	I	A	B	C	D	E
I	I	A	B	C	D	E
A	A	B	I	E	C	D
B	B	I	A	D	E	C
C	C	D	E	I	A	B
D	D	E	C	B	I	A
E	E	C	D	A	B	I

Table 28.8 The group table, under matrix multiplication, for the set $\mathcal{M}$ of six orthogonal 2×2 matrices given by (28.13).

The similarity to table 28.7 is striking. If {R,R',K,L,M} of that table are replaces by {A,B,C,D,E} respectively, the two tables are identical, without even the need to reshuffle the rows and columns. The two groups, one of reflections and rotations of an equilateral triangle, the other of matrices, are isomorphic.

Our second example of a group isomorphic to the same rotation-reflection group is provided by a set of functions of an undetermined variable x. The functions are as follows:

$$\mathrm{f}_1(x) = x, \quad \mathrm{f}_2(x) = 1/(1-x), \quad \mathrm{f}_3(x) = (x-1)/x,$$
$$\mathrm{f}_4(x) = 1/x, \quad \mathrm{f}_5(x) = 1-x, \quad \mathrm{f}_6(x) = x/(x-1),$$

and the law of combination is

$$f_i(x) \bullet f_j(x) = f_i(f_j(x)),$$

i.e. the function on the right acts as the argument of the function on the left to produce a new function of x. It should be emphasised that it is the functions that are the elements of the group. The variable x is the 'system' on which they act, and plays much the same role as the triangle does in our first example of a non-Abelian group.

To show an explicit example, we calculate the product $f_6 \bullet f_3$. The product will be the function of x obtained by evaluating y/(y-1), when y is set equal to (x-1)/x. Explicity

$$f_6(f_3) = \frac{(x-1)/x}{(x-1)/x - 1} = 1 - x = f_5(x).$$

Thus $f_6 \bullet f_3 = f_5$. Further examples are

$$f_2 \bullet f_2 = \frac{1}{1 - 1/(1-x)} = \frac{x-1}{x} = f_3,$$

and

$$f_6 \bullet f_6 = \frac{x/(x-1)}{x/(x-1) - 1} = x = f_1. \tag{28.14}$$