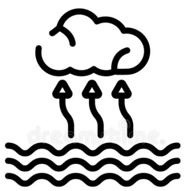
CL246 COURSE PROJECT
GROUP 15



Evaporation

REDUCTION OF HEAT LOAD OF HOUSEHOLDS USING EVAPORATIVE COOLING

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Introduction

As the world continues to urbanise, significant challenges arise in the environment, energy, and water sustainability in cities. Energy is an essential requirement for the existence and development of human life, primarily consisting of domestic sources such as fossil fuels (coal, oil, and natural gas) and electricity. Increasing energy efficiency is necessary to cut carbon emissions, secure energy, and save on energy bills.

User Story

In a tropical country like India, there are large regions that have high temperatures and low humidity, where the solar radiation incident on roofs of domestic and commercial buildings is very high in summer. Buildings in such regions face the problem of excessive heating as an effect of the hot climate. If not taken seriously, **these conditions will waste energy for cooling the room**. Energy consumption will increase when a building is designed without keeping environmental conditions and protection from direct sunlight in mind. The energy cost in such areas increases year on year. Hence, there is a need to find sustainable solutions with low investments, which can help solve this problem by reducing the heat flux inside the house like evaporative cooling.

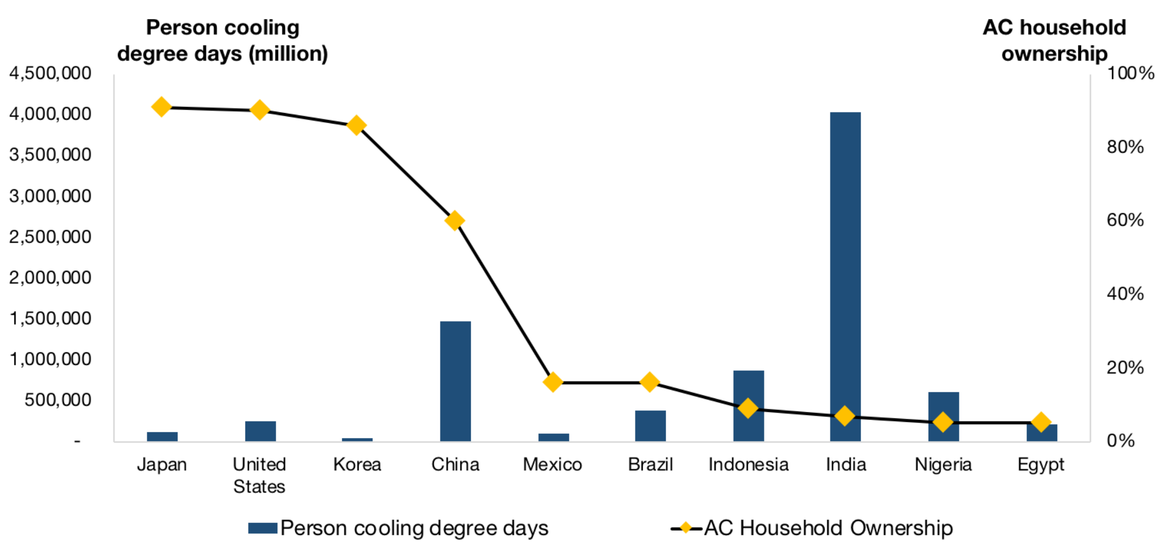


Fig (1) : Shows the highest expenditure of air conditioning in India. Hence, demonstrating the need to develop sustainable methods to reduce heat load of houses

Sources of Heat Gain

Sources of heat gain for the building include:

1. Heat inflow from roof (direct solar radiation)
   The majority of heat gained by houses in hot and dry places is through solar heat flux through the roof. The reason being that the sun is overhead for a longer period, and the intensity of solar radiation during that period is maximum. The major modes of heat transfer involved through the roof are convection through the air, radiation from the solar flux and conduction through the roof.
2. Heat inflow from walls (angled solar radiation) and windows (through hot air flowing in)
   This source of heat gain is majorly due to the temperature gradient between the air outside and that inside. The major modes of heat transfer are convection through air and conduction through the wall envelope.
3. Casual heat gain due to devices operating inside as well as occupants of the space. This type of heat gain is unavoidable and is not a highly significant source of heat gain.

Integral Heat balance in the house:

$$M_R \frac{\mathrm{d}T_R}{\mathrm{d}t} = \dot{Q_W}(t) + \dot{Q_R} + \dot{Q_D}(t) - \dot{Q_F}(t) - \dot{Q}_{inf/ven}(t)$$

where

$\dot{Q_W}(t)$: heat flux through the walls
$\dot{Q_R}$: heat flux through the roof
$\dot{Q_D}(t)$: heat flux through the door
$\dot{Q}_{inf/ven}(t)$: heat flux due to air flow caused by infiltration/ventilation

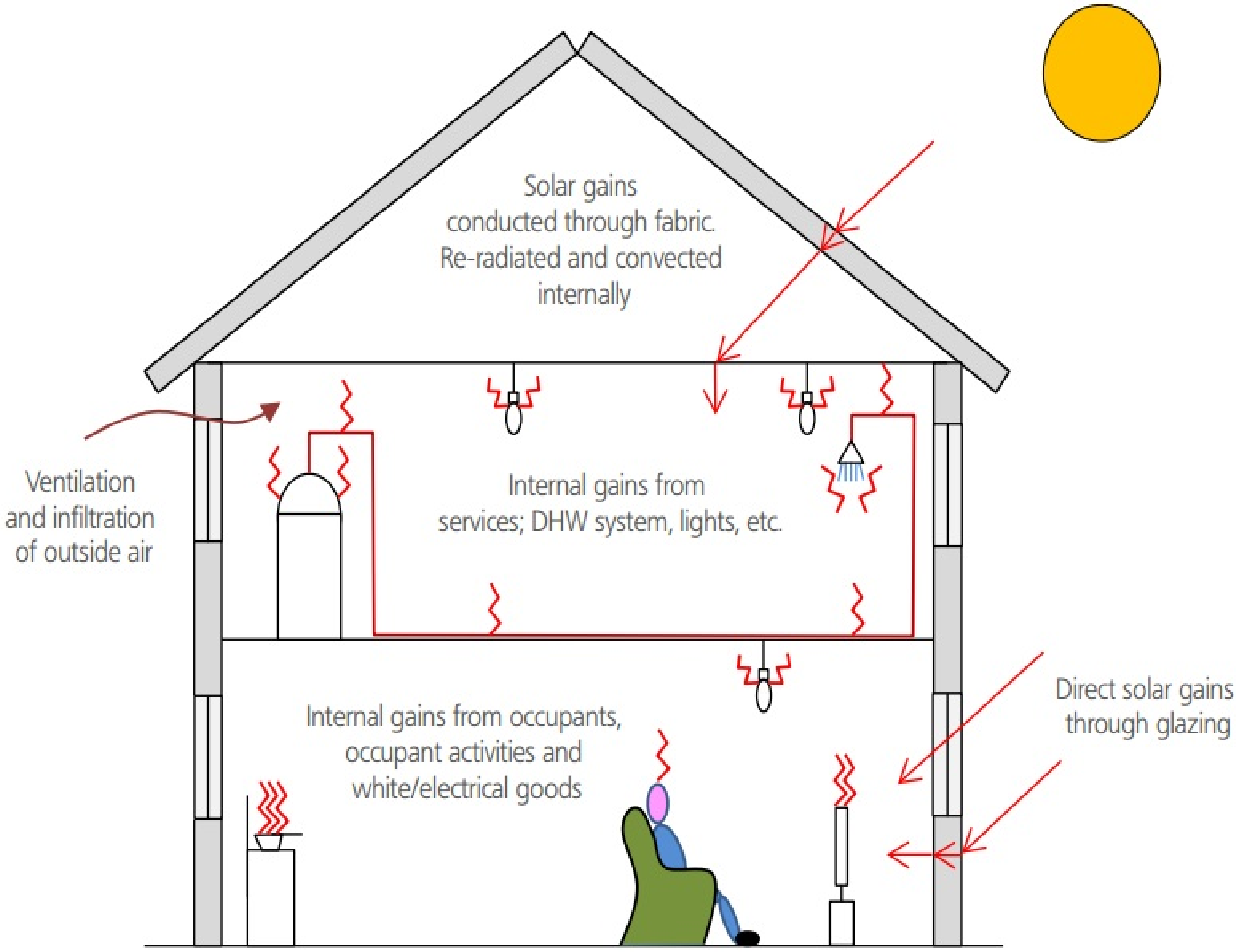


Fig (2): Shows all the three different ways in which heat is gained in the house through the various modes of heat transfer, majorly solar radiation, convection through air and conduction through the building envelope

Proposed Solution

The cost of artificially cooling houses using high-cost air-conditioning systems requires both higher investment and energy costs. Therefore, sustainable solutions like Evaporative cooling, stratified ventilation and geothermal heat pumps are suggested.

We have tried to model evaporative roof cooling for a house in hot and dry places like New Delhi in this project. Some of the incident solar radiation and convection energy is lost in the evaporation of water.

