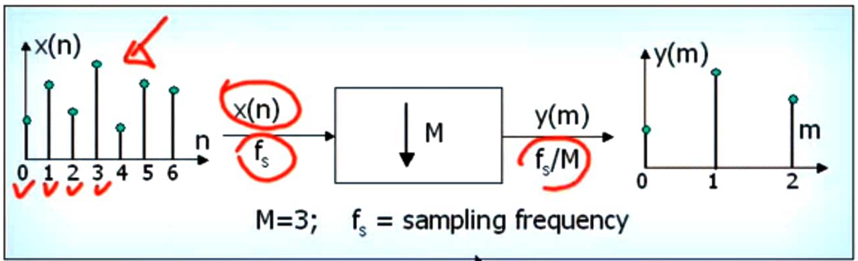
Decimation: A sampling rate decreaser is shown below. We shall confine our attention to a decrease by an integer factor M (e.g. M =3)



The output signal y(m) is obtained by taking every Mth sample of the input signal. If M=3, we should just take every third sample of x(n) to form the desired signal y(m).

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Example: $x[n] = \{1, \cancel{2}, 4, \cancel{3}, 5, \cancel{6}, -8, \cancel{2}, -3, 2\}$

Down sample by 2

$y[m] = \{1,4,5,-8,-3\}$

The output signal $y[n]$ is obtained by taking every M^{th} sample of the input signal. If $M = 3$, we should just take every third sample of $x[n]$ to form the desired signal $y[m]$.

$f_s/3$

Obviously, it only makes sense to reduce the sampling rate if the information constant of the signal we wish to preserve is band limited to $\frac{f_s}{6}$; half the desired sampling rate since the spectral components above this frequency will be aliased into frequencies below $\frac{f_s}{6}$ acco[illegible] sampling rule.

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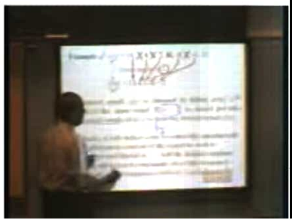
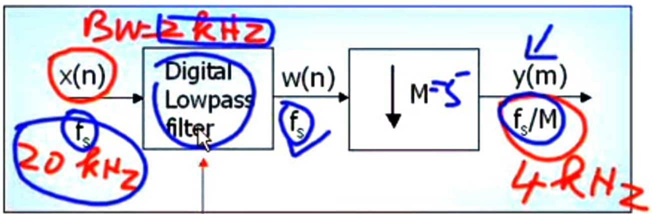
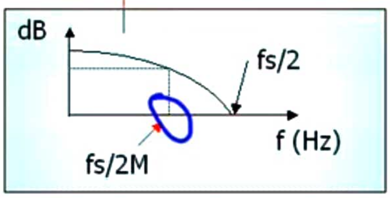


The diagram below shows a block representations of a times M decimator.

The signal x(n) is first passed through a lowpass filter that attenuates the band from {fs/2}/M to fs/2 to prevent aliasing.





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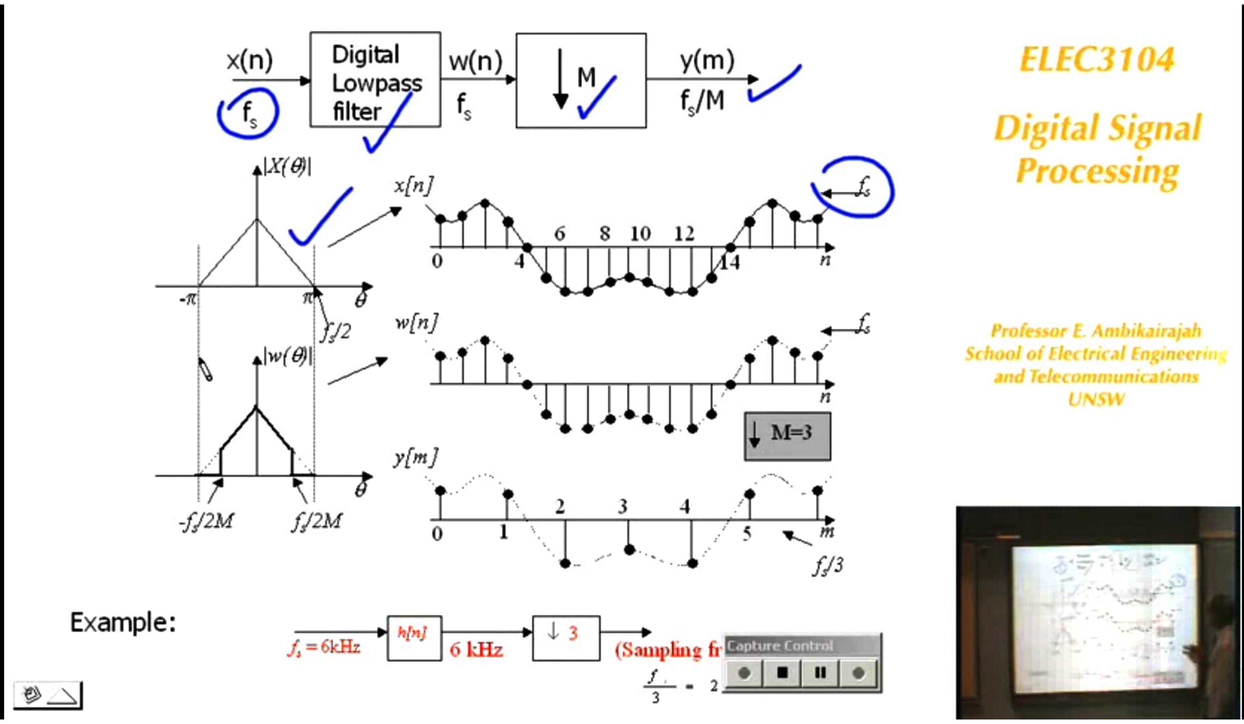

x(n)
f_s
Digital Lowpass filter
w(n)
f_s
M
y(m)
f_s/M
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|X(θ)|
-π
π
θ
$f_s/2$
|w(θ)|
$-f_s/2M$
$f_s/2M$
x[n]
0
4
6
8
10
12
14
n
f_s
w[n]
M=3
y[m]
0
1
2
3
4
5
m
$f_s/3$
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Example:
f_s = 6kHz
h[n]
6 kHz
↓ 3
(Sampling fr
Capture Control

Should we use IIR or FIR for the lowpass filtering required?

