

CONTENTS

PREFACE	VIII
SECTION I: FUNDAMENTALS.....	10
1 BUILDING A RELATIONSHIP: BROUGHT TO YOU BY FUNCTIONS.....	11
1.1 Modeling with Math	12
1.2 Relationship Guide: How to Build a Relationship (Tip: Use a Function)	13
1.3 Visual Model (Mental Model) For a Function	14
1.4 Meet the Everyday Functions You Hardly Think About.....	15
1.5 Function Notation	16
1.5.1 Lazy Mathematicians: Concise Notation.....	20
1.6 Function Definition vs. Function Evaluation (Function Call)	21
1.7 Taking Selfies: Visualizing Functions	23
1.8 The More the Merrier: Functions with More Than One Input.....	24
1.9 Operators Are People, err..., Functions, Too!	26
1.10 It's Complicated: Building Complex Relationships.....	27
1.10.1 Using Function Composition to Build Complex Models.....	27
1.10.2 Adding and Multiplying Functions As Composition	32
1.11 There's More Than One Way to Skin a Cat (or a Relationship)	33
1.11.2 Using a Common Parameter	34
1.11.3 Going Beyond Functions	36
1.12 The Story So Far	37
2 SOLVING RELATIONSHIP PROBLEMS.....	39
2.1 Whodunnit: Which Input Caused That Output?	39
2.1.2 Models for Solutions	42
2.2 Oops! Hit Undo! Hit Undo! Undo with the Inverse of a Function.....	42
2.2.1 When Functions Refuse to Invert.....	44
2.2.2 Finding Inverse Functions.....	45
2.2.3 Using Inverse Functions to Solve Equations.....	47
2.2.4 Graph of Inverse Functions.....	48
2.2.5 Inverse Operators	50
2.3 Putting it All Together: Solving Equations	51
2.3.1 Bad Root! You Produced Nothing!	51
2.3.2 Rooting for the Inverse.....	53
2.5 The Story So Far	55
3 NUMBERS: WHAT FUNCTIONS CONSUME AND PRODUCE.....	56
3.1 Invasive Numbers: How Numbers Breed More Numbers	56
3.1.1 Addition and Multiplication.....	57
3.1.2 First Signs of Trouble: Subtraction.....	57
3.1.3 More Trouble: Division.....	59
3.2 Unwholesome Whole Numbers	59
3.2.1 Poor Abstraction of Reality.....	60

3.3	Operations Revisited (with Negative Numbers)	61
3.3.1	Addition and Subtraction.....	61
3.3.2	Multiplication as Two Operations.....	62
3.4	Go Forth and Multiply, err..., Exponentiate.....	66
3.5	The Story So Far.....	69
4	COMMON RELATIONSHIPS (FUNCTION TOOLBOX)	70
4.1	Power Function Family: How to Grow Your Power.....	71
4.1.1	Linear Term.....	72
4.1.1.1	Reducing Non-Linear Models to Linear Models.....	76
4.1.2	Quadratic Term.....	78
4.1.3	Cubic Term.....	81
4.2	Polynomial Family: How to Accumulate Different Powers.....	82
4.2.1	Linear model (Linear Function).....	83
4.2.1.1	Significance of the Absence of a Constant Term	85
4.2.1.2	Linear Models of Multiple Inputs (Linear Combinations)	86
4.2.2	Quadratic Model.....	87
4.2.3	Cubic Model.....	90
4.2.4	Generalization: Polynomial Functions	90
4.2.4.1	Polynomials as Linear Combinations.....	91
4.2.4.2	The Fundamental Theorem of Algebra	91
4.3	Exponential Family	93
4.3.1	THE (Natural) Exponential Function.....	98
4.4	The Inverse Functions of The Families We Met	99
4.4.1	Inverse of Power Functions	100
4.4.2	Inverse of Polynomials	102
4.4.2.1	Inverse of the Linear Model	103
4.4.2.2	Inverse of Polynomials.....	104
4.4.3	Inverse of Exponential Functions: Logarithmic Model	105
4.5	Reciprocal Functions of the Families We Met.....	109
4.5.1	Reciprocal of Power Functions.....	110
4.5.1.1	Reciprocal of The Linear Term	110
4.5.1.2	Inverse Square Model	112
4.5.2	Reciprocals of Polynomials.....	114
4.5.3	Reciprocal of Exponential Models (Exponential Decay).....	115
4.6	Trigonometric Family	117
4.6.1	Periodic Functions.....	117
4.6.2	Communicating with Waves.....	123
4.6.3	Inverse and Reciprocal Models of Trigonometric Functions.....	128
4.7	Growing Faster than Exponential: The Factorial Function.....	129
4.8	Building Larger Models from Simpler Models	132
4.9	Summary of Function Families	135
4.10	The Story So Far.....	136
5	SERIES: FUNCTIONS UNLIMITED	138
5.1	A Polynomial as a Series	138
5.2	The Sky is the Limit: Power Series	142
5.3	Maclaurin and Taylor Series	144
5.4	Fourier Series.....	149

