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Minireview

Investigation of multifractality in the Brazilian stock market

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ABSTRACT

Many studies point to a possible new stylized fact for financial time series: the multifractality. Several authors have already detected this characteristic in multiple time series in several countries. With that in mind and based on Multifractal Detrended Fluctuation Analysis (MFDFA) method, this paper analyzes the multifractality in the Brazilian market. This analysis is performed with daily data from IBOVESPA index (Brazilian stock exchange's main index) and other four highly marketable stocks in the Brazilian market (VALE5, ITUB4, BBDC4 and CIEL3), which represent more than 25% of the index composition, making up 1961 observations for each asset in the period from June 26 2009 to May 31 2017. We found that the studied stock prices and Brazilian index are multifractal, but that the multifractality degree is not the same for all the assets. The use of shuffled and surrogated series indicates that for the period and the actions considered the long-range correlations do not strongly influence the multifractality, but the distribution (fat tails) exerts a possible influence on IBOVESPA and CIEL3.

Contents

1. Introduction

It is known that companies become successful precisely because by not avoiding the risk, but detecting and exploiting it to their own advantage [1]. The link between risk and reward motivates risk-taking throughout history and a company that decides to protect itself from any kind of risk probably does not generate profit for its shareholders.

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One way to discover and explore the risk is knowing and analyzing the swings of the asset or the group of assets one has. In the financial market, most of the time the risk is measured by the degree of variability of investment returns, that is: the higher the potential oscillations, the greater the risk of the asset.

Many studies have been developed over the years to understand the behavior and fluctuations of assets; several stylized facts for the returns of financial time series were found, such as Clustered volatility, non-normality, heavier tails, stationarity, and non-linearity.

With the evolution of studies on the behavior of prices of financial time series, a new approach appeared: the Fractal Geometry. The first steps on the issue were taken by the Polish mathematician Benoît Mandelbrot. The term "fractal" was coined from the Latin adjective fractus, from the verb frangere, which means to break. When it comes to fractal geometry, it is stated that all objects, phenomena, theories and complex meanings can always be reduced or scaled into smaller units in order to explain them more simply.

The idea of scaling the object into smaller units was initially applied in geometry, as an improvement over the Euclidean geometry. This design was intended to show that things can be seen on ever smaller scales, increasing details and improving viewing.

The fractal scaling behavior has been observed in many areas, such as experimental physics, geophysics, physiology and even in social sciences. However, it is known that most databases generated in nature and social sciences, such as in the financial markets, are complex systems, which are not in equilibrium; therefore, it is not possible to explain their behavior through a single exponent scale.

This is because when there is a database generated by a complex system, we most often find imbalanced data, following scale relationships in several orders of magnitude. These relationships allow a characterization of the data and the complex generator of this series by exponents of fractal or multifractal scale, which can serve as features for comparison with other systems and models.

In many cases, the scaling behavior can be observed by many fractal subsets entwined in the time series. Thus, a multitude of scaling exponents will be needed for a complete description of the behavior of the series and this is called multifractal behavior. The behavior of multifractal scale was first observed by Mandelbrot in the book "Les Objets Fractals: forme, hasard et dimension" [2] and confirmed by Mantegna and Stanley [3]; after that, many authors have tested such behavior in several knowledge areas and have found the existence of multifractality in multiple time series; its application was more successful in the financial market and in the series found in nature.

For the stock market, various authors, such as [4–15] have concluded that stocks and indices of various markets (American, Indian, Polish, among others) exhibit long-range correlations and multifractal characteristics; however, this analysis, as we know, has not been performed for the Brazilian stocks. The specific characteristics of the Brazilian market, associated to the increasing number of researchers interested in its evolution and national and international investors looking for profits in it make its patterns recognition a necessity. This study has also been applied to other financial time series, such as for exchange rate, as in [16–27] and the price of gold [13,28–31], the price of commodities [32], and all concluded that such series have multifractal features.

It is interesting to understand if a financial series has or has not multifractality features because – among others arguments – it can help users to model its volatility. Lux, Morales-Arias and Sattarhoff [25] claim that modeling the volatility of time series is of fundamental importance for both finance professionals as for academics because it allows developing applicable models for risk management, asset pricing, and portfolio allocation.

Thinking about the importance of the subject, the goal of this research was to analyze whether the Ibovespa index and four stocks with high marketability – representing more than 25% of the index itself – have long-range correlations and multifractal features. Besides that, the aim is also to discuss the sources of that multifractality: if they come from the long-range correlations, from the broad fat-tail distributions or both. Finally, we aim to understand if the degree of multifractality is larger for the index or for a specific asset. The results indicate that stock prices and the Brazilian index have multifractal features and this shows that they conform to the stock market's behavior of other countries that were analyzed by other researchers.

2. Research design

Studies on multifractals were initially designed by Mandelbrot in 1975 [2]. In this book, the author understands that many series reviewed in nature cannot be explained by a single exponent of scale and claims that there is multifractality evidence in several cases. Stanley et al. [33] analyzed the dynamics company sizes, using data analysis over many scales of time as their differential work, instead of just a single time interval; Mandelbrot [34] states that the multifractal analysis is a new type of data generation to analyze the financial assets return.

Various models were created over time to analyze the multifractal properties for time series. The most used ones are: the method Multifractal Detrended Fluctuation Analysis (MFDFA), which is a generalization of the DFA method, devised by Kantelhardt et al. [35] and the method Wavelet Transform Modulus Maxima Method (WTMM), an extension of the model described by Holschneider [36], conceived by Calvet and Fisher [37].

Kantelhardt et al. [35] developed the generalization of the method detrended fluctuation analysis (DFA), named Multifractal DFA method (MFDFA) for the multifractal characterization of a time series. This method was compared to the Wavelet Transform Modulus Maxima Method (WTMM). The authors realized that with this new method, you can determine reliably scale multifractal behavior of time series and has results very close to the WTMM besides being simpler to be applied.

The MFDFA is nowadays a standard methodology in the area of econophysics [14,32,38–42]. This method has been widely used in the literature for the multifractality analysis of financial time series. Below is an algorithm of the MFDFA method, as described by Kantelhardt et al. [35].

The generalized multifractal DFA (MFDFA) procedure consists, then, of six steps. The first three steps are essentially identical to the conventional DFA procedure. Let us suppose that x_k is a series of length N, and that this series is of compact support, i.e. $x_k = 0$ for an insignificant fraction of the values only.

