The Interplay Between Product Variety and Customer Retention: Theory and Evidence

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Abstract

The optimal number of products to offer consumers is one of the core strategic problems that firms face. This is increasingly so in digital markets where many firms offer a large product variety. In these markets, consumers purchase products repeatedly, making customer retention an important aspect for firm performance. In this paper, we study the interplay between these two variables, product variety and customer retention. First, we provide a novel game-theoretic model to analyze this interplay and also determine how the optimal product variety depends on the market environment. Second, we empirically test the resulting predictions using data for video games from Steam. Our data set comes with accurate measures of the key variables in our game-theoretic model and additionally contains plausible instrumental variables for empirical identification. We show theoretically that investment in product portfolio size and investment in customer retention are substitutes because the former increases demand from switching consumers whereas the latter increases the repurchase probability of current consumers. We then derive predictions on how market conditions determine a firm's product variety, for which we find ample evidence in our empirical analyses: There is (i) a negative relation between product portfolio size and customer retention, (ii) an inverted u-shape relationship between market value and product variety, and (iii) a positive relation between consumer heterogeneity and product variety. Both our theoretical and empirical results are robust to a wide set of robustness checks.

Keywords: Product Variety, Customer Retention, Multi-Product Competition, Video Games, Product Updates

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1 Introduction

The choice of the product portfolio size (i.e., the number of offered products) is an important strategic decision of firms in most industries. In many digital markets, such as the markets for video games, mobile applications (apps), or electronic books, this decision has become even more prominent than in traditional markets, as the costs of developing and launching new products are relatively small (Waldfogel, 2018). Indeed, in many markets for digital goods, firms offer a large variety of products. At the same time, there is considerable variation within and across market segments. For example, in the video game industry—one of the largest entertainment industries worldwide—some publishers (e.g., Electronic Arts and Ubisoft) offer hundreds of games across genres whereas other publishers (e.g., Rockstar Games and CD Projekt) only have a few titles in their portfolio.[1] These markets are also characterized by repeated and frequent purchase decisions of consumers who usually buy different games or apps over time. Customer retention is therefore an important determinant in these markets, as a consumer's choice whether to buy and/or download a game or an app of the same firm is to large extent driven by whether the previous product has fulfilled or exceeded expectations.

Indeed, in the video game industry, several companies point to the importance of the interplay between variety and retention. For instance, Rockstar Games describes in the annual report its strategy as 'to develop a limited number of titles', while being known to its customers for the 'longevity in the market' and for producing 'add-on content'.[2] This is supported by a Financial Times article explaining that 'gamers keep going back to GTA 5', a Rockstar Games video game released in 2013, as it is frequently 'updated over time'.[3] The major online distribution platform for video games, Steam, also describes updates as a mean to keep customers engaged and retained.[4] By contrast, other video game publishers, such as Electronic Arts, stress primarily their many 'games and services across diverse genres'.[5]

In order to provide more systematic evidence on the importance of the two variables across industries, we determine how often the words "retention" and "variety" have appeared in the annual reports of NASDAQ 100 firms in the last 20 years. We find that both terms are not only nowadays appearing in almost all annual reports, but the number of mentionings

[1] See `https://steamdb.info/publisher/Electronic+Arts/`, `https://steamdb.info/publisher/Ubisoft/`, `https://steamdb.info/publisher/Rockstar+Games/`, and `https://steamdb.info/publisher/CD+PROJEKT+RED/`.

[2] See `https://www.sec.gov/Archives/edgar/data/946581/000162828023019851/ttwo-20230331.htm`.

[3] See `https://www.ft.com/content/93d418c4-ea83-4022-a3db-b155f6c8c33b`.

[4] See `https://partner.steamgames.com/doc/store/updates`.

[5] See `https://www.sec.gov/Archives/edgar/data/712515/000071251522000011/ea-20220331.htm`.

doubled over the time span considered, while mentionings of, for instance, "quality" and other strategic variables have grown less dynamically (see Online Appendix A.1). To assess whether the two variables are also brought up together, we additionally analyze if they are mentioned within the same paragraph. To this end, we focus on the first three pages of a company's business description in the annual reports and query OpenAI's GPT-4 to classify instances in which customer retention and product variety are described. We find that while almost all companies talk about both variables, close to twenty percent do it even within the same paragraph (see Online Appendix A.2). This shows that the two variables not only grew in importance over the last 20 years, but also that their interaction is critical for firms.

The existing literature has provided valuable insights on how product variety and consumer retention can increase firm performance, but has focused on each variable *separately*.[6] However, little is known about the *interplay* between the two and how they are jointly affected by the market environment. This is particularly important as a firm has only limited resources and needs to decide how much to invest in product variety and in customer retention (e.g., via updates). Several interesting questions then arise: Are investments in product portfolio size and in customer retention complementary to each other or are they substitutes? Does the relationship between the two variables depend on the profitability of a market? How does the heterogeneity of consumers drive both investment decisions?

This paper aims to answer these questions. First, we provide a novel game-theoretical model to study the interaction between product variety and consumer retention (e.g., via product updates, add-on content, or loyalty programs). We derive predictions regarding the relationship between the two variables and also how the profitability of a market segment as well as consumer heterogeneity affect product variety. Second, we test our predictions using data from the video game industry and show that our empirical evidence is supportive of the hypotheses derived from the theoretical analysis.

Our theoretical model considers an oligopoly in which each firm has three choice variables—i.e., it invests in customer retention, it chooses the number of products it offers to consumers, and it sets a price for its products. A broader mass of products and a higher customer retention level are both costly but generate higher demand. However, the two variables target different consumer groups. An investment in customer retention increases the repurchase probability from consumers in the firm's customer base, as these consumers are then less likely to switch to an alternative. By contrast, a broader product variety (and a lower price) increase demand from consumers who are willing to switch. By supplying a broader product portfolio, a firm is more likely to offer a match with a consumer's preference and therefore attracts more new customers.

[6] We provide a more detailed literature review at the end of this section.

We first show that there is a negative relationship between investment in customer retention and investment in product portfolio size, that is, the two types of investments are substitutes. If firms invest more in customer retention, consumers are more likely to be satisfied with the product they bought. Therefore, they are less inclined to switch to a product of a competitor, which implies that investment in product variety becomes less profitable.

Second, we determine how the value of a market segment shapes product variety. We find an inverted u-shaped relationship: if the market value is low, optimal product variety increases in the segment's profitability, whereas if it is high, the relationship is negative. The former result is as expected, as investment in product variety is more profitable the larger the market value. Instead, the latter result is perhaps more surprising. Its intuition is rooted in the negative relation between customer retention and product variety. In high-value segments, it is particularly profitable for each firm to retain its previous consumers by investing in customer retention. This leads to few switching consumers, implying that firms optimally offer a smaller product portfolio.

Third, we analyze how the degree of heterogeneity in consumers' preferences affects the product portfolio size of firms and show that the number of products rises with the degree of heterogeneity. The intuition for this result is twofold. First, if consumers are more heterogeneous, their preferences for products are more dispersed. This implies that offering a larger product variety, which increases the probability that a product matches a consumer's taste, is more profitable. In addition, an increase in consumer heterogeneity dampens price competition, which implies that firms can reap larger revenues from additional products.

To the best of our knowledge, the model includes several novel features that are pertinent in industries, but have not been considered together so far. For instance, the model involves the choice of two strategic variables—i.e., product portfolio size and customer retention—in addition to product prices, whereby these variables affect demand differently. Solving such a model is normally complicated; yet, our model is tractable and allows for clear-cut results.

In addition, the main predictions of our model are also novel and differ from those that would be obtained by off-the-shelve models of imperfect competition. E.g., focusing on the choice of product portfolio size in isolation without considering the interaction with customer retention would lead to the prediction that firms offer more products in more valuable markets. Instead, our model generates an inverted u-shaped relation. Moreover, the relationship between product portfolio size and customer retention is not obvious ex ante, and our model provides a clear intuition why it is negative.

To test the predictions of our theoretical model, we need markets that come with both sufficient variation in the relevant variables—i.e., the number of products, consumer retention, segment value, and customer heterogeneity—as well as with accurate measures or at least proxies of these relevant dimensions. Digital markets often fulfill these two main requirements. Their advent has led to increased product variety through lowering costs, but also brought about highly dynamic and innovative markets with short life cycles, which requires firms to invest.

For our empirical analyses, we choose to study the market for online video games. This market does not only play an important role in many consumers' daily routine, it is also economically highly relevant with global turnovers of close to 200 billion USD in 2023.[7] The data for video games is directly retrieved from Steam, the leading platform for the online distribution of video games, and comprises comprehensive information on the universe of more than 47,000 games available in August 2022.

We measure product variety based on the number of titles released by publishers within a year in each segment as defined by the platform, while we infer consumer retention from the updates by a video game. Segment value is approximated by the purchase price. Finally, we measure the degree of consumer heterogeneity by the share of video games that does not have the most popular tag of a focal market among its user-defined tags describing the game.

Besides simple OLS regressions and a series of robustness checks to test the predicted relationships, we also account for endogeneity. First, we employ an instrumental variable approach to account for simultaneity concerns arising from linking product variety with the other core variables retention and segment value. For this, we use changes in exchange rates as cost shifters and the diffusion of tools facilitating updates to instrument prices and updates, respectively. Second, there are potentially unobserved factors when relating product variety to customer heterogeneity, and, hence, we try to control for many segment-specific factors, but also analyze the sensitivity of the baseline regression to varying degrees of possible correlation with the error term.

We find a negative relationship between the measures of product variety and consumer retention; hence, in the video game industry, the two variables are substitutes. We also establish an inverted u-shape for the link between segment value and product variety, while the degree of consumer heterogeneity is positively related with portfolio size. Overall, we therefore find strong empirical support for the hypotheses from the theoretical model. As these hypotheses contradict the conventional wisdom regarding product breadth choice, they underscore the relevance of the novel effects we elicit in our theoretical model.

In addition, the empirical analysis provides novel contributions to the literature. First, it captures updates as a way to retain customers and by that adds retention strategies as a new dimension to research on the determinants of firm performance in digital markets (cf. Ozalp and Kretschmer, 2019). Second, to the best of our knowledge, we are the first to empirically

[7] See `https://www.data.ai/en/insights/mobile-gaming/2023-gaming-spotlight-report/`.

show a trade-off between frequency of product updates and product breadth, which adds to the literature on generational product innovation (Lawless and Anderson, 1996; Chen et al., 2022).

Related literature:

Our paper contributes to several strands of literature.

First, we contribute to the literature on product variety choice. The theoretical literature focuses primarily on how competing firms' product lines differ. Brander and Eaton (1984) consider competition between horizontally differentiated duopolists and determine under which conditions each firm prefers to offer close substitutes or more distant products. Klemperer (1992) as well as Klemperer and Padilla (1997) analyze the effect of shopping costs on product line length and on horizontal differentiation between firms' products. Studying vertical product differentiation, Champsaur and Rochet (1989) find that firms will never choose overlapping product lines, and Johnson and Myatt (2003) develop a model of quality-differentiated products and show how product lines change with entry.[8] None of these papers considers the interplay between investment in customer retention and product variety, which is the focus of our study.[9]

In the empirical literature, several studies analyze the profitability of investing in product variety. In this vein, Sorenson (2000) using data from computer workstation manufacturers, finds that product variety is more valuable in uncertain markets and when firms offer only few products. Eggers (2012) as well as Barroso and Giarratana (2013) analyze breadth versus depth of product lines and determine how experience and product complexity affect the relation between product-line length and firm performance.[10] Cottrell and Nault (2004) show that multi-product firms in the microcomputer software industry benefit from economies of scope in consumption, as consumers prefer to buy products from the same firm. Hui (2004) analyzes the severeness of self cannibalization arising from larger product lines, while Nerkar and Roberts (2004) as well as Bayus and Agarwal (2007) study the effects of the time period a firm is active in the market on product line extensions. We contribute to this literature by studying how optimal product variety is driven by the interaction with customer retention strategies, thereby testing hypotheses derived in our theoretical model.

Second, related to the last point, our paper contributes to the literature stream on gener-

[8]Lancaster (1990) provides a survey, discussing in detail how a firm's optimal product variety depends on demand conditions.

[9]A different strand is the literature on proliferation strategies as entry deterrents. In particular, Schmalensee (1978) demonstrates how an incumbent firm can strategically introduce new products to deter entry. This idea has been generalized and extended by e.g. Judd (1985), Bonanno (1987), Shaked and Sutton (1990), as well as Gilbert and Matutes (1993). For an empirical analysis, see Bayus and Putsis (1999).

[10]Kekre and Srinivasan (1990) find that the benefits of a larger product variety tend to outweigh the associated costs in many manufacturing industries.

ational product innovations, that is, improvements in existing products that result in functional and technical advances (Lawless and Anderson, 1996). This literature focuses on how such innovations allow for better value capture and increased performance—e.g., Banbury and Mitchell (1995) provide evidence for this relation in the cardiac pacemaker industry, Lawless and Anderson (1996) in the microcomputer industry, and Ansari et al. (2016) in the television industry.[11] In our empirical analysis, we use product updates—i.e., an important form of generational product innovations in the video game industry (Chen et al., 2022)—as our measure for retention. We demonstrate a novel trade-off between product updates, which affects the repurchase probability of existing consumers, and product breadth, which aims at attracting new consumers, that has not been analyzed in the literature.[12]

Third, our paper contributes to the mainly empirical literature on customer satisfaction and retention strategies. This literature provides overwhelming evidence for customer satisfaction and retention being closely related as both increase consumers' loyalty to a firm and therefore the repurchase intent and decision.[13] For instance, Anderson and Sullivan (1993) develop a utility-oriented framework to study the main drivers behind this effect. Mittal and Kamakura (2001) as well as Szymanski and Henard (2001) study how the effect is moderated by consumer characteristics. Otto et al. (2020) and Hult et al. (2022) provide recent comprehensive surveys of this literature. In contrast to these papers, we study how consumer retention and product variety relate to each other. To this end, we explicitly model investment in customer retention and analyze the resulting implications on product variety theoretically and empirically.

Finally, our paper relates to the literature on switching costs. The main trade-off in this literature is that switching costs allow firms to price high to existing consumers, but at the same time trigger fierce competition for new consumers (Klemperer, 1987; Beggs and Klemperer, 1992; Shi et al., 2006; Cabral, 2016).[14] In our model, investment in customer retention increases the probability of repurchase from consumers in the customer base, which is similar to an increase in switching costs. However, in contrast to this literature, we consider endogenous investment and also study investment in product variety to attract switching consumers.

In the empirical literature on switching costs, some papers study the effects of product variety. Brush et al. (2012) using data from the banking industry, show that firms benefit

[11]See Helfat and Raubitschek (2000) for a conceptual model and several examples.

[12]Recently, Chen et al. (2022) demonstrate that product updates can increase the learning costs of consumers, which may lead to reduced adoption.

[13]Customer satisfaction is defined, in a broad sense, as a consumer's post-consumption judgment of whether a product provided a pleasurable level of usage-related fulfillment (see e.g. Oliver, 2014).

[14]For models with stochastic switching costs, see e.g. Chen (1997) and Ruiz-Aliseda (2016).

most from switching costs if they offer several complementary products to their base product. Abolfathi et al. (2022a) find that multi-product firms obtain a larger advantage from switching costs compared to single-product firms due to increased flexibility for consumers and test this effect with data from the telecommunication industry. Abolfathi et al. (2022b) show theoretically and empirically that switching costs can lower firms' profits when the conversion from a base product to an advanced product is important for consumers. Different to this literature, we find that a larger product breadth helps to attract switching consumers, thereby providing a new channel how consumer switching and product variety relate to each other.

The paper unfolds as follows: Section 2 sets out the model. Section 3 solves for the equilibrium, presents the theoretical results, and states the hypotheses predicted by the model. Section 4 describes the data, and Section 5 shows the baseline results. Section 6 then lays out the identification strategy and provides corresponding empirical results. Finally, Section 7 concludes. The proofs of our results are in Online Appendix B. Online Appendices C and D present, respectively, technical underpinnings of the equilibrium and several extensions of the theoretical model. Online Appendix E provides details on empirical robustness checks.

2 The Model

We set up a model that allows us to study the interaction between investment in product breadth and investment in customer retention. Both investments are costly for firms but help them to raise demand. The former type of investment increases demand from consumers who look for a new product, whereas the latter aims at keeping current consumers. To capture these pertinent features of several markets, we consider the following model.

Firms. There are M firms, denoted by $i \in \{1, ..., M\}$. Each firm has an existing customer base that is assumed to be of equal size for all firms.[15] Firms simultaneously compete in three strategic variables: (i) each firm chooses the number of products it offers (i.e., its product breadth); (ii) it sets a price for its products; (iii) it invests in customer retention. These choices jointly determine the probability with which each consumer buys from a particular firm, and therefore firms' expected profits. We next describe these choices and introduce the respective notation. When discussing consumers, we explain how the choices generate the buying probabilities.

[15]In Online Appendix D.1, we analyze a two-period model in which a firm's customer base is endogenous and driven by firms' pricing choices in the first period. We show that all our results carry over.

We denote the mass of products of firm i by m_i. To be able to apply differentiation techniques, we treat m_i as a continuous variable.[16] The costs of offering a product breadth m_i is $f(m_i)$, with $f'(m_i) > 0$. We assume that $f(m_i)$ is weakly convex (i.e., $f''(m_i) \geq 0$) to ensure interior solutions. To guarantee that costs are sufficiently low so that equilibrium investment and profit are positive, we assume that $f(m_i)$ goes to 0 as $m_i \to 0$. To simplify the exposition, firm $i \in \{1, ..., M\}$ sets the same price p_i for all of its products. In Online Appendix C.1, we show that this is indeed optimal, that is, even if a firm could charge different prices for its products, it is nevertheless optimal to charge a single one.

We denote the investment in customer retention of firm i by r_i. A higher r_i induces consumers in firm i's customer base to repurchase one of the firm's products with a higher probability. The cost of investment into customer retention is $c(r_i)$, with $c'(r_i) > 0$ and $c''(r_i) > 0$. For the same reason as above, we assume that $c(r_i) \to 0$ as $r_i \to 0$.

Consumers. There is a mass 1 of consumers. Since the customer base of each firm is the same, it is equal to $1/M$.[17] To express in a simple way that a consumer in firm i's customer base buys with a higher probability from firm i again if the latter invests more in customer retention, we assume that the consumer buys a product of firm i with probability r_i.[18] Instead, with probability $1 - r_i$, firm i does not retain the consumer who then (potentially) switches. A switching consumer chooses among all products available in the market. This formulation is consistent with papers modeling the choice of variety-seeking consumers (e.g., Givon, 1984, or Zeithammer and Thomadsen, 2013). If a consumer is satisfied with a product, she buys from the same firm with a high probability, but she may nevertheless occasionally like to try an alternative.[19]

We assume that the probability with which a switching consumer who considers all products in the market buys a product of firm i is

$$\frac{m_i(v - p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v - p_j)^{\frac{1}{\beta}}}, \tag{1}$$

with $v > 0$ and $\beta > 0$. First, the parameter v can be interpreted as the value of the

[16] This is in line with most papers on multi-product firms, such as Champsaur and Rochet (1989), Dewan et al. (2003), or Hamilton (2009).

[17] In Online Appendix D.2, we analyze the case in which firms differ in their customer base and show that our findings also hold in that case.

[18] At the end of this section, we state the assumptions on $c(r_i)$ that guarantee that the investment in customer retention is below 1 (which must hold as r_i is a probability).

[19] In Online Appendix D.3, we present an extension of the model in which an increase in r_i does not only help to retain existing consumers but also to attract switching consumers, and show that our qualitative results still hold.

market. From (1), the probability that a consumer buys a product of firm i is decreasing in p_i. However, if v is larger, this decrease is smaller.[20] Therefore, *ceteris paribus*, firms will set higher prices if v is larger, as consumers still buy higher-priced products with a sizable probability. This resembles markets that are more valuable.

Second, the parameter β can be interpreted as the degree of consumer heterogeneity. If β is close to zero, all consumers buy with a high probability the products from the firm that sets the lowest price. This is equivalent to the case in which consumers are relatively homogeneous in their preferences. Instead, if β tends to infinity, the distribution of how consumers buy products becomes almost uniform, and prices only play a minor role. This represents the case in which consumers are highly heterogeneous in their tastes, as different consumers then buy different products and decide mainly according to their preferences. For intermediate values of β, the model can be interpreted as one in which consumers decide partly based on the price and partly based on their preferences.

