Assignment 1

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Q1. A fair coin is tossed 3 times, and the random variable x equals the total number of heads. Find and sketch $F_x(x)$ and $f_x(x)$.

1. $F_x(x)$ (Cumilative Distribution Function):

- $F_x(x)$ represents the cumilative distribution function (CDF) of the random variable x, which is the total number of heads.

- To find $F_x(x)$, we need to calculate the probability that x is less than or equal to a given value x.

- Since we are dealing with a fair coin, the sample space consists of 8 equwally outcomes (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)

- Let's calculate the probabilities of each value of x:

$(P(x) = 0) = 1/8$

$(P(x) = 1) = 3/8$

$(P(x) = 2) = 3/8$

$(P(x) = 3) = 1/8$

- Now the cumilative probabilities are

$F_x(0)$: Probability that $X \leq 0$ (no heads): $\frac{1}{8}$ (0.125)

$F_x(1)$: $P(x) \leq 1$ (0 or 1 head): $\frac{1}{8} + \frac{3}{8} = \frac{1}{2}$ (0.500)

$F_x(2)$: $P(x) \leq 2$ (0, 1, 2 head): $\frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$ (0.875)

$F_x(3)$: $P(x) \leq 3$ (0, 1, 2, 3 head): 1 (covers all possible outcomes)

Probability Mass Function $f_X(x)$:

- $f_X(x)$ represents the Probability mass function (PMF) of X, which gives the Probability of X taking specific value x.

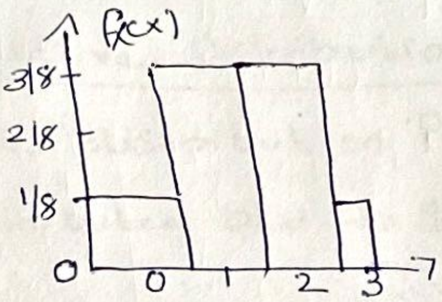
- PMF is simply the Probabilities for each value x.

  - $f_X(0) : P(X=0) = \frac{1}{8}$

    $f_X(1) : P(X=1) = \frac{3}{8}$

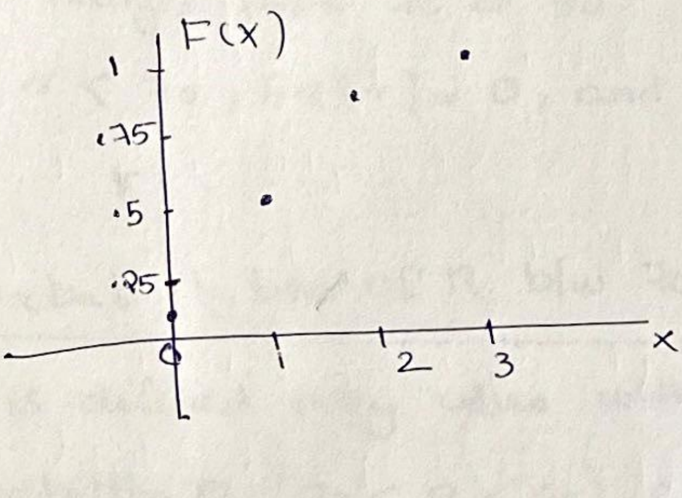
    $f_X(2) : P(X=2) = \frac{3}{8}$

    $f_X(3) : P(X=3) = \frac{1}{8}$



It is symmetrical

Sketch of CDF



Q2. Resistance R of each resister in a production line has been measured and only those resisters are accepted whose resistance lie b/w 69 & 81 ohms. Find out the percentage of the accepted resisters if R is uniform b/w 70 & 80.

Given that R is uniformly distributed b/w 70 & 80 ohms.

Step 1: Probability Density Function (PDF)

~~For~~ For R uniform b/w 70 & 80 ohms, the probability density function $f_R(r)$ is indeed:

$$f_R(r) = \frac{1}{80-70} = \frac{1}{10}$$

Step 2: Cumilative Distribution Function (CDF)

The cumilative distribution function R when R is uniformly distributed b/w 70 & 80 ohms is correctly stated as

$$F_R(r) = \frac{r-70}{80-70} = \frac{r-70}{10}$$

Where r ranges from 70 to 80.

For $r < 70$, $F_R(r) = 0$, and for $r > 80$, ~~$F_R(r)=1$~~

$F_R(r) = 1$

Step 3: Probability ~~b/w~~ of R b/w 70 & 80

Given R is defined only ~~when~~ with in 70 & 80 ohms, the probability $P(70 \leq R \leq 80)$ is

$$P(70 \leq R \leq 80) = F_R(80) - F_R(70)$$

plugging in the CDF

$$P(70 \leq R \leq 80) = \left(\frac{80-70}{10}\right) - \left(\frac{70-70}{10}\right) = 1 - 0 = 1$$

This means 100% of the resistors measured fall between 70 & 80 ohms. Hence, since the resistors must fall b/w 69 & 81 ohms to be accepted and all resistors lie b/w 70 & 80 ohms, 100% of the resistors are accepted.

