Determining Boltzmann constant and an absolute temperature scale from Johnson-Nyquist noise

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Introduction

- Fundamental sources of noise due to probabilistic microscopic processes
- Johnson-Nyquist noise due to thermal excitations of electrons at equilibrium
- Classical effect which is distinct from shot noise due to quantization of electron charge and uncertainty in number states
- Experiments provided verification of parameters in statistical mechanics
- Boltzmann constant is one of seven fundamental constants in modern SI system

Motivation

- Measure Johnson-Nyquist noise in resistors at three temperatures
- Determine the Boltzmann constant k_B and an absolute temperature scale by fitting to theoretical statistical model

Statistical thermodynamics

- Macroscopic quantities in thermodynamics arise from microscopic properties
- Fundamental entropy given by logarithm of number of microstates

$$\sigma = \log g$$

- Related to fundamental temperature via transfer of heat

$$\frac{1}{\tau} = \frac{\partial \sigma}{\partial Q} = \left(\frac{\partial \sigma}{\partial U}\right)_{N,V}$$

- Thermometric temperature requires constant with dimensions of joules $\cdot$ Kelvin^{-1}

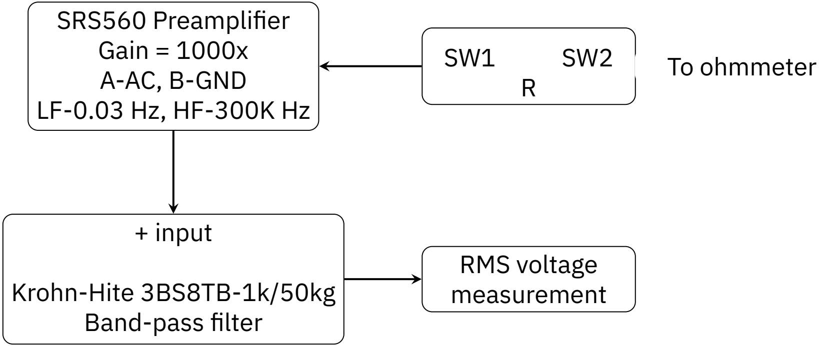
Nyquist theory

- Relates mean square noise to resistance and absolute temperature
- In regime $hf \ll k_B T$, can be derived from classical statistical mechanics without directly referencing properties of microscopic electrons

$$dV^2 = \frac{4Rk_BTdf}{1+(2\pi fRC)^2}$$

- In high frequency and low temperature regimes must use Planck's law

Experimental setup



Experimental setup (cont.)

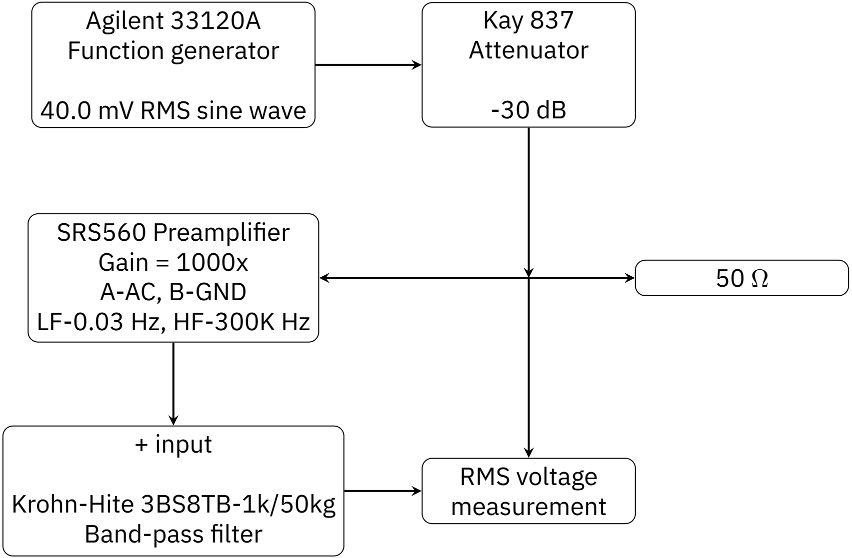
- Measured RMS amplitude of Johnson noise from metal oxide resistors rated at 10, 22, 100, 200, 475, 500, and 806 kΩ, and shorted box at 296 K
- Also measured RMS amplitude of Johnson noise from 475 kΩ metal oxide resistor at around 77 K using liquid nitrogen and 450 K using oven
- Arduino thermocouple system used to determine temperature of oven
- Measured capacitance of system as 79.9 ± 0.6 pF

Experimental setup (cont.)

- Preamplifier allows for more precise measurement of μV signal
- Band-pass filter selects for classical theory regime but produces non-uniform gain
- Trapezoidal integration technique used to calibrate gain curve of setup

$$G = \int \frac{g^2(f)\,df}{1 + (2\pi f RC)^2}$$

Calibration setup



Calibration setup (cont.)

