

# CONTENTS

PREFACE .....	VIII
SECTION I: FUNDAMENTALS.....	1
1 A RELATIONSHIP GUIDE: BROUGHT TO YOU BY FUNCTIONS.....	2
1.1 Modeling with Math .....	3
1.2 Relationship Guide: How to Build a Relationship (Tip: Use a Function) .....	4
1.3 Visual Model (Mental Model) For a Function.....	5
1.4 Meet the Everyday Functions You Hardly Think About.....	6
1.5 Function Notation .....	7
1.5.1 Lazy Mathematicians: Concise Notation.....	11
1.6 Function Definition vs. Function Evaluation (Function Call) .....	12
1.7 Solving Relationship Problems .....	14
1.7.1 Models for Solutions .....	16
1.8 Taking Selfies: Visualizing Functions .....	17
1.9 The More the Merrier: Functions with More Than One Input.....	18
1.10 Operators Are People, err..., Functions, Too! .....	20
1.11 Feeding One Function into Another: Function Composition.....	21
1.11.1 Adding and Multiplying Functions As Composition .....	26
1.12 Oops! Hit Undo! Hit Undo! Undo with the Inverse of a Function.....	27
1.12.1 When Functions Refuse to Invert.....	29
1.12.2 Finding Inverse Functions.....	30
1.12.3 Using Inverse to Solve Equations .....	31
1.12.4 Inverse Operators .....	32
1.13 Putting it All Together: Solving Equations .....	34
1.13.1 Bad Root! You Produced Nothing! .....	34
1.13.2 Rooting for the Inverse.....	36
1.14 One Last Thing: More Ways to Skin a Cat (or a Relationship) .....	37
1.15 The Story So Far.....	41
2 NUMBERS: WHAT FUNCTIONS CONSUME AND PRODUCE .....	43
2.1 Invasive Numbers: How Numbers Breed More Numbers .....	43
2.1.1 Addition and Multiplication.....	44
2.1.2 First Signs of Trouble: Subtraction.....	44
2.1.3 More Trouble: Division.....	46
2.2 Unwholesome Whole Numbers .....	46
2.2.1 Poor Abstraction of Reality.....	47
2.3 Operations Revisited (with Negative Numbers) .....	48
2.3.1 Addition and Subtraction.....	48
2.3.2 Multiplication as Two Operations.....	49
2.4 Go Forth and Multiply, err..., Exponentiate.....	53
2.5 The Story So Far.....	56

3	FUNCTION TOOLBOX.....	57
3.1	Power Function Family: How to Grow Your Power.....	58
3.1.1	Linear Term.....	59
3.1.1.1	Reducing Non-Linear Models to Linear Models.....	63
3.1.2	Quadratic Term.....	65
3.1.3	Cubic Term.....	68
3.2	Polynomial Family: How to Accumulate Different Powers.....	69
3.2.1	Linear model (Linear Function).....	70
3.2.1.1	Significance of the Absence of a Constant Term .....	72
3.2.1.2	Linear Models of Multiple Inputs (Linear Combinations) .....	73
3.2.2	Quadratic Model.....	74
3.2.3	Cubic Model.....	77
3.2.4	Generalization: Polynomial Functions .....	77
3.2.4.1	Polynomials as Linear Combinations.....	78
3.2.4.2	The Fundamental Theorem of Algebra .....	78
3.3	Exponential Family .....	80
3.3.1	THE (Natural) Exponential Function.....	85
3.4	The Inverse Functions of The Families We Met .....	86
3.4.1	Inverse of Power Functions .....	87
3.4.2	Inverse of Polynomials .....	89
3.4.2.1	Inverse of the Linear Model .....	90
3.4.2.2	Inverse of Polynomials .....	91
3.4.3	Inverse of Exponential Functions: Logarithmic Model .....	92
3.5	Reciprocal Functions of the Families We Met.....	96
3.5.1	Reciprocal of Power Functions.....	97
3.5.1.1	Reciprocal of The Linear Term .....	97
3.5.1.2	Inverse Square Model .....	99
3.5.2	Reciprocals of Polynomials.....	101
3.5.3	Reciprocal of Exponential Models (Exponential Decay).....	102
3.6	Trigonometric Family.....	104
3.6.1	Periodic Functions.....	104
3.6.2	Communicating with Waves.....	110
3.6.3	Inverse and Reciprocal Models of Trigonometric Functions.....	115
3.7	Growing Faster than Exponential: The Factorial Function.....	116
3.8	Building Larger Models from Simpler Models.....	119
3.9	Summary of Function Families .....	122
3.10	The Story So Far.....	124
4	SERIES: FUNCTIONS UNLIMITED .....	125
4.1	A Polynomial as a Series.....	125
4.2	The Sky is the Limit: Power Series .....	129
4.3	Maclaurin and Taylor Series .....	131
4.4	Fourier Series.....	136
4.5	The Story So Far.....	137

