1 Q2

Histogram Equalisation is a method to improve image contrast by spreading out the intensity to a higher dynamic range so that the details are more cleraly visible. We apply an histogram equalization intensity tranformation to low contrast image as to convert it into a high contrast image.

Applying the histogram equalization for first time. R : Random variable for intensity of original image R lies from 0 to L-1 (i.e. R in [0,L-1]) R has a probability density : $P_R(r)$ where: $0 \leq P_R(r) \leq 1$

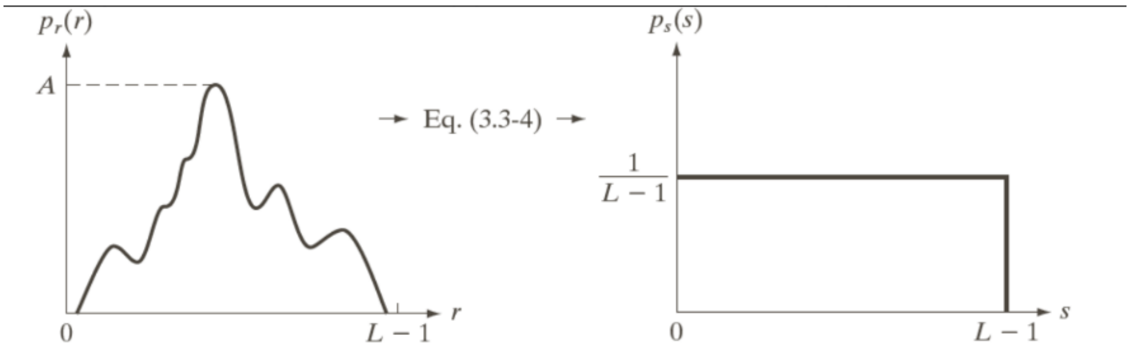
Now we apply T which is the transformation function to R

$S = T(R)$ (where S stands for tranformed random variable after first time applying the histogram equalization to R)

$P_S(s) = P_S(T(r)) = \dfrac{P_R(r)}{|T'(r)|}$

We are assuming cumulative probability distribution $s = T(r) = (L-1)\int_0^r P_R(w)dw$ $P_S(s) = \dfrac{P_R(r)}{|T'(r)|}$ $T'(r) = (L-1)\dfrac{d}{dr}(\int_0^r P_R(w)dw) = (L-1)P_R(r) * P_S(s) = \dfrac{1}{L-1}$

Here, we can see that S has a uniform probability distribution function, which is independent of the original probability distribution function of R



Hypothesis based on above result is that irrespective of number of times the histogram equalization is applied on the image the PDF of the output image will be $\dfrac{1}{L-1}$ as the main aim of the histogram equalization is to uniformly distribute the pixels over the intensity range.

Proof of the above hypothesis: We will apply the histogram equalization second time on the output of the first application of histogram equalization on the original image.

$U = T(S)$ (where U stands for tranformed random variable after second time applying the histogram equalization to R) $P_U(u) = P_U(T(s)) = \dfrac{P_S(s)}{|T'(s)|}$

We are assuming cumulative probability distribution $u = T(s) = (L-1)\int_0^s P_S(w)dw$ $P_U(u) = \dfrac{P_S(s)}{|T'(s)|}$ $T'(s) = (L-1)\dfrac{d}{ds}(\int_0^s P_S(w)dw) = (L-1)P_S(s) * P_U(u) = \dfrac{1}{L-1} * P_U(u) = P_S(s)$

U which we got after second application has the same PDF as S which we got after the first application.

- The hypothesis is therefore justified.

$*$ *Hence Proved*