Step 1: Determine the profile:

$$Y(i) \equiv \sum_{k=1}^{i} \left[x_k - \langle x \rangle\right], \qquad i = 1, \ldots, N. \tag{1}$$

Subtraction of the mean $\langle x \rangle$ is not compulsory, since it would be eliminated by the later detrending in the third step.

Step 2: Divide the profile $Y(i)$ into $N_s \equiv int(N/s)$ non-overlapping segments of equal length s. Since the length N of the series is often not a multiple of the considered time scale s, a short part at the end of the profile may remain. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2N_s$ segments are obtained altogether.

Step 3: Calculate the local trend for each of the $2N_s$ segments by a least-square fit of the series. Then determine the variance.

$$F^2(s, v) \equiv \frac{1}{s} \sum_{i=1}^{s} \left\{Y\left[(v-1)s + i\right] - y_v(i)\right\}^2. \tag{2}$$

For each segment v, $v = 1, \ldots, N_s$ and

$$F^2(s, v) \equiv \frac{1}{s} \sum_{i=1}^{s} \left\{Y\left[N - (v - N_s)s + i\right] - y_v(i)\right\}^2. \tag{3}$$

For $v = N_s + 1, \ldots, 2N_s$. Here, $y_v(i)$ is the fitting polynomial in segment v.

Thus a comparison of the results for different orders of DFA allows one to estimate the type of the polynomial trend in the time series.

Step 4: Average over all segments to obtain the qth order fluctuation function

$$F_q(s) \equiv \left\{\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left[F^2(s, v)\right]^{q/2}\right\}^{1/q} \tag{4}$$

where, in general, the index variable q can take any real value except zero. For $q = 2$, the standard DFA procedure is retrieved. We are in how the generalized q dependent fluctuation functions $F_q(s)$ depend on the time scale s for different values of q. Hence, must repeat steps 2 to 4 for several time scales s.

Step 5: Determine the scaling behavior of the fluctuation functions by analyzing log–log plots $F_q(s)$ versus s for each value of q.

$$F_q(s) \sim s^{h(q)}. \tag{5}$$

The function $h(q)$ is generalized Hurst exponent.

Step 6: using Eq. (5) can be written as $F_q(s) = AS^{h(q)}$, we then have, after taking logarithms of both sides:

$$\log F_q(s) = \log A + h(q) \log s. \tag{6}$$

Then the estimated value of Generalized Hurst exponent $h(q)$ can be obtained.

For monofractal time series characterized by a single exponent over all times scales, $h(q)$ is independent of q. For multifractal time series, $h(q)$ varies with q. The different scaling of small and large fluctuation will yield a significant dependence of $h(q)$ on q. Therefore, for positive value of q, $h(q)$ describes the scaling behavior of the segments with large fluctuations; and for negative q values, the scaling exponent $h(q)$ describes the scaling behavior of segments with small fluctuations.

Another way of confirm multifractality in time series is through multifractal spectrum analysis, which is based on the following relationship between Generalized Hurst exponent $h(q)$ obtained from MFDFA and the Renyi exponent $\tau(q)$:

$$\tau(q) = qh(q) - 1 - q(h(1) - 1). \tag{7}$$

Then, through a Legendre transform, we get

$$\alpha = \frac{d}{dq}\tau(q) \tag{8}$$

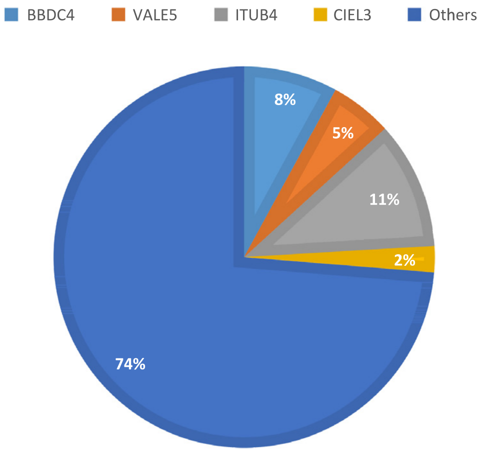

Fig. 1. Composition of the Ibovespa Index.
Source: http://www.bmfbovespa.com.br, January 2017 data.

and

$$f(\alpha) = \alpha(q)\, q - \tau(q). \tag{9}$$

From Eq. (6) we can define the multifractality degree (MF1) [8,16,27,42] as:

$$\Delta h = \max[h(q)] - \min[h(q)]. \tag{10}$$

From Eq. (8) we can define another measure of the multifractality degree (MF2) [10,15,27,29] as:

$$\Delta\alpha = \max[\alpha] - \min[\alpha]. \tag{11}$$

But different authors use different range of q and this results in different values for Δh and for $\Delta\alpha$, producing a slight confusion in the literature. We will address this point again in Section 4.

3. Data

In this study, daily series were analyzed, with values collected through the base of the Thomson Reuters at the closing moment of each day, in the period from June 26, 2009 to May 31, 2017 of 4 Brazilian stocks and the Ibovespa index, making 1961 observations for each asset. This choice was made because the Ibovespa index is the indicator of average performance of the quotations of the assets of greater negotiability and representativeness of the Brazilian market, and the shares chosen represent around 26% of the Ibovespa index. Fig. 1 shows the distribution of these actions in relation to the total index.

It can be noticed that the preferred shares of Itaú/Unibanco represent 11% of the Ibovespa index, while shares of Bradesco (BBDC4), Vale do Rio Doce (VALE5) and Cielo (CIEL3) respectively represent 8%, 5%, and 2%. This makes it possible to verify that the analyzed assets can represent the Brazilian market behavior and that these companies form a very relevant percentage in relation to the marketability and representation in this market.

This section describes and summarizes the statistical properties of the data that were analyzed. From Fig. 2 it is possible to understand the behavior of the prices of the 5 time series daily collected during the period.

The original data $x(t)$ were transformed in logarithmic returns: $r(t) = \ln x(t+1) - \ln x(t)$. Fig. 3 exhibits the behavior of the returns of the 5 financial time series studied in the analyzed period.

It can be seen that the graphs of the returns show an oscillation around zero. It is also observed the presence of volatility clusters at various points in the analyzed period.

Table 1 shows the main descriptive characteristics of the return series of the Ibovespa index and of the shares BBDC4, CIEL3, ITUB4 e VALE5, daily collected at the closing moment in the period from June 26, 2009 to May 31, 2017.