Turning to retained consumers, if a consumer repurchases from firm i, she chooses only among the products offered by firm i. The probability with which a retained consumer buys a particular product of firm i is then

$$\frac{(v-p_i)^{\frac{1}{\beta}}}{m_i(v-p_i)^{\frac{1}{\beta}}} = \frac{1}{m_i}.^{21} \tag{2}$$

Our model of consumer demand is a variant of a standard urn-ball matching function (see e.g., Petrongolo and Pissarides, 2001), where products take the role of balls. However, in contrast to the classic urn-ball matching function, the balls may have different matching probabilities (i.e., probabilities to attract consumers) and firms can influence the matching probability with the price they set.[22] Therefore, firms can attract consumers by a larger product variety and by a lower price.

To guarantee interior solutions, we assume that $\beta > 1$, that is, consumers exhibit some

[20] This can be easily verified by taking the cross-derivative of (1) with respect to p_i and v.

[21] In case firms can set different prices for their products, the probabilities (1) and (2) need to be modified to (denoting the price of product k of firm i by $p_{k,i}$)

$$\frac{\int_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\sum_{j=1}^{M}\int_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell} \quad \text{and} \quad \frac{(v-p_{k,i})^{\frac{1}{\beta}}}{\int_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell},$$

respectively.

[22] See Fainmesser and Galeotti (2021) for a similar formulation in a context where influencers in social media choose the quality of their respective channels, and an influencer probabilistically gains more followers when offering higher quality.

degree of heterogeneity. Otherwise, the second-order conditions with respect to product prices are not satisfied. In fact, if β is sufficiently small, no pure-strategy equilibrium exists. The reason is that the model would then be similar to clearinghouse models, such as Varian (1980) and Narasimhan (1988), in which some consumers are captive to each firm, whereas others buy from the firm with the lowest price. In our model, the captive consumers are those who are retained by a firm, and the non-captive ones are those who choose among all products. Although the mass of retained consumers is endogenous in our case, a similar mechanism as in clearinghouse models applies when consumers' preferences are very homogeneous.

We note that (1) implies that a consumer who is not retained directly by firm i may still buy one of firm i's products, as she chooses among all available products. This assumption is made to ease the exposition, but is not crucial for the result. In Online Appendix D.4, we consider a variant of the model in which consumers who are not retained by a firm do not buy a product from that firm, and show that all our results carry over.

Equilibrium Concept and Technical Assumptions. Since we consider a simultaneous game between firms (i.e., firms choose the variables product variety m_i, product price p_i, and customer retention r_i at the same time), our solution concept is Nash equilibrium. The assumption of simultaneous choices simplifies the presentation of the analysis and is reasonable in digital markets where products can be easily added or withdrawn, and changes in the functionality of products (e.g., via adaptation of the software code) can occur in a fast way, thereby making these choices not necessarily more long-term compared to price choices. However, our main effects carry over to a situation with sequential choices in which firms choose r_i and m_i before setting prices.[23]

Finally, to ensure that solutions are interior, we assume that the cost function for investment in customer retention is sufficiently steep and convex. Specifically, we assume that

$$c'(1) > \frac{v\,(M-1)}{M^2}, \quad c''(\cdot) > v\frac{(M-1)^2}{\beta M^3}, \quad \text{and} \quad c\left((c')^{-1}\,(v/M)\right) \text{ is sufficiently large.}$$

The first assumption ensures that $r_i < 1$ in equilibrium. The second assumption ensures that the Hessian matrix of second derivatives is negative definite at the equilibrium. Finally, the third assumption ensures that a global deviation is not profitable. In particular, this assumption implies that it does not pay off for a firm to focus purely on consumer retention, as the cost of such a strategy would be too high.[24] To simplify some of the proofs we also

[23]We show this in Online Appendix D.5.

[24]Formally, the assumption states that the cost of focusing only on consumer retention, where the optimal

assume that the third derivatives of $c(\cdot)$ and $f(\cdot)$ are either negative or, if they are positive, then small relative to the respective second derivatives. In Online Appendix D.6, we provide an example with concrete cost functions that naturally satisfy these assumptions.

3 Analysis and Results

In this section, we first solve for the Nash equilibrium of the game. We then analyze the interplay between investments in product variety and customer retention and determine how the number of products chosen by firms in equilibrium changes with the value of the market and the degree of heterogeneity.

Denoting by $\mathbf{m} = \{m_1, ..., m_M\}$, $\mathbf{r} = \{r_1, ..., r_M\}$, and $\mathbf{p} = \{p_1, ..., p_M\}$ the vectors of firms' product varieties, investments in customer retention, and product prices, respectively, the profit function of firm i is

$$\Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p}) = \frac{r_i p_i}{M} + \frac{\sum_{j=1}^{M}(1 - r_j)}{M} \left(\frac{p_i m_i (v - p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v - p_j)^{\frac{1}{\beta}}} \right) - f(m_i) - c(r_i).$$

The first term represents the revenue from consumers who firm i retains. Each firm has a symmetric customer base of mass $1/M$, and a consumer in this base buys again from firm i with probability r_i. The second term is the revenue from consumers who are not retained by firms. With probability $1 - r_j$, a consumer in the base of firm j potentially switches. The probability that such a consumer chooses a product of firm i is then given by (1). Finally, the third and the fourth term are the costs of investing into product variety and customer retention, respectively.

Firm i maximizes $\Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})$ with respect to m_i, r_i, and p_i. The solutions are implicitly

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retention level is given by the inverse of the marginal cost function evaluated at v/M, is sufficiently high. A somewhat more precise, but unwieldy and only sufficient condition is

$$c\left((c')^{-1}\left(\frac{v}{M}\right)\right) >> c\left((c')^{-1}\left(\frac{v\beta(M-1)}{M(\beta M + M - 1)}\right)\right),$$

which implies that the cost when focusing only on customer retention is substantially larger than the lowest possible cost in a symmetric equilibrium.

characterized by three first-order conditions, which are given by[25]

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i} = \frac{\sum\limits_{j=1}^{M}(1-r_j)p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\,i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0, \tag{3}$$

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial r_i} = \frac{p_i}{M}\left(1-\frac{m_i(v-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right) - c'(r_i) = 0, \tag{4}$$

and

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial p_{k,i}} = \frac{r_i}{M} + \frac{\sum\limits_{j=1}^{M}(1-r_j)}{M\beta}\frac{m_i(v-p_i)^{\frac{1-\beta}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\left[\beta(v-p_i)-p_i+\frac{p_i m_i\,(v-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right] = 0. \tag{5}$$

The first-order conditions with respect to m_i and r_i—i.e., (3) and (4)—show the trade-off a firm faces when increasing either of the two variables: both an increase in m_i and an increase in r_i raise the demand of firm i (first term in (3) and (4), respectively), but the increase is costly for the firm (second term in (3) and (4), respectively). An important difference between the two strategic variables is that they work in different ways. In particular, investment in customer retention helps a firm to keep a larger mass of its customer base, while raising product variety increases the probability that switching consumers buy from firm i, regardless of whether they belong to the customer base of firm i or of its competitors. Specifically, the first term in (4) is a multiple of firm i's customer base, $1/M$, whereas the first term in (3) is a multiple of the mass of all firms' switching consumers, $\sum\limits_{j=1}^{M}(1-r_j)/M$.

Turning to the first-order condition with respect to price—i.e., (5)—by increasing the price of one of its products, firm i obtains a higher profit from its retained consumers, which is expressed in the first term of (5). Instead, for switching consumers, firm i faces the standard trade-off that a higher price induces fewer consumers to buy from the firm but leads to a larger profit from those consumers who still find one of the firm's products most attractive. This trade-off displays in the second term of (5), which contains both positive and negative elements.

We first determine how investment in customer retention affects a firm's product variety choice, both between firms and within a firm. This leads to the following result:

[25]In Online Appendix C.2, we demonstrate that the second-order conditions for profit maximization are satisfied and that the solutions of the first-order conditions indeed constitute a global maximum.

Result 1. *Holding all other variables constant, an increase in the investment in customer retention, either by a competitor or the own firm—i.e., firm i—leads to a decrease in firm i's product variety, that is,* $dm_i/dr_j < 0$ *and* $dm_i/dr_i < 0$.

The intuition for the result is that investment in product variety and investment in customer retention target different consumer groups.[26] While investment in customer retention allows a firm to keep a larger mass of its existing customer base, a larger product variety aims at attracting more switching consumers. If competitors invest more in customer retention, consumers are less likely to switch. Therefore, a larger product variety is *less* profitable. The same intuition holds if a firm invests more in customer retention on its own, as this also leads to a smaller mass of switching consumers, and the firm therefore benefits less from increasing its product variety.[27]

Result 1 determines the effect of customer retention on product variety, holding all other variables fixed. We now turn to the question of how *in equilibrium*—i.e., when all decisions of firms are taken into account—product variety is affected by the market environment. In the unique symmetric equilibrium of the game, that is, $m_1^* = \cdots = m_M^*$, $r_1^* = \cdots = r_M^*$, and $p_1^* = \cdots = p_M^*$, we denote the equilibrium product portfolio size by m^*, the equilibrium consumer retention level by r^*, and the equilibrium product prices by p^*.[28] These equilibrium values are implicitly characterized by the first-order conditions, which, due to symmetry, can be simplified to[29]

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2} - f'(m^*) = 0, \quad \frac{(M-1)p^*}{M^2} - c'(r^*) = 0, \tag{6}$$

and

$$M\beta\,(v-p^*) - (M-1)p^*\,(1-r^*) = 0. \tag{7}$$

Using these equations, we can now perform comparative-static analyses to determine how the equilibrium product variety $m^\star$ changes with the market environment. We first consider a change in v—i.e., the parameter that measures the market value:

Result 2. *The relation between market value* v *and the equilibrium number of products is non-monotonic, i.e.,* $m^\star$ *is increasing in* v *for low values of* v *(i.e.,* $\partial m^*/\partial v > 0$*) and*

[26]We note that the result determines the relation between product variety and consumer retention, keeping prices fixed. It holds for any constant prices and not just e.g. equilibrium prices.

[27]The latter result depends on the assumption that consumers who are not (directly) retained by firm i still buy one of firm i's product with positive probability. If this would not occur, then $dm_i/dr_i = 0$. However, the relationship $dm_i/dr_j < 0$ still holds.

[28]We discuss the possibility of asymmetric equilibria in Online Appendix C.1.

[29]In Online Appendix D.6, we provide an example with concrete cost functions that allows for a closed-form solution of the model.

decreasing for high values of v (i.e., $\partial m^*/\partial v < 0$).

The result shows that a more valuable market may induce firms to offer a lower variety of products. This is potentially counter-intuitive: if a market is more valuable, firms usually find it more profitable to offer a larger number of products. In particular, if a market is more valuable, the equilibrium price and therefore also the equilibrium profit per product is larger, as p^* is increasing in v. Since the cost of introducing an additional product is not affected by v, this implies that—*ceteris paribus*—firms will optimally expand their product range.

However, in our model there is a strategic countervailing force. In a valuable market segment, each firm has a strong incentive to retain its consumers. It will therefore invest more in consumer retention. This implies that fewer consumers switch away from the firms, which also helps the firm to charge higher prices. Since all firms invest more, the number of switching consumers falls, which implies that each firm can only attract fewer consumers with its product variety. As a consequence, firms respond with a reduction in the number of products they offer. The result therefore occurs because of the interplay between investments in consumer retention and in product variety. In fact, if r_i was fixed for all firms $i \in \{1, ..., M\}$, an increase in v would unambiguously lead to an increase in the number of products. However, this result no longer holds if consumer retention is chosen endogenously.

Result 2 shows that the countervailing effect dominates if v is sufficiently large, leading to an inverted u-shaped relation between m^* and v. The intuition is as follows: If v is large, firms optimally invest a lot to retain a large portion of their consumers. Consequently, investing in a greater number of products does not pay off much. By contrast, if v increases starting from a low level, the effect that the market segment becomes more valuable dominates; hence, each firm optimally increases both its product portfolio size and its investment in consumer retention.

Second, we consider the relation between product variety and consumer heterogeneity β:

Result 3. *The relation between the degree of consumer heterogeneity β and the equilibrium number of products is positive, that is, $m^\star$ is increasing in β (i.e., $\partial m^*/\partial \beta > 0$).*

Result 3 shows that an increase in the heterogeneity of consumer preferences induces firms to expand their portfolio size. Indeed, consumer heterogeneity has a positive effect on product variety via two channels. First, an increase in the number of products generally helps a firm to better cater to consumer tastes. If consumers are more heterogeneous, this effect is more pronounced, which implies that expanding the product portfolio size becomes more profitable. Second, competition between firms is dampened when consumers' tastes are more heterogeneous as the role of prices for competition is diminished. This results in larger revenues and therefore also renders the introduction of additional products profitable.

Due to the latter effect—i.e., prices are larger in equilibrium as consumer heterogeneity increases—firms also invest more in consumer retention. In particular, as prices are higher, retaining a consumer from the customer base is more valuable, which induces firms to invest more in retention. Although this affects product variety in a negative way, the effect is dominated by the two effects described above. Hence, the relation between product variety and consumer heterogeneity is positive in equilibrium.

Finally, we determine how the value of the market and the degree of consumer heterogeneity affect the relationship between consumer retention and product variety *in equilibrium*. To do so, we determine the effect of v and β on $dm_i^*/dr_j^*(< 0)$ and $dm_i^*/dr_i^*(< 0)$.

Result 4. *In equilibrium, the negative effect of consumer retention on product variety is reinforced by an increase in market value v as well as customer heterogeneity β. This holds both between firms and within a firm, that is, $\partial^2 m_i^* / (\partial r_z^* \partial v) < 0$ and $\partial^2 m_i^* / (\partial r_z^* \partial \beta) < 0$, with $z \in \{j, i\}$.*

Result 4 shows that the moderating effect of market value and consumer heterogeneity on the negative relation between consumer retention and product variety is to strengthen this relationship. The intuition is as follows: Both an increase in market value and in consumer heterogeneity allow firms to charge higher prices and to obtain higher revenues. This implies that investing in consumer retention becomes more profitable. The absolute values of the derivatives dm_i^*/dr_j^* and dm_i^*/dr_i^* are then larger. Hence, increases in market value and consumer heterogeneity reinforce the negative relationship between consumer retention and product variety.

Empirical Predictions

We now briefly state the empirical predictions that arise from our theoretical analysis, which we test in the next sections using data from video games. From Result 1, a central effect in our model is that an increase in investment in consumer retention leads to a fall of the investment in product breadth, which can be translated into the following hypothesis:

Hypothesis 1. *There is a negative relation between investment in consumer retention and product variety.*

Results 2 and 3 determine how product variety changes with the value of the market and the degree of consumer heterogeneity. Result 2 states that the value of a market segment affects the equilibrium product portfolio size in a non-monotonic way and can be translated into the following hypothesis:

Hypothesis 2. *The effect of a higher market value on product variety follows an inverted u-shape.*

Result 3 relates the equilibrium product portfolio size to the extent of consumer heterogeneity:

Hypothesis 3. *The effect of a larger degree of consumer heterogeneity on product variety is positive.*

Finally, Result 4 presents the moderating effect of the market value and the degree of consumer heterogeneity on the relationship determined in Result 1:

Hypothesis 4. *The effect of a higher market value or a larger degree of consumer heterogeneity on the relation between investment in consumer retention and product variety is negative.*

4 Data

We test our theoretical predictions using data from the video game industry. In this section, we describe the data set and explain how the variables are mapped from the theory to the empirics along with a provision of descriptive statistics.

4.1 Data Collection and Sample

To assemble our data set, we scraped Steam, the leading platform for online distribution of video games, in August 2022.[30] We collected information on all games listed on the Steam store at that time.[31]

This generated a data set, which we in turn validated by consulting several external sources (SteamDB, SteamSpy, VG Insights) that reported a similar number of games on Steam. This gives us reassurance that our sample indeed constitutes the universe of games. As a next step, we drop video games with missing information on our core variables comprising publisher identity, price, genre, and release date along with games having implausible release dates like ones that lie in the future entirely.[32] In addition, we restrict our sample

[30]See `https://store.steampowered.com/search/?sort_by=Price_ASC&category1=998`. We retrieved the corresponding information from Swiss IP addresses.

[31]In our robustness checks (Section 5.2), we study possible concerns about the underlying data and definitions.

[32]In this step, we drop 11,993 games from our initial 61,427 games.

to games born in the year of 2012 or later.[33] This leaves us with 47,435 video games for our analyses.

4.2 Variables Measurement

Our data set contains either direct measures of the core variables of our model, product portfolio size, retention, segment value, and heterogeneity, or proxies thereof.

Product variety as our first core variable is directly measured by the number of products of a publisher in a market segment in a given release year. We take the publisher as a our focal "producer" making "product portfolio decisions" (see Ozalp and Kretschmer, 2019).[34] We restrict the measure to one year to better reflect the short-lived product cycles in the digital sphere, as in Ozalp and Kretschmer (2019).

The second core variable of our theoretical analysis is retention, which is proxied by the number of updates by a video game on Steam, as motivated in Section 1.

We define segments following Rietveld and Ploog (2021) who also use the Steam classification to distinguish between nine different game genres: action, adventure, casual, massively multiplayer, racing, RPG, simulation, sports, and strategy. This maps into 98 segment-years. In the following, we often use the term "segment" for segment-years to ease readability.

The third core variable is the value of a market segment, which we do not directly observe for games. As an approximation for value, we use the price of the game, since prices should be higher in more valuable segments, deeming both variables to be highly correlated. Indeed, in our theoretical model, equilibrium prices increase in the value of the market v.

Our last core variable from the theoretical model is heterogeneity, for which we exploit user-defined tags of a game. First, we look for the most popular tags in a segment-year (after excluding the segment itself). Second, we define heterogeneity as the share of games within a segment-year not having the most popular tag among the first five (and thereby most important) user-defined tags. This definition is motivated by related research, which defines mainstream (and relatedly niche) based on the popularity of tags a product comes with or gets (see Kerkhof, 2023 as well as Sun and Zhu, 2013).

Table 1 provides an overview of how we empirically measure or proxy the core variables of our theoretical model.

Importantly, our analysis is at the segment-level and aggregates the information per publisher. Our dependent variable is thus the number of products of the own publisher in

[33] This leads to a drop of another 1,999 games.

[34] We elaborate on the distinction between publisher and developer and run robustness analyses in Section 5.2. We note that in two-third of our video games, the two coincide.

Table 1: Mapping Theory to Empirics

		Empirics
	Construct	Games
Theory	Variety	# Own Products
	Retention	# Updates
	Value	Price
	Heterogeneity	% w/o Popular Tag 1

the respective segment and year. The explanatory variables are aggregated to the segment-level by taking averages, implying that dummy variables are translated into shares.

In the regression analyses, we additionally control for the month of release and the year of release as well as segment dummy variables (see Rietveld and Ploog, 2021). We also apply a series of robustness checks in Section 5.2, which involve taking transformed variables, alternative key measures, and more covariates, which we introduce in the following subsection.

4.3 Descriptive Statistics

Table 2 displays descriptive statistics for our four core variables. Portfolio size is low on average with a median of 1. There is, however, substantial variation with the maximum number of products by the publisher in the respective segment-year being 90. Regarding retention, video games have more than 10 updates on average with a large range from 0 to over a thousand. There is also substantial variation in our proxy for segment value. For games, mean segment-specific prices range between 0 and 415CHF ($\approx$ 415USD) with a mean of around 7.42 and a median of 4.64. Heterogeneity is spread across a range from 0 to 1 with a mean and median of a bit more than 0.5. In addition to these four core variables, we also have the previous number of games by the publishers, ratings (if any present), whether the game has a website, and the respective size of the game (if observed). Regarding the number of previous games as an experience measure, we see that the majority of publishers has no previous games (the median is 0), while a few publishers have as many as 307. For ratings, more than 3 out of 4 ratings are on average positive and the amount of ratings is skewed with a mean of more than 2 thousand compared to a median of 59. Finally, the majority of games has a website and the average size is more than 3MB.

