Drifter dispersion in the Adriatic Sea:
Lagrangian data and chaotic model

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Abstract. We analyze characteristics of drifter trajectories from the Adriatic Sea with recently introduced nonlinear dynamics techniques. We discuss how in quasi-enclosed basins, relative dispersion as a function of time, a standard analysis tool in this context, may give a distorted picture of the dynamics. We further show that useful information may be obtained by using two related non-asymptotic indicators, the Finite-Scale Lyapunov Exponent (FSLE) and the Lagrangian Structure Function (LSF), which both describe intrinsic physical properties at a given scale. We introduce a simple chaotic model for drifter motion in this system, and show by comparison with the model that Lagrangian dispersion is mainly driven by advection at sub-basin scales until saturation sets in.

Key words. Oceanography: General (marginal and semi-closed seas) – Oceanography: Physical (turbulence, diffusion, and mixing processes; upper ocean processes)

1 Introduction

Understanding the mechanisms of transport and mixing processes is an important and challenging task which has wide relevance from a theoretical point of view, e.g. for the study of diffusion and chaos in geophysical systems in general or for validating simulation results from a general circulation model. It is also a necessary tool in the analysis of problems of general interest and social impact, such as the dispersion of nutrients or pollutants in sea water with consequent effects on marine life and on the environment (Adler et al., 1996).

Recently, a number of oceanographic programs have been devoted to the study of the surface circulation of the Adriatic Sea by the observation of Lagrangian drifters within the larger framework of drifter-related research in the whole Mediterranean Sea (Poulain, 1999). The Adriatic Sea is a quasi-enclosed basin, about 800 long by 200 km wide, connected to

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the rest of the Mediterranean Sea through the Otranto Strait. From a topographic point of view, three major regions can be considered. The northern part is the shallowest, about 100 m maximum depth, and extends down to the latitude of Ancona. The central part extends down to about 260 m in the Jabuka Pit, and the southern part extends from the Gargano promontory to the Otranto Strait. The southern part is the deepest, reaching about 1200 m in the South Adriatic Pit. Reviews on the oceanography of the Adriatic Sea can be found in Artegiani et al. (1997), Orlic et al. (1992), Poulain (1999) and Zore (1956).

Lagrangian data offer the opportunity to employ techniques of analysis well established in the theory of chaotic dynamical systems to study the behavior of actual trajectories and compare those with a kinematic model.

Let us assume that the Lagrangian drifters are passively advected in a two-dimensional flow, e.g. as would be the case in a frictionless barotropic approximation (Ottino, 1989; Crisanti et al., 1991):

$$\frac{dx}{dt} = u(x, y, t) \quad \text{and} \quad \frac{dy}{dt} = v(x, y, t), \qquad (1)$$

where $(x(t), y(t))$ is the position of a fluid particle at time t in terms of longitude and latitude, and u and v are the zonal and meridional velocity fields respectively.

For the Eulerian description of a geophysical system, one should in principle use numerical solutions of the Navier-Stokes equations (or other suitable equations, e.g. the quasi-geostrophic model) to obtain the velocity fields. In practice, direct numerical simulation of these equations on oceanographic length scales is of course not possible, and one has to invoke approximations, i.e. turbulence modeling. This motivates one to use instead a simplified kinematic approach by adopting a given Eulerian velocity field. The criteria for the construction of such a field follows from phenomenological arguments and/or experimental observation and have recently been reviewed in Yang (1996) and Samelson (1996).

Let us consider the relationship between Eulerian and Lagrangian properties of a system. A wide range of literature on

this topic (e.g. Ottino, 1989; Crisanti et al., 1991) allows us to state that, in general, Eulerian and Lagrangian behaviours are not strictly related to each other. It is not rare to have regular Eulerian behavior, e.g. a time-periodic velocity field co-existing with Lagrangian chaos or vice-versa.

In quasi-enclosed basins like the Adriatic Sea, a characterization of the mechanisms of the mixing is highly non-trivial. We first observe (see below for detailed discussion) that the use of the standard diffusion coefficients can have rather limited applicability (see Artale et al., 1997). Classical studies on Lagrangian particles in ocean models already contain remarks on the intrinsic difficulties in using one-particle diffusion statistics (Taylor, 1921). In situations where the advective time is not much longer than the typical decorrelation time scale of the Lagrangian velocity, the diffusivity parameter related to small-scale turbulent motion cannot converge to its asymptotic value (Figueroa and Olson, 1994). On the other hand, a generalization of the standard Lyapunov exponent, the Finite-Scale Lyapunov Exponent (FSLE), originally introduced for the predictability problem (Aurell et al., 1996, 1997), has been shown to be a suitable tool to describe non-asymptotic properties of transport. This finite-scale approach to Lagrangian transport measures effective rates of particle dispersion without assumptions about small-scale turbulent processes. For an alternative method, see Buffoni et al. (1997) and for a recent review and systematic discussion of non-asymptotic properties of transport and mixing in realistic cases, see Boffetta et al. (2000).

