1 Q1

$$v(x, y) = ax + by + cxy + d \tag{1}$$

To prove: $v(x, y)$ is a linear function of x keeping y constant

$$v \cdot (w \cdot x_1 + x_2, y) = w \cdot v(x_1, y) + v(x_2, y)$$

L.H.S.

$$\begin{align}
v(w \cdot x_1 + x_2, y) &= a \cdot (wx_1 + x_2) + by + c \cdot (wx_1 + x_2) \cdot y + d \tag{2}\\
&= w \cdot (ax_1 + cx_1 y) + ax_2 + by + cx_2 \cdot y + d \tag{3}\\
&= w \cdot (ax_1 + cx_1 y) + v(x_2, y) \tag{4}\\
\tag{5}
\end{align}$$

R.H.S.

$$\begin{align}
w.v(x_1, y) + v(x_2, y) &= w.(ax_1 + by + cx_1 y + d) + v(x_2, y) \tag{6}\\
&= w.(ax_1 + cx_1 y) + w.(by + d) + v(x_2, y) \tag{7}\\
&= w.(ax_1 + cx_1 y) + v(x_2, y) + w.(by + d) \tag{8}\\
R.H.S. &= L.H.S. + w.(by + d) \tag{9}\\
\tag{10}
\end{align}$$

Since L.H.S != R.H.S Therefore, $v(x, y)$ is not linear in x keeping y constant

Similary, To prove: *v(x,y)* is a linear function of y keeping x constant $v(x, w.y_1 + y_2) = w.v(x, y_1) + v(x, y_2)$

L.H.S.

$$\begin{align}
v(x, w.y_1 + y_2) &= ax + b.(wy_1 + y_2) + c.x.(wy_1 + y_2) + d \tag{11}\\
&= w.(by_1 + cxy_1) + ax + by_2 + cxy_2 + d \tag{12}\\
&= w.(by_1 + cxy_1) + v(x, y_2) \tag{13}\\
\tag{14}
\end{align}$$

R.H.S.

$$\begin{align}
w.v(x, y_1) + v(x, y_2) &= w.(ax + by_1 + cxy_1 + d) + v(x, y_2) \tag{15}\\
&= w.(by_1 + cxy_1) + w.(ax + d) + v(x, y_2) \tag{16}\\
&= w.(by_1 + cxy_1) + v(x, y_2) + w.(ax + d) \tag{17}\\
R.H.S. &= L.H.S. + w.(ax + d) \tag{18}\\
\tag{19}
\end{align}$$

Since L.H.S != R.H.S Therefore, $v(x, y)$ is not linear in y keeping x constant

To prove: *v(x,y)* is a linear function of z where $z \triangleq (x, y)$ $v(w.z_1 + z_2) = w.v(z_1) + v(z_2)$ $z_1 = (x_1, y_1)$ $z_2 = (x_2, y_2)$

L.H.S.

$$\begin{aligned}
v(w.z_1 + z_2) &= v(w.(x_1, y_1) + (x_2, y_2)) && (20)\\
&= v((wx_1, wy_1) + (x_2, y_2)) && (21)\\
&= v(wx_1 + x_2, wy_1 + y_2) && (22)\\
&= a.(wx_1 + x_2) + b.(wy_1 + y_2) + c.(wx_1 + x_2).(wy_1 + y_2) + d && (23)\\
&= w.(ax_1 + by_1 + cx_1y_2 + cx_2y_1) + ax_2 + by_2 + cx_2y_2 + d + cw^2x_1y_1 && (24)
\end{aligned}$$

R.H.S.

$$\begin{aligned}
w.v(z_1) + v(z_2) &= w.v(x_1, y_1) + v(x_2, y_2) && (25)\\
&= w.(ax_1 + by_1 + cx_1y_1 + d) + ax_2 + by_2 + cx_2y_2 + d && (26)
\end{aligned}$$

Since L.H.S != R.H.S Therefore, $v(x, y)$ is not linear in z where $z \triangleq (x, y)$

- Hence Proved the function $v(x, y)$ is neither a linear function of x keeping y constant; nor of y keeping x constant; nor of z (where $z \triangleq (x, y)$)