Tasks Performed as a part of the Project

- Modelling the evaporative roof cooling using simple modes of macroscopic heat transfer, mainly conduction, convection and radiation by control volume analysis.
- Control Volume estimation and estimation of heat transfers of the roof in both cases, namely one with water and one without water layer
- Established a complex mathematical and differential equations model using a literature survey
- Solving a simplified steady-state mathematical model of evaporative roof cooling using textbook correlations and equations.

Note: Basic approaches in both the models remain the same but the simplification done in the second model renders the equations solvable.

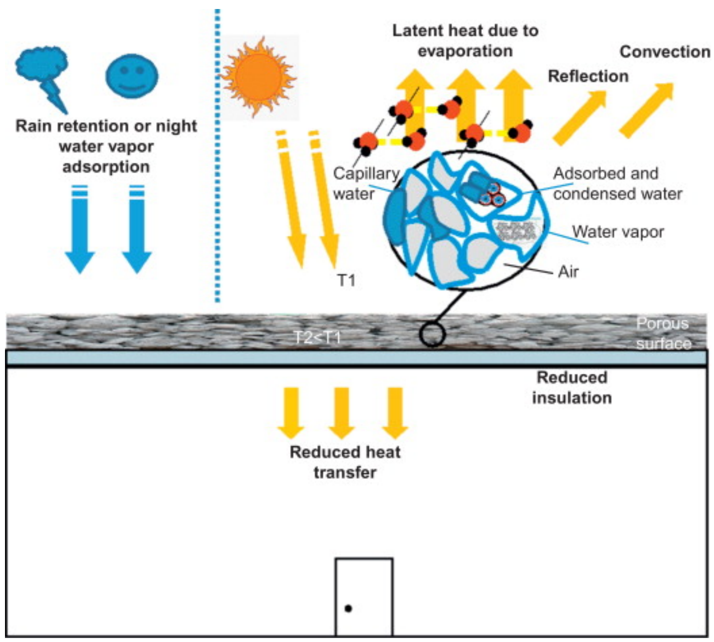


Fig (3) : Shows the schematic of Evaporative roof cooling and reduced heat transfer due to use of a fraction of incident heat in evaporation of water

Final Problem Statement

Estimate and compare the heat flux through the roof of a house in a place with hot and dry conditions in two cases:

A. Normal conditions with just a three-layered roof exposed to the solar radiation and wind convection during the afternoon when the solar radiation can be assumed to constant (invariable with time)
B. A thin water layer is added to the roof (thin enough to neglect temperature variation along the thickness of the water layer), which uses some of the heat incident into evaporation. This phenomenon of reduction of inlet heat flux in the house is known as evaporative cooling.

Considerations:

- Ambient room temperature i.e. inside room temperature: 27 deg centigrade
- Temperature outside vary between 35 deg centigrade to 50 deg centigrade and is constant at any point of time
- Heat transfer majorly involves conduction through roof, convection through air and water and conduction through the roof layers

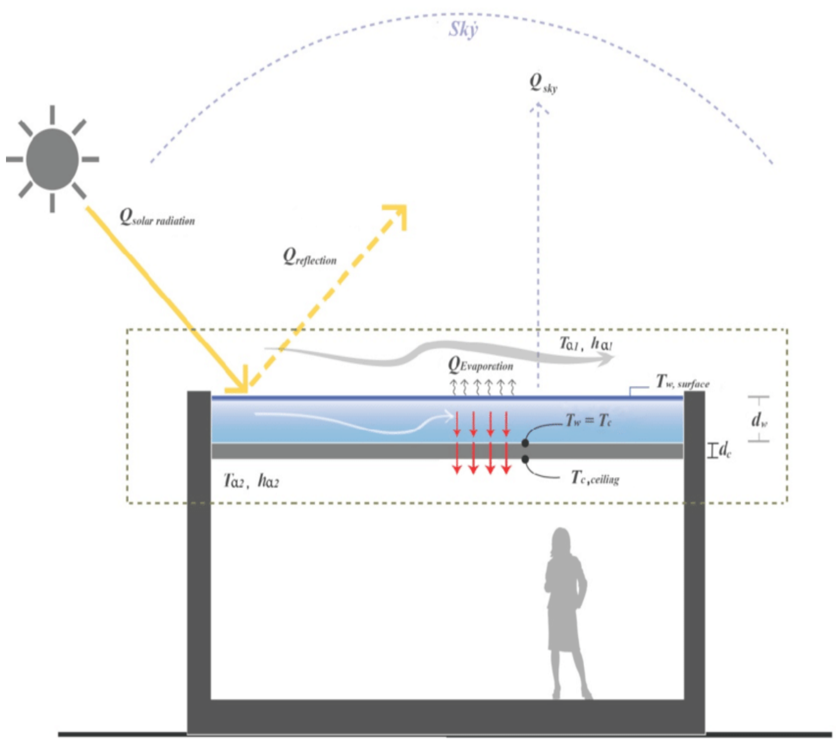


Fig (4): This shows the overall schematic of evaporative cooling problem suggested above.

Evaporative cooling and Status Quo

Air conditioners or HVAC systems are used for cooling and ventilation purposes and generally have high electricity costs associated with them. Design and structural changes such as adding insulation under the roof and walls aim to reduce the rate of incoming heat but do so to a limited extent. Instead, using additional techniques such as evaporative cooling can help reduce the heat transfer rate as well as the overall incoming heat through the roof. Given that the roof of a building is directly exposed to solar radiation and thus forms the bulk of the heat absorption, using this method on the roof would considerably reduce the energy needed to maintain cooler temperatures. Additionally, in places that face frequent power cuts (particularly during the summer months), high load inverters are required to support air conditioners, which would also need to be recharged frequently. This energy requirement is mitigated to some extent by using evaporative cooling, which is a relatively simple, low-cost method.

Further, the cooling effect observed increases with the rise in temperature and drop in humidity, conditions that prevail in large parts of states such as Rajasthan, Maharashtra, Gujarat and Madhya Pradesh.

The main operating cost of an Evaporative Roof Cooling System is water. The evaporation of 1 litre of water will absorb 0.67 kilowatt-hours of heat energy per hour. One ton of air conditioning is the equivalent of 3.517 kilowatt-hours of energy. Therefore, evaporation of fewer than 5.5 litres of water per hour is required to provide the equivalent cooling effect of one ton of air conditioning. For every one ton of heat load removed by an evaporative roof cooling system means one less ton of cooling load placed on your facility's HVAC system.

Other advantages of evaporative cooling systems include a low setup cost and low maintenance requirement - for a correctly set up system, the day-to-day operation only requires replenishment of water and minimal cleaning and practically no operational constraints.

Types of Evaporative Cooling

Active Direct Evaporative Cooling Systems

An active evaporative cooling system uses a system of fans or blowers to drive the ambient air through the wet pad into the system. This system can function against high static pressure, and it can be combined with a heat exchanger (indirect evaporative cooling).

Passive Direct Evaporative Cooling Systems

Passive cooling techniques use natural phenomena, energies, and heat sinks for cooling buildings without the use of mechanical apparatus or consumption of electrical energy. This is the oldest method of evaporative cooling and sometimes referred to as zero-energy cooling as it does not consume any commercial energy.

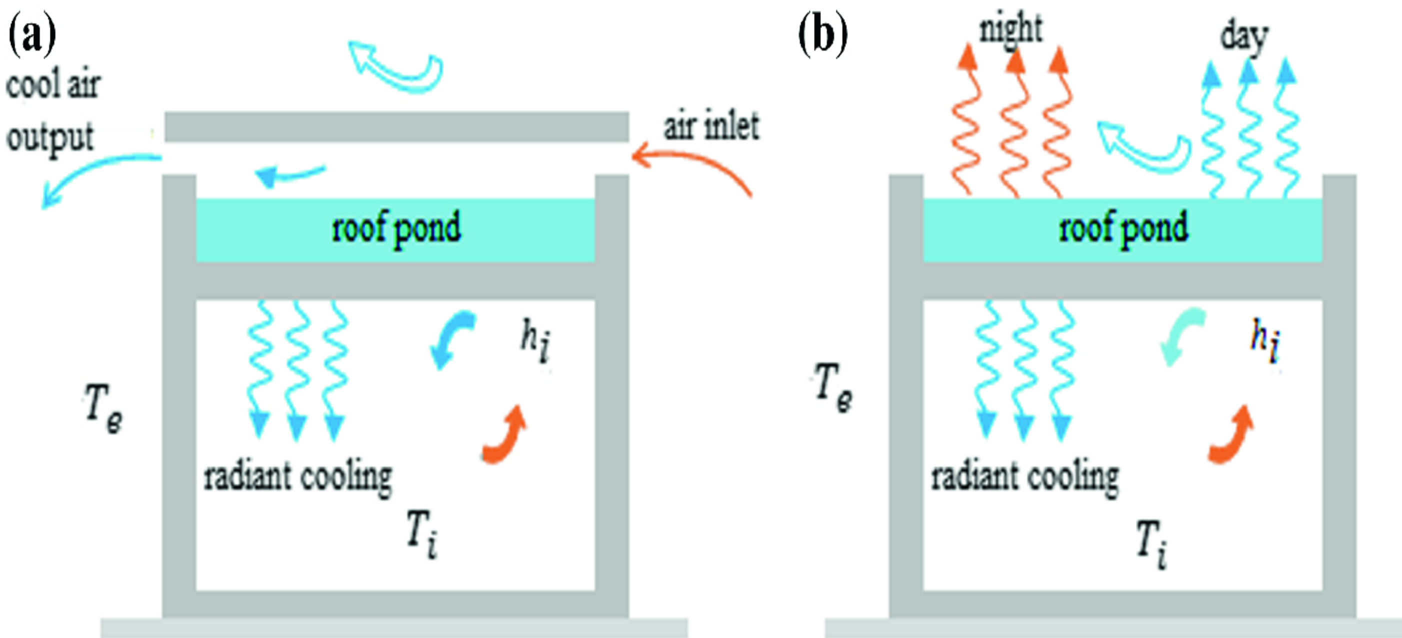


Fig (5): Schematic of Active direct and passive direct evaporative roof cooling systems

Indirect Evaporative Cooling Systems

Indirect Evaporative Cooling Systems consist of heat exchangers used to cool the air supplied to the living space. The evaporative cooling cycle occurs in the heat exchanger.