Using an IIR filter in this case has an obvious shortcoming. We cannot take advantage of the fact that we only have to compute every Nth output, since previous outputs are required to compute the Mth output. Thus no saving is realised.

$$f_s \rightarrow \boxed{\text{filter}} \xrightarrow{f_s} \boxed{\downarrow} \rightarrow$$

On the other hand, using an FIR filter, in this case implies that we can do our computations at the rate of fs/M. Thus , using an FIR filter in the decimation process will lead to a significantly lower computation rate.

Another advantage of using an FIR filter is the fact that we can easily design linear phase filters and is desirable in many applications.

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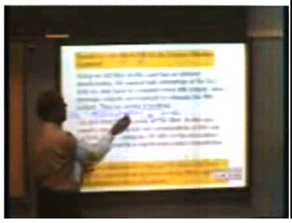
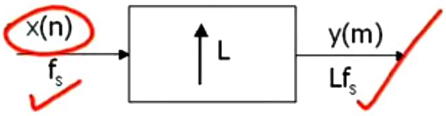


Interpolation

- The process of interpolation involves a sampling rate increase



- The sequence x(n) was derived by sampling x(t) at a sampling rate f_s and we want to obtain a sequence y(n) that approximates as closely as possible the sequence that would have been obtained had we sampled x(t) at the rate Lf_s.
- Interpolation involves inserting between any samples x(n) and x(n-1) and additional L-1 samples.

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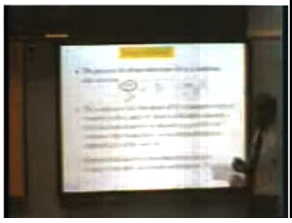

Interpolation Examples

$x[n]=\{1, 2, 4, 3, -5, 6, -7, 2, 4, 3\}$

$\uparrow 2$

$y[m]=\{1,0,2,0,4,0,3,0,-5,0,6,0,-7,0,2,0,4,0,3,0\}$

$x[n]=\{1, 2, 4, 3, -5, 6, -7, 2, 4, 3\}$

$\uparrow 3$

$y[m]=\{1,0,0,2,0,0,4,0,0,3,0,0,-5,0,0,6,0,0,-7,0,0,2,0,0,4,0,0,3,0,0\}$

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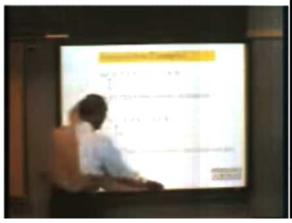
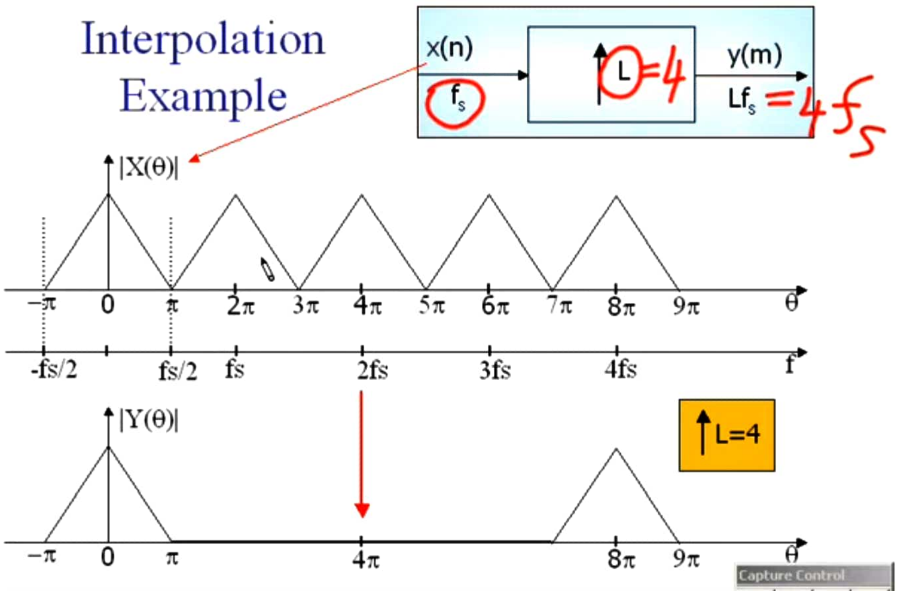