In this paper we report data analysis of surface drifter motion in the Adriatic Sea using relative dispersion, FSLE and Lagrangian Structure Function (LSF), a quantity related to the FSLE. We also introduce a chaotic model for the Lagrangian dynamics and use the FSLE and LSF characteristics to compare model and data. We show that it can be very difficult to obtain an estimate of the diffusion coefficient in a quasi-enclosed basin, and/or to look for deviations from the standard diffusion law. In fact, the time a cluster of particles takes to spread uniformly and reach the boundaries is not much longer than the largest characteristic time of the system. In contrast, the FSLE and the LSF do characterize the transport properties of Lagrangian trajectories at a fixed spatial scale. Finally, we will show that a simple kinematic model reproduces the data.

In Sect. 2 we describe the data set we have used as well as review relevant concepts and analysis techniques for Lagrangian transport and chaos. In Sect. 3 we introduce a kinematic model of the Lagrangian dynamics, and in Sect. 4, we compare the data and the model. Section 5 contains a summary and a discussion of the results.

2   Data set and analysis techniques

2.1   Data set

In a large drifter research program in the Mediterranean Sea, started in the late 80's and continued into the 90's, Lagrang-

ian data from surface drifters deployed in the Adriatic sea have been recorded from December 1994 to March 1996. These drifters are similar to the CODE (COastal Dynamics Experiment) system (Davis, 1985) and they are designed to be sufficiently wind-resistant so as to effectively give a description of the circulation at their actual depth (1 meter). The drifters were tracked by the Argos Data Location and Collection System (DCLS) carried by the NOAA polar-orbiting satellites. It is assumed that after data processing drifter positions are accurate to within 200–300 m, and velocities to within 2–3 cm/s. For a description of the experimental program, see Poulain (1999) and for technical details about the treatment of raw data, see Hansen and Poulain (1996), Poulain et al. (1996) and Poulain and Zanasca (1998).

The data have been stored in separate files, one for each drifter. In the format used by us each file contains: a number of records (i.e. number of points of the trajectory); the time in days; the position of the drifter in longitude and latitude; the velocity of the drifter along the zonal and meridional directions; and the temperature in centigrade degrees. The sampling time is 6 hours. We can identify five main deployments on which we will concentrate our attention. Selecting the tracks by the time of the first record, it is easy to verify that these five subsets consist of drifters deployed in the same area in the Otranto strait near 19 degrees longitude east and 40 degrees latitude north. The experimental strategy of simultaneously releasing the drifters within a distance of some kilometers allows us to study dispersion quantitatively.

From a qualitative point of view, what we observe from the plot of all the trajectories (Fig. 1) is the shape of two (cyclonic) basin-wide gyres, located in the middle and southern regions respectively and, an anti-clockwise boundary current which moves the drifters north-westward along the east coast and south-eastward down the west coast. The latter is a permanent feature of the Adriatic sea (Poulain, 1999). On the other hand, it is known that within a year the pattern of basin-wide gyres may change between one, two and three gyres over a time-scale of months. The southern gyre is the most steady of the three. The data also suggests the presence of small scale structures, even though these are much more likely to be variable in time. The time-scale of the typical recirculation period around a basin-wide gyre is about one month and the time needed to travel along the coasts and complete one lap of the full basin is in the order of a few months.

2.2   Analysis techniques

We recall here some basic concepts about dynamical systems, diffusion and chaos, and the quantities that we shall use to characterize the properties of Lagrangian trajectories.

If we have N_c clusters of initially close particles with each cluster containing n_k elements, relative dispersion can be characterized by the diffusion coefficient

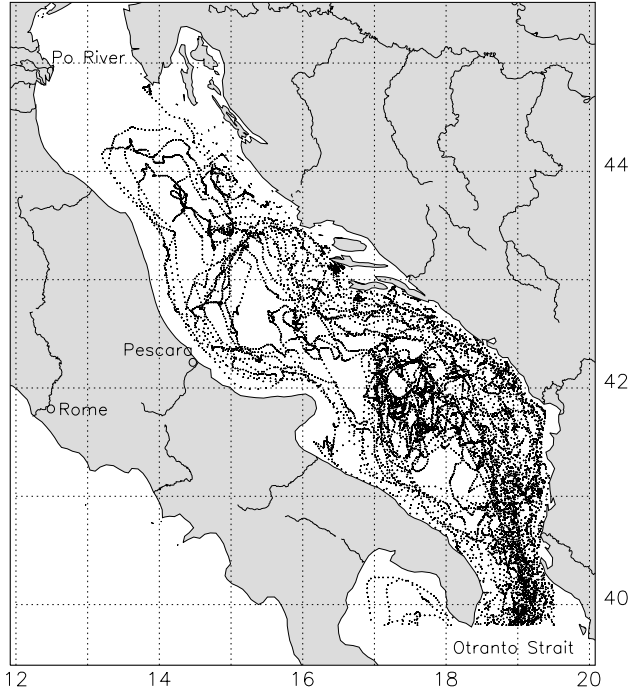$$D_i = \lim_{t \to \infty} \frac{1}{2t} S_i^2(t) \qquad (2)$$

Fig. 1. Plot of the 37 drifter trajectories in the Adriatic Sea used for the data analysis. The longitude east and latitude north coordinates are in degrees. The drifters were deployed on the eastern side of the Otranto strait.

