Equations for SPARTAN Gyroids and Diamonds

This document gives the formal equations to generate each of the diamond and gyroid TMPS structures from seven variables. These seven variables are:

- $\lambda_x, \lambda_y, \lambda_z$: The TPMS "wavelength" in the x, y, and z dimensions (mm).
- $\theta_x, \theta_y, \theta_z$: Rotation of the TPMS lattice about the x, y, and z axes (degrees).
- *p*: The porosity of the structure (float, 0-1). This is the fraction of free volume in the structure.

We generate these structures using Signed Distance Functions (SDF) in three steps. A SDF defines structures as distance fields, where the structure is present wherever the SDF is valued less than or equal to zero; the structure is hollow wherever the SDF is greater than zero.

First, we convert the x, y, and z coordinates to angular wavenumbers. Next, we rotate these coordinates with rotation matrices. These two steps define the TPMS surface. Finally, we control the TPMS wall thickness / structure porosity by the *t* value. We compute the appropriate *t* value for a desired porosity *p* by an empirically determined polynomial (which is unique per TPMS structure).

Math to define structures

1. Rotate the xyz coordinates using rotation matrices.

2. Find our *t* value to offset the signed distance function such that we have a structure with the desired porosity. These polynomial constants were determined empirically by fitting an eight-order polynomial to computed structure porosities as a function of *t* value. Note: *i* starts from zero.

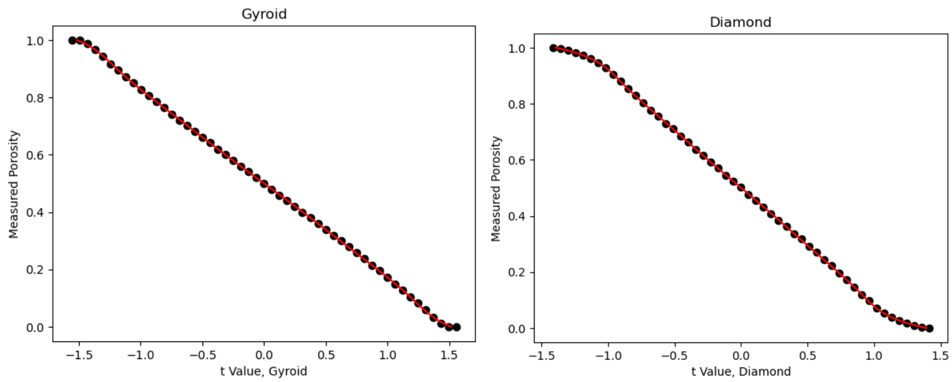

Figure 1: Empirical fits (red lines) to determine the t-value for a desired porosity for gyroid (left) and diamond (right) lattices.

$$p \in [0,1] t_{structure}(p) = \sum_{i=0}^{8} a_i p^{8-i}$$

a_{gyroid} = [-9.86328589e02, 4.43858792e03, -8.42057860e03, 8.75829993e03, -5.43754382e03, 2.05512365e03, -4.60683827e02, 5.58905464e01, -5.79899099e00, 1.51588868e00]

$a_{diamond}$ = [-1.40181203e03, 6.31442849e03, -1.20734024e04,1.27714436e04, -8.16588099e03, 3.23757850e03, -7.84778926e02, 1.09747587e02, -1.01136257e01, 1.39501223e00]

3. Compute a signed distance function (*SDF*) defining the TPMS. The structure is solid where the SDF is less than or equal to zero, and hollow where the SDF is greater than zero.

$$SDF_{gyroid} = cos\, cos\, (x_\lambda{}')\ sin\, sin\, (y_\lambda')\ + cos\, cos\, (y_\lambda{}')\ sin\, sin\, (z_\lambda')\ + cos\, cos\, (z_\lambda')\ sin\, sin\, (x_\lambda{}') - t_{gyroid}(p)$$

$$SDF_{diamond} = sin\, sin\, (x_\lambda{}')\ sin\, sin\, (y_\lambda')\, sin\, sin\, (z_\lambda')\ + sin\, sin\, (x_\lambda{}')\ cos\, cos\, (y_\lambda')\, cos\, cos\, (z_\lambda')\ + cos\, cos\, (x_\lambda{}')\ sin\, sin\, (y_\lambda')\, cos\, cos\, (z_\lambda')\ + cos\, cos\, (x_\lambda{}')\ cos\, cos\, (y_\lambda')\, sin\, sin\, (z_\lambda') - t_{diamond}(p)$$