Interpolation Example



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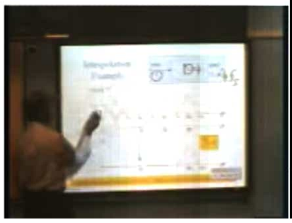
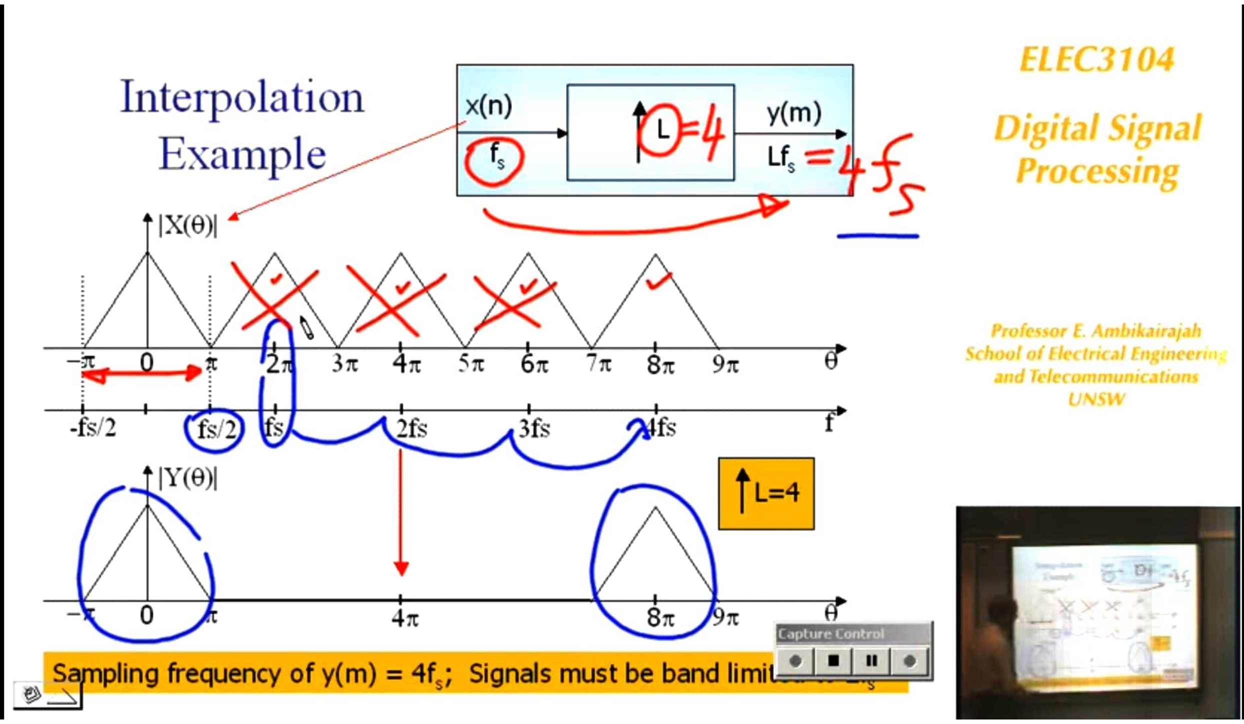


Sampling frequency of y(m) = $4f_s$; Signals must be band limit


Interpolation Example
x(n)
f_s
L
=4
y(m)
Lf_s
=4f_s
|X(θ)|
−π 0 π 2π 3π 4π 5π 6π 7π 8π 9π θ
-fs/2 fs/2 fs 2fs 3fs 4fs f
|Y(θ)|
−π 0 π 4π 8π 9π θ
L=4
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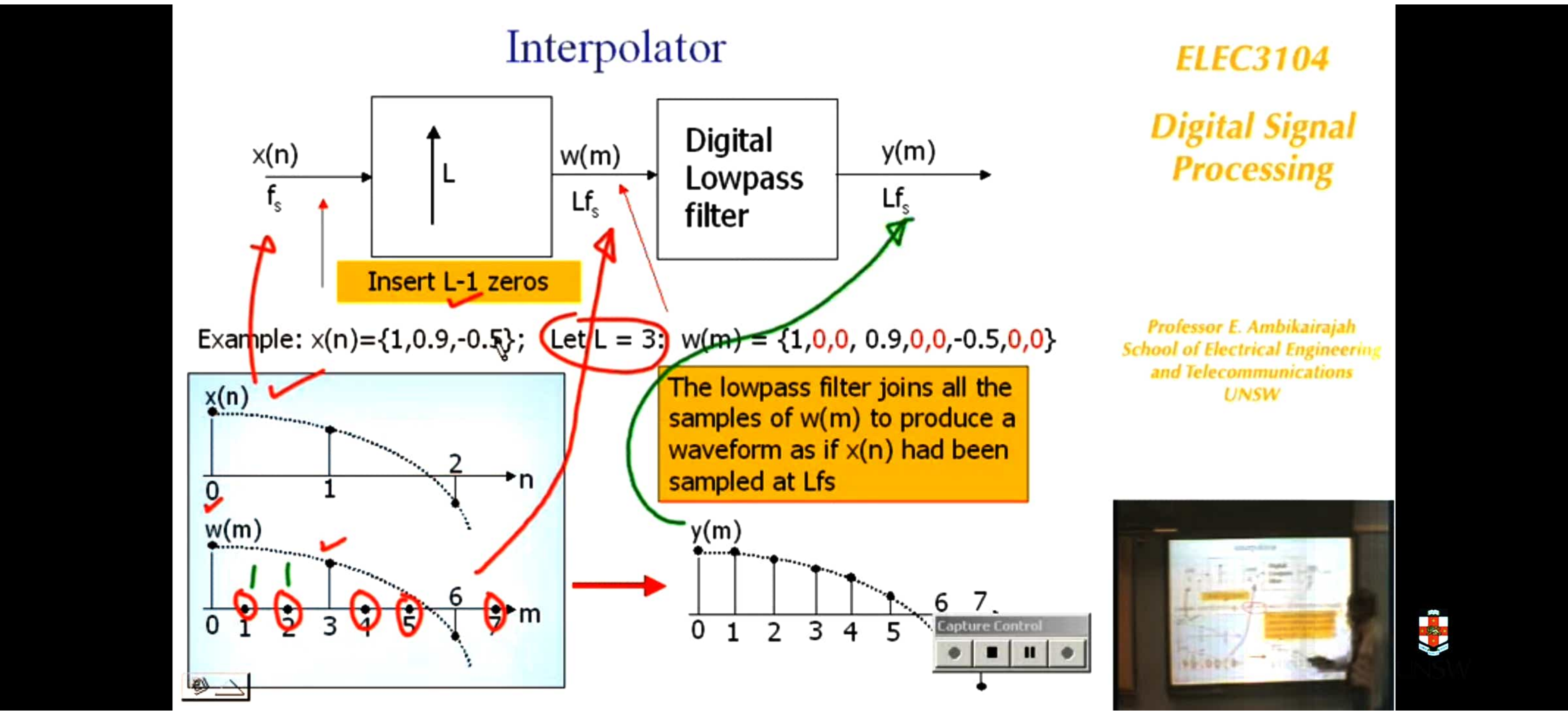
Sampling frequency of y(m) = 4f_s; Signals must be band limit
Capture Control


