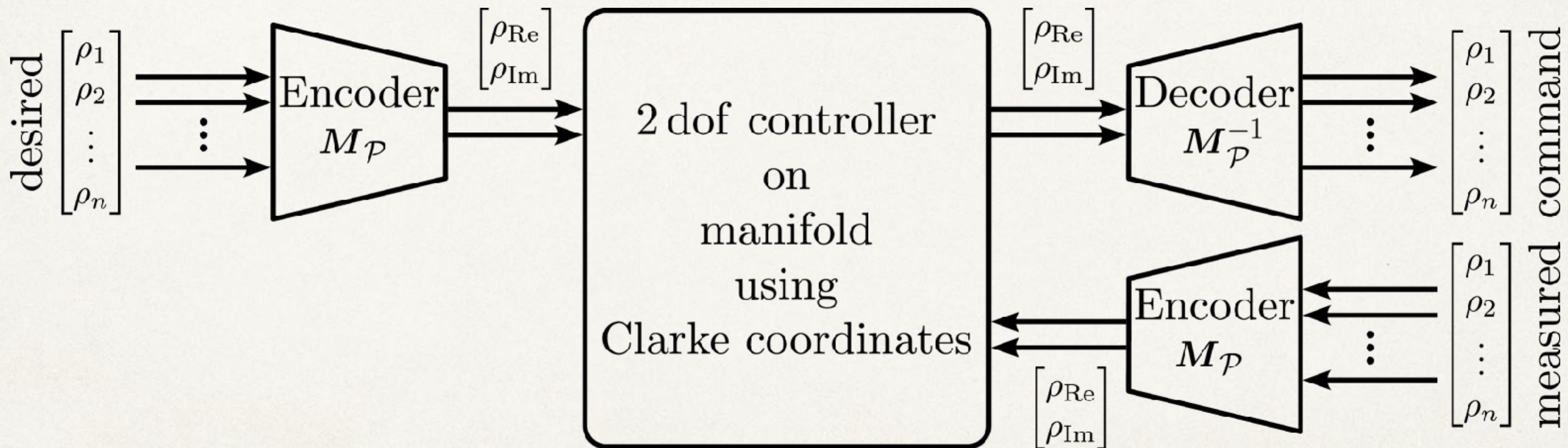




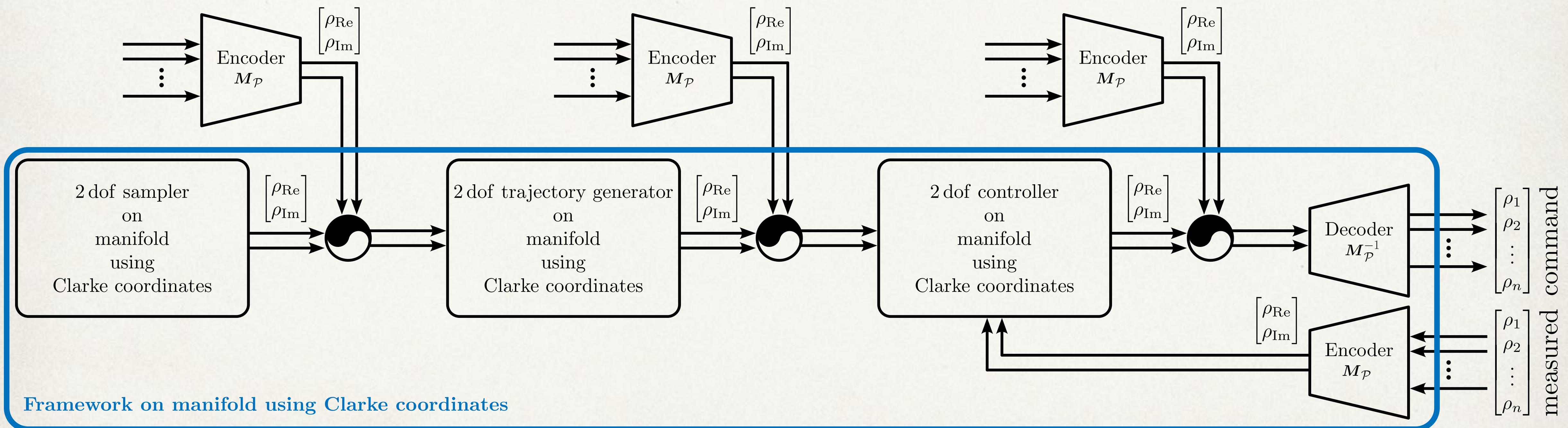
Simple Units



