

# Some Important Corrections About Euler-Type Universal Numerical Integrator (E-TUNI)

Paulo Tasinaffo<sup>1</sup> and Luiz Dias

<sup>1</sup> Instituto Tecnológico de Aeronáutica (ITA), São José dos Campos/SP, Brazil; tasinaffo@ita.br

**Abstract:** A universal numerical integrator (UNI) is a numerical computational structure constructed by coupling a conventional numerical integrator (e.g., Euler, Runge-Kutta, Adams-Bashforth, etc.) with some kind of universal approximator of functions (e.g., feed-forward neural networks, fuzzy inference systems, etc.). A UNI can be divided into three types, thus giving rise to three types of distinct methodologies, namely: (i) the NARMAX methodology (Nonlinear Auto Regressive Moving average with eXogenous input), (ii) the mean derivatives methodology (E-TUNI), and (iii) the instantaneous derivatives methodology. The major contribution of this article is to correct some mathematical developments of E-TUNI that appear wrongly in articles already published about it. Besides this, a bibliographic review of the E-TUNI (Euler-Type Universal Numerical Integrator) is carried out, which is a special type of UNI that works exclusively with mean derivatives functions. The E-TUNI is based entirely on the first-order Euler-type integrator and serves for modelling non-linear dynamic systems of real-world with high numerical accuracy. Furthermore, the E-TUNI is designed here following the supervised learning paradigm. Finally, the NARMAX and mean derivatives (E-TUNI) methodologies are both fixed-step in the simulation phase. On the other hand, the instantaneous derivatives methodology (e.g., Runge Kutta neural network, etc.) is of varied-step in the simulation phase. However, all these three methodologies are of fixed-step in the training phase.

**Keywords:** Universal Numerical Integrator (UNI); Euler-Type Universal Numerical Integrator (E-TUNI); Runge-Kutta Neural Network (RKNN); Adams-Bashforth Neural Network (ABNN); Nonlinear Auto Regressive Moving average with eXogenous input (NARMAX); neural differential equations

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3.1.1. Subsubsection

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	Data	Data	Data
	Data	Data	Data
Entry 2	Data	Data	Data
	Data	Data	Data
	Data	Data	Data
Entry 3	Data	Data	Data
	Data	Data	Data
	Data	Data	Data
Entry 4	Data	Data	Data
	Data	Data	Data
	Data	Data	Data

\* Tables may have a footer.

Text.75

Text.76

3.3. *Formatting of Mathematical Components*77

This is the example 1 of equation:78

$$a = 1,$$

(1)

the text following an equation need not be a new paragraph. Please punctuate equations as regular text.7980

This is the example 2 of equation:81

$$a = b + c + d + e + f + g + h + i + j + k + l + m + n + o + p + q + r + s + t + u + v + w + x + y + z$$

(2)

Please punctuate equations as regular text. Theorem-type environments (including propositions, lemmas, corollaries etc.) can be formatted as follows:8283

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Authors should discuss the results and how they can be interpreted from the perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.90919293

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This section is not mandatory, but can be added to the manuscript if the discussion is unusually long or complex.9596

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## Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute	
DOAJ	Directory of open access journals	
TLA	Three letter acronym	151
LD	Linear dichroism	

Appendix A

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Appendix A.1

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Entry 2	Data	Data

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5. Author 1, A.B. (University, City, State, Country); Author 2, C. (Institute, City, State, Country). Personal communication, 2012. 168

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