1-D ramp Image form $I(x) = cx + d$

Case 1: Image convoluted with gaussian filter of size 2a+1 with standard deviation σ and mean zero.

Convolution:

$$J(x) = \frac{1}{K} \sum_{i=-a}^{a} I(x+i).\, e^{\frac{-i^2}{2\sigma^2}}$$

$$J(x) = \frac{1}{K} \sum_{i=-a}^{a} (c(x+i)+d).\, e^{\frac{-i^2}{2\sigma^2}}$$

$$J(x) = \frac{1}{K} \sum_{i=-a}^{a} (cx+d).\, e^{\frac{-i^2}{2\sigma^2}} + \frac{1}{K} \sum_{i=-a}^{a} (ci).\, e^{\frac{-i^2}{2\sigma^2}}$$

$\frac{1}{K} \sum_{i=-a}^{a} (ci).\, e^{\frac{-i^2}{2\sigma^2}=0}$ .... Since its an odd function of i

$$J(x) = \frac{1}{K} \sum_{i=-a}^{a} (cx+d).\, e^{\frac{-i^2}{2\sigma^2}}$$

$$J(x) = \frac{1}{K} \sum_{i=-a}^{a} I(x).\, e^{\frac{-i^2}{2\sigma^2}} = \frac{I(x)}{K} \sum_{i=-a}^{a} e^{\frac{-i^2}{2\sigma^2}} \ldots \sum_{i=-a}^{a} e^{\frac{-i^2}{2\sigma^2}=K} \ldots$$

$$J(x) = \frac{I(x)}{K}.K = I(x)$$

Since $J(x) = I(x)$, therefore the image is unfiltered.

Case 2: Iamage convoluted with bilateral filter of size 2a+1 with parameters σ_s, σ_r

Convolution:

$$J(x) = \frac{1}{K_1} \sum_{i=-a}^{a} I(x+i).\, e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(I(x+i)-I(x))^2}{2\sigma_r^2}}$$

$$J(x) = \frac{1}{K_1} \sum_{i=-a}^{a} (cx+ci+d).\, e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(cx+ci+d-cx+d)^2}{2\sigma_r^2}}$$

$$J(x) = \frac{1}{K_1} \sum_{i=-a}^{a} (cx+ci+d).\, e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(ci)^2}{2\sigma_r^2}}$$

$\frac{1}{K_1} \sum_{i=-a}^{a} ci.\, e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(ci)^2}{2\sigma_r^2}=0}$ .... Since its an odd function of i

$$J(x) = \frac{1}{K_1} \sum_{i=-a}^{a} (cx+d).\, e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(ci)^2}{2\sigma_r^2}}$$

$$J(x) = \frac{1}{K_1} \sum_{i=-a}^{a} I(x).\, e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(ci)^2}{2\sigma_r^2}}$$

$$J(x) = \frac{I(x)}{K_1} \sum_{i=-a}^{a} e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(ci)^2}{2\sigma_r^2}} \ldots \sum_{i=-a}^{a} e^{\frac{-i^2}{2\sigma_s^2}}.\, e^{\frac{-(ci)^2}{2\sigma_r^2}=K_1} \ldots$$

$$J(x) = \frac{I(x)}{K_1}\, .K_1 = I(x)$$

Since $J(x) = I(x)$, therefore the image is unfiltered.

For both the filters the output image remains unfiltered and is same as the input image, so the output of both the filters is same.

For both the filters we got the same image back.

Hence Proved