Interpolator
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x(n)
fs
L
w(m)
Lfs
Digital Lowpass filter
y(m)
Lfs
Insert L-1 zeros
Example: x(n)={1,0.9,-0.5}; Let L = 3: w(m) = {1,0,0, 0.9,0,0,-0.5,0,0}
The lowpass filter joins all the samples of w(m) to produce a waveform as if x(n) had been sampled at Lfs
x(n)
0
1
2
n
w(m)
0 1 2 3 4 5 6 7 m
y(m)
0 1 2 3 4 5 6 7
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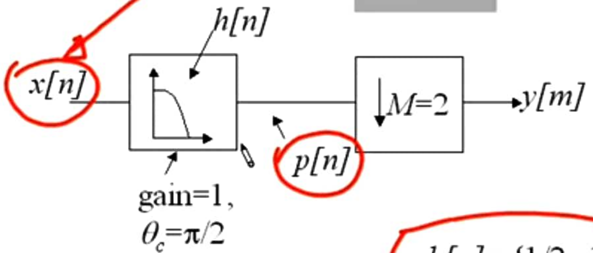
Example: Decimation of $x[n] = \{2, 6, 4, 2, 6, 8, 4, 2, 4, 4\}$

$\downarrow M=2$



$h[n] = \{1/2,\ 1/2\}$

$h[0]=1/2,\ h[1]=1/2$

$$p[n] = x[n] * h[n] = \{4,5,3,4,7,6,3,3,4,2\}$$

$$\downarrow 2$$

$$y[n] = \{4,3,7,3,4\}$$

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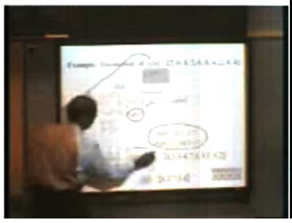
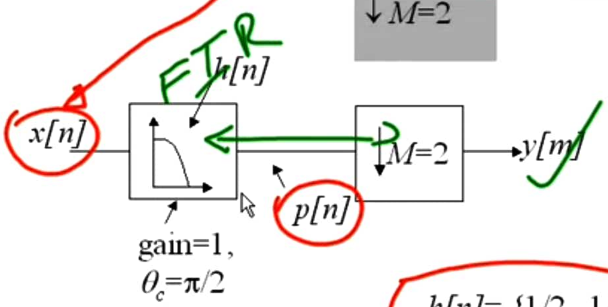


Example: Decimation of $x[n]$ ={2, 6, 4, 2, 6, 8, 4, 2, 4, 4}

$\downarrow M$=2



$h[n]$= {1/2, 1/2}

$h[0]$=1/2, $h[1]$=1/2

$$p[n] = x[n] * h[n] = \{4,\cancel{5},3,\cancel{4},7,6,3,3,4,2\}$$

$$\downarrow 2$$

$$y[n] = \{4,3,7,3,4\}$$

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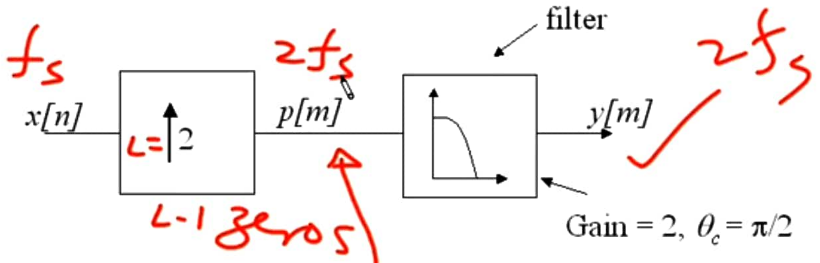
Example: Linear interpolation of $x[n] = \{1,3,5,3,7\}$



$x[n] = \{1,3,5,3,7\}$

$p[n] = \{1,0,3,0,5,0,3,0,7,0\}$ (insert zeros)

$h[n] = \left\{\frac{1}{2}, 1, \frac{1}{2}\right\}$

$y[n] = p[n] * h[n] = \{1,2,3,4,5,4,3,5,7,3.5\}$

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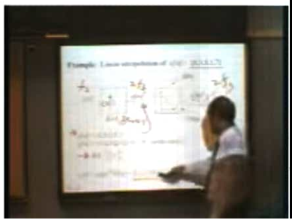
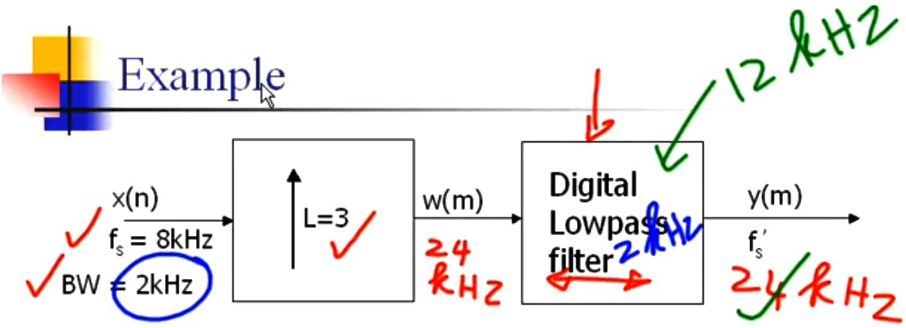


Example



- What should be the cut-off frequency of the digital lowpass filter?

- What should be the value of f_s'?

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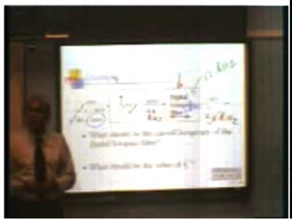
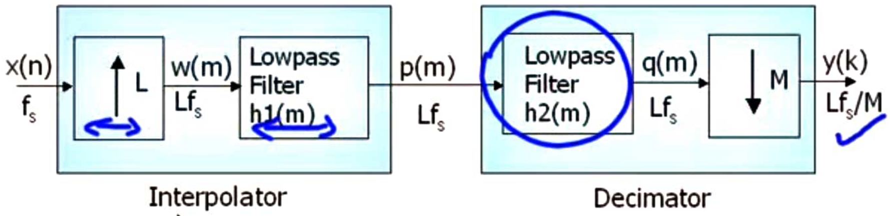


Figure below shows that the sampling frequency change is achieved by first interpolating the data by L and then decimating by M.