Table 2: Descriptive Statistics

	Mean	Median	Min	Max	N
# Own Products	1.24	1.00	1.00	90.00	38387
# Updates	10.27	2.50	0.00	1064.00	38353
Price	7.42	4.64	0.00	415.99	38387
% w/o Popular Tag 1	0.57	0.55	0.00	0.94	38387
Prev. # Own Products	0.79	0.00	0.00	307.00	38387
% Positive Ratings	76.41	80.00	0.00	100.00	23088
# Ratings	2283.12	59.00	10.00	6.6e+06	23088
Website Dummy	0.55	1.00	0.00	1.00	38387
Size (in KB)	3128.29	800.00	0.64	8.4e+06	36358

Notes: Observations are on a publisher segment-year level. Rating measures are missing if there are no ratings. Missings in size are due to inconclusive values.

5 Baseline Results

5.1 Baseline Estimations

In order to test our empirical predictions, we first run simple OLS regressions relating portfolio size to our three measures approximating retention, value, and heterogeneity. Given the skewness of some core variables, we apply the inverse hyperbolic sine (IHS) transformation instead of the still more popular logarithmization since it does not imply adding an arbitrary positive number to the zeroes in the data, while still retaining the more tractable interpretation of the log transformation (Bellemare and Wichman, 2020).

Table 3 shows the results, where in columns 1 to 3 each core variable is included separately, while controlling for segment, year, and month fixed effects in all specifications.[35] The respective coefficients show the hypothesized direction and are statistically significant.[36] In column 4, we include all of our measures in one specification and the results remain. The patterns amplify once we exclude publishers with only one observation in column 5 of Table 3 as these may be less professional. As a result, Hypotheses 1 to 3 are supported by the empirical evidence. Table 6 (in Online Appendix E) repeats the baseline regressions without a transformation and shows that the results are not sensitive to the IHS transformation.[37]

In order to test Hypothesis 4, we augment the baseline regressions by interacting the retention variable with value and heterogeneity. Table 4 shows the corresponding results, where the baseline coefficients from Table 3 remain. In columns 1 and 3, the interaction

[35]The results remain irrespective of taking the mean or median for the month fixed effect.

[36]Normalizing the update measure by dividing through the age of a game yields similar results.

[37]Since the dependent variable can be interpreted as a count variable, we also employ a negative binomial approach that leads to similar results.

Table 3: Baseline Estimations: H1-H3

	# Own Products				
# Updates (ihs)	-0.012***			-0.014***	-0.027***
	(0.001)			(0.001)	(0.002)
Price (ihs)		0.155***		0.117***	0.236***
		(0.011)		(0.011)	(0.025)
$Price^2$ (ihs)		-0.073***		-0.051***	-0.109***
		(0.006)		(0.006)	(0.013)
% w/o Popular Tag 1 (ihs)			0.050*	0.051*	0.112***
			(0.027)	(0.027)	(0.043)
Constant	0.960***	0.871***	0.909***	0.860***	0.767***
	(0.019)	(0.019)	(0.025)	(0.025)	(0.035)
Segment FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
No. of Obs.	38353	38387	38387	38353	17717

Notes: Column 5 only includes publishers with more than one observation. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

between updates and prices suggest a statistically significant and negative relationship. The same holds for the interaction effect between updates and the share of games without the most popular tag, reported in columns 2 and 4. To conclude, both interaction effects have the hypothesized sign and are statistically significant. Hence, Hypothesis 4 is supported as well.

5.2 Robustness

In order to check the robustness and sensitivity of our OLS baseline estimations, we employ a series of auxiliary analyses. They comprise more control variables, sample restrictions, and different sets of fixed effects. The results remain qualitatively unchanged throughout the different specifications. Finally, we also address some concerns about the underlying data. We relegate the robustness check results to Online Appendix E.

Table 7 repeats the baseline regressions and adds control variables with the caveat of less observations due to missings in the extra covariates. Specifically, we include the previous number of products of a publisher as a proxy for experience. We also include the number of ratings and the share of positive ratings, thus approximating popularity and quality. Moreover, we account for the level of professionalism as well as the complexity through the presence of a website and the code size.

Table 8 employs different fixed effects. In column 1, we additionally include Segment x Year fixed effects, i.e., interactions between both variables. The interactions make the

Table 4: Baseline Estimations: H4

	# Own Products			
# Updates (ihs)	-0.006***	-0.005**	-0.019***	-0.010**
	(0.001)	(0.002)	(0.003)	(0.005)
Price (ihs)	0.129***	0.119***	0.250***	0.238***
	(0.011)	(0.011)	(0.025)	(0.025)
# Updates (ihs) × Price (ihs)	-0.004***		-0.003***	
	(0.001)		(0.001)	
$Price^2$ (ihs)	-0.053***	-0.052***	-0.113***	-0.110***
	(0.006)	(0.006)	(0.013)	(0.013)
% w/o Popular Tag 1 (ihs)	0.051*	0.085***	0.111***	0.181***
	(0.027)	(0.028)	(0.043)	(0.047)
# Updates (ihs) × % w/o Popular Tag 1 (ihs)		-0.018***		-0.035***
		(0.004)		(0.008)
Constant	0.842***	0.840***	0.750***	0.728***
	(0.026)	(0.026)	(0.036)	(0.037)
Segment FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes
No. of Obs.	38353	38353	17717	17717

Notes: Columns 3 and 4 only include publishers with more than one observation. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

heterogeneity measure statistically insignificant. In column 2, we include publisher fixed effects and our initial patterns remain, though statistically less significant. The weaker significance in both cases is most likely due to the lack of variation.

Having only a snapshot from August 2022 for the underlying data raises multiple concerns. First, the year of 2022 is incomplete and thus we re-run our analysis without games from 2022 and get the qualitatively almost identical results. Second, having data from one period in time may lead to missing firms (and products) from the past that exited until that date. We address this so-called survivorship bias in Online Appendix E.2 and show that the vast majority of video games stays in the market. In that appendix, we also discuss and show that our results do not depend on choosing publishers instead of developers as the unit of observation.

6 Identification

6.1 Motivation

Identification problems loom large in our empirical specification, since all three explanatory variables comprising retention, segment value, and heterogeneity are either endogeneous via our theoretical model or because we suspect correlation with the error term.

Factors that are important for consumer retention and value of a segment (e.g., segment popularity) usually also determine portfolio size, which leads to an endogeneity problem that is also reflected in the theoretical model—i.e., portfolio size and retention level are a function of segment value. Hence, we employ an instrumental variable approach that exploits exogenous variation. This requires us to find suitable instrumental variables—i.e., variables that are correlated with the endogenous variables ("relevance") but uncorrelated with the error term of our estimating equation ("exogeneity"). Given that the squared instrumental variables for value are appropriate instruments for value squared, only two instruments are necessary.

Additionally, the heterogeneity of consumers in a segment is likely to be endogenous via the correlation with the error term of our equation of interest as potentially unobserved (to us) factors that drive portfolio size will be correlated with heterogeneity. Therefore, we try to include relevant factors for heterogeneity and assess how much unobserved heterogeneity is necessary to rule out a statistically significant relationship.

Exchange rates as an instrument for prices: Prices (as a proxy for value of a segment-year) are strategic variables and thus simultaneity concerns arise with the decision of portfolio size. In the past, exchange rates have been used frequently as instruments for prices, also in video games settings (see Kretschmer and Claussen, 2016). The idea is that fluctuations in exchange rates may act as a cost shifter for prices as many publishers do not reside in the U.S. but are paid in USD given Steam's pricing policy.[38]

Indeed, there are anecdotes of countries with higher inflation (e.g., Turkey and Argentina), where both game publishers as well as Steam reacted by adjusting prices.[39] This motivates the "relevance" criterion of exchange rates as an instrument, i.e., a correlation with the endogenous variable. As for "exogeneity", we hypothesize that variations in the exchange rates affect portfolio size decisions only through the channel of price changes.

Still, it begs the question, which exchange rates are relevant for a publisher as this heavily depends on its location and the location of its users. In order to locate a publisher, we use the languages supported by the video game. We hypothesize that catering to a specific user group necessitates the support of its language, and this support is more likely if the developer itself resides in that country. The basic approach involves 1) taking all supported languages for the interface of a game, 2) identify the biggest countries that speak those languages, and

[38]See `https://partner.steamgames.com/doc/store/pricing`.

[39]See `https://www.vgchartz.com/article/456957/playstation-increases-steam-game-prices-in-several-countries/` and `https://www.pcgamer.com/steam-is-making-it-harder-to-get-cheap-games-from-other-regions/`.

3) take the yearly average exchange rate of the currency the countries have to the USD. The detailed procedure is provided in Online Appendix E.3.

Diffusion of update tools as an instrument for updates: Investment in customer retention and, hence, the propensity to update is also a strategic variable that is decided jointly with portfolio size. The diffusion of technologies that specifically reduce the costs to update may sway developers to update more. In general, there are Software Development Kits (SDKs), which are standardized tools that facilitate software development of particular functionalities and decrease the corresponding costs (see Miric et al., 2023). We are interested in SDKs that particularly decrease costs to produce updates, which in turn generates incentives for publishers to update games more frequently. Hence, we leverage differences in the adoption and diffusion of the particular technology over time and segments (see Miric et al., 2023). We scrape information on the SDKs from SteamDB.[40] Specifically, we look at the SDK called SteamWorks, which helps along the whole production chain from testing a game—i.e., before deploying the update—uploading to Steam, to even posting patch notes.[41] Having the SDK makes updates easier and thus we expect a positive correlation with the number of updates, thereby motivating the "relevance" criterion. With respect to "exogeneity", since we are focusing on an SDK that particularly helps with updates, we hypothesize that the effect on portfolio size is only governed through the variation in updates.[42]

Regression sensitivity of consumer heterogeneity: The last explanatory variable motivated by our theoretical model is consumer heterogeneity, approximated by the share of games without the most popular tag. This measure is not endogenous in the theoretical model, but potentially endogenous unobserved heterogeneity might affect both this variable and the error term of our equation of interest. An obvious approach would be to include all relevant factors, and, indeed, we account for segment-specific effects already through segment fixed effects in the baseline regressions along with other fixed effects specifications in the robustness checks (see Section 5.2). Still, there might be unobserved factors partly determining the customer base. To assess the degree to which this possible correlation might influence our parameter estimates, we use the kinky least square estimator (KLS, Kripfganz

[40] We retrieve the corresponding list of games (see `https://steamdb.info/tech/`) to relate it back to our sample.

[41] See `https://partner.steamgames.com/doc/sdk/updating`. The tool also offers other services to engage with customers through Steam, which makes it even more attractive to adopt when following a retention strategy.

[42] This is backed up by anecdotal evidence on reddit, see `https://www.reddit.com/r/gamedev/search/?q=steamworks`.

and Kiviet, 2021).[43]

The basic idea behind this estimator is to allow for a range of correlations between the potentially endogenous variable and the error term of the equation on interest. Since KLS assesses the possible bias of the OLS estimates conditional on a given degree of correlation on a grid, this bias is best analyzed by a plot of coefficient estimates generated under varying degrees of correlation. The corresponding search grid over these correlations in principle ranges from -1 to 1, but both extreme values make little intuitive sense in real world applications. Kripfganz and Kiviet (2021) indeed consider an interval of [-.75 and .75] in their empirical examples. Moreover, economic theory may help to finding out whether the correlation of the error term and the endogenous variable is positive or negative, as will be outlined in the following section on results.

6.2 IV and Regression Sensitivity Results

We start by presenting results from our instrumental variable estimations and then proceed with results from sensitivity checks of the baseline regression.

Instrumental variable results: Table 5 reports IV estimation results. Columns 1 and 3 show the first stage for instrumenting updates with the share of games by a publisher equipped with the SteamWorks SDK and prices with exchange rates, respectively.[44] In Subsection 6.1, we provided arguments why our set of instruments should be exogenous and also motivated their relevance. In Table 5, we now provide tests on the relevance criterion. We can see that both types of instruments are highly correlated with the corresponding endogenous variable. The relevance is properly assessed by F tests for joint significance of the instruments in the first-stage equations, which we display at the bottom of Table 5 in columns 1 and 3. Our F tests are all substantially larger than the rule-of-thumb value of 10 suggested by Staiger and Stock (1997) and also substantially larger than the more conservative critical value of 104.7 advocated for more recently by Lee et al. (2022). We also find the instrumental variable estimations to be statistically different from the baseline OLS results (as displayed by the p-values of the endogeneity test), suggesting the coefficients in the OLS to be endogenous conditional on our choice of instruments. Finally, we see that in each second stage (columns 2 and 4), the results from our baseline estimations

[43] Another approach to bound the causal effect of an endogenous variable are 'Oster (2019) bounds', which are a special case of the KLS model (i.e, they are nested within KLS). Oster (2019) bounds make, however, strong assumptions on the relative correlation of unobserved and observed heterogeneity, which is why we resort to KLS instead.

[44] Using the share of games with the SDK in the segment also works as an instrument, but is weaker due to less variation.

hold after instrumenting. Table 9 in Online Appendix E.4 additionally shows an approach to instrument the price and its squared term, thereby confirming both the relevance and necessity of exchanges rates as an instrument, while the baseline results—including the inverted u-shaped relationship—remain qualitatively unchanged.

Table 5: Instrumental Variable Estimations

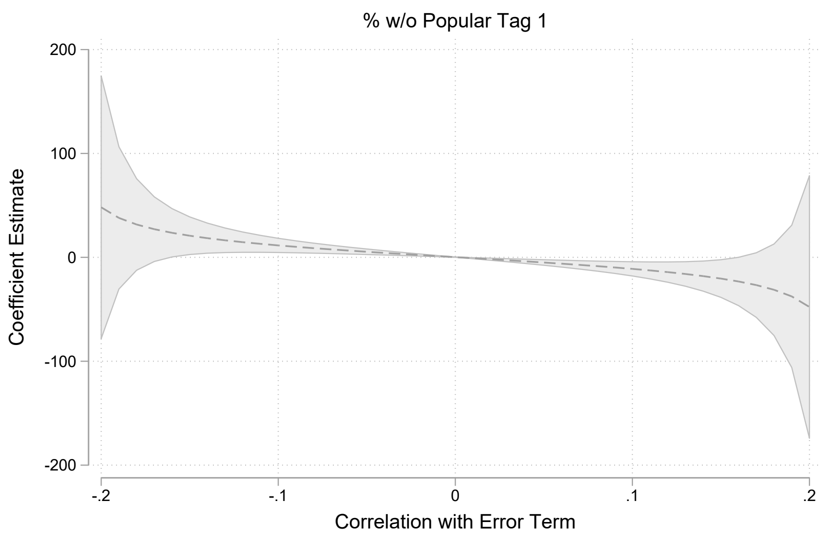
	1st Stage # Updates	2nd Stage # Own Products	1st Stage Price	2nd Stage # Own Products
# Updates		-0.014***	0.031***	-0.002***
		(0.004)	(0.002)	(0.000)
Price	0.572***	0.014***		0.032***
	(0.022)	(0.003)		(0.007)
$Price^2$	-0.002***	-0.000***		
	(0.000)	(0.000)		
% w/o Popular Tag 1	13.359***	0.379***	0.525	0.198*
	(2.570)	(0.131)	(0.772)	(0.115)
% w/ SteamWorks	8.266***			
	(0.774)			
Avg. Exchange Rate to USD			-5.243***	
			(0.231)	
Constant	4.353*	0.967***	13.390***	0.726***
	(2.587)	(0.120)	(0.820)	(0.126)
Segment FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes
1st Stage F-Test	114.14		514.20	
Endogeneity Test p-value	0.00		0.00	
No. of Obs.	38353	38353	38353	38353

Notes: Columns 1 and 3 represent first-stage regressions, while columns 2 and 4 show the second-stage regressions. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Regression sensitivity results: Combinations of the possible correlations of the endogenous variable "heterogeneity" and the error term as well as the resulting estimate on the coefficient of interest are shown in Figure 1, along with their corresponding confidence bounds.

The figure displays the coefficient estimates on the heterogeneity variable if we let the correlation with the error term vary in the range [-.2,.2]. In fact, the analysis actually does not consider correlations much beyond that range because confidence bounds become very large for higher correlations, which is a direct consequence of the relative imprecision with which the coefficient is estimated given its low variation within and across observations. It is clear, however, that our parameter of interest is positive over the whole range of negative correlations we deem to be economically meaningful. It does, however, become statistically insignificant at the 95 percent level if the correlation is below -.17. Clearly, the estimated coefficients for the variable become excessively large even for small values of correlations. The economic effects of the theory-motivated variables are, however, of secondary order since the theoretical model only predicts effect signs and is uninformative regarding effect sizes. Overall, the KLS robustness check indicates that the sign of the coefficient remains positive as in the main model even for higher values of possible negative correlations, indicating that the true coefficient estimates indeed are positive, which is consistent with our theoretical model.

Figure 1: Regression Sensitivity of Heterogeneity



7 Conclusion

This paper studies the interaction between the strategic choices of investment in product breadth and investment in customer retention, theoretically and empirically. The relation between these two choices is particularly important in digital markets, as for digitized products the offered variety can be increased at relatively low costs and consumers frequently purchase goods, which makes customer retention an important dimension of a firm's demand.

We first develop a novel theoretical model in which firms in an oligopoly have three strategic variables: investment in customer retention, investment in the mass of offered products, and pricing of the offered products. Although analyzing such a multi-dimensional game can be challenging, the model is highly tractable and provides new result. First, we show that the relation between customer retention and product breadth is negative—i.e., the two types of investments are substitutes. While customer retention aims at keeping existing consumers, a larger product line helps to increase demand from switching consumers. We also show how the market environment affects a firm's product variety choice. We find that the value of a segment has a non-monotonic effect on product variety—i.e., there is an inverted u-shaped relationship—which occurs genuinely because of the interplay between product variety and customer retention. Moreover, we show that the degree of consumer heterogeneity has a positive effect on product variety, and that both market value and consumer heterogeneity exacerbate the negative relation between product variety and consumer retention.

In our empirical analyses, we test the predictions of our theoretical model using a data set from the video games industry. The data is rich in information and bears a lot of heterogeneity in all relevant dimensions, thus providing us with either direct measures or at least proxies for our key variables. The empirical methods involve basic OLS regressions, a series of robustness checks, as well as dealing with endogeneity problems through an instrumental variable approach and regression sensitivity analyses. We find a negative relationship between product variety and consumer retention, which demonstrates that the two variables are substitutes. We also find an inverted u-shaped relationship between segment value and product variety, while customer heterogeneity is positively related with product line length. Therefore, there is strong empirical support for the hypotheses from the theoretical model, which emphasizes the relevance of the model's novel effects. Moreover, our empirical analysis is the first to document a trade-off between product updates and product breadth.

An important managerial implication of our analysis is that investment in product breadth and investment in customer retention are substitutes. Therefore, these two investments should not be considered in isolation. Specifically, the relation between the two variables is negative, because customer retention strategies mainly increase the repurchasing rate, whereas a larger product line helps to attract new consumers. Consequently, if, for instance, a manager contemplates about reducing the product line (e.g., to save costs), a profitable complementary strategy could be to increase measures that make consumers more satisfied with their existing products, such as providing better functionality of products or an increased updating frequency.

Another implication is that the optimal adjustment in product variety to changing market conditions can be opposite to first-glance intuition; therefore, such decisions should be made with care. Suppose that a market segment becomes more valuable (e.g., because of fashion). The standard reaction would most likely be to increase product portfolio size. However, a more profitable response might be to invest in customer retention. This strategy is even more valuable if competitors react in the same way and fewer consumers are willing to switch.

We conclude by discussing some avenues for future research, both theoretical and empirical. In the theoretical analysis, we consider a static model to keep the analysis simple (i.e., each firm is endowed with an exogenous customer base). It might be interesting to consider a dynamic model, which explicitly analyzes how the possibility of gaining demand in several periods shapes the incentives to invest in portfolio size and in customer retention. This could provide meaningful insights into the dynamic interplay between the two decisions. Second, we consider a simple demand structure by assuming that products are differentiated in a symmetric way. A richer framework could allow for vertical differentiation between products, implying that some products receive more demand than others.

In the empirical analysis, it would be interesting to study whether our results extend to other (e.g., non-digital) markets, and how they potentially need to be modified. This could provide insights into determining how traditional markets and digital markets differ with respect to the interplay between product variety and customer retention. In addition, in the video game industry, consumers repeatedly buy or download games, which is in line with our theoretical model. In other markets, the repurchasing decision occurs less frequently. An interesting question is how findings might differ for such markets.

