1 Case 1: No Water Layer

- Author: **Team G15**
- Attempt: 3

1.1 Analysis

1.1.1 To find

1. Temperature of Roof Surface (T_s)
2. Total heat flux entering the house through the roof, (q_t) when no water layer is present

1.1.2 Nomenclature

- T_s = roof surface temperature (outside)
- T_a = ambient air temperature (outside)
- T_r = room temperature (inside)
- Nu_a = Nusselt number of air
- Ra_a = Rayleigh number of air
- Re_a = Reynolds number of air
- Pr_a = Prandtl number of air
- α_a = thermal diffusivity of air
- k_a = thermal conductivity of air
- h_r = free convection coefficient of room air
- ν_a = dynamic Viscosity of air
- Roof layers:
  - 1: Concrete
  - 2: Brick
  - 3: Lime
- k_i = thermal conductivity of i^{th} roof layer
- L_i = length of i^{th} roof layer
- q_r = radiative heat transfer (per unit area)
- q_c = convective heat transfer (per unit area)
- q_t = net heat transfer into the room (per unit area)
- β = coefficient of thermal expansion
- S = Intensity of Solar Radiation (i.e. solar constant)

1.1.3 Assumptions

- Steady state with room maintained at fixed ambient temperature

1.1.4 Equations

Energy balance,

$$q_t = q_c + q_r$$

Radiation heat transfer,

$$q_r = \tau_s \cdot S - h_r \cdot (T_a - T_s)$$

$$h_r = \epsilon_s \cdot \sigma \cdot \frac{(\overline{T}_s)^4 - (\overline{T}_a - 12)^4}{\overline{T}_a - \overline{T}_s}$$

Convection heat transfer,

$$q_c = h_c \cdot (T_a - T_w)$$

$$h_c = \frac{k_a}{L_s} \cdot Nu_a$$

$$Nu_a = 0.15 \cdot Ra_a^{1/3} + 0.664 \cdot Re_a^{1/2} \cdot Pr_a^{1/3}$$

$$Re_a = \frac{v_a \cdot L_s}{\nu_a}$$

$$Ra_L = \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L_s^3}{\nu_a \cdot \alpha_a}$$

Total heat transfer,

$$q_t = \frac{T_w - T_r}{R_{net}}$$

$$R_{net} = \frac{1}{h_r} + \sum_{i=1}^{3} \frac{L_i}{k_i}$$

1.1.5 Properties

Outside Air

- Mild breeze $v_a = 2.78\ m/s$
- $T_a \in [305, 320]K$
- $T_f = 320K$
- $\beta = \frac{1}{T_f} = 0.0031\ K^{-1}$
- Table A.4, air (T_f):
    - $\nu = 18 \cdot 10^{-6}\ m^2/s$
    - $\alpha = 25 \cdot 10^{-6}\ m^2/s$
    - $Pr = 0.702$
    - $k = 27.7 \cdot 10^{-3}\ W/m \cdot K$
- $S = 1366\ W/m^2$

Roof

- $L_s = 5\ m$ (approx thickness of water layer)
- $\epsilon_s = 0.9$ (concrete surface)
- $\tau_s = 0.9$
- $t = 0.2\ m$ thick with,
  - Cement = 5 cm
  - Brick = 10 cm
  - Lime = 5 cm
- K_i, Conductivity of each layer,
  - Cement = 0.72 $W/m \cdot K$
  - Brick = 0.71 $W/m \cdot K$
  - Lime = 0.73 $W/m \cdot K$

Inside air

- $T_r = 300K$ (Room Temperature)
- $h_r = 8.4\ W/m^2 \cdot K$

1.1.6 Tools used

- **Python**
- **SymPy** for creating symbolic equations and solving them
- **NumPy**
- **Matplotlib** for plotting results

1.2 Solving (Python Code)

1.2.1 Initialize Values

[1]:
```
%matplotlib inline
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

# Initialize matplotlib
plt.rc('text', usetex=True) # Unnecessary
plt.style.use('ggplot')
plt.rcParams['grid.color'] = '#C0C0C0'
```

Outside Air

- Table A.4 used (from reference #2)

