Design of Distributed Systems [1]

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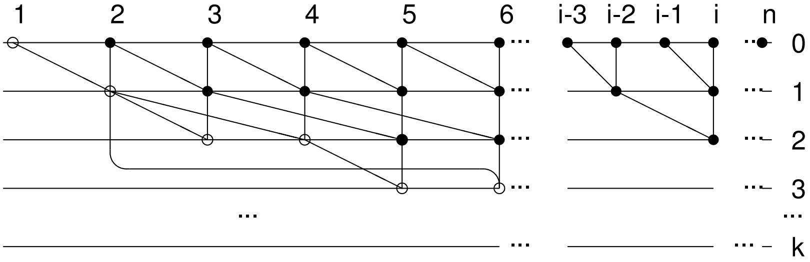
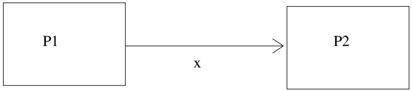
Contents

Computation of the Value of an Associative Function



$\circ : k(i) = \lceil log(i) \rceil$

Channels

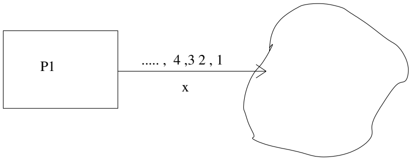


- $x : Ch(Int)$ – queue, buffer for one directional communication
- Error-free, unbounded or bounded
- $\overline{x}$ – the history of the channel
- Operations:
  - $x := x; e = hiext(x, e)$ (P1)
  - $x := lorem(x)$, if $x \neq <>$ (P2)
  - $e := lov(x) = x.lov$, if $x \neq <>$
  - $x := <>$
  - $n := length(x) = |x|$

Semantics of Operations

- $wp(x := x; e,\ R) = R^{x \leftarrow x;e,\ \overline{x} \leftarrow \overline{x};e}$
- $wp(x := lorem(x),\ \text{if } x \neq <>,\ R) =$
  $$(x \neq <> \rightarrow R^{x \leftarrow lorem(x)}) \wedge (x = <> \rightarrow R).$$
- $wp(x := <>,\ R) = R^{x \leftarrow <>, \overline{x} \leftarrow <>}$.
- **Locality:** any property P of P1 is stable in the other process(es), if $V(P)$ contains local variables and outgoing channels variables of P1 only.
- For any property P, if $P \Rightarrow P^{\overline{x} \leftarrow \overline{x};e}$ and $V(P) = \{\overline{x}\}$, then P is stable in the system.

Example – Natural Number Generator (NNG)



$$
\begin{array}{rll}
A = & Ch(Int)\times & Ch(Int) \\
 & x & \overline{x} \\
B = & Ch(Int)\times & Ch(Int) \\
 & x' & \overline{x}'
\end{array}
$$

$$(x = x' = <> \wedge \overline{x} = \overline{x}' = <>) \in INIT_{x',\overline{x}'} \tag{1}$$

$$\overline{x} \leq [1, 2, ..] \in \mathrm{inv}_{x',\overline{x}'} \tag{2}$$

$$\forall k \in N_0 : |\overline{x}| = k \hookrightarrow_{x',\overline{x}'} |\overline{x}| = k + 1 \tag{3}$$

NNG –Refinement of the Problem

$$
\begin{array}{llll}
A = & Ch(Int)\times & Ch(Int)\times & N_0 \\
 & x & \overline{x} & i \\
B = & Ch(Int)\times & Ch(Int) & \\
 & x' & \overline{x}' &
\end{array}
$$

$$(x = x' =<> \wedge \overline{x} = \overline{x}' =<>) \in INIT_{x',\overline{x}'} \tag{4}$$

$$i \in N_0 \in \mathrm{inv}_{x',\overline{x}'} \tag{5}$$

$$((i = 0 \wedge \overline{x} =<>) \vee (i > 0 \wedge \overline{x} = [1,..i])) \in \mathrm{inv}_{x',\overline{x}'} \tag{6}$$

$$\forall k \in N_0 : |\overline{x}| = k \mapsto_{x',\overline{x}'} |\overline{x}| = k + 1 \tag{7}$$

NNG –Solution

$$
\begin{aligned}
&S:\\
&\quad(\quad s_0 : i := 0,\\
&\\
&\qquad\quad \{s_1 : x, i := x;(i+1), i+1\}\\
&\quad)
\end{aligned}
$$

The Program Solves the Problem

Proof.