with

$$S_i^2(t) = \frac{1}{N_c} \sum_{k=1}^{N_c} \frac{1}{n_k} \sum_{j=1}^{n_k} (x_i^{(k,j)}(t) - \langle x_i(t) \rangle^{(k)})^2 \qquad (3)$$

where

$$\langle x_i(t) \rangle^{(k)} = \frac{1}{n_k} \sum_{j=1}^{n_k} x_i^{(k,j)}(t) \qquad (4)$$

$x_i^{(k,j)}$ is the i-th spatial coordinate of the j-th particle in the k-th cluster; $S^2 = \sum_i S_i^2$ is the mean square displacement of the particles relative to their time evolving mean position. If $\delta(t)$ is the distance between two trajectories $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$ in a cluster at time t, relative dispersion is defined as

$$\langle \delta^2(t) \rangle = \langle ||\boldsymbol{x}^{(1)}(t) - \boldsymbol{x}^{(2)}(t)||^2 \rangle \qquad (5)$$

where the average is over all pairs of trajectories in the cluster. In a standard diffusive regime, $\boldsymbol{x}^{(1)}(t)$ and $\boldsymbol{x}^{(2)}(t)$ become independent variables and, for $t \to \infty$, we have $\langle \delta^2(t) \rangle = 2S^2(t)$. In the following, we shall consider the cluster mean square radius $S^2(t)$ as a measure of relative dispersion. Absolute dispersion, which is defined as the mean square displacement from an initial position, will not be taken into account in our analysis.

If, in the asymptotic limit, $S_i^2(t) \sim t^{2\alpha}$ with $\alpha = 1/2$, we have the linear law of standard diffusion for the mean square displacement and the D_i's are finite. If $\alpha \neq 1/2$, we have

a so-called anomalous diffusion (Bouchaud and Georges, 1990).

The difficulty which often arises when measuring the exponent α is that, due to the finite size of the domain, dispersion cannot reach its true asymptotic behavior. In other words, diffusion may not be observable over sufficiently large scales, i.e. much larger than the largest Eulerian length scale. Therefore, we cannot have a robust estimate of the exponent of the asymptotic power law. Moreover, the relevance of asymptotic quantities, like the diffusion coefficients, is questionable in the study of realistic cases concerning the transport problem in finite-size systems (Artale et al., 1997).

The diffusion coefficients characterize long-time (large-scale) dispersion properties. In contrast, at short times (small scales) the relative dispersion is related to the chaotic behavior of the Lagrangian trajectories.

A quantitative measure of instability for the time evolution of a dynamical system (Lichtenberg and Lieberman, 1992) is commonly given by the Maximum Lyapunov Exponent (MLE) λ, which gives the rate of exponential separation of two nearby trajectories

$$\lambda = \lim_{t \to \infty} \lim_{\delta(0) \to 0} \frac{1}{t} \ln \frac{\delta(t)}{\delta(0)} \qquad (6)$$

where $\delta(t) = ||\boldsymbol{x}^{(1)}(t) - \boldsymbol{x}^{(2)}(t)||$ is the distance between two trajectories at time t. When $\lambda > 0$ the system is said to be chaotic. There exists a well-established algorithm to numerically compute the MLE introduced by Benettin et al. (1980).

A characteristic time, T_λ, associated to the MLE is the predictability time, defined as the minimum time after which the error on the state of the system becomes larger than a tolerance value Δ, if the initial uncertainty is δ (Lichtenberg and Lieberman, 1992):

$$T_\lambda = \frac{1}{\lambda} \ln \frac{\Delta}{\delta} \qquad (7)$$

Let us recall that λ is a mathematically well-defined quantity which measures the growth of infinitesimal errors. In physical terms, at any time, δ has to be much less than the characteristic size of the smallest relevant length of the velocity field. For example, in 3D fully developed turbulence, δ has to be much smaller than the Kolmogorov length.

When the uncertainty reaches non-infinitesimal sizes, i.e. macroscopic scales, the perturbation δ is governed by the nonlinear terms and that renders its growth rate a scale-dependent index (Aurell et al., 1996, 1997; Artale et al., 1997). It is useful to introduce the Finite Scale Lyapunov Exponent (FSLE), $\lambda(\delta)$. Assuming $r > 1$ is a fixed amplification ratio and $\langle \tau_r(\delta) \rangle$ the mean time that δ takes to grow to $r \cdot \delta$, we have:

$$\lambda(\delta) = \frac{1}{\langle \tau_r(\delta) \rangle} \ln r \qquad (8)$$

The average $\langle \cdot \rangle$ is performed over all the trajectory pairs in a cluster. We note the following properties of the FSLE:

a) in the limit of infinitesimal separation between trajectories, $\delta \to 0$, the FSLE tends to the maximum Lyapunov exponent (MLE);

b) in case of standard diffusion, $\langle \delta(t)^2 \rangle \sim t$, we find that $\lambda(\delta) \sim \delta^{-2}$ and the proportionality constant is of the order of the diffusion coefficient;

c) any slope > -2 for $\lambda(\delta)$ vs δ indicates super-diffusive behavior, i.e. non-neglectable correlations persist at long times and advection is still relevant;

d) in particular, when $\lambda(\delta) = $ constant over a range of scales, we have exponential separation between trajectories at a constant rate within that range of scales (chaotic advection).