The two Digital Lowpas Filters, h1(m) and h2(m) can be combined into a single filter since they are in cascade and have a common sampling frequency

If $M > L$ the resulting operation is a decimation process by a

$M < L$ the resulting operation is an interpolation

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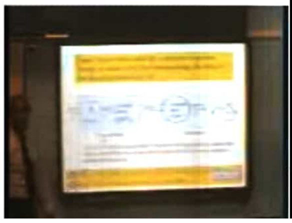
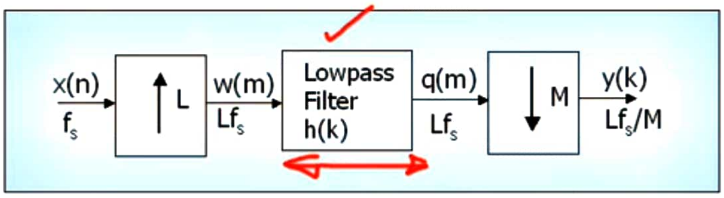


Summary: Sampling Rate Conversion by Non-Integer Factors



The lowpass filter that we require is the one that has a cut-off frequency:

$$\boxed{\theta_c = \min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)}$$

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Decimation by 2

x(n) → [↓2] → y(n)=x(2n)

Case 1

Aliasing term

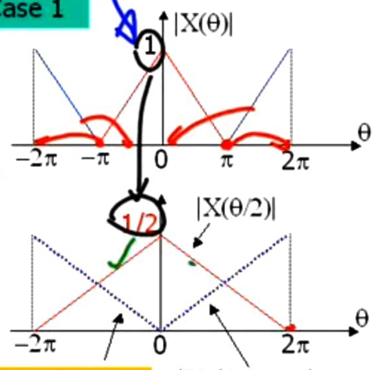
$$Y(z)=\frac{1}{2}[X(z^{\frac{1}{2}})+X(-z^{\frac{1}{2}})]$$

$$Y(\theta)=\frac{1}{2}[X(e^{\frac{j\theta}{2}})+X(-e^{\frac{j\theta}{2}})]$$

$$=\frac{1}{2}[X(e^{\frac{j\theta}{2}})+X(e^{j(\frac{\theta}{2}-\pi)})]$$

$$=\frac{1}{2}[X(\frac{\theta}{2})+X(\frac{\theta}{2}-\pi)]$$

Aliasing term



Extreme Aliasing

Stretch X(θ) by a factor 2 to obtain X(θ/2)

The spectrum is s[...] down sampling

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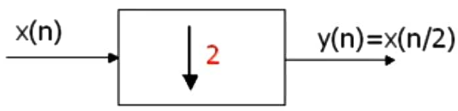
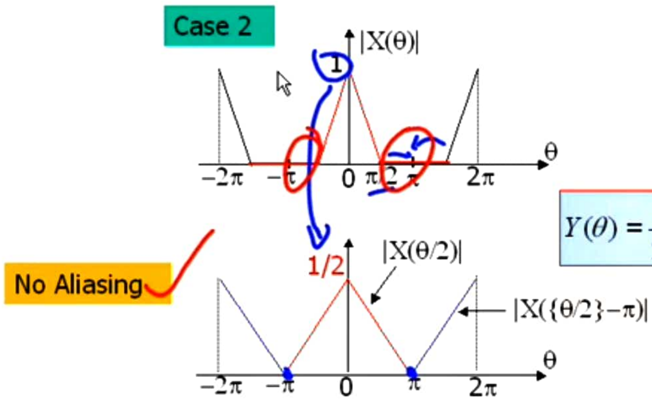
Decimation by 2





$$Y(\theta) = \frac{1}{2}[X(\frac{\theta}{2}) + X(\frac{\theta}{2} - \pi)]$$

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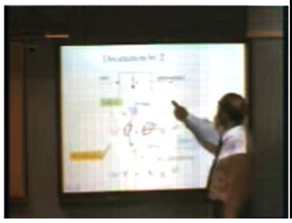