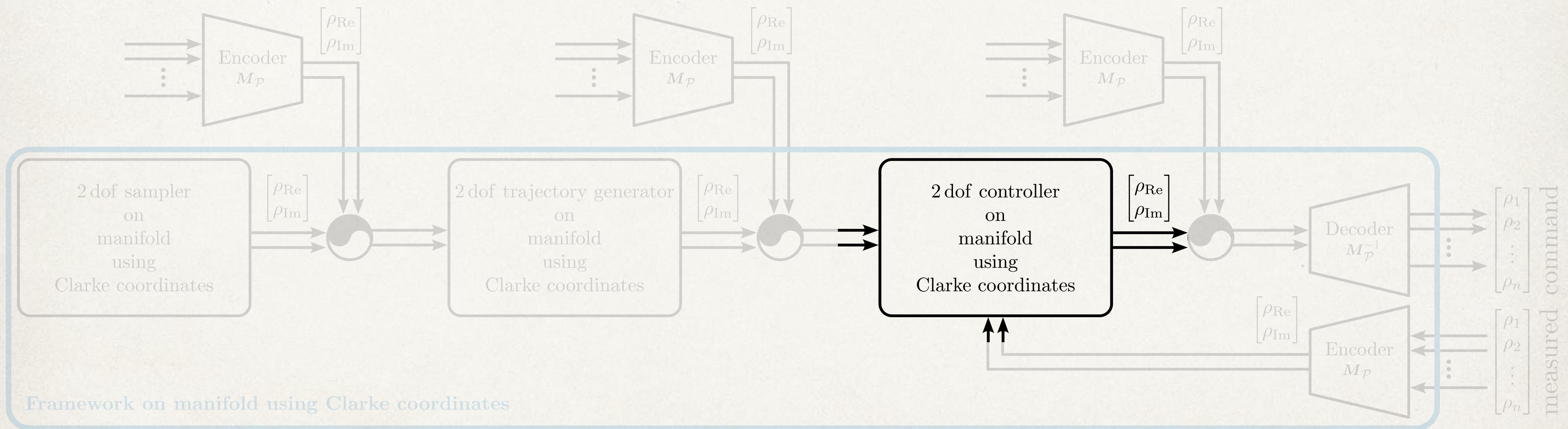
Simple Units



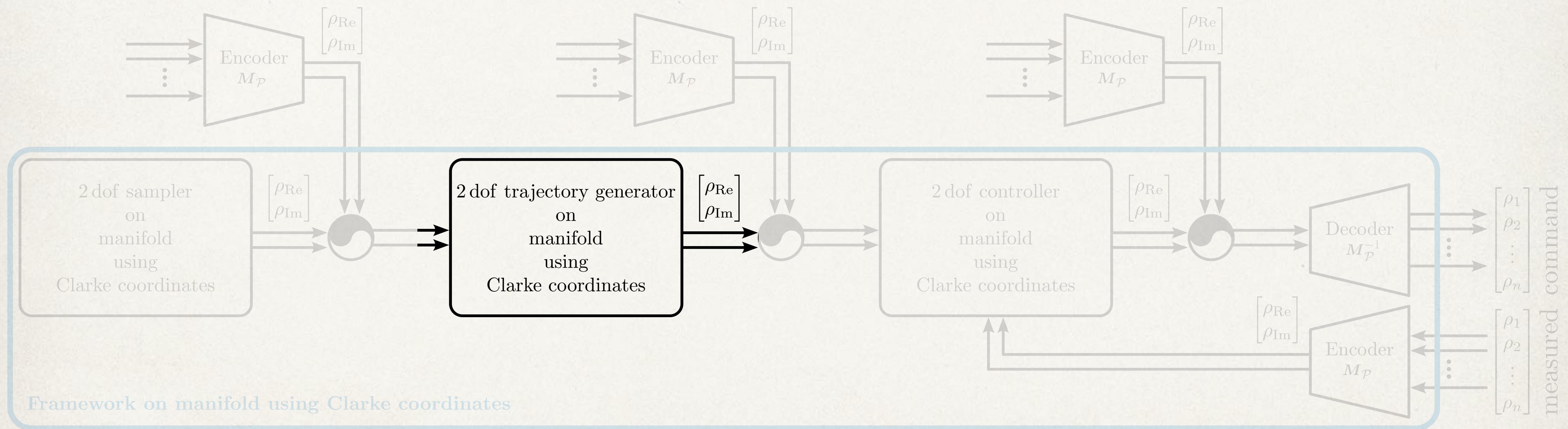
Concatenation of Units



