Equations for SPARTAN Gyroids and Diamonds

This document gives the formal equations to generate each of the diamond and gyroid TMPS structures from seven variables. These seven variables are:

- $\lambda_x, \lambda_y, \lambda_z$: The TPMS "wavelength" in the x, y, and z dimensions (mm).
- $\theta_x, \theta_y, \theta_z$: Rotation of the TPMS lattice about the x, y, and z axes (degrees).
- *p*: The porosity of the structure (float, 0-1). This is the fraction of free volume in the structure.

We generate these structures using Signed Distance Functions (SDF). An SDF defines structures as distance fields, where the structure is present wherever the SDF is valued less than or equal to zero and hollow wherever the SDF is greater than zero.

First, we convert the x, y, and z coordinates to angular wavenumbers. Next, we rotate these coordinates with rotation matrices. These two steps define the TPMS surface. Finally, we control the TPMS wall thickness / structure porosity by the *t* value. We compute the appropriate *t* value for a desired porosity *p* by an empirically determined polynomial (which is unique per TPMS structure).

Equations to define our structures

1. Convert the coordinate system *X* to angular wavenumbers.

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in R^3 \tag{1.1}$$

$$X_\lambda = X \begin{bmatrix} 2\pi/\lambda_x \\ 2\pi/\lambda_y \\ 2\pi/\lambda_z \end{bmatrix}^T \tag{1.2}$$

Rotate X_λ using rotation matrixes for X, Y, and Z rotations.

$$\mathrm{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix} \tag{1.3}$$

$$\mathrm{R}_y = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix} \tag{1.4}$$

$$R_z = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.5}$$

Applying the rotations with (1.6) yields our effective coordinates $x'_\lambda, y'_\lambda, z'_\lambda$ that account for both the lattice periodicity and rotation.

$$X'_\lambda = X_\lambda R_x R_y R_z = \begin{bmatrix} x'_\lambda \\ y'_\lambda \\ z'_\lambda \end{bmatrix} \tag{1.6}$$

2. Let *p* be the desired porosity of the structure, with a value between zero and one.

$$p \in [0,1] \tag{2.1}$$

Porosity is controlled in the SDF using a *t* value. Larger *t* values generate structures with more free space (*ie* higher porosity).

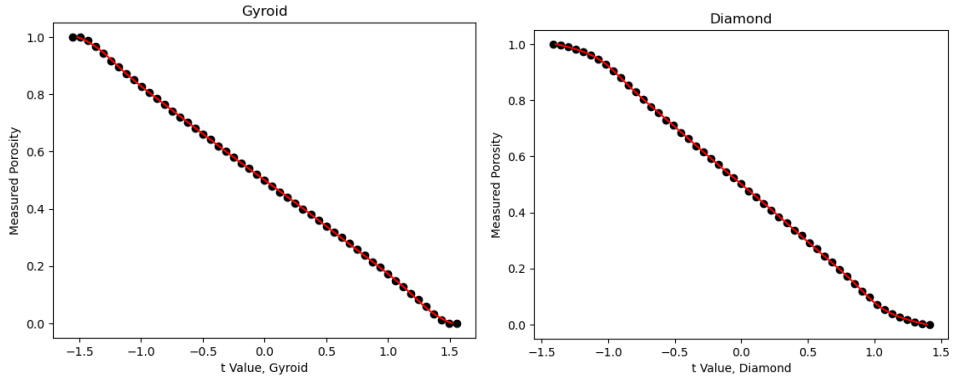

Figure 1: Empirical fits (red lines) to determine the t-value for a desired porosity for gyroid (left) and diamond (right) lattices.

We must find a *t* value to offset the signed distance function such that we have a structure with the desired porosity. The relationship between *t* and porosity varies per TPMS structure (Figure 1). We empirically fit eighth-order polynomials to computed structure porosities as a function of *t* value; these polynomials are used to enable direct control of porosity by *p* instead of *t*.

$$t_{structure}(p) = \sum_{i=0}^{8} a_i p^{8-i} \tag{2.2}$$

a_{gyroid} = [-9.86328589e02, 4.43858792e03, -8.42057860e03, 8.75829993e03, -5.43754382e03,

2.05512365e03, -4.60683827e02, 5.58905464e01, -5.79899099e00, 1.51588868e00]

$a_{diamond}$ = [-1.40181203e03, 6.31442849e03, -1.20734024e04,1.27714436e04, -8.16588099e03, 3.23757850e03, -7.84778926e02, 1.09747587e02, -1.01136257e01, 1.39501223e00]

3. Compute the signed distance function (*SDF*) defining the TPMS. The structure is solid where the SDF is less than or equal to zero, and hollow where the SDF is greater than zero.

$$SDF_{gyroid} = cos(x_\lambda')sin(y_\lambda') + cos(y_\lambda')sin(z_\lambda') + cos(z_\lambda')sin(x_\lambda') - t_{gyroid}(p)$$

$$SDF_{diamond} = sin(x_\lambda')sin(y_\lambda')\sin(z_\lambda') + sin(x_\lambda')cos(y_\lambda')cos(z_\lambda') + cos(x_\lambda')sin(y_\lambda')cos(z_\lambda') + cos(x_\lambda')cos(y_\lambda')sin(z_\lambda') - t_{diamond}(p)$$