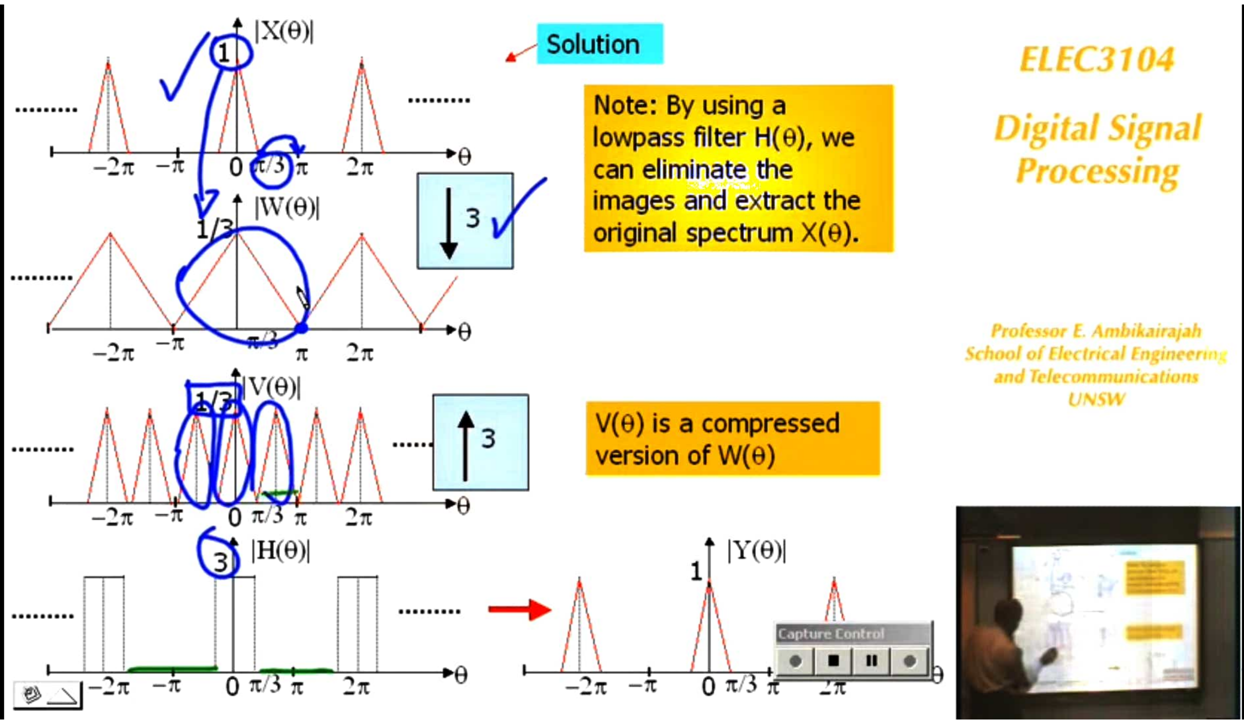

Solution
Note: By using a lowpass filter H(θ), we can eliminate the images and extract the original spectrum X(θ).
V(θ) is a compressed version of W(θ)
|X(θ)|
|W(θ)|
|V(θ)|
|H(θ)|
|Y(θ)|
1/3
3
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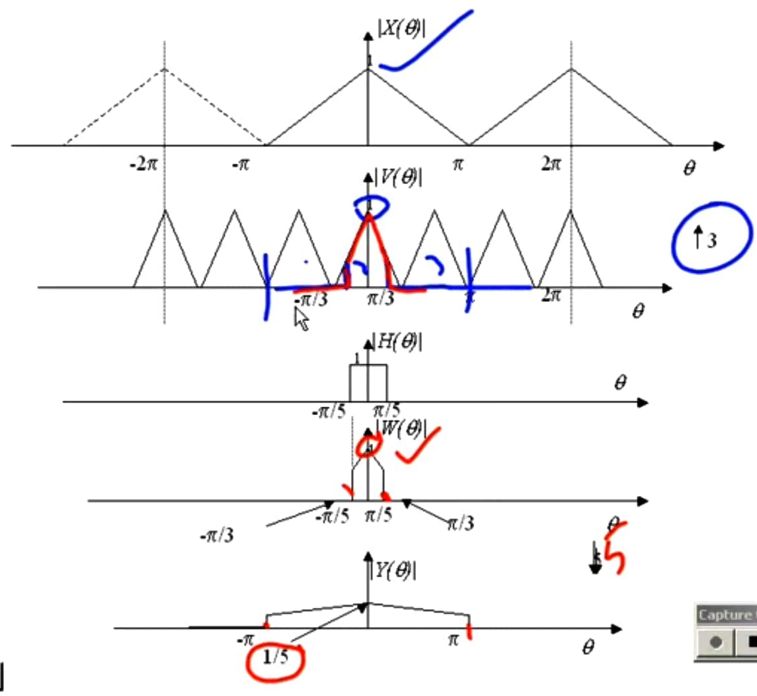

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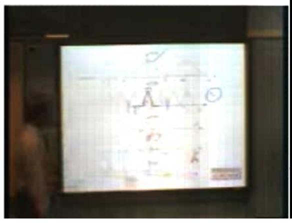
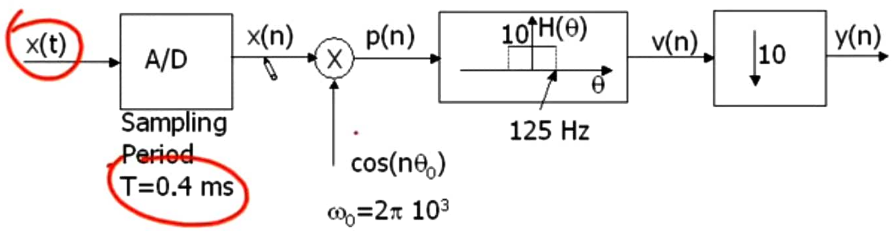


Example: x(t) is the input signal for the system shown below. The analogue signal x(t) has the spectrum X(f) given by:

$$X(f) = \begin{cases} 1 & 0.9kHz \le f \le 1.1kHz \\ 1 & -1.1kHz \le f \le -0.9kHz \\ 0 & elsewhere \end{cases}$$



H(θ) is an ideal lowpass filter (gain=10) with cut-off frequency f_c = 125 Hz. Sketch, one above another, $|X(\theta)|$, $|P(\theta)|$, $|V(\theta)|$, $|Y(\theta)|$ against θ.

$$\text{Note: } P(\theta) = \frac{1}{2}[X(\theta + \theta_0) + X(\theta - \theta_0)]$$

