Determining chirp mass and distance of gravitational wave sources in LIGO data

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Introduction

- Gravitational waves originally predicted by Einstein in 1916
- First direct detection of gravitational waves by LIGO in 2015
- 2017 Nobel Prize in Physics awarded to Rainer Weiss, Barry Barish, and Kip Thorne
- New opportunities to probe strong field phenomena

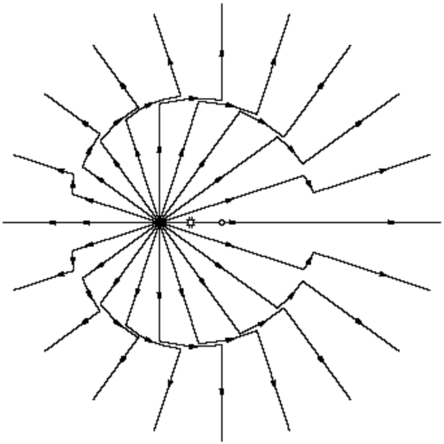
Motivation

- Reproduce LIGO event detections using simplified processing pipeline
- Determine chirp mass and distance to event from strain data

Source of gravitational waves

- Accelerating masses emit gravitational radiation which carries away energy
- Analogous to electromagnetic radiation from classical acceleration electron
- Chirp waves produced by inspiraling black hole or neutron star mergers
- Other signals produced by supernovae, rotating neutron stars, early universe

Sources of gravitational waves (cont.)



Shockwaves in field from classical accelerating charge

Theoretical model

- For binary system with masses m_1, m_2, chirp mass given by

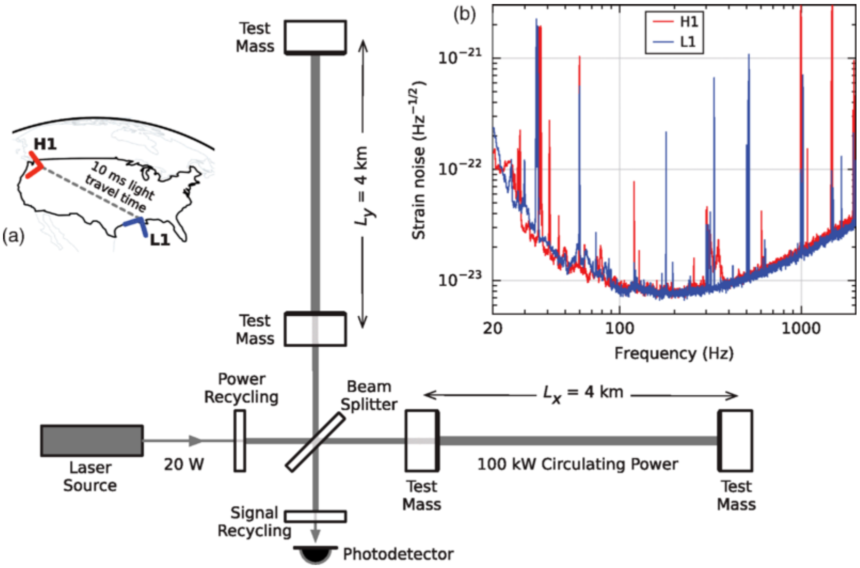
$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

- Using the Newtonian approximation for circular orbits, we obtain

$$\omega(t) \approx 948.5 \left(\frac{M_\odot}{M_c}\right)^{5/8} \left(\frac{1\ \mathrm{s}}{\Delta t}\right)^{3/8} [\mathrm{Hz} \cdot \mathrm{rad}]$$

$$\frac{dE}{dt} \propto (M_c \omega(t))^{10/3}$$

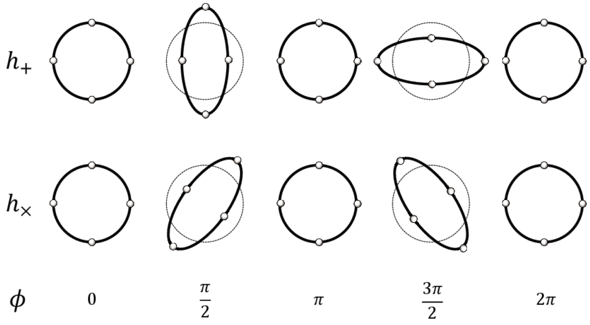
LIGO experiment



LIGO experiment (cont.)