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The Interplay Between Product Variety and Customer Retention: Theory and Evidence Online Appendix

This Online Appendix consists of five appendices (A-E).

Online Appendix A presents details on the analyses of the annual reports of NASDAQ 100 firms (pp. 2-7). It first explains the keyword analysis (Online Appendix A.1) and then the text analysis (Online Appendix A.2).

Online Appendix B presents the proofs of Results 1-4 (pp. 8-12).

Online Appendix C provides technical results (pp. 13-21). First, it shows the optimality of symmetric prices and discusses potential multiplicity of equilibria (Online Appendix C.1). Second, it demonstrates that the first-order conditions are sufficient (Online Appendix C.2).

Online Appendix D presents several model extensions (pp. 22-39). Online Appendix D.1 considers an extended model in which the customer base of each firm is endogenous. Online Appendix D.2 analyzes the case of asymmetric firms. Online Appendix D.3 extends the model to the situation in which investment in customer retention not only has a positive effect on the repurchase probability but also on attracting switching consumers. Online Appendix D.4 considers the case in which unsatisfied consumers do not buy from the same firm. Online Appendix D.5 presents a model with sequential instead of simultaneous decisions where investments in product variety and customer retention precede the pricing decision. Online Appendix D.6 considers a concrete example with specific cost functions, which allows to obtain a closed-form solution.

Finally, Online Appendix E provides details on empirical robustness checks (pp. 40-45). It presents additional OLS estimation tables (Online Appendix E.1), an explanation of further robustness checks (Online Appendix E.2), details on the usage of exchange rates as an instrument (Online Appendix E.3), as well as an additional IV estimation table (Online Appendix E.4).

Online Appendix A: Details on Text Analyses

The goal of the text analyses is to understand systematically whether there is evidence for both the importance of the two strategic variables product variety and customer retention as well as their interplay being mentioned and talked about specifically in firms' annual reports.

Sample: In the following text analyses, we consider the annual reports filed by firms publicly listed in the NASDAQ 100 as of the beginning 2024. These reports are usually called "10-K". The stock symbols assigned to the above companies are retrieved from NASDAQ's website and then matched to the central index key (CIK) uniquely identifying companies in the documents filed on the EDGAR platform provided by the US Securities and Exchange Commission (SEC).[45]

Online Appendix A.1: Keyword Analysis

The data for the keyword analysis is based on the 10-X[46] Document Dictionaries provided by the Software Repository for Accounting and Finance at the University of Notre Dame.[47] The corresponding file contains counts of words from a dictionary for each 10-X filing of the last 30 years and allows a simple keyword analysis without going through the annual reports themselves. We take the so-called Loughran-McDonald Master dictionary and infer our words of interest "retention" (ID: 64349) and "variety" (ID: 82824) along with alternative strategic variables of a company as a benchmark ("quality": 60038, "version": 83137 & 83139).[48]

As outlined above, we restrict attention to NASDAQ 100 firms, of which 95 are included in the database.[49] Additionally, we restrict attention to the last 20 years, as this gives us sufficient coverage with at least 60 firms in each period. For each firm and period, we look for the count of the respective word. Accordingly, we can measure both the share of annual reports containing the word at least once as well as how often the word comes up on average across the annual reports. Figure 2 shows the corresponding graphs over time and aggregated across firms. The upper panel shows that "quality" is always mentioned, while the words "variety" and "retention" experience a growth in importance over the years as opposed to "version", which roughly remained at its initial level that was comparable to "retention" at

[45] See `https://www.nasdaq.com/market-activity/quotes/nasdaq-ndx-index` and `https://raw.githubusercontent.com/jadchaar/sec-cik-mapper/main/mappings/stocks/ticker_to_cik.json`.

[46] The "X" in 10-X represents all different form types a company needs to file, e.g., 10-K, 10-Q. We focus on 10-K as these are the most comprehensive reports about a company.

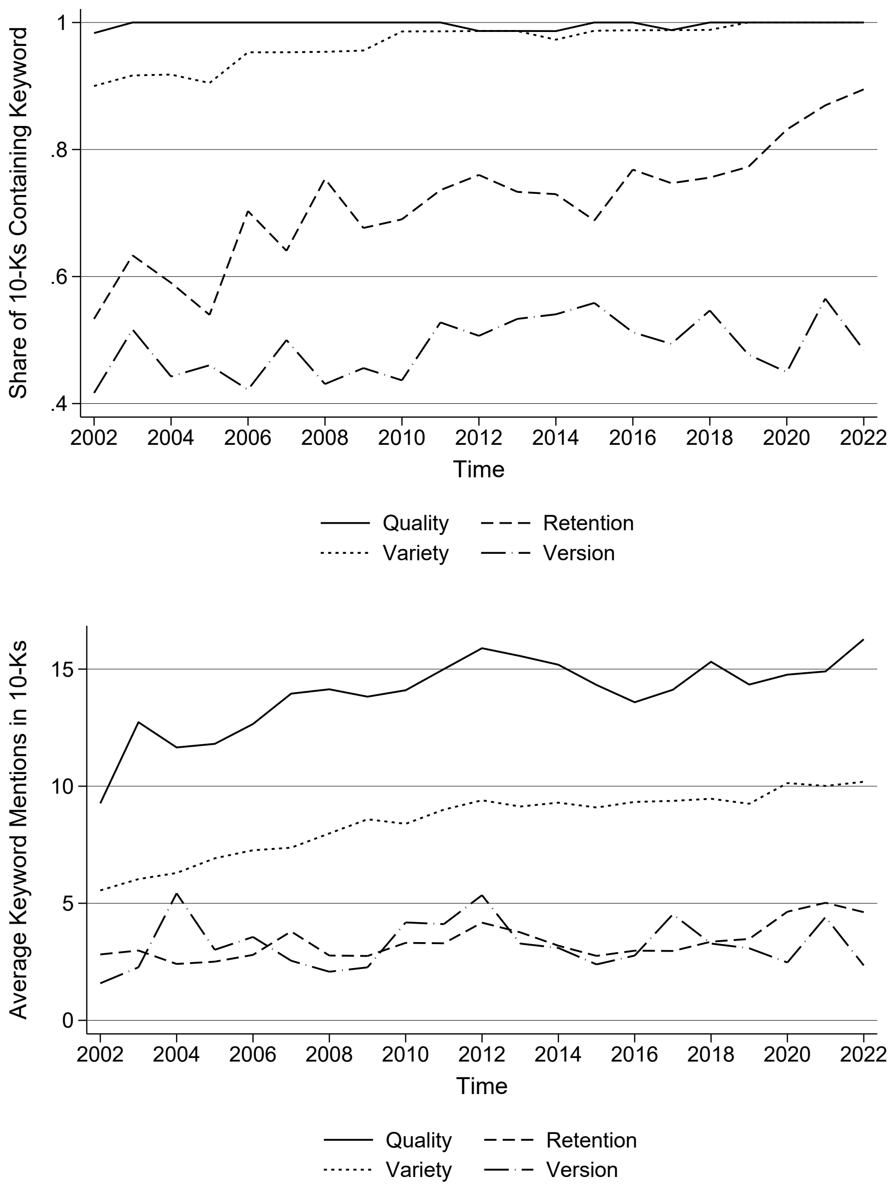
[47] See `https://sraf.nd.edu/sec-edgar-data/10x-document-dictionaries/`.

[48] See `https://sraf.nd.edu/loughranmcdonald-master-dictionary/`.

[49] The CIKs of AstraZeneca, ASML Holding, Coca-Cola, GlobalFoundries, and PDD Holdings cannot be found.

the beginning of the observation period. The lower panel displays the average amount of mentions. It shows a similar dynamic with "variety" and "retention" roughly doubling the corresponding frequency, whereas this does not hold for "quality" and "version".

Figure 2: Frequency of Words in 10-K Reports of NASDAQ 100 Firms



Notes: Based on 10-K reports from 2002-2022 of NASDAQ 100 firms, N = 1,564.

Despite the illustrative evidence, the analysis does neither pick up synonyms nor paraphrases of specific concepts and does not elaborate on where it was mentioned. While this is a drawback, the words "variety" and "retention" are quite specific on its own and the reports predominantly revolve around the firm.

More importantly, however, the analysis remains silent about the context of the mentions, which is necessary to classify it as a firm's strategy and whether the mentions of the specific variables go hand in hand, thereby displaying an interplay. Hence, the next subsection digs deeper into an actual text analysis of the annual reports.

Online Appendix A.2: Text Analysis

The data for this text analysis is based on the Stage One 10-X Parse Data provided by the Software Repository for Accounting and Finance at the University of Notre Dame.[50] This data base contains all 10-X filings available on the EDGAR platform provided by the U.S. SEC as raw documents and is only cleaned of all extraneous materials (e.g., HTML code, PDF's, jpg's etc.).

As outlined above, we restrict attention to NASDAQ 100 firms and annual reports ("10-K"). Moreover, we focus on the section "PART I Item 1 - Business" within a 10-K as this has been found indicative for describing a firm's strategy, also through the previously mentioned anecdotes.[51] As these descriptions vary in size, we focus on roughly the first three pages (i.e., first 15,000 characters) and to the last 13 years (2011-2023) to ease the computational burden of the analysis.

This prepared text of the company's business description serves as the "input" for the query process via OpenAI's GPT-4 in order to classify instances in the text, where information describing product variety and/or customer retention is contained.

In the following, the query process via GPT-4 is described, which involves both the prompt (including the instruction) as well as technical details (that allow replicability):

1. Preparation of the following prompt given to GPT-4 (turbo) without a specific role assigned:

    - Please review the company description provided below. Your task is to analyze the text and compile two separate lists. Each list should contain the top two sentences that best support a classification of the company's business strategy into one of two categories: "product variety" and "customer retention".

    - Instructions:

        (a) Identify and list the top two sentences that support the classification of the company's business strategy as "product variety".

[50] See `https://sraf.nd.edu/data/stage-one-10-x-parse-data/`.

[51] The extraction of the specific section 'PART I Item 1 - Business' is done through the use of regular expressions.

    (b) Identify and list the top two sentences that support the classification of the company's business strategy as "customer retention".
    (c) If no relevant sentences can be found for a category, provide an empty list for that category.

2. Dispatch requests to OpenAI API in batch sizes of 20, employ settings as in Gilardi et al. (2023), and catch different API errors through a maximum of three retries. Parse "answers" in a json format.

The "output" by GPT-4 is thus for each input up to two sentences classified being about product variety and customer retention, respectively. We focus on the first (i.e., top) sentence for each measure and look at the position of each sentence in the initial text. To illustrate the process from starting with the text and classifications to ending up with our measure of an interplay between variety and retention, we use the annual report from 2023 by Netflix (i.e., its section "PART I Item 1 - Business") and the corresponding first five sentences:

- about us netflix, inc. **netflix, the company, registrant, we, or us is one of the world s leading entertainment services with approximately 231 million paid memberships in over 190 countries enjoying tv series, films and games across a wide variety of genres and languages.** members can play, pause and resume to watch as much as they want, anytime, anywhere, and can change their plans at any time. our core strategy is to grow our business globally within the parameters of our operating margin target. *we strive to continuously improve our members' experience by offering compelling content that delights them and attracts new members.* we seek to drive conversation around our content to further enhance member joy, and we are continuously enhancing our user interface to help our members more easily choose content that they will find enjoyable.

In the example, OpenAI's GPT-4 classified the "product variety" sentence (highlighted in bold) starting at character position 23 and the "customer retention" sentence (highlighted in italics) starting at character position 503 out of the whole text comprising 15,000 characters. We then compute the difference between the two positions to measure the distance in characters between the sentences. Hence, it is 0 for the very same sentence and increases the further away the sentences are.

As our goal is to assess the interplay between the two measures, we want to know whether the sentences describing them appear close to each other. For this, we define sentences being close if the respective sentences are not further apart than 500 characters (in the space of

15,000 characters). This can be thought of as one paragraph.[52] The Netflix example above can be thus considered as one with relatively distant sentences but still making it into our definition. As a result, we can look for how many texts the two measures are described at all. If both are described, we can count how often the respective parts are "close" to each other to assume an interplay.

Getting to the data, the input are 977[53] texts, of which GPT-4 classified 99.9 (98.7) percent of these as containing sentences that reflect product variety (customer retention).[54] Hence, almost all companies seemingly talk about the two measures somewhere in the first three pages of their company description and this may indicate that keywords themselves only serve as a lower bound as they do not pick up synonyms or paraphrases.

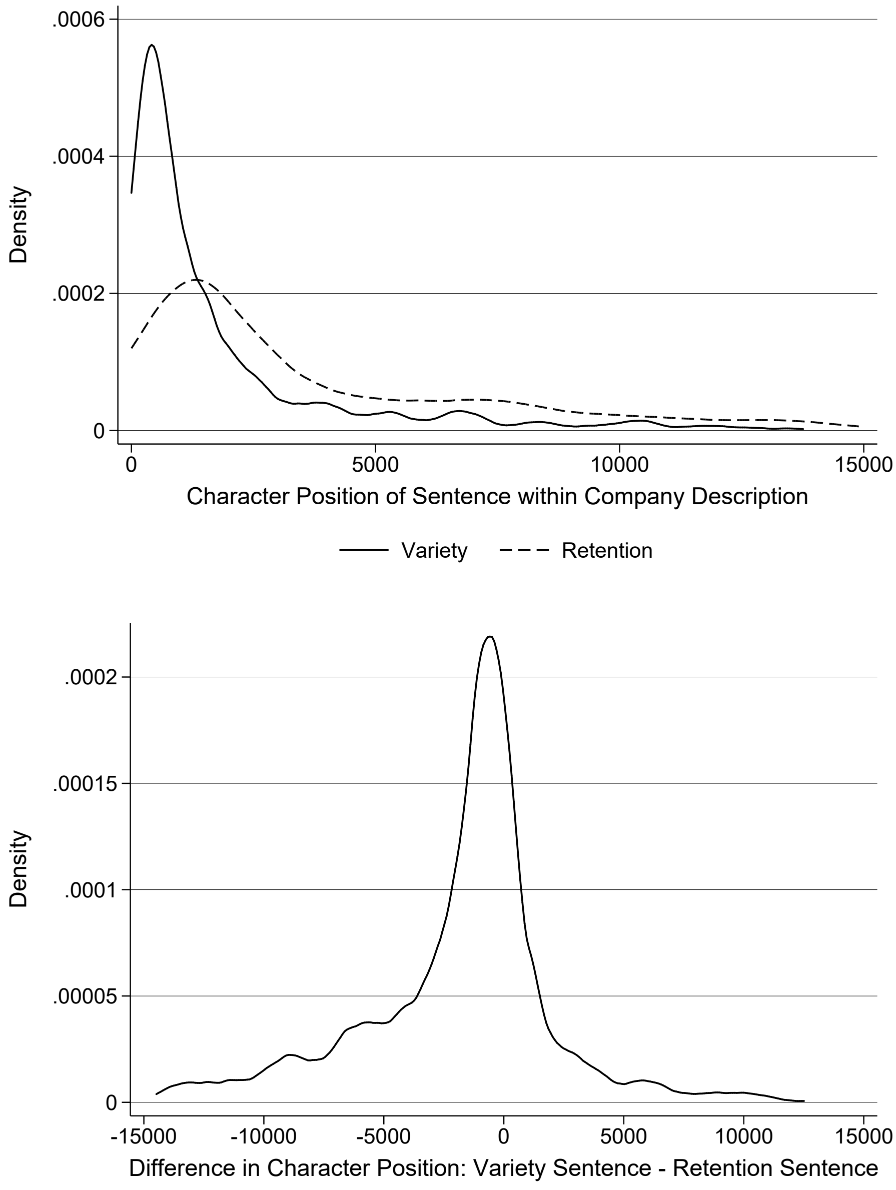
More interestingly, we turn toward the positioning of the respective sentences. The upper panel in Figure 3 shows that both measures are described in the beginning of the company description, with product variety having an especially early (and high-level) peak compared to customer retention. While these patterns may already hint at an overlap, the lower panel in Figure 3 shows the difference when the respective sentences come up and shows some concentration around 0, with the majority being negative (i.e., "variety" appearing before "retention"), which resonates well with the upper panel. In numbers, 19.3 percent (or 189 of 977) of texts describe product variety and customer retention within one paragraph (+/- 500 characters). The figure also shows that the threshold at 500 characters is not crucial.

[52]There is no unique definition for how many sentences or words constitute a paragraph. However, estimates range between 100 to 300 words (see `https://libguides.hull.ac.uk/writing/paras` and `https://warwick.ac.uk/fac/soc/al/globalpad-rip/openhouse/academicenglishskills/writing/paragraphing/`). Thus, we take the lower bound and translate 100 words into 500 characters.

[53]We depart from 1,014 texts, but in 37 instances, the texts may violate the terms and services according to OpenAI and hence there is no output.

[54]As this classification is only for illustrative purposes, we do not resort to validations. However, the anecdotes from the introduction were successfully classified and we do not expect a systematic bias with respect to the difference measure.

Figure 3: Distribution and Distance of Sentence Positions



Notes: Difference (in the lower panel) means starting position in characters (variety) minus starting position in characters (retention). Based on ‘PART I Item 1 - Business’ within a 10-K from 2011-2023 of NASDAQ 100 firms, N = 977.

Online Appendix B: Proofs of Results 1 - 4

Proof of Result 1

Using the Implicit-Function Theorem to (3) yields

$$\frac{dm_i}{dr_j} = -\frac{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i \partial r_j}}{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial (m_i)^2}}.$$

Due to the fact that $\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})/\partial (m_i)^2 < 0$—i.e., second-order conditions are satisfied—the sign of dm_i/dr_j is determined by the sign of $\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})/\partial m_i \partial r_j$. Differentiating $\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\partial m_i$ with respect to r_j, we obtain

$$\operatorname{sign}\left\{\frac{dm_i}{dr_j}\right\} = \operatorname{sign}\left\{-\frac{p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\,i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}\right\} < 0. \tag{8}$$

Similarly, the sign of dm_i/dr_i is also determined by the sign of the term in curly brackets in (8), which implies that $dm_i/dr_i < 0$ as well. Therefore, holding all other variables constant, an increase in r_j and r_i both lead to a decrease in m_i.

Proof of Result 2

The symmetric equilibrium is implicitly characterized by the three equations given in (6) and (7) of the main text. Totally differentiating (6) and (7) with respect to m^*, r^*, p^*, and v, we obtain

$$-\left(\frac{p^*(M-1)(1-r^*)}{M^2\,(m^*)^2} + f''(m^*)\right)dm^* - \left(\frac{p^*(M-1)}{M^2 m^*}\right)dr^* + \left(\frac{(M-1)(1-r^*)}{M^2 m^*}\right)dp^* = 0,$$

$$-c''(r^*)ds - + \left(\frac{(M-1)}{M^2}\right)dp^* = 0,$$

and

$$\left(-\frac{1}{M\,(m^*)^2} + \frac{p^*(M-1)(1-r^*)}{(v-p^*)\beta M^2\,(m^*)^2}\right)dm^* - \left(\frac{p^*(M-1)}{M^2 m^*\beta(v-p^*)}\right)dr^* -$$
$$-\left(\frac{v(M-1)((1-r^*)}{M^2 m^*\beta((v-p^*)^2}\right)dp^* + \left(\frac{p^*(M-1)(1-r^*)}{M^2 m^*\beta(v-p^*)^2}\right)dv = 0.$$

From these expressions, we can solve for dm^*/dv, dr^*/dv, and dp^*/dv. Doing so and focusing on dm^*/dv (as we are interested in how the equilibrium number of products changes with

v), we obtain

$$\frac{dm^*}{dv} = \frac{p^* m^*(M-1)(1-r^*)\left(p^*(M-1) - M^2(1-r^*)c''(r^*)\right)}{\begin{aligned} M\Big[&v\left(p^*(M-1) - c''(r^*)M^2(1-r^*)\right)\left(v\beta - 2\beta p^* + M(n^*)^2 f''(n^*)\right) + \\ &+ (p^*)^2\left((M-1)\left(\beta p^* - M(n^*)^2 f''(n^*)\right) - M(1-r^*)c''(r^*)\left((M-1)(1-r^*) + \beta M\right)\right)\Big]\end{aligned}}. \tag{9}$$

Solving (7) for p^* yields

$$p^* = \frac{v\beta M}{\beta M + (M-1)(1-r^*)}.$$

Inserting the last expression into (9) and simplifying, we obtain

$$\frac{dm^*}{dv} = \frac{(M-1)m^*\beta\mu\left(c''(r^*)M(1-r^*)\mu - v\beta(M-1)\right)}{\left(c''(r^*)M\mu^2 - v\beta\left(M-1\right)^2\right)\left(v\beta(M-1)(1-r^*) + M\left(m^*\right)^2\mu f''(m^*)\right)}, \tag{10}$$

with $\mu \equiv (M-1)(1-r^*) + \beta M > 0$.