The model outlined in this project utilises passive direct cooling where the roof is covered in water (which has a high latent heat of evaporation) to minimise heat inflow to the building envelope - solar radiation is consumed for the evaporation of this water, keeping the building cool.

In this project we have majorly focused on Direct Evaporative cooling.

Qualitative Temperature Profile of Direct Evaporative roof cooling systems

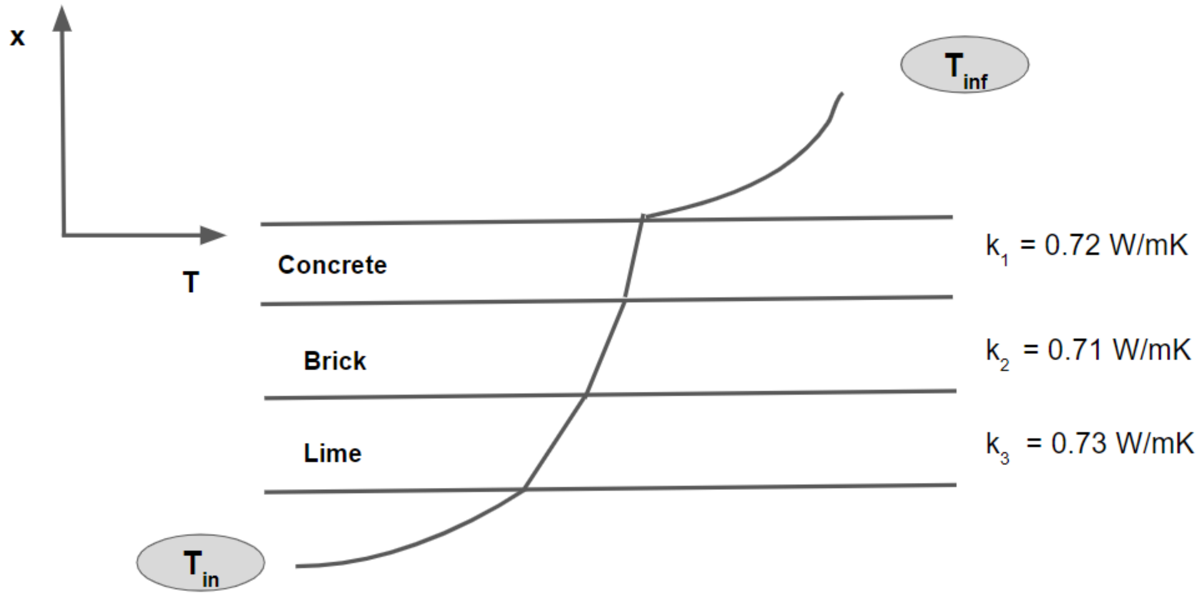


Fig (6): This is the qualitative temperature profile for the first case when the roof is directly exposed to the radiation

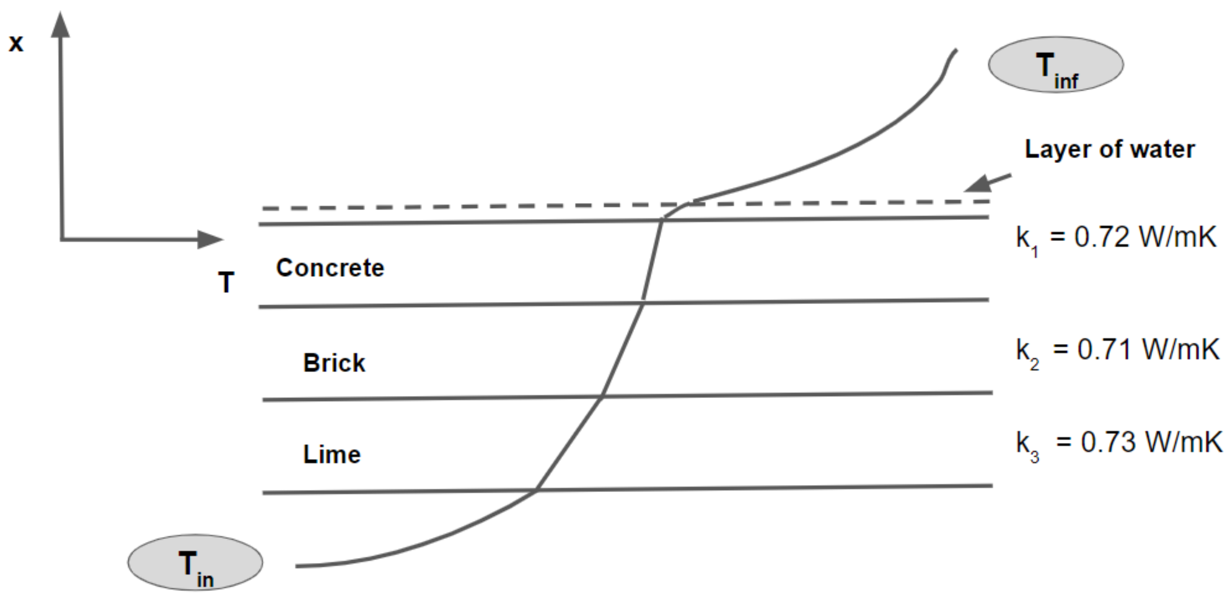


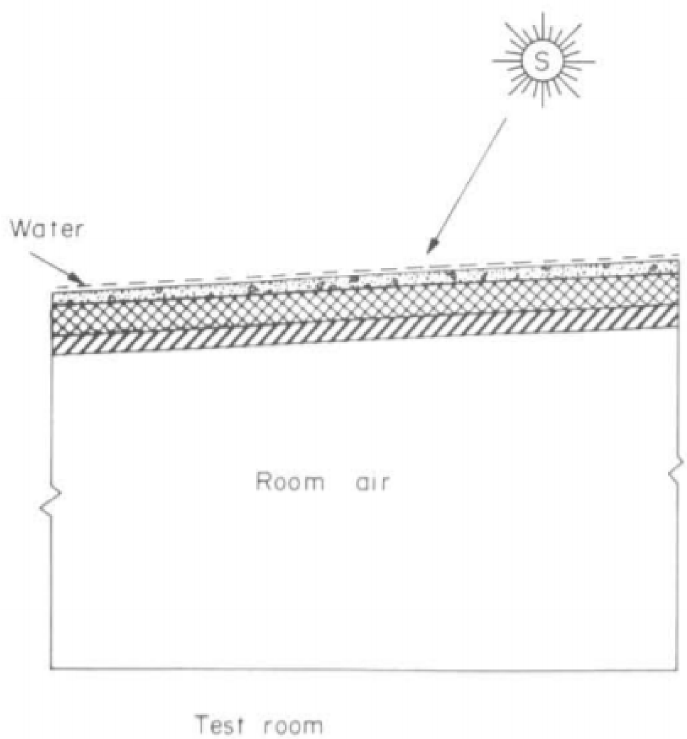
Fig (7): This is the qualitative temperature profile of the second case when there is a layer of water over the roof surface to allow evaporative cooling

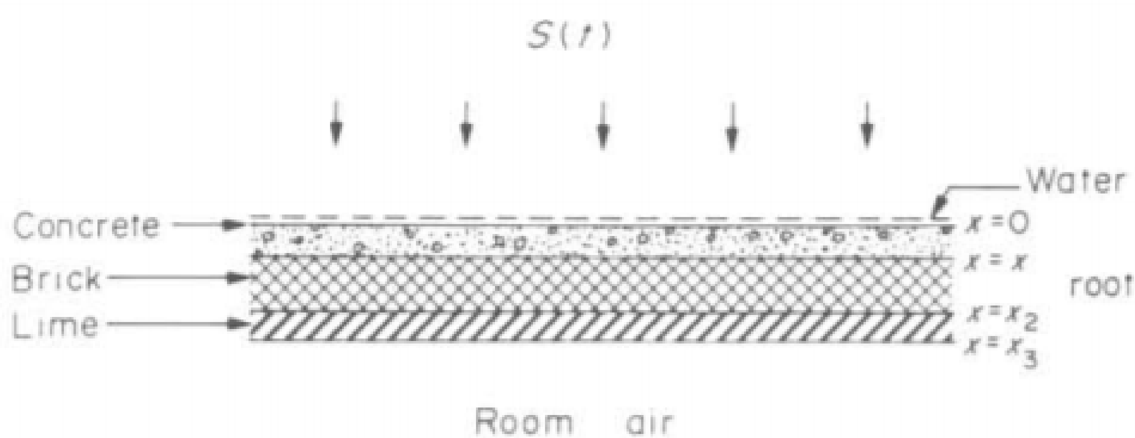
Overall Assembly of roof and house

The basic overall assembly of the house consists of walls, roof and floor, whose schematic representations are mentioned below. This report majorly concerns the heat flux through the roof with and without a layer of water over it.
The roof contains a three-layered plaster of concrete, brick and lime. For case 1, we have a bare roof, while for case 2, we have a thin layer of water over the roof.