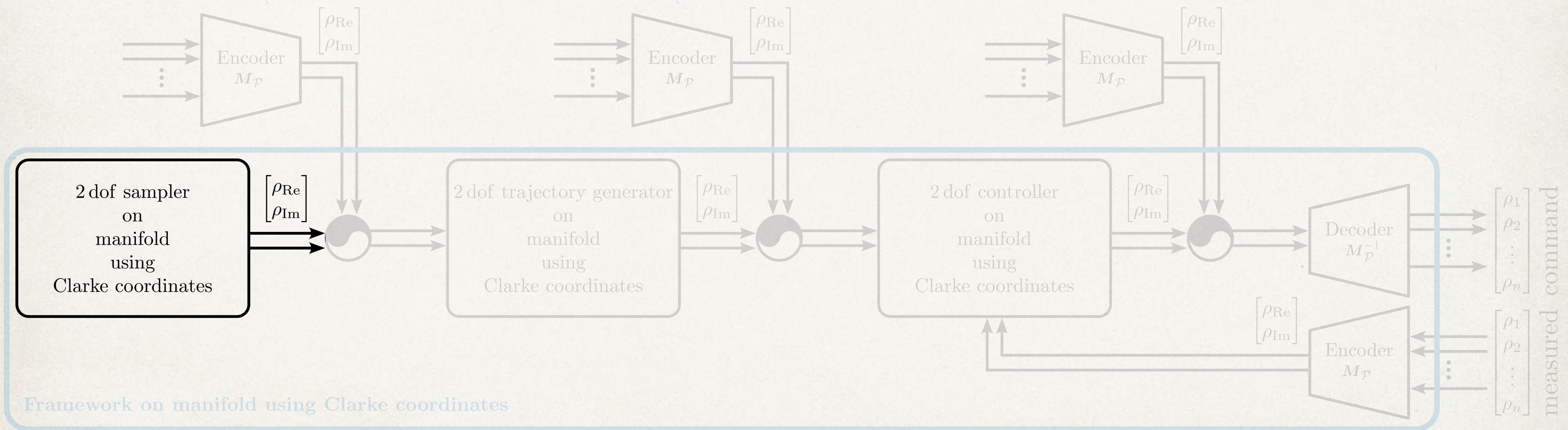
Concatenation of Units



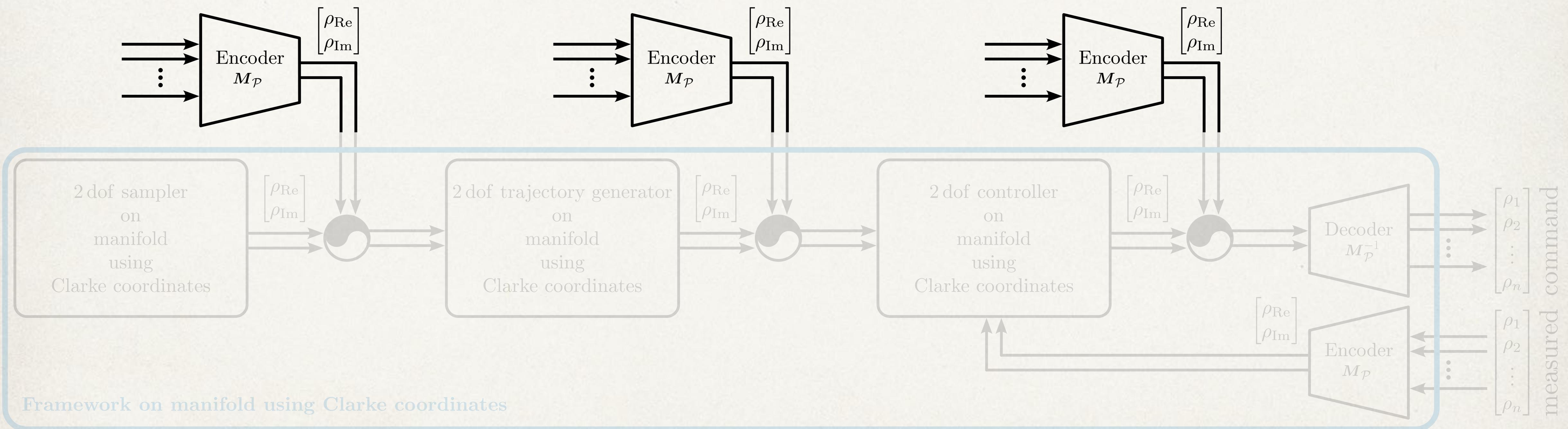
Concatenation of Units