The daily time series of each asset in the period from June 26, 2009 to May 31, 2017 have 1961 observations and 1960 logarithmic returns, quoted at the closing of each day and already filtered by dividends. The table exhibits the mean, median, maximum and minimum values, the standard deviation, the measures of asymmetry, kurtosis, the value of Jarque–Bera and the probability for the normality test for the returns of the analyzed assets. It can be seen that the BBDC4 share presented the highest daily return, and in one day its price came to vary 12.25%. The price of CIEL3, ITUB4 and VALE5 were not far behind reaching a maximum return of just over 10%. The Ibovespa, with the lowest maximum value, presented the highest daily variation, 6.39% in the analyzed period.

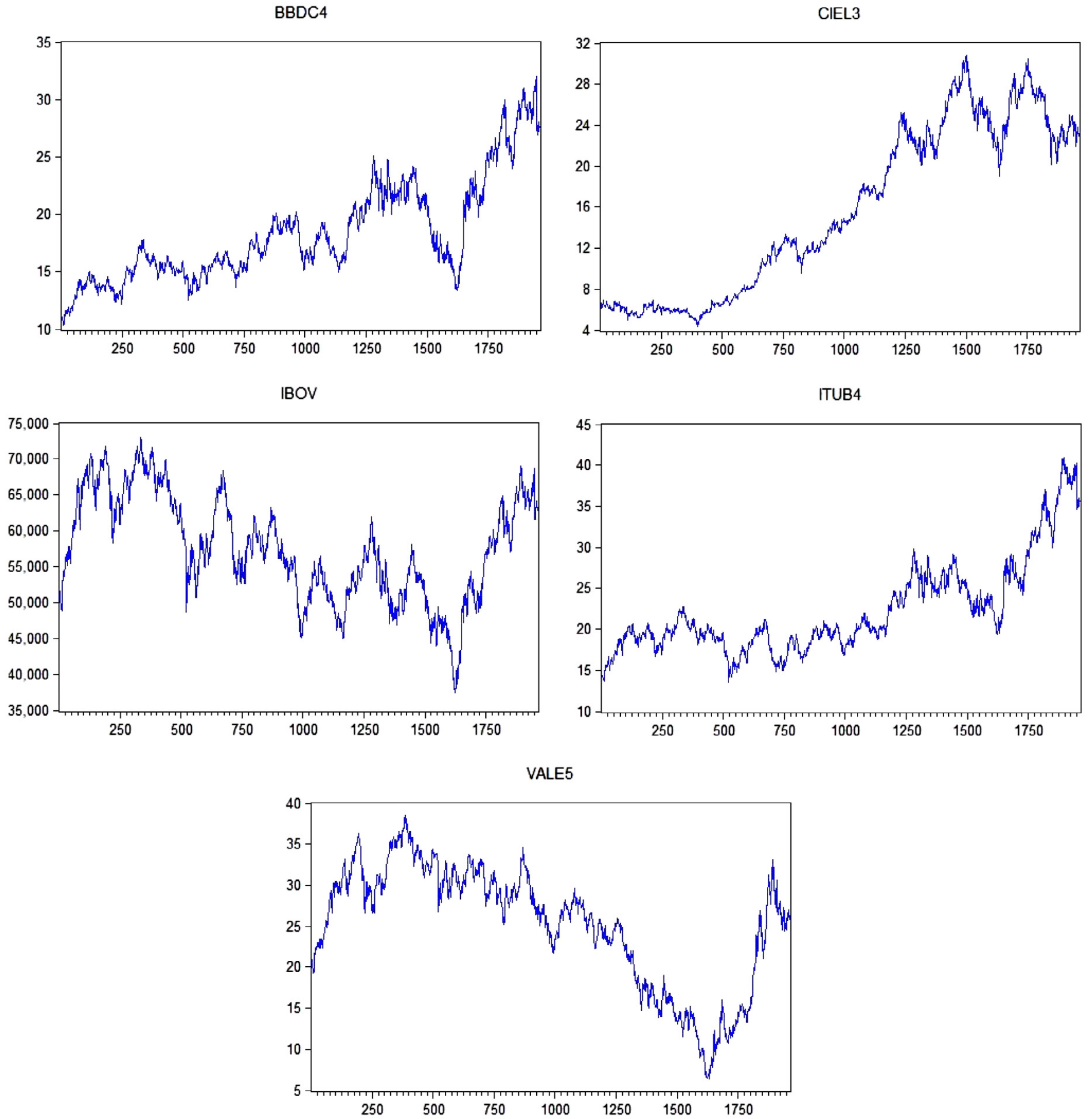

Fig. 2. Daily closing price (y-axis) of the series studied versus time in working days (x-axis), starting from June 26, 2009 and ending in May 31, 2017.

Regarding the minimum values, CIEL3 and BBDC4 shares had the largest drop in the period, reaching a negative oscillation of more than 14% in one day, followed by ITUB4 and VALE5 with minimum variations of almost 13% in one day and the Ibovespa, with 9.2%. It is evident that even with companies that are highly representative in the Ibovespa index, fluctuating more than 10% at their highs and more than 12% at their lows, the index oscillation is much lower.

The results of the standard deviation of each asset were compatible with their maximum and minimum values: all stocks were valued at around 0.02 and the Ibovespa index had the smallest deviation, showing the importance of diversifying a portfolio: as the Ibovespa index is the average performance indicator of the most representative assets and marketable assets of the Brazilian market, its risk ends up being diversified and being the lowest. This information is in agreement with the portfolio diversification theory affirmed by [43].

As regarding the asymmetry, it is known that a distribution is perfectly symmetric, when this value is equal to zero. And the further away from zero, the more asymmetrical is the distribution. The asymmetry values presented in the table indicate that the distributions of the returns of the analyzed series are not symmetrical. However, it can be considered