Another interesting quantity related to the FSLE is the Lagrangian Structure Function (LSF) $\nu(\delta)$, defined as

$$\nu(\delta) = \left\langle \left\| \frac{d\boldsymbol{x'}}{dt} - \frac{d\boldsymbol{x}}{dt} \right\| \right\rangle_\delta \tag{9}$$

where the value of the velocity difference is taken at the times for which the distance between the trajectories enters the scale δ and the average is performed over a large number of realizations. The LSF, $\nu(\delta)$, is a measure of the velocity at which two trajectories depart from each other, as a function of scale. By dimensional arguments, we expect that the LSF is proportional to the scale of the separation and to the FSLE:

$$\nu(\delta) \sim \delta \, \lambda(\delta) \tag{10}$$

so that we should find similar behavior for $\lambda(\delta)$ and $\nu(\delta)/\delta$, if independently measured.

In order to study the transport properties of the drifter trajectories, we have focused our interest on the measurement of $S_i^2(t)$, $\lambda(\delta)$ and $\nu(\delta)$.

With regards to the practical definitions of the FSLE and the LSF, we have chosen a range of scales $\delta = (\delta_0, \delta_1, ..., \delta_n)$ separated by a factor $r > 1$ such that $\delta_{i+1} = r \cdot \delta_i$ for $i = 0, n - 1$. The ratio r is often referred to as the "doubling" factor even though it is not necessarily equal to 2, e.g. in our case we fixed it at $\sqrt{2}$. The r value has naturally an inferior bound because of the temporal finite resolution of the trajectories (i.e. it cannot be arbitrarily close to 1) and it must be not much larger than 1, if we want to resolve scale separation in the system.

The smallest threshold δ_0 is placed just above the initial mean separation between two drifters, ~ 10 km, and the largest one δ_n is naturally selected by the finite size of the domain, ~ 500 km.

Following the same procedure, it is straightforward to compute the LSF as the mean velocity difference between two trajectories at the moment in which the separation reaches a scale δ:

$$\nu(\delta) = \left\langle \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2} \right\rangle_\delta \tag{11}$$

where the average is performed over the number of all the pairs within a set of particles, at the time in which $||\boldsymbol{x'} - \boldsymbol{x}|| = \delta$.

In Sect. 4 below we shall show the results of our data analysis and compare them with the simulations from our chaotic model for the Lagrangian dynamics of the Adriatic drifters.

3 The chaotic model

In phenomenological kinematic modeling of geophysical flows, two possible approaches can be considered: stochastic and chaotic. Both procedures generally involve a mean velocity field, which gives the motion over large scales, and a perturbation which describes the action of the small scales. The model is stochastic or chaotic if the perturbation is a random process or a deterministic time-dependent function, respectively. Examples on kinematic mechanisms proposed to model the mixing process can be found in Bower (1991), Samelson (1992), Bower and Lozier (1994), Cencini et al. (1999).

The choice of one or the other depends on what one is interested in and what experimental information is available. In our case, we have opted for a deterministic model since there are indications that, at the sea surface, the instabilities of the Eulerian structures are primarily due to air-sea interactions, which are nearly periodic perturbations.

We want to consider a simple model. So let us assume as main features of the surface circulation the following elements: an anti-clockwise coastal current; two large cyclonic gyres; and some natural irregularities in the Lagrangian motion induced by the small scale structures.

Let us notice that the actual drifters may leave the Adriatic sea through the Otranto Strait, but we model our domain with a closed basin in order to study the effects of the finite scales on the transport and treat it like a 2D system, since the drifters explore the circulation in the upper layer of the sea within the first meters of water.

On the basis of the previous considerations, we introduce our kinematic model for the Lagrangian dynamics. Under the incompressibility hypothesis we write a 2D velocity field in terms of a stream function:

$$u = -\frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \Psi}{\partial x}. \tag{12}$$

Let us write our stream function as a sum of three terms:

$$\Psi(x, y, t) = \Psi_0(x, y) + \Psi_1(x, y, t) + \Psi_2(x, y, t) \tag{13}$$

defined as follows:

$$\Psi_0(x, y) = \frac{C_0}{k_0}[-\sin(k_0(y + \pi)) + \cos(k_0(x + 2\pi))] \tag{14}$$

