Problem Sheet 1

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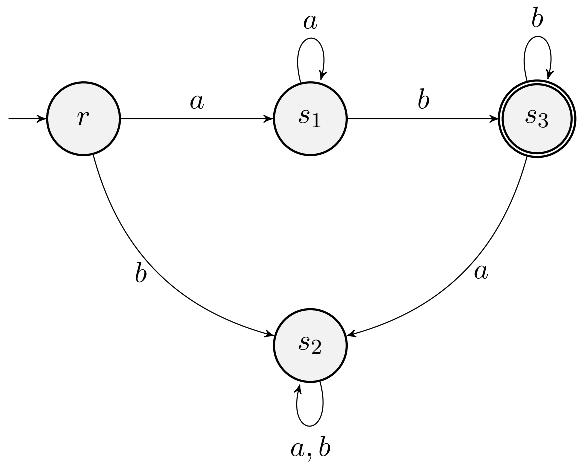
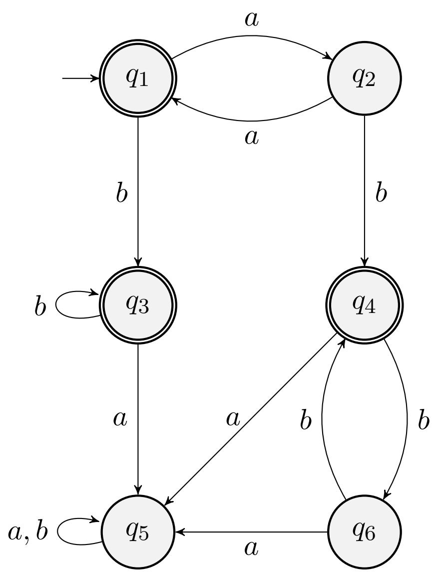
1 Question 1

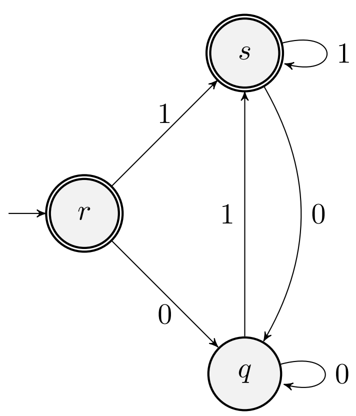
1. $\{a^n b^m | n \geq 1, m \geq 1\}$



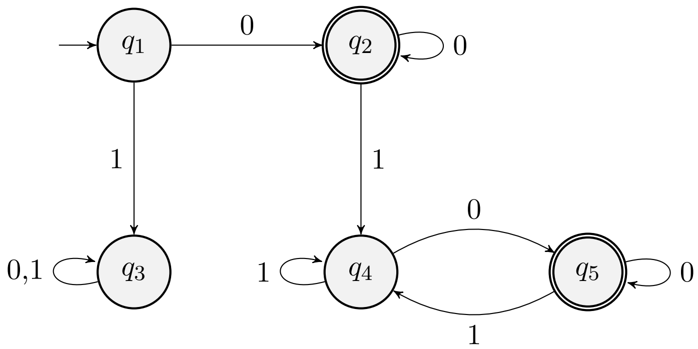
2. $\{a^n b^m | n$ is even OR m is odd $\}$



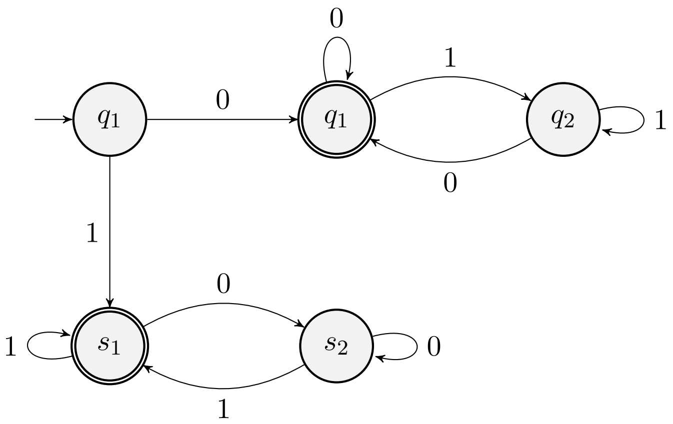
3. $\{w \in \{0,1\}* | w = \varepsilon$ or w ends in 1$\}$



4. $\{w \in \{0,1\}* | w$ starts and ends with 0$\}$



5. $\{w \in \{0,1\}* | w$ starts and ends with the same letter$\}$



2 Question 2

From diagram 3: $\{w \in \{0,1\}* | w = \varepsilon$ or w ends in 1$\}$

Let

$$M = (Q, \Sigma, \delta, r, F)$$

be a finite automaton where:

$$Q = \{r, s, q\}$$

$$\Sigma = \{0, 1\}$$

δ is described as:

δ	0	1
r	q	s
s	q	s
q	q	s

r is the start state

$$F = \{r, q\}$$

Alternatively δ can be written as:

$$\delta(r, 0) = q$$

$$\delta(r,1) = s$$
$$\delta(q,0) = q$$
$$\delta(q,1) = s$$
$$\delta(s,0) = q$$
$$\delta(s,1) = s$$

3 Question 3

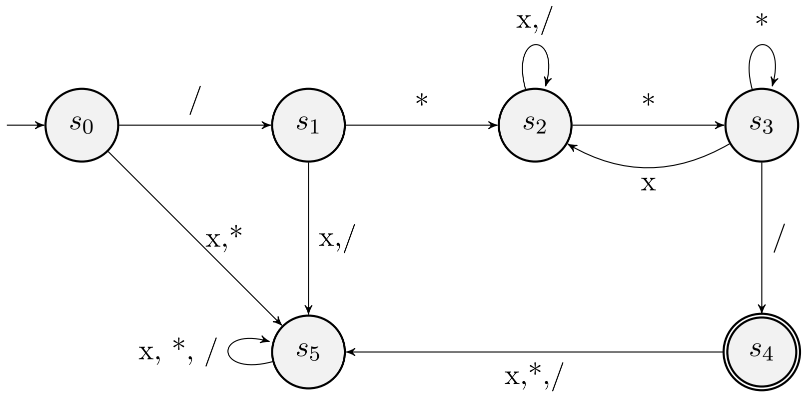


Figure 1: FSM for Java's multiline comment engine

4 Question 4

a) $\Sigma = \{a, b\}$

Accepted:

- (b, a)
- (a, b, a)

Rejected:

- (b, a, a, b)
- (a, a, a, b, b, a)

$$L(M) = \{a^n b a^m | n \geq 0, m \geq 0\}$$

b) $\Sigma = \{-,), :\}$

Accepted:

- $(:, -, -,))$

- $(:,))$

Rejected:

- $(-, :,))$
- $(:, -, -,),))$

$$L(M) = \{: -^n) | n \geq 0\}$$

5 Finite Automata Notes

A finite automata is a 5-tuple $(Q, \Sigma, \delta, s_0, F)$ where Q is a finite set of states, Σ is the input alphabet, δ is the transition function, $s_0 \in Q$ is the start state, $F \subseteq Q$ is the set of accept states.

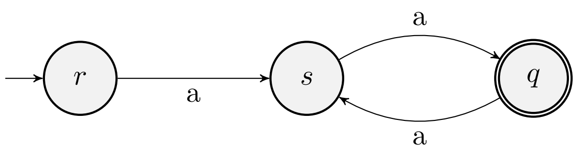


Figure 2: FSM for $\{a^n | n = 2m, m > 0, m \in \mathbb{N}\}$

For the FSM in *Figure 2*:

- $Q = \{r, s, q\}$
- $\Sigma = \{a\}$
- $\delta : Q \times \Sigma \to Q$
- $s_0 = r$
- $F = \{q\}$

The combination of a state and a string is a configuration. A configuration is a pair: $(q, w) \in Q \times \Sigma*, (q \in Q, w \in \Sigma*)$. With an original string of $aaaa$ the list of configurations for the FSM in *Figure 2* is:

- $(r, aaaa)$
- (s, aaa)
- (q, aa)
- (s, a)
- (q, ε)

Let (q, w) and (q', w') be configurations, where:

$$w = aw'$$

for some letter $a \in \Sigma$, and

$$\delta(q, a) = q'$$

Then we say that (q, w) yields (q', w') in one step.

Therefore, (q, aa) yields (s, a) in one step in the example in *Figure 2*.

For configurations (q, w) and (q', w') we say that (q, w) yields $(q', w'$ if there is a finite sequence of configurations

$$(q_1, w_1), (q_2, w_2), ..., (q_k, w_k)$$

Such that $(q_1, w_1) = (q, w), (q_k, w_k) = (q', w')$, and (q_i, w_i) yields (q_{i+1}, w_{i+1}) in one step for all $i = 1, 2, ..., k - 1$

The sequence of configurations defined above is called a **computation**

The finite automaton $M = (Q, \Sigma, \delta, s_0, F)$ **accepts** the string $w \in \Sigma*$ if (s_0, w) yields (q, ε) where $q \in F$.

We say that the finite automaton M **recognises** the language A if $A = \{w | M \quad \text{accepts} \quad w\}$.

The language **recognised by** a finite automaton M is denoted $L(M)$.

The language A is called regular if there exists some finite automaton M such that $A = L(M)$. (i.e. a finite automaton exists that recognises it)