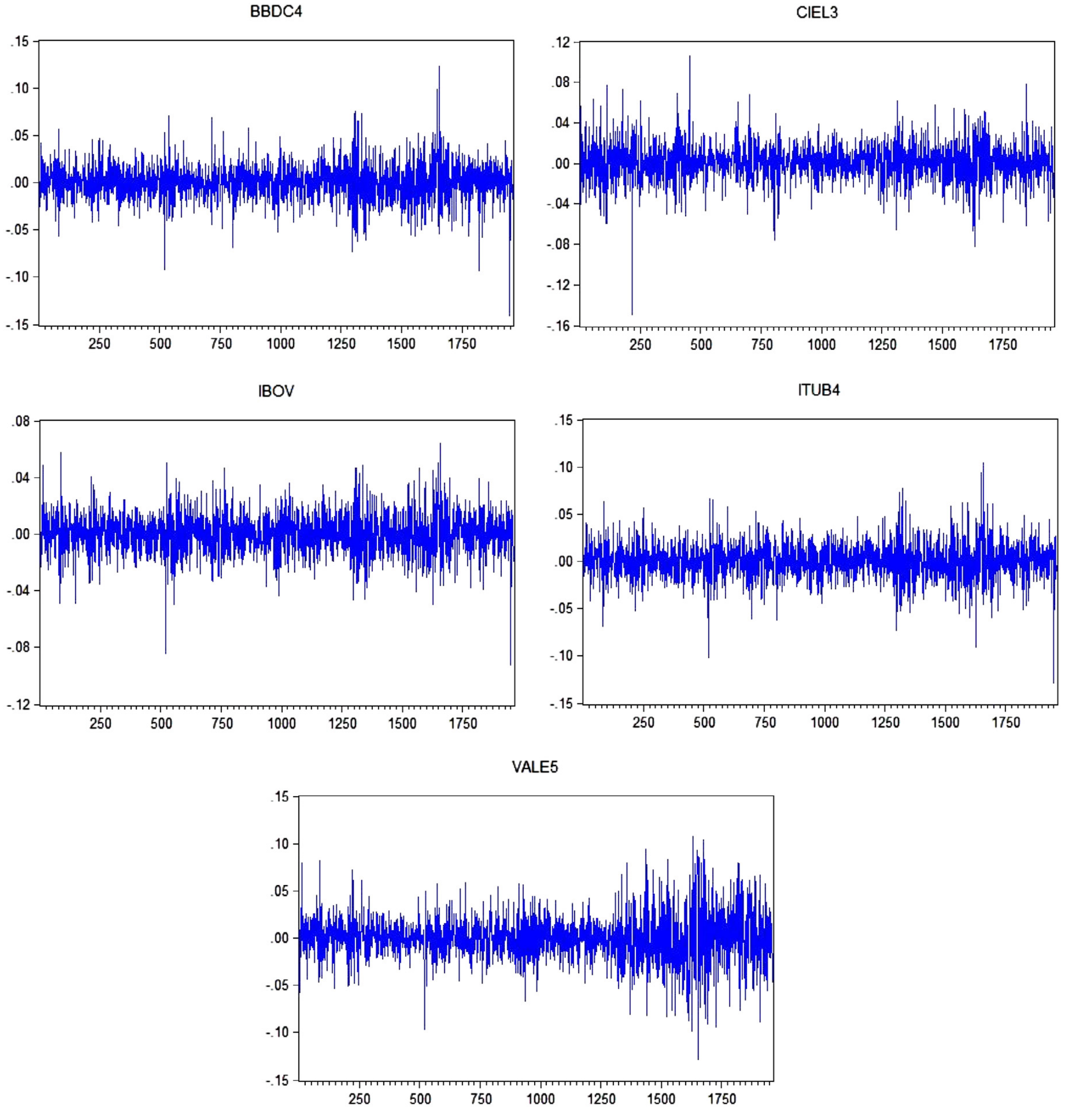

Fig. 3. Returns of stock indexes (y-axis) of the series studied versus time in working days (x-axis), starting from June 26, 2009 and ending in May 31, 2017.

that the analyzed series are reasonably symmetrical, since they have coefficients in the range from −1 to 1, or close to these values. When the asymmetry is characterized by a shift to the left, it can be said that this characterization is given by negative returns, which is the case of BBDC4, CIEL3, Ibovespa index and ITUB4. The opposite is also true, that is, when the asymmetry is characterized by a rightward shift, it can be said that this characterization is given by positive returns, which is the case of VALE5.

The kurtosis values indicate that the return series are not normal, since the kurtosis value of a normal distribution is equal to 3. In the series presented in the table, the values vary between 4.95 and 6.62. This means that there is an excess of kurtosis ($K > 3$) in all distributions, that is, they are higher (tapered) and concentrated than normal. They are also called leptokurtic and are characterized by typical fat tails or "heavy tails" a known stylized fact of financial time series. The values of Jarque–Bera and the rejection of null hypothesis are in agreement with the results of asymmetry and kurtosis, indicating the non-normality of the series studied.

Table 1
Descriptive analysis of the return series.

	BBDC4	CIEL3	IBOV	ITUB4	VALE5
Mean	0.000464	0.000668	0.000101	0.000465	0.000111
Median	0.000569	0.000848	0.000252	0.000338	0.00000
Maximum	0.122462	0.106023	0.063873	0.103684	0.107513
Minimum	−0.140558	−0.149532	−0.092110	−0.128364	−0.128403
Std. Dev.	0.019570	0.018894	0.014718	0.019361	0.024921
Skewness	−0.041023	−0.186684	−0.146527	−0.039401	0.046993
Kurtosis	6.441180	6.625920	4.954086	5.740286	5.101723
Jarque–Bera	967.1300	1084.527	318.6910	613.4429	361.2783
Probability	0.000000	0.000000	0.000000	0.000000	0.000000
Observations	1961	1961	1961	1961	1961

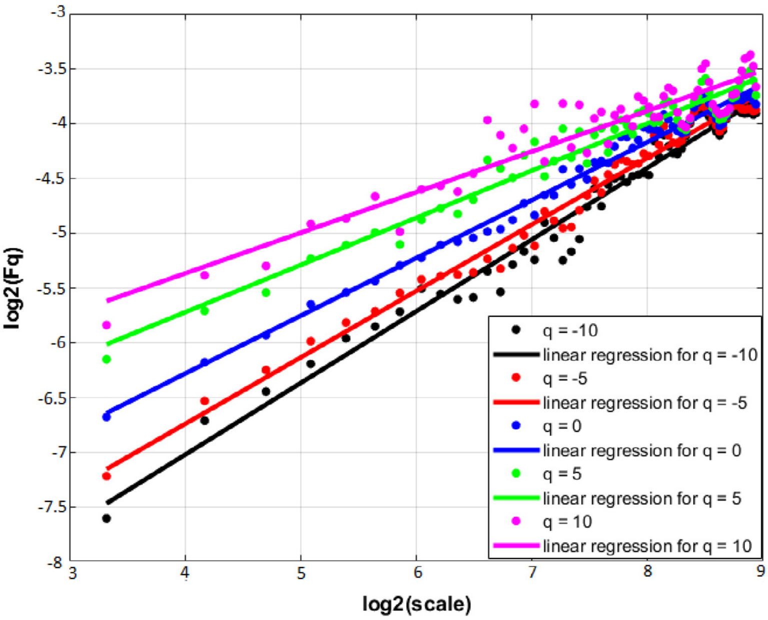

Fig. 4. Log–log plot of the *Fq* curve for scale.

