eksāmens 3

Reinis Lācis, REBC01

May 2019

Orģinālā lapa:

where

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{1-\rho} \right]^{-1}.$$

Performance Measures for the *M/M/c* Queue

It is clearly easier to begin by computing the expected queue length L_q rather than the mean number in the system, since the only p_n's involved are those for which $n \geq c$. These are the only states for which customers must wait in the queue. We have

$$L_q = \sum_{n=c}^{\infty} (n-c) p_n$$

with

$$p_n = \frac{(\rho c)^n}{c^{n-c} c!} p_0 \quad \text{for } n \geq c,$$

which implies that

$$L_q = \sum_{n=c}^{\infty} \frac{n}{c^{n-c} c!} (\rho c)^n p_0 - \sum_{n=c}^{\infty} \frac{c}{c^{n-c} c!} (\rho c)^n p_0.$$

Let us consider each of the terms on the right-hand side separately. The first term is given by

$$\frac{p_0}{c!} \sum_{n=c}^{\infty} \frac{n(\rho c)^n}{c^{n-c}}.$$

Notice that, since

$$\frac{n\rho^n c^n}{c^{n-c}} = n\rho^n c^c = \frac{n\rho^{n-c-1} \rho^{c+1} c^{c+1}}{c} = \frac{(\rho c)^{c+1}}{c} n\rho^{n-c-1},$$

we have

$$\begin{aligned}
\frac{p_0}{c!} \sum_{n=c}^{\infty} \frac{n(\rho c)^n}{c^{n-c}} &= \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c} \right] \sum_{n=c}^{\infty} \left[(n-c)\rho^{n-c-1} + c\rho^{n-c-1} \right] \\
&= \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c} \right] \left\{ \sum_{n=c}^{\infty} (n-c)\rho^{n-c-1} + \sum_{n=c}^{\infty} c\rho^{n-c-1} \right\} \\
&= \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c} \right] \left\{ \frac{1}{(1-\rho)^2} + \frac{c}{\rho} \frac{1}{1-\rho} \right\},
\end{aligned}$$

using derivatives of the geometric series,

$$= \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c} \right] \left\{ \frac{1}{(1-\rho)^2} + \frac{c/\rho}{1-\rho} \right\}.$$

Now for the second term. We have

$$\begin{aligned}
\frac{p_0}{c!} \sum_{n=c}^{\infty} \frac{c(\rho c)^n}{c^{n-c}} &= \frac{p_0}{c!} \sum_{n=c}^{\infty} c\rho^c \rho^{n-c} c^c = \frac{p_0}{c!} c(\rho c)^c \sum_{n=c}^{\infty} \rho^{n-c} \\
&= \frac{p_0}{c!} \frac{c(\rho c)^c}{1-\rho} = \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c} \right] \frac{c/\rho}{1-\rho}.
\end{aligned}$$

Manis izveidotā lapa:

where

$$p_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!} \frac{1}{1-\rho}\right]^{-1}$$

Performance Measures for the $M/M/c$ Queue

It is clearly easier to begin by computing the expected quene lenghy L_q rather than the mean number in the system, since the only ρ_n's involved are those for wich n $\geqslant$ c. These are the only states for which customers must wait in the queue. We have

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with

$$p_n = \tfrac{(\rho c)^n}{c^{n-c} c!} p_0 \quad \text{for n} \geqslant \text{c}$$

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we have

$$\frac{p_0}{c!} \sum_{n=c}^{\infty} \frac{n(\rho c)^n}{c^{n-c}} = \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c}\right] \sum_{n=c}^{\infty} \left[(n-c)\rho^{n-c-1} + c\rho^{n-c-1}\right]$$

$$= \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c}\right] \left\{\sum_{n=c}^{\infty} (n-c)\rho^{n-c-1} + \sum_{n=c}^{\infty} c\rho^{n-c-1}\right\}$$

$$= \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c}\right] \left\{\frac{1}{(1-\rho)^2} + \frac{c}{\rho} \frac{1}{1-\rho}\right\},$$

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$$\frac{p_0}{c!} \sum_{n=c}^{\infty} \frac{c(\rho c)^n}{c^{n-c}} = \frac{p_0}{c!} \sum_{n=c}^{\infty} c\rho^c \rho^{n-c} c^c = \frac{p_0}{c!} c(\rho c)^c \sum_{n=c}^{\infty} \rho^{n-c}$$

$$= \frac{p_0}{c!} \frac{c(\rho c)^c}{1-\rho} = \frac{p_0}{c!} \left[\frac{(\rho c)^{c+1}}{c}\right] \frac{c/\rho}{1-\rho}.$$

Kods:

```
\documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage[english]{babel}
\usepackage{fancyhdr}
\usepackage{amssymb}
\usepackage{graphicx}
\usepackage{geometry}
 \geometry{
 a4paper,
 total={210mm,350mm},
 left=29mm,
 right=29mm,
 top=30mm,
 }

\title{eksāmens 3}
\author{Reinis Lācis, REBC01 }
\date{May 2019}

\begin{document}

\maketitle

\newpage
\textbf{Orģinālā lapa:}

\includegraphics[scale=0.15, angle=-90]{orginals3.jpg}

\vspace{40mm}
\textbf{Manis izveidotā lapa:}
\newpage

\begin{flushright}
\textbf{11.4 Multiserver Systems 421}
\end{flushright}
\noindent\makebox[\linewidth]{\rule{\textwidth}{0.1pt}}

\noindent where
$$p_0=\Bigg[\sum_{n=0}^{c-1}
\frac{(c\rho)^n}{n!}+
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\vspace{5mm}
\noindent\textbf{Performance Measures for the \emph{M/M/c} Queue}

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\noindent with


\begin{center}
\begin{tabular}{ ccc }
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\noindent Notice that, since

$$\frac{n\rho ^nc^n}{c^{n-c}}
=n\rho ^nc^c=\frac{n\rho ^{n-c-1}\rho
^{c+1}c^{c+1}}{c}=\frac{(\rho c)^{c+1}}{c}n\rho ^{n-c-1},$$

\noindent we have

$$\frac{p_0}{c!}\sum_{n=c}^{\infty}\frac{n(\rho c)^n}{c^{n-c}}=\frac{p_0}{c!}\Bigg
[\frac{(\rho c)^{c+1}}{c}\Bigg]\sum_{n=c}
^{\infty}\big[(n - c)\rho
^{n-c-1}+c\rho ^{n-c-1}\big]$$
$$=\frac{p_0}{c!}\Bigg[\frac{(\rho c)^{c+1}}{c}\Bigg]\Bigg
\{\sum_{n=c}^{\infty}(n - c)\rho ^{n-c-1}+
\sum_{n=c}^{\infty}c\rho ^{n-c-1}\Bigg\}$$
$$=\frac{p_0}{c!}\Bigg[\frac{(\rho c)
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\{\frac{1}{(1 - \rho )^2}+
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\noindent Now for the second term. We have

$$\frac{p_0}{c!}\sum_{n=c}^{\infty}\frac{c(\rho c)^n}{c^{n-c}}=\frac{p_0}{c!}\sum_{n=c}
^{\infty}c\rho ^c\rho ^{n-c}c^c=\frac{p_0}{c!}c(\rho c)
^c\sum_{n=c}^{\infty}\rho ^{n-c}$$

$$=\frac{p_0}{c!}\frac{c(\rho c)^c}
{1 - \rho}=\frac{p_0}{c!}
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\newpage
\textbf{Kods:}
\begin{verbatim}
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