## SECTION II: BEYOND FUNDAMENTALS ..... 138

### 5 FUNCTIONS THAT CAUSE (YOUR HEAD TO) SPIN..... 139

5.1	A “Rotated” Number.....	142
5.2	Functions with Complex (“Rotated”) Input and Output.....	144
5.2.1	Basic Arithmetic Operators (Functions) .....	144
5.2.2	Complex Multiplication as Two Operations.....	146
5.3	Complex Roots of Polynomials.....	147
5.3.1	Why do They Come in Pairs?.....	149
5.4	Graph of a Complex Function .....	150
5.5	Complex Numbers as “Complete” Numbers.....	153
5.6	Non-real Exponents (Advanced Topic).....	156
5.6.1	Exponentiation with an Imaginary Input.....	157
5.6.2	Exponentiation with a Complex Input.....	161
5.7	The Story So Far.....	163

### 6 FUNCTIONS IN 3D SPACE ..... 165

6.1	Vectors: Representing Objects in Space .....	165
6.2	Where do Babies, err..., Vectors, Come From?.....	166
6.3	Vector Difference (Subtraction) .....	167
6.4	Vector Addition .....	168
6.5	Multiplication by a Scalar (Scaling).....	169
6.6	Examples of Vector Addition and Subtraction.....	169
6.7	Vector Functions You Meet Every Day.....	171
6.8	Product Between Two Vectors.....	173
6.8.1	Dot Product (Scalar Product).....	173
6.8.2	Cross Product (Vector Product).....	177
6.8.3	Why is Vector Product Not Commutative? .....	181
6.8.4	2D Vectors vs. Complex Numbers .....	182
6.9	Linear Combinations of Vectors .....	182
6.10	Component Representation and Algebraic Vectors.....	184
6.10.1	Dot Product in Component Representation.....	185
6.10.1.1	Dot Product as a Linear Combination of Components .....	186
6.10.2	Cross Product in Component Representation.....	189
6.11	What Unites Algebraic and Geometric Vectors?.....	191
6.12	Modeling with Vectors (Scalar Fields and Vector Fields).....	193
6.12.1	Scalar Fields .....	193
6.12.2	Vector Fields .....	195
6.12.3	Vector Valued Functions of a Parameter.....	199
6.13	Function Space .....	200
6.14	The Story So Far.....	202

### 7 MATRICES: EXTENDING LINEAR FUNCTIONS..... 204

7.1	Linear Combination Revisited.....	204
7.2	Multiple Linear Combinations.....	205
7.3	What Does a Matrix Represent? .....	210

7.4	Modeling with a Matrix .....	214
7.4.1	Extending Linear Models .....	216
7.5	Matrix Multiplication .....	216
7.6	Matrix Multiplication as Function Composition .....	218
7.7	Row View vs. Column View of a Matrix .....	221
7.8	Linear Transformations .....	223
7.8.1	Performing Multiple Transformations at Once .....	225
7.8.2	Multiplying by a Matrix as Two Operations in One .....	226
7.9	Systems of Linear Equations .....	227
7.10	The Story So Far .....	229
8	SUMMARY: FUNCTIONS IN PERSPECTIVE .....	231
8.1	The Meaning of Life, err..., Math .....	231
8.2	Objects .....	231
8.3	Functions .....	232
8.3.1	Linear Models .....	233
8.3.2	Non-Linear Models .....	236
8.4	The Story .....	238
	EPILOGUE .....	241
	SUPPLEMENT: PRELUDE TO CALCULUS .....	243
	ACKNOWLEDGEMENTS .....	250
	INDEX .....	251
	ABOUT THE AUTHOR .....	253