4. Results

4.1. The multifractal characteristics of stock indexes studied

Fig. 4 is the double logarithmic plot of $F_q(s)$ versus s, which are described in Eqs. (1) to (7) ($s = 10, 18, 26, \ldots, 490$; where $490 \sim N/4$ and N is the number of data in the series). The upper and lower curves correspond to the values of $q = 10$ and $q = -10$, respectively, for the Ibovespa index. The slopes of the regression curves give the generalized Hurst exponents, $h(q)$.

Fig. 5 presents the local derivatives of Fig. 4 [44]. They were computed (for $q = 0$) from every 15 points in Fig. 4 and the goal is to narrow the interval where the results are significant, as the scale range can affect multifractality of stock returns [27]. A small variation is tolerated, as we are dealing with experimental data and performing a statistical analysis [44]. So, after Fig. 5, we set the scale vector from 10 to 370, with steps of 8.

Fig. 6 shows the generalized Hurst exponents $h(q)$ for different values of q ranging from −10 to 10 ($q = -10, -9, \ldots -1, 0, 1, \ldots, 9, 10$) [45] for all series analyzed, identifying their respective error bars (plus or minus one standard deviation, obtained in this case directly from the theory of linear regression and for the other parameters from moving block bootstrap with 50 adjacent windows of 1911 data points each [46]) for the data and period here considered. From Eq. (4) is possible to observe that for negative values of q small fluctuations will produce larger values of the qth order fluctuation function than the values for positive q. But, for positive q, large fluctuations will produce larger values of the qth order fluctuation function than that for negative q. So, the values of the generalized Hurst exponents are not constant, but, instead, are decreasing with increasing values of q, identifying a multifractal behavior of the series. This implies that it is not possible to use only fractal models for the analysis of these financial time series.

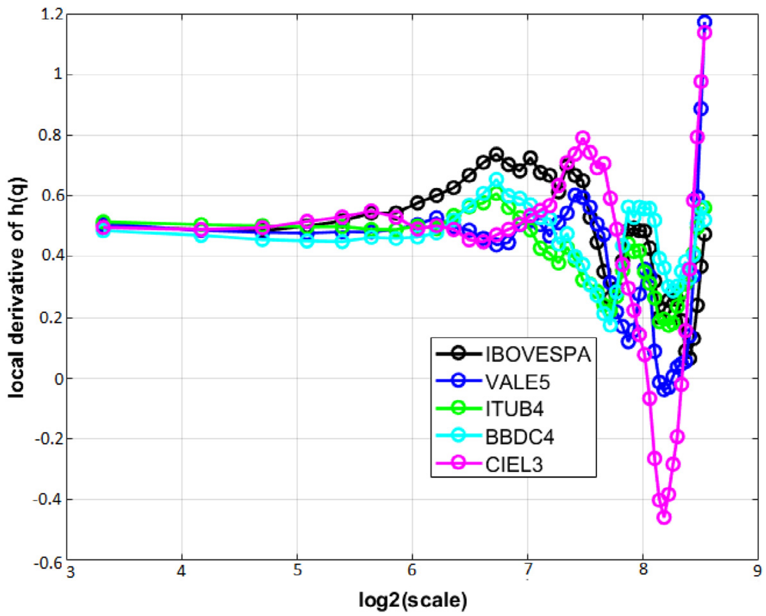

Fig. 5. Local derivatives (for $q = 0$) for the data displayed in graphics like Fig. 4, where 15 points where used to compute the local slopes.

Table 2
Multifractality degrees (MF1) of Brazilian's stock index returns.

	h max	h min	Δh
Ibovespa	0.7325 (0.0237)	0.3179 (0.0264)	0.4146 (0.0501)
VALE5	0.6607 (0.0246)	0.2874 (0.0350)	0.3733 (0.0596)
ITUB4	0.7096 (0.0198)	0.2774 (0.0184)	0.4322 (0.0382)
BBDC4	0.6553 (0.0192)	0.3595 (0.0360)	0.2958 (0.0552)
CIEL3	0.7077 (0.0231)	0.2887 (0.0184)	0.4190 (0.0315)

Note: Standard deviations are in parentheses. For Δh the sum of standard deviations of max(h) and min(h) are in parentheses.

A first measure of the multifractality, the multifractality degree MF1, defined in Eq. (10), was then computed. But, here, there is a confusion in the literature, because different authors use different ranges of q, some going from $q = -5$ to $q = 5$ [12,45], others from $q = -10$ to $q = 10$ [16,27,42], including $q = -50$ to $q = 50$ [44] and, even, $q = -100$ to $q = 100$ [15]. The problem arises because the maximum value of $h(q)$ (or the minimum value of α) is bounded by physical reasons, as the physical signal does not diverge to infinity at any point, while the opposite side of the interval needs not to be bounded [47]. In order to avoid this kind of problem some authors suggest to use de interval for the Δh and for $\Delta\alpha$ associated to a fractal dimension equal to zero [47,12]. But not every curve reaches $f(\alpha) = 0$, so we have two options: or extrapolate the curve (fitting it with another one) or consider small values around zero as being reasonable. A data exploration revealed that a range for q from $q = -50$ to $q = 50$ produces that small values for $f(\alpha)$ and, so, we fixed the computations of the deltas between these two extreme values of q [44]. The values of the multifractality degree MF1 found (Δh) are displayed in Table 2.

It is important to note that for stationary time the generalized Hurst exponent $h(2)$ obtained through the MFDFA method is identical to the Hurst exponent obtained by the DFA method. As the $h(2)$ presented for Ibovespa index are greater than 0.5, we conclude that the returns of the Ibovespa index show some degree of persistence. Instead, for the four stocks the $h(2)$ values are smaller than 0.5, indicating, for the stocks, some degree of anti-persistent behavior.

Fig. 7 presents the multifractal spectrum MF2 of the series studied with respective error bars, for q ranging from -10 to 10. This graph identifies the maximum $\alpha(q)$ parameter and the minimum $\alpha(q)$ parameter of the sample. The curves in Fig. 7 are shifted to the right in comparison to other graphics like this found in the literature and this is a consequence of the definition of Eq. (7) here used [12,48,49]. The width (difference between the maximum and minimum parameters) is one of the indicators of the multifractal behavior of the data [50].