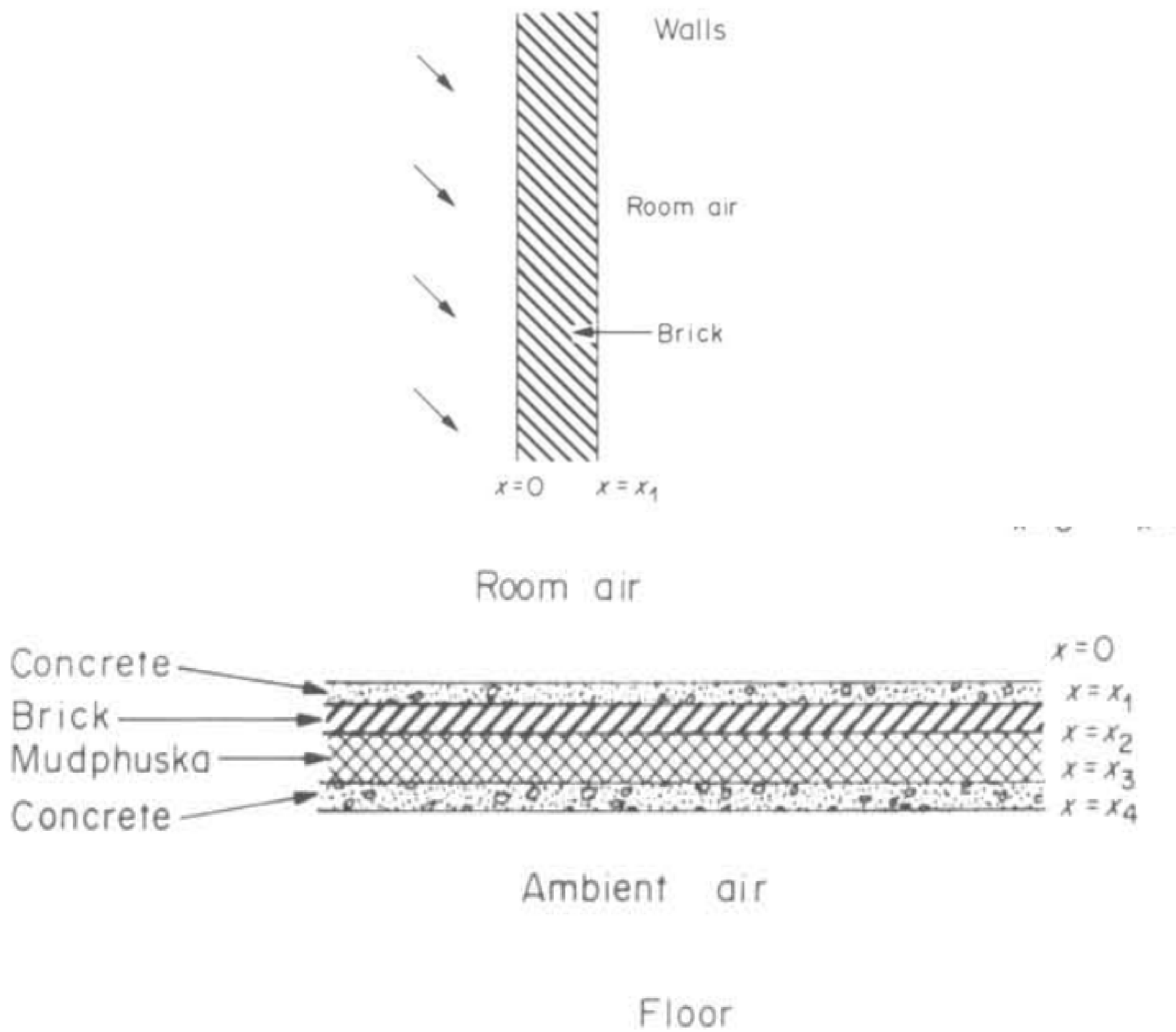
Fig 3:

A)





B)



C)

Fig (8) :

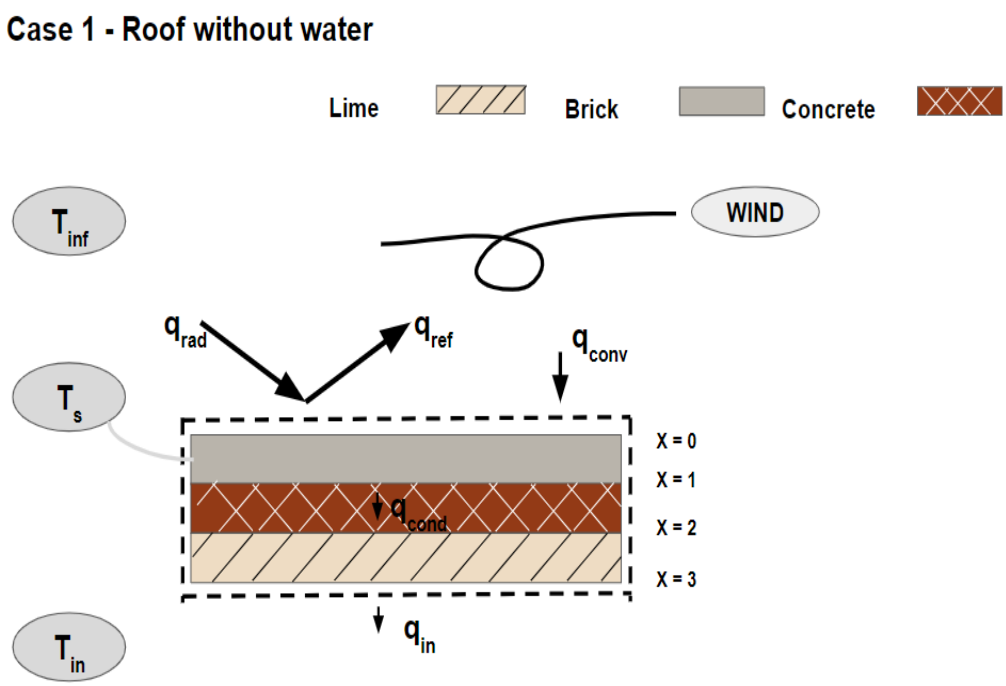
A) Shows the schematic of house explicitly mentioning the various layers of roof and the water layer

B) Schematic of roof having 3 solid layers with a layer of water on it

C) Schematic of the wall and floor of the house showing it various solid layers

Control Volume Analysis

Case 1: Roof without without



Fig(9): This is control volume and overall assembly of roof for the first case when the roof is directly exposed to sunlight

Basic Integral Balance Equation for case 1:

