Labsheet 1.3

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1 Exercise 1

$$
\begin{aligned}
S &= \lambda xyz.xz(yz) \\
K &= \lambda xy.x \\
I &= \lambda x.x
\end{aligned}
$$

1.1 a)

$$
\begin{aligned}
SKK &= (\lambda xyz.xz(yz))(\lambda xy.x)(\lambda xy.x) \\
&\rightarrow_\alpha (\lambda xwz.xz(wz))(\lambda xy.x)(\lambda xy.x) \\
&\rightarrow_\beta (\lambda wz.(\lambda xy.x)z(wz))(\lambda xy.x) \\
&\rightarrow_\beta (\lambda z.(\lambda xy.x)z((\lambda xy.x)z)) \\
&\rightarrow_\beta (\lambda z.(\lambda xy.x)z(\lambda y.z)) \\
&\rightarrow_\beta (\lambda z.(\lambda y.z)(\lambda y.z)) \\
&\rightarrow_\beta (\lambda z.z)
\end{aligned}
$$

1.2 b)

$$
\begin{aligned}
SIK =&(\lambda xyz.xz(yz))IK \\
\rightarrow_\beta&(\lambda yz.Iz(yz))K \\
\rightarrow_\beta&(\lambda z.Iz(Kz)) \\
=&(\lambda z.Iz((\lambda xy.x)z)) \\
\rightarrow_\beta&(\lambda z.Iz(\lambda y.z)) \\
=&(\lambda z.(\lambda x.x)z(\lambda y.z)) \\
\rightarrow_\beta&(\lambda z.z(\lambda y.z))
\end{aligned}
$$

1.3 c)

$$\begin{aligned}
SSS =& (\lambda xyz.xz(yz))SS \\
\rightarrow_\beta & (\lambda yz.Sz(yz))S \\
\rightarrow_\beta & (\lambda z.Sz(Sz)) \\
=& (\lambda z.(\lambda xyw.xw(yw))z(Sz)) \\
\rightarrow_\beta & (\lambda z.(\lambda yw.zw(yw))(Sz)) \\
\rightarrow_\beta & (\lambda z.(\lambda w.zw((Sz)w))) \\
=& (\lambda z.(\lambda w.zw(((\lambda xyk.xk(yk))z)w))) \\
\rightarrow_\beta & (\lambda z.(\lambda w.zw((\lambda yk.zk(yk))w))) \\
\rightarrow_\beta & (\lambda z.(\lambda w.zw(\lambda k.zk(wk)))) \\
=& (\lambda zw.zw(\lambda k.zk(wk)))
\end{aligned}$$

2 Exercise 2

Let W be the term:

$$\lambda x.\lambda y.xyy$$

2.1 a)

$$\begin{aligned}
WW =& (\lambda xy.xyy)(\lambda xy.xyy) \\
\rightarrow_\alpha & (\lambda zw.zww)(\lambda xy.xyy) \\
\rightarrow_\beta & (\lambda w.(\lambda xy.xyy)ww) \\
\rightarrow_\beta & (\lambda w.(\lambda y.wyy)w) \\
\rightarrow_\beta & (\lambda w.www)
\end{aligned}$$

2.2 b)

The term WWW can be reduced to $(\lambda w.www)W$ which with one further beta reduction reduces back to WWW This tells us that WWW does not have a normal form and will only ever reduce to itself.