COST FUNCTION

SHOW THAT: $\frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)})-y^{(i)}\right)^2$

equals

$$\frac{1}{2m}(\vec{X}\vec{\theta}-\vec{y})^T(\vec{X}\vec{\theta}-\vec{y})$$

PROOF

$\underbrace{\begin{bmatrix} 1 & x_1^0 & y_2^0 \\ 1 & x_1^1 & x_2^1 \\ \vdots & \vdots & \vdots \\ 1 & & \end{bmatrix}}_{m\times n+1}$ $\underbrace{\begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \vdots \end{bmatrix}}_{n+1\times 1}$

$\sum_{i=1}^{m}\theta_i x_i^0$

$\sum^{m}\theta_i x_i^1$

$$\vec{X}\vec{\theta}=\overbrace{\begin{bmatrix} \boxed{\theta_0+\theta_1 x_1^0+\theta_2 x_2^0} \\ \boxed{\theta_0+\theta_1 x_2^1+\theta_2 x_2^1} \\ \theta+\theta_1 x_2^1+\theta_3 x_3^2 \end{bmatrix}}^{m\times 1}$$

$$\Rightarrow \underbrace{\vec{X}\vec{\theta}}_{m\times 1}-\underbrace{\vec{y}}_{m\times 1}=\begin{bmatrix} \theta_0+\theta_1 x_1^1+\theta_2 x_2^1-y_1 \\ \theta_0+\theta_1 x_2^1+\theta_2 x_2^2-y_2 \\ \dots \end{bmatrix}$$

$\sum_{i=1}^{m}(\theta^T \vec{x}^{(i)}-\vec{y}^{(i)})$

$$=\underbrace{\begin{bmatrix} error_1 \\ error_2 \\ \vdots \\ error_m \end{bmatrix}}_{m\times 1}$$ so for $\frac{1}{2m}(X\theta)(X\theta)^T$

$$\Rightarrow \frac{1}{2m}\underbrace{\begin{bmatrix} error_1, error_2, \dots error_m \end{bmatrix}}_{1\times m}\times\underbrace{\begin{bmatrix} error_1 \\ \vdots \\ error_m \end{bmatrix}}_{m\times 1}=$$

$$=\frac{1}{2m}\Big[\underbrace{error_1^2+\dots+error_m^2}_{\sum_{i=1}^{m}(\theta^T\vec{x}^{(i)}-\vec{y}^{(i)})^2}\Big]=\frac{1}{2m}\,error_{total}^2 \in \mathbb{R}$$

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