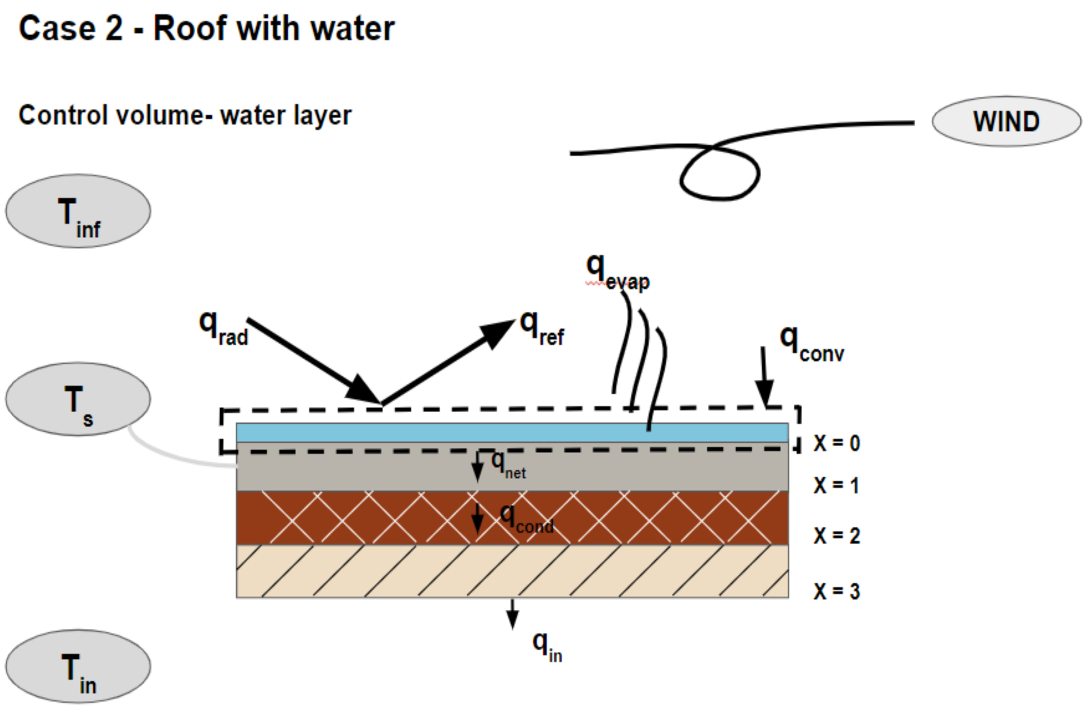
1) $q_{rad} + q_{conv} - q_{ref} = q_{cond}$

2) $q_{cond} = q_{in}$

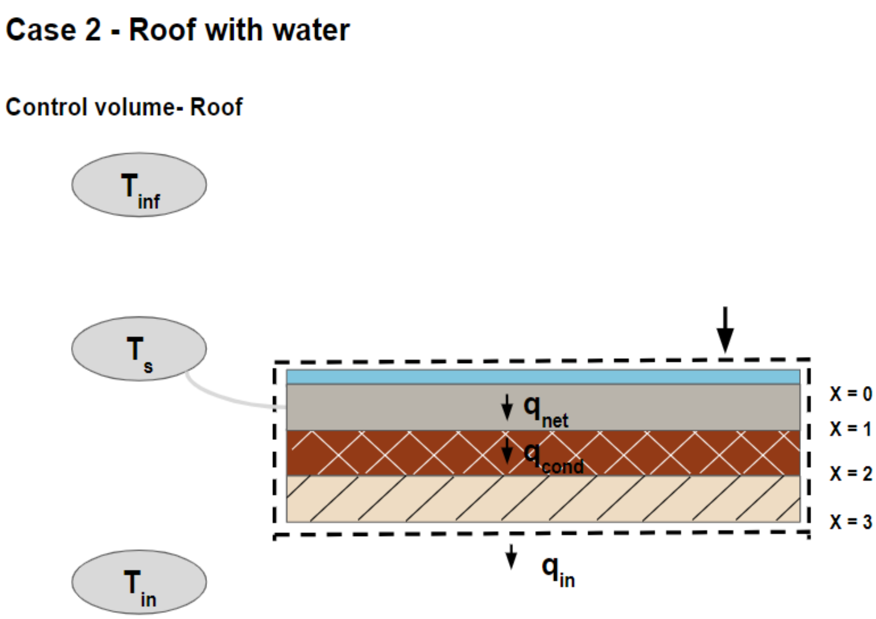
Where,

- q_{rad} - radioactive flux
- q_{conv} - convective flux
- q_{net} - net heat flux
- q_{cond} - conductive heat flux
- q_{in} - influx of heat

Case 2: Roof with a layer of water



(A)



(B)

Basic integral balance for case II:

$q_{net} = q_{cond} = q_{in}$
$q_{net} = q_{rad} + q_{conv} - q_{ref} - q_{evap}$

Where,

- q_{rad} - radioactive flux
- q_{conv} - convective flux
- q_{net} - net heat flux
- q_{cond} - conductive heat flux
- q_{in} - influx of heat
- q_{evap} - evaporative heat flux
- q_{ref} - reflective heat flux

Basic Mathematical Model and Common equations

Case 1: Without a layer of Water:

Assumption:

1. Steady state with room maintained at fixed ambient temperature.
2. Dry surroundings

Unknowns: Temperature of roof surface and heat flux entering the room

In general:

$$k\frac{\partial^2 \theta_j}{\partial x^2} = \rho c \frac{\partial \theta_j}{\partial t} \tag{2.1}$$

Boundary conditions:

$$-k\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = h_0[\theta_{SA}(t) - \theta|_{x=0}] \quad \text{(BC 1)}$$

$$-k\left.\frac{\partial \theta}{\partial x}\right|_{x=\mathrm{X}} = h_i[\theta(x=\mathrm{X}, t) - T_R] \quad \text{(BC 2)}$$

q

where:

$$\theta_{SA} = \frac{\alpha_S S(t)}{h_0} + T_A - \frac{\varepsilon \Delta R}{h_0} \tag{2.2}$$

θ_{SA} : sol air temperature
$S(t)$: intensity of solar radiation
α_S : absorptivity of surface
ΔR : difference between radiation from surrounding and that emmited by a blackbody at ambient air temperature

$$\theta_j(x = \mathrm{X}_1, t) = \theta_{j+1}(x = \mathrm{X}_1, t) \tag{3.1}$$

$$-k_j \left.\frac{\partial \theta_j}{\partial x}\right|_{x=\mathrm{X}_1} = -k_{j+1} \left.\frac{\partial \theta_{j+1}}{\partial x}\right|_{x=\mathrm{X}_1} \tag{3.2}$$

where:

k_i : conductivity of i^{th} layer
T_A : ambient air temperature

$$S(t) = S_0 \qquad\qquad \dot{Q}_W(t) = A(u(\theta_S - T_R))$$

$$\frac{1}{u} = \frac{1}{h_0} + \sum_j \frac{X_j}{k_j} + \frac{1}{h_i}$$

Eq 2.1: Describes the fundamental laws governing conduction in solids

BC1 and BC2 : Are the Boundary conditions between the solid layer exposed to inner and outer walls layers and air

3.1 and 3.2: Are the equations of continuity of heat flux and temperature between layers
U is the cumulative heat transfer coefficient for the system

Case 2: Roof with the layer of water

Assumptions:

1. Steady state with room maintained at fixed ambient temperature
2. Water is still (vw = 0) but gentle breeze is present (va = 10 km/h)
3. Dry Surroundings

Unknowns: Temperature of water surface and heat flux entering the room

In this case all the above equations are required along with the equation of heat balance over the water layer.

Eq 4.1: Gives the fundamental heat transfer equation for water layer taking into consideration the convection through the air and roof surface as well as the solar radiation.
Eq 4.2/4.3/4.4: Describes the total radiative, convective and evaporative heat flux having specific heat transfer coefficients and their correlations mentioned below obtained from references.

(3) Evaporative cooling:

$$(bd\rho_w C_w \frac{\partial T_w}{\partial t} + \dot{m}_w C_w \frac{\partial T_w}{\partial y})dy = [\tau_1 S(t) \\ - Q_r - Q_c - Q_e + h_1(\theta_1|_{x=0} - T_w)]bdy \quad (4.1)$$

$$Q_r = h_r(T_w - T_A) \quad (4.2)$$

$$Q_c = h_c(T_w - T_A) \quad (4.3)$$

$$Q_e = 0.013h_e[p(\bar{T}_w - \gamma p(\bar{T}_A)] \quad (4.4)$$

where:

τ_1 : fraction of solar radiation absorbed by water
Q_r : radiative heat
Q_c : convective heat flux
ρ : saturated partial pressure
$\left.\begin{matrix} h_r \\ h_e \\ h_c \end{matrix}\right\}$: respective heat transfer coefficient

$$P(T) = R_1 T + R \quad (4.5)$$

on simplification,

$$M_w \frac{\partial T_w}{\partial t} + \dot{m}_w C_w \frac{\partial T_w}{\partial y} = bH(T_S - T_w) + bh_1(\theta_1|_{x=0} - T_w) \quad (4.6)$$

$$T_s = 1/H(\tau_1 S(t) + H_1 T_A(t) - R_0 R_2(1-r)) \quad (4.7)$$

$$H = h_r + h_c + R_0 R_1 \quad (4.8)$$

$$H_1 = h_r + h_c + rR_0 R_1 \quad (4.9)$$

$$R_0 = 0.013h_c \quad (4.10)$$

$$M_w = bdC_w S_w \quad (4.11)$$

$$\dot{Q}_R(y,t) = h_i[\theta|_{x=x_4} - T_R] \quad (4.12)$$

Conclusion

The models we have made indicate a drastic decrease in incoming heat flux to the building roof when the water layer is present. This indicates that evaporative cooling may be a viable solution that can solve the problem of high electricity costs, particularly in high-temperature areas in the drier parts of the Indian subcontinent.

Though we have made several assumptions and simplifications while solving this problem, we obtain a high reduction in heat flux (50-75%). This indicates that there will be considerable reduction even assuming that not all of these simplifications hold.