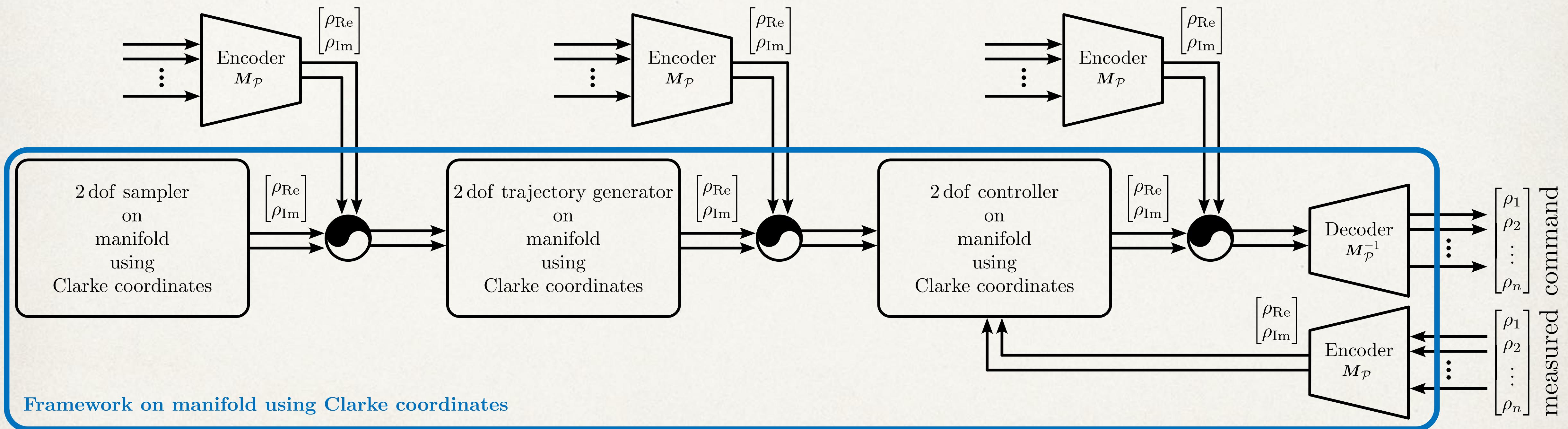
Concatenation of Units



Concatenation of Units