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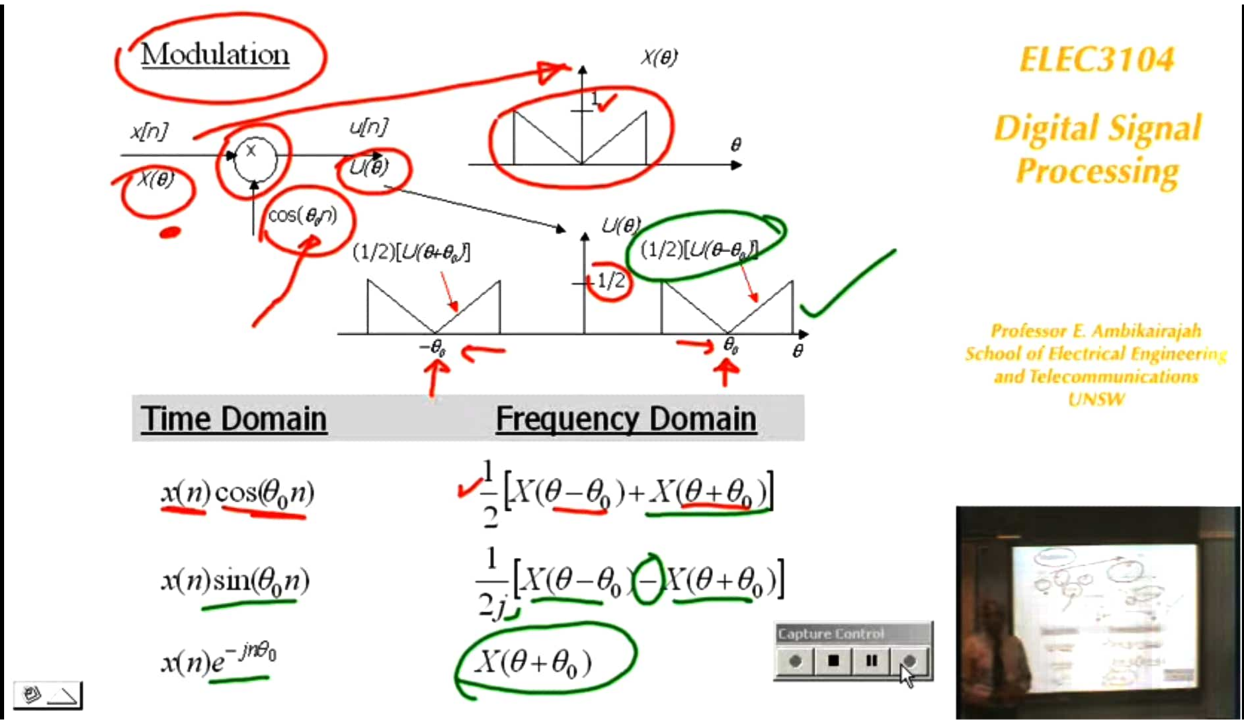
Modulation
x[n]
u[n]
X(θ)
U(θ)
cos(θ0n)
X(θ)
1
θ
U(θ)
(1/2)[U(θ+θ0)]
(1/2)[U(θ-θ0)]
1/2
-θ0
θ0
θ
Time Domain
Frequency Domain
$x(n)\cos(\theta_0 n)$
$\frac{1}{2}[X(\theta-\theta_0)+X(\theta+\theta_0)]$
$x(n)\sin(\theta_0 n)$
$\frac{1}{2j}[X(\theta-\theta_0)-X(\theta+\theta_0)]$
$x(n)e^{-jn\theta_0}$
$X(\theta+\theta_0)$
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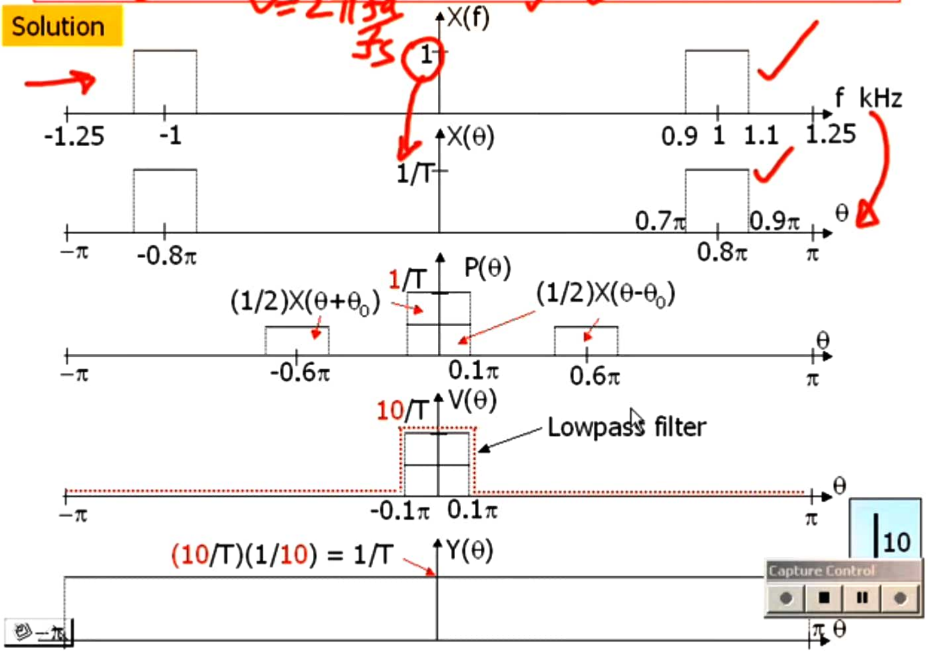
$$f_S = \frac{1}{T} = 2.5\,kHz;\quad \theta_0 = 2\pi\,10^3\,(0.4\,10^{-3}) = 0.8\pi;\quad \theta_C = 2\pi\,(125\,/(2.5\,10^3)) = 0.1\pi$$

$\theta = 2\pi \frac{f_a}{f_s}$

Solution



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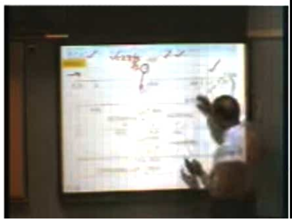
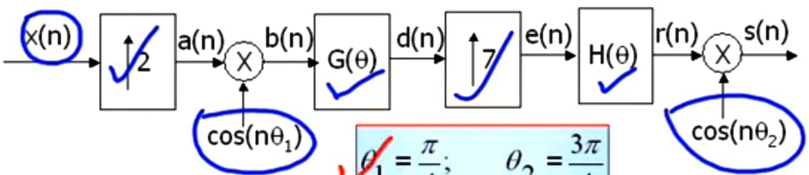


Example: x(n) is the input signal for the system shown below.