(5):

- We show $i \in N_0 \in \mathrm{inv}_S(x = x' =<> \wedge \overline{x} = \overline{x}' =<>)$
- $sp(i := 0, x = x' =<> \wedge \overline{x} = \overline{x}' =<>) =$
  $i = 0 \wedge x = x' =<> \wedge \overline{x} = \overline{x}' =<>$
  $\Rightarrow i \in N_0$
- $i \in N_0 \Rightarrow (wp(x, i := x; (i+1), i+1), i \in N_0) = i + 1 \in N_0$

$\square$

The Program Solves the Problem

Proof.

(6):

- $sp(i := 0, x = x' =<> \wedge \overline{x} = \overline{x}' =<>) =$
  $i = 0 \wedge x = x' =<> \wedge \overline{x} = \overline{x}' =<>$
  $\Rightarrow i = 0 \wedge x' =<>$
- $((i = 0 \wedge \overline{x} =<>) \vee (i > 0 \wedge \overline{x} = [1, .., i])) \Rightarrow$
  $(wp(x, i := x; (i + 1), i + 1), ((i = 0 \wedge \overline{x} =<>) \vee (i > 0 \wedge \overline{x} = [1, ..i]))) =$
  $((i{+}1 = 0 \wedge \overline{x}; (i{+}1) =<>) \vee (i{+}1 > 0 \wedge \overline{x}; i = [1, .., i{+}1])) =$
  $\overline{x}; i = [1, .., i + 1]))$

$\square$

The Program Solves the Problem

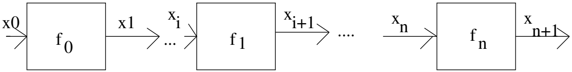
Proof.

(7):

- $\forall k \in N_0$:
- $|\overline{x}| = k \rhd_{x',\overline{x}'} |\overline{x}| = k + 1 \Leftrightarrow$
  $|\overline{x}| = k \Rightarrow wp(S, |\overline{x}| = k \vee |\overline{x}| = k + 1) =$
  $|\overline{x}; (i + 1)| = k \vee |\overline{x}; (i + 1)| = k + 1$ and
- $|\overline{x}| = k \Rightarrow wp(s, |\overline{x}| = k + 1) = |\overline{x}; (i + 1)| = k + 1$

$\square$

Pipeline



- $F = f_n \circ ... \circ f_0$.
- $D =< d_1, .., d_m >$.
- $m >> n$

Specification of Pipeline

$$\begin{array}{rcccccccc} A & = & Ch(a)_{x_0} & \times & Ch(a)_{\overline{x_0}} & \times & Ch(a)_{x_{n+1}} & \times & Ch(a)_{\overline{x_{n+1}}} \\ B & = & Ch(a)_{x'_0} & \times & Ch(a)_{\overline{x_0}'} & \times & Ch(a)_{x'_{n+1}} & \times & Ch(a)_{\overline{x_{n+1}}'} \end{array}$$

$$\begin{aligned} Q ::= (x_0 = \overline{x_0} = x'_0 = \overline{x_0}' = D \wedge \\ \wedge x_{n+1} = \overline{x_{n+1}} = x'_{n+1} = \overline{x_{n+1}}' = <>) \\ Q \in INIT_{\underbrace{x'_0 \overline{x_0}' x'_{n+1} \overline{x_{n+1}}'}_{h}} \qquad (8) \\ \mathrm{FP}_h \Rightarrow \overline{x_{n+1}} = F(\overline{x_0}') = F(D) \qquad (9) \\ Q \in \mathrm{TERM}_h \qquad (10) \\ (\overline{x_0} = \overline{x_0}' = D) \in inv_h \text{ for the whole system} \qquad (11) \end{aligned}$$

Refinement of the Problem

$$\mathrm{FP}_h \Rightarrow \forall i \in [0..n] : x_i = <> \tag{12}$$

$$\forall i \in [0..n] : (f_i(\overline{x_i} - x_i) = \overline{x_{i+1}}) \in \mathrm{inv}_h \tag{13}$$

$$\text{Variant function: } (|x_0|, ..., |x_n|) \tag{14}$$

Refinement of the Problem

Proof.

- By fixed point refinement it is sufficient: $(12) \wedge (13) \wedge (11) \Rightarrow (9)$.
- Proof by using the lemma: $(12) \wedge (13) \wedge (11) \Rightarrow (\overline{x_{i+1}}) = f^i(D)$.
- The lemma is proved by induction.

$\square$

Solution

$$
\begin{aligned}
S: \quad & (\|_{i=1}^{n}\ x_i :=<>, \\
\{ \quad & \Box_{i=0}^{N} x_i, x_{i+1} := \mathit{lorem}(x_i), \mathit{hiext}(x_{i+1}, f_i(x_i.\mathit{lov})), \\
& \quad \text{if } x_i \neq <> \})
\end{aligned}
$$