[2]:
```
v_a = 2.78 # Velocity (m / s)

# Temperatures
T_f = 320.0 # (K)
beta = 1/T_f # (K)
```

```
T_a = np.array([305.0, 310.0, 315.0, 320.0]) # (K)
T_a_avg = 273 + 37 # (K)

# Universal Constants
sigma = 5.67e-8 # Stefan Boltzmann constant (W / m^2 * K^4)
g = 9.8 # (m / s^2)
S = 1366 # Solar constant

# Table A.6, air @ T = T_f
nu_a = 18e-6 # dynamic visosity (m^2 / s)
alpha_a = 25e-6 # (m^2 / s)
k_a = 27.7e-3 # thermal conductivity (W / m * K)
Pr_a = 0.702
```

Roof Layers

[3]:
```
# Temperatures
T_s = sp.symbols('T_s') # Roof surface temp (K)
T_s_avg = 273.0 + 35.0 # (K)

# Surface
L_s = 5 # Dimensions (m)
tau_s = 0.9 # Roof's solar absorbtivity
epsilon_s = 0.9 # Emissivity of roof surface (concrete)

# Layer 1: Concrete
k_1 = 0.72 # (W / m * K)
L_1 = 0.05 # (m)

# Layer 2: Brick
k_2 = 0.71 # (W / m * K)
L_2 = 0.10 # (m)

# Layer 3: Lime
k_3 = 0.73 # (W / m * K)
L_3 = 0.05 # (m)
```

Inside Air

[4]:
```
h_r = 8.4 # (W / m^2 * K)
T_r = 300 # (K)
```

1.2.2 Equations

Radiation Heat

[5]:
```
h_r = epsilon_s * sigma * (T_s_avg**4 - (T_a_avg - 12)**4)/(T_a_avg - T_s_avg) #␣
 ↪(W / m^2 * K)
q_r = tau_s * S - h_r * (T_a - T_s) # (W / m^2)
```

```python
# Example at T_a = 310K and T_s = 314K
q_r_test = q_r[1].replace(T_s, 314)
print('Approximate value of q_r = %.2f W/m^2' % (q_r_test))
```

```
Approximate value of q_r = 1343.00 W/m^2
```

Convection Heat

- From below analysis, we can neglect free convection in comparison to forced convection

Free Convection

[6]:
```python
Ra_a = (g * beta * (T_s - T_a) * L_s**3) / (nu_a * alpha_a)
Nu_a_fr = 0.15 * Ra_a**(1/3)
h_c_fr = k_a / L_s * Nu_a_fr

# Example at T_a = 310K and T_s = 314K
h_c_fr_test = h_c_fr[1].replace(T_s, 314)
print('Approximate value of free convection coefficient = %.2f W/K*m^2' %
 ↪(h_c_fr_test))
```

```
Approximate value of free convection coefficient = 2.69 W/K*m^2
```

Forced Convection

[7]:
```python
Re_a = v_a * L_s / nu_a
Nu_a_fo = 0.664 * Re_a**1/2 * Pr_a**1/3
h_c_fo = k_a / L_s * Nu_a_fo

# Example at T_a = 310K and T_s = 314K
print('Approximate value of forced convection coefficient = %.2f W/K*m^2' %
 ↪(h_c_fo))
```

```
Approximate value of forced convection coefficient = 332.36 W/K*m^2
```

Total Convection

[8]:
```python
h_c = h_c_fo # Neglicting free convection
q_c = h_c * (T_a - T_s) # (W / m^2)

# Example at T_a = 310K and T_s = 314K
q_c_test = q_c[1].replace(T_s, 314)
print('Approximate value of q_c = %.2f W/m^2' % (q_c_test))
```

```
Approximate value of q_c = -1329.43 W/m^2
```

Total Heat:

[9]:
```python
R = 1/h_r + L_1/k_1 + L_2/k_2 + L_3/k_3 # (m^2 * K / W)

q_t = (T_s - T_r) / R # (W / m^2)

# Example at T_a = 310K and T_s = 314K
```

```
q_t_test = q_t.replace(T_s, 314)
print('Approximate value of q_t = %.2f W/m^2' % (q_t_test))
```

```
Approximate value of q_t = 44.59 W/m^2
```

1.2.3 Solving

$$q_c + q_r = q_t$$
$$\therefore\ q_c + q_r - q_t = 0$$

Calculate T_s

```
[10]: eq = q_c + q_r - q_t

n = len(eq)
T_s_calc = np.empty(n, dtype=object)

for i in range(n):
    T_s_calc[i] = round(sp.solve(eq[i], T_s)[0], 2)

for i in range(n):
    print('T_s = %.1f K for T_a = %.1f K' % (T_s_calc[i], T_a[i]))
```

```
T_s = 309.0 K for T_a = 305.0 K
T_s = 313.9 K for T_a = 310.0 K
T_s = 318.9 K for T_a = 315.0 K
T_s = 323.8 K for T_a = 320.0 K
```