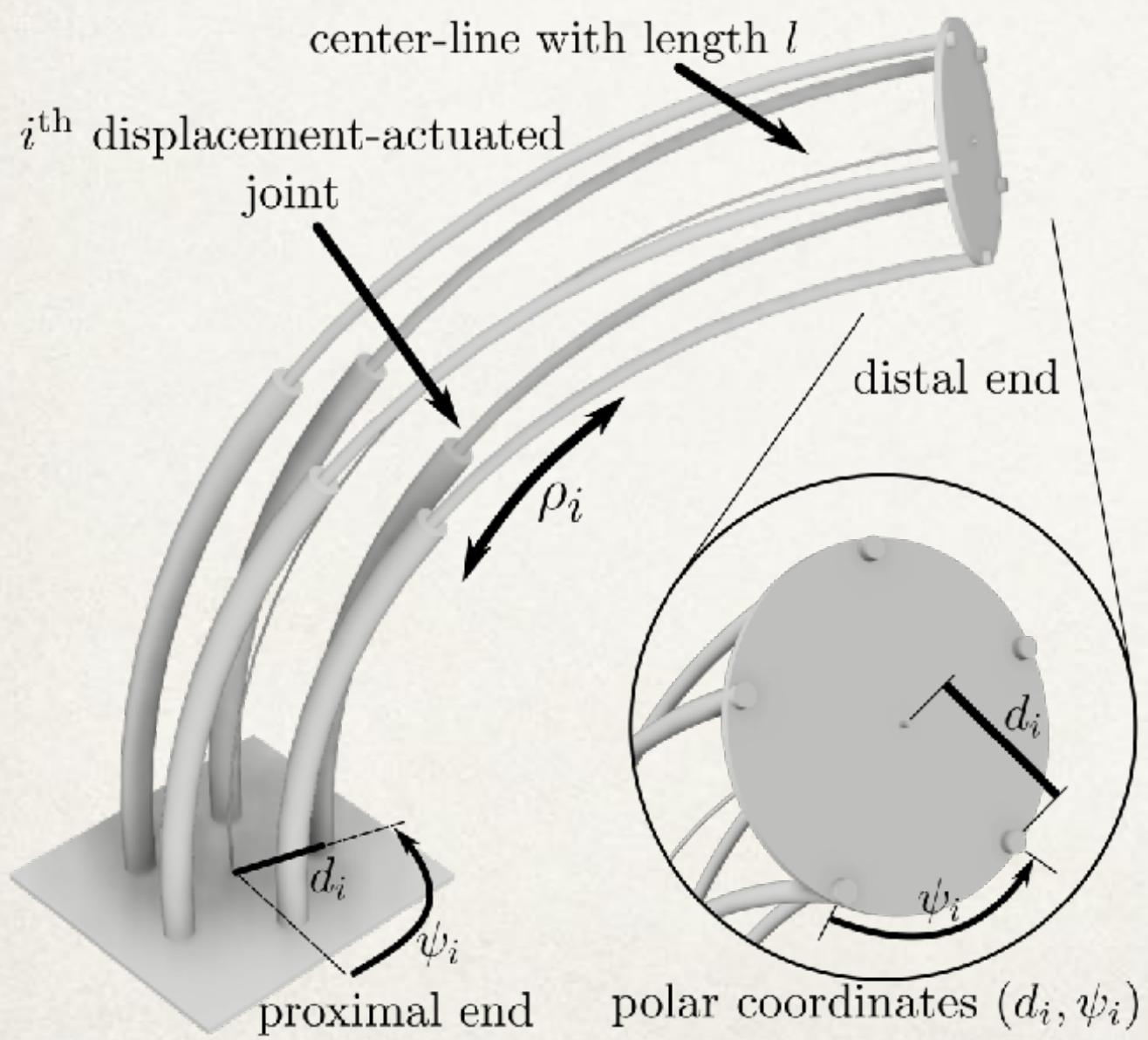
Concatenation of Units



Takeaway: Prelude of Benchmarking

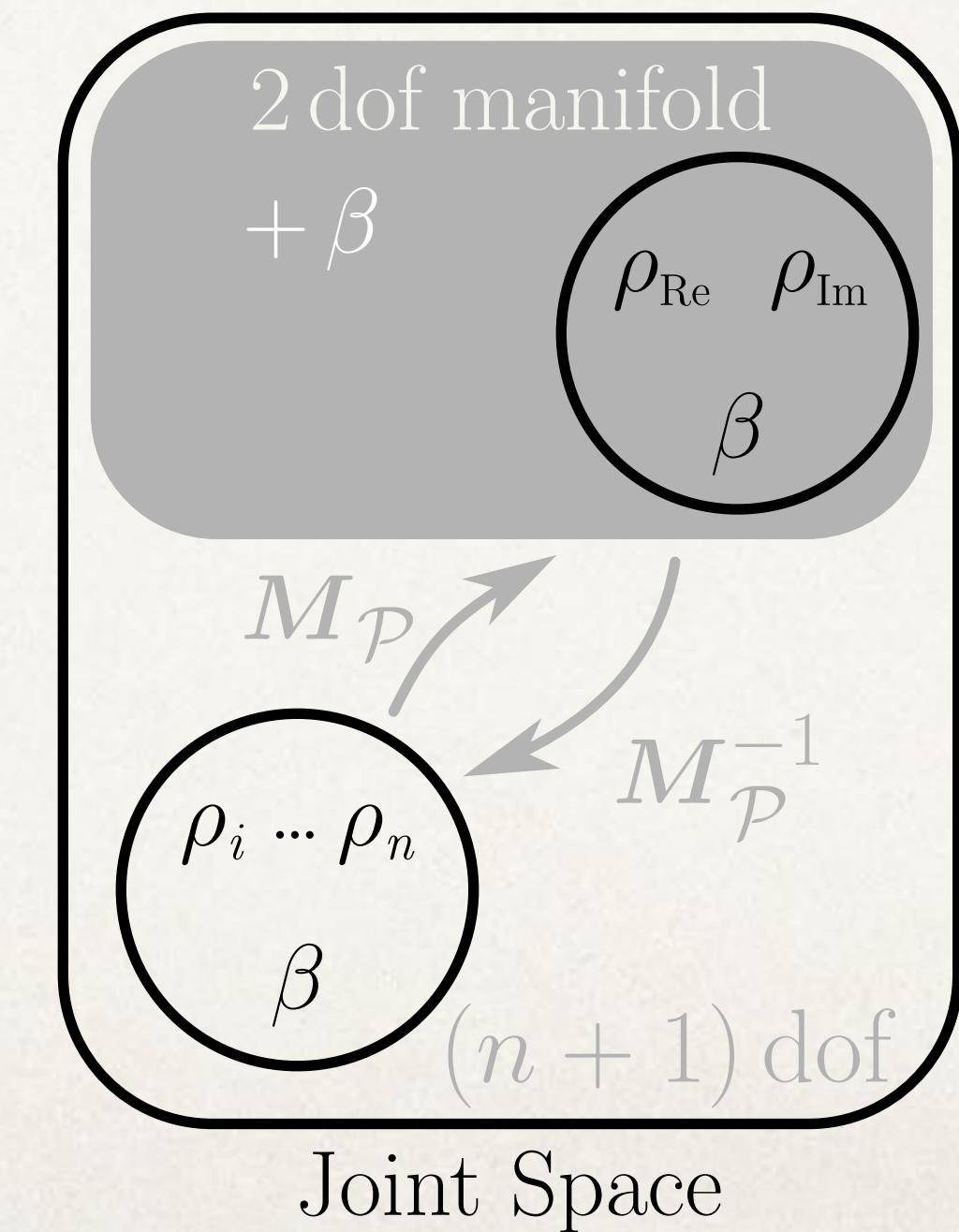
I.

right definition and abstraction



II.

manifolds and parameterization



III.

testable units