The assumption $c''(\cdot) > v\left(M-1\right)^2 / \left(\beta M^3\right)$ implies that the term in the first parentheses of the denominator of the right-hand side of (10)—i.e., $c''(s^*)M\mu^2 - v\beta\left(M-1\right)^2$ is strictly positive. As the term in the second parentheses is also strictly positive, the denominator is strictly positive.

Therefore, the sign of dm^*/dv depends on the sign of the numerator, which is determined by the sign of $c''(r^*)M(1-r^*)\mu - v\beta(M-1)$. This term is positive if and only if

$$v < \frac{c''(r^*)M(1-r^*)\mu}{\beta(M-1)}. \tag{11}$$

For $v \to 0$, this inequality holds, as the left-hand side goes to zero whereas the right hand side is strictly positive. As v increases, the slope of the left-hand side of (11) equals 1. We next determine the slope of the right-hand side. Taking the derivative of the right-hand side with respect to v yields

$$\frac{-c''(r^*)M\left(\beta M + 2(M-2)(1-r^*)\right) + c'''(r^*)M(1-r^*)\mu}{\beta(M-1)}\frac{dr^*}{dv}, \tag{12}$$

where dr^*/dv can be determined in the same way as dm^*/dv above and is given by

$$\frac{dr^*}{dv} = \frac{(M-1)\beta\mu}{c''(r^*)M\mu^2 - v\beta\left(M-1\right)^2} > 0.$$

Due to the assumption that $c'''(\cdot)$ is either negative or, in case it is positive, small relative to $c''(\cdot)$, it follows that the expression in (12) is negative. Therefore, the right-hand side of (11) is negative, which implies that the inequality in (11) does no longer hold if v is sufficiently large. Therefore, the numerator of dm^*/dv is negative for v large enough.

It remains to show that the assumption $c''(\cdot) > v\,(M-1)^2\,/\,(\beta M^3)$ allows for a v large enough so that the numerator can be negative. Inserting the upper bound for v that results from the assumption above into the numerator and simplifying yields that the sign of the numerator equals the sign of the expression

$$\frac{-c''(r^*)M\left(\beta^2M^2 - \beta M(M-1) + r^*(M-1)\left(\beta M + (M-1)(2-r^*)\right)\right)}{M-1},$$

which is negative for all $\beta > 1$. Therefore, the numerator is indeed negative for v large enough but still in the admissible bounds. It follows that $dm^*/dv > 0$ for v below a threshold, but $dm^*/dv < 0$ for v above the threshold.

Proof of Result 3

In the same way as in the proof of Result 2, we can totally differentiate (6) and (7) with respect to m^*, r^*, p^*, and β, and solve for $dm^*/d\beta$, $dr^*/d\beta$, and $dp^*/d\beta$. Focusing on $dm^*/d\beta$ and using $p^* = (v\beta M)\,/\,(\beta M + (M-1)(1-r^*))$, we obtain

$$\frac{dm^*}{d\beta} = \frac{vm^*(1-r^*)\mu(M-1)^2\left(v\beta(M-1) + c''(r^*)M(1-r^*)\mu\right)}{\left(c''(r^*)M\mu^2 - v\beta\,(M-1)^2\right)\left(v\beta(M-1)(1-r^*) + M\,(m^*)^2\,\mu f''(m^*)\right)}.$$

By the same argument as in the proof of Result 2, the term in the first parentheses in the denominator is strictly positive. As all other terms are strictly positive as well, $dm^*/d\beta > 0$.

Proof of Result 4

To determine how dm_i/dr_j is affected by v and β in equilibrium, we first use that, at a symmetric equilibrium, $m_1^* = \cdots = m_M^* = m^*$, $r_1^* = \cdots = r_M^* = r^*$, and $p_1^* = \cdots = p_M^* = p^*$. Plugging this into the expression for dm_i/dr_j, which is given by

$$\frac{dm_i}{dr_j} = -\frac{\frac{\partial^2\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i \partial r_j}}{\frac{\partial^2\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial (m_i)^2}},$$

and simplifying yields

$$\frac{dm_i^*}{dr_j^*} = -\frac{p^*m^*(M-1)}{2p^*(1-r^*)(M-1) + f''(m^*)\,(m^*)^2\,M^3}.$$

Inserting $p^* = v\beta M / (\beta M + (M-1)(1-r^*))$ into the last equation, we obtain

$$\frac{dm_i^*}{dr_j^*} = -\frac{\beta v m^* (M-1)}{2\beta v (1-r^*)(M-1) + f''(m^*)\left(m^*\right)^2 M^2 \left(\beta M + (M-1)(1-r^*)\right)}. \tag{13}$$

We first take the derivative of the right-hand side of (13) with respect to v. This yields

$$-\frac{\beta(M-1)}{\left(2\beta v(M-1)(1-r^*) + f''(m^*)\left(m^*\right)^2 M^2(\beta M + (M-1)(1-r^*))\right)^2} \times \tag{14}$$

$$\times \begin{Bmatrix} f''(m^*)\left(m^*\right)^3 M^2 \left(\beta M + (M-1)(1-r^*)\right) + \left(2\beta v(M-1) + f''(m^*)\left(m^*\right)^2 M^2\right) v m^*(M-1)\frac{dr^*}{dv} \\ \left(2\beta v^2(M-1)(1-r^*) - \left(f''(m^*) + f'''(m^*) v m^*\right)\left(m^*\right)^2 M^2(\beta M + (M-1)(1-r^*)\right)\frac{dm^*}{dv} \end{Bmatrix}.$$

Inserting dr^*/dv and dv^*/dv from Result 2, we obtain that the sign of (14) depends on the sign of

$$-\Big[\left(\left(f''(m^*)\right)^2 \left(m^*\right)^4 M^3 \left(\beta M + (M-1)(1-r^*)\right)^2 + 2\beta^2 v^2 (M-1)^2 (1-r^*)^2\right) c''(r^*)$$

$$+ f''(m^*)\beta^2 v^2 (m^*)^2 (M+2)(M-1)^2 -$$

$$- f'''(m^*)\beta v (m^*)^3 M(M-1)\left(\beta v(M-1) - c''(r^* M(1-r^*)(\beta M + (M-1)(1-r^*)\right).\Big]$$

The terms in the first two lines of the square bracket are all positive due to the fact that $c''(\cdot) > 0$ and $f''(\cdot) \geq 0$. Moreover, since $f'''(\cdot)$ is negative or, if positive, rather small compared to $f''(\cdot)$, the sign of the term in the last line is also positive or only slightly negative. Therefore, the expression in the square brackets is positive, which implies that the entire term is negative. This implies that

$$\frac{\partial\left(\frac{dm_i^*}{dr_j^*}\right)}{\partial v} < 0.$$

Hence, an increase in v amplifies the negative effect of consumer retention on variety.

Proceeding in the same way for β yields that the sign of

$$\frac{\partial\left(\frac{dm_i^*}{dr_j^*}\right)}{\partial \beta}$$

is given by the sign of

$$-\Big[\left(\left(f''(m^*)\right)^2 v\beta m^* M^2 + 2(M-1)^2(1-r^*)^2 \left(\beta M + (M-1)(1-r^*)\right)\right) c''(r^*)$$

$$+f''(m^*)(m^*)^2(M+2)(M-1)\left(\beta M+(M-1)(1-r^*)\right)^2-$$
$$\left.-f'''(m^*)\beta v(m^*)^3 M(M-1)\left(\beta v(M-1)-c''(r^*M(1-r^*)(\beta M+(M-1)(1-r^*)\right).\right]$$

By the same arguments as above, this term is strictly negative, which implies that an increase in β also amplifies the negative effect of consumer retention on variety. Finally, since dm_i^*/dr_i^* is determined by the same term as dm_i^*/dr_j^*, the result also holds for dm_i^*/dr_i^*.

Online Appendix C: Heterogeneous Prices and Second-Order Conditions

Online Appendix C.1: Optimality of Symmetric Prices and Discussion of Equilibrium Symmetry

In the model of the main text, we assumed that each firm sets the same price for all its products. In this appendix, we first show that this pricing strategy is indeed optimal even if a firm could set different prices. As stated in footnote 21 of the main text, if a firm could set different prices for its products, the probability with which a switching consumer buys a product of firm i is

$$\frac{\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\sum_{j=1}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell},$$

and the probability with which a retained consumer of firm i buys product k is

$$\frac{(v-p_{k,i})^{\frac{1}{\beta}}}{\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}.$$

The maximization problem of firm i is then given by

$$\max_{m_i, r_i, p_{k,i} \forall k \in [0,m_i]} \frac{r_i}{M} \left(\frac{\int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell} \right) + \frac{\sum_{j=1}^{M} (1-r_j)}{M} \left(\frac{\int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\sum_{j=1}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell} \right) - f(m_i) - c(r_i). \tag{15}$$

From Leibniz's rule, the derivative of $\int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell$ with respect to m_i is $p_{m_i,i}(v-p_{m_i,i})^{\frac{1}{\beta}}$. We can use this in the derivative of (15) with respect to m_i to obtain that the first-order condition with respect to m_i is

$$\frac{(v-p_{m_i,i})^{\frac{1}{\beta}}}{M} \left(\frac{r_i \left(p_{m_i,i} \int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell - \int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell \right)}{\left(\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell \right)^2} + \right.$$

$$\left. + \frac{\sum_{j=1}^{M} (1-r_j) p_{m_i,i} \left(\sum_{j=1, i\neq j}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell \right)}{\left(\sum_{j=1}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell \right)^2} \right) - f'(m_i) = 0. \tag{16}$$

Second, the first-order condition with respect to r_i is given by

$$\left(\frac{\int\limits_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{M}\right)\left(\frac{\sum\limits_{j=1,\,i\neq j}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell}{\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell\left(\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell\right)}\right)-c'(r_i)=0. \qquad (17)$$

Third, we take the derivative of (15) with respect to $p_{k,i}$. This can be done by considering that firm i charges $p_{k,i}$ for a set $n>0$ out of its m_i products, and then let $n\to 0$. To do this, we can write the function to be maximized as

$$\frac{r_i}{M}\left(\frac{np_{k,i}(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{n(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}\right)+$$

$$+\frac{\sum\limits_{j=1}^{M}(1-r_j)}{M}\left(\frac{np_{k,i}(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\sum\limits_{j=1,i\neq j}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell+n(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}\right)-f\left(m_i\right)-c\left(r_i\right).$$

Taking the derivative with respect to $p_{k,i}$, setting the result equal to zero, and letting n go to zero then yields

$$r_i\frac{(\beta(v-p_{k,i})-p_{k,i})\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell+\int\limits_0^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell\right)^2}+ \qquad (18)$$

$$+\sum_{j=1}^{M}(1-r_j)\frac{(\beta(v-p_{k,i})-p_{k,i})\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell+\int\limits_0^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell\right)^2}=0,\quad \forall k\in[0,m_i].$$

Using (18), we can now show that it is optimal for each firm i to set the same prices for all its products, given m_i, r_i, and the choices of competitors. To see this, we can rewrite (18) as

$$(\beta(v-p_{k,i})-p_{k,i})\left[\frac{r_i}{\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}+\frac{\sum\limits_{j=1}^{M}(1-r_j)}{\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell}\right]+ \qquad (19)$$

$$+\left[\frac{r_i\int\limits_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell\right)^2}+\frac{\sum\limits_{j=1}^{M}(1-r_j)\int\limits_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell\right)^2}\right]=0,\quad \forall k\in[0,m_i].$$

Denoting

$$X\equiv\left[\frac{r_i}{\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}+\frac{\sum\limits_{j=1}^{M}(1-r_j)}{\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell}\right]$$

and

$$Y\equiv\frac{r_i\int\limits_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell\right)^2}+\frac{\sum\limits_{j=1}^{M}(1-r_j)\int\limits_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell\right)^2},$$

it follows from (19), that the first-order conditions for all $p_{k,i}$, $k\in[0,m_i]$, have the structure $(\beta(v-p_{k,i})-p_{k,i})\,X+Y=0$. Specifically, X and Y are the same in all first-order conditions for $p_{k,i}, k\in[0,m_i]$. As the term in parentheses is linear in $p_{k,i}$, the solution to (19) must be the same for all $p_{k,i}$. Therefore, firm i optimally sets the prices of all its products equal to each other, that is, $p_{k,i}=p_i$ for all $k\in[0,m_i]$. Using this for all firms, the first-order conditions just derived—i.e., (16), (17), and (18)—can be simplified to (3), (4), and (5).

We next discuss equilibrium symmetry and the possibility of an asymmetric equilibrium to exist. In a symmetric equilibrium, where $m_1^*=\cdots=m_M^*\equiv m^*$, $r_1^*=\cdots=r_M^*\equiv r^*$, and $p_1^*=\cdots p_M^*\equiv p^*$, conditions (3), (4), and (5) can then be simplified to obtain (6) and (7). Solving (7) for p^* yields

$$p^*=\frac{\beta vM}{\beta M+(M-1)(1-r^*)}.$$

Inserting this into the second equation of (6) yields

$$\frac{\beta v(M-1)}{M\left(\beta M+(M-1)(1-r^*)\right)}=c'(r^*)\tag{20}$$

It is easy to see from (20) that the assumption $c'(1)>v\,(M-1)\,/M^2$ ensures that the solution for r^* is below 1.

Moreover, our assumptions also ensure uniqueness of a symmetric equilibrium. First, uniqueness of r^* follows from (20): The left-hand side is increasing in r^* and strictly convex. Instead, due to the assumption that $c'''(\cdot)$ is negative or only slightly positive, the right-hand side, although also increasing in r^*, is concave or only slightly convex. As the left-hand side is larger than the right-hand side at $r=0$ and the reverse holds for $r=1$, there is a unique intersection point between the two. Similarly, inserting $p^* = \beta v M / \left(\beta M + (M-1)(1-r^*)\right)$ into the first equation of (6) yields

$$\frac{\beta v(M-1)(1-r^*)}{Mm^*\left(\beta M + (M-1)(1-r^*)\right)} = f'(m^*). \tag{21}$$

Since (20) does not depend on m^*, the left-hand side is strictly decreasing in m^*, whereas the right-hand side is weakly increasing. This again implies that there is a unique value for m^* that satisfies (21). Therefore, there exists a unique symmetric equilibrium.

Finally, we turn to the question whether an asymmetric equilibrium in which e.g. some firms set a relatively high product variety but invest only a small amount in customer retention, whereas for others the opposite holds, can exist. The first observation is that such an equilibrium can only occur if different firms charge different prices. This can be seen from (3). Potential differences in customer retention levels of firms affect all firms in the same way as the first term of (3) depends on $\sum_{j=1}^{M}(1-r_j)$. Therefore, any difference in the number of products for firms can only be due to different prices. It follows that, if prices of all firms are the same, then also $m_1^* = \cdots = m_M^*$. Using this in (4), the first term in this equation is then also the same for all firms, which implies that $r_1^* = \cdots = r_M^*$.

A consequence of this result is that for $\beta \to \infty$, only a symmetric equilibrium exists. The reason is that the optimal prices for all firms are $p_j^* = v$, $j \in \{1, ..., M\}$, as $\beta \to \infty$. Since prices are the same for all firms in that case, the arguments above imply that also m_j^* and r_j^* must be the same for all firms.

Instead, if prices of firms were different, the equilibrium conditions (3) and (4) can in principle be satisfied with different values for m_i and r_i for firms that charge different prices. Condition (5) can then also be satisfied for all firms although firms set different strategic variables. We performed several numerical simulation with different parameters and cost functions to check whether asymmetric equilibria can arise. However, in all of the simulations we found that the only equilibrium is the symmetric one. Although we could not show analytically that no asymmetric equilibrium exists, the simulations strongly point to the uniqueness of the symmetric equilibrium.

There are indeed forces in the model that push toward a symmetric equilibrium. To see this, consider (3)—i.e., the first-order condition for m_i—and suppose there are two firms, j and k, that set a different number of products in equilibrium—e.g., $m_j > m_k$. For both firms, (3) must hold. Since $f''(\cdot) \geq 0$ and $m_j > m_k$, it follows that $f'(m_j) \geq f'(m_k)$. Therefore, the first term of (3) must be larger in the condition for firm j than in the condition for firm k. Because $\sum\limits_{j=1}^{M}(1-r_j)$ and also the denominator is the same in both conditions, it must hold that

$$p_j\left(v-p_j\right)^{\frac{1}{\beta}}\times\left(\sum_{i=1,\,i\neq j}^{M}m_i(v-p_i)^{\frac{1}{\beta}}\right)\geq p_k\left(v-p_k\right)^{\frac{1}{\beta}}\times\left(\sum_{i=1,\,i\neq k}^{M}m_i(v-p_i)^{\frac{1}{\beta}}\right).$$

The first term on each side of the inequality is $p_h\left(v-p_h\right)^{\frac{1}{\beta}},\ h\in\{j,k\}$. The sign of the derivative of $p_h\left(v-p_h\right)^{\frac{1}{\beta}}$ is equal to the sign of $\beta(v-p_h)-p_h$, which is strictly negative, as otherwise (5) cannot be satisfied. Therefore, the first term points to $p_j\leq p_k$ to fulfill (3) for both firms. However, for the second term, the opposite holds. Since $m_j\geq m_k$, if p_j would be larger than p_k, then

$$\left(\sum_{i=1,\,i\neq j}^{M}m_i(v-p_i)^{\frac{1}{\beta}}\right)\leq\left(\sum_{i=1,\,i\neq k}^{M}m_i(v-p_i)^{\frac{1}{\beta}}\right).$$

This demonstrates that there are balancing forces in the conditions (3) and (5) that make it difficult to satisfy (3) for different values of m_i.[55]

We note that we focused on potential asymmetric equilibria that are interior—i.e., equilibria where the first-order conditions are satisfied. This is indeed sufficient because we show in the next online appendix that, given our assumptions, it cannot not be optimal for a firm to set some of its strategic variables at extremal values.

Online Appendix C.2: Sufficiency of First-Order Conditions

In this appendix, we determine the conditions so that the first-order conditions (3), (4), and (5) of the main text are indeed sufficient for a maximum. To do so, we first show that the Hessian matrix given by[56]

$$\begin{pmatrix}\frac{\partial^2\Pi_i}{\partial(m_i)^2} & \frac{\partial^2\Pi_i}{\partial m_i\partial r_i} & \frac{\partial^2\Pi_i}{\partial m_i\partial p_i}\\ \frac{\partial^2\Pi_i}{\partial r_i\partial m_i} & \frac{\partial^2\Pi_i}{\partial(r_i)^2} & \frac{\partial^2\Pi_i}{\partial r_i\partial p_i}\\ \frac{\partial^2\Pi_i}{\partial p_i\partial m_i} & \frac{\partial^2\Pi_i}{\partial p_i\partial r_i} & \frac{\partial^2\Pi_i}{\partial(p_i)^2}\end{pmatrix}$$

is negative definite at the symmetric equilibrium.

We start with the entries on the diagonal to determine the first-order principal minors.

[55] This argument holds independently of the values of r_i as these values enter (3) for all firms in the same way.

[56] To reduce the notational burden, we omit the arguments of the function Π_i.

Taking the derivative of the right-hand side of (3) with respect to m_i yields

$$\frac{\partial^2 \Pi_i}{\partial (m_i)^2} = -\left(\frac{2\,(v-p_i)^{\frac{2}{\beta}}}{M}\right)\left(\frac{\sum\limits_{j=1}^{M}(1-r_j)p_i\left(\sum\limits_{j=1,\, i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^3}\right) - f''(m_i) < 0, \qquad (22)$$

where the inequality is due to the fact that the first term is negative and $f''(m) \geq 0$.

Next, taking the derivative of the right-hand side of (4) with respect to r_i yields $\partial^2\Pi_i/\partial(r_i)^2 = -c''(r_i) < 0$.