Temperature of outside air (T_{inf} in K)	Without Water Layer (kW/m^2)	With Water Layer (kW/m^2)
305	28.5	6.0
310	44.3	16.2
315	60.0	26.3
320	75.8	36.5

This experiment can be extended further by considering incident solar radiation to be a function of time, as an average, varying over 24 hours of time.

Therefore, we use the simple technique of evaporation to effectively reduce heat and make cooling more efficient and cost effective.

The following sections are the mathematical computations for both cases:

A) Heat Influx in roof without water layer
B) Heat Influx in roof with a water layer

These cases include all the simplifications, assumptions and conditions used for solving along with the proper references.

We will now model and solve the equations in python using the SymPy module.

1 Case 1: No Water Layer

- Author: **Team G15**
- Attempt: 3

1.1 Analysis

1.1.1 To find

1. Temperature of Roof Surface (T_s)
2. Total heat flux entering the house through the roof, (q_t) when no water layer is present

1.1.2 Nomenclature

- T_s = roof surface temperature (outside)
- T_a = ambient air temperature (outside)
- T_r = room temperature (inside)
- Nu_a = Nusselt number of air
- Ra_a = Rayleigh number of air
- Re_a = Reynolds number of air
- Pr_a = Prandtl number of air
- α_a = thermal diffusivity of air
- k_a = thermal conductivity of air
- h_r = free convection coefficient of room air
- ν_a = dynamic Viscosity of air
- Roof layers:
  - 1: Concrete
  - 2: Brick
  - 3: Lime
- k_i = thermal conductivity of i^{th} roof layer
- L_i = length of i^{th} roof layer
- q_r = radiative heat transfer (per unit area)
- q_c = convective heat transfer (per unit area)
- q_t = net heat transfer into the room (per unit area)
- β = coefficient of thermal expansion
- S = Intensity of Solar Radiation (i.e. solar constant)

1.1.3 Assumptions

- Steady state with room maintained at fixed ambient temperature

1.1.4 Equations

Energy balance,

$$q_t = q_c + q_r$$

Radiation heat transfer,

$$q_r = \tau_s \cdot S - h_r \cdot (T_a - T_s)$$

$$h_r = \epsilon_s \cdot \sigma \cdot \frac{(\overline{T}_s)^4 - (\overline{T}_a - 12)^4}{\overline{T}_a - \overline{T}_s}$$

Convection heat transfer,

$$q_c = h_c \cdot (T_a - T_w)$$

$$h_c = \frac{k_a}{L_s} \cdot Nu_a$$

$$Nu_a = 0.15 \cdot Ra_a^{1/3} + 0.664 \cdot Re_a^{1/2} \cdot Pr_a^{1/3}$$

$$Re_a = \frac{v_a \cdot L_s}{\nu_a}$$

$$Ra_L = \frac{g \cdot \beta \cdot (T_s - T_a) \cdot L_s^3}{\nu_a \cdot \alpha_a}$$

Total heat transfer,

$$q_t = \frac{T_w - T_r}{R_{net}}$$

$$R_{net} = \frac{1}{h_r} + \sum_{i=1}^{3} \frac{L_i}{k_i}$$

1.1.5 Properties

Outside Air

- Mild breeze $v_a = 2.78\ m/s$
- $T_a \in [305, 320]K$
- $T_f = 320K$
- $\beta = \frac{1}{T_f} = 0.0031\ K^{-1}$
- Table A.4, air (T_f):
  - $\nu = 18 \cdot 10^{-6}\ m^2/s$
  - $\alpha = 25 \cdot 10^{-6}\ m^2/s$
  - $Pr = 0.702$
  - $k = 27.7 \cdot 10^{-3}\ W/m \cdot K$
- $S = 1366\ W/m^2$

Roof

- $L_s = 5\ m$ (approx thickness of water layer)
- $\epsilon_s = 0.9$ (concrete surface)
- $\tau_s = 0.9$
- $t = 0.2\ m$ thick with,
  - Cement = 5 cm
  - Brick = 10 cm
  - Lime = 5 cm
- K_i, Conductivity of each layer,
  - Cement = 0.72 $W/m \cdot K$
  - Brick = 0.71 $W/m \cdot K$
  - Lime = 0.73 $W/m \cdot K$

Inside air

- $T_r = 300K$ (Room Temperature)
- $h_r = 8.4\ W/m^2 \cdot K$

1.1.6 Tools used

- **Python**
- **SymPy** for creating symbolic equations and solving them
- **NumPy**
- **Matplotlib** for plotting results

1.2 Solving (Python Code)

1.2.1 Initialize Values

[1]:
```
%matplotlib inline
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

# Initialize matplotlib
plt.rc('text', usetex=True) # Unnecessary
plt.style.use('ggplot')
plt.rcParams['grid.color'] = '#C0C0C0'
```

Outside Air

- Table A.4 used (from reference #2)

[2]:
```
v_a = 2.78 # Velocity (m / s)

# Temperatures
T_f = 320.0 # (K)
beta = 1/T_f # (K)
```

```python
T_a = np.array([305.0, 310.0, 315.0, 320.0]) # (K)
T_a_avg = 273 + 37 # (K)

# Universal Constants
sigma = 5.67e-8 # Stefan Boltzmann constant (W / m^2 * K^4)
g = 9.8 # (m / s^2)
S = 1366 # Solar constant

# Table A.6, air @ T = T_f
nu_a = 18e-6 # dynamic visosity (m^2 / s)
alpha_a = 25e-6 # (m^2 / s)
k_a = 27.7e-3 # thermal conductivity (W / m * K)
Pr_a = 0.702
```

Roof Layers

[3]:
```python
# Temperatures
T_s = sp.symbols('T_s') # Roof surface temp (K)
T_s_avg = 273.0 + 35.0 # (K)

# Surface
L_s = 5 # Dimensions (m)
tau_s = 0.9 # Roof's solar absorbtivity
epsilon_s = 0.9 # Emissivity of roof surface (concrete)

# Layer 1: Concrete
k_1 = 0.72 # (W / m * K)
L_1 = 0.05 # (m)

# Layer 2: Brick
k_2 = 0.71 # (W / m * K)
L_2 = 0.10 # (m)

# Layer 3: Lime
k_3 = 0.73 # (W / m * K)
L_3 = 0.05 # (m)
```

Inside Air

[4]:
```python
h_r = 8.4 # (W / m^2 * K)
T_r = 300 # (K)
```

1.2.2 Equations

Radiation Heat

[5]:
```python
h_r = epsilon_s * sigma * (T_s_avg**4 - (T_a_avg - 12)**4)/(T_a_avg - T_s_avg) #␣
 ↪(W / m^2 * K)
q_r = tau_s * S - h_r * (T_a - T_s) # (W / m^2)
```

```
# Example at T_a = 310K and T_s = 314K
q_r_test = q_r[1].replace(T_s, 314)
print('Approximate value of q_r = %.2f W/m^2' % (q_r_test))
```

```
Approximate value of q_r = 1343.00 W/m^2
```

Convection Heat

- From below analysis, we can neglect free convection in comparison to forced convection

Free Convection

[6]:
```
Ra_a = (g * beta * (T_s - T_a) * L_s**3) / (nu_a * alpha_a)
Nu_a_fr = 0.15 * Ra_a**(1/3)
h_c_fr = k_a / L_s * Nu_a_fr

# Example at T_a = 310K and T_s = 314K
h_c_fr_test = h_c_fr[1].replace(T_s, 314)
print('Approximate value of free convection coefficient = %.2f W/K*m^2' %
 ↪(h_c_fr_test))
```

```
Approximate value of free convection coefficient = 2.69 W/K*m^2
```

Forced Convection

[7]:
```
Re_a = v_a * L_s / nu_a
Nu_a_fo = 0.664 * Re_a**1/2 * Pr_a**1/3
h_c_fo = k_a / L_s * Nu_a_fo

# Example at T_a = 310K and T_s = 314K
print('Approximate value of forced convection coefficient = %.2f W/K*m^2' %
 ↪(h_c_fo))
```

```
Approximate value of forced convection coefficient = 332.36 W/K*m^2
```

Total Convection

[8]:
```
h_c = h_c_fo # Neglicting free convection
q_c = h_c * (T_a - T_s) # (W / m^2)

# Example at T_a = 310K and T_s = 314K
q_c_test = q_c[1].replace(T_s, 314)
print('Approximate value of q_c = %.2f W/m^2' % (q_c_test))
```

```
Approximate value of q_c = -1329.43 W/m^2
```

Total Heat:

[9]:
```
R = 1/h_r + L_1/k_1 + L_2/k_2 + L_3/k_3 # (m^2 * K / W)

q_t = (T_s - T_r) / R # (W / m^2)

# Example at T_a = 310K and T_s = 314K
```

```python
q_t_test = q_t.replace(T_s, 314)
print('Approximate value of q_t = %.2f W/m^2' % (q_t_test))
```

```
Approximate value of q_t = 44.59 W/m^2
```