Table 3 shows the intermittency degree, $\Delta\alpha$, which shows the difference between the maximum of α and the minimum of α of the series analyzed (Eq. (11)), with q varying between -50 and 50 [44]. The greater the value of $\Delta\alpha$, the more uneven the time series distribution will be and stronger the multifractality. Among the analyzed cases, a stronger degree of multifractality is observed for the ITUB4 series.

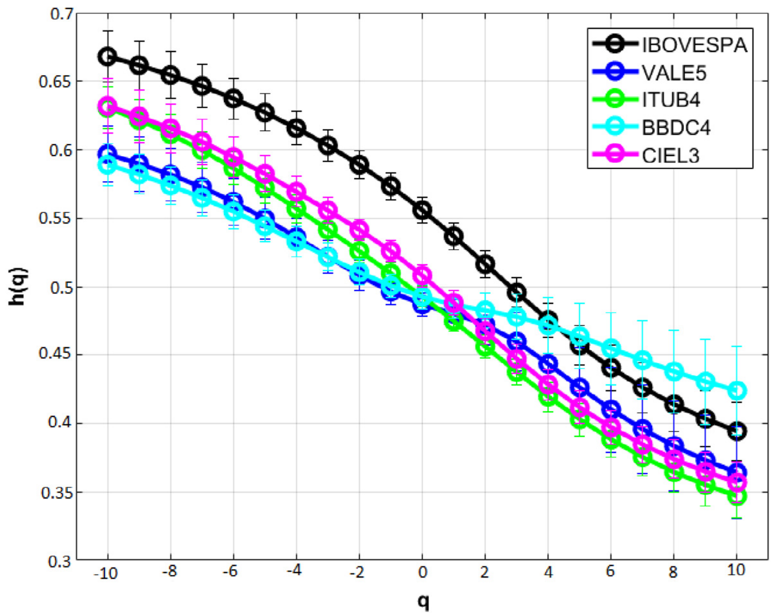

Fig. 6. Generalized Hurst exponent $h(q)$ of Brazilian's stock indexes analyzed.

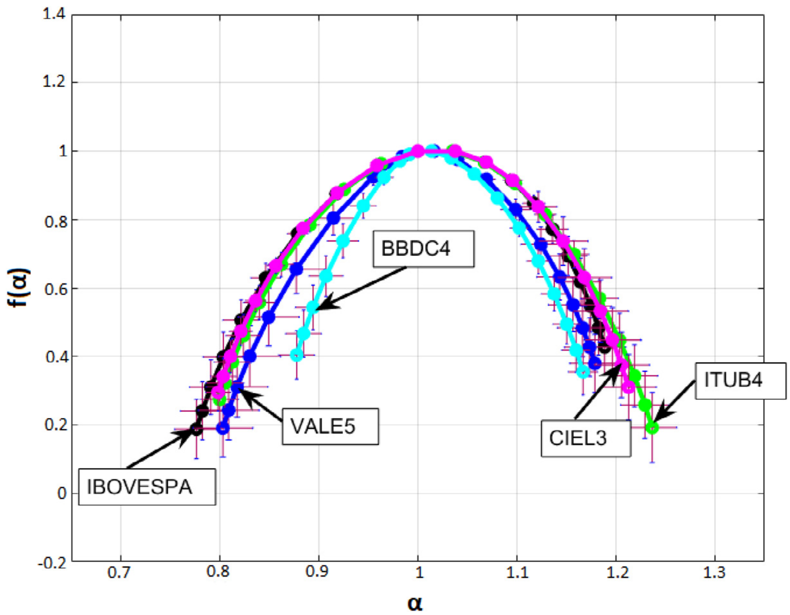

Fig. 7. Multifractal spectra of Brazilian's stock indexes analyzed.

Fig. 8 presents the scale exponents, also called mass exponents, of order q; they are obtained through the modified Eq. (7). If the $h(q)$ were constant (that is, not multifractal), we should get a straight line for all the assets. As we can see, this is not the case, providing a further indication of the multifractal character of these series.

Fig. 9 exhibits how the singularity exponents α varies with q. As for the $h(q)$, there is a strong variation across the values of q, indicating a multifractal pattern for all the series here considered.

Fig. 10 shows the singularity dimension $f(\alpha)$. It should be observed that the values of $f(\alpha)$ converge to $f(0)\,(=1)$ for all the series and decreases to zero as the values of q depart from zero. For a monofractal $h(q)$ should be constant, the q-order singularity exponent α should be equal to $h(q)$ and, consequently, $f(\alpha)$ should be a constant equal to 1. This is another evidence of the multifractal behavior of the series here analyzed.

Table 3
Intermittency degrees of Brazilian's stock index returns.

	α max	α min	$\Delta\alpha$
Ibovespa	1.2154 (0.0184)	0.7606 (0.0188)	0.4548 (0.0372)
VALE5	1.1999 (0.0217)	0.7868 (0.0268)	0.4131 (0.0485)
ITUB4	1.2564 (0.0220)	0.7833 (0.0167)	0.4731 (0.0387)
BBDC4	1.1880 (0.0132)	0.8532 (0.0159)	0.3348 (0.0291)
CIEL3	1.2408 (0.0252)	0.7807 (0.0106)	0.4601 (0.0358)

Note: Standard deviations are in parentheses. For $\Delta\alpha$ the sum of standard deviations of max(α) and min(α) are in parentheses.

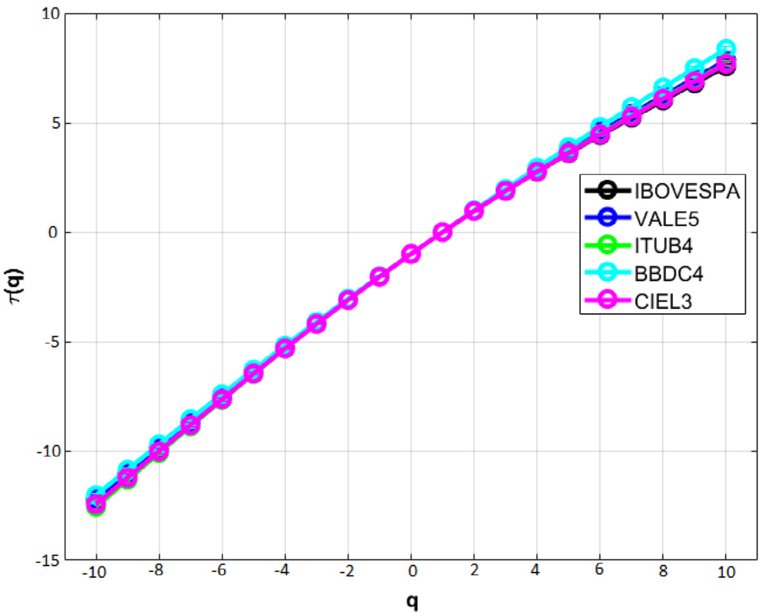

Fig. 8. Mass exponents of Brazilian's stock indexes analyzed.

