Labsheet 1.3

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1 Exercise 1

a) x is a variable

b) $\lambda x.x$ is an abstraction

c) $(\lambda a.z)a$ is an application

d) $\lambda a.za$ is an abstraction

e) $(\lambda n.n)z$ is an application

f) $\lambda z.(\lambda y.(\lambda x.x)y)z$ is an abstraction

g) $(\lambda t.((\lambda t.(\lambda t.t)t)t))t$ is an application

2 Exercise 2

a) $(\lambda x.xy)[\lambda z.z/y] == (\lambda x.x(\lambda z.z))$

b) $(\lambda x.xy)[\lambda z.zx/y] \rightarrow_\alpha (\lambda w.wy)[\lambda z.zx/y] == \lambda w.w(\lambda z.zx)$

c) $(f(\lambda x.yx)yx)[fy/x] \rightarrow_\alpha (f(\lambda z.yz)yx)[fy/x] == (f(\lambda z.yz)y(fy))$

d) $(\lambda f.f(\lambda x.yx)yx)[fy/x] \rightarrow_\alpha (\lambda z.z(\lambda w.yw)yx)[fy/x] == (\lambda z.z(\lambda w.yw)y(fy))$

3 Exercise 3

a) $(\lambda x.\lambda y.x)yx \rightarrow_\beta (\lambda z.y)x$

b) $(\lambda f.f(\lambda x.x))(\lambda y.z) \rightarrow_\beta (\lambda y.z)(\lambda x.x)$

c) $(\lambda x.\lambda y.yx)(\lambda x.xy) \rightarrow_\alpha (\lambda w.\lambda z.zw)(\lambda x.xy) \rightarrow_\beta \lambda z.z(\lambda x.xy)$

d) $(\lambda x.xx)((\lambda y.y)(\lambda x.x)) \rightarrow_\beta (\lambda x.xx)(\lambda x.x) \rightarrow_\beta (\lambda x.x)(\lambda x.x) \rightarrow_\beta \lambda x.x$

e) $(\lambda x.xx)(\lambda y.y)(\lambda x.x) \rightarrow_\beta (\lambda y.y)(\lambda y.y)(\lambda x.x) \rightarrow_\beta (\lambda y.y)(\lambda x.x) \rightarrow_\beta (\lambda x.x)$

f) $(\lambda x.xx)(\lambda x.xx)((\lambda y.y)(\lambda x.x)) \rightarrow_\beta (\lambda x.xx)(\lambda x.xx)(\lambda x.x) \rightarrow_\beta (\lambda x.xx)(\lambda x.xx)(\lambda x.x) \rightarrow_\beta (\lambda x.xx)(\lambda x.xx)(\lambda x.$
...

4 Exercise 4

4.1 a)

$$M = (\lambda x.\lambda y.xy)z) \rightarrow_\beta (\lambda y.zy)$$
$$N = (\lambda x.x)(\lambda y.zy) \rightarrow_\beta (\lambda y.zy)$$
$$P = (\lambda y.zy)$$

4.2 b)