1.2.3 Solving

$$q_c + q_r = q_t$$
$$\therefore \; q_c + q_r - q_t = 0$$

Calculate T_s

[10]:
```python
eq = q_c + q_r - q_t

n = len(eq)
T_s_calc = np.empty(n, dtype=object)

for i in range(n):
    T_s_calc[i] = round(sp.solve(eq[i], T_s)[0], 2)

for i in range(n):
    print('T_s = %.1f K for T_a = %.1f K' % (T_s_calc[i], T_a[i]))
```

```
T_s = 309.0 K for T_a = 305.0 K
T_s = 313.9 K for T_a = 310.0 K
T_s = 318.9 K for T_a = 315.0 K
T_s = 323.8 K for T_a = 320.0 K
```

Calculate q_t

[11]:
```python
q_t_calc_1 = np.empty(n, dtype=object)

for i in range(n):
    q_t_calc_1[i] = q_t.replace(T_s, T_s_calc[i])

for i in range(n):
    print('Heat entering = %.1f W/m^2 for T_a = %.1f K' % (q_t_calc_1[i],␣
 ↪T_a[i]))
```

```
Heat entering = 28.5 W/m^2 for T_a = 305.0 K
Heat entering = 44.3 W/m^2 for T_a = 310.0 K
Heat entering = 60.0 W/m^2 for T_a = 315.0 K
Heat entering = 75.8 W/m^2 for T_a = 320.0 K
```

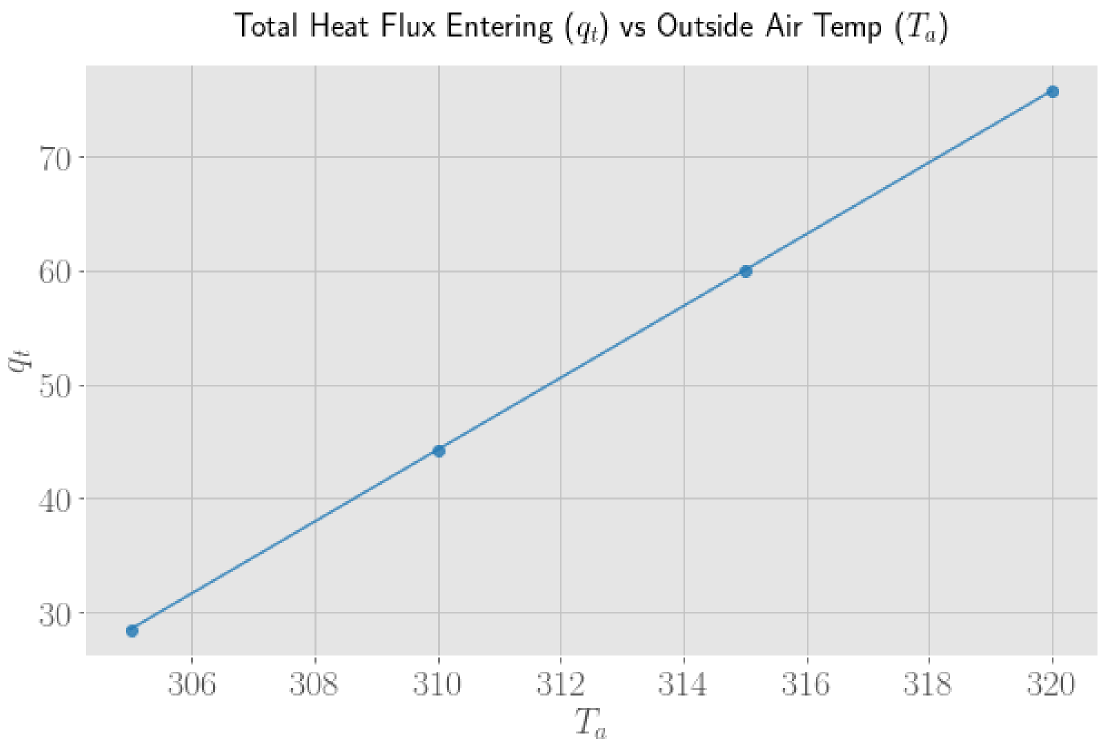
1.2.4 Plot

- Total Heat Flux Entering (q_t) vs Outside Air Temp (T_a)

[12]:
```python
def make_plot(x, y, xlabel, ylabel, title):
    plt.plot(x, y, color='#1F77B4cc', marker='o')
```

```
    plt.xticks(fontsize=20)
    plt.yticks(fontsize=20)
    plt.xlabel(xlabel, fontsize=20)
    plt.ylabel(ylabel, fontsize=20)
    plt.title(title, fontsize=18, pad=15)
```

[13]:
```
fig = plt.figure(figsize=(10, 6))
make_plot(x=T_a, y=q_t_calc_1, xlabel='$T_a$', ylabel='$q_t$',
          title='Total Heat Flux Entering ($q_t$) vs Outside Air Temp ($T_a$)')
```



2 Case 2: Water Layer

- Author: **Team G15**
- Attempt: 3

2.1 Analysis

2.1.1 To find

1. Temperature of Water Surface (T_w)
2. Total heat flux entering the house through the roof, (q_t) when a water layer is present

2.1.2 Nomenclature

- S = Intensity of Solar Radiation (i.e. solar constant)
- v_w = water velocity
- v_a = wind velocity
- ϵ_w = emissivity of water surface
- σ = Stefan-Boltzmann constant $(5.67 * 10^{-8}\ W/m^2K^4)$
- T_r = room temperature (inside)
- T_w = water surface temperature (outside)
- T_a = ambient air temperature (outside)
- $\overline{T}_w$ = average water surface temperature (outside)
- $\overline{T}_a$ = average air temperature (outside)
- τ_w = fraction of solar radiation absorbed by water
- k_w = thermal conductivity of water
- L_w = length of water layer
- h_w = convection coefficient of water layer
- h_r = radiative heat transfer coefficient
- h_c = convective heat transfer coefficient
- h_e = evaporative heat transfer coefficient

2.1.3 Assumptions

1. Steady state with room maintained at fixed ambient temperature
2. Water is still ($v_w = 0$) but gentle breeze is present ($v_a = 10\ km/h$)
3. Dry Surroundings

2.1.4 Equations

Energy balance,

$$q_t = q_c + q_r - q_e$$

Radiation heat transfer,

$$q_r = \tau_w \cdot S - h_r \cdot (T_a - T_w)$$

$$h_r = \epsilon_w \cdot \sigma \cdot \frac{(\overline{T}_w)^4 - (\overline{T}_a - 12)^4}{\overline{T}_a - \overline{T}_w}$$

Convection heat transfer,

$$q_c = h_c \cdot (T_a - T_w)$$

$$h_c = 5.678 \cdot (1 + 0.85 \cdot (v_a - v_w))$$

Evaporative heat transfer,

$$q_e = 0.013 \cdot h_c \cdot (p(\overline{T}_w) - \gamma \cdot p(\overline{T}_a))$$

$$p(T) = R_1 \cdot T + R_2$$

Total heat transfer,

$$q_t = \frac{T_w - T_r}{R_{net}}$$

$$R_{net} = \frac{1}{h_r} + \sum_{i=1}^{3} \frac{L_i}{k_i} + \frac{1}{h_w}$$

$$h_w = \frac{k_w}{L_w} \cdot (0.14 \cdot (Gr \cdot Pr)^{1/3} + 0.644 \cdot (Pr \cdot Re)^{1/3})$$

$$Gr = \frac{g \cdot \beta \cdot (T_w - T_a) \cdot (L_w)^3}{\nu^2}$$

2.1.5 Properties

Outside Air

- Mild breeze $v_a = 2.78\ m/s$
- $T_a \in [305, 320]K$
- $T_f = 320K$
- $\beta = \frac{1}{T_f} = 0.0031\ K^{-1}$
- Table A.4, air (T_f):
  - $\nu = 18 \cdot 10^{-6}\ m^2/s$
  - $\alpha = 25 \cdot 10^{-6}\ m^2/s$
  - $Pr = 0.702$
  - $k = 27.7 \cdot 10^{-3}\ W/m \cdot K$
- $S = 1366\ W/m^2$
- $R_1 = 325\ Pa/°C$ and $R_2 = -5155\ Pa$ (*from reference* **#1**)
- $\gamma = 0.27$ (approx average over a day)

Water layer

- $L_w = 0.1\ m$ (approx thickness of water layer)
- Table A.6, water (T_w):
  - $\nu = 18 \cdot 10^{-6}\ m^2/s$
- Still water $v_w = 0$
- $\epsilon_w = 0.95$
- $\tau_w = 0.6$

Roof

- $t = 0.2\ m$ thick with,
  - Cement = 5 cm
  - Brick = 10 cm
  - Lime = 5 cm
- K_i, Conductivity of each layer,
  - Cement = 0.72 $W/m \cdot K$

- Brick = 0.71 $W/m \cdot K$
- Lime = 0.73 $W/m \cdot K$

Inside air

- $T_r = 300K$ (Room Temperature)
- $h_r = 8.4\ W/m^2 \cdot K$

2.1.6 Tools used

- **Python**
- **SymPy** for creating symbolic equations and solving them
- **NumPy**
- **Matplotlib** for plotting results