Calculate q_t

```
[11]: q_t_calc_1 = np.empty(n, dtype=object)

for i in range(n):
    q_t_calc_1[i] = q_t.replace(T_s, T_s_calc[i])

for i in range(n):
    print('Heat entering = %.1f W/m^2 for T_a = %.1f K' % (q_t_calc_1[i],␣
 ↪T_a[i]))
```

```
Heat entering = 28.5 W/m^2 for T_a = 305.0 K
Heat entering = 44.3 W/m^2 for T_a = 310.0 K
Heat entering = 60.0 W/m^2 for T_a = 315.0 K
Heat entering = 75.8 W/m^2 for T_a = 320.0 K
```

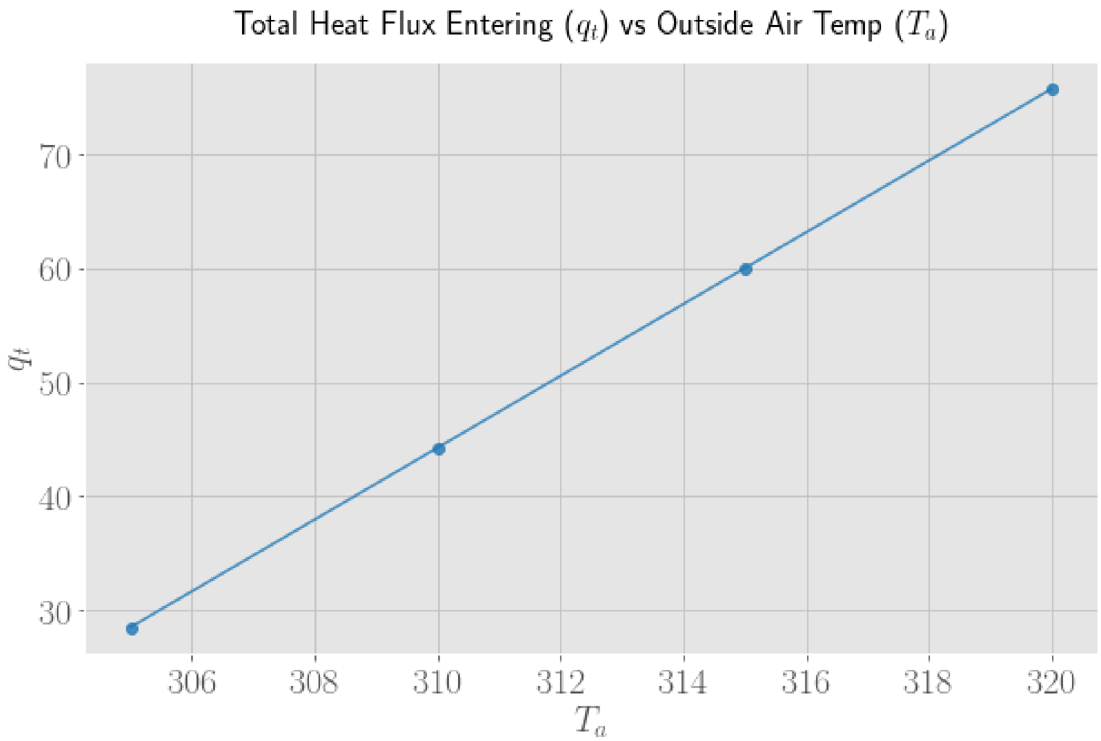
1.2.4 Plot

- Total Heat Flux Entering (q_t) vs Outside Air Temp (T_a)

```
[12]: def make_plot(x, y, xlabel, ylabel, title):
    plt.plot(x, y, color='#1F77B4cc', marker='o')
```

```
    plt.xticks(fontsize=20)
    plt.yticks(fontsize=20)
    plt.xlabel(xlabel, fontsize=20)
    plt.ylabel(ylabel, fontsize=20)
    plt.title(title, fontsize=18, pad=15)
```

[13]:
```
fig = plt.figure(figsize=(10, 6))
make_plot(x=T_a, y=q_t_calc_1, xlabel='$T_a$', ylabel='$q_t$',
          title='Total Heat Flux Entering ($q_t$) vs Outside Air Temp ($T_a$)')
```