Proceeding in the same way for the derivative with respect to p_i yields that $\partial^2\Pi_i/\partial(p_i)^2$ evaluated at a symmetric equilibrium is

$$\frac{p(M-1)(1-r) - \beta(2Mv - p(M+1+M(1-r)))}{M^2 m\beta^2(v-p)^2}.$$

This term is negative if and only if

$$\beta > \frac{p(M-1)(1-r)}{2Mv - p(M+1+M(1-r))} \qquad (23)$$

This threshold value of β is increasing in p and decreasing in r. As p can never be above v in any equilibrium, the highest value of p equals v. As r is a probability, the lowest value of r equals 0. Inserting $p = v$ and $r = 0$ into the threshold value and simplifying, we obtain that the right-hand side of (23) is 1. Therefore, for β larger than 1, also $\partial\Pi_i/\partial p_i^2 < 0$. It follows that all first-order principal minors are negative.

We next consider the second-order principal minors. We start with the one for m_i and r_i, where the respective determinant is given by $\left(\partial^2\Pi_i/\partial\,(m_i)^2\right)\left(\partial^2\Pi_i/\partial\,(r_i)^2\right) - \left(\partial^2\Pi_i/\,(\partial m_i \partial r_i)\right)^2$. Evaluating this determinant at a symmetric equilibrium, we obtain

$$\frac{c''(r)\,(2M^3(M-1)p(1-r) + M^6 m^2 f''(m)) - (M-1)^2 p^2}{M^6 m^2}.$$

Inserting the equilibrium value of p given by $p = \beta M v/(\beta M + (M-1)(1-r))$ into this expression and simplifying yields

$$\frac{c''(r)(\beta M + (M-1)(1-r))\,(2\beta v M^2(M-1)(1-r) + M^4 m^2 f''(m)(\beta M + (M-1)(1-r)))}{M^4 m^2(\beta M + (M-1)(1-r))^2}$$
$$- \frac{\beta^2 v^2 (M-1)^2}{M^4 m^2(\beta M + (M-1)(1-r))^2}. \qquad (24)$$

This term is increasing in $f''(m)$. Therefore, if it is positive for $f''(\cdot) = 0$, it is also positive if $f''(\cdot) \geq 0$. Inserting $f''(m) = 0$ and solving for $c''(r)$ yields that (24) is positive if

$$c''(r) > \frac{\beta v(M-1)}{2(1-r)M^2(\beta M + (M-1)(1-r))}.$$

From the first-order conditions, r is implicitly given by $r = (\beta M + M - 1)/(M-1) - \beta v/c'(r)$. Inserting this into the last inequality and simplifying yields

$$c''(r) > \frac{v(M-1)\left(c'(r)\right)^2}{2\beta(M-1+M^2 c'(r))}.$$

For any value of $c'(r) \in (0, \infty)$, the condition $c''(r) > v(M-1)^2/(\beta M^3)$ ensures that the inequality holds. It follows that the determinant of the Hessian matrix for m_i and r_i is positive.

We next turn to the determinant of the second-order principal minor with respect to m_i and p_i, which is given by $\left(\partial^2 \Pi_i/\partial \left(m_i\right)^2\right)\left(\partial^2 \Pi_i/\partial \left(p_i\right)^2\right) - \left(\partial^2 \Pi_i/\left(\partial m_i \partial p_i\right)\right)^2$. At a symmetric equilibrium, it is equal to

$$\frac{\left[f''(m)M^2m^2 + 2p(M-1)(1-s)\right]\left[\beta(2Mv - p(M+1+M(1-s))) - p(M-1)(1-s)\right]}{M^5 m^3 \beta^2 (v-p)^2}$$

As $f''(m) \geq 0$, the sign of this determinant is positive if (23) holds. It is therefore fulfilled for $\beta > 1$.

Similarly, the determinant of the second-order principal minor with respect to r_i and p_i—i.e., $\left(\partial^2 \Pi_i/\partial \left(r_i\right)^2\right)\left(\partial^2 \Pi_i/\partial \left(p_i\right)^2\right) - \left(\partial^2 \Pi_i/\left(\partial r_i \partial p_i\right)\right)^2$—evaluated at a symmetric equilibrium is

$$\frac{c''(r)\left[\beta(2Mv - p(M+1+M(1-s))) - p(M-1)(1-s)\right]}{M^2 m \beta^2 (v-p)^2}.$$

Due to the fact that $c''(r) > 0$, this expression is positive if the term in square brackets in the numerator is positive. This is again true if (23) holds. It is therefore fulfilled for $\beta > 1$.

Finally, we turn to the determinant of the third-order principal minor, which is given by

$$\begin{aligned}
&\left(\frac{\partial^2 \Pi_i}{\partial \left(p_i\right)^2}\right)\left[\left(\frac{\partial^2 \Pi_i}{\partial \left(m_i\right)^2}\right)\left(\frac{\partial^2 \Pi_i}{\partial \left(r_i\right)^2}\right) - \left(\frac{\partial^2 \Pi_i}{\partial m_i \partial r_i}\right)^2\right] + \\
&+\left(\frac{\partial^2 \Pi_i}{\partial p_i \partial r_i}\right)\left[\left(\frac{\partial^2 \Pi_i}{\partial \left(m_i\right)^2}\right)\left(\frac{\partial^2 \Pi_i}{\partial p_i \partial r_i}\right) - \left(\frac{\partial^2 \Pi_i}{\partial m_i \partial r_i}\right)\left(\frac{\partial^2 \Pi_i}{\partial m_i \partial p_i}\right)\right] + \\
&+\left(\frac{\partial^2 \Pi_i}{\partial p_i \partial m_i}\right)\left[\left(\frac{\partial^2 \Pi_i}{\partial m_i \partial r_i}\right)\left(\frac{\partial^2 \Pi_i}{\partial p_i \partial r_i}\right) - \left(\frac{\partial^2 \Pi_i}{\partial (r_i)^2}\right)\left(\frac{\partial^2 \Pi_i}{\partial m_i \partial p_i}\right)\right].
\end{aligned} \tag{25}$$

Inserting the relevant second-order derivatives into this term and evaluating it at the symmetric equilibrium yields that the sign of this term is given by the sign of

$$-\left[c''(r)\left(2M^3p(M-1)(1-s)+f''(m)M^6m^2\right)+p^2(M-1)^2\right]\times$$
$$\times\left[2M\beta v-p\left(\beta(M(1-s)+M+1)+(M-1)(1-s)\right)\right]. \tag{26}$$

The first term in square brackets is positive due to the fact that $c''(r)>0$ and $f''(m)\geq 0$. The second term is strictly decreasing in p. Inserting the highest possible value for p in any equilibrium, which is v, into the second term and simplifying, we obtain that it is given by $v(M-1)(1-s)(\beta-1)$, which is strictly positive as $\beta>1$. Therefore, (25) is negative.

As a consequence, if the assumptions of Section 2 are fulfilled, the first-order principal minors are negative, the second-order principal minors are positive, and the third-order principal minor is again negative. As a consequence, the Hessian matrix is negative definite, which implies that the first-order conditions constitute an interior local maximum. Therefore, if all firms $j\neq i$ choose its strategic variables as prescribed by the first-order conditions, doing so as well constitutes a local maximum for firm i.[57]

We next check under which conditions this interior local maximum is also a global maximum. First, our assumptions on $c(r_i)$ ensure that the optimal value of r_i is interior. Second, the structure of the maximization problem implies that for any value of r_i and p_i, there is a single value of m_i that constitutes a local maximum. This is due to the fact that $\partial^2\Pi_i/\partial(m_i)^2<0$—i.e., the demand function is strictly concave in m_i and $f''\geq 0$. Third, if firm i sets the price for all products equal to v, the derivative of $\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})$ with respect to m_i is negative. In fact, the first term in (3) is zero, whereas the second term is negative as $f''(\cdot)\geq 0$.

Taking these arguments together, if the local maximum does not constitute a global maximum, this global maximum cannot be interior, but must occur at a point at which some of the variables take extreme values. Since the optimal value of r_i is necessarily interior, the only candidate for the potential global maximum is the one in which firm i focuses only on consumer retention but does not invest in product variety. It then optimally sets $m_i=0$ and $p_i=v$, that is, m_i and p_i are set at extreme values. The resulting optimization problem

[57] If a firm could set different prices for its products, the second-order conditions are more complicated, as a firm has a continuum of choice variables. However, the structure of the first derivative for any two prices is the same. It is then possible to check the second-order conditions by focusing on the derivatives of two prices—e.g., $p_{k,i}$ and $p_{k',i}$—where $p_{k,i}$ applies to a set n of products and $p_{k',i}$ to a set n' of products. The resulting Hessian is then a 4x4 matrix. Following the same procedure as above, and letting n and n' go to zero after derivatives are taken, we can show that the Hessian is negative definite if the assumptions of Section 2 are fulfilled.

of firm i is

$$\max_{r_i} \frac{r_i v}{M} - c(r_i),$$

yielding an optimal customer retention level, denoted by r_i^{ext}, that is implicitly defined by

$$r_i^{ext} = (c')^{-1}\left(\frac{v}{M}\right). \tag{27}$$

Inserting this optimal solution into the profit function, we obtain that firm i's profit is then given by

$$(c')^{-1}(v/M)\left(\frac{v}{M}\right) - c\left((c')^{-1}(v/M)\right).$$

Instead, firm i's profit at the symmetric equilibrium can be written as

$$\frac{\beta v}{\beta M + (M-1)(1-r^*)} - c(r^*) - f(m^*).$$

Since firm i charges the maximal price to retained consumers and sets $m_i = 0$ at the extremal solution, $r_i^{ext} > r^*$. This implies that its cost of investment in customer retention are larger in the extremal solution compared to the interior local maximum. It therefore follows that the extremal solution cannot be a global maximum if the cost at this solution—i.e., $c\left((c')^{-1}(v/M)\right)$—are sufficiently large compared to the costs at lower values of r_i.

Expressing this via the primitives of the model, we can first determine the implicit solution for r^*. Solving (7) for p^* and inserting the respective value in the second equation of (6), we obtain that r^* is implicitly given by

$$\frac{v\beta(M-1)}{M(\beta M + (M-1)(1-r^*))} = c'(r^*).$$

The left-hand side is increasing in r^* and is equal to $v\beta(M-1)/\left(M(\beta M + (M-1))\right)$ at $r^* = 0$. Therefore, a sufficient (but not necessary) condition for the interior local maximum to be a global maximum is that

$$c\left((c')^{-1}\left(\frac{v}{M}\right)\right) >> c\left((c')^{-1}\left(\frac{v\beta(M-1)}{M(\beta M + M - 1)}\right)\right).$$

In words, the costs at the extremal solution are substantially larger than the lowest possible cost at the interior solution, which implies that a firm is better off by choosing a low value of r_i than choosing $r_i = r_i^{ext}$. Therefore, the extremal solution is not optimal.

Online Appendix D: Extensions

Online Appendix D.1: Endogenous Customer Base

In the main model, we considered a scenario in which the customer base of each firm is exogenous—i.e., each firm has the same base of size $1/M$. In this appendix, we analyze an extended scenario in which each firm can influence its customer base via setting the prices of its existing products.

To incorporate this in a simple way, suppose that each firm $i = 1, ..., M$ has a mass $\tilde{m}$ of existing products and sets a price $\tilde{p}_i$ for these products.[58] Consistent with the main model, the resulting demand of firm i for its existing products is then

$$\frac{\tilde{m}(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}\tilde{m}(v-\tilde{p}_j)^{\frac{1}{\beta}}} = \frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}.$$

Each firm obtains profits for its existing products, which are given by the respective price times the demand. The aggregate demand of firm i then determines firm i's customer base. As in the main model, each firm can then invest to retain consumers who are in the customer base, but can also invest in product variety to attract potential switching consumers.

Following the notation of the main model, the vector of all firms' prices for the existing products is denoted by $\tilde{\mathbf{p}}$, whereas the respective vectors for the other strategic variables are still denoted by $\mathbf{m}$, $\mathbf{r}$, and $\mathbf{p}$, respectively. Firm i's profit function in this extended model can then be written as

$$\Pi_i(\mathbf{m}, \mathbf{r}, \tilde{\mathbf{p}}, \mathbf{p}) = \frac{\tilde{p}_i(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}} + r_i p_i \left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right) +$$

$$+\left(\frac{\sum\limits_{j=1}^{M}(1-r_j)(v-\tilde{p}_j)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\left(\frac{p_i m_i (v-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}}\right) - f(m_i) - c(r_i).$$

In contrast to the main model, the profit function now consists of an additional term—i.e., the first term in $\Pi_i(\mathbf{m}, \mathbf{r}, \tilde{\mathbf{p}}, \mathbf{p})$—which gives the profit that firm i obtains from the existing products. Moreover, in the second and the third term, the customer base is now no longer

[58] We focus on the case in which firms are symmetric with respect to the mass of their existing products. Considering asymmetric masses would complicate the analysis, but would not change the main insights, as the analysis follows similar lines as considered in Online Appendix D.2.

$1/M$ but

$$\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}.$$

Firm i maximizes this profit function with respect to $\tilde{p}_i$, m_i, r_i, and p_i. To simplify the exposition, we consider a game in which firms choose their maximization variables at the same time.[59]

The first-order condition with respect to $\tilde{p}_i$ is

$$(\beta(v-\tilde{p}_i)-\tilde{p}_i)\left(\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}\right)+\tilde{p}_i(v-\tilde{p}_i)^{\frac{1}{\beta}}-r_ip_i\sum_{j=1,\jmath\neq i}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}$$

$$+\frac{p_im_i(v-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\left(\sum_{j=1}^{M}(r_i-r_j)(v-\tilde{p}_j)^{\frac{1}{\beta}}\right)=0.$$

Turning to the variables that are also present in the main model, we obtain that the respective first-order conditions need to be slightly modified. Specifically, the first-order condition with respect to m_i is

$$\left(\frac{\sum\limits_{j=1}^{M}(1-r_j)(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\frac{p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\,i\neq j}^{M}m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}-f'(m_i)=0,$$

the one with respect to r_i is

$$p_i\left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\left(1-\frac{m_i(v-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\right)-c'(r_i)=0,$$

and the one with respect to p_i is

$$r_i\left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)-$$

[59] The analysis of sequential decisions in which e.g. $\tilde{p}_i$ is chosen before the other variables, is considerably more complicated. However, it proceeds along the same lines as in Online Appendix D.5. We numerically analyzed this case and obtained that our main results continue to hold.

$$+\frac{\sum_{j=1}^{M}(1-r_j)}{\beta}\left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\left(\frac{m_i\,(v-p_i)^{\frac{1-\beta}{\beta}}}{\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\right)\left(\beta(v-p_i)-p_i+\frac{p_i m_i\,(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\right)=0.$$

In a symmetric equilibrium, where we denote the equilibrium variables by $\tilde{p}^*$, p^*, m^*, and r^*, these conditions can be simplified to

$$M\beta\,(v-\tilde{p}^*)-(M-1)\tilde{p}^*-(M-1)p^*r^*=0, \tag{28}$$

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2}-f'(m^*)=0, \quad \frac{(M-1)p^*}{M^2}-c'(r^*)=0, \tag{29}$$

$$\text{and} \quad M\beta\,(v-p^*)-(M-1)p^*\,(1-r^*)=0. \tag{30}$$

The new condition (28) is the simplified first-order condition with respect to $\tilde{p}_i$ and determines $\tilde{p}^*$. However, it is straightforward to see that (29) and (30) are equivalent to the conditions in the main model. Therefore, all results of the main model carry over to the baseline model.

The intuition behind this result is that in a symmetric equilibrium, all firms charge the same prices for their existing products, which implies that in equilibrium, the customer base of each firm is $1/M$. The maximization with respect to m_i, r_i, and p_i then yields the same results as in the main model.

Although this intuition is simple, the analysis shows that the main model can be extended to allow for an endogenous customer base. For instance, a similar result would apply if firms can influence their customer base not only via setting prices for their existing products, but also if the mass of existing products was a strategic variable of each firm. Therefore, our results are robust to these extensions.

Online Appendix D.2: Firms are Asymmetric with respect to Customer Base

In the main model, we considered the case in which all firms have the same customer base, that is, each firm's customer base is $1/M$. We now demonstrate that our results carry over to the situation in which firms have asymmetric customer bases. To simplify the exposition, we consider a situation with two different types of firms, where one type of firms has a smaller customer base than the other.[60] As will become clear, the situation with M different firms can be tackled in the same way and delivers qualitatively the same results.

[60] Firms may also differ in other dimensions. For example, they may have different costs to offer a larger product variety (i.e. a different $f(\cdot)$-function) or differ in their investment cost of customer retention (i.e., a different $c(\cdot)$-function). Solving for the equilibrium in these situations can be done in the same way as in the case in which firms differ in their customer bases.

Suppose that, among the M firms, $K < M$ firms have a customer base of $\underline{b}/\left(K\underline{b} + (M-K)\overline{b}\right)$, whereas $M - K$ firms have a customer base of $\overline{b}/\left(K\underline{b} + (M-K)\overline{b}\right)$, with $\underline{b} \neq \overline{b}$. The profit function of firm i, with $b_i \in \left\{\underline{b}, \overline{b}\right\}$ is then given by

$$\Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p}) = \frac{b_i r_i p_i}{K\underline{b} + (M-K)\overline{b}} + \frac{\sum_{j=1}^{M} b_j(1-r_j)}{K\underline{b} + (M-K)\overline{b}} \left(\frac{p_i m_i (v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v - p_{\ell,j})^{\frac{1}{\beta}}} \right) - f(m_i) - c(r_i).$$

Determining the first-order conditions, we obtain[61]

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M} b_j (1-r_j) p_i (v-p_i)^{\frac{1}{\beta}} \left(\sum_{j=1,\, i\neq j}^{M} m_j (v-p_j)^{\frac{1}{\beta}} \right)}{\left(K\underline{b} + (M-K)\overline{b}\right) \left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}} \right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})}{\partial r_i} = \frac{p_i b_i}{K\underline{b} + (M-K)\overline{b}} \left(1 - \frac{m_i (v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} \right) - c'(r_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})}{\partial p_i} = \frac{b_i r_i}{\left(K\underline{b} + (M-K)\overline{b}\right)} -$$

$$+ \frac{\sum_{j=1}^{M} b_j (1-r_j)}{\left(K\underline{b} + (M-K)\overline{b}\right) \beta} \frac{m_i (v-p_i)^{\frac{1-\beta}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} \left(\beta(v-p_i) - p_i + \frac{m_i p_i (v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} \right) = 0.$$

It is straightforward to check that the signs of the cross-derivatives dm_i/dr_j and dm_i/dr_i are the same as in the main model. This is because the terms in the first-order conditions (with the exception of the cost functions that do not affect the cross derivatives) are only multiplied by a different parameter compared to the main model (i.e., $b_j/\left(K\underline{b} + (M-K)\overline{b}\right)$ instead of $1/M$), but are not affected otherwise. Therefore, Result 1 also holds with asymmetric firms.

Proceeding in the same way as in the main model, we obtain the equilibrium conditions from these first-order conditions. In the unique symmetric equilibrium, all firms of the same type set the same equilibrium values. Denoting by $\underline{m}^*$, $\underline{r}^*$, and $\underline{p}^*$ the equilibrium levels of firms with a smaller product variety and by $\overline{m}^*$, $\overline{r}^*$, and $\overline{p}^*$ as those of firms with a larger

[61] As in the main model, it is optimal for each firm to set the same price for all its products.

product variety, we can write the equilibrium conditions for the latter three variables as

$$\left(K\underline{b}\,(1-\underline{r}^*)+(M-K)\overline{b}\,(1-\overline{r}^*)\right)\times$$
$$\times\frac{\overline{p}^*\,(v-\overline{p}^*)^{\frac{1}{\beta}}\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K-1)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}{\left(K\underline{b}+(M-K)\overline{b}\right)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)^2}-f'\left(\overline{m}^*\right)=0,$$

$$\frac{\overline{b}\overline{p}^*\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K-1)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}{\left(K\underline{b}+(M-K)\overline{b}\right)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}-c'\left(\overline{r}^*\right)=0,$$

and

$$\frac{\overline{r}^*\overline{b}\beta\,(v-\overline{p}^*)^{\frac{1}{\beta}}}{\overline{m}^*}-\frac{K\underline{b}\,(1-\underline{r}^*)+(M-K)\overline{b}\,(1-\overline{r}^*)}{\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}}\times$$
$$\times\left(\overline{p}^*-\beta(v-\overline{p}^*)-\frac{\overline{m}^*\overline{p}^*\,(v-\overline{p}^*)^{\frac{1}{\beta}}}{\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}}\right)=0.$$

In a similar way, we can write the equilibrium conditions for $\underline{m}^*$, $\underline{r}^*$, and $\underline{p}^*$. This provides us with six equations for the six equilibrium values.

