

## CSCI 678: Statistical Analysis of Simulation Models

### Homework 5

1. The program `asm5a.c` (attached at end) was used for all calculations and analytic work. It produces the following output:

Simulating 10,000 times...

(a) Variates:

0.0553, 0.1416, 1.2354, 0.3623, 1.9618, 0.1460, 1.0917, 0.9178, 5.2585, 1.8794

(b) Exact confidence interval:

$\Pr(0.763826 < \text{mean} < 2.721327) = .95$

(c) Approximate confidence interval:

$\Pr(0.816510 < \text{mean} < 1.793455) = .95$

(d) K-S test results with theoretical distribution:

ats 0.997139 <= crit 1.358000 -- accept null hypothesis

(e) K-S test results with fitted distribution:

ats 0.662582 <= crit 1.094000 -- accept null hypothesis

[9,999 more simulations]

(f) Summary:

Percentage of exact confidence intervals that contain 1.0:

0.9487

Percentage of approximate confidence intervals that contain 1.0:

0.9542

Percentage of accepted hypotheses with theoretical distribution:

0.8746

Percentage of accepted hypotheses with fitted distribution:

0.7435

Binary	101101
Octal	55
Decimal	45
Hecadecimal	2D

Figure 1: (a)

Binary	11110010111
Octal	3627
Decimal	1943
Hecadecimal	797

Figure 2: (b)

Binary	11100
Octal	34
Decimal	28
Hecadecimal	1C

Figure 3: (c)

Binary	11000101
Octal	305
Decimal	197
Hecadecimal	C5

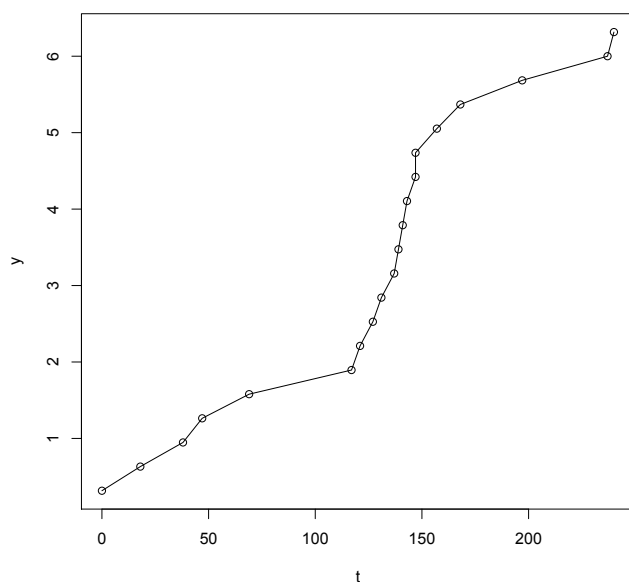
Figure 4: (d)

- 2.

3. The notes tell us that as long as  $m$  is prime and  $a$  is a primitive root of  $m$  (that is, if the smallest integer  $l$  for which  $a^l - 1$  is divisible by  $m$  is  $l = m - 1$ ), then  $a$  will have a full period. We see that 7 is indeed prime. The only integers which satisfy the second condition are 3 and 5 ( $7 \mid 3^6 - 1 = 728, 7 \mid 5^6 - 1 = 15624$ ). The program `asm5b.c` (attached at end) was used to verify these results. It produces the output:

```
a = 3
    3, 2, 6, 4, 5, 1, 3
    period = 5
a = 5
    5, 4, 6, 2, 3, 1, 5
    period = 5
```

4.



5. The following array gives us the values:

sacf	acf
0.900721	0.901
0.8208184	0.821
0.7509325	0.751
0.689028	0.689
0.6363274	0.636
0.5912949	0.591
0.5500705	0.550
0.5120536	0.512
0.4786871	0.478
0.4496129	0.449

I don't know about you, but I think that's pretty good.

## Paper Summary

The paper describes a nonparametric technique for estimating the cumulative intensity function  $\Lambda(t)$  for a non-stationary Poisson process on a finite time interval  $(0, S]$ . The author starts by reviewing several proposed solutions for simulating NHPPs, such as thinning, assuming  $\lambda(t)$  is of the form  $(\alpha t)^\beta$ , and estimating with a piecewise constant function. Some flaws with these is that they are either computationally expensive or require arbitrary decisions from the modeler. The author then suggests a procedure in which  $\Lambda(t)$  is estimated with the following piecewise linear function determined by the order statistics of the superpositions of the  $k$  realizations of the process:

$$\hat{\Lambda}(t) = \frac{in}{k(n+1)} + \frac{n(t - t_{(i)})}{(n+1)k(t_{(i+1)} - t_{(i)})}$$

The author then justifies the use of the proposed estimator, and proceeds to variate generation. After that, the author describes examples.