2 Case 2: Water Layer

- Author: **Team G15**
- Attempt: 3

2.1 Analysis

2.1.1 To find

1. Temperature of Water Surface (T_w)
2. Total heat flux entering the house through the roof, (q_t) when a water layer is present

2.1.2 Nomenclature

- S = Intensity of Solar Radiation (i.e. solar constant)
- v_w = water velocity
- v_a = wind velocity
- ϵ_w = emissivity of water surface
- σ = Stefan-Boltzmann constant $(5.67 * 10^{-8}\ W/m^2K^4)$
- T_r = room temperature (inside)
- T_w = water surface temperature (outside)
- T_a = ambient air temperature (outside)
- $\overline{T}_w$ = average water surface temperature (outside)
- $\overline{T}_a$ = average air temperature (outside)
- τ_w = fraction of solar radiation absorbed by water
- k_w = thermal conductivity of water
- L_w = length of water layer
- h_w = convection coefficient of water layer
- h_r = radiative heat transfer coefficient
- h_c = convective heat transfer coefficient
- h_e = evaporative heat transfer coefficient

2.1.3 Assumptions

1. Steady state with room maintained at fixed ambient temperature
2. Water is still ($v_w = 0$) but gentle breeze is present ($v_a = 10\ km/h$)
3. Dry Surroundings

2.1.4 Equations

Energy balance,

$$q_t = q_c + q_r - q_e$$

Radiation heat transfer,

$$q_r = \tau_w \cdot S - h_r \cdot (T_a - T_w)$$

$$h_r = \epsilon_w \cdot \sigma \cdot \frac{(\overline{T}_w)^4 - (\overline{T}_a - 12)^4}{\overline{T}_a - \overline{T}_w}$$

Convection heat transfer,

$$q_c = h_c \cdot (T_a - T_w)$$

$$h_c = 5.678 \cdot (1 + 0.85 \cdot (v_a - v_w))$$

Evaporative heat transfer,

$$q_e = 0.013 \cdot h_c \cdot (p(\overline{T}_w) - \gamma \cdot p(\overline{T}_a))$$

$$p(T) = R_1 \cdot T + R_2$$

Total heat transfer,

$$q_t = \frac{T_w - T_r}{R_{net}}$$

$$R_{net} = \frac{1}{h_r} + \sum_{i=1}^{3} \frac{L_i}{k_i} + \frac{1}{h_w}$$

$$h_w = \frac{k_w}{L_w} \cdot (0.14 \cdot (Gr \cdot Pr)^{1/3} + 0.644 \cdot (Pr \cdot Re)^{1/3})$$

$$Gr = \frac{g \cdot \beta \cdot (T_w - T_a) \cdot (L_w)^3}{\nu^2}$$

2.1.5 Properties

Outside Air

- Mild breeze $v_a = 2.78\ m/s$
- $T_a \in [305, 320]K$
- $T_f = 320K$
- $\beta = \frac{1}{T_f} = 0.0031\ K^{-1}$
- Table A.4, air (T_f):
  - $\nu = 18 \cdot 10^{-6}\ m^2/s$
  - $\alpha = 25 \cdot 10^{-6}\ m^2/s$
  - $Pr = 0.702$
  - $k = 27.7 \cdot 10^{-3}\ W/m \cdot K$
- $S = 1366\ W/m^2$
- $R_1 = 325\ Pa/{}^\circ C$ and $R_2 = -5155\ Pa$ (*from reference* **#1**)
- $\gamma = 0.27$ (approx average over a day)

Water layer

- $L_w = 0.1\ m$ (approx thickness of water layer)
- Table A.6, water (T_w):
  - $\nu = 18 \cdot 10^{-6}\ m^2/s$
- Still water $v_w = 0$
- $\epsilon_w = 0.95$
- $\tau_w = 0.6$

Roof

- $t = 0.2\ m$ thick with,
  - Cement = 5 cm
  - Brick = 10 cm
  - Lime = 5 cm
- K_i, Conductivity of each layer,
  - Cement = 0.72 $W/m \cdot K$

- Brick = 0.71 $W/m \cdot K$
- Lime = 0.73 $W/m \cdot K$

Inside air

- $T_r = 300K$ (Room Temperature)
- $h_r = 8.4\ W/m^2 \cdot K$

2.1.6 Tools used

- **Python**
- **SymPy** for creating symbolic equations and solving them
- **NumPy**
- **Matplotlib** for plotting results