Proceeding in the same way as in the proofs of Results 2, and 3 we can totally differentiate these equations with respect to the six equilibrium variables as well as v and β, which allows us to determine $d\overline{m}^*/dv$, $d\underline{m}^*/dv$, $d\overline{m}^*/d\beta$, and $d\underline{m}^*/d\beta$. The resulting expressions are very long compared to the main model due to the additional parameters $\underline{b}$, $\overline{b}$, and K. However, we can show that the results are akin to those of the main model. In particular, $d\overline{m}^*$ and $d\underline{m}^*$ change non-monotonically with v, that is, $d\overline{m}^*/dv$ and $d\underline{m}^*/dv$ are both positive for small values of v, but negative for large values of v and there is a unique value of v at which both derivatives are zero. Instead, $d\overline{m}^*/d\beta$, and $d\underline{m}^*/d\beta$ are positive for all values of v. Therefore, Results 2 and 3 also hold with asymmetric firms.

In addition, evaluating dm_i/dr_j and dm_i/dr_i at the equilibrium, and differentiating with respect to v and β yields that the respective derivatives are negative, regardless of whether firm i or firm j are those with a high or a low customer base. This implies that Result 4 holds as well.

Finally, we note that considering more than two types of firms leads to similar results. The model becomes more complicated to solve, as, given that there then are k different types of firms, with $2 \leq k \leq M$, there are $3 \times k$ unknowns. However, the method is the same and the results are qualitatively similar.

Online Appendix D.3: Investment in Customer Retention has Positive Effects on Switching Consumers

In the main text, we considered the situation in which investment in customer retention affects the probability that a consumer of firm i repurchases from firm i. This is consistent with the findings of many papers in the Marketing literature. However, since a firm can achieve a larger customer retention e.g. by a rise in the quality of its products, it may therefore also affect the demand from switching consumers, that is, an increase r_i may also help firm i to attract a larger mass of switching consumers.

In this appendix, we consider the above scenario. To model this effect in a simple way, suppose that the probability that a switching consumer buys a product of firm i is

$$\frac{m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}, \tag{31}$$

where $\psi'(\cdot)>0$ and $\psi''(\cdot)<0$. This formulation implies that an investment in r_i does not only increase firm i's retention rate, but also the value that consumers attribute to firm i's products. Because $\psi(\cdot)$ is increasing but concave, a consumer who buys a product from firm i benefits if r_i is larger, but at a decreasing rate. This assumption also ensures that second-order conditions are satisfied.

Firm i's profit function can then be written as

$$\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p}) = \frac{r_i p_i}{M} + \frac{\sum\limits_{j=1}^{M}(1-r_j)}{M}\left(\frac{p_i m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}\right) - f\left(m_i\right) - c\left(r_i\right).$$

The resulting the first-order conditions are (using again that each firm i sets the same price for all of its products)

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i} = \frac{\sum\limits_{j=1}^{M}(1-r_j)p_i\left(v+\psi(r_i)-p_i\right)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial r_i} = \frac{p_i}{M}\left(1-\frac{m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}\right) + \sum_{j=1}^{M}(1-r_j)\times$$

$$\times \frac{m_i p_i \psi'(r_i)(v+\psi(r_i)-p_i)^{\frac{1-\beta}{\beta}} \left(\sum\limits_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)}{\beta M \left(\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)^2} - c'(r_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial p_i} = \frac{r_i}{M} + \frac{\sum\limits_{j=1}^{M}(1-r_j)}{M\beta} \frac{m_i\,(v+\psi(r_i)-p_i)^{\frac{1-\beta}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}} \times$$

$$\times \left(\beta\,(v+\psi(r_i)-p_i) - p_i + \frac{p_i m_i\,(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}\right) = 0.$$

From the first-order conditions for m_i and r_i, we now determine the relation between m_i and r_j as well as m_i and r_i, as in Result 1 of the main model. Using the first-order condition for m_i, we obtain that the sign of dm_i/dr_j is given by the sign of

$$-\frac{p_i\,(v-p_i)^{\frac{1}{\beta}} \left(\sum\limits_{j=1,\, i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} -$$

$$-\sum_{j=1}^{M}(1-r_j) \frac{p_i\psi'(r_i)(v+\psi(r_i)-p_i)^{\frac{1}{\beta}} m_j(v+\psi(r_j)-p_j)^{\frac{1-\beta}{\beta}} \sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}{M\beta\left(\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)^4} \times$$

$$\times \left(2\sum_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}} - \sum_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right).$$

The term in the first line is equivalent to that in the main model. Instead, the term in the second and third line is new and arises because of $\psi'(r_i) > 0$. Its sign depends again on the sign of the term in parentheses in the third line. For symmetric firms, this term is positive, which implies that the second term is negative overall and therefore the entire expression is strictly negative. The effect of the main model regarding dm_i/dr_j is then reinforced. In particular, if a rival firm j increases r_j, this does not only imply that fewer consumers switch, but also that the rival's products become more attractive. Investing in product portfolio size then becomes less profitable for firm i, as the firm can attract fewer consumers with each of its products.

Turning to dm_i/dr_i, we obtain that the sign of this term is given by the sign of

$$-\frac{p_i\left(v+\psi(r_i)-p_i\right)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_i)-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}+$$

$$+\sum_{j=1}^{M}(1-r_j)\frac{p_i\psi'(r_i)\sum\limits_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}(v+\psi(r_i)-p_i)^{\frac{1-\beta}{\beta}}}{M\beta\left(\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)^4}\times$$

$$\times\left(\sum_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}-m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}\right).$$

Again, the term in the first line is the same as that in the main model, whereas the new term is that in the second and third line. The sign of this term is again determined by the sign of the term in parentheses in the third line. If, for instance, there are more than two firms and firms are symmetric, this term is positive. It follows that the sign of dm_i/dr_i is no longer clear. The reason is that offering a larger product portfolio size becomes more valuable for firm i if r_i increases the value of each product. However, for this effect to dominate the effect of the main model, which is represented by the first line, $\psi'(r_i) > 0$ must be sufficiently large. If $\psi'(r_i)$ is rather small, $dm_i/dr_i < 0$ as in the main model.

Overall, this analysis shows that Result 1 of the main model tends to carry over to this extension. Although the sign of dm_i/dr_i, which is certainly negative in the main model, is now no longer clear, the term dm_i/dr_j remains negative and is even exacerbated. If $\psi'(r_i)$ is rather small, the sign of sign of dm_i/dr_i remains negative and so we get the same results as in the main model. However, even if it is large, the negative effect is likely to be dominating overall, as dm_i/dr_j applies to all competitors whereas dm_i/dr_i applies only to firm i. Hence, there is a strong indication that the relation between product variety and customer retention is negative.

We now turn to the equilibrium of the game. Denoting the equilibrium values again by m^*, r^*, and p^*, as in the main model, the three conditions determining the symmetric equilibrium can be written as

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2}-f'(m^*)=0, \quad \frac{(M-1)p^*\beta\left(v+\psi\left(r^*\right)-p^*\right)+\psi'\left(r^*\right)\left(1-r^*\right)}{M^2\beta\left(v+\psi\left(r^*\right)-p^*\right)}-c'(r^*)=0,$$

and

$$M\beta\left(v+\psi\left(r^*\right)-p^*\right)-(M-1)p^*\left(1-r^*\right)=0.$$

Following the proof of Result 2 and totally differentiating these first-order conditions with respect to m^*, r^*, p^*, and v, we can determine $dm^\star/dv$. Tedious but otherwise routine calculations show that

$$\text{sign}\left\{\frac{dm^\star}{dv}\right\} = \text{sign}\left\{\beta m^*(M-1)\mu\left[c''\left(r^*\right)M(1-r^*)\mu - \beta(M-1)\left(v+\psi\left(r^*\right)\right) + \psi''\left(r^*\right)\mu(1-r^*)\right]\right\},$$

with $\mu \equiv (M-1)(1-r^*) + \beta M$, as above. In the same way as in the proof of Result 2, we can show that this expression is positive if $v \to 0$, decreasing in v, and turning negative as v gets large. Hence, Result 2 of the main model also holds in this extension.

Similarly, totally differentiating these first-order conditions with respect to m^*, r^*, p^*, and β, we can solve for $dm^\star/d\beta$ to get

$$\text{sign}\left\{\frac{dm^\star}{d\beta}\right\} = \text{sign}\Big\{\left(v+\psi\left(r^*\right)\right)m^*\mu(M-1)^2 \times$$

$$\times\left[\beta(M-1)\left(v+\psi\left(r^*\right)+\psi''\left(r^*\right)\mu(1-r^*)+c''\left(r^*\right)M(1-r^*)\mu\right)\right]\Big\},$$

which is strictly positive.

Finally, to determine how dm_i/dr_j and dm_i/dr_i are affected by v and β in equilibrium, we can proceed in the same way as in the proof of Result 4 of the main model. This yields that $\partial\left(dm_i^*/dr_j^*\right)/\partial v < 0$ and $\partial\left(dm_i^*/dr_j^*\right)/\partial\beta < 0$. Therefore, the result that v and β reinforce the negative effect of r_j^* on m_i^* carries over to this extended model. For dm_i^*/dr_i^*, we obtain that $\partial\left(dm_i^*/dr_i^*\right)/\partial v < 0$ and $\partial\left(dm_i^*/dr_i^*\right)/\partial\beta < 0$ if $\psi'(r^*)$ is sufficiently small. This is in line with the previous result that $dm_i/dr_i < 0$ also only holds if $\psi'(r^*)$ rather small.

Online Appendix D.4: Switching Consumers do not Buy from the Same Firm

In the main model, we assumed that switching consumers make their choice among the products of all firms. This implies that a consumer who belongs to the customer base of firm i but is not retained by firm i, nevertheless takes the products of firm i into account and may buy one of its products. In this extension, we now consider a modification of our main model in which switching consumers do not buy from their respective original firm—e.g., because they were not satisfied with the product of that firm and therefore do not consider this firm's products in their choice set.

The profit function of firm i then takes the following form:

$$\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p}) = \frac{r_i p_i}{M} + \frac{\sum\limits_{j=1,j\neq i}^{M}(1-r_j)}{M}\left(\frac{p_i m_i (v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j (v-p_{\ell,j})^{\frac{1}{\beta}}}\right) - f\left(m_i\right) - c\left(r_i\right). \qquad (32)$$

The difference to the main model is only in the second term of the profit function, where the sum over the unsatisfied consumers now only considers firms $j = 1, ..., i-1, i+1, ..., M$, but no longer firm i. Otherwise, the profit function is unchanged.

In the same way as in Online Appendix C.1, we can show that a firm optimally charges the same prices for all of its products. The counterpart to the first-order conditions of the main text are the first-order conditions resulting from the maximization of (32). They are given by

$$\frac{\sum\limits_{j=1,j\neq i}^{M}(1-r_j)p_i\left(v-p_i\right)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{p_i}{M} - c'(r_i) = 0,$$

and

$$r_i + \frac{\sum\limits_{j=1,j\neq i}^{M}(1-r_j)}{\beta}\frac{m_i\left(v-p_i\right)^{\frac{1-\beta}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\left(\beta(v-p_i) - p_i + \frac{p_i m_i\left(v-p_i\right)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right) = 0.$$

It is easy to see that $dm_i/dr_j < 0$ still holds. However, as mentioned in the main text, $dm_i/dr_i = 0$ now. As consumers who switch away from firm i do not consider firm i's product in their choices, firm i cannot gain any of these consumers when offering a larger product variety. This implies that the choices m_i and r_i are no longer interdependent.

As in the main model, in a symmetric equilibrium, the three first-order conditions can be simplified to get

$$\frac{(1-r^*)(M-2)p^*}{m^* M(M-1)} - f'(m^*) = 0, \quad \frac{p^*}{M} - c'(r^*) = 0,$$

and

$$\frac{(M-1)\beta\left(v-p^*\right) - (M-1)p^*\left(1-r^*\right)}{m^* M(M-1)\beta\left(v-p^*\right)} = 0.$$

We can now follow the same procedure as in the proof of Results 2 and 3. First, determining dm^*/dv yields

$$\frac{dm^*}{dv}=\frac{(M-2)m^*\beta\hat{\mu}\left(c''(r^*)M(1-r^*)\hat{\mu}-v\beta(M-1)\right)}{\left(c''(r^*)M\hat{\mu}^2-v\beta(M-1)(M-2)\right)\left(v\beta(M-2)(1-r^*)+M\left(m^*\right)^2\hat{\mu}f''(m^*)\right)},$$

with $\hat{\mu}\equiv(M-2)(1-r^*)+M(\beta-1)>0$. In the same way as in Online Appendix B, we can show that it is non-monotonic in v, that is, the sign of dm^*/dv is determined by the sign of the numerator, which is positive for v small, but negative if v is above a threshold value. Second, determining $dm^*/d\beta$, we obtain

$$\frac{dm^*}{d\beta}=\frac{vm^*(1-r^*)\hat{\mu}(M-1)(M-2)\left(v\beta(M-1)+c''(r^*)M(1-r^*)\hat{\mu}\right)}{\left(c''(r^*)M\hat{\mu}^2-v\beta(M-1)(M-2)\right)\left(v\beta(M-2)(1-r^*)+M\left(m^*\right)^2\hat{\mu}f''(m^*)\right)}.$$

which is strictly positive, as the numerator is strictly positive.

Finally, we turn to Result 4. Evaluating dm_i/dr_j at the equilibrium yields

$$\frac{dm_i^*}{dr_j^*}=-\frac{\beta vm^*(M-2)}{2\beta v(1-r^*)(M-2)+f''(m^*)\left(m^*\right)^2M(M-1)\left(\beta(M-1)+(M-2)(1-r^*)\right)}.$$

Taking the derivative with respect to v, we obtain

$$-\Big[\Big(\left(f''(m^*)\right)^2\left(m^*\right)^4M^2(M-1)\left(M(\beta-1)+(M-2)(1-r^*)\right)^2+2\beta^2v^2(M-2)^2(1-r^*)^2\Big)c''(r^*)$$
$$+f''(m^*)\beta^2v^2(m^*)^2(M+1)(M-1)(M-2)-$$
$$-f'''(m^*)\beta v(m^*)^3(M-1)(M-2)\left(\beta v(M-1)-c''(r^*M(1-r^*)(M(\beta-1)+(M-2)(1-r^*))\right)\Big].$$

Similarly, taking the derivative with respect to β, we obtain

$$-\Big[\Big(\left(f''(m^*)\right)^2v\beta m^*M^2(M-1)+2(M-2)^2(1-r^*)^2\left(M(\beta-1)+(M-2)(1-r^*)\right)\Big)c''(r^*)$$
$$+f''(m^*)(m^*)^2(M+1)M\left(M(\beta-1)+(M-2)(1-r^*)\right)^2-$$
$$-f'''(m^*)\beta v(m^*)^3(M-1)(M-2)\left(\beta v(M-1)-c''(r^*M(1-r^*)(M(\beta-1)+(M-2)(1-r^*))\right)\Big].$$

Because $c''(\cdot)>0$, $f''(\cdot)\geq 0$, and $f'''(\cdot)$ is negative, or if positive, then small compared to the second derivatives, both of these expressions are negative. This shows that also Result 4 holds in case switching consumers do not buy from the same firm.

Online Appendix D.5: Customer Retention and Product Variety Choices Precede Price Choices

In the main model, we considered the case in which firms simultaneously choose product variety, customer retention, and product prices. In this section, we analyze the case in which the first two variables—i.e., product variety and customer retention—are chosen before product prices are set. A natural reason for such a sequential timing could be that product variety and customer retention are more long-term decisions than product prices. In particular, in some industries product prices can be changed at a relatively fast speed, whereas changing the product portfolio size or improving the functionality of products may take longer.[62]

The sequential game therefore unfolds as follows. In the first, stage, each firm $i = 1, ..., M$ chooses the mass of its products, m_i, and the customer retention level, r_i. Given these choices, in the second stage, each firm i sets prices for its products p_i. We analyze the game by backward induction and solve for the subgame-perfect equilibrium of the game.

In the second stage, the first-order condition with respect to p_i is the same as in the main model—i.e., (5)—taking m_i and r_i, $i = 1, ..., M$, from the previous stage as given.

In the first stage, envoking the Envelope-Theorem, the two first-order conditions for m_i and r_i can be written as

$$\frac{\partial \Pi_i}{\partial m_i} = \left.\frac{\partial \Pi_i}{\partial m_i}\right|_{sim} + \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial \Pi_i}{\partial p_j}\left(-\frac{\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2 \Pi_i}{\partial (p_j)^2}}\right) = 0 \tag{33}$$

and

$$\frac{\partial \Pi_i}{\partial r_i} = \left.\frac{\partial \Pi_i}{\partial r_i}\right|_{sim} + \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial \Pi_i}{\partial p_j}\left(-\frac{\frac{\partial^2 \Pi_j}{\partial p_j \partial r_i}}{\frac{\partial^2 \Pi_i}{\partial (p_j)^2}}\right) = 0, \tag{34}$$

respectively, where we used that $dp_j/dm_i = -\left(\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i}\right) / \left(\frac{\partial^2 \Pi_i}{\partial (p_j)^2}\right)$ and $dp_j/dr_i = -\left(\frac{\partial^2 \Pi_j}{\partial p_j \partial r_i}\right) / \left(\frac{\partial^2 \Pi_i}{\partial (p_j)^2}\right)$. In these equations, $\left.\frac{\partial \Pi_i}{\partial m_i}\right|_{sim}$ and $\left.\frac{\partial \Pi_i}{\partial r_i}\right|_{sim}$ are the respective first-order conditions from the simultaneous game and are given by (3) and (4). In addition to the first-order conditions of the simultaneous game, those of the sequential game also consider the effect that a change in m_i and r_i has on the prices chosen by rival firms in the second stage. This is represented by the second term in the two conditions (33) and (34).

[62] Nevertheless, as mentioned in the main text, in many digital markets, it is relatively simple and takes little time to add or withdraw products, and changes in the software code to improve the functionality can also be introduced at a fast rate. Therefore—and also to bring out our effects in the simplest way—we consider a simultaneous timing in the main model.

From firm i's profit function, we obtain

$$\frac{\partial \Pi_i}{\partial p_j} = \frac{\sum\limits_{j=1}^{M}(1-r_j)}{M}\left(\frac{m_i p_i (v-p_i)^{\frac{1}{\beta}}(v-p_j)^{\frac{1-\beta}{\beta}}}{\beta\left(\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right) > 0. \tag{35}$$

We now turn to the signs of the terms for dp_j/dm_i and dp_j/dr_i. The terms in the respective numerators are the cross derivatives of firm j's profit function with respect to p_j and m_i in (33) and with respect to p_j and r_i in (34). These terms are given by

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i} = -\frac{\sum\limits_{j=1}^{M}(1-r_j)}{M}\left(\frac{(v-p_j)^{\frac{1-\beta}{\beta}}(v-p_i)^{\frac{1}{\beta}}}{\beta\left(\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right)\left(2\frac{m_j p_j (v-p_j)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} - (p_j - \beta(v-p_j))\right) \tag{36}$$

and

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial r_i} = -\left(\frac{(v-p_j)^{\frac{1-\beta}{\beta}}}{M\beta\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}}\right)\left(\frac{m_j p_j (v-p_j)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} - (p_j - \beta(v-p_j))\right) > 0, \tag{37}$$

respectively. While we cannot determine the sign of (36), the sign of (37) is strictly positive. This is due to the fact that the first term in parentheses is positive, while the second term in parentheses is negative. The latter follows from the first-order condition for the prices, as given by (5). This first-order condition can only be satisfied if the following inequality holds:

$$p_j - \beta(v-p_j) > \frac{m_j p_j (v-p_j)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}}. \tag{38}$$

The reason is that the first term in (5)—i.e., $r_j \beta (v-p_j)^{(\beta-1)/\beta}/m_j$—is strictly positive, which implies that the second term must be negative. This is only true if (38) holds. The implication of this argument is that, in (37), the second term in parentheses is negative; hence, $\partial^2 \Pi_j / (\partial p_j \partial r_i)$ is positive. Finally, turning to (34) again, the denominator of the term in the large parentheses $\partial^2 \Pi_i / \partial (p_j)^2$, which is strictly negative because of the second-order condition. As a consequence, $dp_j/dr_i > 0$ then implies that the second term in (34) is positive.