5.5	The Story So Far.....	150
SECTION II: BEYOND FUNDAMENTALS		151
6	FUNCTIONS THAT CAUSE (YOUR HEAD TO) SPIN.....	152
6.1	A “Rotated” Number.....	155
6.2	Functions with Complex (“Rotated”) Input and Output.....	157
6.2.1	Basic Arithmetic Operators (Functions)	157
6.2.2	Complex Multiplication as Two Operations.....	159
6.3	Complex Roots of Polynomials.....	160
6.3.1	Why do They Come in Pairs?.....	162
6.4	Graph of a Complex Function	163
6.5	Complex Numbers as “Complete” Numbers.....	166
6.6	Non-real Exponents (Advanced Topic).....	169
6.6.1	Exponentiation with an Imaginary Input.....	170
6.6.2	Exponentiation with a Complex Input.....	174
6.7	The Story So Far.....	176
7	FUNCTIONS IN 3D SPACE	178
7.1	Vectors: Representing Objects in Space	178
7.2	Where do Babies, err..., Vectors, Come From?	179
7.3	Vector Difference (Subtraction)	180
7.4	Vector Addition	181
7.5	Multiplication by a Scalar (Scaling).....	182
7.6	Examples of Vector Addition and Subtraction.....	182
7.7	Vector Functions You Meet Every Day.....	184
7.8	Product Between Two Vectors.....	186
7.8.1	Dot Product (Scalar Product).....	186
7.8.2	Cross Product (Vector Product).....	190
7.8.3	Why is Vector Product Not Commutative?	194
7.8.4	2D Vectors vs. Complex Numbers	195
7.9	Linear Combinations of Vectors	195
7.10	Component Representation and Algebraic Vectors.....	197
7.10.1	Addition and Subtraction in Component Representation.....	198
7.10.2	Dot Product in Component Representation.....	198
7.10.2.1	Dot Product as a Linear Combination of Components	200
7.10.3	Cross Product in Component Representation.....	203
7.11	What Unites Algebraic and Geometric Vectors?.....	204
7.12	Modeling with Vectors (Scalar Fields and Vector Fields).....	206
7.12.1	Scalar Fields	206
7.12.2	Vector Fields	209
7.12.3	Vector Valued Functions of a Parameter.....	213
7.13	Function Space	214
7.14	The Story So Far.....	216
8	MATRICES: EXTENDING LINEAR FUNCTIONS.....	218
8.1	Linear Combination Revisited.....	218

8.2	Multiple Linear Combinations.....	219
8.3	What Does a Matrix Represent?	224
8.4	Modeling with a Matrix	228
8.4.1	Extending Linear Models	230
8.5	Matrix Multiplication	230
8.6	Matrix Multiplication as Function Composition	232
8.7	Row View vs. Column View of a Matrix	235
8.8	Linear Transformations.....	237
8.8.1	Performing Multiple Transformations at Once	239
8.8.2	Multiplying by a Matrix as Two Operations in One.....	240
8.9	Systems of Linear Equations.....	241
8.10	The Story So Far.....	243
9	SUMMARY: FUNCTIONS IN PERSPECTIVE.....	245
9.1	The Meaning of Life, err..., Math.....	245
9.2	Objects	246
9.3	Functions	246
9.3.1	Linear Models.....	247
9.3.2	Non-Linear Models.....	250
9.4	The Story.....	252
	EPILOGUE.....	255
	SUPPLEMENT: PRELUDE TO CALCULUS.....	257
	ACKNOWLEDGEMENTS	264
	INDEX	265
	ABOUT THE AUTHOR.....	267