To prove:

$$\mathscr{F}(\mathscr{F}(\mathscr{F}(\mathscr{F}(f(t))))) = f(t)$$

where $\mathscr{F}$ is the continuous Fourier operator

We know:

Fourier Tansform:

$$F(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$$

Inverse Fourier Tansform:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega$$

$$\mathscr{F}(f(t)) = F(w)$$

$$\begin{aligned}
\mathscr{F}(\mathscr{F}(f(t))) &= \mathscr{F}^2 f(t) && (1)\\
&= \mathscr{F}F(w) && (2)\\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{-i\omega t}d\omega && (3)
\end{aligned}$$

(From inverse fourier transform equation)

$$\begin{aligned}
\mathscr{F}(\mathscr{F}(f(t))) &= \mathscr{F}^2(f(t)) && (4)\\
&= f(-t) && (5)\\
&\to \mathscr{F}(F(\omega)) && (6)\\
&= f(-t) && (7)
\end{aligned}$$

$$\begin{aligned}
\mathscr{F}(\mathscr{F}(\mathscr{F}(f(t)))) &= \mathscr{F}^3 f(t) && (8)\\
&= \mathscr{F}f(-t) && (9)\\
&= F(-w) && (10)\\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt && (11)
\end{aligned}$$

$$\begin{aligned}
\mathscr{F}(\mathscr{F}(\mathscr{F}(\mathscr{F}(f(t))))) &= \mathscr{F}^4(f(t)) && (12)\\
&= \mathscr{F}(F(-w)) && (13)\\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}dw && (14)
\end{aligned}$$

(From inverse fourier transform equation)

Therefore,

$$\mathscr{F}(\mathscr{F}(\mathscr{F}(\mathscr{F}(f(t))))) = \mathscr{F}^4(f(t)) = f(t)$$

$\therefore$ Hence Proved