$$\Psi_1(x, y, t) = \frac{C_1}{k_1} \sin(k_1(x + \epsilon_1 \sin(\omega_1 t)))$$
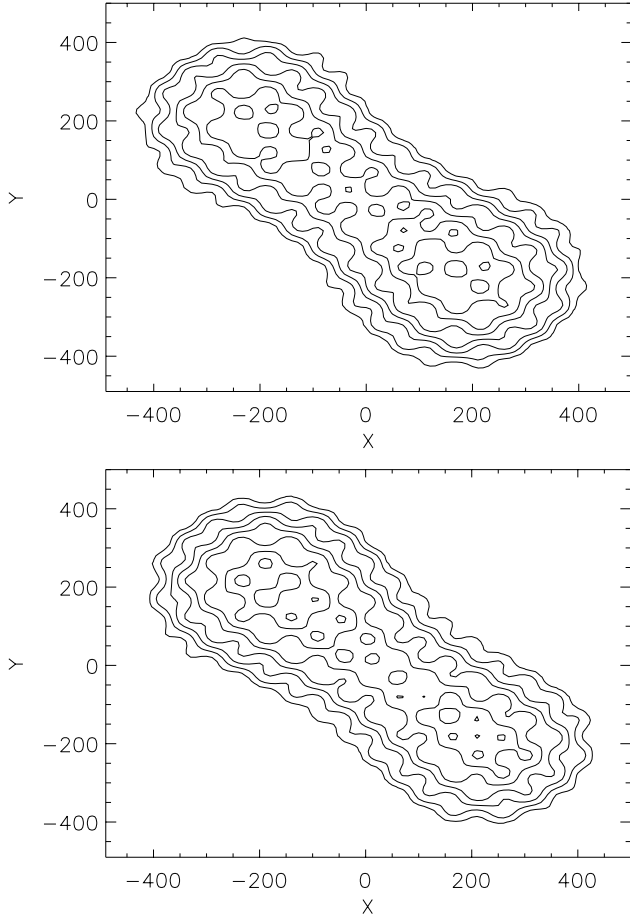$$\cdot \sin(k_1(y + \epsilon_1 \sin(\omega_1 t + \phi_1))) \tag{15}$$

Fig. 2. Model stream function isolines at **(a)** $t = 0$ and **(b)** $t = T_1/2$, with $T_1 \sim 30$ days. The boundary of the domain is the zero isoline. The coordinates (x, y) are in km.

$$\Psi_2(x, y, t) = \frac{C_2}{k_2} \sin(k_2(x + \epsilon_2 \sin(\omega_2 t)))$$
$$\cdot \sin(k_2(y + \epsilon_2 \sin(\omega_2 t + \phi_2))) \qquad (16)$$

where $k_i = 2\pi/\lambda_i$, for $i = 0, 1, 2$, the λ_i are the wavelengths of the spatial structure of the flow; analogously $\omega_j = 2\pi/T_j$, for $j = 1, 2$, and the T_j are the periods of the perturbations. In the non-dimensional expression of the equations, the units of length and time have been set to 200 km and 5 days, respectively. The choice of the values of the parameters is discussed below.

The stationary term Ψ_0 defines the boundary large scale circulation with positive vorticity. Ψ_1 contains the two cyclonic gyres and it is explicitly time-dependent through a periodic perturbation of the streamlines. The term Ψ_2 gives the motion over scales smaller than the size of the large gyres and it is time-dependent as well. A plot of the Ψ-isolines at fixed time is shown in Fig. 2. The actual basin is the inner region with negative Ψ values and the zero isoline is taken as a dynamical barrier which defines the boundary of the domain.

The main difference with reality is that the model domain is strictly a closed basin, whereas the Adriatic Sea communicates with the rest of the Mediterranean through the Otranto Strait. This is not crucial as long as we observe the two evolutions of experimental and model trajectories within time scales smaller than the mean exit time from the sea, typically of the order of a few months. Furthermore, the presence of the quasi-steady cyclonic coastal current is compatible with the interplay between the Po river southward inflow at the north-western side and the Otranto channel northward inflow at the south-eastern side of the sea.

The non-stationary of the stream function is a necessary feature of a 2D velocity field in order to have Lagrangian chaos and mixing properties. That is, so that a fluid particle will visit any portion of the domain after a sufficiently long interval of time.

We have chosen the parameters as follows. The velocity scales C_0, C_1 and C_2 are all equal to 1 which, in physical dimensions, corresponds to $\sim$0.5 m/s. The wave numbers k_0, k_1 and k_2 are fixed at $1/2$, 1 and 4π, respectively. In Fig. 2a,b we can see two snapshots of the streamlines at fixed time. The length scales of the model Eulerian structures are of $\sim$1000 km (coastal current), $\sim$200 km (gyres) and $\sim$50 km (eddies). The typical recirculation times for gyres and eddies turns out to be of the order of 1 month and a few days, respectively.

With regards to the time-dependent terms in the stream function, the pulsations are $\omega_1 = 1$ and $\omega_2 = 2\pi$, which determine oscillations of the two large-scale vortices over a period $T_1 \simeq 30$ days and oscillations of the small-scale vortices over a period $T_2 \simeq 5$ days. The respective oscillation amplitudes are $\epsilon_1 = \pi/5$ and $\epsilon_2 = \epsilon_1/10$ which correspond to $\sim$100 km and $\sim$10 km.

The choice of the phase factors ϕ_1 and ϕ_2 determines how much the vortex pattern changes during a perturbation period. We have chosen to set both ϕ_1 and ϕ_2 to $\pi/4$. This choice of the parameters for the time-dependent terms in the stream function is only supposed to be physically reasonable, for the experimental data give us limited information about the time variability of the Eulerian structures.

The chaotic advection (Ottino, 1989; Crisanti et al., 1991), occurring in our model makes an ensemble of initially close trajectories spread apart from one another, until the size of the mean relative displacement reaches a saturation value corresponding to the finite length scale of the domain.

