f is the original image

f_1 is the image after applying the filter for the first time

The filter is the mean filter of size **(2a+1)x(2a+1)** where a>0

f_2 is the image after applying the filter for the second time

f_k is the image after applying the filter for the kth iteration/time

Let the filter be g

So we will convolve the image with the mean filter to get output images

$f_1 = g * f$

$f_2 = g * f_1$

$f_2 = g * (g * f)$

Convolution is an associative operation i.e.:

$a * (b * c) = (a * b) * c$

$f_2 = (g * g) * f$

$f_2 = (g^2) * f$ ... For kth iteration ...

$f_k = (g^k) * f$

On multiple passes of mean/average kernel the resulting kernel tends to be gaussian kernel due to **central limit theorem**.

If the process is repaeted that is mean filter is applied again and again on an image then the equivalent kernel/filter tends to be an gaussian filter.

Therefore, yes f_k can be expressed as an convolution of f with some kernel and that kernel is *Gaussian Kernel*.