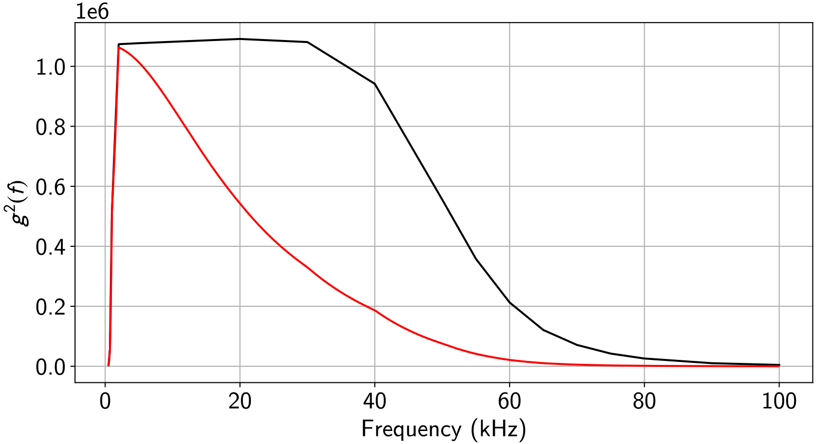


Figure 1: Example with $g^2(f)$ in black and gain integral curve $R = 100\ \mathrm{k\Omega}$ in red

Results and analysis

- For each resistor we use the measured resistance and voltage to compute

$$k_B = \frac{V^2}{4RTG} = \frac{V_R^2 - V_S^2}{4RTG}$$

- Statistical error determined from measurement uncertainty on V_R, V_S, R, C, $g(f)$
- Systematic error estimated due to calibration cutoff at 1e-5, trapezoidal integration at 1e-6, measurement readings at 1e-5, and temperature fluctuations of 3 K at 296 K

Results and analysis (cont.)

Table 1: Calculated value of k_B for each resistor. Accepted value is 1.38×10^{-23} J · K^{-1}.

Resistance (kΩ)	k_B (10^{-23} J · K^{-1})
10.0088	1.26 ± 0.05 (stat) $\pm$ 0.01 (sys)
22.268	1.29 ± 0.05 (stat) $\pm$ 0.01 (sys)
99.988	1.56 ± 0.06 (stat) $\pm$ 0.02 (sys)
199.03	1.51 ± 0.06 (stat) $\pm$ 0.02 (sys)
478.53	2.51 ± 0.11 (stat) $\pm$ 0.03 (sys)
498.09	2.57 ± 0.11 (stat) $\pm$ 0.03 (sys)
815.68	3.41 ± 0.14 (stat) $\pm$ 0.03 (sys)

Results and analysis (cont.)

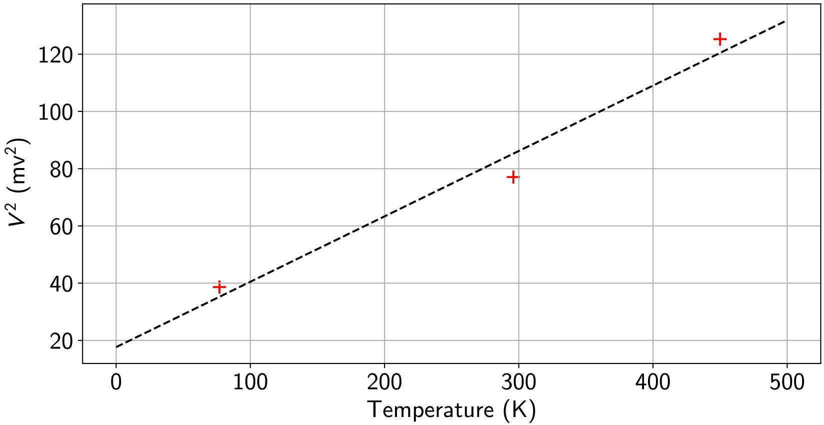


Figure 2: Relationship between Johnson noise and temperature for 475 kΩ resistor. Line of best fit with $R^2 = 0.973$ gives absolute zero at -77 $\pm$ 12 K or -350 $\pm$ 12 °C.

Time constant dependence

- Positive trend in calculated value of k_B for increasing resistance
- May be accounted for by discrepancies in measurement of capacitance
- Determined that capacitance of approximately 37 pF minimized spread of $\log \frac{(k_B)_{\text{exp}}}{k_B}$
- Average adusted value of k_B is $1.11 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$, relative error of 0.193

Conclusions

- Possible to estimate k_B and thermodynamic absolute zero using Johnson noise
- Requires reliable temperature control and measurement
- Demonstrates validity of classical statistical mechanics approximation in this regime

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