2.2 Solving (Python Code)

2.2.1 Initialize Values

Outside Air

- Saturation pressure of water p = R_1*T + R_2

[14]:
```
v_a = 2.78 # Velocity (m / s)

# Temperatures
T_f = 320 # (K)
beta = 1/T_f # (K)
T_a = np.array([305.0, 310.0, 315.0, 320.0]) # (K)
T_a_avg = 273 + 37 # (K)

# Constants
sigma = 5.67e-8 # Stefan Boltzmann constant (W / m^2 * K^4)
g = 9.8 # (m / s^2)
R_1 = 325 # (N / m^2 °C)
R_2 = -5155 # (N / m^2)
gamma = 0.27
S = 1366 # Solar constant

def p(T): # Saturation pressure of water as a function of temperature (N / m^2)
    return R_1 * (T-273) + R_2
```

Water Layer

[15]:
```
v_w = 0 # Velocity (m / s)
L_w = 5 # Dimensions (m)

# Temperatures
T_w = sp.symbols('T_w') # (K)
T_w_avg = 273 + 32 # (K)
```

```python
# Constants
epsilon_w = 0.95 # Emissivity of water surface
tau_w = 0.6 # Water's solar absorbtivity
```

- Table A.6 used (*from reference* **#2**)
- Upon analysing the below data, we can approximate h_w to 950 W/m^2

[16]:
```python
rho_w = 990 # density (kg / m^3)
k_w = 0.63 # thermal conductivity (W / m * K)
mu_w = 1e-6 * np.array([769, 695, 631, 577]) # viscosity (N * s / m^2)
nu_w = mu_w / rho_w # dynamic visosity (m^2 / s)

Pr_w = np.array([5.20, 4.62, 4.16, 3.77]) # Prandtl number
Re_w = 0 # Reynolds number, still water
Gr_w = g * beta * (T_a - T_w) * L_w**3 / nu_w**2 # Grashof number

# Water free convection coeffecient
h_w = (k_w/L_w) * (0.14 * (Gr_w*Pr_w)**(1/3) + 0.644 * (Pr_w*Re_w)**(1/3))

# Example at T_a = 310K and T_w = 306K
h_w_test = h_w[1].replace(T_w, 306)
print('Approximate min value of h_w = %.2f W/K*m^2' % (h_w_test))
```

```
Approximate min value of h_w = 923.62 W/K*m^2
```

Roof Layers

[17]:
```python
# Layer 1: Concrete
k_1 = 0.72 # (W / m * K)
L_1 = 0.05 # (m)

# Layer 2: Brick
k_2 = 0.71 # (W / m * K)
L_2 = 0.10 # (m)

# Layer 3: Lime
k_3 = 0.73 # (W / m * K)
L_3 = 0.05 # (m)
```

Inside Air

[18]:
```python
h_r = 8.4 # (W / m^2 * K)
T_r = 300 # (K)
```

2.2.2 Equations

Radiation Heat

[19]:
```
h_r = epsilon_w * sigma * (T_w_avg**4 - (T_a_avg - 12)**4)/(T_a_avg - T_w_avg) #␣
↪(W / m^2 * K)
q_r = tau_w * S - h_r * (T_a - T_w) # (W / m^2)

# Example at T_a = 310K and T_w = 306K
q_r_test = q_r[1].replace(T_w, 306)
print('Approximate value of q_r = %.2f W/m^2' % (q_r_test))
```

```
Approximate value of q_r = 786.53 W/m^2
```

Convection Heat

- Forced convection and free convection both have been used

[20]:
```
h_c = 5.678 * (1 + 0.85 * (v_a - v_w))
print('h_c = %.2f W/K*m^2' % (h_c))

q_c = h_c * (T_a - T_w) # (W / m^2)

# Example at T_a = 310K and T_w = 306K
q_c_test = q_c[1].replace(T_w, 306)
print('Approximate value of q_c = %.2f W/m^2' % (q_c_test))
```

```
h_c = 19.10 W/K*m^2
Approximate value of q_c = 76.38 W/m^2
```

Evaporation Heat:

[21]:
```
q_e = 0.013 * h_c * (p(T_w_avg) - gamma * p(T_a_avg)) # function p defined␣
↪above, (W / m^2)

# Example at T_a = 310K and T_w = 306K
print('Approximate value of q_e = %.2f' % (q_e))
```

```
Approximate value of q_e = 841.55
```

Total Heat:

[22]:
```
h_w = 1200 # from above approximation (W / m^2 * K)
R = 1/h_r + L_1/k_1 + L_2/k_2 + L_3/k_3 + 1/h_w # (m^2 * K / W)

q_t = (T_w - T_r) / R # (W / m^2)

# Example at T_a = 310K and T_w = 306K
q_t_test = q_t.replace(T_w, 306)
print('Approximate value of q_t = %.2f W/m^2' % (q_t_test))
```

```
Approximate value of q_t = 14.98 W/m^2
```

2.2.3 Solving

$$q_c + q_r - q_e = q_t$$
$$\therefore\ q_c + q_r - q_e - q_t = 0$$

Calculate T_w

[23]:
```
eq = q_c + q_r - q_e - q_t

n = len(eq)
T_w_calc = np.empty(n, dtype=object)

for i in range(n):
    T_w_calc[i] = round(sp.solve(eq[i], T_w)[0], 2)

for i in range(n):
    print('T_w = %.1f K for T_a = %.1f K' % (T_w_calc[i], T_a[i]))
```

```
T_w = 302.4 K for T_a = 305.0 K
T_w = 306.5 K for T_a = 310.0 K
T_w = 310.5 K for T_a = 315.0 K
T_w = 314.6 K for T_a = 320.0 K
```

Calculate q_t

[24]:
```
q_t_calc_2 = np.empty(n, dtype=object)

for i in range(n):
    q_t_calc_2[i] = q_t.replace(T_w, T_w_calc[i])

for i in range(n):
    print('Heat entering = %.1f W/m^2 for T_a = %.1f K' % (q_t_calc_2[i],␣
 ↪T_a[i]))
```

```
Heat entering = 6.0 W/m^2 for T_a = 305.0 K
Heat entering = 16.2 W/m^2 for T_a = 310.0 K
Heat entering = 26.3 W/m^2 for T_a = 315.0 K
Heat entering = 36.5 W/m^2 for T_a = 320.0 K
```

2.2.4 Plot

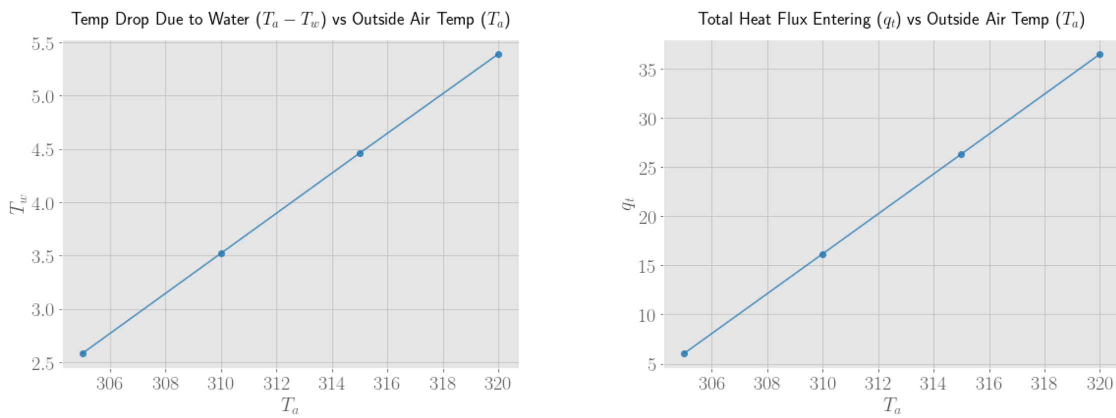
- Temp Drop Due to Water ($T_a - T_w$) vs Outside Air Temp (T_a)
- Total Heat Flux Entering (q_t) vs Outside Air Temp (T_a)

[25]:
```
fig = plt.figure(figsize=(16, 6))

ax1 = fig.add_subplot(121)
make_plot(x=T_a, y=T_a-T_w_calc, xlabel='$T_a$', ylabel='$T_w$',
          title='Temp Drop Due to Water ($T_a - T_w$) vs Outside Air Temp␣
 ↪($T_a$)')
```

```python
ax2 = fig.add_subplot(122)
make_plot(x=T_a, y=q_t_calc_2, xlabel='$T_a$', ylabel='$q_t$',
          title='Total Heat Flux Entering ($q_t$) vs Outside Air Temp ($T_a$)')

fig.tight_layout(w_pad=10)
```



2.3 References

1. A. Shrivastava *et al*. "Evaporative cooling model…" (1984)
2. F. Incropera *et al*. "Fundamentals of Heat and Mass Transfer"

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1) A. Shrivastava et al. “Evaporative cooling model…” (1984)
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3) M. Ndukuwu et al. “Mathematical model for direct…” (2013)
4) G. Pagliarini et al. “Dynamic Thermal Simulation…” (2011)
5) Xuan et. al. “Research and application of evaporative…” (2012)