- Amplitude of strain signal dependent on polarization of wave
- Consider projection of quadrupole moment onto plane of interferometer

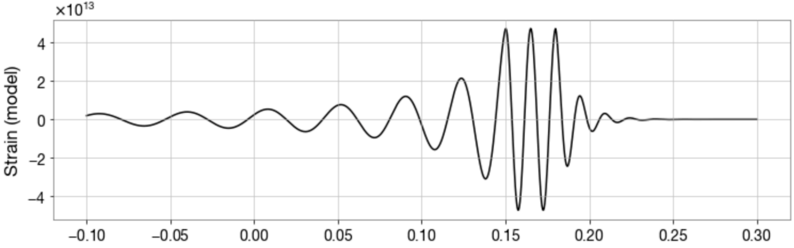
$$\Delta l = \Delta l_1 - \Delta l_2 = hL$$



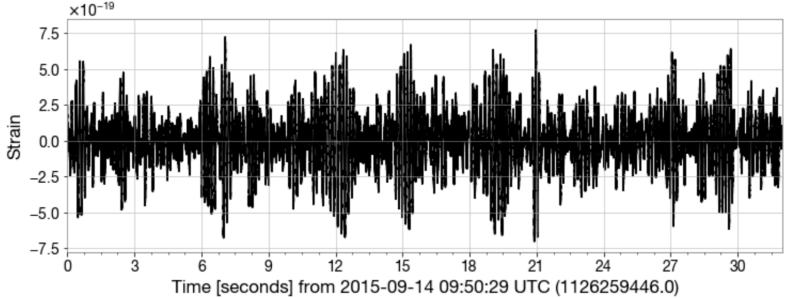
Processing techniques

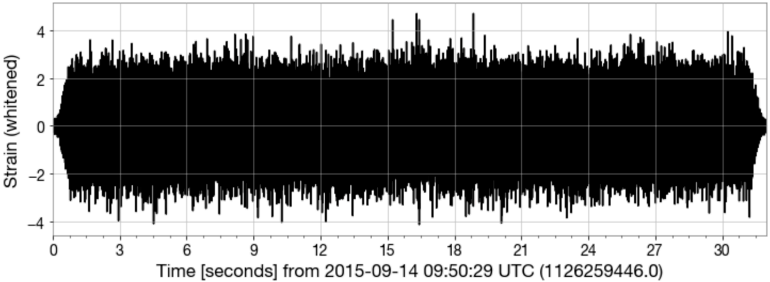
- Used `gwpy` package adopted from LIGO library to whiten data
- Fit to waveform using `lmfit`, used cutoff $\Delta t = 3 \times 10^{-2}$ and exponential dropoff
- Reported event time only has precision of 0.1, search for most likely value of t_0

$$f(t) = C(M_c\omega(t))^{10/3}\cos(\omega(t)\Delta t + \phi)$$

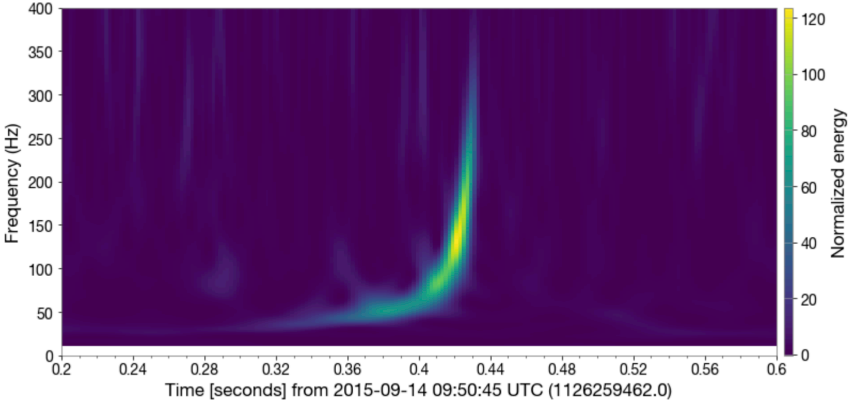


Processing techniques (cont.)





Processing techniques (cont.)



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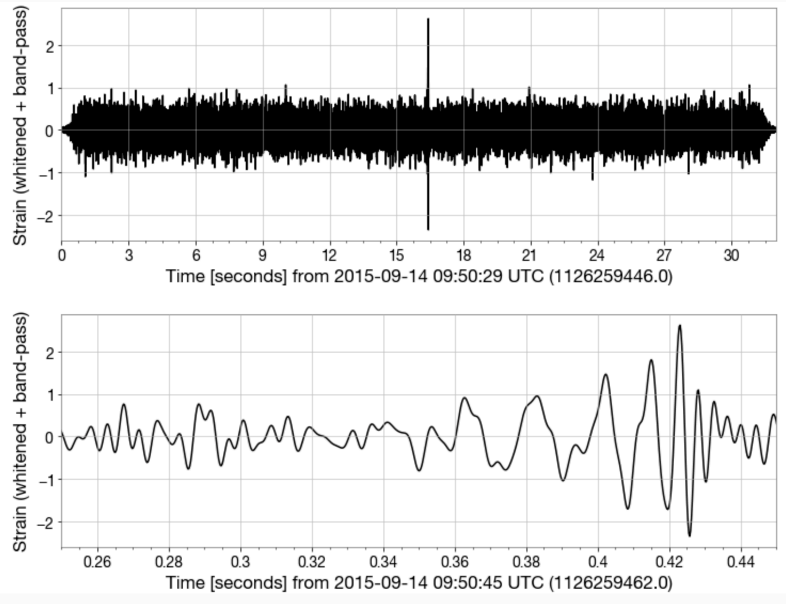