Taken these results together, it follows that investment in customer retention is larger in the sequential timing as compared to the simultaneous timing. Due to the fact that the second term of (34) is positive, at the point of r_i at which the first-order condition in the simultaneous timing is fulfilled (i.e., the equilibrium value of r_i in the simultaneous timing), the first-order condition in the sequential timing is positive. It follows that the maximum in the sequential timing must lie to the right of the maximum of the simultaneous timing, which implies that the equilibrium investment in customer retention is larger in the sequential timing. The intuition is that investing more in customer retention by firm i induces fewer consumers to switch, which implies that competition for switching consumers is reduced. Hence, the pricing pressure on products is lower. This induces all rival firms to increase their prices, which is beneficial for firm i. Therefore, firm i has a stronger incentive to raise r_i.

Instead, for the optimal number of products, the direction of the change between sequential and simultaneous timing is not clear. This is because the sign of $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ is not clear-cut. It depends on the sign of the last term in parentheses. In contrast to the term in (37), this term has two times the positive expression $\left(p_j(v-p_j)^{\frac{1}{\beta}}\right)/\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)$ instead of only once. Therefore the term can either be positive or negative. The intuition is that an increase in m_i has a twofold effect on pricing incentives of rivals. First, each rival will serve fewer consumers with its products, which, similar to the effect outlined in the previous paragraph, induces rivals to increase prices. Second, a larger number of products by firm i enhances competition for switching consumers, which leads to downward pressure in prices. Therefore, the overall effect is ambiguous.

We now turn to the interaction between the number of products and customer retention, both between firms and within a firm. To determine these effects, we need to take the derivative of the first-order condition (33) with respect to r_j and r_i. Unfortunately, the resulting expressions are rather unwieldy without clear results. However, we performed numerous numerical simulations with different functions and verified that in almost all of them, the direction of the interaction between the number of products and customer retention is the same as in Result 1.[63]

We provide two reasons for the findings that we obtained in our simulations: First, in (33), the first term is the same as in the first-order condition for the simultaneous case, which by itself is responsible for Result 1. In the simulations, we obtain that for many parametrizations, the effect resulting from the first term is dominating the effect resulting from the second term. Second, the effect of the second term often also goes in the same direction as that of the first term. To see this, note that the second term in (33) is the

[63] The numerical simulations are available from the authors.

fraction between $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ and $\partial^2\Pi_i/\partial\left(p_j\right)^2$. In both terms, the levels of customer retention show up only in the term $\sum_{j=1}^{M}(1-r_j)$, which implies that they cancel out in the fraction. Therefore, the derivative of the second term of (33) with respect to r_j and r_i is equal to

$$\sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial^2\Pi_i}{\partial p_j \partial r_j}\left(-\frac{\frac{\partial^2\Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2\Pi_i}{\partial(p_j)^2}}\right) \quad \text{and} \quad \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial^2\Pi_i}{\partial p_j \partial r_i}\left(-\frac{\frac{\partial^2\Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2\Pi_i}{\partial(p_j)^2}}\right),$$

respectively. From (35), it is easy to see that $\partial^2\Pi_i/\left(\partial p_j \partial r_j\right) < 0$ and $\partial^2\Pi_i/\left(\partial p_j \partial r_i\right) < 0$. Moreover, as explained above, the sign of $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ in these expressions is determined by the sign of

$$\beta\left(v-p_j\right)-p_j+2\frac{p_j(v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}. \tag{39}$$

Inspection of (5) shows that for small values of r_i, the first term of (5) is small, which implies that the second term in the large parentheses in (5) must be close to zero, so that (5) holds. This, however, implies that the expression in (39) is positive. Taken together, for small values of customer retention, the derivative of the second term in (33) with respect to r_j and r_i, respectively, is negative, and therefore goes in the same direction as the first term, which amplifies Result 1.

We now turn to Results 2 and 3. In the symmetric equilibrium, all firms choose the same number of products, denoted by $m^\star$, and set the same customer retention level, denoted by $r^\star$, in the first stage, and, in the second stage, set the same price $p^\star$ for their products. As the first-order conditions in the second stage are the same as in the simultaneous game, the equation for the equilibrium price is the same as that shown in the proof of Result 2 and is given by

$$p^* = \frac{v\beta M}{\beta M + (M-1)(1-r^*)}.$$

Using it in the two first-order conditions (33) and (34) and simplifying, these first-order conditions are given by

$$\frac{v\beta(1-r^*)(M-1)}{Mm^\star\mu} - f'(m^\star) - \frac{v\beta(1-r^*)\left(r^*(M-1)-1\right)}{M\mu\left(Mm^\star(1+\beta)-2r^*(M-1)\right)} = 0, \tag{40}$$

and

$$\frac{v\beta(M-1)}{M\mu} - c'(r^\star) - \frac{v\beta m^\star r^*(M-1)}{M\mu\left(Mm^\star(1+\beta)-2s^*(M-1)\right)} = 0, \tag{41}$$

where, as above, $\mu \equiv (M-1)(1-r^*)+\beta M$.

Totally differentiating (40) and (41) with respect to m^*, r^*, p^*, and v, is tedious but standard calculations show that the sign of $m^\star/dv$ is given by the sign of

$$c''(s^\star)M(1-r^*)\mu\eta\left(m^\star(\beta M(M-1)+M^2-(M-1)(1-r^*))-2r^*(M-1)^2\right)-$$

$$-v\beta(M-1)\Big\{(m^\star)^2\left[\beta^2M^2(M-1)+\beta\left(2r^\star M(M-1)+2M^2(M-1)-M\right)+r^\star(M-1)(2M-r^\star)+\right.$$

$$\left.+(M^2-1)(M-1)\right]-2m^\star(M-1)\left(2\beta r^\star(M-1)+2r^\star(M-1)(M-r^\star)-1\right)+4\left(r^*\right)^2(M-1)^3\Big\}.$$

with $\eta \equiv Mm^\star(1+\beta)-2r^*(M-1)$. Following the method of the proof of Result 2, this expression is positive for $v \to 0$, but strictly decreasing in v and becomes negative as v gets large. Therefore, $m^\star$ changes non-monotonically with v—i.e., it is increasing in v if v is below a certain level, but decreasing for v above this level. Proceeding in the same way for $m^\star/d\beta$ yields that its sign is strictly positive. Therefore, Results 2 and 3 of the simultaneous model also hold with sequential decisions.

As we could show Result 1 on the interaction between the number of products and customer retention only numerically, we can determine Result 4 only numerically as well. Considering several concrete functions for $f(m_i)$ and $c(r_i)$, we numerically solve (40) and (41) for m^* and r^* and plug it into the expressions for dm_i/dr_j and dm_i/dr_i. Taking the respective derivatives with respect to v and β, we obtain that the result is negative, that is, an increase in the value of the market as well as in the degree of consumer heterogeneity strengthens the negative effect of consumer retention on a firm's product portfolio.

Online Appendix D.6: Concrete Example

In this appendix, we provide a concrete example that allows for closed-form solutions. Consider the following functional forms for the firms' cost functions: $f(m_i) = fm_i$ and $c(r_i) = cr_i^2$, that is, marginal costs for investment in product variety are constant and marginal costs for investment in customer retention are increasing. These assumptions seem reasonable in many industries: once a firm has entered a market segment (and incurred the respective fixed costs), the cost for launching an additional set of products in this segment is usually independent of the number of products. Instead, raising customer retention becomes increasingly costly as different instruments to do so have different costs (e.g., providing an update is usually cheaper than general improvements in product quality).

With this formulation, the three conditions that determine a symmetric equilibrium, are given by

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2}-f=0, \quad \frac{(M-1)p^*}{M^2}-2cr^*=0,$$

and

$$M\beta\left(v-p^{*}\right)-(M-1)p^{*}\left(1-r^{*}\right)=0.$$

Solving the last condition for p^* yields

$$p^{*}=\frac{M\beta v}{(M-1)(1-r^{*})+\beta M}. \tag{42}$$

Inserting this into the first condition and solving for m^*, we obtain

$$m^{*}=\frac{\beta v\left((M-1)(1-r^{*})\right)}{fM\left((M-1)(1-r^{*})+\beta M\right)}. \tag{43}$$

Inserting (42) and (43) into the second condition and solving for r^*, we obtain two solutions:

$$\frac{cM\left(M-1+\beta M\right)+\sqrt{\psi}}{2cM(M-1)} \quad \text{and} \quad \frac{cM\left(M-1+\beta M\right)-\sqrt{\psi}}{2cM(M-1)},$$

with $\psi\equiv cM\left(cM\left(M-1+\beta M\right)^{2}-2v\beta(M-1)^{2}\right)$. It is easy to check that only the second solution is in the admissible range as the first solution is above 1 for all admissible values, which is not possible due to the fact that r^* is a probability. Inserting the resulting solution for r^* back into the equations for m^* and p^*, the resulting equilibrium expressions for the three variables are

$$m^{*}=\frac{v\beta\left(\sqrt{\psi}+cM\left(M-1-\beta M\right)\right)}{fM\left(cM\left(M-1+\beta M\right)+\sqrt{\psi}\right)}, \quad r^{*}=\frac{cM\left(M-1+\beta M\right)-\sqrt{\psi}}{2cM(M-1)}, \tag{44}$$

and

$$p^{*}=\frac{2cM^{2}\beta v}{cM\left(M-1+\beta M\right)+\sqrt{\psi}}.$$

The resulting equilibrium profit in a symmetric equilibrium is

$$\frac{\beta v\left(cM^{2}(1+\beta)+3cM-\sqrt{\psi}\right)}{2M\left(cM^{2}(1+\beta)-cM+\sqrt{\psi}\right)}. \tag{45}$$

We can now determine the conditions so that our assumptions spelled out at the end of Section 2 are satisfied. First, from the second expression of (44), we obtain that $r^* < 1$ if $c < v(M-1)/(2M^2)$.[64] Second, because $c''(\cdot) = 2c$, the assumption that guarantees that the Hessian is negative definite is $c < v(M-1)^2/(2\beta M^3)$. Finally, to ensure that no global deviation exists, the profit in (45) must be larger than the profit from setting all prices equal

[64]This is also obtained from the assumption given as the end of Section 2, as $c'(1) = 2c$.

to v and not investing in product variety (i.e., $m_i = 0$). If a firm would follow this strategy, its optimal value of customer retention would be $v/(2cM)$, resulting in a profit of $v^2/(4cM^2)$. Subtracting this profit from the one in (45), we obtain that the difference is positive if

$$cM\left(2c\beta M(3+M+\beta M) - v(M-1+\beta M)\right) - (2c\beta M + v)\sqrt{\psi} > 0.$$

It is easy to show numerically that this inequality is satisfied as long as c is large enough. For instance, considering the values $M = 7$, $v = 5$, and $\beta = 2$, the inequality is satisfied for c approximately larger than 0.965. If β gets larger, which implies that product variety becomes more important, the inequality naturally becomes less tight—e.g. if $\beta = 4$, it is satisfied for c approximately larger than 0.556.

We now show Results 2-4 with the concrete example. Taking the derivative of m^* with respect to v yields that it is positive if and only if

$$v < \frac{cM\left(M-1+2\beta M\right)}{2\beta(M-1)},$$

thereby confirming Result 2.

Taking the derivative of m^* with respect to M yields

$$2c^2M^2(M-1)^2\left[cM\left(M(1+\beta)-1\right) + v\beta\left(\beta M + 2(M-1)\right)\right] +$$

$$+c^2M^2\left((M-1)^2 - \beta M\left(\beta M - 2(M-1)\right)\right)\sqrt{\psi} + (\psi)^{\frac{3}{2}},$$

which is strictly positive, due to the assumption that $c > v\left(M-1\right)^2/\left(2\beta M^3\right)$—i.e., the assumption that guarantees an interior solution. This confirms Result 3.

Finally, dm_i/dr_j (and dm_i/dr_i) are given by (13). Using the concrete example, we can set $f''(\cdot)$ equal to 0, and insert m^* and r^* determined above into (13). This yields

$$-\frac{cv\beta(M-1)}{f\left(cM\left(M-1+\beta M\right)+\sqrt{\psi}\right)}. \tag{46}$$

Taking the derivative of (46) with respect to v and simplifying, we obtain that the sign of this derivative is given by the sign of

$$-\left((\beta M + M - 1)\sqrt{\psi} + c\beta^2M^3 + cM(M-1)^2 + (M-1)(2cM^2 - v(M-1))\right),$$

which is strictly negative for all $c > v(M-1)^2/(2\beta M^3)$. Similarly, differentiating (46) with respect to β and simplifying yields that the sign of this derivative is given by the sign of

$$-\left(\sqrt{\psi} + cM(M-1) + \beta(2cM^2 - v(M-1))\right),$$

which is again strictly negative for all $c > v(M-1)^2/(2\beta M^3)$.

Online Appendix E: Details on Robustness

Online Appendix E.1: Additional OLS Tables

Table 6: Baseline Estimations without Transformations

	# Own Products				
# Updates	-0.001***			-0.001***	-0.002***
	(0.000)			(0.000)	(0.000)
Price		0.006***		0.007***	0.005***
		(0.001)		(0.001)	(0.001)
$Price^2$		-0.000***		-0.000***	-0.000***
		(0.000)		(0.000)	(0.000)
% w/o Popular Tag 1			0.200***	0.213***	0.418***
			(0.076)	(0.076)	(0.123)
Constant	1.106***	1.039***	0.949***	0.913***	0.677***
	(0.051)	(0.051)	(0.074)	(0.074)	(0.109)
Segment FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
No. of Obs.	38353	38387	38387	38353	17717

Notes: Column 5 only includes publishers with more than one observation. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 7: Baseline Estimations With More Controls

	# Own Products	
# Updates (ihs)	-0.020***	-0.030***
	(0.002)	(0.003)
Price (ihs)	0.288***	0.390***
	(0.020)	(0.033)
$Price^2$ (ihs)	-0.147***	-0.199***
	(0.011)	(0.018)
% w/o Popular Tag 1 (ihs)	0.068**	0.098**
	(0.034)	(0.048)
Prev. # Own Products	0.029***	0.026***
	(0.004)	(0.004)
% Positive Ratings	-0.000***	-0.001***
	(0.000)	(0.000)
# Ratings	0.000	0.000
	(0.000)	(0.000)
Website Dummy	-0.043***	-0.077***
	(0.005)	(0.008)
Size (in KB)	0.000**	0.000***
	(0.000)	(0.000)
Constant	0.920***	0.920***
	(0.033)	(0.044)
Segment FE	Yes	Yes
Year FE	Yes	Yes
Month FE	Yes	Yes
No. of Obs.	21959	12252

Notes: Column 2 only include publishers with more than one observation. The number of observations is lower due to missing information for ratings. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 8: Baseline Estimations with Other FE

	# Own Products	
# Updates (ihs)	-0.030***	-0.001
	(0.002)	(0.004)
Price (ihs)	0.267***	0.367***
	(0.021)	(0.065)
$Price^2$ (ihs)	-0.128***	-0.192***
	(0.012)	(0.036)
% w/o Popular Tag 1 (ihs)	-0.136	0.064
	(0.513)	(0.056)
% Positive Ratings	-0.000**	0.000
	(0.000)	(0.000)
# Ratings	0.000	0.000
	(0.000)	(0.000)
Website Dummy	-0.048***	-0.035***
	(0.005)	(0.013)
Size (in KB)	0.000***	0.000
	(0.000)	(0.000)
Constant	1.035**	0.801***
	(0.409)	(0.059)
Segment FE	Yes	Yes
Year FE	Yes	Yes
Month FE	Yes	Yes
Segment x Year FE	Yes	No
Publisher FE	No	Yes
No. of Obs.	21959	11078

Notes: Column 1 additionally includes Segment x Year FE and Column 2 includes publisher FE. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Online Appendix E.2: Further Robustness Checks

Addressing Survivorship Bias: Having data from one period in time may lead to missing firms (and products) that were active in the past but exited until that date. However, for video games the costs to stay in the market are very low as there are e.g. no recurring store fees or maintenance costs. One can see this, for example, by comparing video games with a release date from 2014 in our data (1,522) to the total of video games published on Steam in 2014 (1,772).[65] This means that the majority of video games from 2014 (about 85 percent) are still in the market. Furthermore, we crawled all games available on Steam exactly one year after the initial data collection in the same way. It shows that more than 94 percent of the games in the estimation sample are still available in August 2023.

Of course, there are shocks affecting the market environment by removals of firms or increasing costs/decreasing revenues through new policies, thus creating larger exits. However, given the low costs to stay in the market, one can presume that only very low-value video games leave the market. These games do not play a crucial role for the market. Finally, we also account for year-specific effects by including release year fixed effects in our regressional analyses.

Publisher vs. Developer: An important distinction is the one between developers and publishers in digital markets, especially for video games where the development and distribution is more expensive. A (too) simple way of describing the roles would be that a developer programs a software, while the publisher is in charge of selling it. However, the actual relationship between the two parties is more complicated and the degree of influence varies. Following the classification on Steam, we take the field "Publisher" on each page of a video game to identify a producer. In case this information is missing, we take the field "Developer". In two-third of the cases for our video games, the two fields coincide and thus it is the same entity. Re-running our analysis with developers instead of publishers leads to qualitatively very similar results.

[65] See `https://www.polygon.com/platform/amp/2016/12/1/13807904/steam-releases-2016-growth`.

Online Appendix E.3: Details on Exchange Rates as an Instrument

Procedure to match supported languages on Steam with exchange rates:

1. Map language data from Steam to the official names of the main national languages[66] and those that are not official national languages directly to the ISO country codes.

2. Assign currencies (from Western Union) and languages to the ISO country codes and thus have a mapping between currencies and languages.

3. Find and take the most relevant currencies, i.e. currencies of the largest economic areas (based on total GDP, World Bank).

4. Some manual corrections are made, as Steam has some peculiarities for specific languages: For "Spanish - Spain", only Spain is taken into account; for "Spanish - Latin America", only the largest Latin American countries are taken into account; similar procedures for "Portuguese - Portugal" & "Portuguese - Brazil".

5. Correct country code (if unique) or the 3 most relevant country codes (if not unique) are assigned to the language information from Steam.

6. Currency rates (scraped via Yahoo Finance API) are then assigned via these country codes. Currency pair X - USD in the period 2012-2022 are queried.

[66] See https://plos.figshare.com/articles/dataset/List_of_official_and_most_spoken_languages_for_each_country_in_the_world_/19622821/1.

Online Appendix E.4: Additional IV Table

Table 9: Additional Instrumental Variable Estimations

	1st Stage Price	1st Stage $Price^2$	2nd Stage # Own Products
# Updates	0.029***	0.178***	-0.002***
	(0.001)	(0.024)	(0.000)
Price			0.034***
			(0.007)
$Price^2$			-0.000***
			(0.000)
% w/o Popular Tag 1	0.484	-3.962	0.198*
	(0.630)	(12.001)	(0.115)
Avg. Exchange Rate to USD	-5.112***	-86.539***	
	(0.189)	(3.595)	
1st Stage $Residuals^2$	0.003***	1.040***	
	(0.000)	(0.000)	
Constant	13.256***	160.729***	0.713***
	(0.670)	(12.754)	(0.126)
Segment FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Month FE	Yes	Yes	Yes
1st Stage F-Test	369.21		
Endogeneity Test p-value	0.00		
No. of Obs.	38353	38353	38353

Notes: Columns 1 and 2 represent first-stage regressions, while column 3 shows the second-stage regression. 1st Stage $Residuals^2$ is a variable corresponding to the squared residuals of a regression of prices on the instrument, Avg. Exchange Rate to USD, along with all the other explanatory variables of the baseline estimation. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

References

[1] Gilardi F, Alizadeh M, Kubli M (2023) ChatGPT outperforms crowd workers for text-annotation tasks. *Proceedings of the National Academy of Sciences* 120(30), e2305016120.