4.2. The sources of multifractality

There are two main factors that contribute to the multifractal properties of a time series: temporal correlation and probability distributions, that is, fat tails. In order to identify the sources of multifractality of the Brazilian series returns we calculated the generalized Hurst exponents $h(q)$ for the original and for the shuffled and surrogated series of returns (Fig. 11). It is observed that the procedure of shuffling the series destroys any temporal correlations, preserving the flotation distribution, and the phase randomization procedure weakens the non-Gaussian distribution of the time series. If the shuffled curve of $h(q)$ keeps close to the curve for the original time series, this indicates that destroying the long-range correlations does not affect the multifractality, which, by its side, implies that the long-range correlations are not the cause of the multifractality. The same interpretation can be made for the surrogated curve of $h(q)$: if it keeps close to the curve of the original time series, this indicates that the fat tails are not the cause of the multifractality (because fat tails are destroyed in surrogate time series). Therefore, this observation allows us to visualize the possible sources of multifractality for the Brazilian market.

Table 4 exhibits the multifractality degrees MF1 and Table 5 exhibits the multifractality degrees MF2 for the Ibovespa index and for the VALE5, ITUB4, BBDC4 and CIEL3 stocks. As we can see in both tables, there is a reduction in the values of Δh and $\Delta\alpha$ for the Ibovespa index as well for three of the assets: VALE5, ITUB4 and CIEL3, both for shuffled and surrogated series. The BBDC4 is an exception: just the multifractality degree for shuffled series diminished.

These results are not conclusive, due to the large error bars (compatible with others found in the literature). There is, generally speaking, a reduction in most of the values, but this reduction cannot be significant due to the error bars. So, it seems that long-range correlations are not significant in the origins of the multifractality in the Brazilian stock market, but the fat tails can have a significant role for the IBOVESPA and for the CIEL3. This can be a local result, because the values change as the time spanned by the series changes.

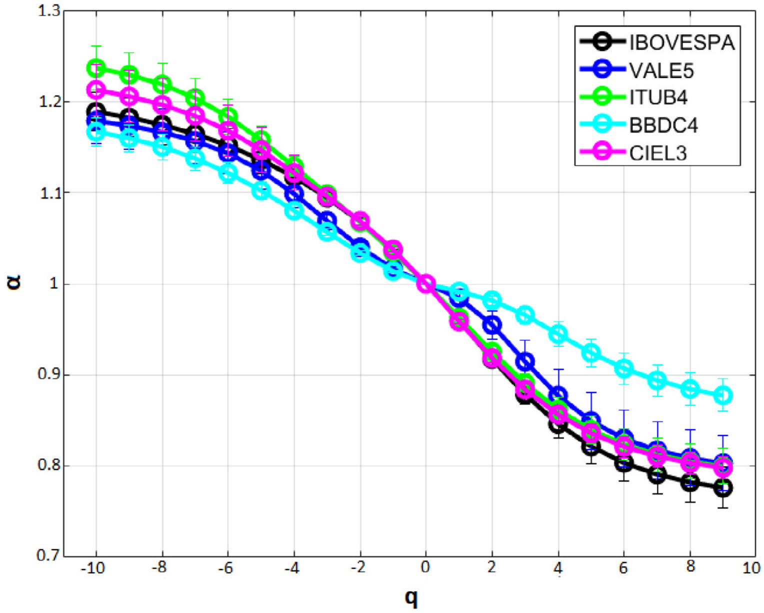

Fig. 9. The singularity exponents α versus q.

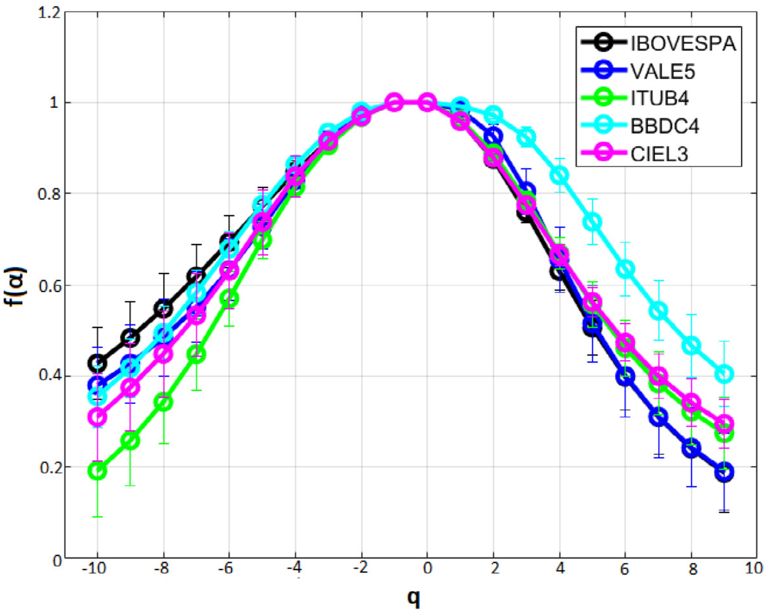

Fig. 10. The singularity dimension $f(\alpha)$ versus q.

5. Conclusions

Analyzing and understanding the behavior of financial market assets is of great importance to the area of financial accounting since it helps users to take better-informed decisions. The first thing is to try to understand the behavior and then try to quantify it in something useful to the market, like forecasting. In this paper, we addressed the first try, leaving the second to a next work.

This study examined the shares of Bradesco, Cielo, Itaú, Vale do Rio Doce and the Ibovespa index, quoted by daily, amounting to 1961 observations, in order to verify if the financial series are multifractal, through the MFDFA (Multifractal Detrended Fluctuation Analysis) method, designed by Kantelhardt [33].