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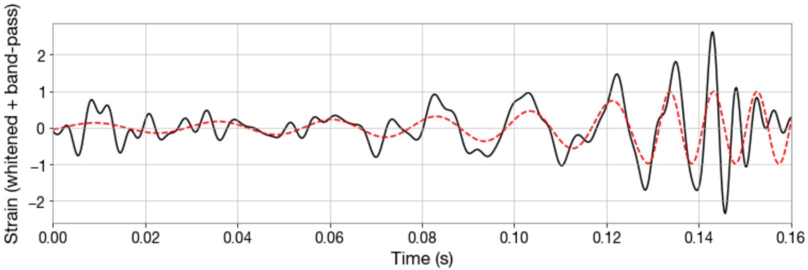
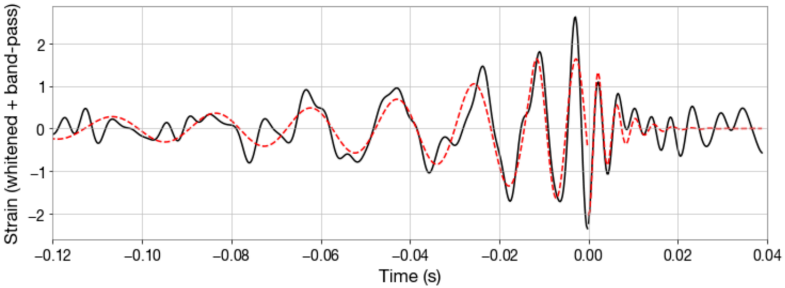


$$\chi^2 = 0.451$$

Piecewise fit

- Fit $t < t_0$ and $t > t_0$ pieces separately to address issues with initial model
- Fit to GW150914 template to determine cutoffs and exponential dropoff
- Used $t > t_0$ cutoff of 1.26×10^{-3} and exponential factor $(10^{-90})^{\Delta t}$

Piecewise fit (cont.)



$$\chi^2 = 0.219$$

Estimates for chirp mass

Event	Detector	Reported M_c ($M_\odot$)	$t < t_0$ estimate	$t > t_0$ estimate
GW1501914	H1	$28.6^{+1.7}_{-1.5}$	13.0 ± 0.1	26.1 ± 0.8
GW170104	H1	$21.1^{+2.4}_{-2.7}$	20.3 ± 0.8	35.7 ± 1.0
GW170814	L1	$24.1^{+1.4}_{-1.1}$	20.5 ± 0.3	28.5 ± 0.8
GW170818	L1	$26.5^{+2.1}_{-1.7}$	27.8 ± 0.9	33.5 ± 0.7
GW190412	L1	$15.2^{+0.2}_{-0.2}$	17.8 ± 0.3	52.9 ± 2.3
GW190421_213856	H1	$46.6^{+6.6}_{-6.0}$	16.4 ± 0.3	60.3 ± 6.2
GW190521_074359	L1	$39.9^{+2.2}_{-2.9}$	32.6 ± 0.6	60.8 ± 1.4
GW190630_185205	L1	$29.4^{+1.6}_{-1.5}$	14.7 ± 0.2	32.9 ± 1.1
GW190727_060333	H1	$44.7^{+5.3}_{-5.7}$	24.9 ± 0.7	44.1 ± 1.6

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Systematic error analysis

- Only considered $t < t_0$ estimates for M_c within 20% or 3σ of reported value
- Outliers appear to be caused by convergence of model to incorrect t_0
- Average systematic error of about 3.0%

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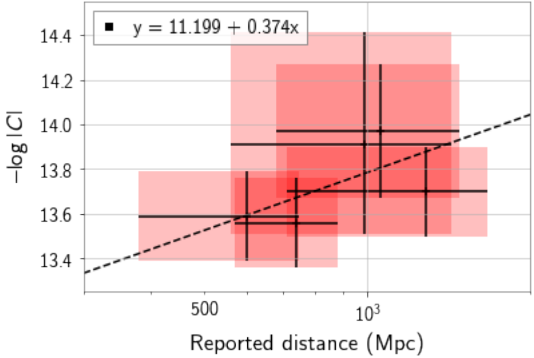
Inferring distance

- Chirp mass determines both waveform and luminosity of event
- Possible to infer distance from strain amplitude and triangulation
- Analyzed relationship between reported distance and constant factor in model

Event	Reported distance (Mpc)	Detector	$-\log \lvert C_{t<t_0} \rvert$
GW170104	990^{+440}_{-430}	H1	$13.91^{+0.05}_{-0.04}$
GW170814	600^{+150}_{-220}	L1	13.59 ± 0.02
GW170818	1060^{+420}_{-380}	L1	13.97 ± 0.03
GW190412	740^{+140}_{-170}	L1	13.56 ± 0.02
GW190521_074359	1280^{+380}_{-570}	L1	13.70 ± 0.02

Inferring distance



$$R^2 = 0.368, R = 0.607$$

$$y = 11.199 + 0.374 \log x$$

Conclusions

- Simplified pipeline can be used to detect transient events and reliably estimate chirp mass, after controlling for estimate of t_0
- Predictive relationship with $R^2 = 0.368$ observed between distance and luminosity
- LIGO A+ upgrades to decrease noise expected to be complete in 2024

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