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CSCI 678: Statistical Analysis of Simulation Models
Homework 7

1. (a)
$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & a \le x < c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & c \le x < b \\ 1 & x \ge b \end{cases}$$

   (b)
$$F^{-1}(u) = \begin{cases} a + \sqrt{u(b-a)(c-a)} & 0 \le u < (c-a)/(b-a) \\ b - \sqrt{(1-u)(b-a)(b-c)} & (c-a)/(b-a) \le u \le 1 \end{cases}$$

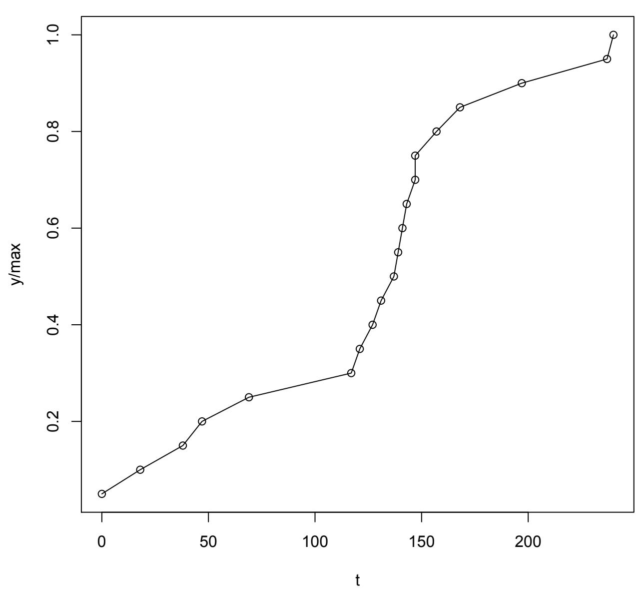
   (c) The program `asm7a.c` contains the function and the handwritten analytic work, and is attached at the end. Variates, along with the mean are printed on a separate sheet.

   (d) The program `asm7a.c` contains the function and the handwritten analytic work, and is attached at the end. Variates, along with the mean are printed on a separate sheet.

   (e) The program `asm7a.c` contains the function and the handwritten analytic work, and is attached at the end. Variates, along with the means, are printed on a separate sheet.

   (f) Written on the source code is a count of computations for each algorithm.

2. (a)



   (b) The realization of the simulation yields the following variates:

   `0.672, 0.344, 0.016, 5.607, 11.508, 17.410, 23.902, 30.459, 37.016, 40.508, 43.459, 46.410, 52.770, 59.984, 67.197, 80.803, 96.541, 112.279, 117.918, 119.230, 120.541, 122.279, 124.246, 126.213, 127.787, 129.098, 130.410, 132.082,`

```
134.049,136.016, 137.328, 137.984, 138.639, 139.295, 139.951, 140.607, 141.262,
141.918, 142.574, 143.459, 144.770, 146.082, 147.000, 147.000, 147.000, 147.820,
151.098, 154.377, 157.721, 161.328, 164.934, 169.426, 178.934, 188.443, 198.311,
211.426, 224.541, 237.049, 238.033, 239.016
```

Visual representation of how the variates were obtained is found on the above diagram in (a).

3. Let $Y = \min\{u_1, u_2, \dots, u_n\}$. We see that

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(\min\{u_1, \dots u_n\} \leq y) \\ &= 1 - P(\min\{u_1, u_2, \dots u_n\} > y) \\ &= 1 - P(u_1 > y, u_2 > y, \dots u_n > y) \\ &= 1 - P(u_1 > y) * P(u_2 > y) * \dots * P(u_n > y) \\ &= 1 - (1 - P(u_1 \leq y)) * (1 - P(u_2 \leq y)) * \dots * (1 - P(u_n \leq y)) \\ &= 1 - (1 - F(u_1)) * (1 - F(u_2)) * \dots * (1 - F(u_n)) \\ &= 1 - (1 - F(u))^n \end{aligned}$$

Taking the inverse, we get $Y = F^{-1}(u) = 1 - (1 - u)^{1/n}$

4. As the acceptance-rejection method is a trial-until-first-success process, it's logical that we should use a geometric (I don't remember which parameterization is capitalized, but I'm talking about the distribution of the trial number of the first success) distribution. The parameter $p = f(x)/f^*(x)$, where f^* is the majorizing function and f is the pdf will be p in our geometric distribution.

5. If we take T to be nonnegative, then the inverse-chf method gives us that $T = H^{-1}(-\ln(1 - U))$. From this, we see that $H(T) = -\ln(1 - U)$. Applying a probability integral transformation, we can express U as $F_X(X)$, where F_X is the cdf for an exponential(1) random variable. We see then that $H(T) = -\ln(1 - F_X(X)) = -\ln(e^{-X}) = X$, which we know is exponential(1).

6. The program `asm7e.r` was used to obtain the following results:

```
"Estimate:"
8.165347
"Exact Result:"
8.144789
```