Q3 Given that x is uniformly distributed b/w 0 and 5, the CDF of x, denoted $F(x)$, is calculated as follows:

$$F(x) = \frac{x-a}{x-b}$$

where

- $a = 0$ (lower bound of the distribution)
- $b = 5$ (upper bound of the distribution)

Thus the formula simplifies to

$$F(x) = \frac{x}{5}$$

To find the value of x that corresponds to a specific percentile u, you use the formula:

$$u = F(x_u) = \frac{x_u}{5}$$

$$x_u = 5u$$

Let's calculate x_u for the specified values of u:

1. $u = 0.5$

$$x_{0.5} = 5 \times 0.5 = 2.5$$

2. $u = 1.25$

$$x_{1.25} = 5 \times 1.25$$

$$= 6.25$$

3. $U = 0.73$

$$x_{0.73} = 5 \times 0.73 = 3.65$$

④ $u = 1.56$

$x_{1.56} = 5 \times 1.56$

$= 7.8$

⑤ $u = 2.56$

$x_{2.56} = 5 \times 2.56$

$= 12.8$

⑥ $u = 3.28$

$x_{3.28} = 5 \times 3.28$

$= 16.4$

Therefore the (X_u) values for the given percentile as follows

$X_{-}(0.5) = 2.5$

$X-(1.25) = 6.25$

$X-(0.73) = 3.65$

$X-(1.56) = 7.8$

$X-(2.56) = 12.8$

$X_{-}(3.28) = 16.4$

Q4. Probability of Selecting a Red chalk:

- probability of picking from each box:

 - P of picking the circular box, $P(\text{Circular}) = 0.3$

 - Probability of picking the Square box, $P(\text{square}) = 0.7$

Probability of picking a red chalk from each box:

circular box: 4 red out of 11 total chalks

$\rightarrow P(\text{red} \mid \text{circular}) = \frac{4}{11}$

• Square box: 5 red out of 7 total chalks

↳ $P(\text{red} \mid \text{square}) = \frac{5}{7}$

Using the law of total probability:

$P(\text{red}) = P(\text{red} \mid \text{circular}) \times P(\text{circular}) + P(\text{red} \mid \text{square}) \times P(\text{square})$

$P(\text{red}) = \left(\frac{4}{11}\right) \times 0.3 + \left(\frac{5}{7}\right) \times 0.7$

$P(\text{red}) = \frac{4}{11} \times 0.3 + \left(\frac{5}{7}\right) \times 0.7$

$= \frac{1.2}{11} + \frac{3.5}{7}$

$= 0.1091 + 0.5$

$= 0.6091$

So the probability of selecting a red chalk is approximately 0.6091 or 60.91%.

Probability the chalk from the square box given it is green

1. Probability of picking a green chalk from each box:

• circular box: 7 green out of 11 total chalks

↳ $P(\text{green} \mid \text{circular}) = \frac{7}{11}$

• Square box: 2 green out of 7 total chalks

↳ $P(\text{green} \mid \text{square}) = \frac{2}{7}$

Use the law of total probability for selecting a green chalk:

$P(\text{green}) = P(\text{green} \mid \text{circular}) \times P(\text{circular}) + P(\text{green} \mid \text{square}) \times P(\text{square})$

$P(\text{green}) = \frac{7}{11} \times 0.3 + \left(\frac{2}{7}\right) \times 0.7$

$P(\text{green}) = 0.1909 + 0.2$

$= 0.3909$

Using Baye's theorem

$$P(\text{square} \mid \text{green}) = \frac{P(\text{green} \mid \text{square}) \times P(\text{square})}{P(\text{green})}$$

$$= \frac{\left(\frac{2}{7}\right) \times 0.7}{0.3909}$$

$$= \frac{0.2}{0.3909}$$

$$\approx 0.5117$$

There for the probability that the chalk came from the square box is given that it is green is approximately 0.5117 or 51.17 %.

Given:

Q5) The random variable 'X' is normally distributed with a mean (μ) of 0 and a standard deviation (σ) of 2.

So, $X \sim N(0.2)$.

a) $P(1 \leq x \leq 3)$

To calculate this, we convert X to the standard normal variable Z using the formula:

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{X - 0}{2}$$

$$= \frac{X}{2}$$

Now, we need to find $P(1 \leq x \leq 3)$, which translates to

$$P\left(\frac{1}{2} \leq \frac{x}{2} \leq \frac{3}{2}\right)$$

$$P\left(\frac{1}{2} \leq Z \leq \frac{3}{2}\right)$$

The probabilites of Z can be found using the standard normal distribution table or a calculator:

$$P\left(\frac{1}{2} \leq Z \leq \frac{3}{2}\right) = P(Z \leq 1.5) - P(Z \leq 0.5)$$

$$P(Z \leq 1.5) \approx 0.9332$$

$$P(Z \leq 0.5) \approx 0.6915$$

Thus

$$P(1 \leq X \leq 3) = 0.9332 - 0.6915$$

$$= 0.2417$$

b, $P(1 \leq x \leq 3 | x \geq 1)$

This is the conditional probability of x being b/w 1 & 3 given that x is at least 1. By the definition of conditional probability:

$$P(1 \leq x \leq 3 | x \geq 1) = \frac{P(1 \leq x \leq 3 \cap x \geq 1)}{P(x \geq 1)}$$

since $1 \leq x \leq 3$ is a subset of $x \geq 1$, the numerator simply becomes $P(1 \leq x \leq 3)$, which is 0.2417

For $P(x \geq 1)$:

$$P(x \geq 1) = 1 - P(x < 1)$$

$$P(x < 1) = P(z < 0.5)$$

$$\approx 0.6915$$

$$P(x \geq 1) = 1 - 0.6915$$

$$= 0.3085$$

Thus

$$P(1 \leq x \leq 3 | x \geq 1)$$

$$= \frac{0.2417}{0.3085}$$

$$\approx 0.7835$$

Therefore:

- $P(1 \leq x \leq 3) \approx 0.2417$
- $P(1 \leq x \leq 3 | x \geq 1) \approx 0.7835$