The scale-dependent degree of chaos is given by the FSLE. Due to the relatively sharp separation between large and small scales in the model, we expect $\lambda(\delta)$ to display a step-like behavior with two plateaus, one for each characteristic time, and a cut-off at scales comparable with the size of the domain. In the limit of small perturbations, the FSLE gives an estimate of the MLE of the system.

The LSF $\nu(\delta)$, on the other hand, is expected to be proportional to the size of the perturbation and to λ, as discussed in the introduction. Therefore, the quantity $\nu(\delta)/\delta$ is expected to be qualitatively proportional to $\lambda(\delta)$, in the sense that the mean slopes have to be compatible with each other.

In the following section we will show results of our simulations together with the outcome of the data analysis.

4   Comparison between data and model

The statistical quantities relative to the drifter trajectories
have been computed according to the following prescription.
The number of selected drifters for the analysis is 37, dis-
tributed in 5 different deployments in the Strait of Otranto,
containing, respectively, 4, 9, 7, 7 and 10 drifters. These are
the only drifter trajectories out of the whole data set which
are long enough to study the Lagrangian motion on the basin
scale. To obtain the highest statistics possible, at the price
of losing information on the seasonal variability, the times of
all of the 37 drifters are measured as $t - t_0$, where t_0 is the
time of deployment. Moreover, to restrict the analysis only
to the Adriatic basin, we impose the condition that a drifter
is discarded as soon as its latitude moves south of 39.5 N or
its longitude exceeds 19.5 E. Let us consider the reference
frame in which the axes are aligned, respectively, with the
short side, orthogonal to the coasts, which we call the trans-
verse direction and the long side, along the coasts, which we
call the longitudinal direction.

Before the presentation of the data analysis, let us briefly
discuss the problem of finding characteristic Lagrangian
times. A first obvious candidate is

$$\tau_L^{(1)} = \frac{1}{\lambda} \tag{17}$$

Of course $\tau_L^{(1)}$ is related to small scale properties. Another
characteristic time, at least if the diffusion is standard, is the
so-called integral time scale (Taylor, 1921)

$$\tau_L^{(2)} = \frac{1}{\langle v^2 \rangle} \int_0^\infty C(\tau) d\tau \tag{18}$$

where $C(\tau) = \sum_{i=1}^d \langle v_i(t) v_i(t+\tau) \rangle$ is the Lagrangian veloc-
ity correlation function and $\langle v^2 \rangle$ is the velocity variance. We
want to stress that it is always possible (at least in principle)
to define $\tau_L^{(1)}$ while to compute $\tau_L^{(2)}$ (the integral time scale) it
is necessary to be in a standard diffusion case (Taylor, 1921).

The relative dispersion curves along the two natural direc-
tions of the basin for data and model trajectories are shown
in Figs. 3a and 3b. The curves from the numerical simula-
tion of the model are computed observing the spreading of a
cluster of 10^4 initial conditions. When a particle reaches the
boundary ($\Psi = 0$), it is eliminated. Along with observational
and simulation data, we also plot a straight line correspond-
ing to a standard diffusion with coefficient 10^3 m²/s, (Falco
et al., 2000). We discuss this comparison below. Consider-
ing the effective diffusion properties, one should expect that
the shape of $S_i^2(t)$, before the saturation regime, can still be
affected by the action of the coherent structures. Actually,
neither the data nor the model dispersion curves display a
clear power-law behavior, and are indeed quite irregular. The
growth of the mean square radius of a cluster of drifters ap-
pears still strongly affected from the details of the system and
the saturation begins no later than ∼1 month (∼ the largest
characteristic Lagrangian time). This prevents any attempt
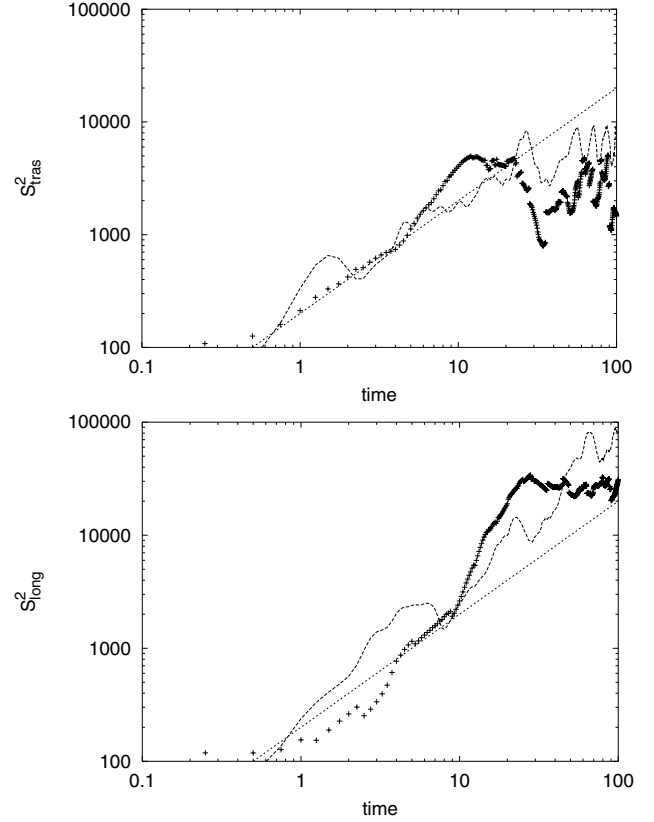at defining a diffusion coefficient for the effective dispersion