If $x(n) = 0.5\,\delta(n+1)+\delta(n)+0.5\delta(n-1)$, Show that

$$S(\theta) = \begin{cases} 1/2 & 10\pi/4 \le \theta \le 11\pi/4 \\ 1/2 & -11\pi/4 \le \theta \le -10\pi/4 \\ 0 & \textit{elsewhere} \end{cases}$$



$$\theta_1 = \frac{\pi}{4}; \qquad \theta_2 = \frac{3\pi}{4}$$

$$G(\theta) = \begin{cases} 1 & 0 \le |\theta| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\theta| \le \pi \end{cases}$$

$$H(\theta) = \begin{cases} 1 & 0 \le |\theta| \le \frac{\pi}{7} \\ 0 & \frac{\pi}{7} < |\theta| \le \end{cases}$$

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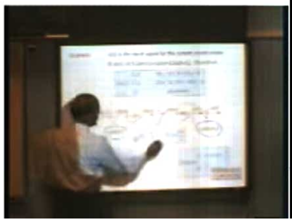
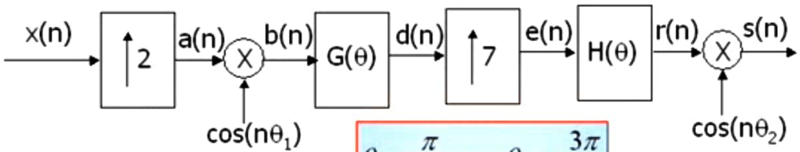


Solution



$$\theta_1 = \frac{\pi}{4}; \qquad \theta_2 = \frac{3\pi}{4}$$

$$x(n) = \frac{1}{2}\delta(n+1) + \delta(n) + \frac{1}{2}\delta(n-1) \quad \Rightarrow X(z) = \frac{1}{2}z^{+1} + 1 + \frac{1}{2}z^{-1}$$

$$X(\theta) = \frac{1}{2}e^{j\theta} + 1 + \frac{1}{2}e^{-j\theta} = 1 + \cos(\theta) \qquad -\pi \le \theta \le \pi$$

$$A(\theta) = 1 + \cos(2\theta)$$

$$B(\theta) = \frac{1}{2}\left\{A(\theta - \theta_1) + A(\theta + \theta_1)\right\} = \frac{1}{2}\left\{1 + A\cos 2(\theta - \frac{\pi}{4}) + 1 + A\cos 2(\theta + \frac{\pi}{4})\right\}$$

$$= 1$$

$$D(\theta) = B(\theta).G(\theta) = \begin{matrix} 1 & -\pi/4 \le \theta \le \pi/4 \\ 0 & elsewhere \end{matrix}$$

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Digital Signal Processing

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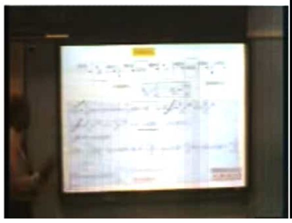
UNSW



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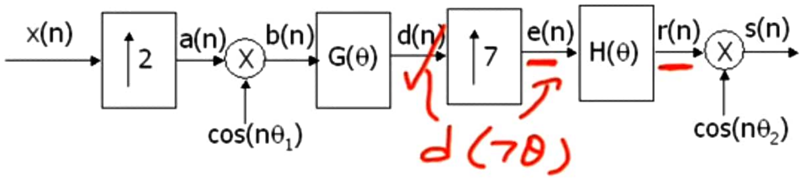
Solution……



$$R(\theta) = E(\theta).H(\theta) = D(7\theta).H(\theta) \quad \{H(\theta)=1\}$$

$$= \begin{cases} D(7\theta) & -\pi/7 \le \theta \le \pi/7 \\ 0 & elsewhere \end{cases}$$

$$R(\theta) = \begin{cases} \begin{cases} 1 & -\pi/4 \le \theta \le \pi/4 \\ 0 & \pi/4 \le |\theta| \le \pi \end{cases} \\ \\ 0 & \pi/7 \le |\theta| \le \pi \end{cases} = \begin{cases} 1 & -\pi/28 \le \theta \le \pi/28 \\ \\ 0 & \pi/28 \le |\theta| \le \pi \end{cases}$$

$$H(\theta) = \begin{cases} 1 & 0 \le |\theta| \le \frac{\pi}{7} \\ 0 & \frac{\pi}{7} < |\theta| \le \pi \end{cases}$$

$$G(\theta) = \begin{cases} 1 & 0 \le |\theta| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\theta| \le \pi \end{cases}$$

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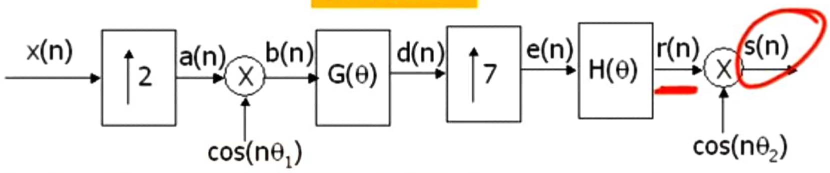
Solution….



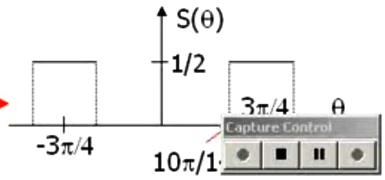
$$\theta_2 = \frac{3\pi}{4}$$

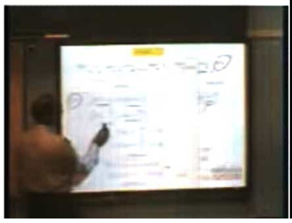
$$S(\theta) = \frac{1}{2}\left\{R(\theta-\theta_2) + R(\theta+\theta_2)\right\}$$

$$= \frac{1}{2}\left\{R\left(\theta-\frac{3\pi}{4}\right) + R\left(\theta+\frac{3\pi}{4}\right)\right\}$$

$$S(\theta) = \begin{cases} \tfrac{1}{2} & -\pi/28 \le \theta - \frac{3\pi}{4} \le \pi/28 \\ \tfrac{1}{2} & -\pi/28 \le \theta + \frac{3\pi}{4} \le \pi/28 \\ 0 & \textit{elsewhere} \end{cases}$$

$$S(\theta) = \begin{cases} \tfrac{1}{2} & 10\pi/14 \le \theta \le 11\pi/14 \\ \tfrac{1}{2} & -11\pi/14 \le \theta \le -10\pi/14 \\ 0 & \textit{elsewhere} \end{cases}$$





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