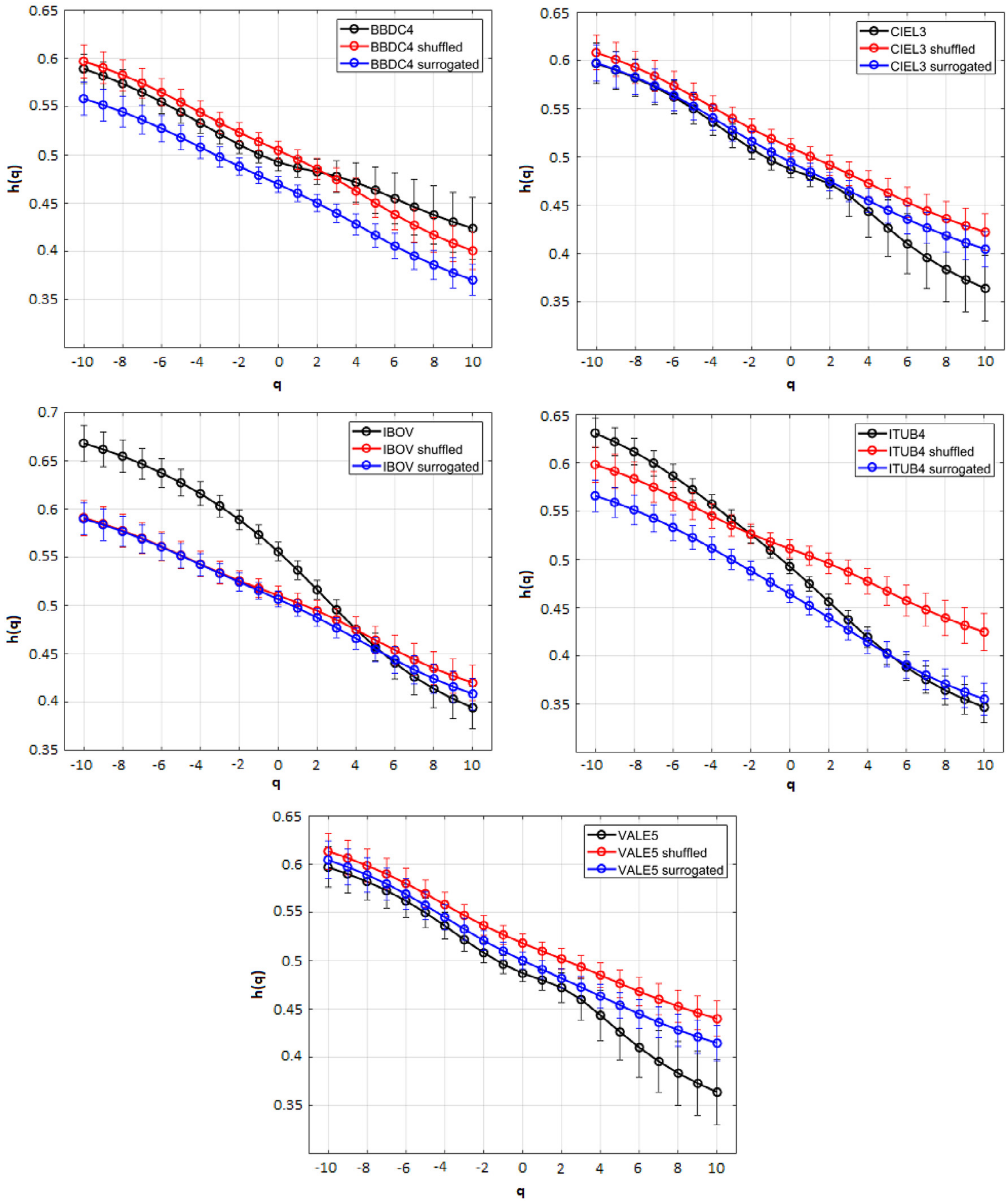

Fig. 11. Generalized Hurst exponents $h(q)$ for different time series: the original (black), the shuffled (red) and the surrogated (green). The values and the standard deviations are in Table 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

It was evident at the end that all series studied have multifractal characteristics, but the degree of multifractionality, although not the same for all assets, does not seem to originate from long-range correlations, at least for the period considered and for the Brazilian market. The influences of long-range correlations and fat tails seem to have a small influence on the multifractality of these series. Regarding the distributions, IBOVESPA and CIEL3 seem to be affected by their influence. Among the possible reasons for this weak influence of both the main factors that cause multifractality may be the large concentration of shares in the hands of a few investors in most companies that own shares in the Brazilian stock market and the huge control of the Brazilian government over the economy, both factors harming free market influences.

Table 4
Multifractality degrees of the 4 Brazilian's stock and Ibovespa index returns for the original, shuffled and surrogated time series.

	Δh Original data	Δh Shuffled data	Δh Surrogate data
Ibovespa	0.4146 (0.0501)	0.2980 (0.0851)	0.2978 (0.0732)
VALE5	0.3733 (0.0596)	0.3117 (0.0779)	0.3233 (0.0688)
ITUB4	0.4322 (0.0382)	0.3215 (0.0826)	0.3439 (0.0640)
BBDC4	0.2958 (0.0552)	0.3133 (0.0738)	0.3362 (0.0701)
CIEL3	0.4190 (0.0315)	0.3377 (0.0829)	0.2987 (0.0754)

Note: for shuffled and surrogated the numbers are the average over the 50 realizations. The sum of standard deviations of max(h) and min(h) are in parentheses.

Table 5
Intermittency degrees of the 4 Brazilian's stock and Ibovespa index returns for the original, shuffled and surrogated time series.

	$\Delta\alpha$ Original data	$\Delta\alpha$ Shuffled data	$\Delta\alpha$ Surrogate data
Ibovespa	0.4548 (0.0372)	0.3376 (0.0864)	0.3370 (0.0745)
VALE5	0.4136 (0.0485)	0.3510 (0.0786)	0.3626 (0.0701)
ITUB4	0.4731 (0.0387)	0.3609 (0.0839)	0.3837 (0.0653)
BBDC4	0.3348 (0.0291)	0.3526 (0.0750)	0.3760 (0.0715)
CIEL3	0.4601 (0.0358)	0.3778 (0.0844)	0.3381 (0.0772)

Note: for shuffled and surrogated the numbers are the average over the 50 realizations. The sum of standard deviations of max(α) and min(α) are in parentheses.

With this survey, a small gap on the multifractal characteristics of the time series of the Brazilian financial market has been healed. However, it is known that this area still needs to be further explored. For future research, it is suggested that the multifractality should be designed for daily data for longer periods of time, for high frequency data, to other financial series of the Brazilian market (such as the exchange rates), for attempts to look for differentiation in the multifractal behavior in normal periods and periods of crisis and for forecasting.

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