Fig. 3. Relative dispersion curves for data (plus symbol) and model
(dashed) trajectories along the two natural directions in the basin
geometry: **(a)** transverse component, **(b)** longitudinal component.
The time is measured in *days* and the dispersion in km². The exper-
imental curve is computed over the 37 drifter trajectories; the model
curve is computed over a cluster of 10^4 particles, initially placed at
the border of the southern gyre with a mean square displacement
of ∼50 km². The straight line with slope 1 has been plotted for
comparison with a standard diffusive scaling with a corresponding
diffusion coefficient ∼10^3 m²/s, typical of marine turbulent mo-
tions.

in this system. Although the saturation values are very simi-
lar, we can see that, in the intermediate range, the agreement
between observation and simulation is not good. We point
out that the trouble in reproducing the drifter dispersion in
time does not depend much on the statistics. It makes no dif-
ference whether there are 37 (data) or 10^4 (model) trajecto-
ries. The problem is that the classic relative dispersion is not
the most suitable quantity to be measured (irregular behavior
even at high statistics).

Let us now discuss the FSLE results. The curve measured
from the data has been averaged over the total number of
pairs out of 37 trajectories (∼700), under the condition that
the evolution of the distance between two drifters is no longer
followed when any of the two exits the Adriatic basin (see
above). In Fig. 4 the FSLE's for data and model are plotted.
Phenomenologically, fluid particle motion is expected to be
faster at small scales and slower at large scales. The decrease
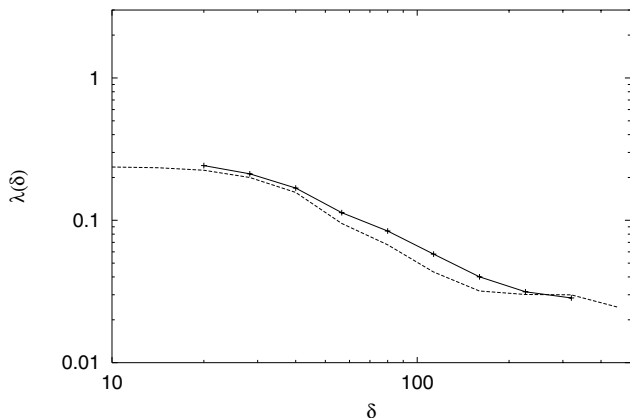of $\lambda(\delta)$ at increasing δ reflects the presence of several scales

Fig. 4. Finite-Scale Lyapunov Exponent for data (continuous line) and model (dashed line) trajectories. δ is in km and $\lambda(\delta)$ is in day^{-1}. The experimental FSLE is computed over all the pairs of trajectories out of 37 drifters; the FSLE from the model is averaged over 10^4 simulations. The simulated $\lambda(\delta)$ has a step-like behavior with one plateau at small scales (<50 km) and one at basin scales (>100 km), corresponding to doubling times of $\sim$3 days and $\sim$30 days, respectively.

Fig. 5. Lagrangian Structure Function for data (continuous line) and model (dashed line) trajectories. δ is in km and $\nu(\delta)/\delta$ is in day^{-1}. The experimental LSF is computed over all the pairs of drifters; the LSF from the model is averaged over 10^4 simulations.

of motions (at least two) involved in the dynamics. In particular, looking at the values of $\lambda(\delta)^{-1}$ at the extreme points of the δ-range, we see that small-scale (mesoscale) dispersion has a characteristic time $\sim$4 days and the large-scale (gyre scale) dispersion has a characteristic time $\sim$1 month. The ratio between gyre scale and mesoscale is of the same order as the ratio between the inverse of their respective characteristic times ($\sim$10), so the slope of $\lambda(\delta)$ at intermediate scales is about -1. The fact that the slope is larger than -2 indicates that relative dispersion is faster than standard diffusion up to sub-basin scales, i.e. Lagrangian correlations are non-vanishing due to coherent structures. It is interesting to compare this Lagrangian technique of measuring the effective Lagrangian dispersion on finite scales to the more traditional technique of extracting a (standard) diffusivity parameter from the reconstruction of the small-scale anomalies in the velocity field (Falco et al., 2000). Estimates of the zonal and meridional diffusivity in Falco et al. (2000) are of the order of $\sim10^3$ m^2/s and are compatible with the value of the effective finite-scale diffusive coefficient given by the FSLE, defined as $\lambda(\delta) \cdot \delta^2$, computed at $\delta = 20$ km ($\sim$ the mesoscale).

The FSLE computed in the numerical simulations shows two plateaus, one at small scales and the other at large scales, describing a system with two characteristic time scales and presents the same behavior, both qualitative and quantitative, as the FSLE is computed for the drifter trajectories.

It is worth noting that it is much simpler for the model to reproduce, even quantitatively, the relation between characteristic times and scales of the drifter dynamics (FSLE) rather than the behavior of the relative dispersion in time.