2.2 Solving (Python Code)

2.2.1 Initialize Values

Outside Air

- Saturation pressure of water p = R_1*T + R_2

[14]:
```
v_a = 2.78 # Velocity (m / s)

# Temperatures
T_f = 320 # (K)
beta = 1/T_f # (K)
T_a = np.array([305.0, 310.0, 315.0, 320.0]) # (K)
T_a_avg = 273 + 37 # (K)

# Constants
sigma = 5.67e-8 # Stefan Boltzmann constant (W / m^2 * K^4)
g = 9.8 # (m / s^2)
R_1 = 325 # (N / m^2 °C)
R_2 = -5155 # (N / m^2)
gamma = 0.27
S = 1366 # Solar constant

def p(T): # Saturation pressure of water as a function of temperature (N / m^2)
    return R_1 * (T-273) + R_2
```

Water Layer

[15]:
```
v_w = 0 # Velocity (m / s)
L_w = 5 # Dimensions (m)

# Temperatures
T_w = sp.symbols('T_w') # (K)
T_w_avg = 273 + 32 # (K)
```

```
# Constants
epsilon_w = 0.95 # Emissivity of water surface
tau_w = 0.6 # Water's solar absorbtivity
```

- Table A.6 used (*from reference* **#2**)
- Upon analysing the below data, we can approximate h_w to 950 W/m^2

[16]:
```
rho_w = 990 # density (kg / m^3)
k_w = 0.63 # thermal conductivity (W / m * K)
mu_w = 1e-6 * np.array([769, 695, 631, 577]) # viscosity (N * s / m^2)
nu_w = mu_w / rho_w # dynamic visosity (m^2 / s)

Pr_w = np.array([5.20, 4.62, 4.16, 3.77]) # Prandtl number
Re_w = 0 # Reynolds number, still water
Gr_w = g * beta * (T_a - T_w) * L_w**3 / nu_w**2 # Grashof number

# Water free convection coeffecient
h_w = (k_w/L_w) * (0.14 * (Gr_w*Pr_w)**(1/3) + 0.644 * (Pr_w*Re_w)**(1/3))

# Example at T_a = 310K and T_w = 306K
h_w_test = h_w[1].replace(T_w, 306)
print('Approximate min value of h_w = %.2f W/K*m^2' % (h_w_test))
```

```
Approximate min value of h_w = 923.62 W/K*m^2
```

Roof Layers

[17]:
```
# Layer 1: Concrete
k_1 = 0.72 # (W / m * K)
L_1 = 0.05 # (m)

# Layer 2: Brick
k_2 = 0.71 # (W / m * K)
L_2 = 0.10 # (m)

# Layer 3: Lime
k_3 = 0.73 # (W / m * K)
L_3 = 0.05 # (m)
```

Inside Air

[18]:
```
h_r = 8.4 # (W / m^2 * K)
T_r = 300 # (K)
```

2.2.2 Equations

Radiation Heat

[19]:
```
h_r = epsilon_w * sigma * (T_w_avg**4 - (T_a_avg - 12)**4)/(T_a_avg - T_w_avg) #␣
↪(W / m^2 * K)
q_r = tau_w * S - h_r * (T_a - T_w) # (W / m^2)

# Example at T_a = 310K and T_w = 306K
q_r_test = q_r[1].replace(T_w, 306)
print('Approximate value of q_r = %.2f W/m^2' % (q_r_test))
```

```
Approximate value of q_r = 786.53 W/m^2
```

Convection Heat

- Forced convection and free convection both have been used

[20]:
```
h_c = 5.678 * (1 + 0.85 * (v_a - v_w))
print('h_c = %.2f W/K*m^2' % (h_c))

q_c = h_c * (T_a - T_w) # (W / m^2)

# Example at T_a = 310K and T_w = 306K
q_c_test = q_c[1].replace(T_w, 306)
print('Approximate value of q_c = %.2f W/m^2' % (q_c_test))
```

```
h_c = 19.10 W/K*m^2
Approximate value of q_c = 76.38 W/m^2
```

Evaporation Heat:

[21]:
```
q_e = 0.013 * h_c * (p(T_w_avg) - gamma * p(T_a_avg)) # function p defined␣
↪above, (W / m^2)

# Example at T_a = 310K and T_w = 306K
print('Approximate value of q_e = %.2f' % (q_e))
```

```
Approximate value of q_e = 841.55
```

Total Heat:

[22]:
```
h_w = 1200 # from above approximation (W / m^2 * K)
R = 1/h_r + L_1/k_1 + L_2/k_2 + L_3/k_3 + 1/h_w # (m^2 * K / W)

q_t = (T_w - T_r) / R # (W / m^2)

# Example at T_a = 310K and T_w = 306K
q_t_test = q_t.replace(T_w, 306)
print('Approximate value of q_t = %.2f W/m^2' % (q_t_test))
```

```
Approximate value of q_t = 14.98 W/m^2
```

2.2.3 Solving

$$q_c + q_r - q_e = q_t$$
$$\therefore\ q_c + q_r - q_e - q_t = 0$$

Calculate T_w

[23]:
```
eq = q_c + q_r - q_e - q_t

n = len(eq)
T_w_calc = np.empty(n, dtype=object)

for i in range(n):
    T_w_calc[i] = round(sp.solve(eq[i], T_w)[0], 2)

for i in range(n):
    print('T_w = %.1f K for T_a = %.1f K' % (T_w_calc[i], T_a[i]))
```

```
T_w = 302.4 K for T_a = 305.0 K
T_w = 306.5 K for T_a = 310.0 K
T_w = 310.5 K for T_a = 315.0 K
T_w = 314.6 K for T_a = 320.0 K
```

Calculate q_t

[24]:
```
q_t_calc_2 = np.empty(n, dtype=object)

for i in range(n):
    q_t_calc_2[i] = q_t.replace(T_w, T_w_calc[i])

for i in range(n):
    print('Heat entering = %.1f W/m^2 for T_a = %.1f K' % (q_t_calc_2[i],␣
 ↪T_a[i]))
```

```
Heat entering = 6.0 W/m^2 for T_a = 305.0 K
Heat entering = 16.2 W/m^2 for T_a = 310.0 K
Heat entering = 26.3 W/m^2 for T_a = 315.0 K
Heat entering = 36.5 W/m^2 for T_a = 320.0 K
```

2.2.4 Plot

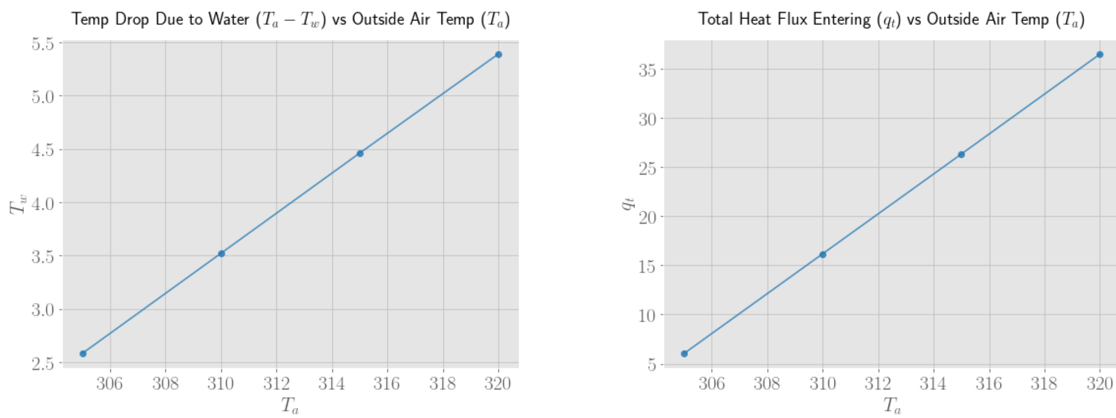
- Temp Drop Due to Water ($T_a - T_w$) vs Outside Air Temp (T_a)
- Total Heat Flux Entering (q_t) vs Outside Air Temp (T_a)

[25]:
```
fig = plt.figure(figsize=(16, 6))

ax1 = fig.add_subplot(121)
make_plot(x=T_a, y=T_a-T_w_calc, xlabel='$T_a$', ylabel='$T_w$',
          title='Temp Drop Due to Water ($T_a - T_w$) vs Outside Air Temp␣
 ↪($T_a$)')
```

```
ax2 = fig.add_subplot(122)
make_plot(x=T_a, y=q_t_calc_2, xlabel='$T_a$', ylabel='$q_t$',
          title='Total Heat Flux Entering ($q_t$) vs Outside Air Temp ($T_a$)')

fig.tight_layout(w_pad=10)
```



2.3 References

1. A. Shrivastava *et al*. "Evaporative cooling model…" (1984)
2. F. Incropera *et al*. "Fundamentals of Heat and Mass Transfer"