The LSV in Fig. 5 shows that the behavior of $\nu(\delta)$, the mean velocity difference between two particle trajectories as
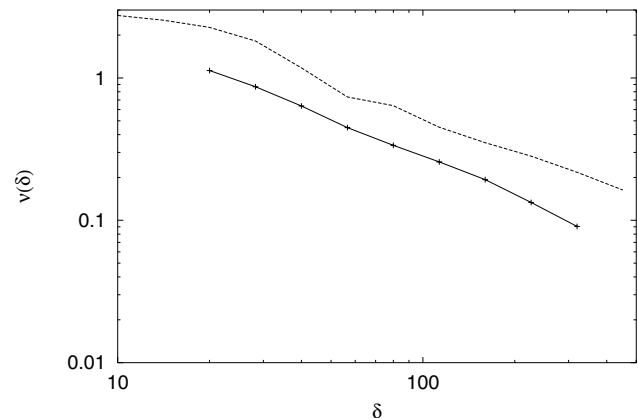
the scale of the separation varies, is compatible with the behavior of the FSLE as expected by dimensional arguments, i.e. $\nu(\delta)/\delta$, the LSF divided by the scale at which it is computed, has the same slope as $\lambda(\delta)$. This accounts for the robustness of the information given by the finite-scale analysis.

We see the theoretical predictions of FSLE and LSV are fairly well comparable with the corresponding quantities observed from the data, if we consider the relatively simple model which we used for the numerical simulations. Of course, an agreement exists due to the appropriate choice of the parameters of the model, capable of reproducing the correct relation between scales of motion and characteristic times, and because large-scale ($\sim$ sub-basin scales) Lagrangian dispersion is weakly dependent on the small-scale ($\sim$ mesoscale) details of the velocity field.

5 Discussion and conclusions

In this paper we have analyzed an experimental data set recorded from Lagrangian surface drifters deployed in the Adriatic sea. The data span the period from December 1994 to March 1996, during which five sets of drifters were released at different times in the vicinity of the same point on the eastern side of the Otranto Strait. Adopting a technique borrowed from the theory of dynamical systems, we studied the Lagrangian transport properties by measuring relative dispersions S_i^2, finite-scale Lyapunov exponents $\lambda(\delta)$, and Lagrangian structure function $\nu(\delta)$. Relative dispersion, as function of time, does not provide much information, except for an idea of the size of the domain where saturation sets in at long times. The behavior of S_i^2 looks quite irregular and this is due not to poor statistics but rather to intrinsic reasons. In contrast, the results obtained with the FSLE, i.e. dispersion rates at different scales of motion, give a more useful description of the properties of the drifter spreading. In particular, $\lambda(\delta)$ detects the characteristic times associated with

the Eulerian characteristic lengths of the system.

We have also introduced a simple chaotic model of the La-grangian evolution and compared it with the observations. In our point of view, the actual meaning of the chaotic model, in relation to the behavior of the drifters, is not the best-fitting model. We do not claim that the quite difficult task of modeling the marine surface circulation driven by wind forcing can be exploited by a simple dynamical system. But a simple dynamical system can give satisfactory results if we are interested in large-scale properties of Lagrangian disper-sion, since they depend much on the topology of the velocity field and only slightly on the small-scale details of the veloc-ity structures. In this respect, chaotic advection, very likely present in every geophysical fluid flow, is crucial for what concerns tracer dispersion, since it can easily overwhelm the effects of small-scale turbulent motions on large-scale trans-port (Crisanti et al., 1991). Even when a standard diffusiv-ity parameter can be computed from the variance and the self-correlation time of the Lagrangian velocity, its relevance for reproducing the effective dispersion on finite scales, in presence of coherent structures, is questionable. In addition, practical difficulties arising from both finite resolution and boundary effects suggest a revision of the analysis techniques to be used for studying Lagrangian motion on finite scales, i.e. in non-asymptotic conditions. Considering that, gener-ally, there is more physical information in a scale-dependent indicator ($\lambda(\delta)$) rather than in a time function ($S^2(t)$), we come to the conclusion that the FSLE is a more appropriate tool of investigation of finite-scale transport properties. It is important to remark (Aurell et al, 1996; Artale et al., 1997; Boffetta et al., 2000) that, in realistic cases, $\lambda(\delta)$ is not just another way to look at $S^2(t)$ vs t, in particular it is not true that $\lambda(\delta)$ behaves like $(d \ln S^2(t)/dt)_{S^2=\delta^2}$. This is because $\lambda(\delta)$ is a quantity which characterizes Lagrangian properties at the scale δ in a non ambiguous way. On the contrary, $S^2(t)$ can depend strongly on $S^2(0)$, so that, in non-asymptotic conditions, it is relatively easy to get erroneous conclusions only by looking at the shape of $S^2(t)$. In fact, at a given time, relative dispersion inside a sub-cluster of drifters can be rather different from other sub-clusters, e.g. due to fluc-tuations in the cross-over time between exponential and dif-fusive regimes. Therefore, when performing an average over the whole set of trajectories, one may obtain a quite spuri-ous and inconclusive behavior. On the other hand, we have seen that the analysis of FSLE (and LSF) and the studying of the transport properties at a given spatial scale, rather than at a given time, can provide more reliable information on the relative dispersion of tracers.

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